MITP workshop

Massification & next-to-soft stabilisation

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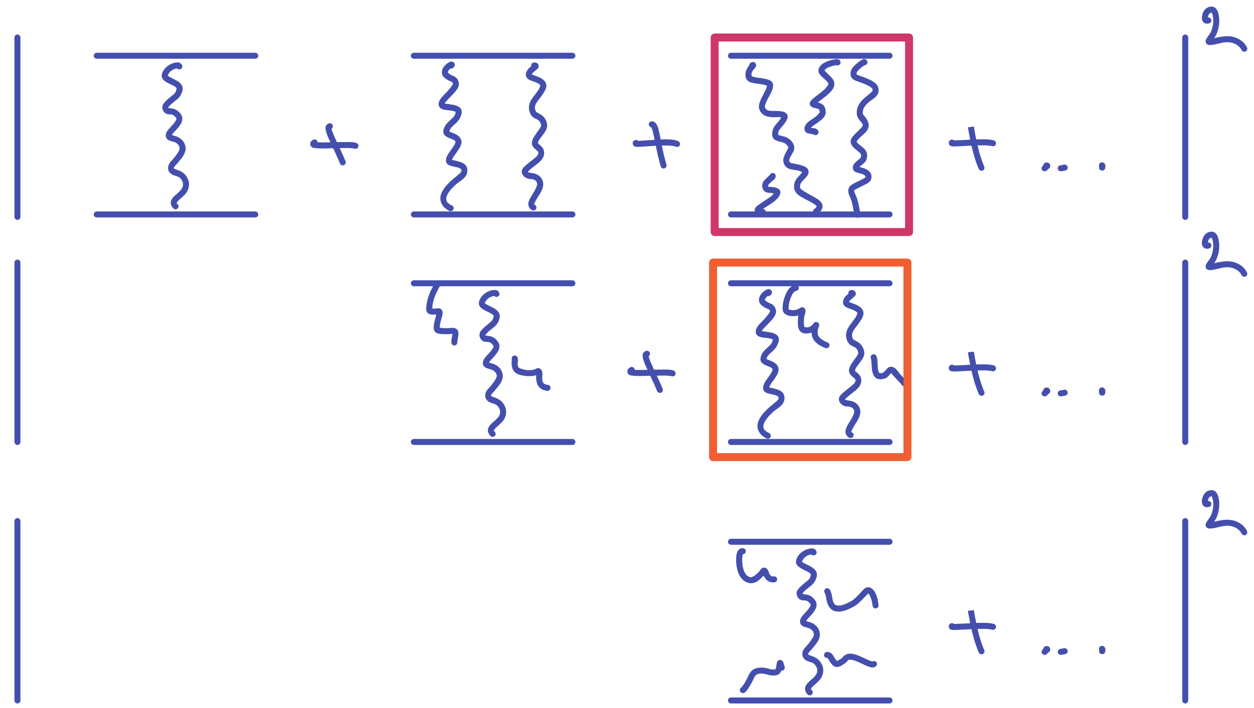
QED vs. QCD

cross section

$$\sigma = \int d\Phi_n \,|\ldots|^2 + \int d\Phi_{n+1} \,|\ldots|^2 + \int d\Phi_{n+2} \,|\ldots|^2 + \ldots$$



$m \neq 0$

⊕ simple IR structure ⇝ **FKS$^\ell$ subtraction scheme** → Marco

⊖ loops with mass ⇝ **massification**

⊖ numerical instabilities ⇝ **next-to-soft stabilisation**

} now

! process-independent solutions

Massification [Becher, Melnikov 07; McMule 18]

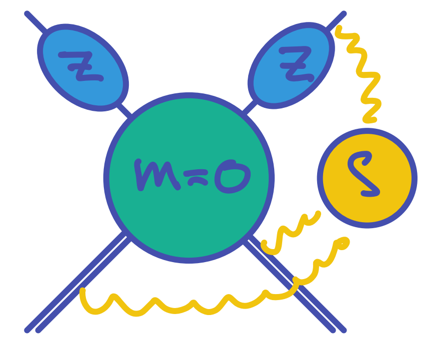
@ 2 loop ↗ heavy particle

↳ ⊖ loops with masses

hierarchy

$$p_j^2 = m_j^2 \sim \lambda^2 \ll Q^2$$

$$p_j \sim (1, \vec{e}_j \beta_j), \quad \beta_j = 1 - \mathcal{O}(\lambda^2)$$



process indep. 1+ fermion loops (QED)

factorisation

$$\mathcal{A}_n = \prod_j Z(m_j)\, S(Q^2, \{m_j\})\, \mathcal{A}_n(Q^2, \{m_j = 0\}) + \mathcal{O}(\lambda)$$

! $\mathcal{O}(\log m)$ <u>and</u> $\mathcal{O}(1)$ included → Marco

SCET

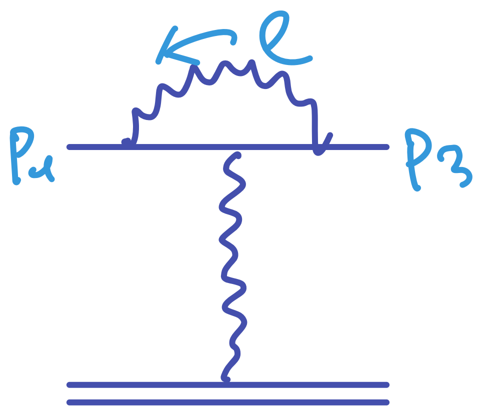
$$\mathcal{L}_n^{LP} = \sum_j \mathcal{L}_{SCET}(\xi_j, A_j^\mu) + C_n O_n(\{\xi_j\}, \{A_j^\mu\}, A_s^\mu)$$

! ∄ soft-collinear IA (decoupling trsf.)

Method of regions

[Beneke, Smirnov 98] } convenient tool to extract Z

expand @ integrand



$$\rightsquigarrow \quad A^{(1)} \sim \int \frac{dl}{[l^2][l^2+2l\cdot p_1][l^2+2l\cdot p_3]}$$

light-cone basis

decomp. into small & large

$$\{e_j = (1,\vec{e}_j),\ \bar{e}_j = (1,-\vec{e}_j)\}$$

$$p_j = \underbrace{(e_j\cdot p_j)\,\bar{e}_j}_{p_j^+} + \underbrace{(\bar{e}_j\cdot p_j)\,e_j}_{p_j^-} + p_j^\perp \sim (\lambda^2, 1, \lambda)$$

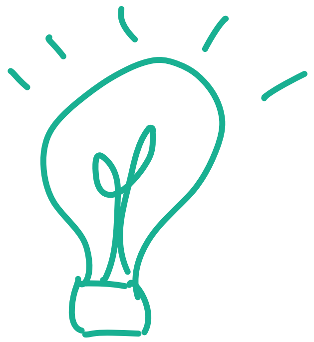
"hard" $(l \sim \lambda^0)$ "collinear"

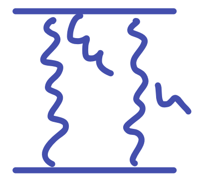
$$A^{(1)} = \int \frac{dl}{[l^2][l^2+2l\cdot p_1^-][l^2+2l\cdot p_3^-]} + (l\sim p_1) + (l\sim p_3) + \mathcal{O}(\lambda)$$

$A^{(1)}(m=(p_j^-)^2=0)$ $\qquad 2Z^{(1)}(m)A^{(0)}(m=0)$

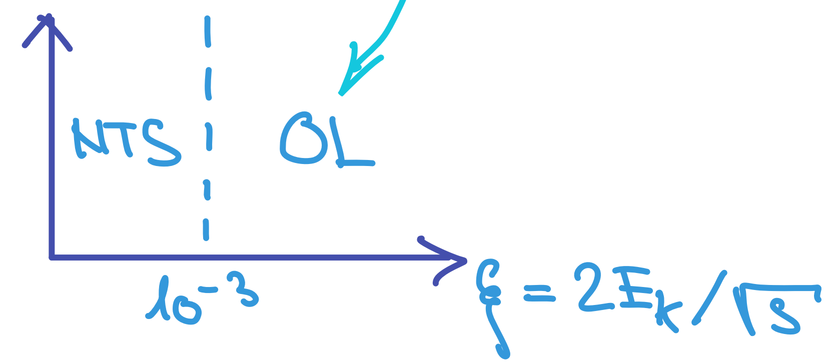
Next-to-soft stabilisation

↳ ⊖ numerical instabilities $E_k \to 0$ [Buccioni et.al. 19]





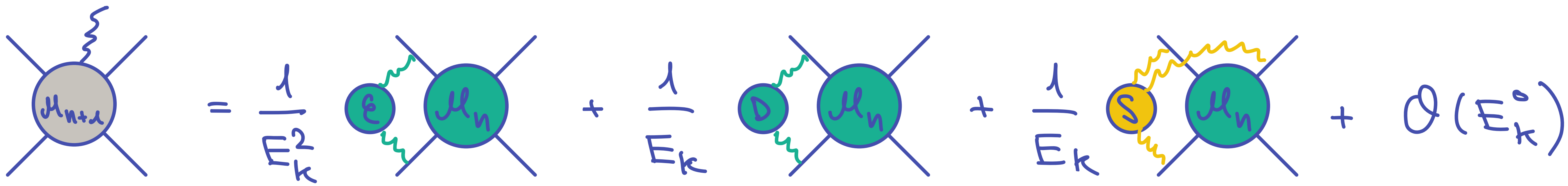
$$ {}_k \overset{MoR}{=} \frac{A}{E_k^2} + \frac{B}{E_k} + \mathcal{O}(E_k^0) \rightsquigarrow $$

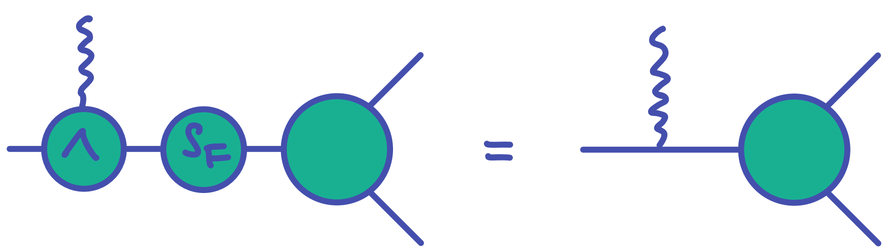


LBK theorem

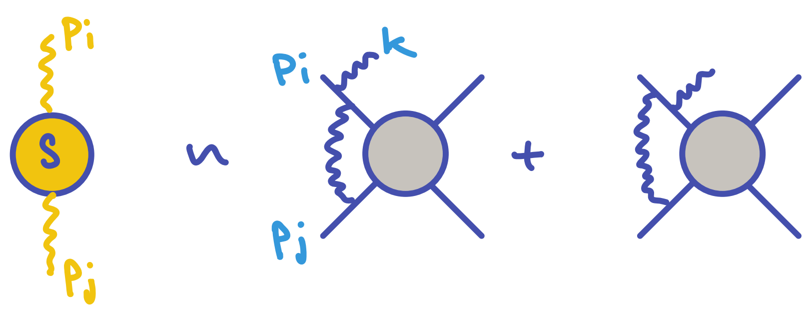
[Low 58; Burnett, Kroll 68] ⇝ tree level

[McMule 21] ⇝ one loop





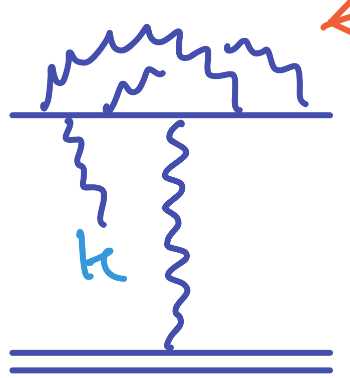
∘ $+ \mathcal{O}(E_k^0)$ @ squared amplitude ⚠



∘ $\sim (p_i \cdot p_j\, I_1 + m_j^2\, k \cdot p_i\, I_2)$

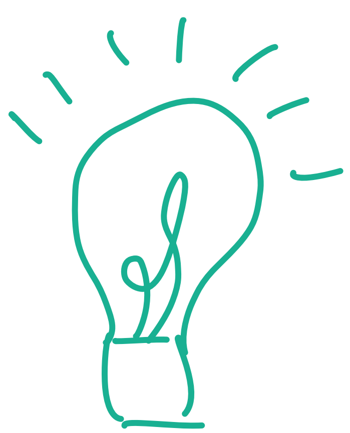
$$ \int \frac{d\ell}{[\ell^2][-\ell\cdot p_j][\ell\cdot p_i - k\cdot p_i]} $$

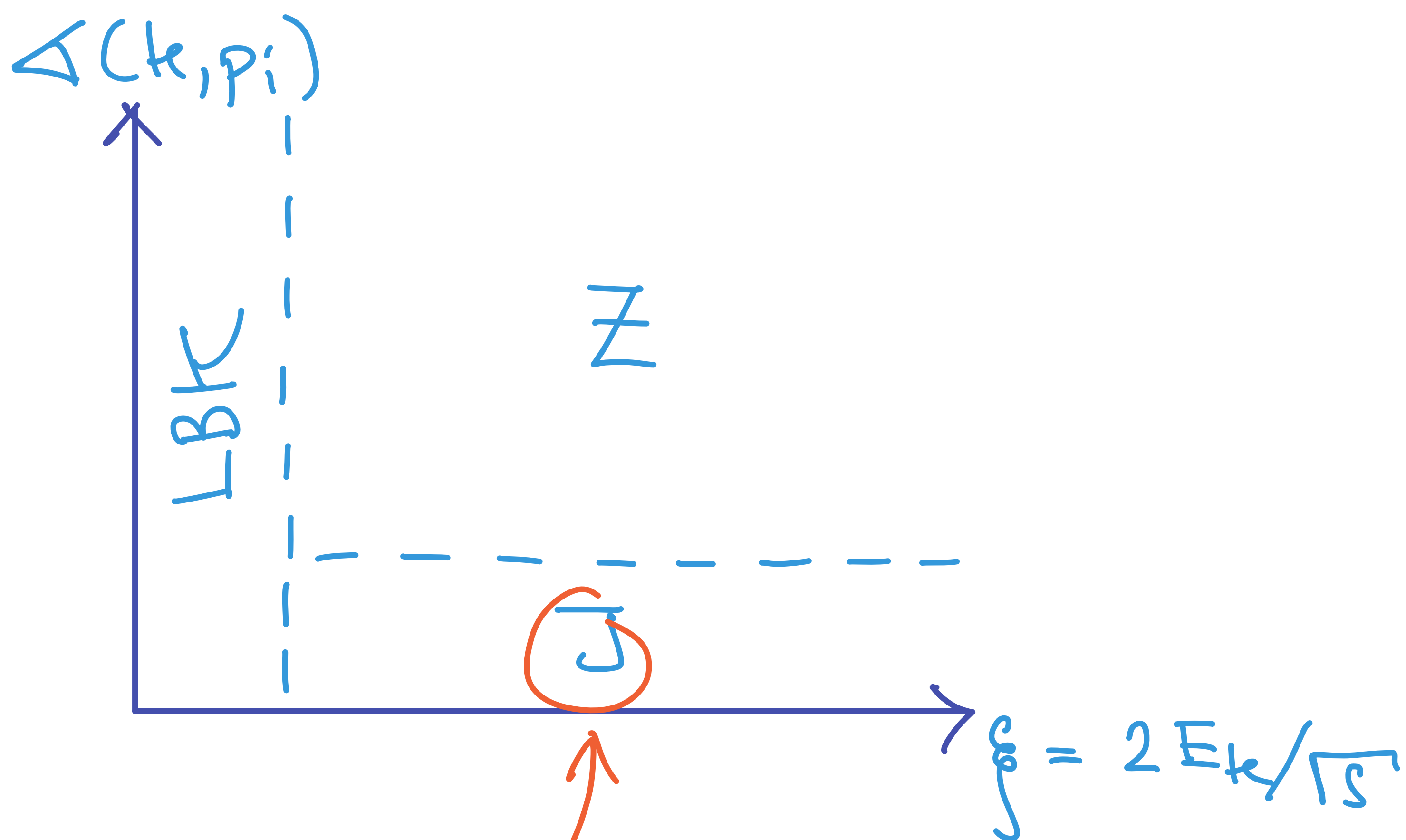
Outlook: massification of real corrections



only known for $m = 0$ → Yannick (N^3LO)

massification breaks down for $\underbrace{k \to 0}_{\text{soft}}$ or $\underbrace{k \cdot p_i \sim m^2}_{\text{collinear}}$





massive splitting function
@ one loop [McMule 21]

Conclusions

- $m \neq 0$ ⇝ loops harder ⇝ massification ($m^2 \ll Q^2$)

  ↕ process-indep. solutions

- $m \neq 0 \ll Q^2$ ⇝ num. instabilities ⇝ next-to-soft stabilisation

- massification of real-virtual-virtual
  ⇝ LBK @ 2 loop ✓
  ⇝ massive splitting function @ 2 loop ○

- LBK @ all orders ○