Recent calculations for MUonE with MESMER

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MUONE 2022
15 November 2022

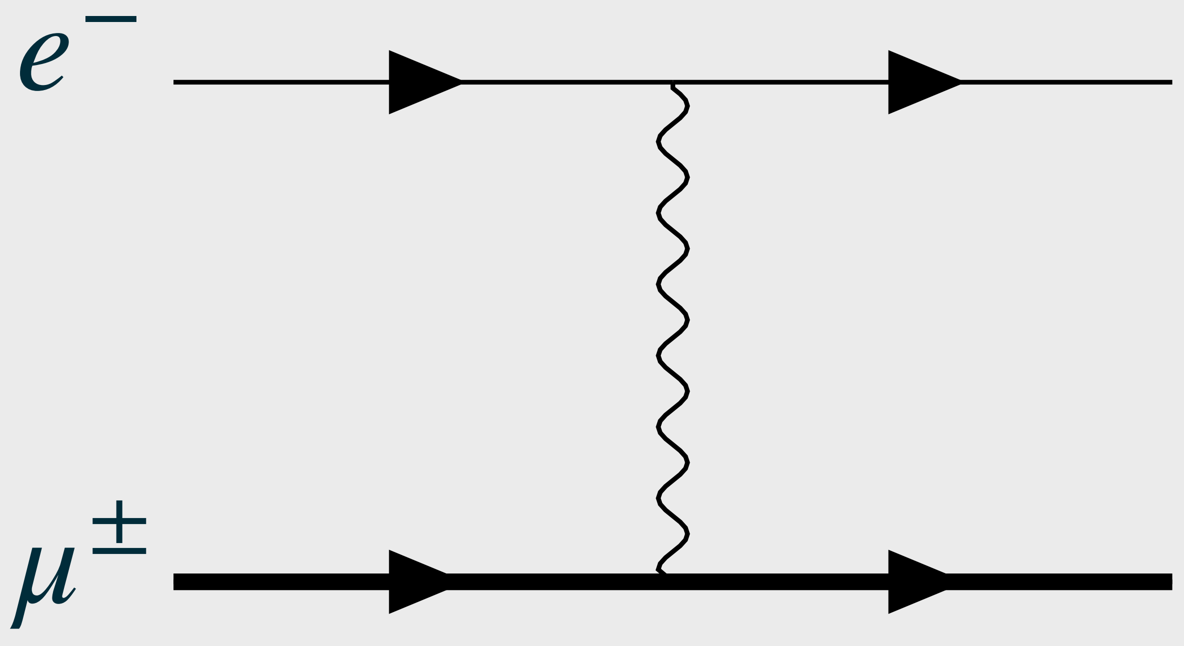
NNLO LEPTONIC CORRECTIONS

E. Budassi et al., JHEP (2021) 098,
arXiv:2109.14606v2 [hep-ph]

Motivation

Calculation of complete fixed-order NNLO QED corrections that include one virtual/real leptonic pair

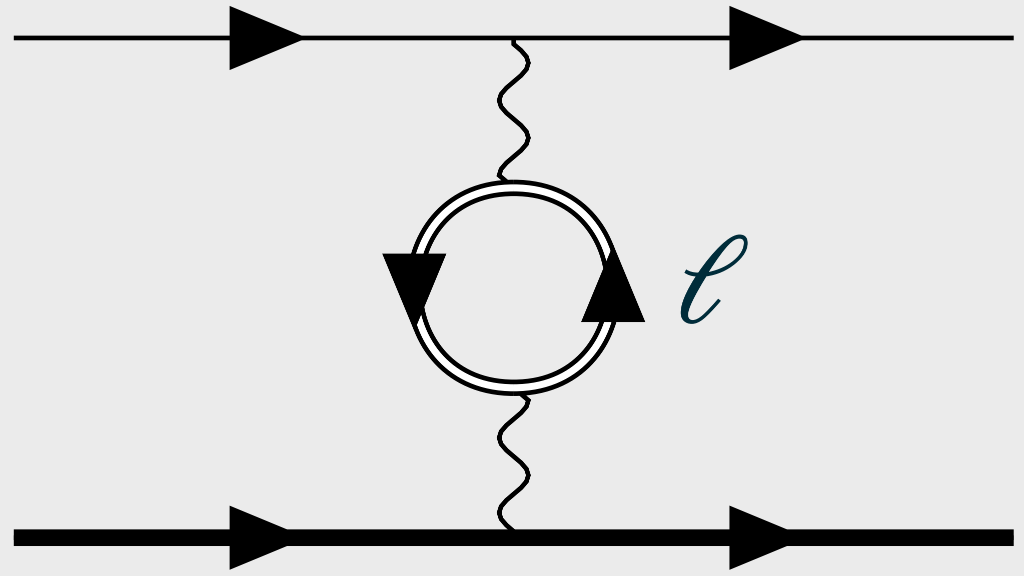
$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$



(a)

Real pair emission particularly important:

- Reducible background to elastic process $\mu^\pm e^- \to \mu^\pm e^-$
- Partial cancellation between virtual diagrams and interference of real ones

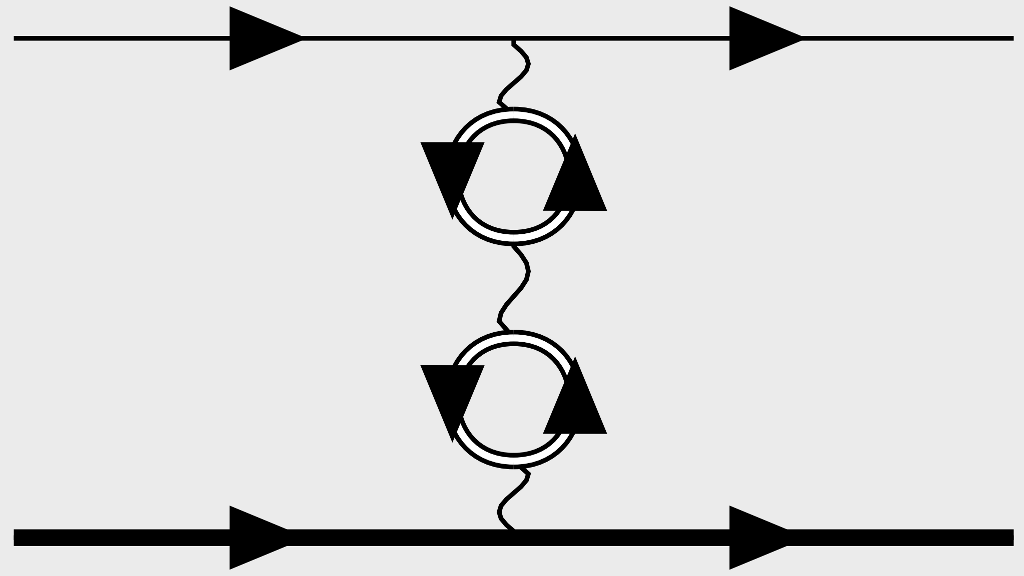


(b)

NNLO leptonic virtual contributions

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

1. Virtual two-loop contribution $d\sigma^{\alpha^2}_{\text{virt}}$
   - Squared absolute value of NLO photon VP insertion
   - NNLO iterated VP insertion in photon propagator
   - Interference of NLO photon VP insertion with NLO corrections
   - NNLO photonic corrections with VP insertion not in loop-photon propagator
   - NNLO irreducible vertex and box corrections



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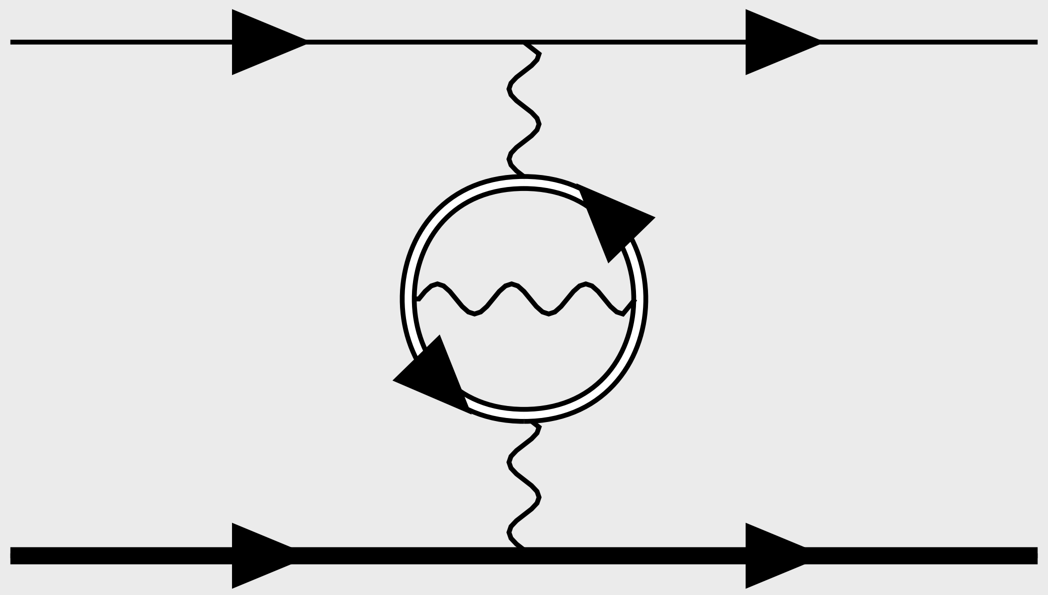
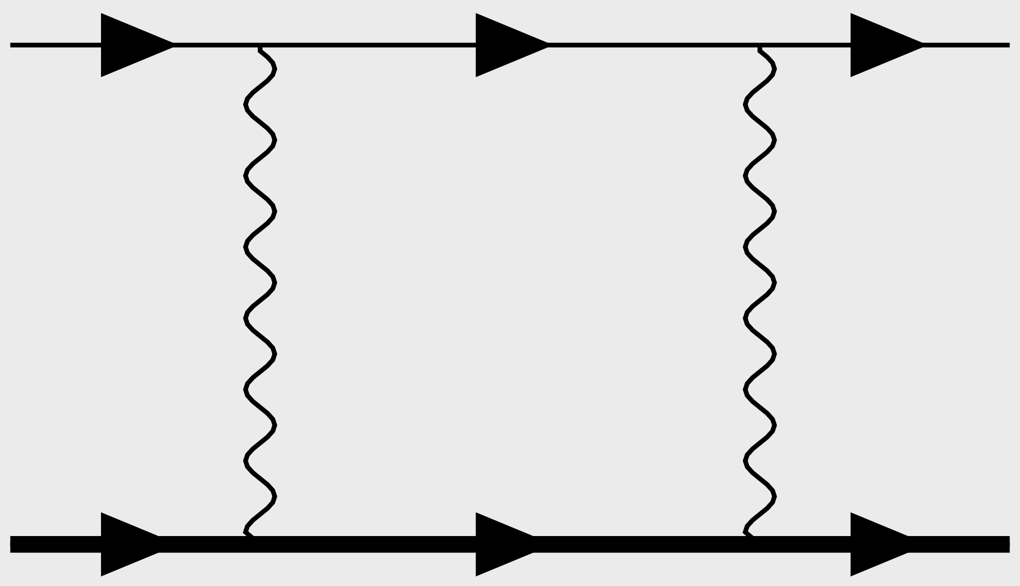


(b)

NNLO leptonic virtual contributions

$$d\sigma_{N_f}^{\alpha^2} = d\sigma_{\text{virt}}^{\alpha^2} + d\sigma_{\gamma}^{\alpha^2} + d\sigma_{\text{real}}^{\alpha^2}$$

1. Virtual two-loop contribution $d\sigma_{\text{virt}}^{\alpha^2}$
   - Squared absolute value of NLO photon VP insertion
   - NNLO iterated VP insertion in photon propagator
   - Interference of NLO photon VP insertion with NLO corrections
   - NNLO photonic corrections with VP insertion not in loop-photon propagator
   - NNLO irreducible vertex and box corrections



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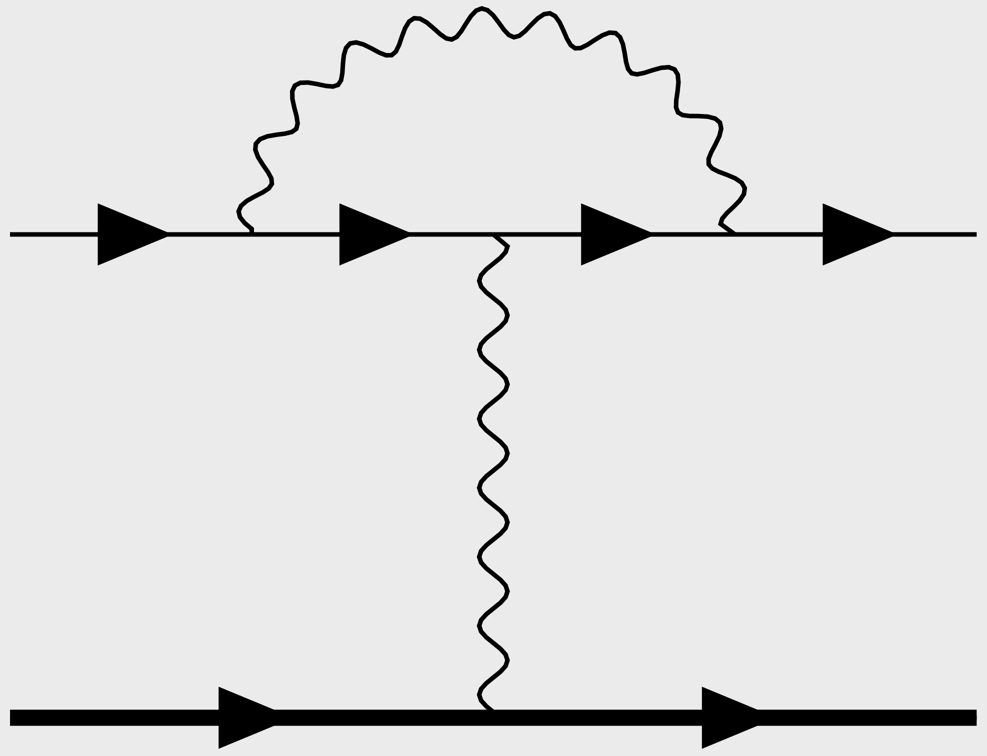

(b)

NNLO leptonic virtual contributions

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

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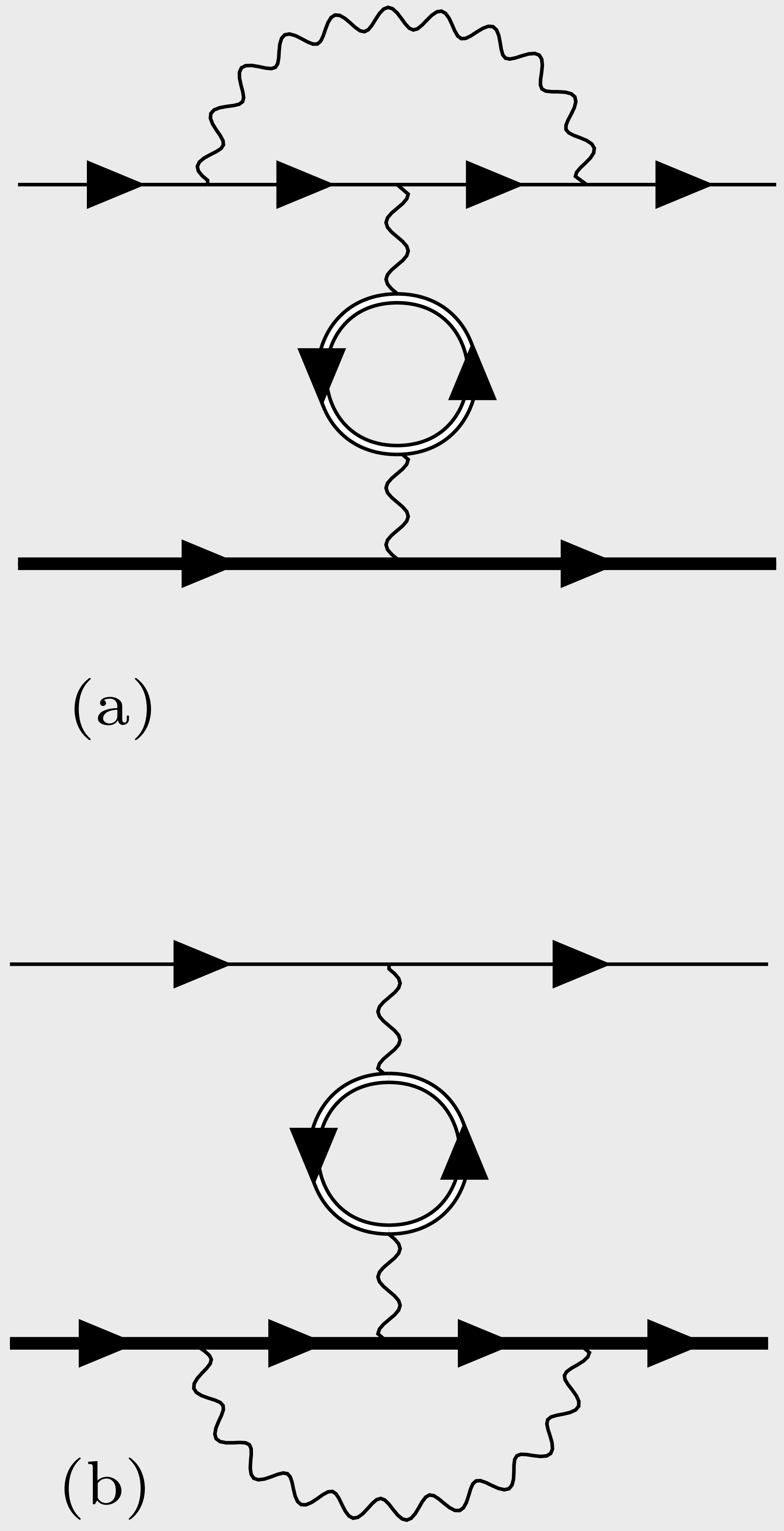

(a)

(b)

NNLO leptonic virtual contributions

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

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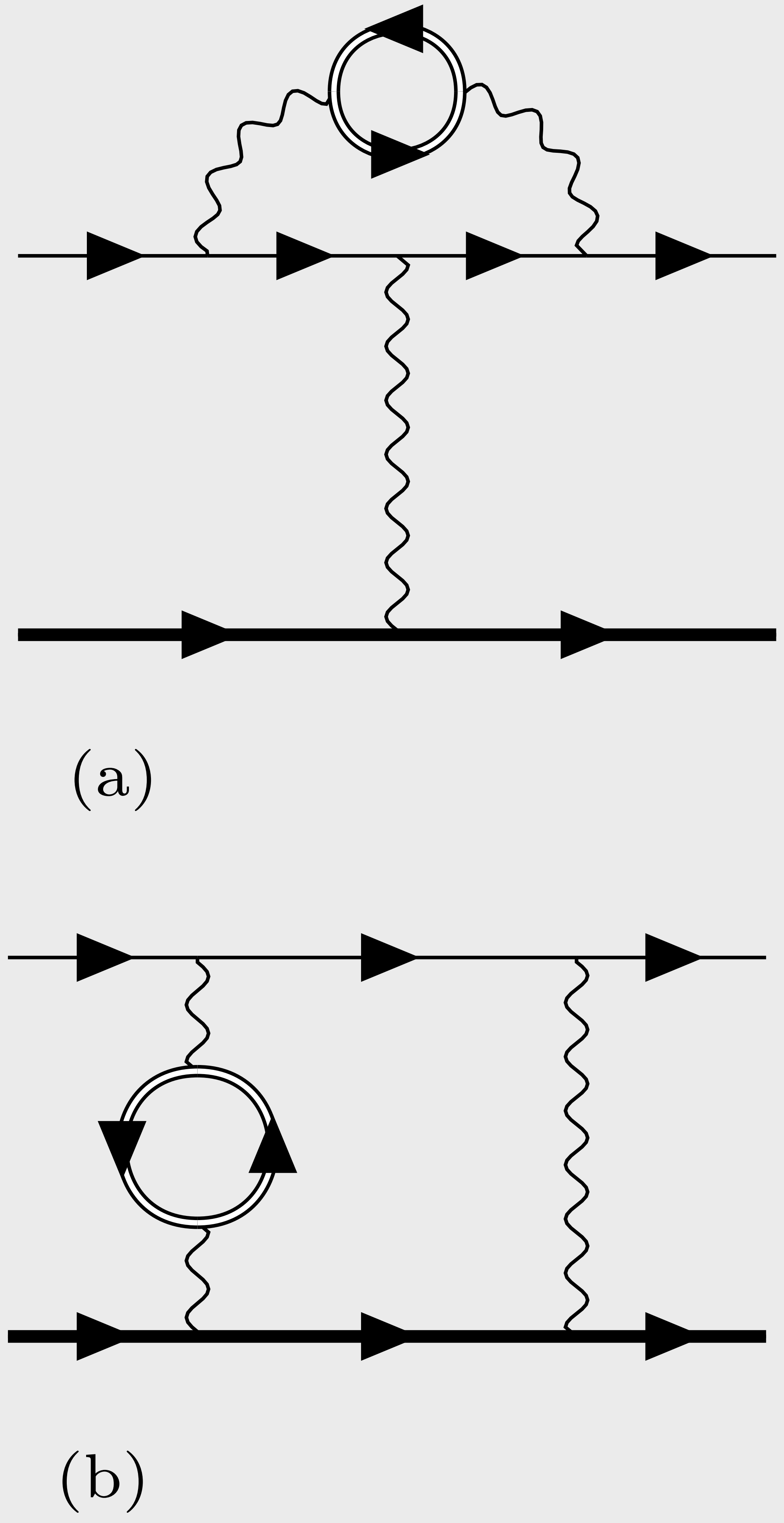


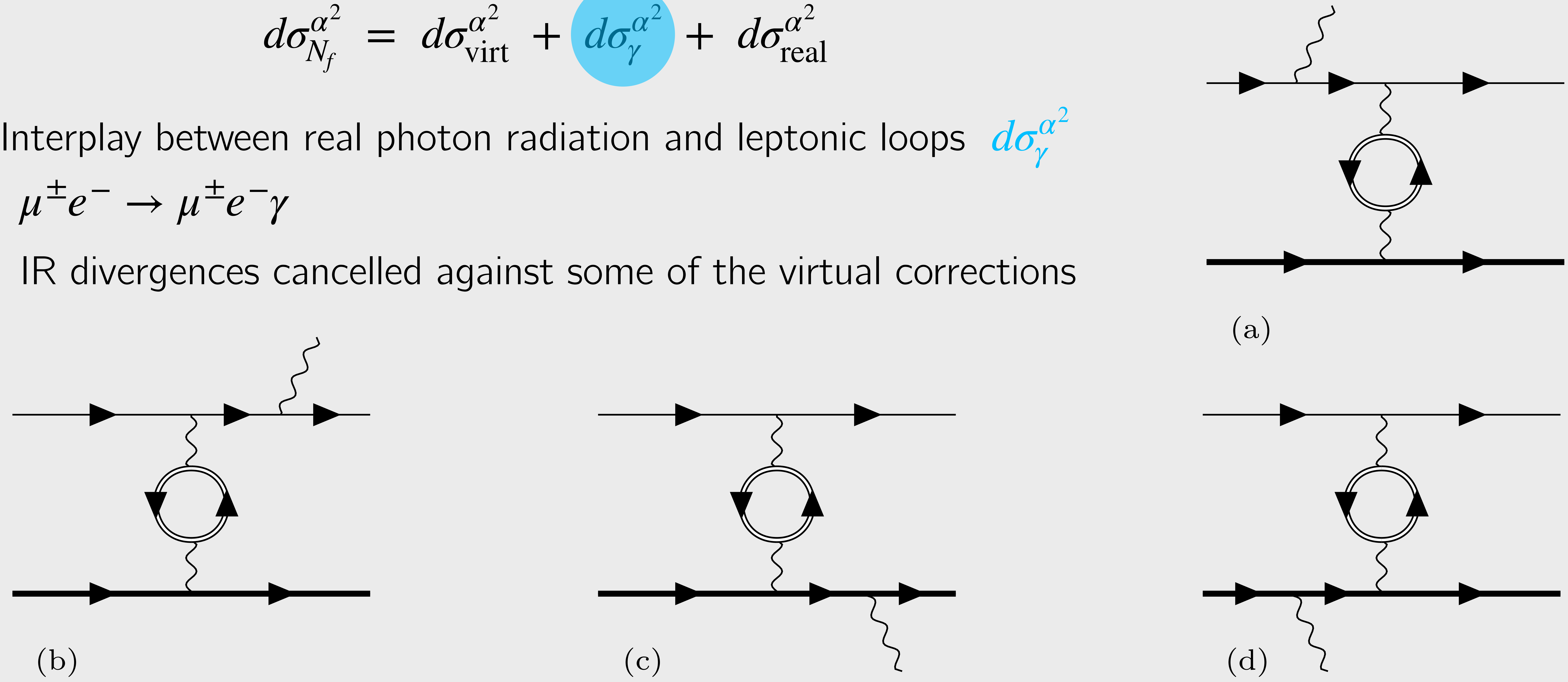
Real photon contributions

$$d\sigma_{N_f}^{\alpha^2} = d\sigma_{\text{virt}}^{\alpha^2} + d\sigma_{\gamma}^{\alpha^2} + d\sigma_{\text{real}}^{\alpha^2}$$

2. Interplay between real photon radiation and leptonic loops $d\sigma_{\gamma}^{\alpha^2}$

   $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}\gamma$

   IR divergences cancelled against some of the virtual corrections



Virtual and real-virtual calculation

Starting from photonic NLO vertex and box diagrams, replace photon propagator:

$$\frac{-ig_{\mu\nu}}{q^2+i\epsilon} \to \frac{-ig_{\mu\delta}}{q^2+i\epsilon} i\left(q^2 g^{\delta\lambda} - q^\delta q^\lambda\right) \Pi_\ell(q^2) \frac{-ig_{\lambda\nu}}{q^2+i\epsilon}$$

S. Actis et al., Nucl. Phys. B Proc. Suppl. 183 (2008) 174

S. Actis et al., Phys. Rev. Lett. 100 (2008) 131602

J. H. Kuhn et al., Nucl. Phys. B 806 (2009) 300

A. H. Hoang et al., Nucl. Phys. B 452 (1995) 173

R. Barbieri et al., Nuovo Cim. A 11 (1972) 824

R. Barbieri et al., Nuovo Cim. A 11 (1972) 825

The renormalised VP function can be obtained by a dispersion relation, so

$$\frac{-ig_{\mu\nu}}{q^2+i\epsilon} \to -ig_{\mu\nu}\left(\frac{\alpha}{3\pi}\right)\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{1}{q^2-z+i\epsilon}\underbrace{\left(1+\frac{4m_\ell^2}{2z}\right)\sqrt{1-\frac{4m_\ell^2}{z}}}_{R_\ell(z)}$$

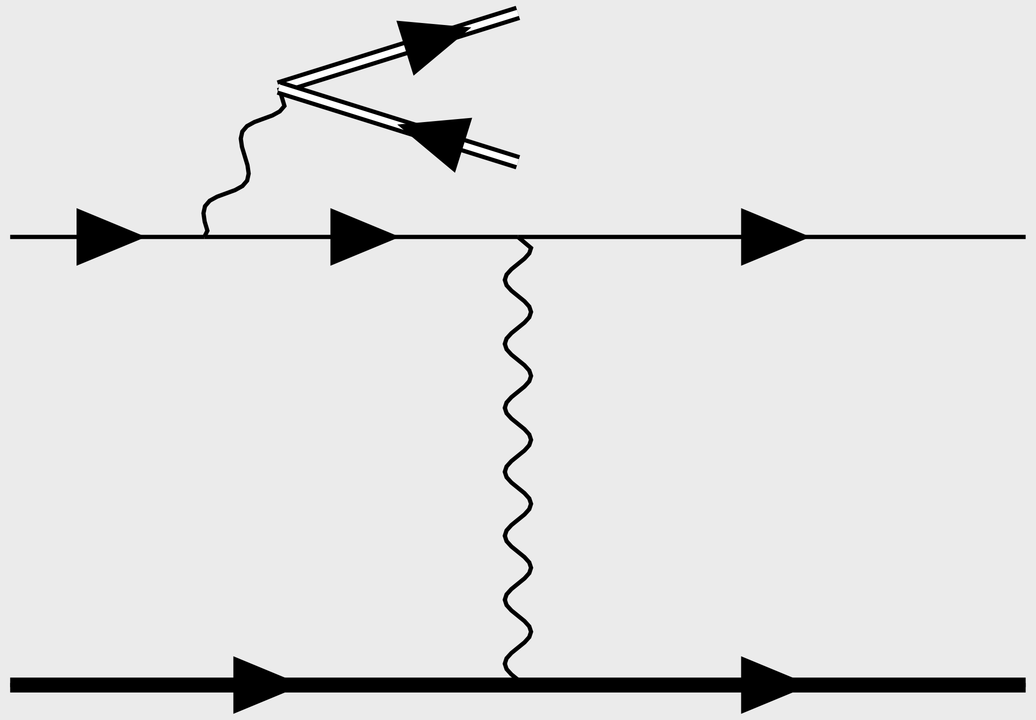
$$\mathcal{A}_{2L}(\mu^\pm e^- \to \mu^\pm e^-) = \left(\frac{\alpha}{3\pi}\right)\int_{4m_\ell^2}^{\infty}\frac{dz}{z} R_\ell(z)\, \mathcal{A}_{1L}(\mu^\pm e^- \to \mu^\pm e^-; z)$$

Real pair production

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

3. Real pair production $d\sigma^{\alpha^2}_{\text{real}}$

   $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ $\qquad \ell = e, \mu$



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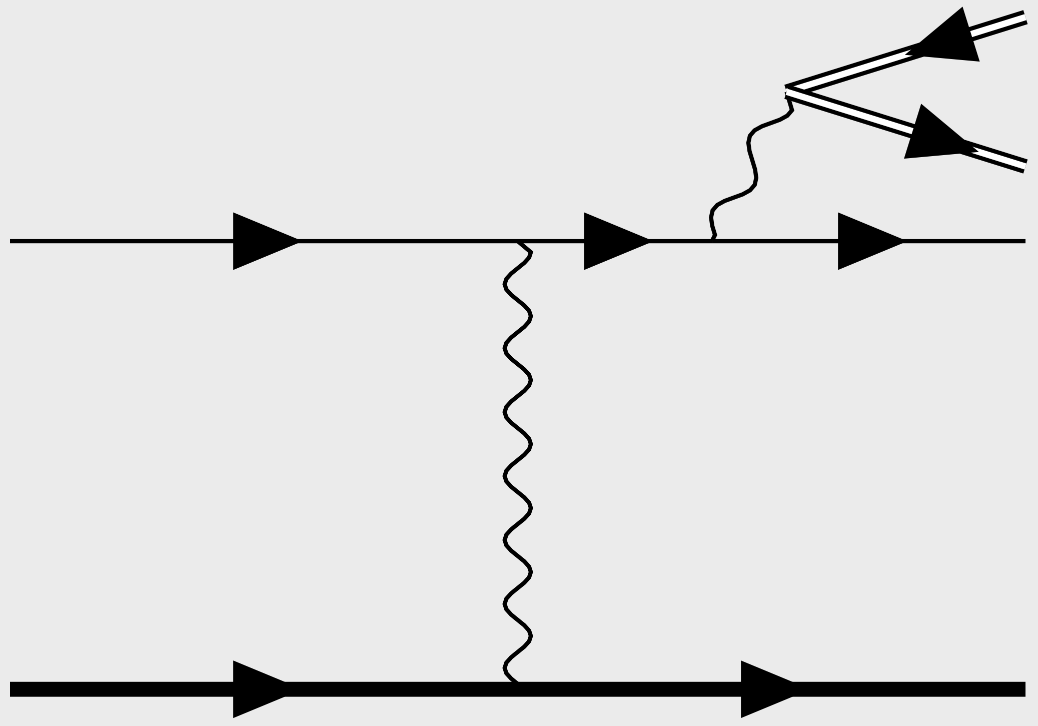

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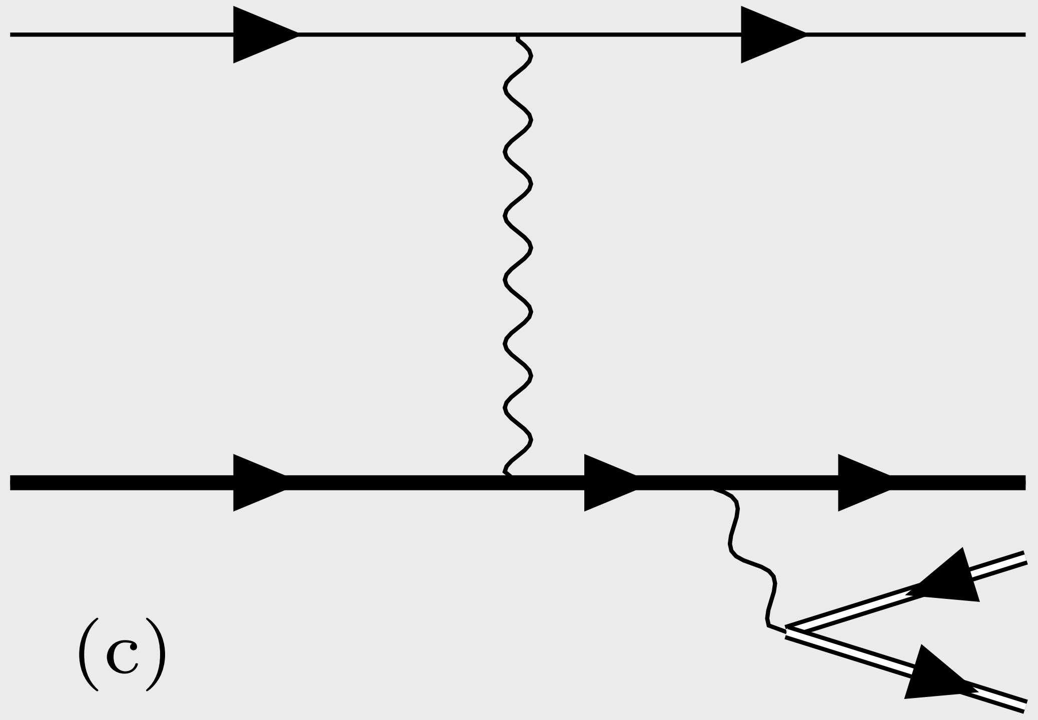

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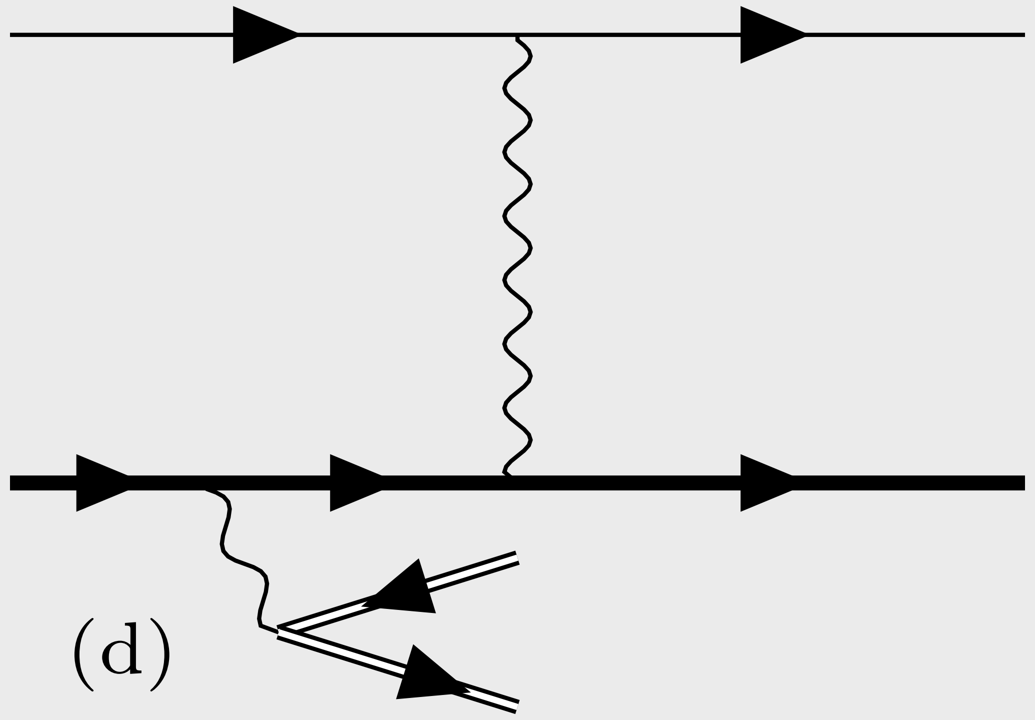

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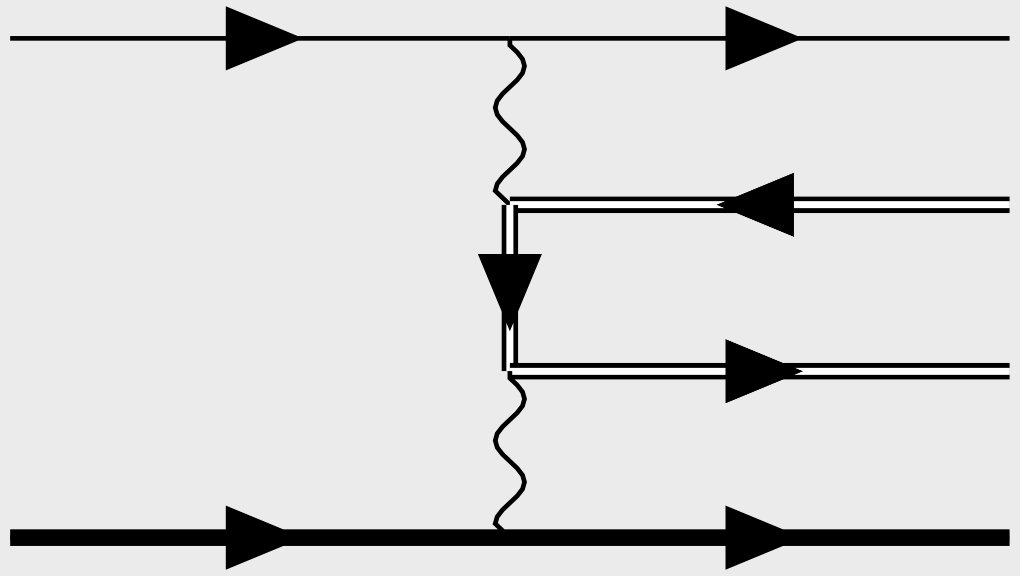

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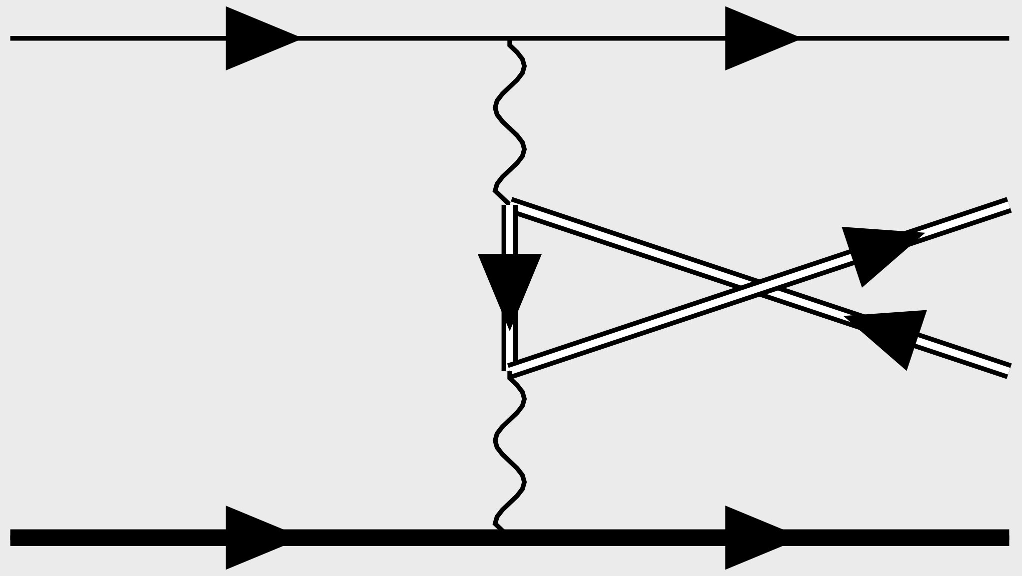


(f)

Real pair emission calculation

Matrix element implemented in MESMER - cross-check with RECOLA

S. Actis et al., Comput. Phys. Comm. 214 (2017) 140-173

A. Denner et al., Comput. Phys. Comm. 224 (2018) 346-361

$$dLips = \int \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} \frac{d^3p_5}{(2\pi)^3 2E_5} \frac{d^3p_6}{(2\pi)^3 2E_6} \delta^4\left(p_1 + p_2 - \sum_{i=3}^{6} p_i\right)$$

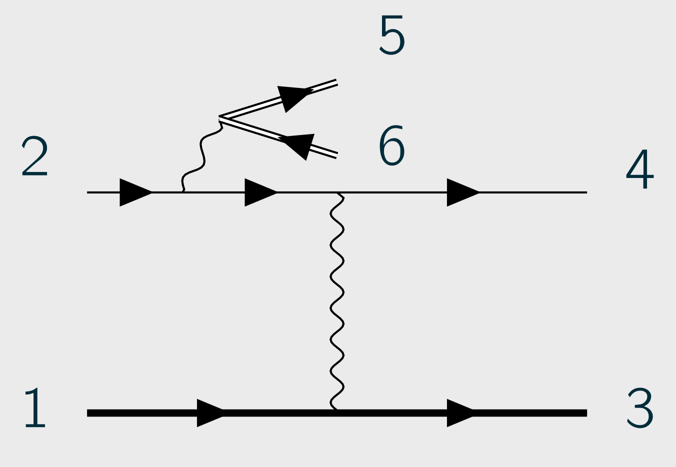
$$d\Phi_n = \int \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3 2E_i} \delta^4\left(P - \sum_{i=1}^{n} p_i\right)$$

$2 \to 4$ phase-space in MESMER decomposed with multi-channel approach:

$$dLips = (2\pi)^6 \int dQ^2_{456}\, dQ^2_{56}\, d\Phi_2(P \to p_3 + Q_{456})\;\; d\Phi_2(Q_{456} \to p_4 + Q_{56})\;\; d\Phi_2(Q_{56} \to p_5 + p_6)$$

Identical particles $p_{4(3)} \leftrightarrow p_6$

Numerical approach cross-checked with analytical formulae



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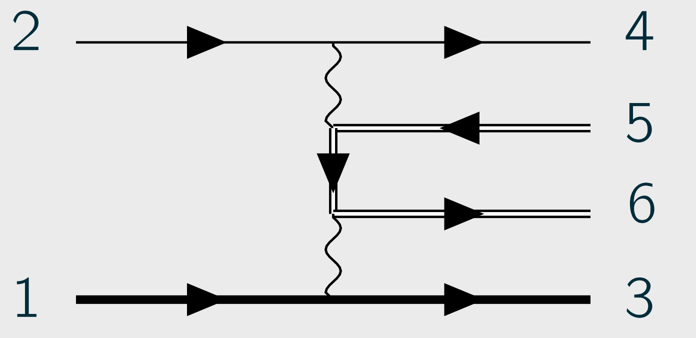


(e)

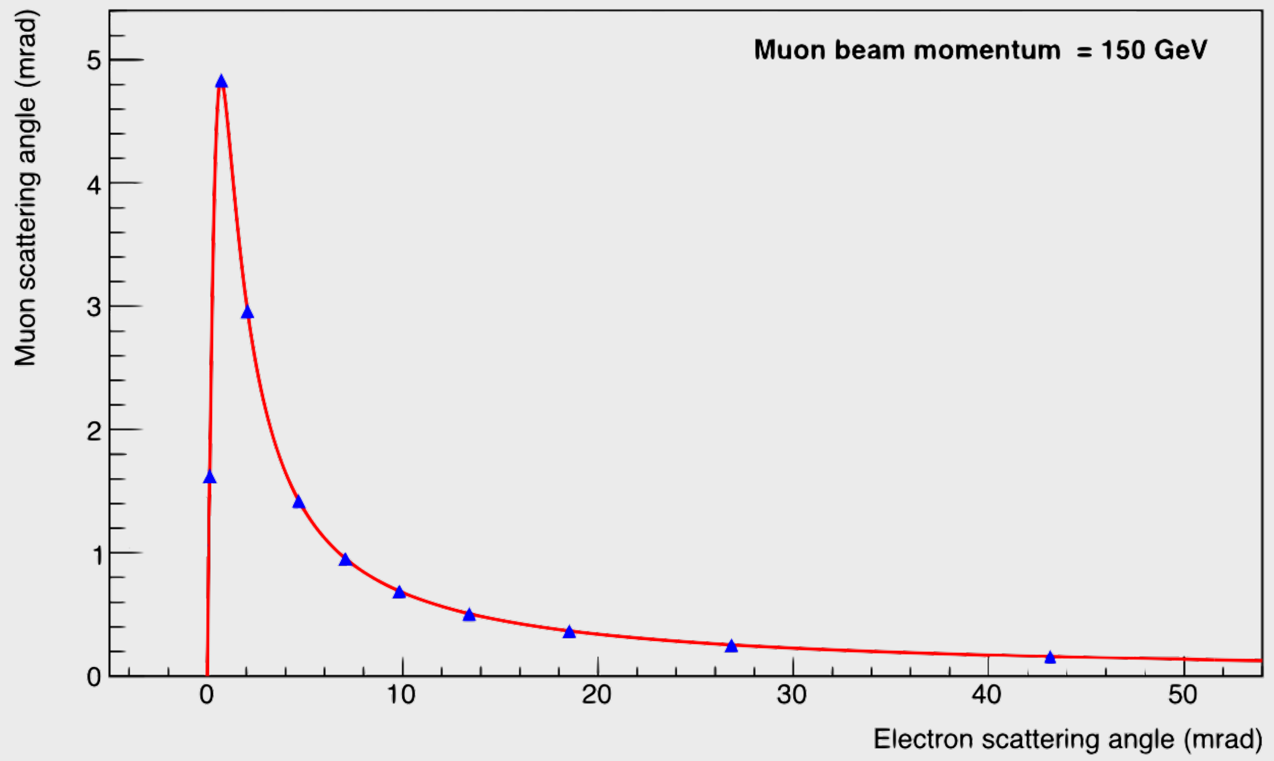
MESMER is available at: www.github.com/cmcc/mesmer

Event selection

Muon beam 150 GeV - basic acceptance cuts for $2 \to 4$ process:

- one muon-like track $\theta_\mu < \bar{\theta}_\mu \simeq 4.84$ mrad , $E_\mu > \bar{E}_\mu \simeq 10.28$ GeV
- one electron-like track $\theta_e < 100$ mrad , $E_e > 1$ GeV

G. Abbiendi et al., arXiv:1609.08987 [hep-ph]



Three elasticity selection cuts

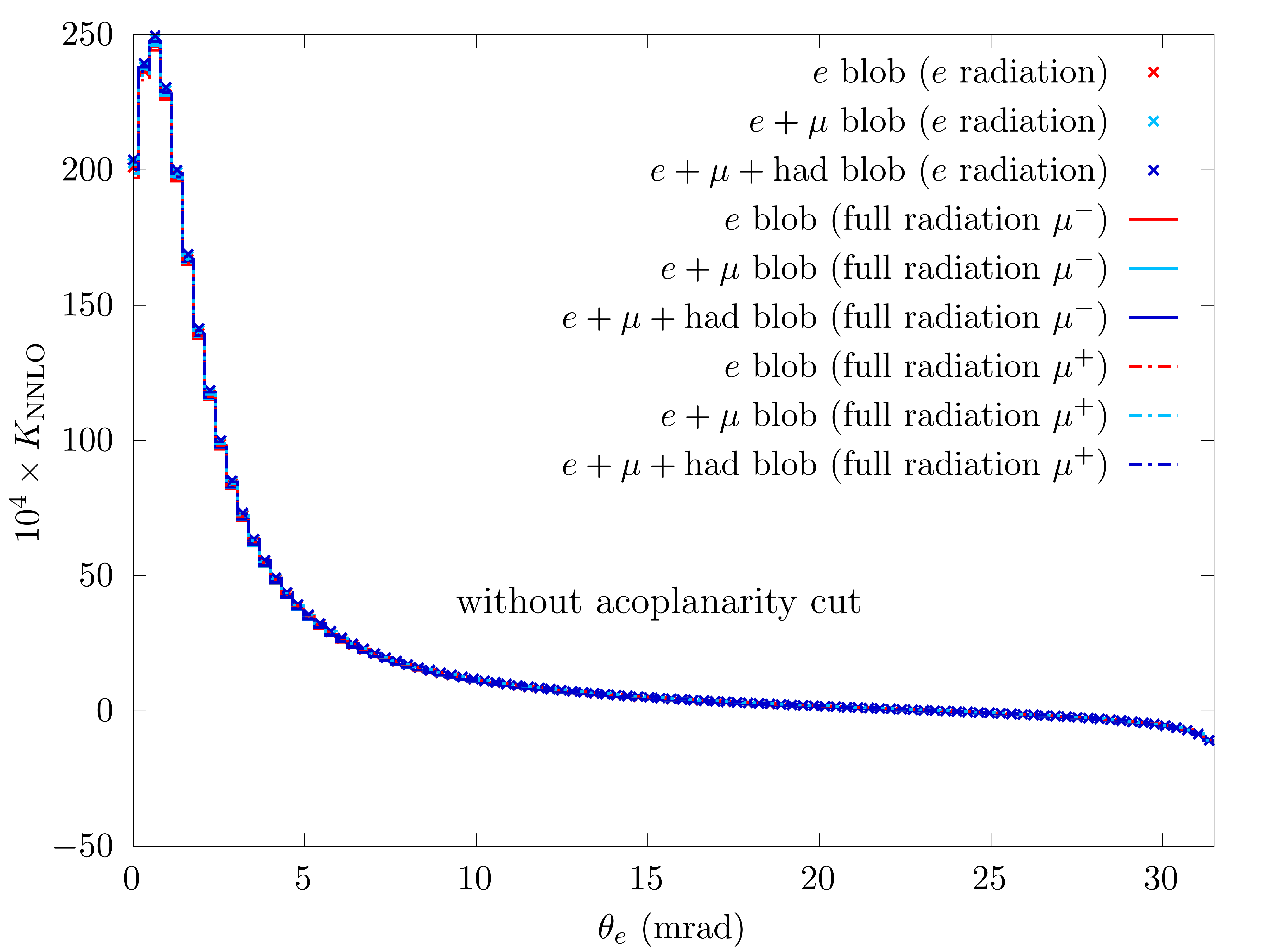
1. to reduce impact of peripheral diagrams $\theta_e, \theta_\mu > \theta_c = 0.2$ mrad
2. acoplanarity cut $\xi = \left| \pi - \left| \phi_e - \phi_\mu \right| \right| < \xi_c = 3.5$ mrad

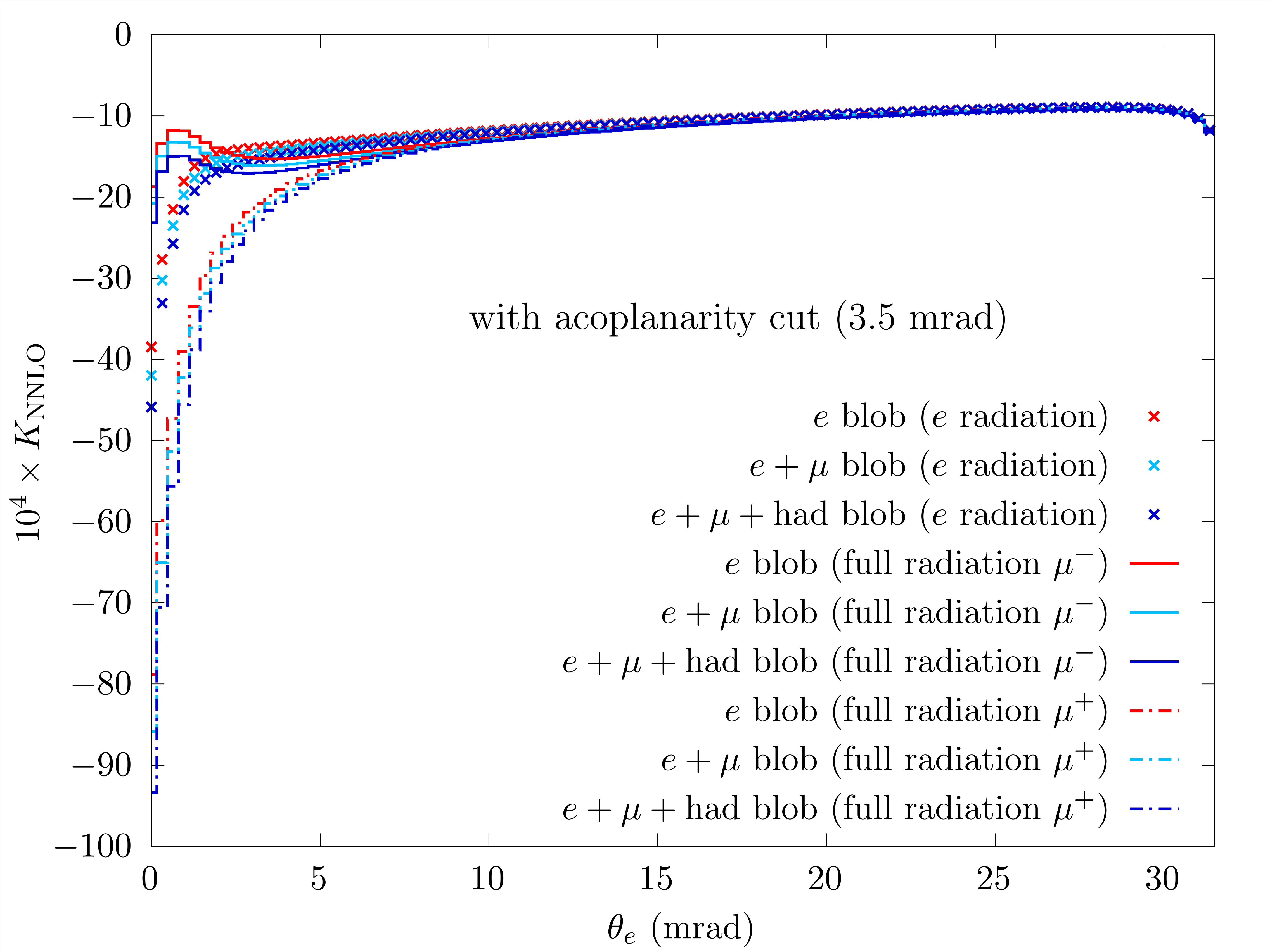
$$\theta_\mu(\theta_e) = \arctan\left[\frac{2m_e r \cos\theta_e \sin\theta_e}{E^i_\mu - r\left(rE^i_\mu + 2m_e\right)\cos^2\theta_e}\right]$$

3. elasticity distance $\delta < \delta_c = 0.2$ mrad with $\delta = \min_{\theta_e} \sqrt{(\theta_e - \theta_e^0)^2 + (\theta_\mu(\theta_e) - \theta_\mu^0)^2}$

Numerical results: virtual and real-virtual

Electron scattering angle $K_{\text{NNLO}} = \dfrac{d\sigma^{\alpha^2}_{N_f}}{d\sigma_{\text{LO}}}$





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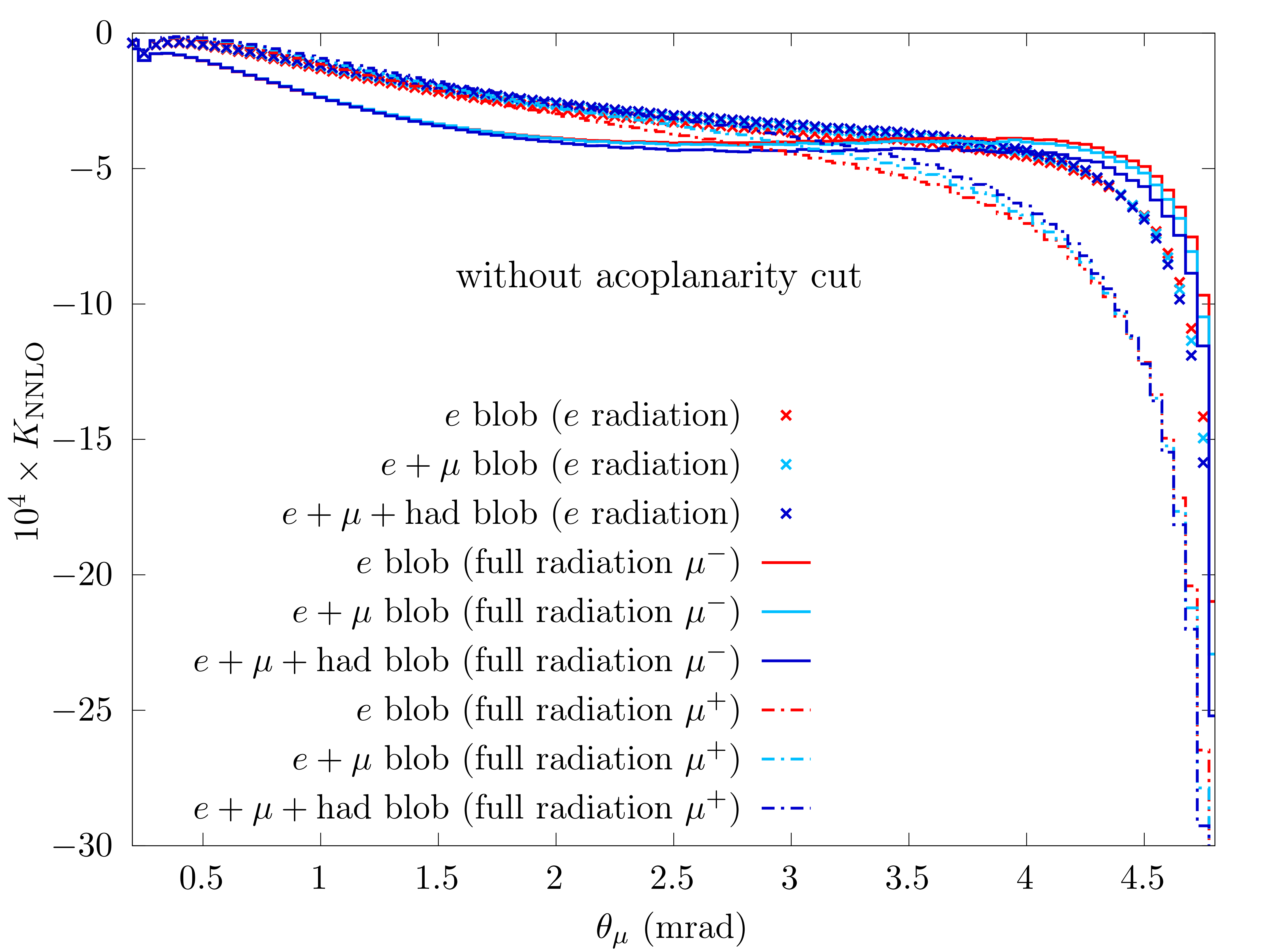
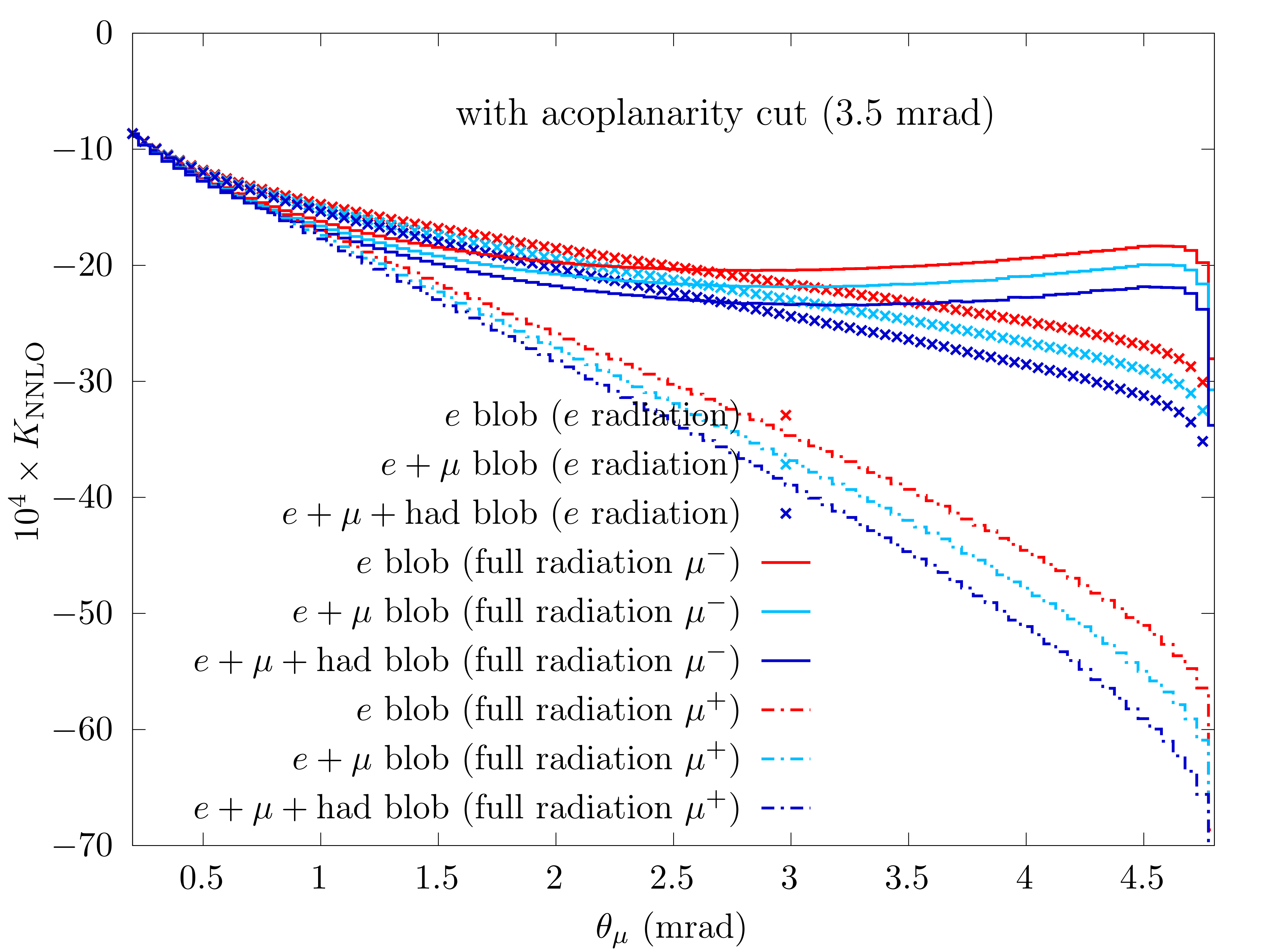
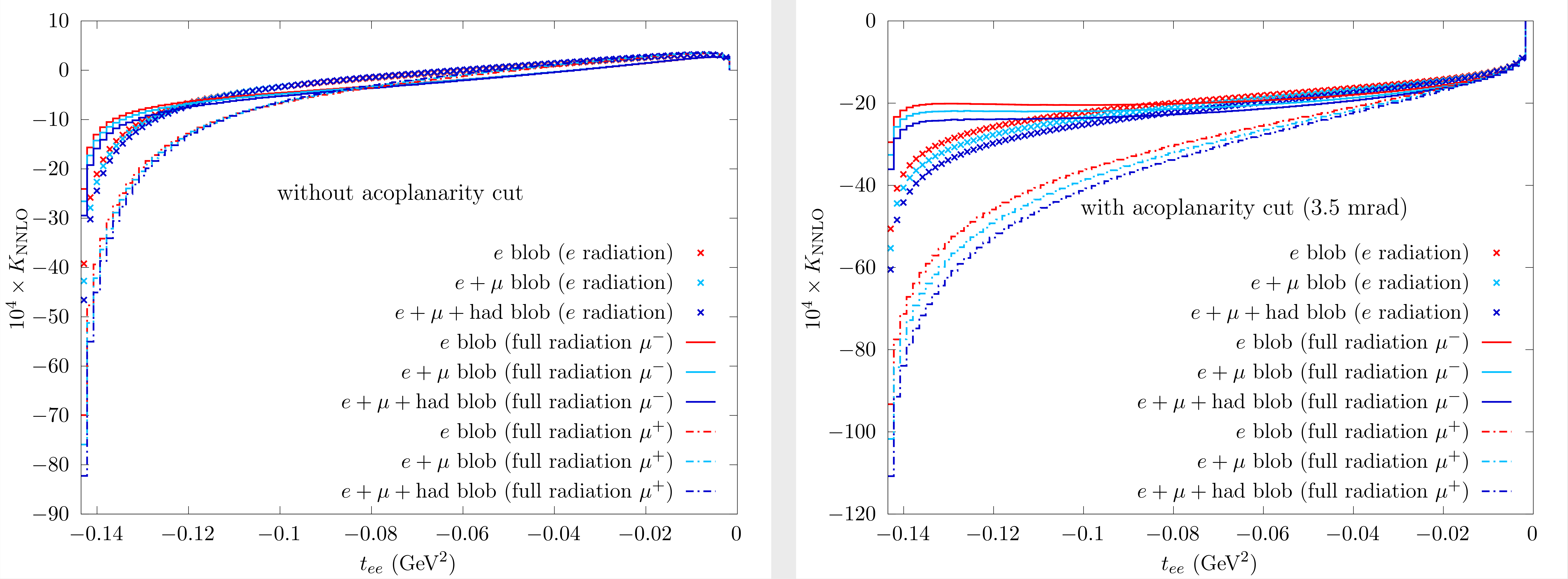


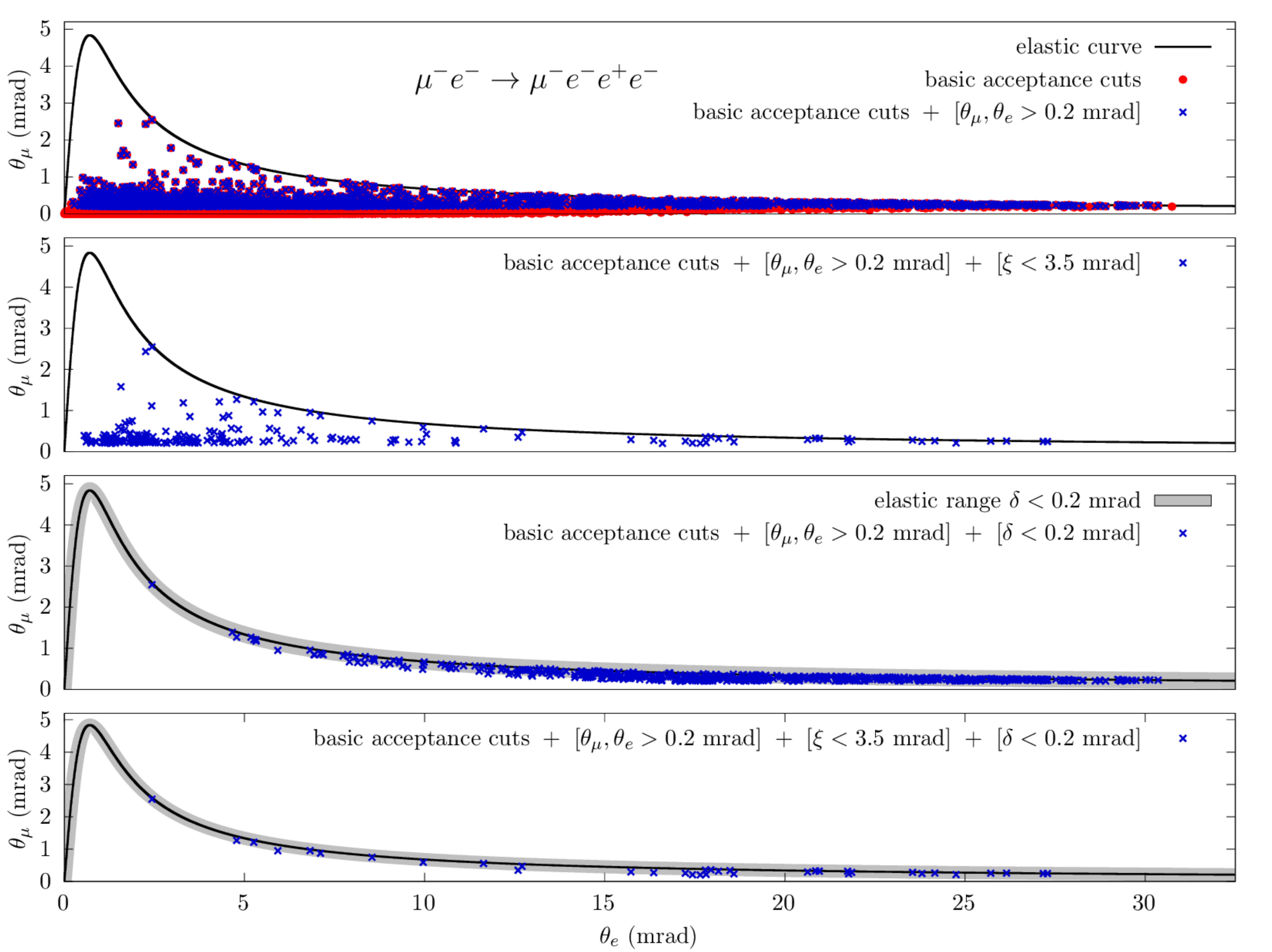


Numerical results: virtual and real-virtual

Momentum transfer on electron line $K_{\text{NNLO}} = \frac{d\sigma^{\alpha^2}_{N_f}}{d\sigma_{\text{LO}}}$



Numerical results: real pair emission



Numerical results: real pair emission

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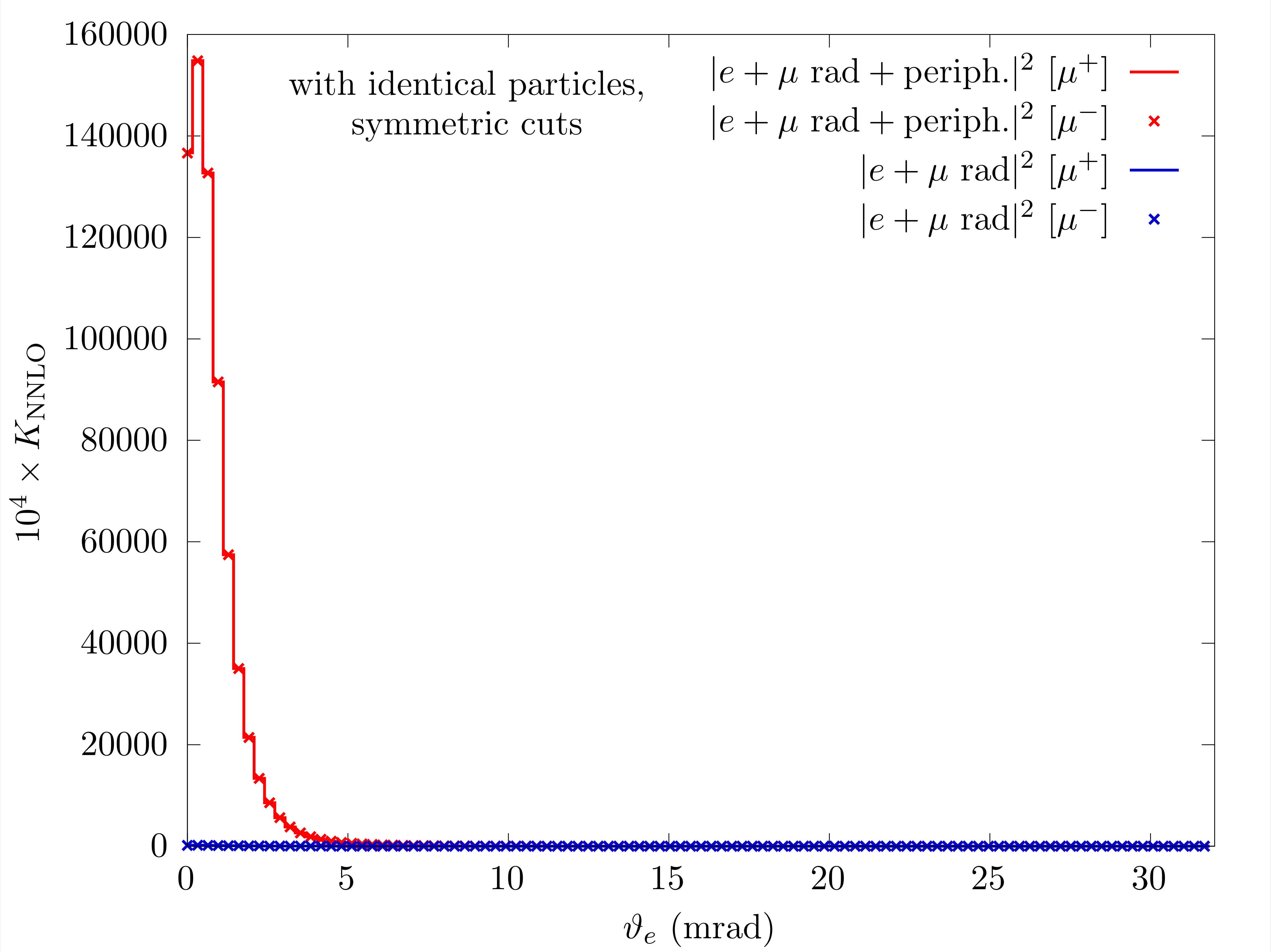
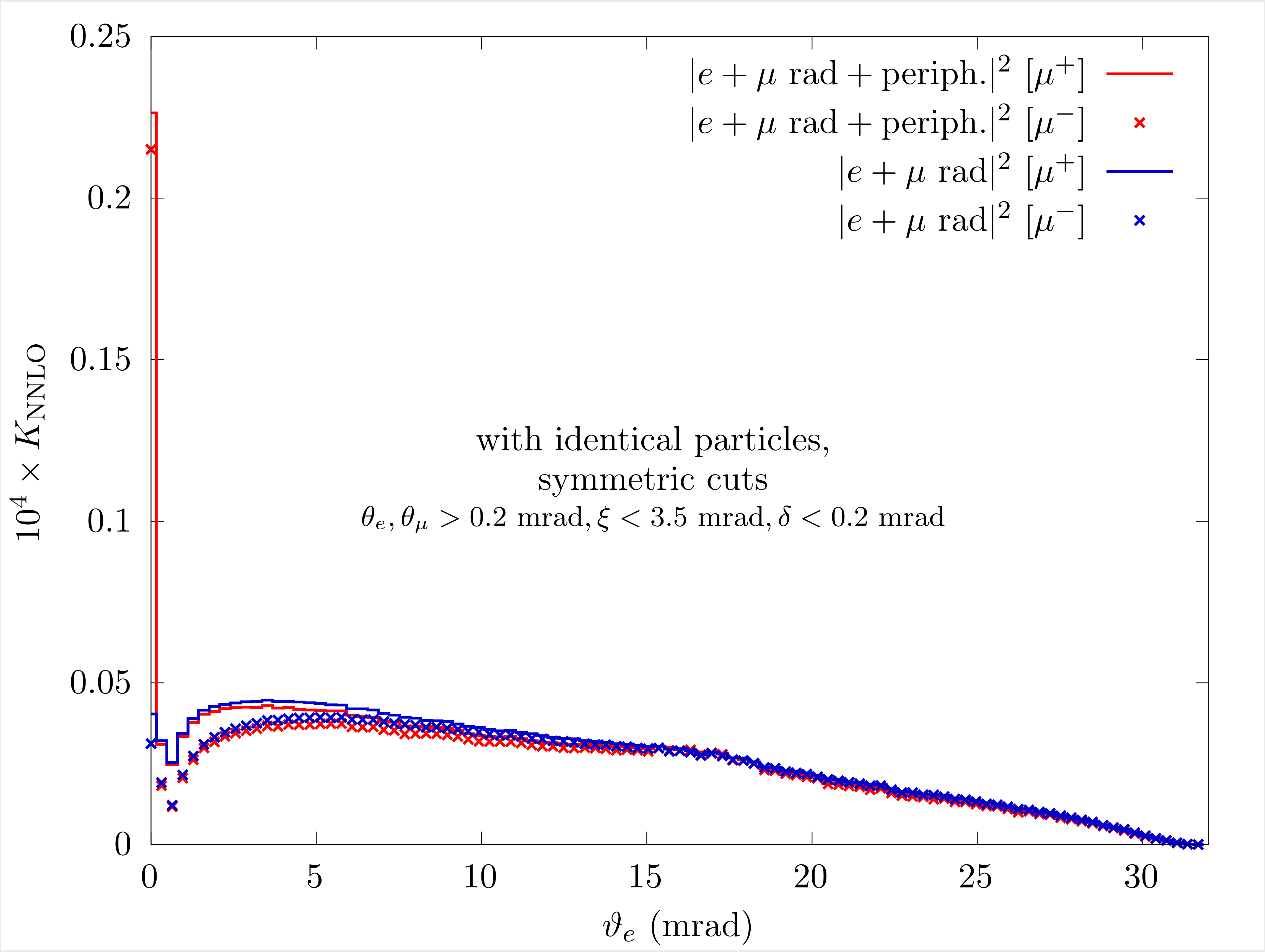
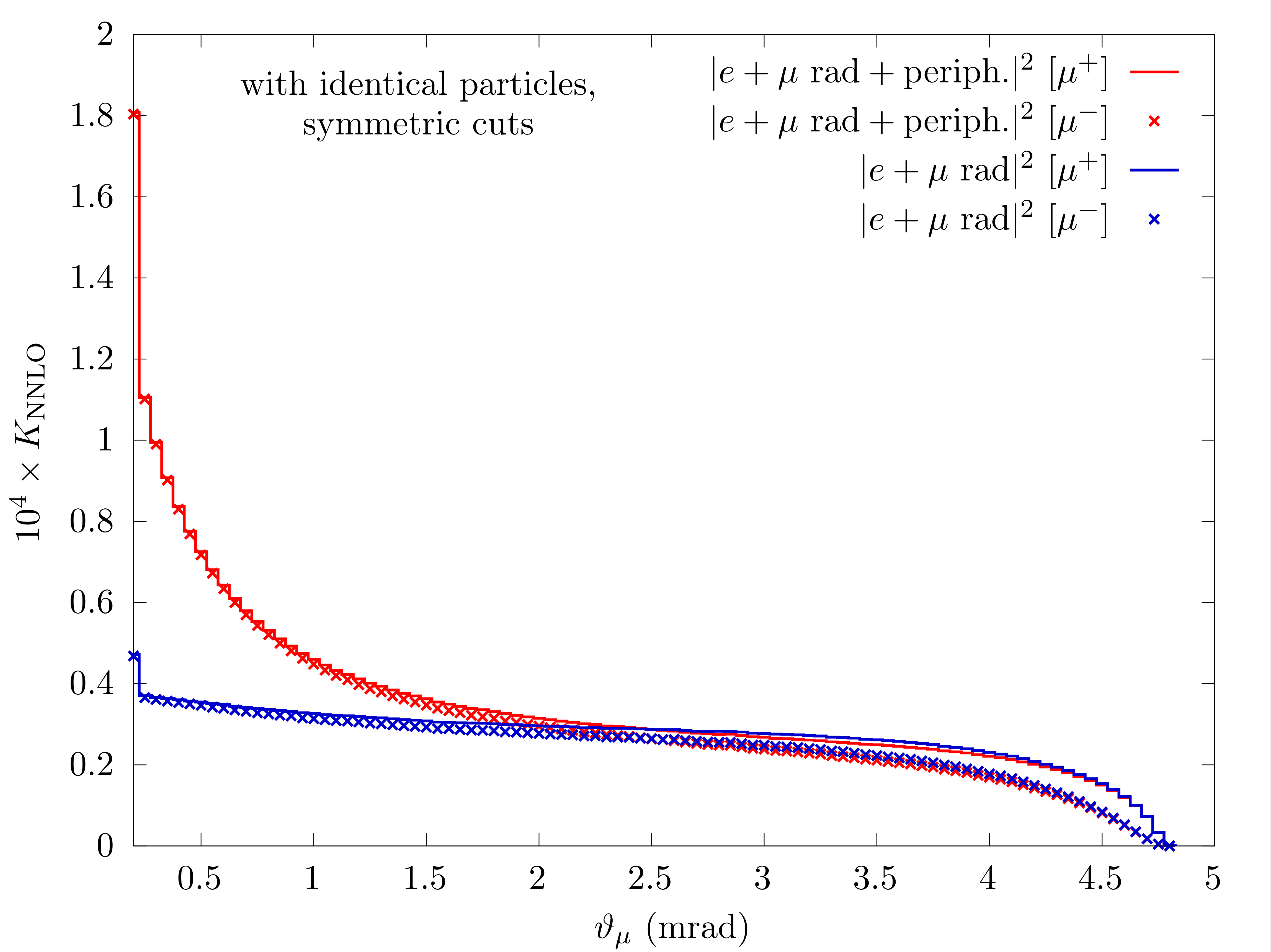
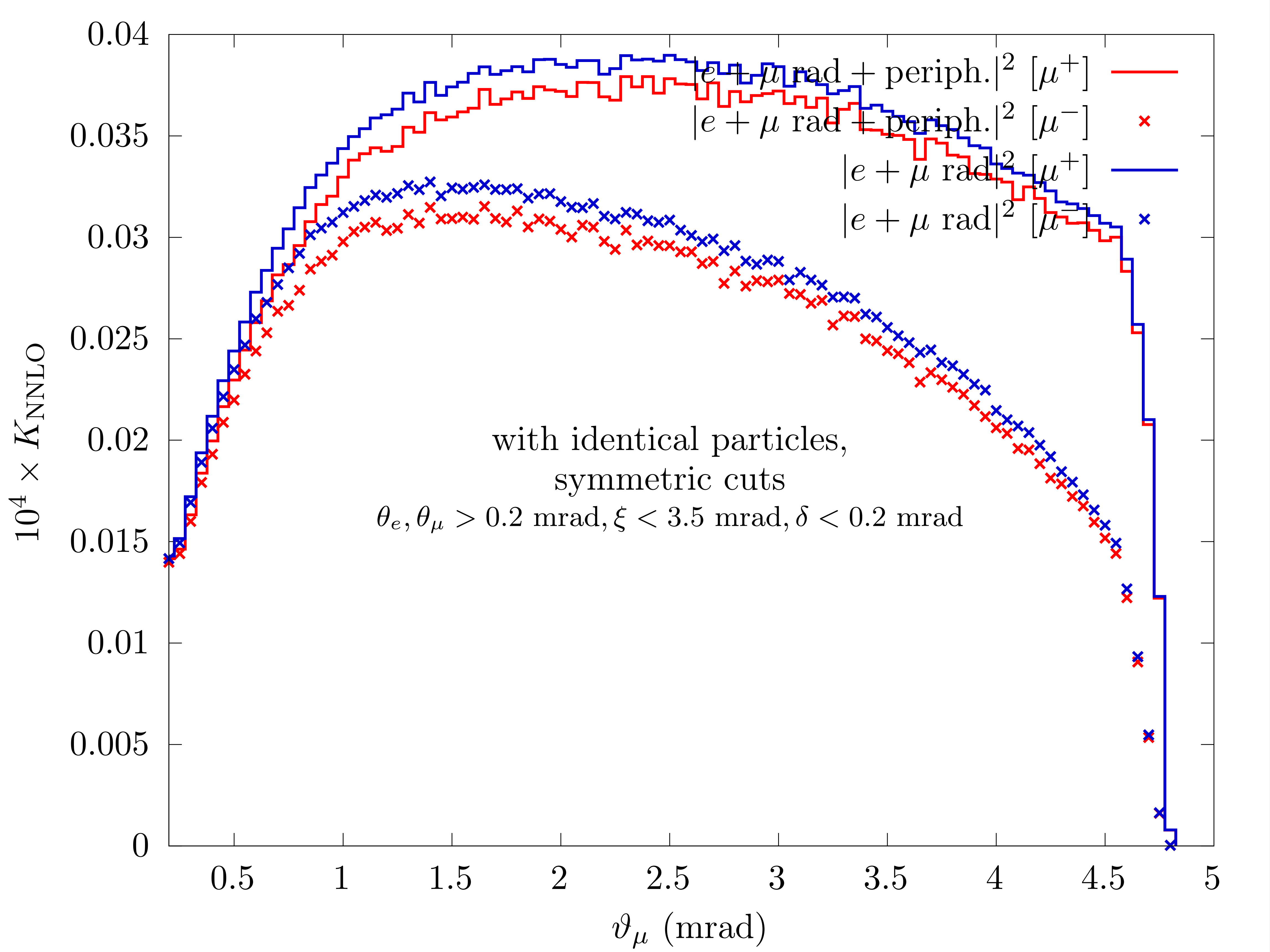




Numerical results: real pair emission

Muon scattering angle $K_{\mathrm{NNLO}} = \dfrac{d\sigma^{\alpha^2}_{N_f}}{d\sigma_{\mathrm{LO}}}$





Numerical results: real pair emission

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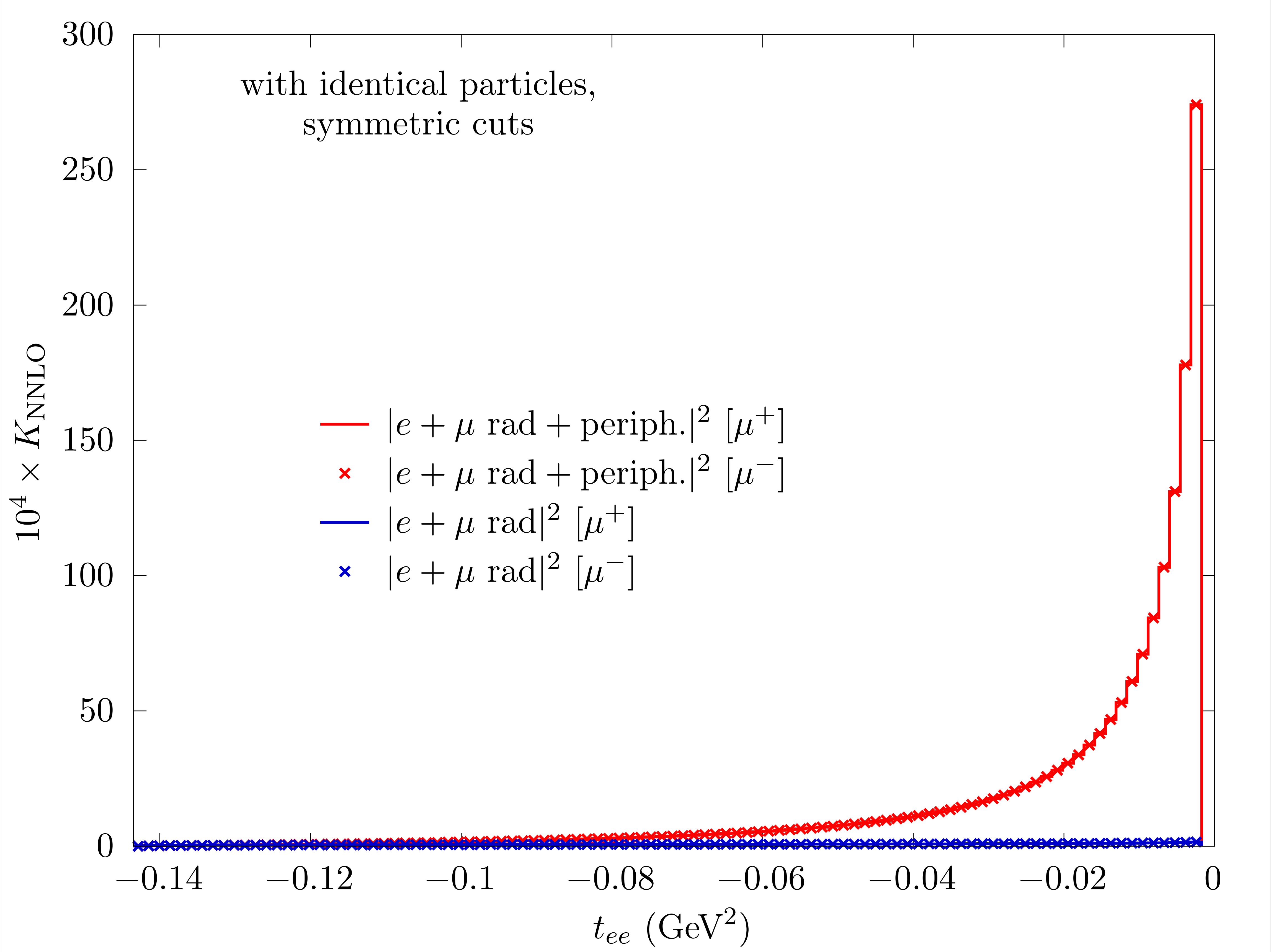
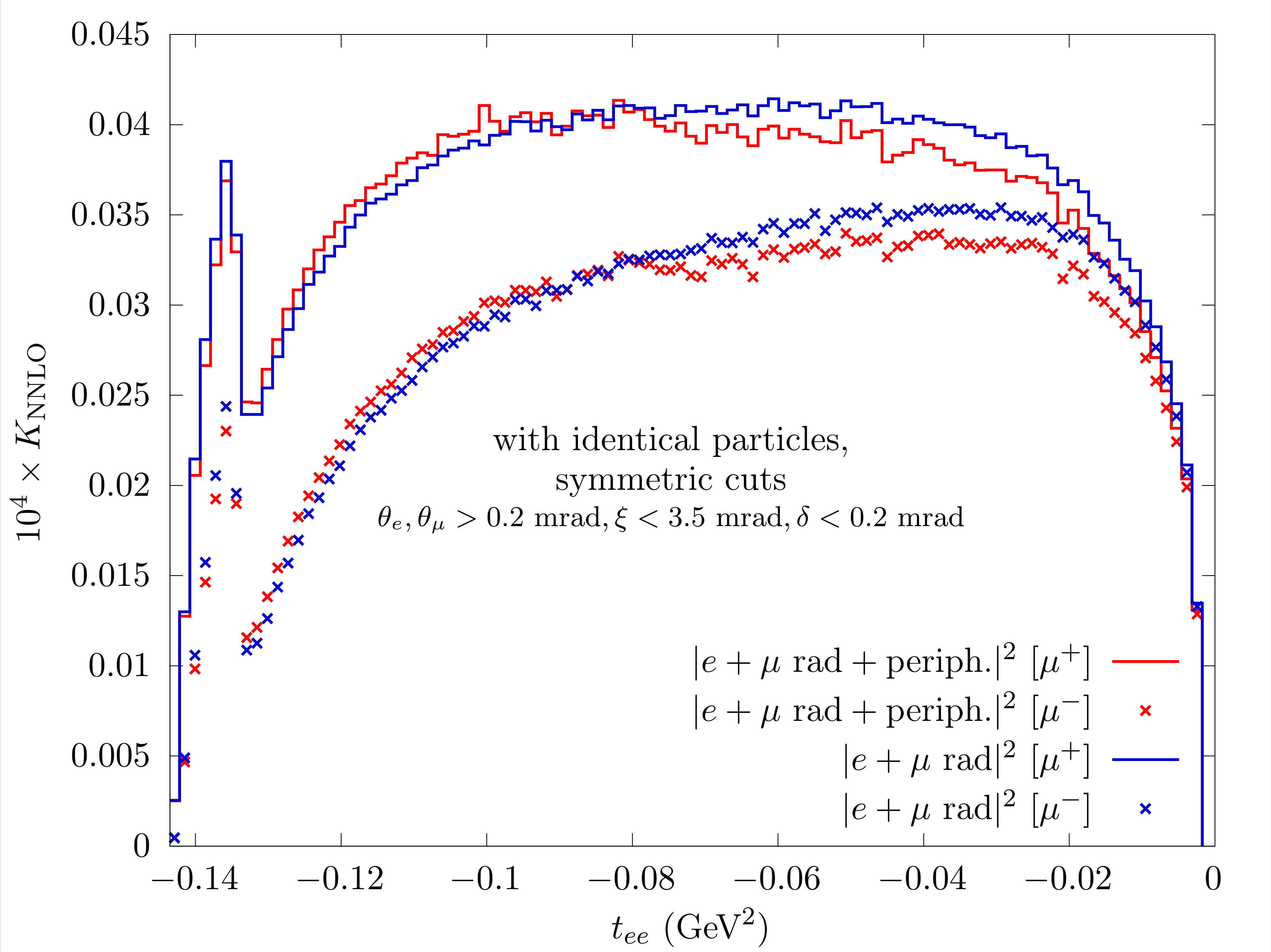




SINGLE PION PRODUCTION

E. Budassi et al., Phys. Lett. B 829 (2022) 137138, arXiv:2203.01639 [hep-ph]

Motivation

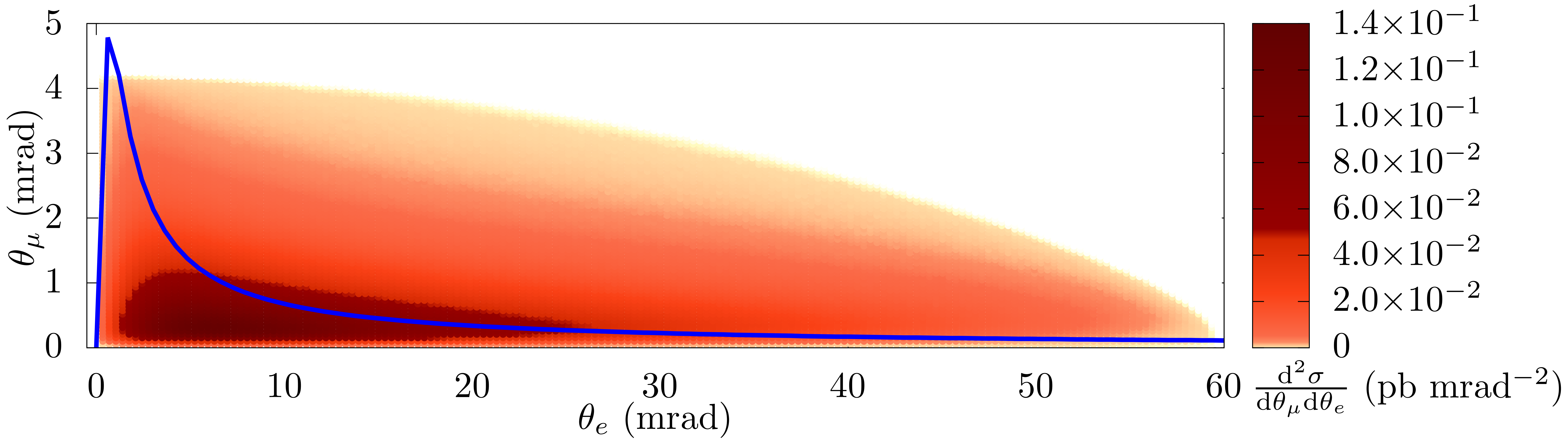
- Reliable estimates of possible backgrounds such as real and virtual hadronic contributions are needed
- Virtual hadronic contributions have been already discussed

  M. Fael et al., JHEP 02 (2019) 027

  M. Fael et al., Phys. Rev. Lett. 122 (2019) 192001

- Real-pair production does not contribute due to MUonE phase-space
- Potential important reducible background: $\mu^{\pm}e^{-} \rightarrow \mu^{\pm}e^{-}\pi^{0} \rightarrow \mu^{\pm}e^{-}\gamma\gamma$

Single pion production



Single pion production

$$\mathscr{L}_{\mathrm{I}} = \frac{g}{2!}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}F_{\kappa\lambda}\varphi_\pi$$

*Brodsky, S. J., et al., Phys. Rev. D 4 (1971) 1532-1557

Czyz, H., et al., Phys. Rev. D 97 (1) (2018) 016006

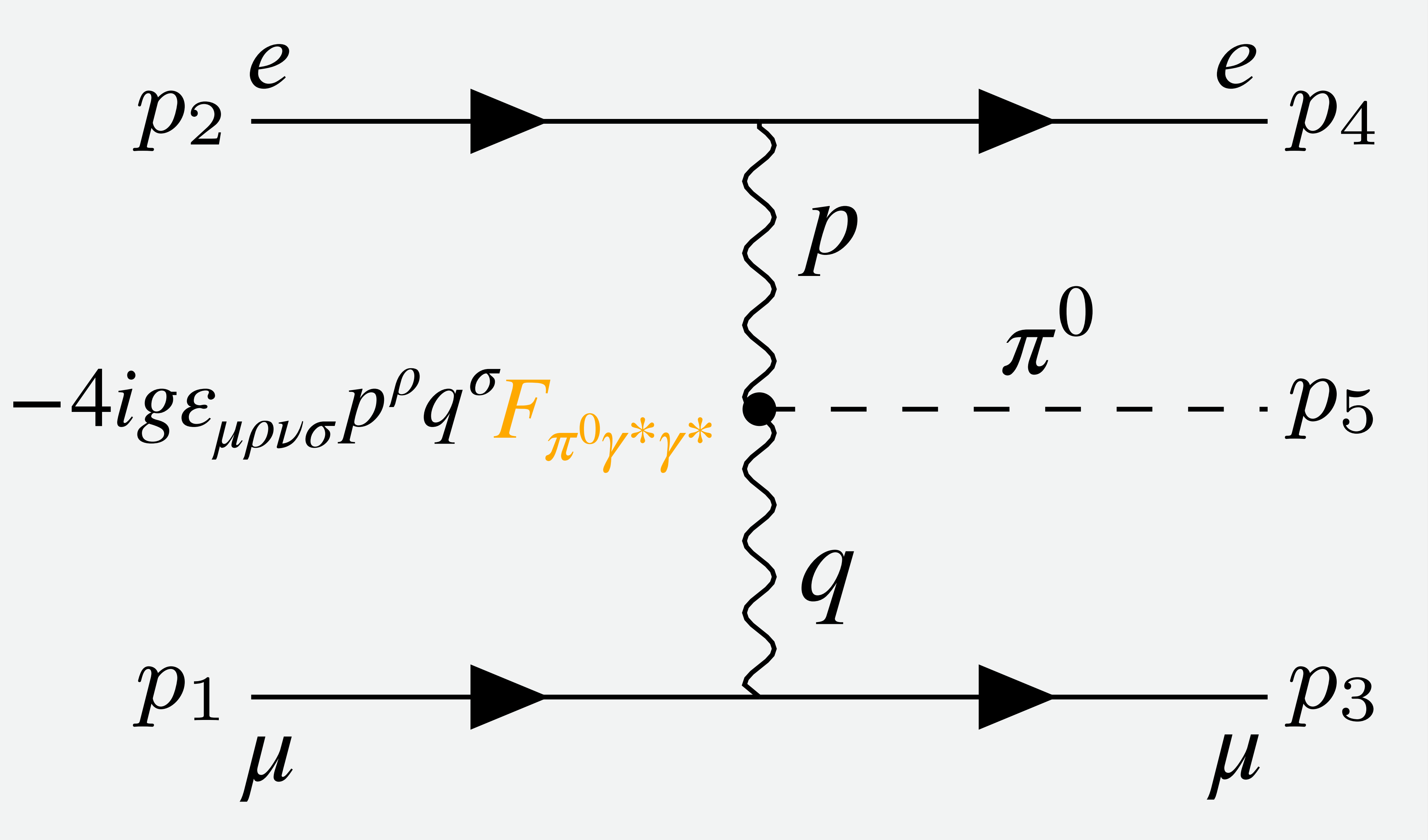
$$g^2 = \frac{4\pi\Gamma_{\pi^0\to\gamma\gamma}}{m_{\pi^0}^3}$$

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\alpha^2 m_{\pi^0}^3}{64\pi^3 f_\pi^2}$$

$$f_\pi = 0.092388 \text{ GeV}$$

$$\Gamma_{\pi^0\to\gamma\gamma} = 7.731 \text{ eV}$$

$$m_\pi = 134.9766 \text{ MeV}$$



ME and PS implemented in MESMER - Table 1 from * perfectly reproduced

Comparison with $e^+e^- \to e^+e^-\pi^0$ (t-channel only) with EKHARA

Czyz, H., et al., Comput. Phys. Commun. 182 (2011) 1338-1349

Czyz, H., et al., Comput. Phys. Commun. 234 (2019) 245-255

Numerical results

Total cross section: $\sigma_{\mu e \pi^0} = 6.53589(6)$ pb

Basic acceptance cuts
$\vartheta_\mu \leq 4.84$ mrad
$\vartheta_e < 100$ mrad
$E_\mu \geq 10.28$ GeV
$E_e > 0.2$ GeV

- basic acceptance cuts:

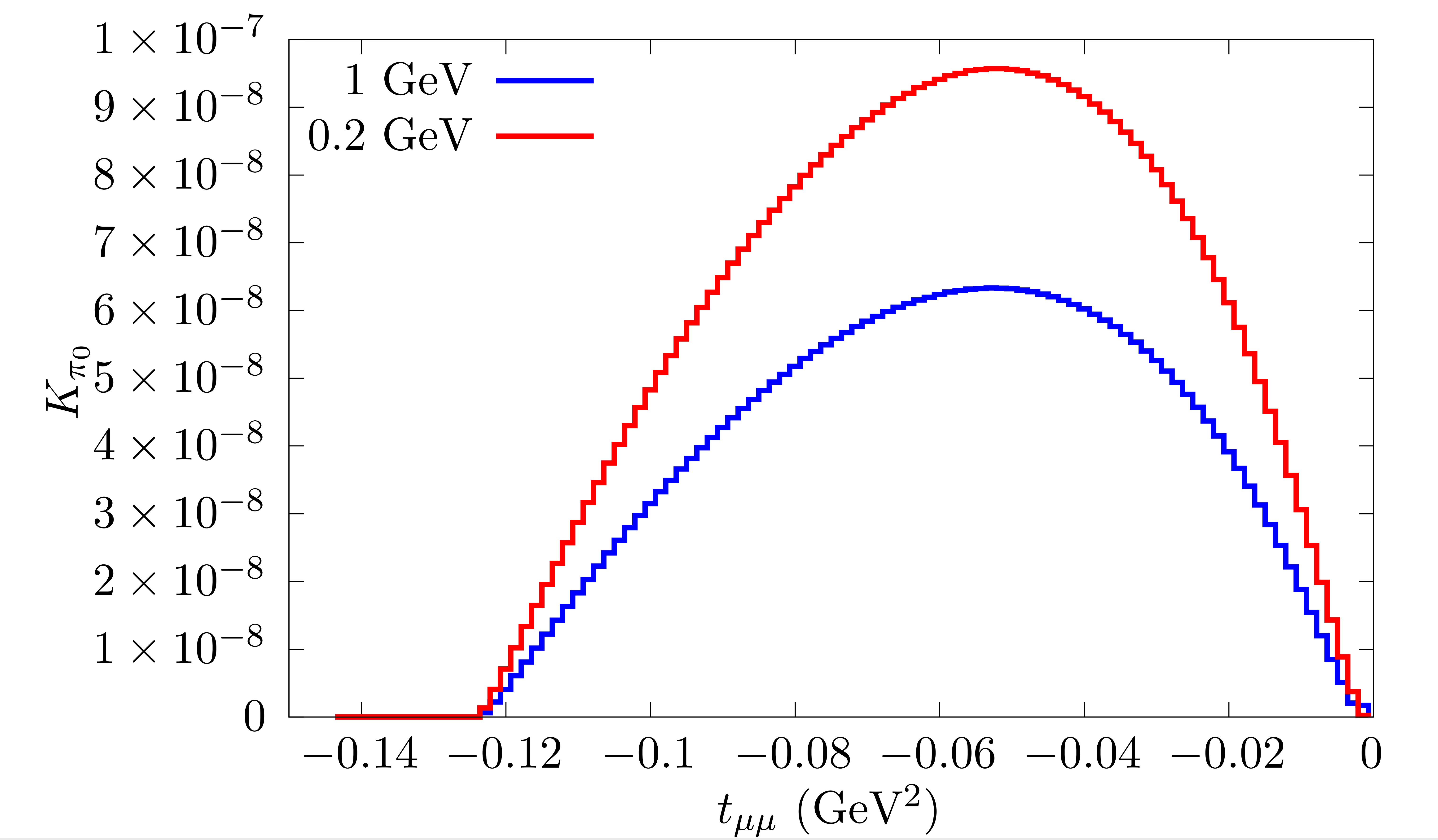
$$\sigma^{0.2\,\text{GeV}}_{\mu e \pi^0} = 2.69836(4)\ \text{pb} \qquad \text{w.r.t.} \qquad \sigma^{0.2\,\text{GeV}}_{\text{LO}} \sim 1265\ \mu\text{b}$$

- basic acceptance cuts + $E_e > 1$ GeV

$$\sigma^{1\,\text{GeV}}_{\mu e \pi^0} = 1.61597(3)\ \text{pb} \qquad \text{w.r.t.} \qquad \sigma^{1\,\text{GeV}}_{\text{LO}} \sim 245\ \mu\text{b}$$

Numerical results

Momentum transfer along muon line



Negligible contribution in differential distributions

$$K_{\pi^0} = \frac{d\sigma_{\pi^0}}{d\sigma_{\mathrm{LO}}}$$

New physics searches @ MUonE

Asai, K., et al., arXiv:2109.10093 [hep-ph]
Galon, I., et al., arXiv:2202.08843 [hep-ph]
Grilli di Cortona, G., Nardi, E., Phys.Rev.D 105 (2022) 11, L111701

Background for possible NP searches at MUonE in $2 \to 3$ processes: $\mu e \to \mu e Z' \to \mu e \nu \bar{\nu}$

$L_\mu - L_\tau$ gauge model with $m_{Z'} = 10 \sim 200\ \mathrm{MeV}$

Selection criteria: $\theta_\mu > 1.5\ \mathrm{mrad}$

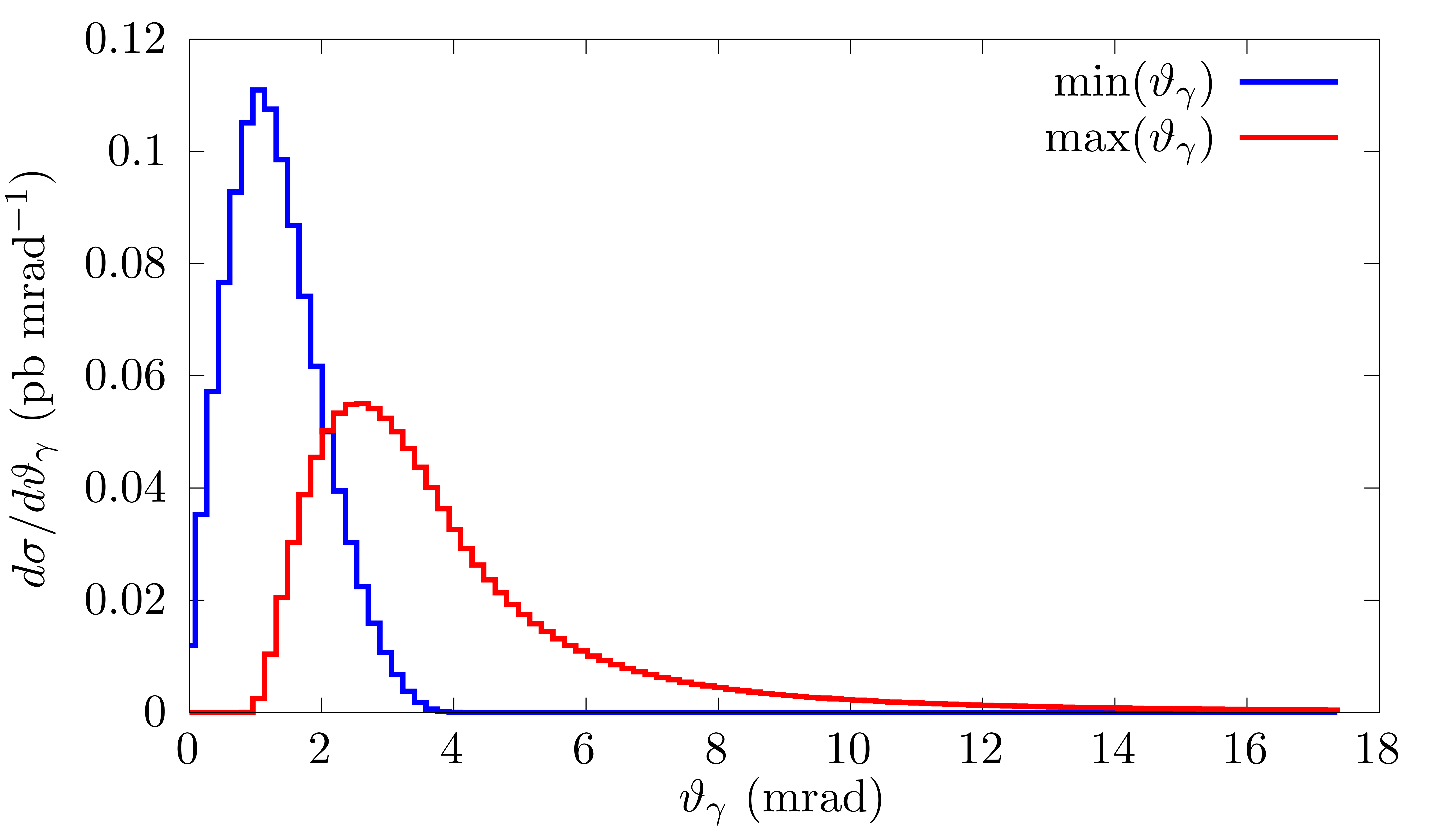
$1\ \mathrm{GeV} < E_e < 25\ \mathrm{GeV}$

$\sigma_{\mu e \pi^0} = 0.19210(1)\ \mathrm{pb}$

Integrated luminosity $= 15\ \mathrm{fb}^{-1}$ $\rightarrow$ $N_{\pi^0} \sim N_{Z'} \sim 3 \times 10^3$ $\rightarrow$ photon veto strategy

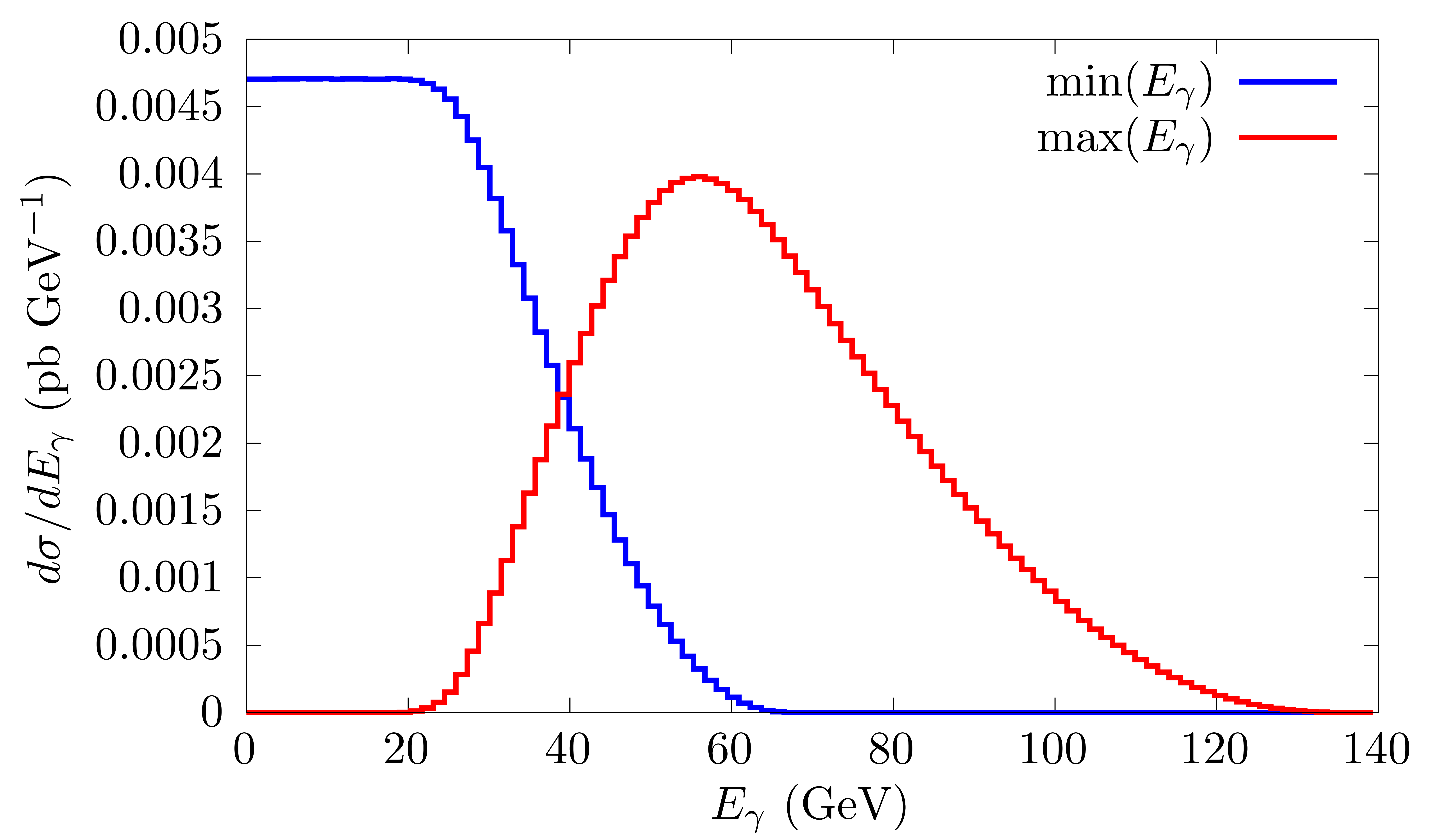
New physics searches @ MUonE

Decay photon angle



New physics searches @ MUonE

Decay photon energy



Summary and prospects

- NNLO leptonic corrections add an important piece to full NNLO calculation
- Quantification of this contribution important for background control at MUonE
- Single pion production has been excluded as a possible background for MUonE

- Now the effort is towards a fully fledged NNLO QED Monte Carlo generator
- Include also a matching of fixed-order calculation to multiple photon emission

Thank you!

BACK-UP

Recent calculations for MUonE with MESMER

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MUONE 2022
15 November 2022

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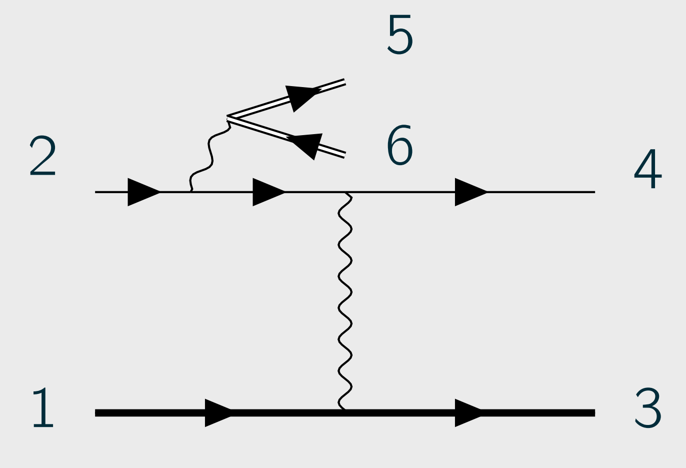
Real pair emission calculation

Matrix element implemented in MESMER - cross-check with RECOLA

Phase space cross-checked with independent code

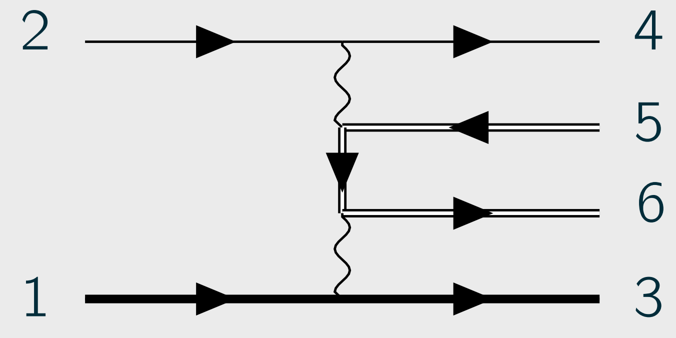
2 → 4 phase-space in independent code decomposed in:

- $$dLips = (2\pi)^3 \int dQ^2\, d\Phi_3(P \to p_3 + p_4 + Q)\, d\Phi_2(Q \to p_5 + p_6)$$



(a)

- No dedicated parametrisation for emission from muon line because $\sqrt{s} \sim m_\mu$
- $$dLips = (2\pi)^6 \int dQ_{356}^2\, dQ_{56}^2\, d\Phi_2(P \to p_4 + Q_{356})\, d\Phi_2(Q_{356} \to p_3 + Q_{56})\, d\Phi_2(Q_{56} \to p_5 + p_6)$$



(e)

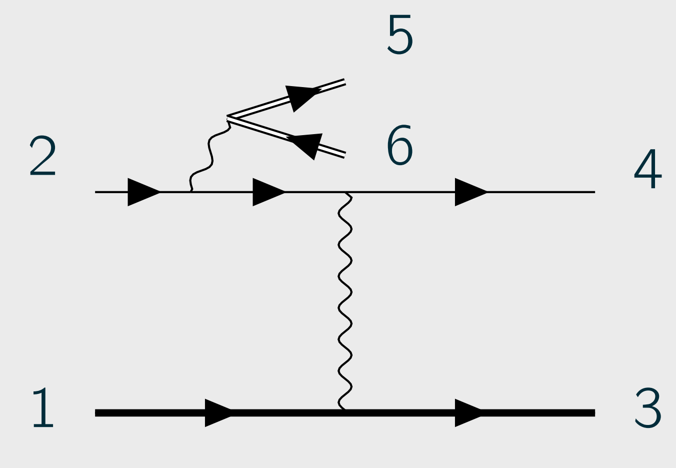
Real pair emission calculation

Matrix element implemented in MESMER - cross-check with RECOLA

Phase space cross-checked with independent code

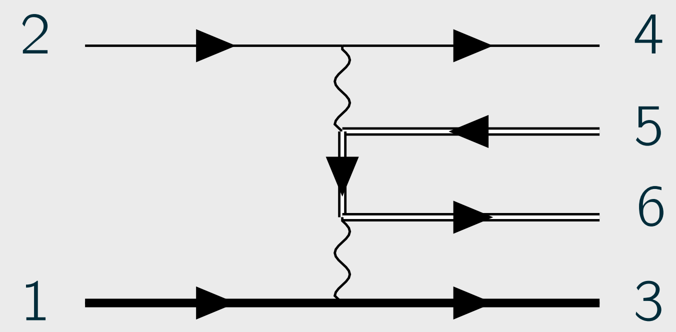
$2 \to 4$ phase-space in independent code decomposed in:

$\star \ \to \ p_5 + p_6$ rest frame



(a)

- $Q^2, \ \cos\theta_4, \phi_4, \phi_3, \ \cos\theta_{56}, \phi_{56}, \ \cos\theta_5^\star, \phi_5^\star$
- No dedicated parametrisation for emission from muon line because $\sqrt{s} \sim m_\mu$
- $Q_{356}^2, Q_{56}^2 \ \cos\theta_4, \phi_4, \phi_3^{\star\star}, \ \cos\theta_3^{\star\star}, \ \ \cos\theta_5^\star, \phi_5^\star$



(e)

$\star\star \to \ p_1 - p_4 + p_2$ rest frame

Real pair emission phase-space

Multi-channel approach

Identical particles $p_4 \leftrightarrow p_6$

$2 \to 4$ phase-space in MESMER parametrised by:

$$Q^2_{456},\ Q^2_{56},\ t_{13},\ \phi_3,\ \cos\theta^\dagger_{56},\ \phi^\dagger_{56},\ \cos\theta^\star_5,\ \phi^\star_5$$

$\dagger \to p_1 - p_3 + p_2$ rest frame

$\star \to p_5 + p_6$ rest frame

$Q^2_{456} = (p_4 + p_5 + p_6)^2$ sampled as $1/Q^2_{456}$

$Q^2_{56} = (p_5 + p_6)^2$ sampled as $1/Q^2_{56}$

$t^2_{13} = (p_1 - p_3)^2$ sampled as $1/t_{13}$

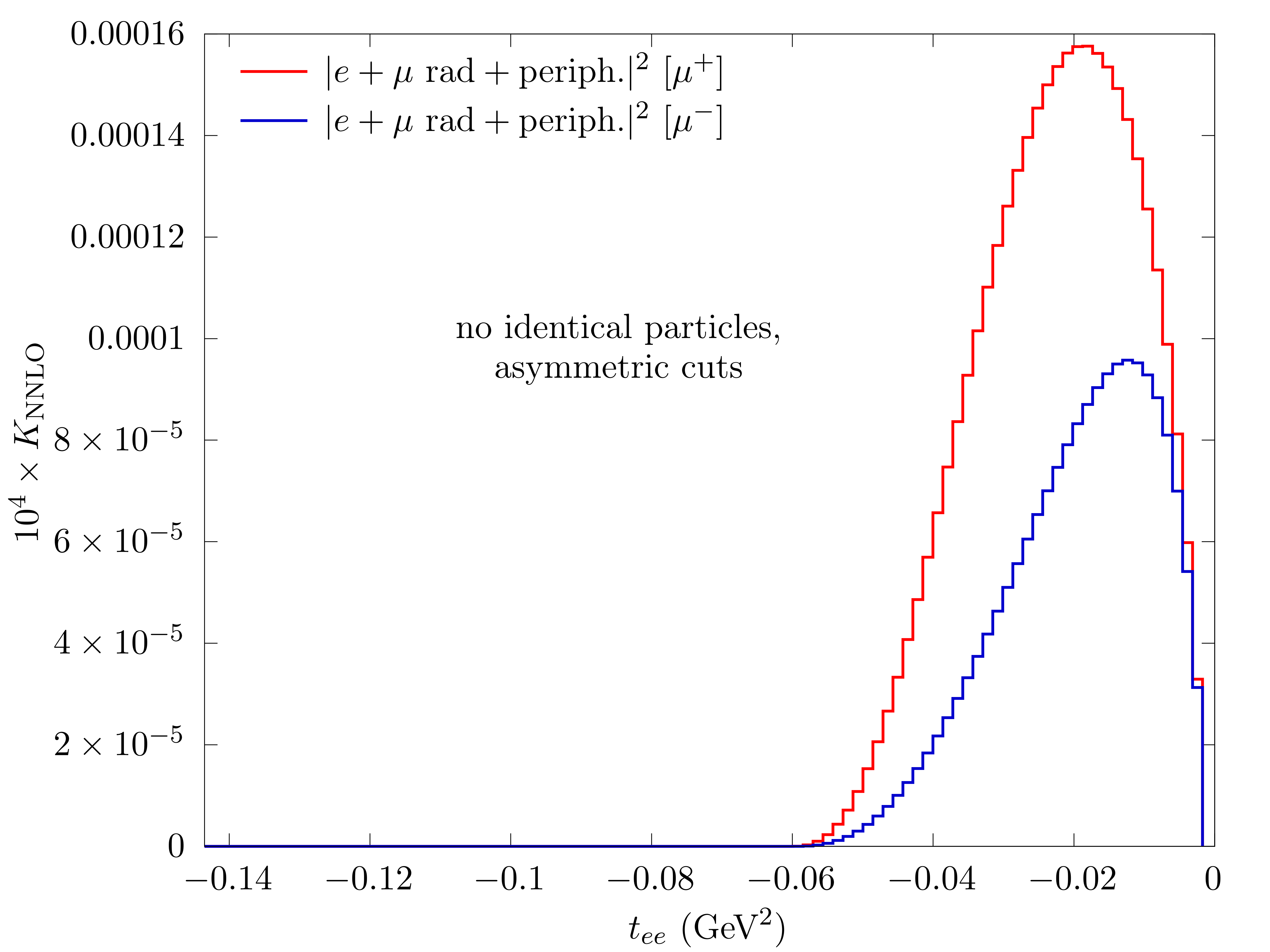
$\phi_3, \phi^\dagger_{56}, \phi^\star_5$ sampled uniformly

$\cos\theta^\dagger_{56}$ sampled as $1/(1-\beta_2\cos\theta^\dagger_{56})$ or $1/(1-\beta_{13}\cos\theta^\dagger_{56})$

$\cos\theta^\star_5$ sampled as $1/(1-\beta_{13}\cos\theta^\star_5)$ or $1/(1-\beta_{24}\cos\theta^\star_5)$

Numerical results: real pair emission

Momentum transfer on electron line



Negligible contribution due to $\mu^{\pm} e \to \mu^{\pm} e \mu^{+} \mu^{-}$