

MITP workshop

The Evaluation of the Leading Hadronic Contribution to the Muon  $g-2$ : Toward the MUonE Experiment

## Report from the N<sup>3</sup>LO Workstop






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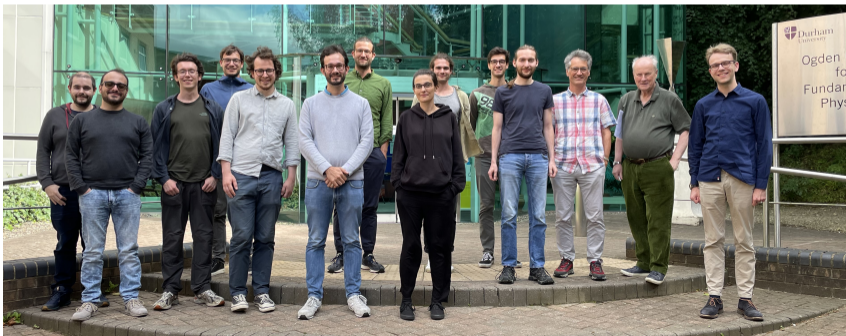
physical  $2 \rightarrow 2$  cross section

$$\begin{aligned}
 \sigma = & \int d\Phi_2 \left| \begin{array}{c} \text{tree} \\ + \text{NLO} \\ + \text{NNLO} \\ + \text{N}^3\text{LO} \\ + \dots \end{array} \right|^2 \\
 & + \int d\Phi_3 \left| \begin{array}{c} \text{tree} \\ + \text{NLO} \\ + \text{NNLO} \\ + \dots \end{array} \right|^2 \\
 & + \int d\Phi_4 \left| \begin{array}{c} \text{tree} \\ + \text{NLO} \\ + \dots \end{array} \right|^2 \\
 & + \int d\Phi_5 \left| \begin{array}{c} \text{tree} \\ + \dots \end{array} \right|^2
 \end{aligned}$$

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 •  $\sim$  three-loop heavy quark form factor  $\rightarrow$  Matteo's talk
- 
 •  $\sim$  really difficult, see later
- 
 •  $\sim$  OpenLoops $\oplus$ NTS stabilisation  $\rightarrow$  nasty but doable
- 
 •  $\sim$  tree level but difficult phase space  $\rightarrow$  (hopefully) easy enough
- 
 •  $\int d\Phi_5 \left| \dots \right| \sim$  FKS<sup>3</sup> subtraction

## small workstop in Durham (3 – 5 Aug)

- VVV: Lange, Schönwald, Fael
- RVV: Ronca, Moodie
- RRV: Lindert, Schär
- assembly & dirty tricks: Engel, Rocco



the most difficult part: four scales ( $m$ ,  $Q^2$ ,  $s_{12}$ ,  $s_{1\gamma}$ )

### using approximations

- massless integrals known (three-jet @ NNLO)
- massification, NTS stabilisation, jettification
- + fast evaluation (fully analytic), stable by construction, not too difficult
- introduces difficult-to-quantify systematic error

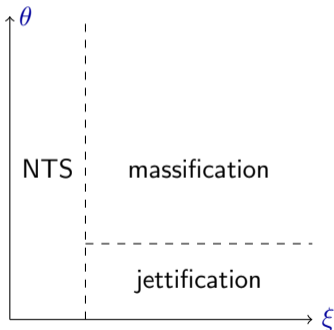
### using brute-force

- numerical evaluation of full integrals (eg. DiffExp)
- ⇒ need to find master integral basis (KIT group)
- + no new systematic error
- slow evaluation, potentially very unstable, extremely difficult

the most difficult part: four scales ( $m, Q^2, s_{12}, s_{1\gamma}$ )

using approximations

using brute-force



- starting point:  $\frac{d\vec{F}}{dx_i} = A_i \vec{F}$
- know integrals at some point (eg. boundary conditions)
- find a line to where you want it  $0 < y < 1$
- solve  $d\vec{F}/dy = A_y \vec{F}$  using power series in  $y$
- repeat
- highly efficient implementation in DiffExp [Hidding 20]

- the  $N^3$  LO calculation will be very difficult
  - ... but it will be doable for the dominant corrections
  - ... using **all** the tools
- ⇒ whitepaper will be written to demonstrate this to the community
- ⇒ provide input for Strong2020 Workstop in Zurich (Summer 23)