The Hadronic Vacuum Polarization from the lattice

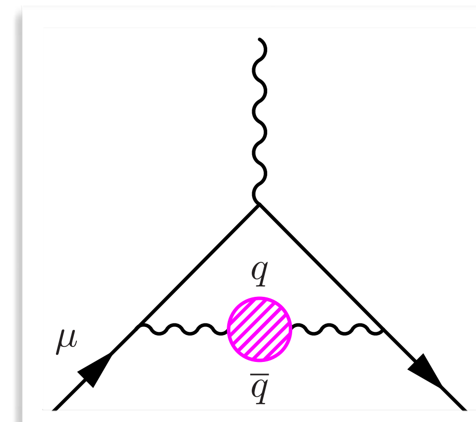
Davide Giusti

Universität Regensburg

Toward the MUonE Experiment
Mainz

17th November 2022

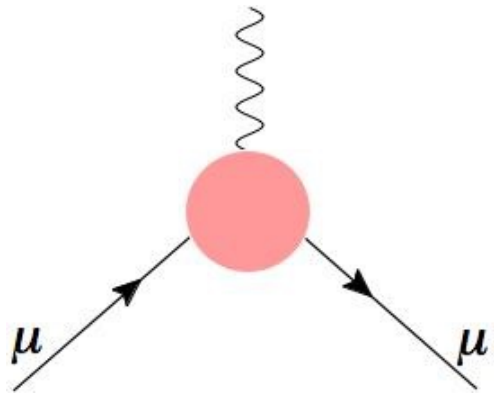
OUTLINE



- Introduction
- HVP from the lattice
- Window observables
- Connections to the MUonE experiment

Introduction

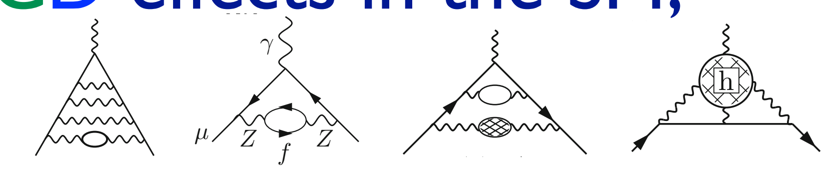
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

<u>muon anomalous magnetic moment:</u> $a_\mu \equiv \dfrac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



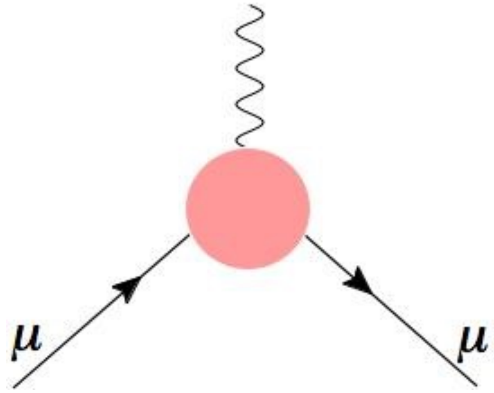
PDG 2021

$$a_\mu^{\mathrm{exp}} = 116\ 592\ 061\,(41)\cdot 10^{-11} \quad 0.35\mathrm{ppm}$$

$$a_\mu^{\mathrm{SM}} = 116\ 591\ 810\,(43)\cdot 10^{-11} \quad 0.37\mathrm{ppm}$$

$\sim 4.2\sigma$

Muon magnetic anomaly



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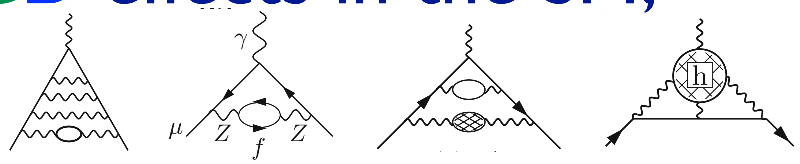
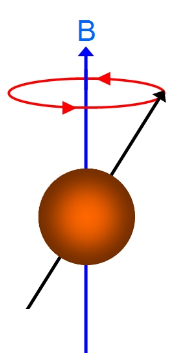


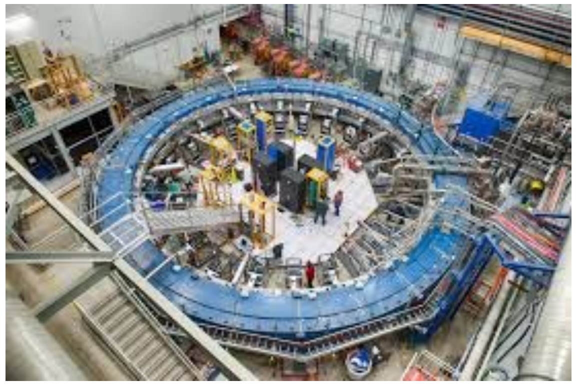
Muon g-2 2021

$$a_{\mu^+}^{\text{exp,FNAL}} = 116\ 592\ 040\,(51)(19)\cdot 10^{-11}$$

$$\vec{M} = g_{\mu}\frac{e}{2m_{\mu}}\vec{S}$$

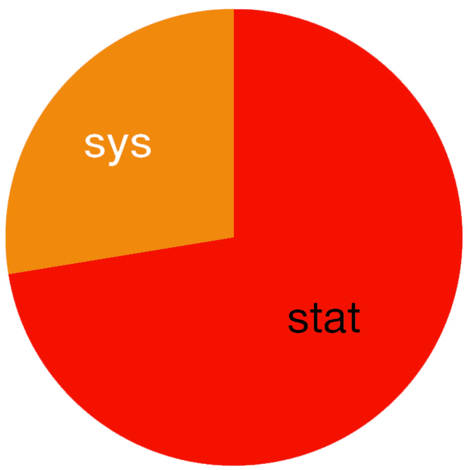


target 0.14ppm

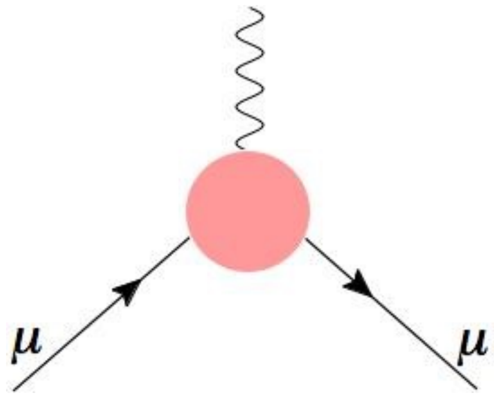


FermiLab (E989)

Run-1



Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

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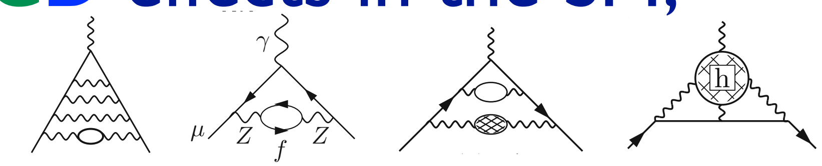


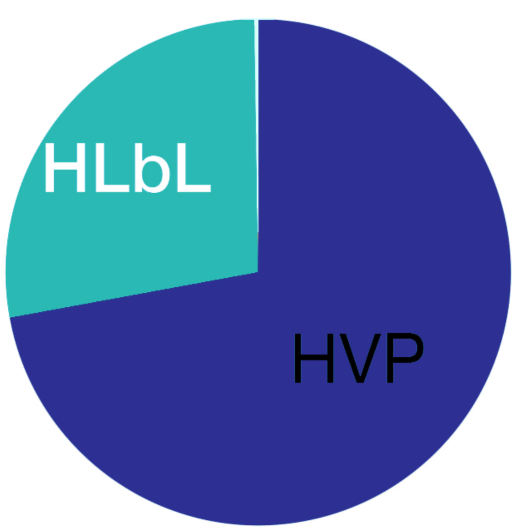
PDG 2021

$$a_\mu^{\mathrm{SM}} = 116\;591\;810\,(1)(40)(18)\cdot 10^{-11}$$

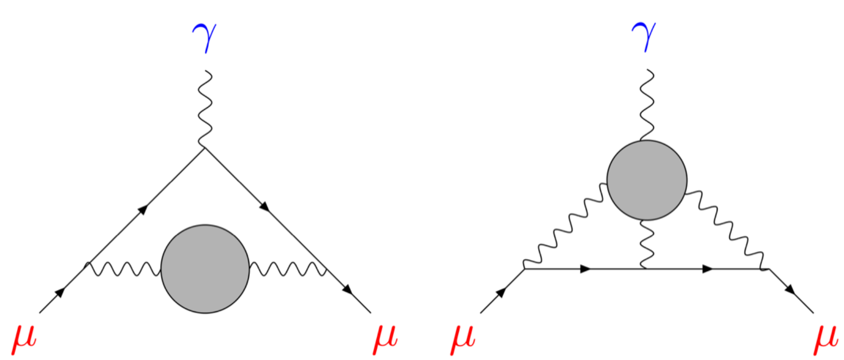
QED+EW

LO Had.

NLO/NNLO Had.



error budget



- dispersion relations using exp. cross sections
- *ab-initio* **LQCD**

Standard Model prediction vs Experiment

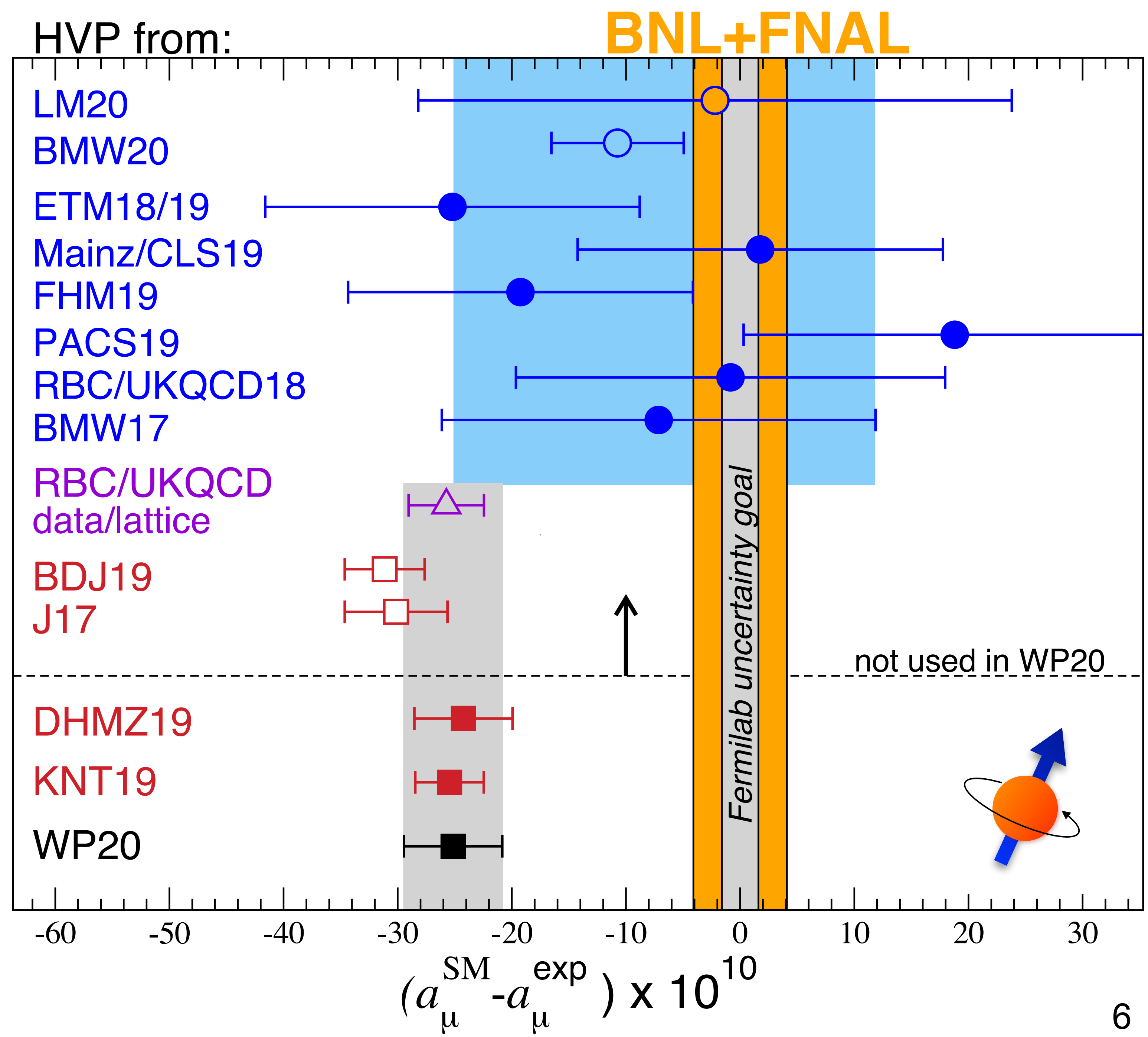
$$a_\mu^{\mathrm{SM}} \quad a_\mu^{\mathrm{HVP}} + \left[a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Weak}} + a_\mu^{\mathrm{HLbL}}\right]$$

Lattice QCD + QED

hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints



In numbers…

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
HVP LO (R-ratio)	692.8 ± 2.4	[KNT '19]
	694.0 ± 4.0	[DHMZ '19]
	692.3 ± 3.3	[CHHKS '19]
HVP LO (R-ratio, avg)	693.1 ± 4.0	[WP '20]
HVP LO (lattice<2021)	711.6 ± 18.4	[WP '20]
HVP NLO	-9.83 ± 0.07	[Kurz et al '14, Jegerlehner '16, WP '20]
HVP NNLO	1.24 ± 0.01	[Kurz '14, Jeger. '16]
HLbyL LO (pheno)	9.2 ± 1.9	[WP '20]
HLbyL LO (lattice<2021)	$7.8 \pm 3.1 \pm 1.8$	[RBC '19]
HLbyL LO (avg)	9.0 ± 1.7	[WP '20]
HLbyL NLO (pheno)	0.2 ± 0.1	[WP '20]
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP Tot. (R-ratio)	684.5 ± 4.0	[WP '20]
HLbL Tot.	9.2 ± 1.8	[WP '20]
SM [0.37 ppm]	11659181.0 ± 4.3	[WP '20]
Exp [0.35 ppm]	11659206.1 ± 4.1	[BNL '06 + FNAL '21]
Exp $-$ SM	25.1 ± 5.9 [4.2σ]	

In numbers...

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
HVP LO (R-ratio)	692.8 ± 2.4	[KNT '19]
	694.0 ± 4.0	[DHMZ '19]
	692.3 ± 3.3	[CHHKS '19]
HVP LO (R-ratio, avg)	693.1 ± 4.0	[WP '20]
HVP LO (lattice)	707.5 ± 5.5	[BMWc '20]
HVP NLO	-9.83 ± 0.07	
		[Kurz et al '14, Jegerlehner '16, WP '20]
HVP NNLO	1.24 ± 0.01	[Kurz '14, Jeger. '16]
HLbyL LO (pheno)	9.2 ± 1.9	[WP '20]
HLbyL LO (lattice<2021)	$7.8 \pm 3.1 \pm 1.8$	[RBC '19]
HLbyL LO (lattice 2021)	$10.7 \pm 1.1 \pm 0.9$	[Mainz '21]
HLbyL LO (avg)	9.0 ± 1.7	[WP '20]
HLbyL NLO (pheno)	0.2 ± 0.1	[WP '20]
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP Tot. (lat. + R-ratio)	698.9 ± 5.5	[WP '20, BMWc '20]
HLbL Tot.	9.2 ± 1.8	[WP '20]
SM [0.49 ppm]	11659195.4 ± 5.7	[WP '20 + BMWc '20]
Exp [0.35 ppm]	11659206.1 ± 4.1	[BNL '06 + FNAL '21]
Exp $-$ SM	10.7 ± 7.0 $[1.5\sigma]$	

Hadronic contributions

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

Huge challenge: theory of strong interaction between quarks and gluons, QCD, hugely nonlinear at energies relevant for a_μ

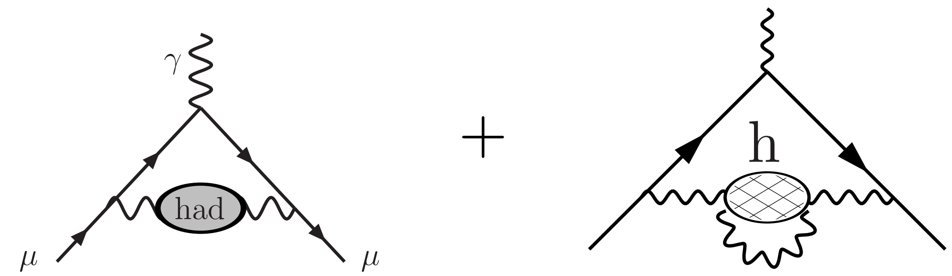
$\rightarrow$ perturbative methods used for electromagnetic and weak interactions do not work

$\rightarrow$ need nonperturbative approaches

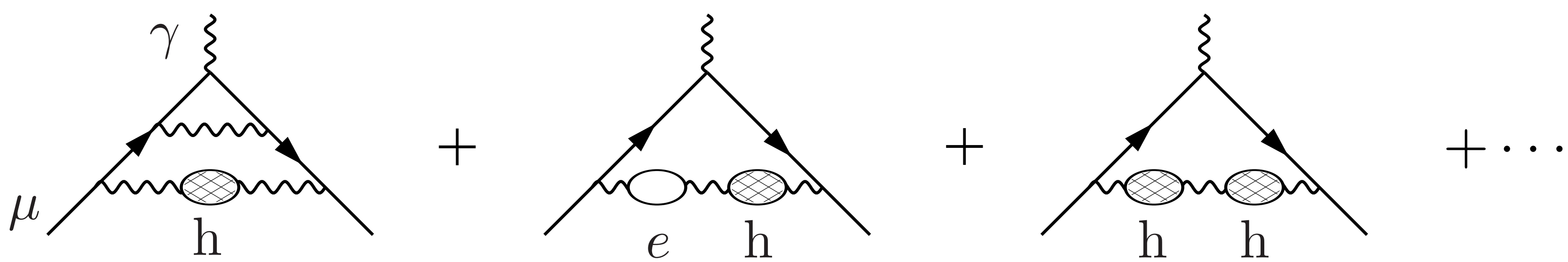
Write

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

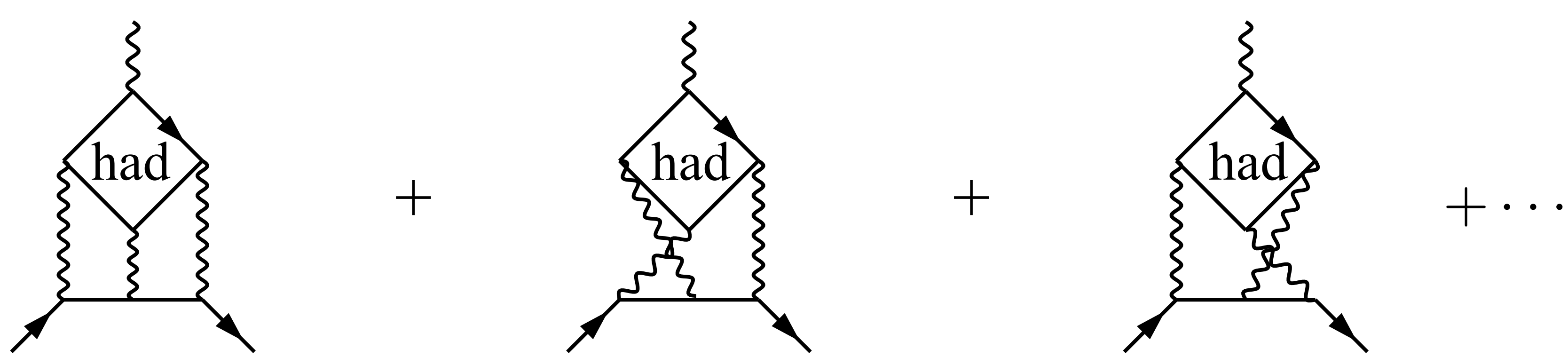
Hadronic contributions: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

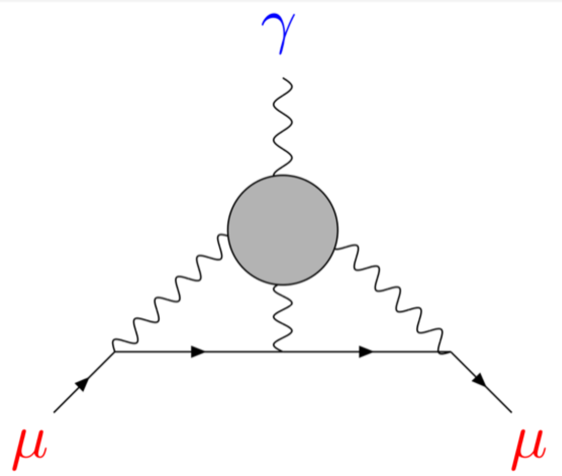


$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

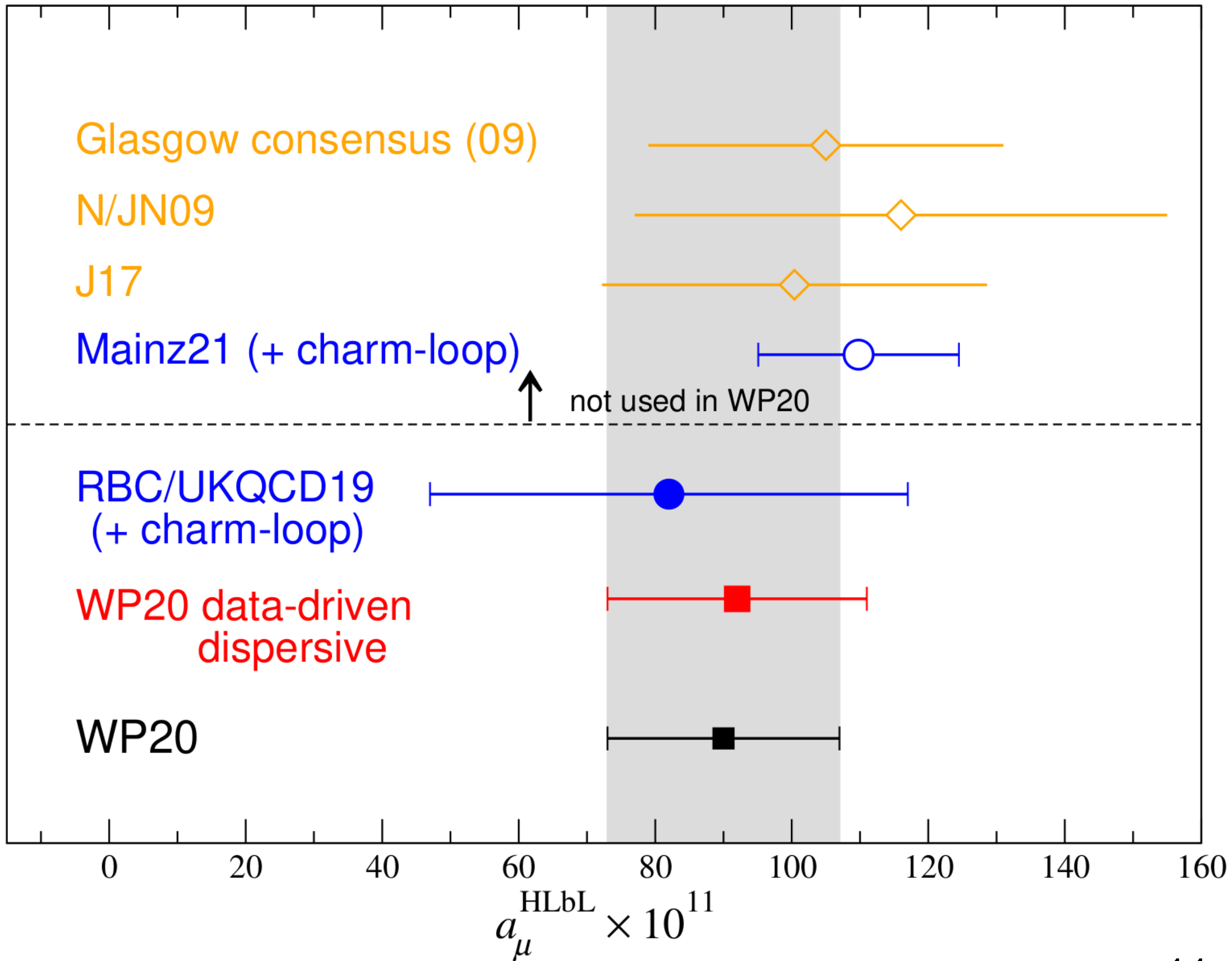


$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq$ 10% instead of $\simeq$ 0.2%
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - $\rightarrow$ Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,... '15-'20]
  - $\rightarrow$ Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average w/ NLO HLbL and conservative error estimates [WP '20]
- $a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} - a_\mu^{\text{HLbL}} = 709.7(4.5) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{HVP}}$



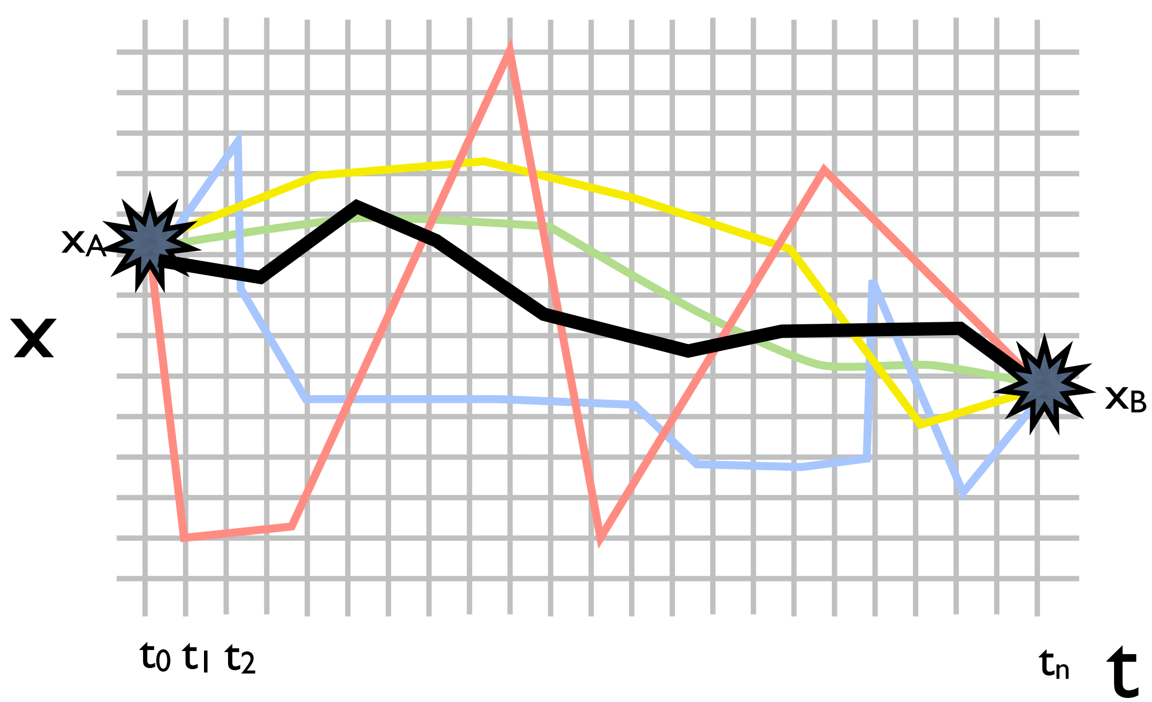
[Colangelo '21]

Small interlude:
Lattice QCD

Lattice QCD

Numerical first-principles approach to
non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- QCD equations ⟷ integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- ~10^{12} variables (for state-of-the-art)

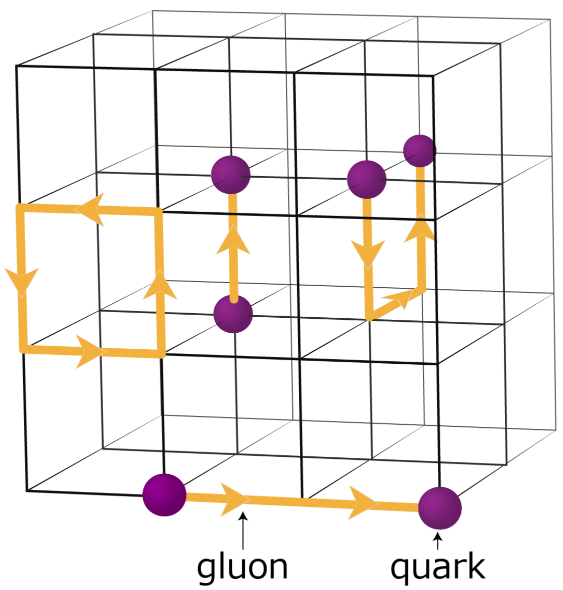


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time $t \to i\tau$
- Finite lattice spacing a
- Volume $L^3 \times T = 64^3 \times 128$
- Boundary conditions



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

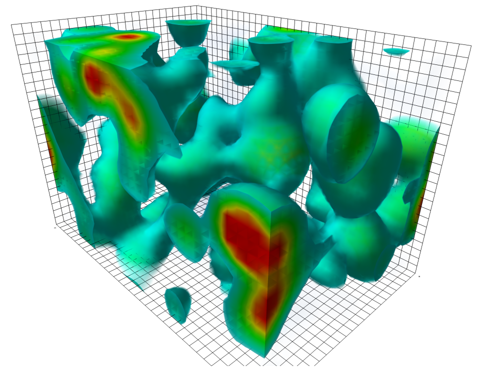
Lattice QCD

Workflow of a lattice QCD calculation

1. Generate field configurations via Hybrid Monte Carlo
   - Leadership-class computing
   - ~100K cores or 1000GPUs, 10's of TF-years
   - O(100-1000) configurations, each ~10-100GB

2. Compute propagators
   - Large sparse matrix inversion
   - ~few 100s GPUs
   - 10x field config in size, many per config

3. Contract into correlation functions
   - ~few GPUs
   - O(100k-1M) copies



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Hadrons are emergent phenomena of statistical average over background gluon configurations

- 1 year on supercomputer ~ 100k years on laptop

Challenges of a full lattice calculation

To make contact with experiment need:

- **A valid approximation to the SM**
  - $\rightarrow$ at least u, d, s in the sea w/ $m_u = m_d \ll m_s$ (N_f=2+1) $\Rightarrow \sigma \sim 1\%$
  - $\rightarrow$ better also include c (N_f=2+1+1) & $m_u \leq m_d$ & EM $\Rightarrow \sigma \sim 0.1\%$

- **u & d w/ masses well w/in $SU(2)$ chiral regime** : $\sigma_\chi \sim (M_\pi/4\pi F_\pi)^2$
  - $\rightarrow$ $M_\pi \sim 135\,\mathrm{MeV}$ or many $M_\pi \leq 400\,\mathrm{MeV}$ w/ $M_\pi^{\mathrm{min}} < 200\,\mathrm{MeV}$ for $M_\pi \rightarrow 135\,\mathrm{MeV}$

- **a $\rightarrow$ 0** : $\sigma_a \sim (a\Lambda_{\mathrm{QCD}})^n,\ (am_q)^n,\ (a|\vec{p}|)^n$ w/ $a^{-1} \sim 2 \div 4\,\mathrm{fm}$
  - $\rightarrow$ at least 3 a's $\leq 0.1\,\mathrm{fm}$ for $a \rightarrow 0$

- **L $\rightarrow \infty$** : $\sigma_L \sim (M_\pi/4\pi F_\pi)^2 \times e^{-LM_\pi}$ for stable hadrons, $\sim 1/L^n$ for resonances, QED, ...
  - $\rightarrow$ many L w/ $(LM_\pi)^{\mathrm{max}} \gtrsim 4$ for stable hadrons & better otherwise to allow for $L \rightarrow \infty$

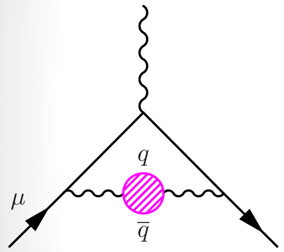
- These requirements $\Rightarrow O(10^{12})$ **dofs** that have to be integrated over

- **Renormalization** : best done nonperturbatively

- **A signal** : $\sigma_{\mathrm{stat}} \sim 1/\sqrt{N_{\mathrm{meas}}}$, reduce w/ $N_{\mathrm{meas}} \rightarrow \infty$

Credits: L. Lellouch

HVP from the lattice

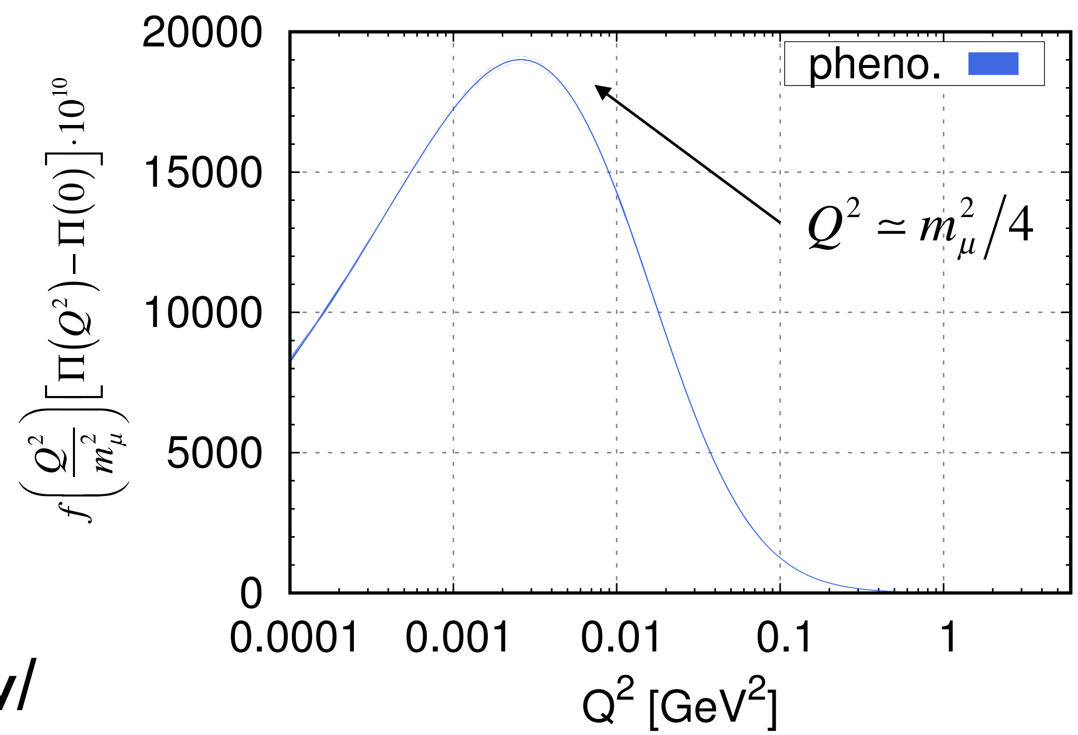


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi(Q^2)$$

$$a_\mu^{\text{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right) \left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972



F. Jegerlehner, "alphaQEDc17"

FV & $a \neq 0$: A. discrete momenta ($Q_{\min} = 2\pi/T > m_\mu/2$); B. $\Pi_{\mu\nu}(0) \neq 0$ in FV contaminates $\Pi(Q^2) \sim \Pi_{\mu\nu}(Q)/Q^2$ for $Q^2 \to 0$ w/ very large FV effects; C. $\Pi(0) \sim \ln(a)$

Time-Momentum Representation

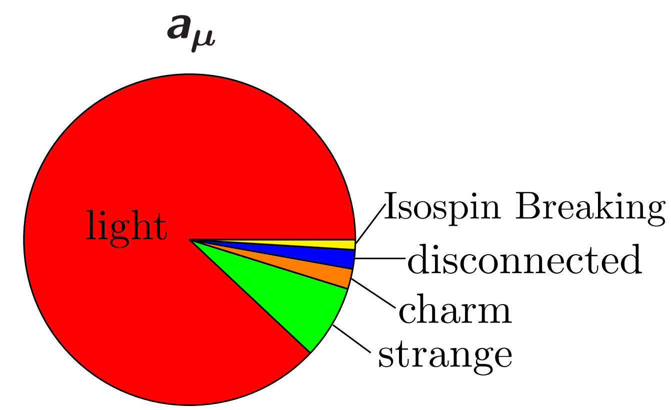
$$a_\mu^{\text{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dt \; \tilde{f}(t)\, V(t)$$

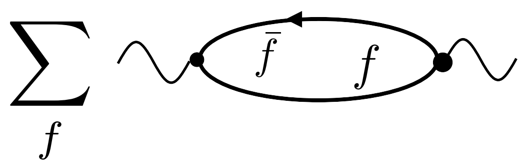
$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \, \langle J_i(\vec{x},t) J_i(0) \rangle$$

D. Bernecker and H. B. Meyer, 2011

Time-Momentum Representation

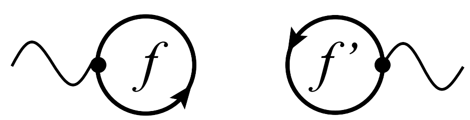
- No reliance on exp. data, except for hadronic quantities used to calibrate the simulation ($M_\pi, M_K, M_{nucl}, \ldots$)
- Can perform an explicit quark flavor separation of $a_\mu^{\rm HVP,LO}$



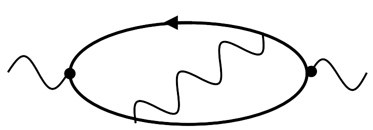


light-quark connected $a_\mu^{\rm HVP,LO}({\rm ud}) \sim 90\,\%$ of total

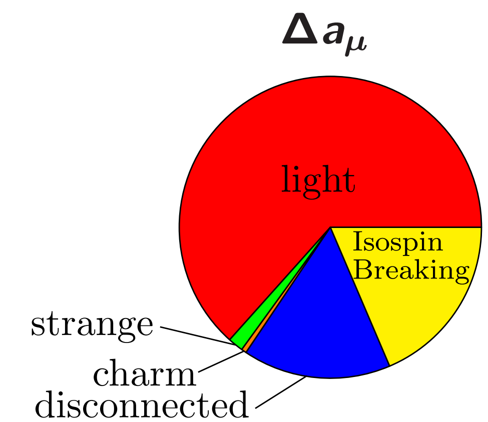
s,c-quark connected $a_\mu^{\rm HVP,LO}({\rm s,c}) \sim 8\,\%\,,2\,\%$ of total



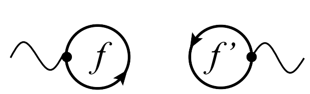
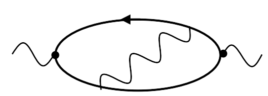
disconnected $a_{\mu,disc}^{\rm HVP,LO} \sim 2\,\%$ of total

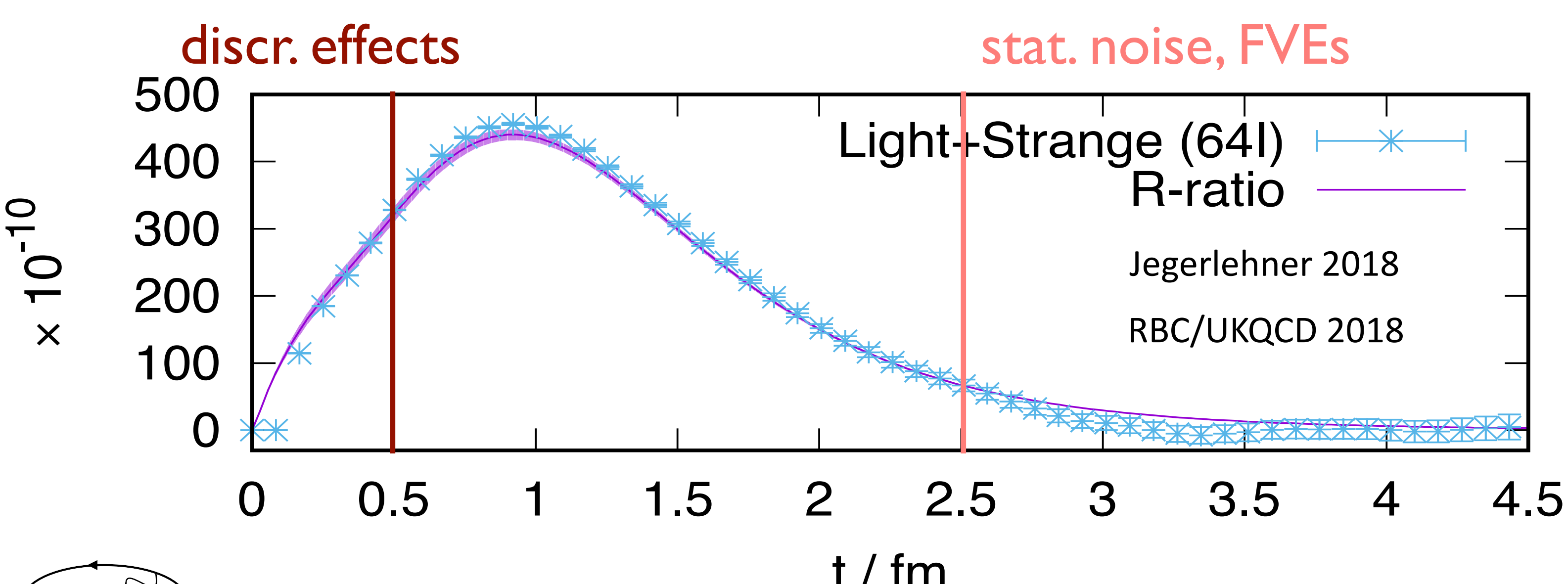


IB ($m_u \neq m_d + {\rm QED}$) $\delta a_\mu^{\rm HVP,LO} \sim 1\,\%$ of total



Challenges:

- sub-percent stat. precision
  exp. growing StN ratio in $V(t)$ as $t \to \infty$
- correct for FVEs, control discr. effects (scale setting and continuum extrap.)
- quark-disconn. diagrams 
  control stat. & stochastic noise
- isospin-breaking: $m_u \neq m_d$, $\alpha_{em} \neq 0$ 

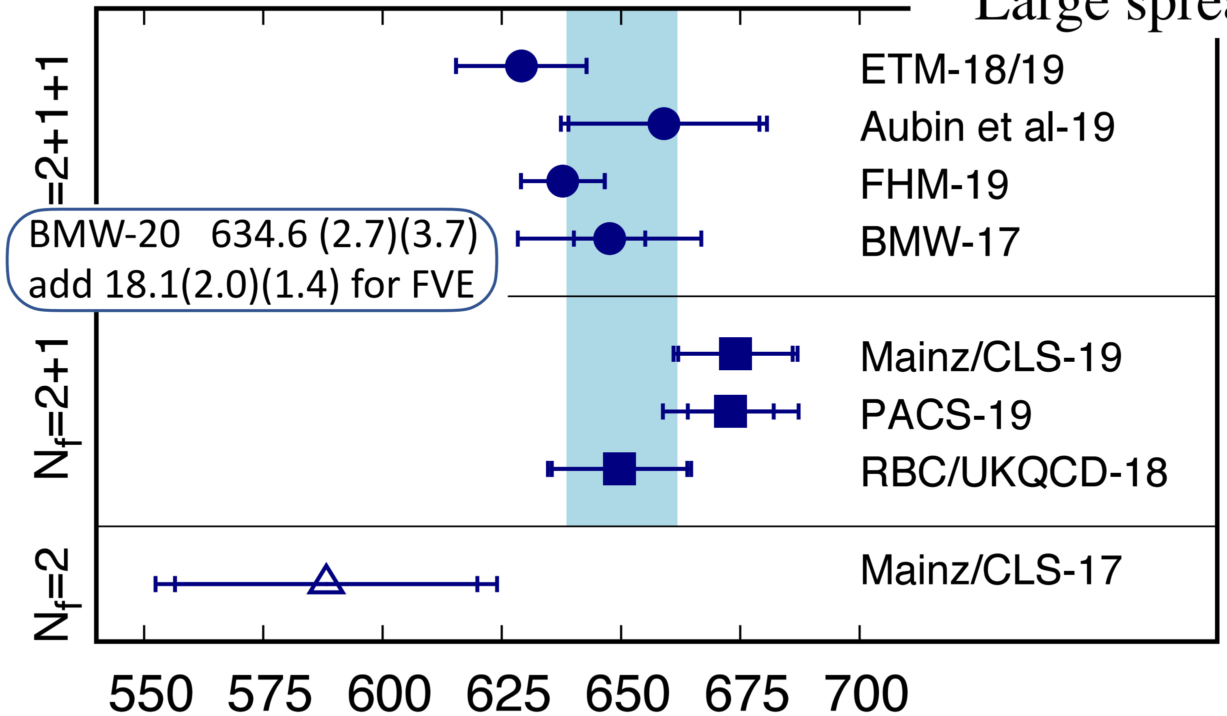


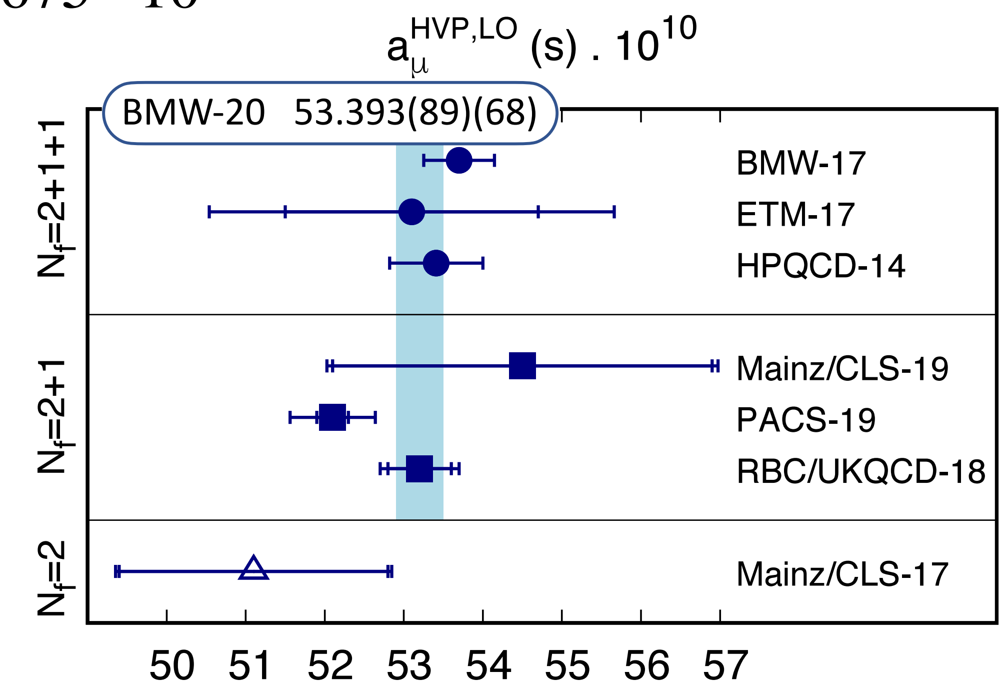
Results for each contribution

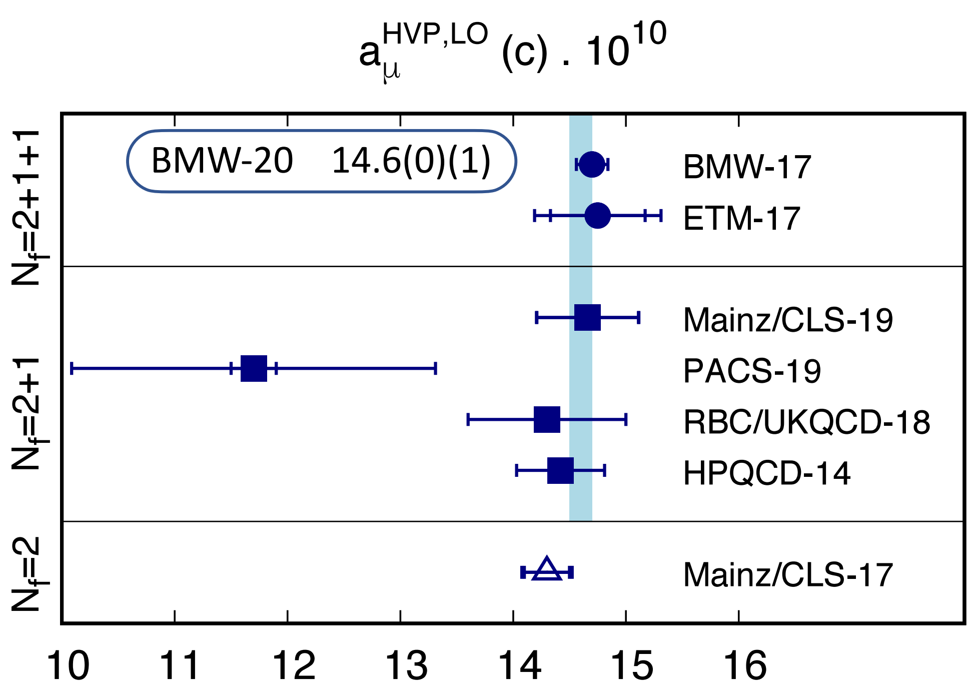
WP '20

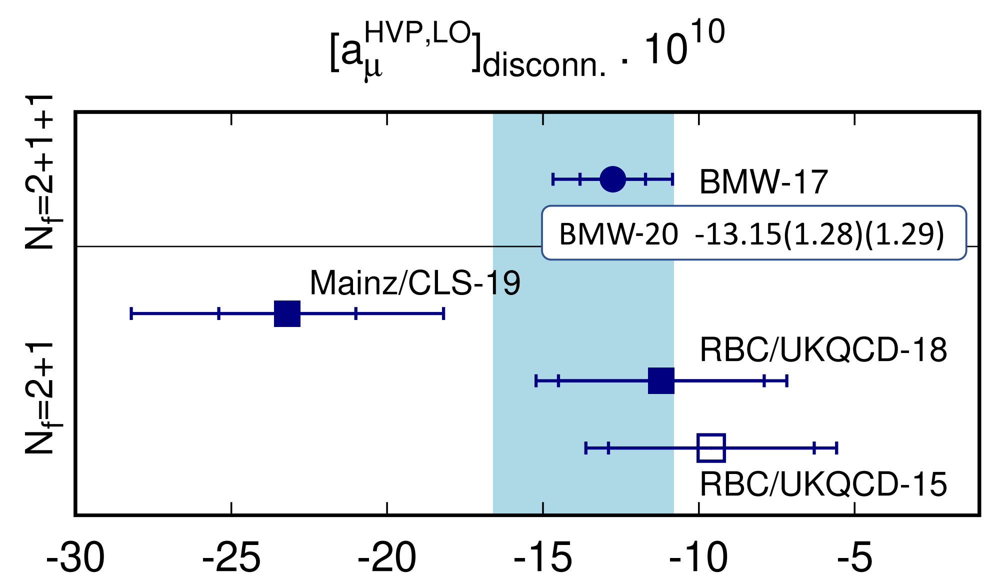
$$a_\mu^{\mathrm{HVP,LO}}(\mathrm{ud}) = 650.2(11.6) \cdot 10^{-10}$$

Large spread $630 \div 675 \cdot 10^{-10}$

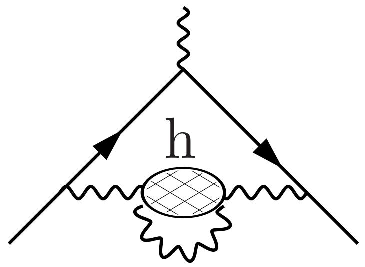


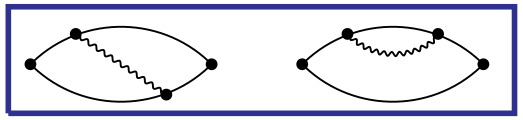




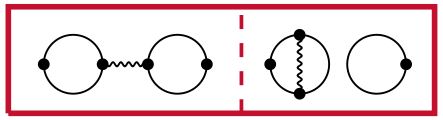


Isospin-breaking contributions

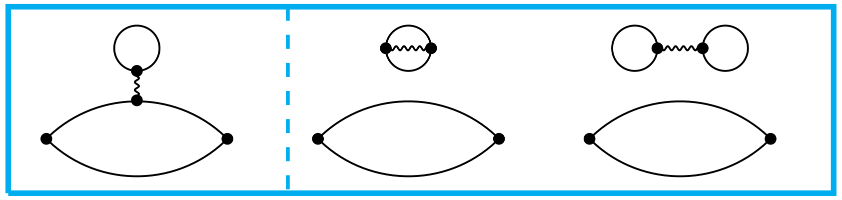




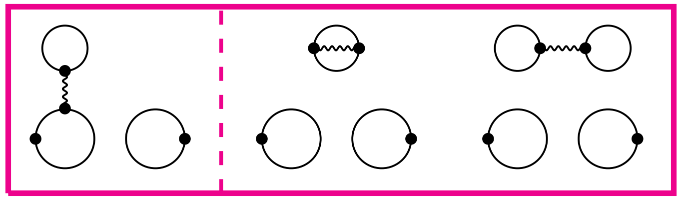
BMW −1.27(40)(33)
RBC/UKQCD 5.9(5.7)(1.7)
ETM 1.1(1.0)



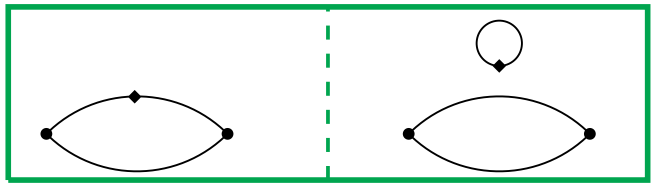
−0.55(15)(11) BMW
−6.9(2.1)(2.0) RBC/UKQCD



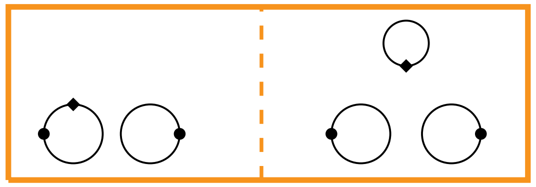
−0.0095(86)(99) 0.42(20)(19) BMW



0.011(24)(14) −0.047(33)(23) BMW

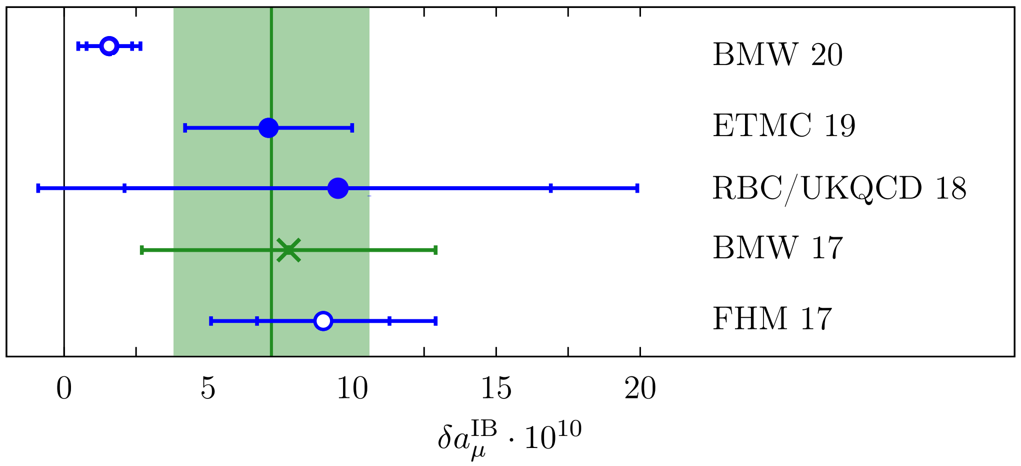


6.59(63)(53) BMW
10.6(4.3)(6.8) RBC/UKQCD
6.0(2.3) ETM
7.7(3.7) 9.0(2.3) FHM
9.0(0.8)(1.2) LM



−4.63(54)(69) BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]



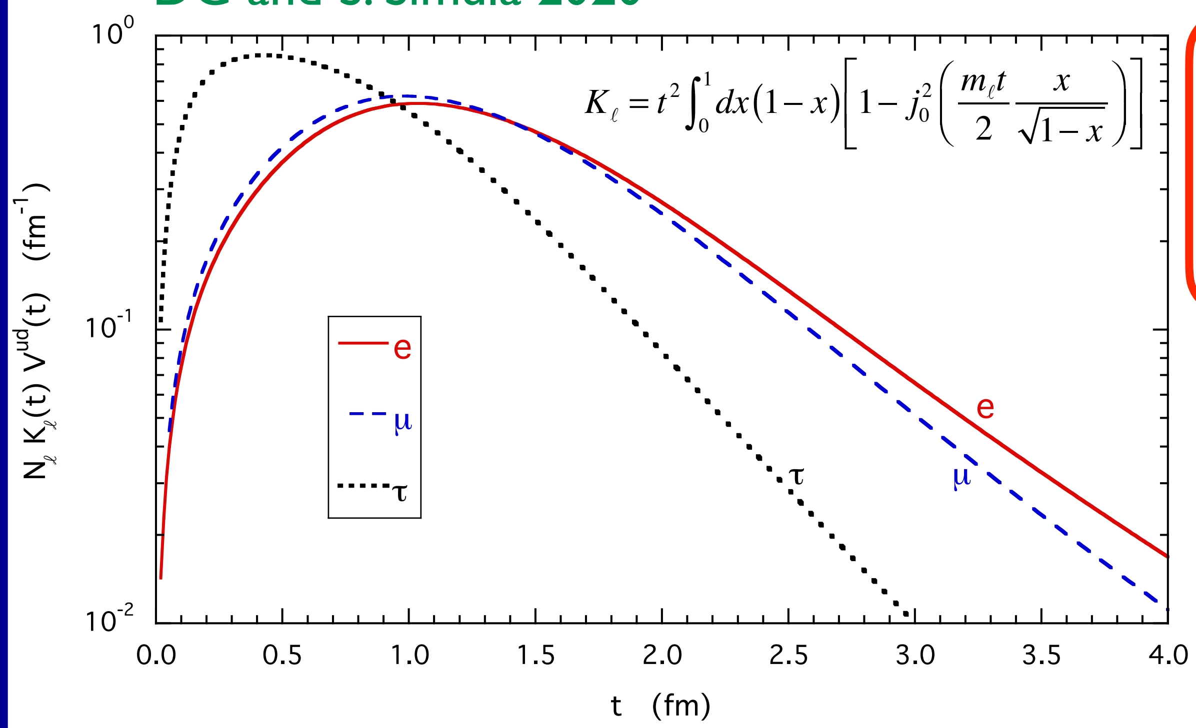
- Small overall value due to large cancellations
- Large statistical uncertainties
- More precise calculations are in progress

Plots by V. Gülpers and H. Wittig

Ratios of the HVP contributions to the lepton g-2

Ratio electron/muon

DG and S. Simula 2020



$$\boxed{R_{e/\mu} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\mathrm{HVP}}}{a_\mu^{\mathrm{HVP}}}}$$

- numerator and denominator share the same hadronic input
- hadronic uncertainties strongly correlated (~ 98%) and largely cancel out

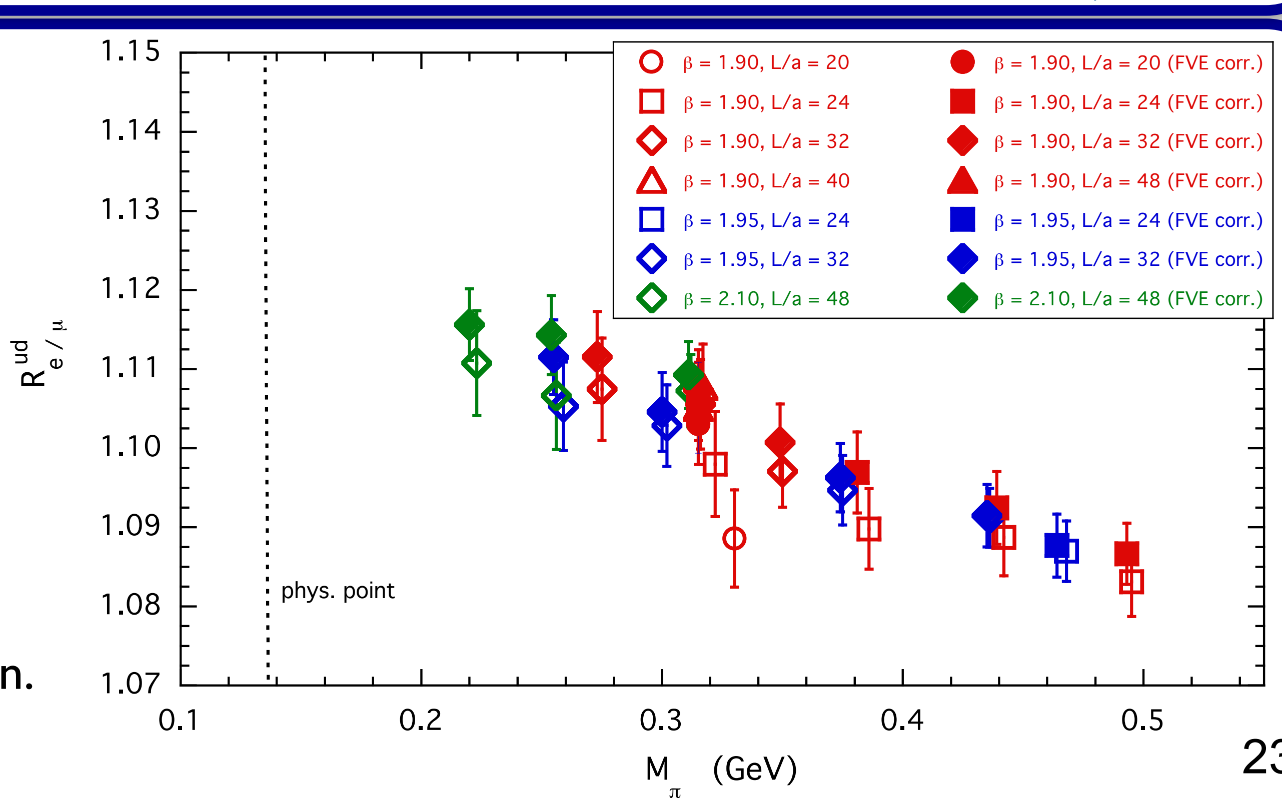
$$\boxed{R_{e/\mu} \equiv R_{e/\mu}^{ud} \cdot \widetilde{R}_{e/\mu}}$$

$$R_{e/\mu}^{ud} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\mathrm{HVP}}(ud)}{a_\mu^{\mathrm{HVP}}(ud)}$$

$$\widetilde{R}_{e/\mu} \equiv \frac{1 + \sum_{j=s,c,IB,disc} \frac{a_e^{\mathrm{HVP}}(j)}{a_e^{\mathrm{HVP}}(ud)}}{1 + \sum_{j=s,c,IB,disc} \frac{a_\mu^{\mathrm{HVP}}(j)}{a_\mu^{\mathrm{HVP}}(ud)}}$$

$R_{e/\mu}^{ud}$

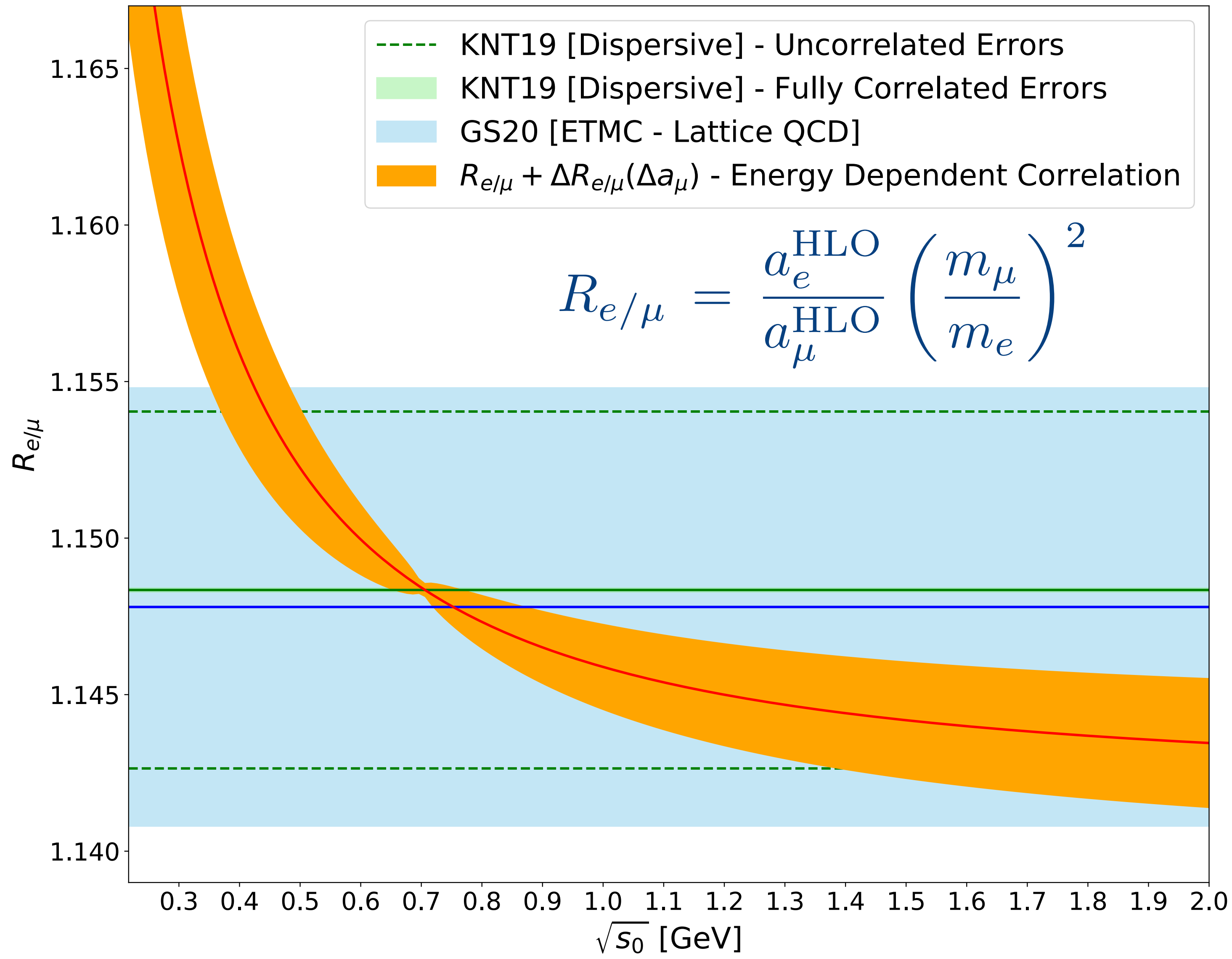
- Precision of the data ≈ 4 times better than the individual HVP terms
- Discretization and scale setting errors play a minor role
- Non-trivial pion mass dependence
- Visible FVEs, removed using the analytic representation. The correction does not exceed ~ 1.3%



Trying to accommodate the g-2 discrepancy

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19.
Possible future bounds on very low energy shifts Δσ(s)?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Window observables

Windows "on the g-2 mystery"

Restrict integration over Euclidean time to sub-intervals

→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f; t_0, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f; t_0, t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f; t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

"Standard" choice:

$t_0 = 0.4$ fm $\quad t_1 = 1.0$ fm

$\Delta = 0.15$ fm

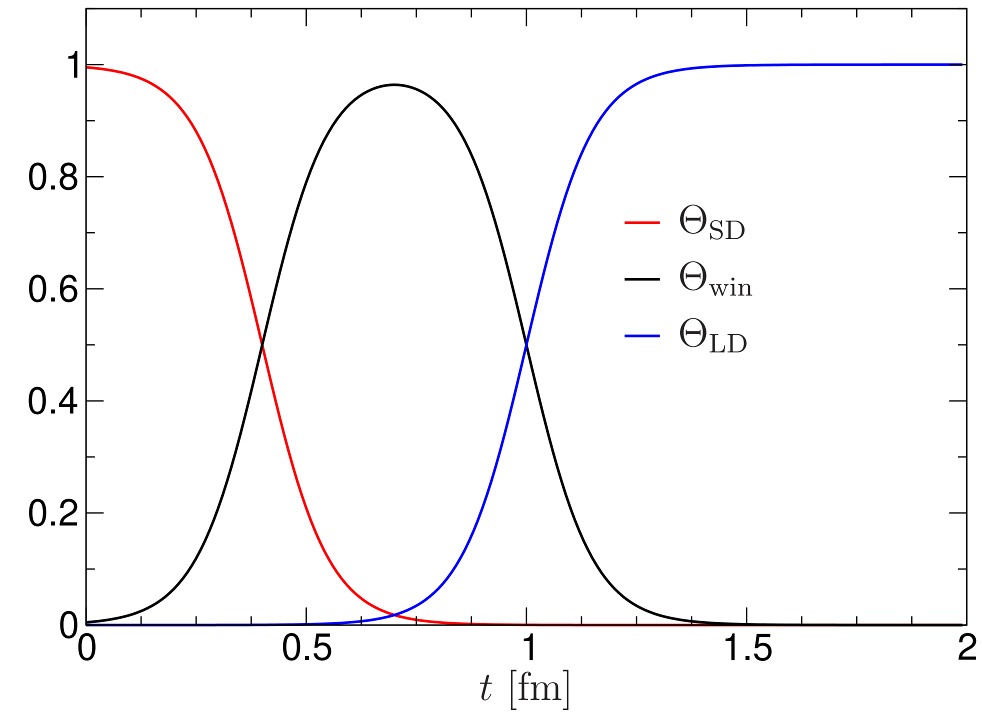
RBC/UKQCD 2018

Intermediate window

- Reduced FVEs
- Much better StN ratio

→ Precision test of different lattice calculations

→ Commensurate uncertainties compared to dispersive evaluations

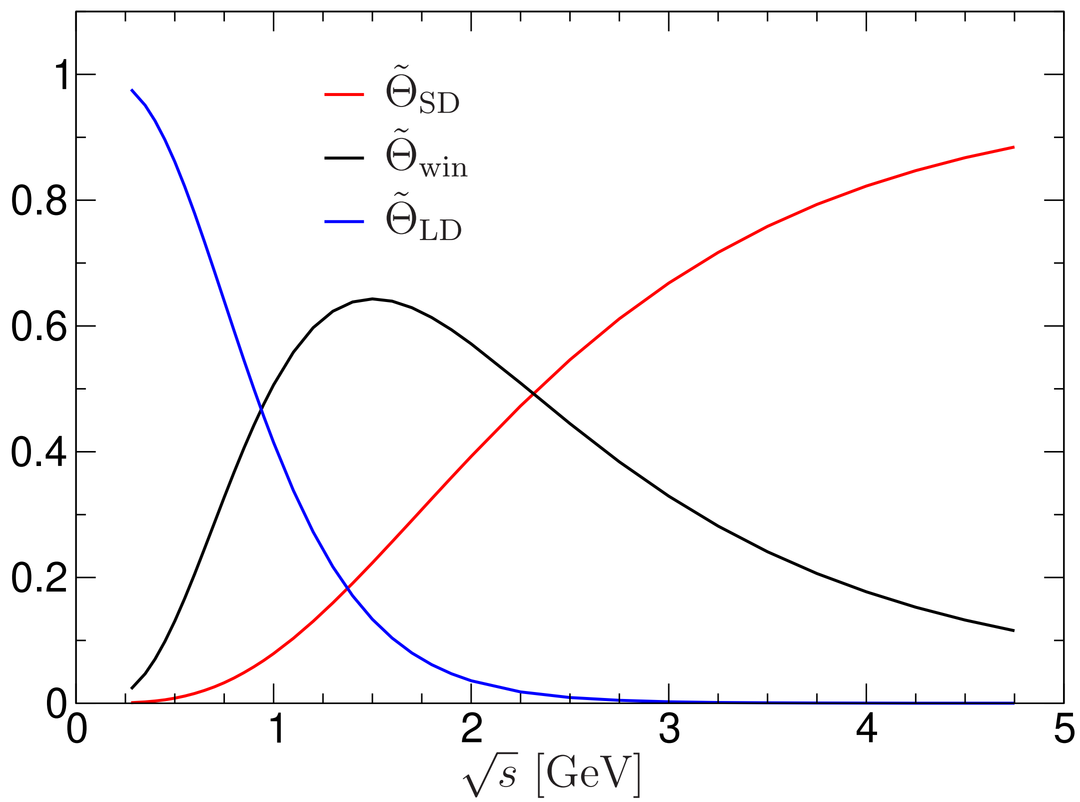


Comparison with *R*-ratio

$$V(t) = \frac{1}{12\pi^2}\int_{M_{\pi^0}}^{\infty} d(\sqrt{s})\, R(s)\, s\, e^{-\sqrt{s}t} \qquad R(s) = \frac{3s}{4\pi\alpha_{em}^2}\sigma(s, e^+e^- \to hadrons)$$

Insert $V(t)$ into the expression for TMR

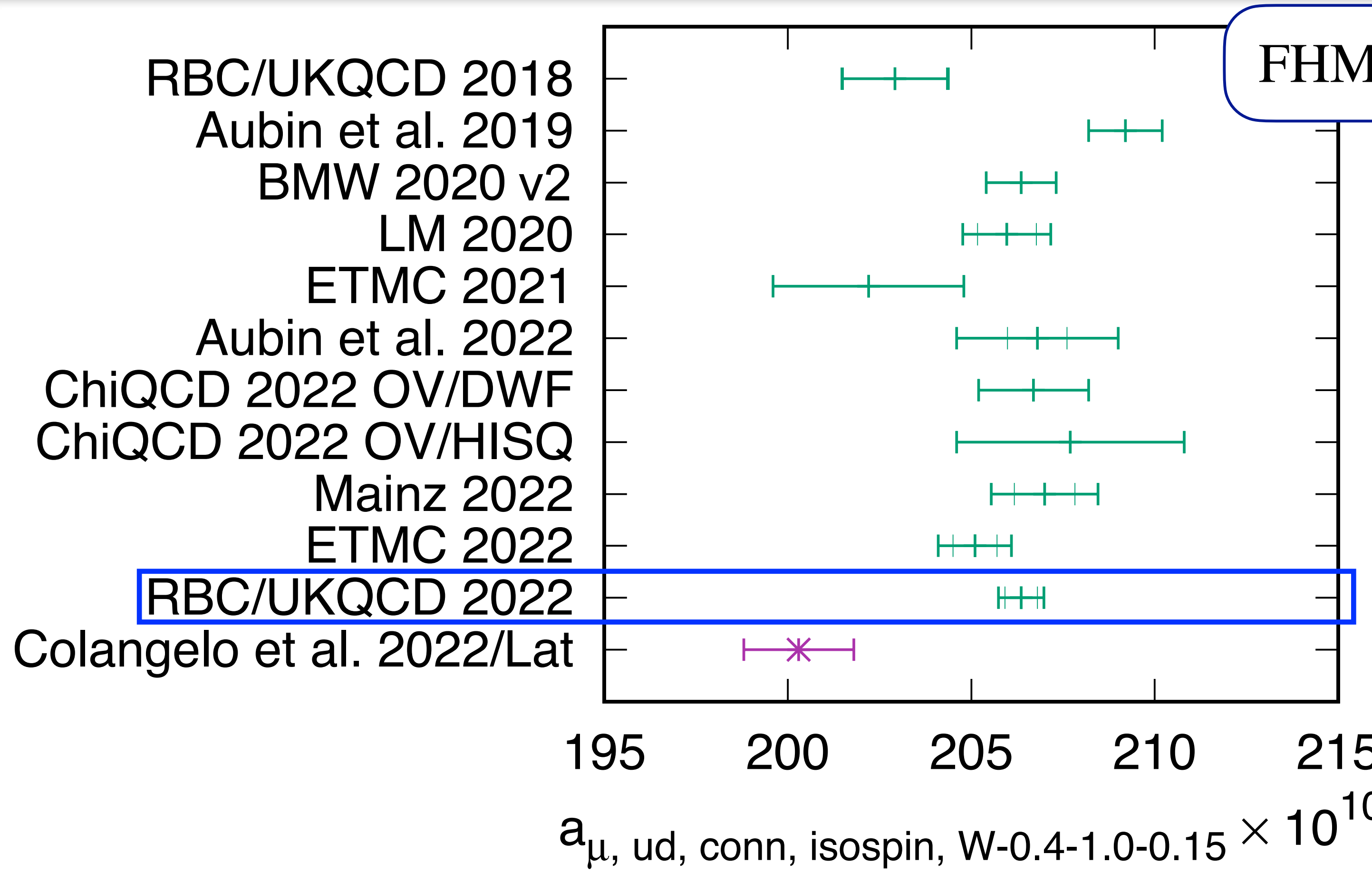
$$a_{\mu,win}^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_{M_{\pi^0}}^{\infty} d(\sqrt{s}) R(s)\, \frac{1}{12\pi^2}\, s \int_0^{\infty} dt\, \tilde{f}(t)\, \Theta_{win}(t)\, e^{-\sqrt{s}t}$$



Colangelo *et al.* 2022

	$a_{\mathrm{SD}}^{\mathrm{HVP}}$	$a_{\mathrm{int}}^{\mathrm{HVP}}$	$a_{\mathrm{LD}}^{\mathrm{HVP}}$	$a_{\mathrm{total}}^{\mathrm{HVP}}$
All channels	68.4(5)	229.4(1.4)	395.1(2.4)	693.0(3.9)
	[9.9%]	[33.1%]	[57.0%]	[100%]
2π below 1.0 GeV	13.7(1)	138.3(1.2)	342.3(2.3)	494.3(3.6)
	[2.8%]	[28.0%]	[69.2%]	[100%]
3π below 1.8 GeV	2.5(1)	18.5(4)	25.3(6)	46.4(1.0)
	[5.5%]	[39.9%]	[54.6%]	[100%]
White Paper [1]	–	–	–	693.1(4.0)
RBC/UKQCD [24]	–	231.9(1.5)	–	715.4(18.7)
BMWc [36]	–	236.7(1.4)	–	707.5(5.5)
BMWc/KNT [7, 36]	–	229.7(1.3)	–	–
Mainz/CLS [99]	–	237.30(1.46)	–	–
ETMC [100]	69.33(29)	235.0(1.1)	–	–

Results for the intermediate window



FHM 2022 $206.1(1.2) \cdot 10^{-10}$

Talk by S. Gottlieb
@ LatticeNET 2022

Talk by C. Lehner
@ Muon g-2 TI 2022

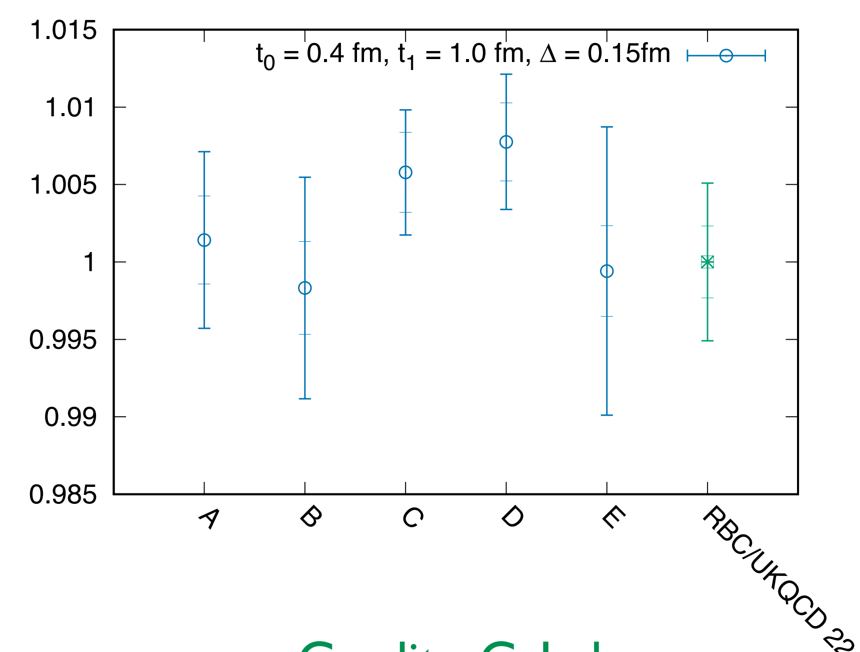
Blinding

- 2 analysis groups for ensemble parameters (not blinded)
- 5 analysis groups for vector-vector correlators (blinded, to avoid bias towards other lattice/R-ratio results)
- Blinded vector correlator $C_b(t)$ relates to true correlator $C_0(t)$ by

$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t) \tag{1}$$

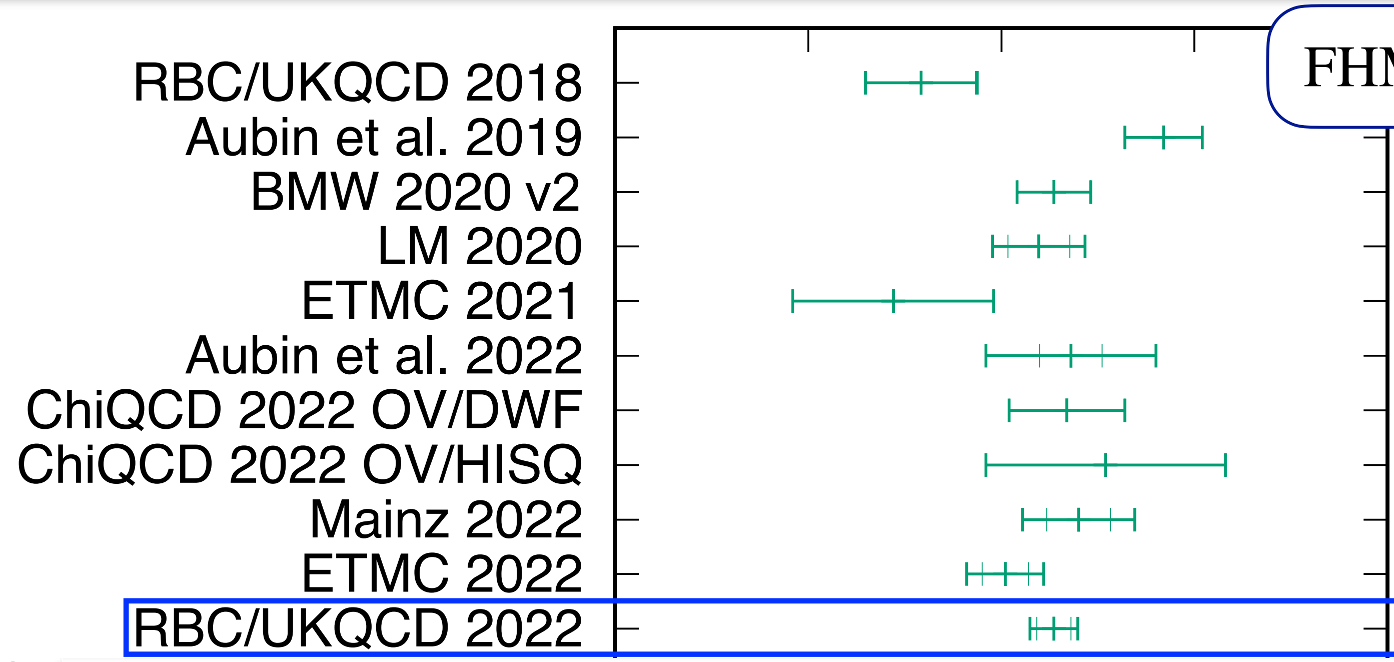
with appropriate random b_0, b_1, b_2, different for each analysis group. This prevents complete unblinding based on previously shared data on coarser ensembles.

Relative unblinding (standard window)



Credits: C. Lehner

Results for the intermediate window



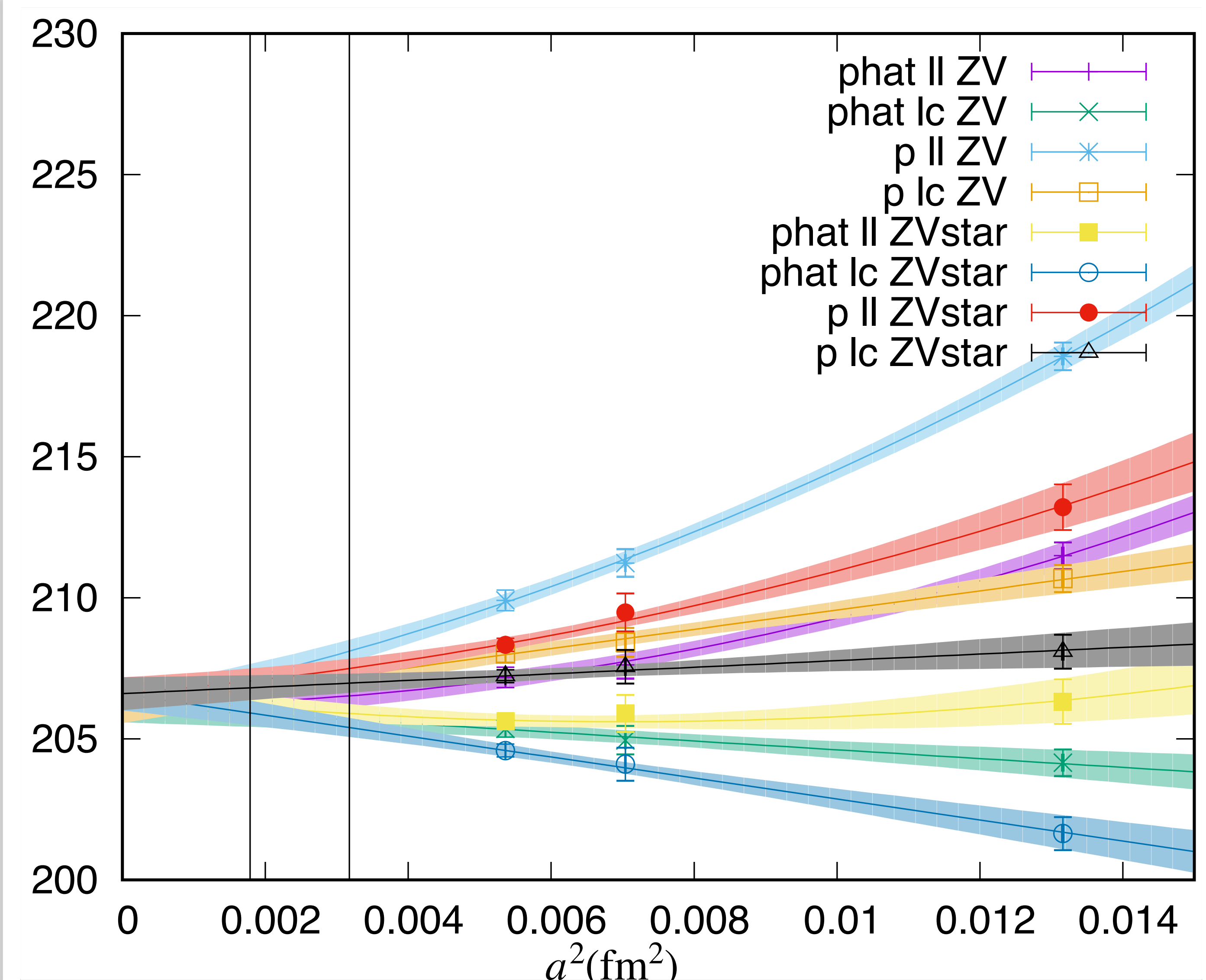
FHM 2022 $206.1(1.2) \cdot 10^{-10}$

Talk by S. Gottlieb @ LatticeNET 2022

RBC/UKQCD 2022

- tot. err. $\sim 0.3\,\%$ of which $\sim 50\,\%$ comes from $a \to 0$
- 3.9σ tension w/ Colangelo *et al.* 22/ Lat

Co



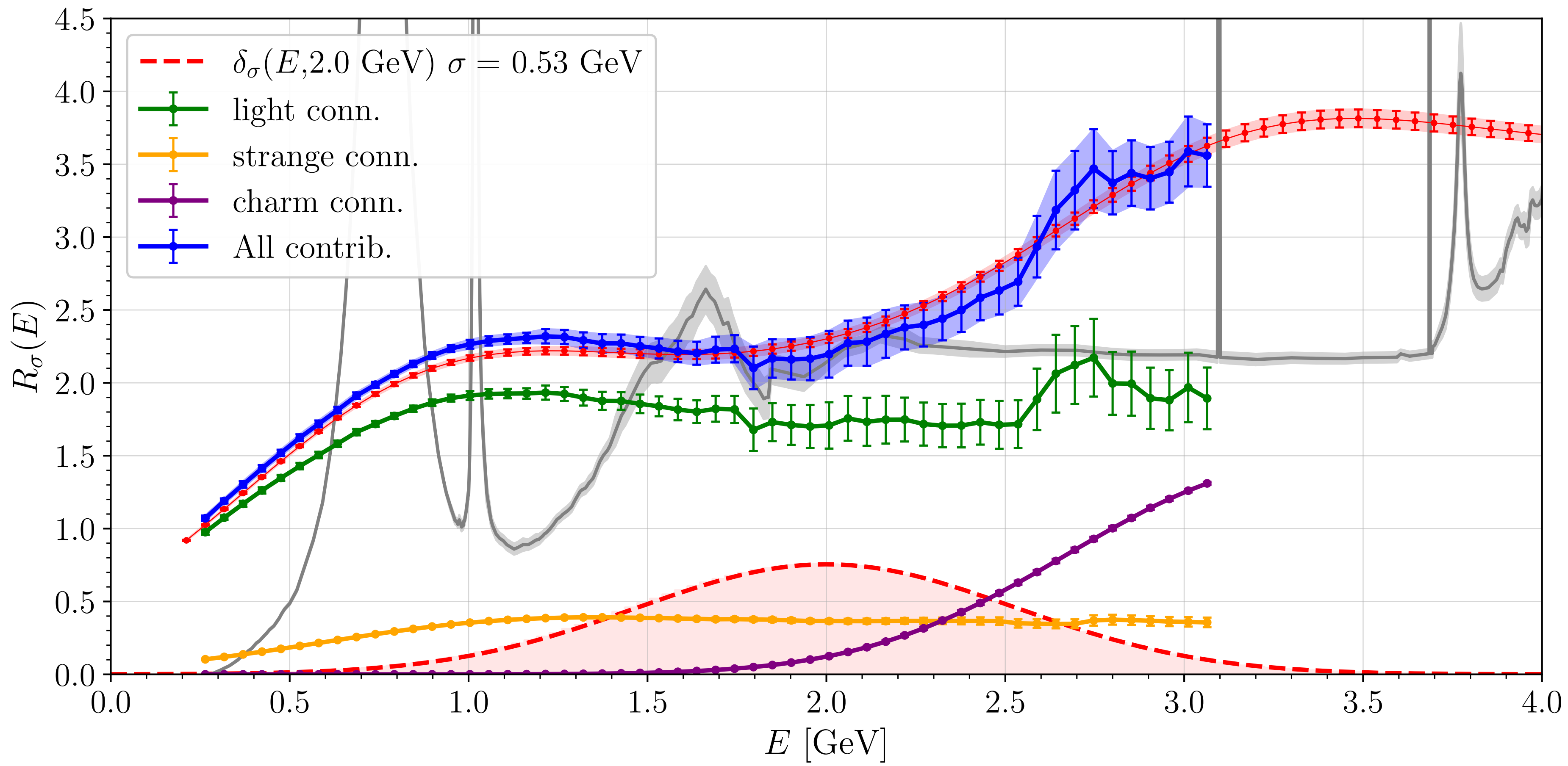
Talk by C. Lehner @ Muon g-2 TI 2022

Probing the R-ratio on the lattice

$R_\sigma(E)$: preliminary results

$$R_\sigma(E) = \int_{2M_\pi}^{\infty} \mathrm{d}\omega \delta_\sigma(\omega, E) R(\omega) \qquad \delta_\sigma(\omega, E) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(\omega-E)^2}{2\sigma^2}}$$

$R_\sigma(E)$ from e^+e^- data



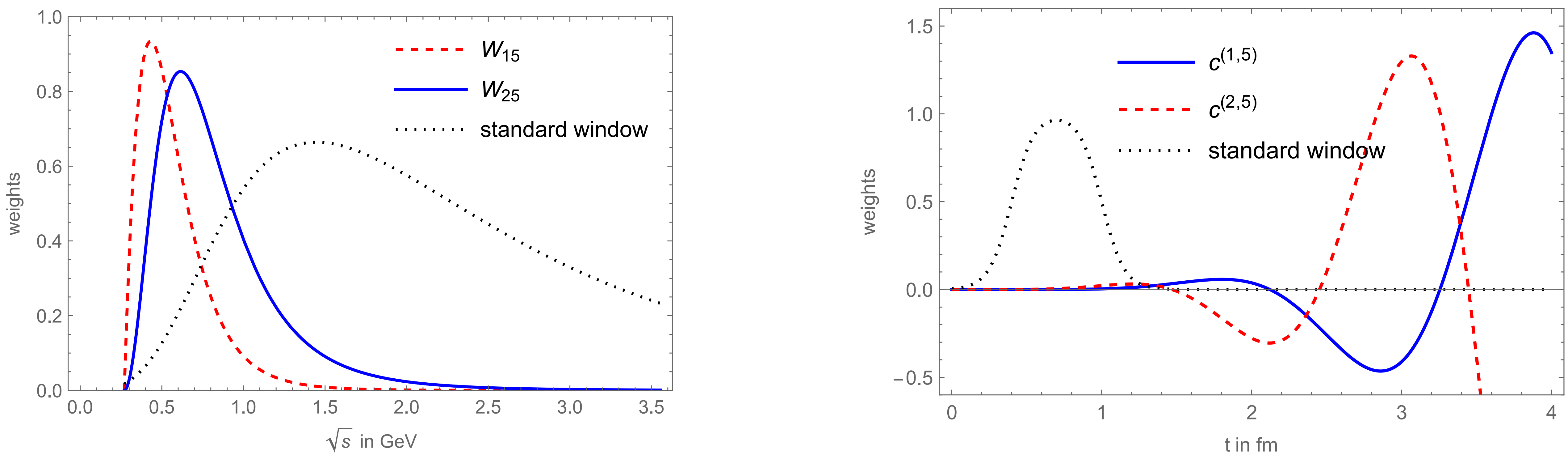
- Uncertainty coming mostly from light quark contributions, strange & charm ones are very precise
- Disconnected contributions are tiny and cannot be appreciated on this scale

Spectral-weight sum rules

This can be recast in terms of $C(t) = \frac{1}{3}\sum_{\vec{x},i}\langle j_i^{\rm EM}(\vec{x},t)j_i^{\rm EM}(0)\rangle = -\int \frac{dQ}{2\pi}\,e^{iQt}\,Q^2\,\hat{\Pi}(Q^2)$ as

$$\int_{s_{\rm th}}^{\infty} ds\, W_{m,n}(s)\,\rho(s) = \int_0^{\infty} dt\, \underbrace{(-1)^m \sum_{k=1}^{n} \frac{(Q_k^2+s_{\rm th})^m}{\prod_{\ell\neq k}(Q_\ell^2-Q_k^2)}\left(\frac{4\sin^2(Q_k t/2)}{Q_k^2}-t^2\right)}_{c^{(m,n)}}\, C(t)$$

Examples: choose $Q_k^2 = 0.25,\ 0.325,\ 0.4,\ 0.475,\ 0.55\ {\rm GeV}^2$ and $n=1,\ 2$:

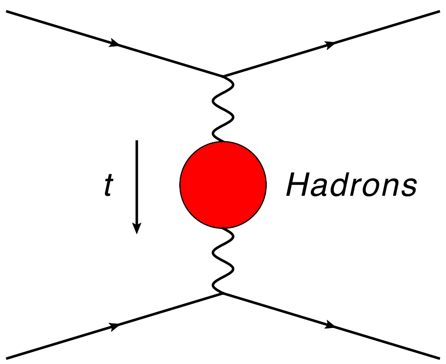


Note: arbitrary vertical scales!

Talk by M. Golterman @ Muon g-2 TI 2022; D. Boito *et al.* 2022

Connections to the MUonE Experiment

MUonE

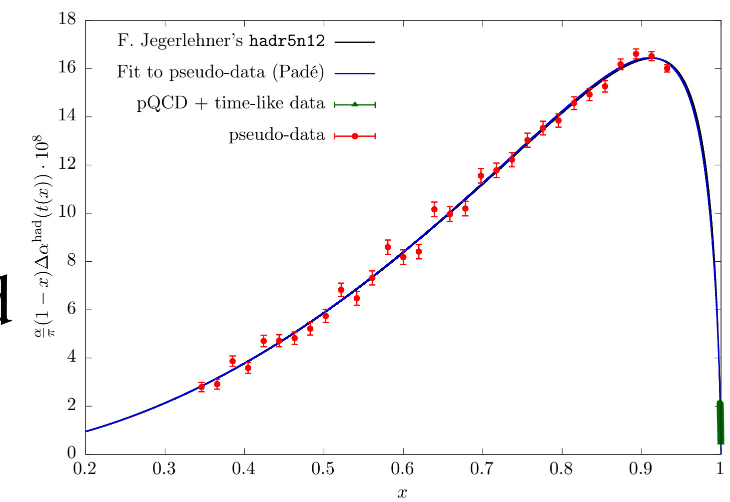


$$t(x) \equiv -\frac{x^2}{1-x} m_\mu^2$$

B. E. Lautrup *et al.* 1972

$$\boxed{a_\mu^{\mathrm{HVP}} = \frac{\alpha_{em}}{\pi} \int_0^1 dx (1-x)\, \Delta\alpha_{em}^{\mathrm{HVP}} [t(x)]}$$

$$\sigma(\mu e \to \mu e)$$



$x \in [0.93,\ 1]$ not experimentally reached

LQCD

DG and S. Simula 2019

$$\left[a_\mu^{\mathrm{HVP}}\right]_> = 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}_>(t) V(t) \quad \longrightarrow \quad \left[a_\mu^{\mathrm{HVP}}\right]_> = 92(2) \cdot 10^{-10}$$

quark-connected terms only

Uncertainty $(\simeq 2 \cdot 10^{-10})$ close to the experimental statistical target $(\simeq 0.3\%)$ of $\left[a_\mu^{\mathrm{HVP}}\right]_<$

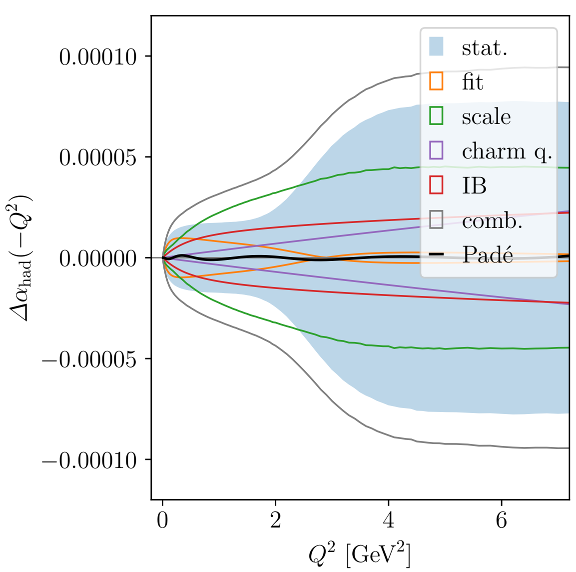
Hadronic running of α_{em} from the lattice

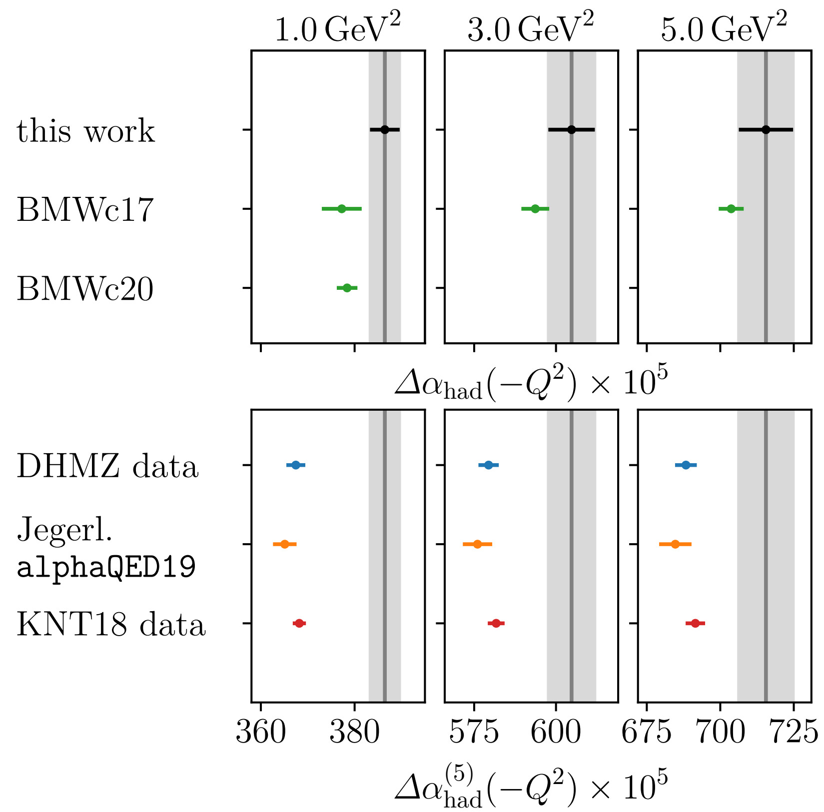
Lattice result for the hadronic running of α

[Cè et al., arXiv:2203.08676]

Starting point: Results for $\Delta\alpha_{\rm had}(-Q^2)$ for Euclidean momenta $0 \leq Q^2 \leq 7\,{\rm GeV}^2$ **[T. San José, TUE 17:10]**

Rational approximation:





- Mainz/CLS and BMWc (2017) differ by 2–3% at the level of 1–2σ
- Tension between Mainz/CLS and phenomenology by $\sim 3\sigma$ for $Q^2 \gtrsim 3\,{\rm GeV}^2$
- Tension increases to $\gtrsim 5\sigma$ for $Q^2 \lesssim 2\,{\rm GeV}^2$ (smaller statistical error due to ansatz for continuum extrapolation)

Systematic uncertainties from fit ansatz, scale setting, charm quenching, isospin-breaking and missing bottom quark contribution (five flavour theory) included in error budget

Summary and Outlook

- Tremendous progress in lattice calculations of HVP (and HLbL!) contributions
- Sub-percent calculation by BMW must be checked and impressive efforts from various lattice collaborations are in progress (stay tuned!)
- An update of the White Paper will be released by the first quarter of 2023
- Benchmark quantities (windows) crucial for checking the internal consistency of lattice calculations
- Extend calculation of window quantities to individual flavor and quark-disconnected contributions
- Extend comparison with phenomenological analyses to understand discrepancies
- $\mu e \to \mu e$ experiment MUonE very important for experimental cross-check and complementarity w/ LQCD