PAUL SCHERRER INSTITUT

The Evaluation of the Leading Hadronic Contribution to a_μ

Towards the MUonE Experiment

Results at NNLO from McMule – Subtraction & Results

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- McMule methods: IR subtraction with FKS$^\ell$
- $\mu - e$ scattering @ NNLO with McMule
  - implementation, internal & external checks
  - massification for mixed VV corrections
  - next-to-soft stabilisation for mixed RV corrections
- finally, results, i.e. colourful plots

Handling IR divergences at NNLO

handling IR divergences – a comparison

$$\mathrm{d}\Phi_{n+r} \quad \equiv \quad \mathrm{d}\Phi_n \prod_{i=1}^{r} \mathrm{d}\Phi_{i,\gamma} = \mathrm{d}\Phi_n \prod_{i=1}^{r} \mathrm{d}\Phi_{i,\gamma}^{d=4-2\epsilon}\, \xi^2\, \xi_i^{-1-2\epsilon}\, \mathrm{d}\xi_i\, \mathrm{d}\Upsilon_i^{d=4-2\epsilon}$$

$$\int \mathrm{d}\Phi_n \left\{ \textcolor{red}{\times} + \int \mathrm{d}\Phi_\gamma\, \textcolor{red}{\tilde{\times}} \right\}$$

$$① \quad = \quad \int \mathrm{d}\Phi_n\, \underbrace{\mathrm{d}\Phi_\gamma \left\{ \textcolor{red}{\tilde{\times}} - \textcolor{green}{\times} \right\}}_{\langle \left(\frac{1}{\xi^{1+2\epsilon}}\right)_c, \cdot \rangle} + \int \mathrm{d}\Phi_n \left\{ \textcolor{red}{\times} + \underbrace{\int \mathrm{d}\Phi_\gamma\, \textcolor{green}{\times}}_{\langle -\frac{\xi_c^{-2\epsilon}}{2\epsilon}\,\delta(\xi), \cdot \rangle} \right\}$$

$$② \quad = \quad \int \mathrm{d}\Phi_n \left\{ \textcolor{red}{\times} + \left[\int^{\omega_s} \mathrm{d}\Phi_\gamma\, \textcolor{green}{\times} + \int_{\omega_s} \mathrm{d}\Phi_\gamma\, \textcolor{red}{\tilde{\times}} \right] \right\}$$

two methods

① FKS SUBTRACTION + DIMREG

1. reproduce and isolate IR behaviour from regions of the phase space where (one or more) real photons are soft:

$$\lim_{\xi\to 0} \xi^2\, \mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\, M_n^{(\ell)}$$

2. isolate IR-divergent behaviour from virtual amplitudes:

$$\sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = e^{-\alpha\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)\,f}$$

3. cancel analytically IR divergences and then integrate numerically in $d = 4$ over the non-radiative phase space

② SLICING + m_γ (to my understanding)

1. choose the resolution parameter for the photon energy, ω_s, as slicing parameter:

   for $E_\gamma < \omega_s$ a real-emission contribution is *degenerate* with the one *without* the emitted photon

2. regulate IR divergences from virtual amplitudes with a non-zero photon mass, λ

3. integrate numerically the contributions regrouped according to rule 1; for soft regions use eikonal approximation

- ① [Engel, Signer, Ulrich, 1909.10244]

$$\begin{aligned}
\sigma^{(2)} &= \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c) \\
\sigma_n^{(2)}(\xi_c) &= \int \mathrm{d}\Phi_n^{d=4} \mathcal{M}_n^{(2),f} \\
\sigma_{n+1}^{(2)}(\xi_c) &= \int \mathrm{d}\Phi_{n+1}^{d=4} \frac{1}{1!} \left(\frac{1}{\xi_1}\right)_c \xi_1 \, \mathcal{M}_{n+1}^{(2),f} \\
\sigma_{n+2}^{(2)}(\xi_c) &= \int \mathrm{d}\Phi_{n+2}^{d=4} \frac{1}{2!} \left(\frac{1}{\xi_1}\right)_c \left(\frac{1}{\xi_2}\right)_c \xi_1 \, \xi_2 \, \mathcal{M}_{n+2}^{(0),f}
\end{aligned}$$

- ②

$$\sigma^{(2)} = \sigma_{0\gamma,\,\mathrm{h}}(\omega_s) + \sigma_{1\gamma,\,\mathrm{h}}(\omega_s) + \sigma_{2\gamma,\,\mathrm{h}}(\omega_s)$$

$\mu - e$ SCATTERING AT NNLO WITH McMULE

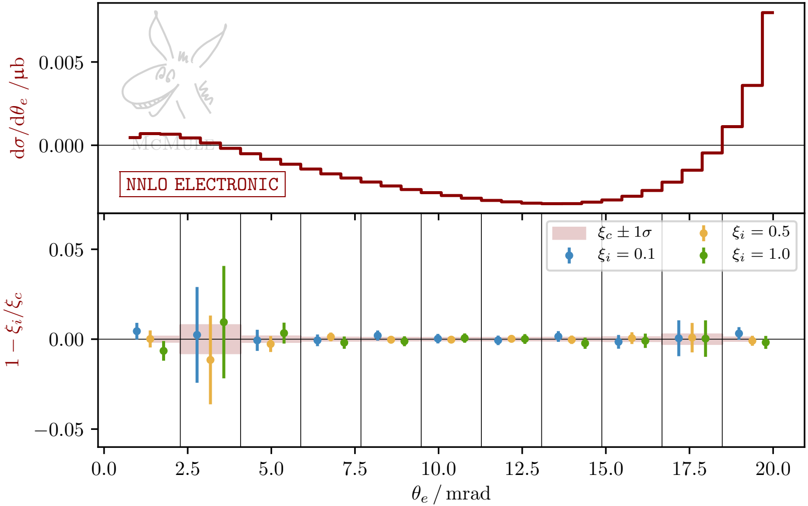
- massless photons
- massive leptons
  - at 1 loop [RV]
    - $\rightarrow$ use OpenLoops [Buccioni et al. 18, 19] for remarkable stability
    - $\rightarrow$ instabilities from exclusivity in collinear regions $\rightarrow$ NTS stabilisation [Engel, Signer, Ulrich 22]
  - at 2 loops [VV]
    - $\rightarrow$ 2-loop amplitude with massless electrons from [Bonciani et al. 21]
    - $\rightarrow$ 2-loop massification [McMule 18], adding (missing) leading mass effects (miss $\propto (\alpha/\pi)^2\, m_e^2/q^2$ when VV is known with $m_e = 0$)
    - $\rightarrow$ numerical evaluation of GPLs with handyG [McMule 20] returns 1 ev/s

- at NNLO the calculation is split into photonic [MUonE TH initiative 20]

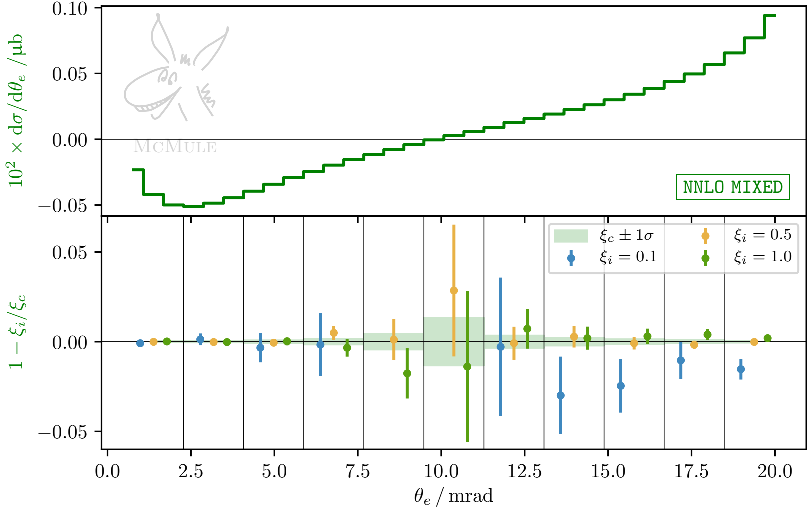
$$\begin{aligned}\sigma^{(2)} &= \sigma^{(\mathrm{VV})} + \sigma^{(\mathrm{RV})} + \sigma^{(\mathrm{RR})} \\ &= \int \mathrm{d}\Phi_n \, \mathcal{M}_n^{(2)} + \int \mathrm{d}\Phi_{n+1} \, \mathcal{M}_{n+1}^{(1)} + \int \mathrm{d}\Phi_{n+2} \, \mathcal{M}_{n+2}^{(0)}\end{aligned}$$

  and fermionic (leptonic and hadronic) contributions
- for each part identify gauge-invariant subsets based on lepton charges (q for electron, Q for muon)
  - q^4 : emission from e-line only [Carloni et al. 20, McMule 20]
  - Q^4 : emission form μ-line only $\rightarrow$ trivial from e-only
  - $\{q^3 Q,\ q^2 Q^2,\ q Q^3\}$: mixed diagrams

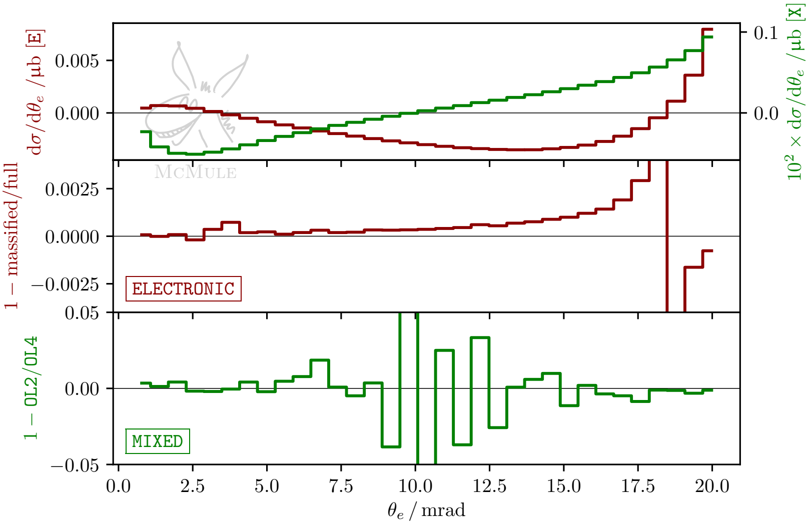
validation: ξ_c (in)dependence for electronic corrections



validation: ξ_c (in)dependence for mixed corrections



validation: massification and NTS stabilisation



validation: massification and fermionic corrections

- Bhabha-inspired [McMule 21] check for massification

$$\text{massified}\ \Big[\mu\, e \to \mu\, e\, (m_e = 0,\, m_\mu = 0)\Big]$$

vs

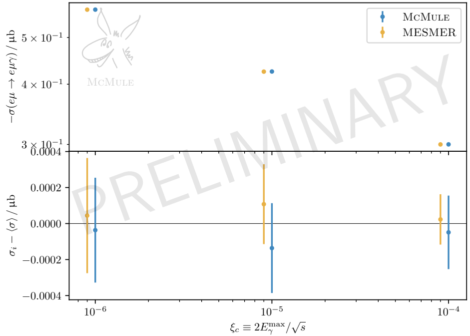
$$\text{small-}m_\mu\ \text{expansion for massified}\ \Big[\mu\, e \to \mu\, e\, (m_e = 0,\, m_\mu)\Big]$$

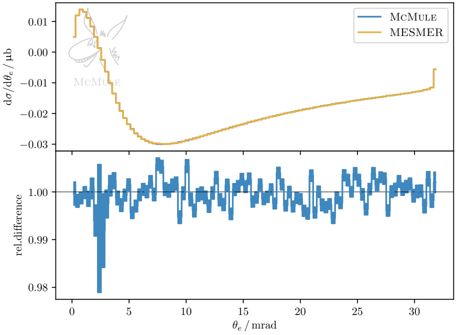
- cross-check for leptonic and hadronic corrections, computed with hyperspherical method [Fael 18], with [Fael and Passera 19]

comparison McMule vs Mesmer: radiative (photonic) $\mu - e$ @NLO

$e\,\mu \to e\,\mu\,\gamma$ @ NLO with $\xi_c = \omega_s = 10^{-\{6,5,4\}}$

(Mesmer as in [Carloni et al. 20])





- mixed RR and mixed RV coincide from the radiative comparison
- total xsec for mixed $\mu - e$ (NNLO correction):
  - McMule + massified VV ($m_e = 0$): $-2.50(1) \cdot 10^{-2}\,\mu\text{b}$
  - McMule + YFS approximation for VV (courtesy of Carlo): $6.45(1) \cdot 10^{-2}\,\mu\text{b}$
  - Mesmer: $6.47(3) \cdot 10^{-2}\,\mu\text{b}$

    [LO $= 1.214(1) \cdot 10^{2}\,\mu\text{b}$]

$$\mu\, e \to \mu\, e \quad @\ \mathrm{NNLO}$$

- kinematical setup mimics MUonE:

$$E_{\mu,i} = 160\,\mathrm{GeV} \qquad E_{e,f} > 1\,\mathrm{GeV} \qquad \theta_{\mu,f} > 0.3\,\mathrm{mrad}$$

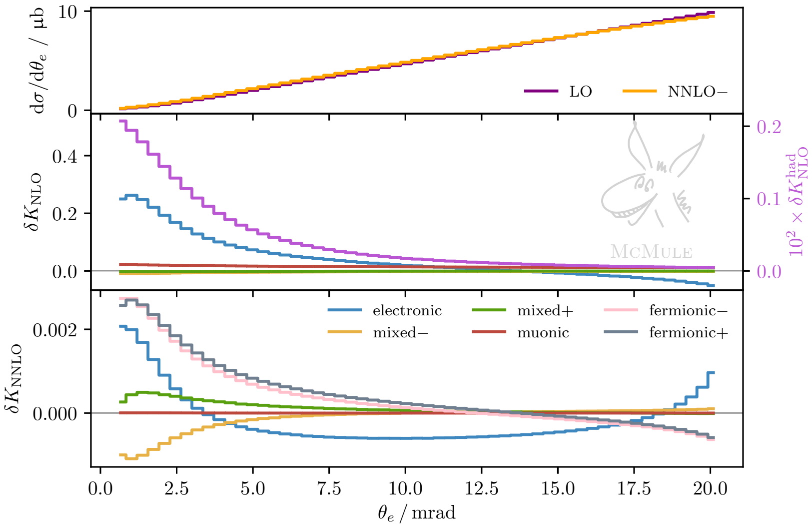
- results for two scenarios, S1 and S2, i.e. w/ or w/out elasticity cut:

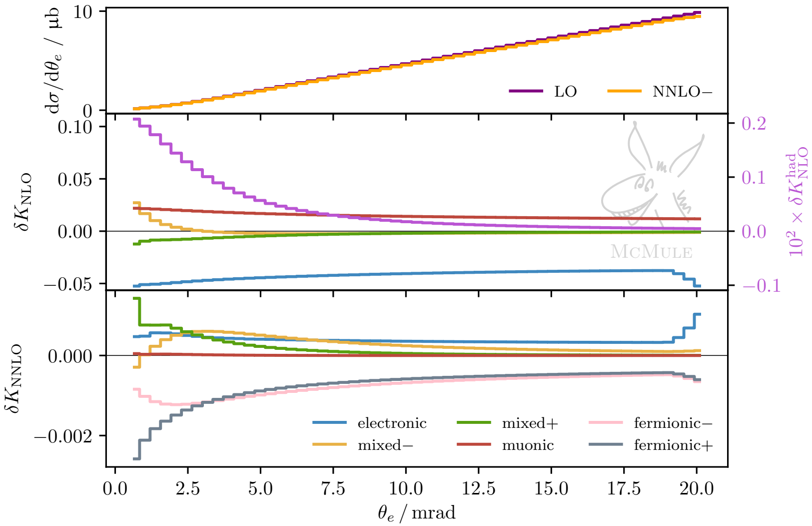
$$0.9 < \frac{\theta_{\mu,f}}{\theta^{\mathrm{el}}_{\mu,f}} < 1.1$$

total cross sections

	$\sigma/\mu\mathrm{b}$		$\delta K^{(i)}/\%$	
	S1	S2	S1	S2
σ_0	106.44356	106.44356		
σ_1 $-$	106.99038(3)	102.86304(3)	0.51372(3)	−3.36377(3)
σ_1 $+$	107.41847(3)	103.18338(3)	0.91589(3)	−3.06283(3)
$\sigma_e^{(2)}$	0.00090	0.06595	0.00084	0.06411
$\sigma_{e\mu}^{(2)}$ $-$	0.00097(1)	0.01926	0.00091(1)	0.01872
$\sigma_{e\mu}^{(2)}$ $+$	0.00328(1)	−0.01768	0.00305(1)	−0.01713
$\sigma_\mu^{(2)}$	−0.00005	0.00002	−0.00005	0.00002
$\sigma_{\mathrm{lep}}^{(2)}$ $-$	−0.01195	−0.06568	−0.01117	−0.06385
$\sigma_{\mathrm{lep}}^{(2)}$ $+$	−0.00424	−0.05959	−0.00395	−0.05775
$\sigma_{\mathrm{had}}^{(2)}$ $-$	−0.00045	−0.00104	−0.00042	−0.00101
$\sigma_{\mathrm{had}}^{(2)}$ $+$	−0.00004	−0.00068	−0.00004	−0.00066
σ_2 $-$	106.97977(3)	102.88154(3)	−0.00992(4)	0.01799(4)
σ_2 $+$	107.41832(3)	103.19386(3)	−0.00013(4)	−0.01016(4)

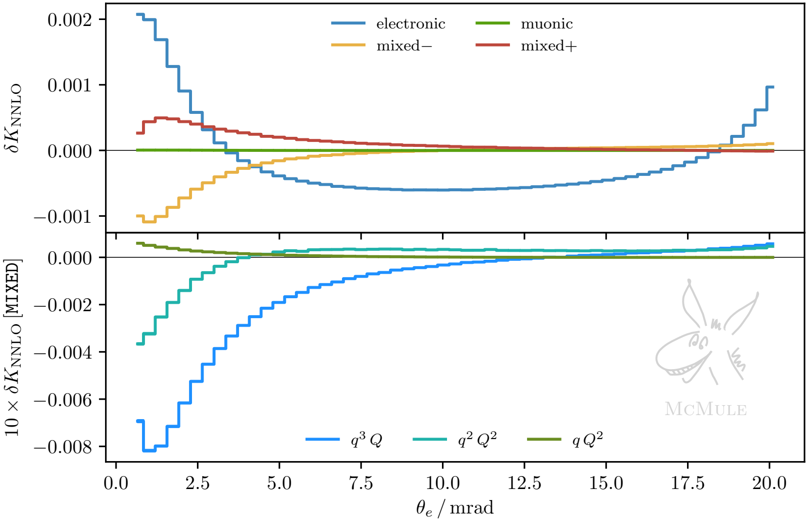
results: θ_e in S1





$d\sigma/d\theta_e$ / μb
10
0
LO
NNLO−
$\delta K_{\rm NLO}$
0.10
0.05
0.00
−0.05
0.2
0.1
0.0
−0.1
$10^2 \times \delta K^{\rm had}_{\rm NLO}$
McMule
$\delta K_{\rm NNLO}$
0.000
−0.002
electronic
mixed+
fermionic−
mixed−
muonic
fermionic+
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

results: θ_e in `S1` (zoom)



short term

- all these results in a paper, very soon! [McMule & Padova+]
- we have started thinking about $\mathrm{N^3LO}$ dominant corrections (see Tim's and Yannick's talks) in Durham (Workstop – 3-5 August 2022)

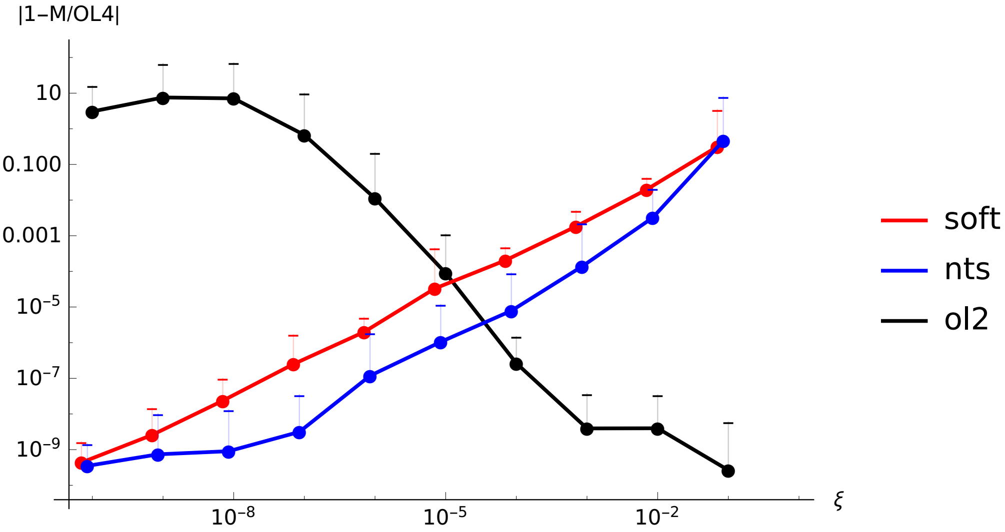
  → subtraction scheme is ready: FKS^3

  → $\mathrm{N^3LO}$ form factor [Fael et al. 22]; higher-order massification and NTS

long term

- parton-shower merging to fixed-order results to cure small-angle behaviour
- STRONG 2020 Theory Workstop in Zurich – June 2023

OpenLoops vs NTS



OpenLoops vs NTS (collinear)