Radiative corrections to μe scattering in MESMER

MITP, Mainz

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In collaboration with: C. M. **Carloni Calame**, M. **Chiesa**, S. M. **Hasan**, G. **Montagna**, O. **Nicrosini** and F. **Piccinini**

Overview

- Photonic NNLO contributions
- MESMER

} Ettore Budassi

- NNLO lepton pair contributions
- π^0 production

} Clara Lavinia del Pio

Numerical results for μe scattering

μe scattering at NLO

- Both real and virtual QED corrections to μe scattering are included.

- Weak effects are known, at the level of $\sim 10^{-5}$ (LO) and $\lesssim 10^{-6}$ (NLO).

- Full dependence on masses and Radiative Corrections have been studied with specific kinematical cuts that mimic the experimental setup of MUonE:

  - **Basic acceptance cuts:** $\vartheta_e, \vartheta_\mu < 100$ mrad and $E_e > 1$ GeV;

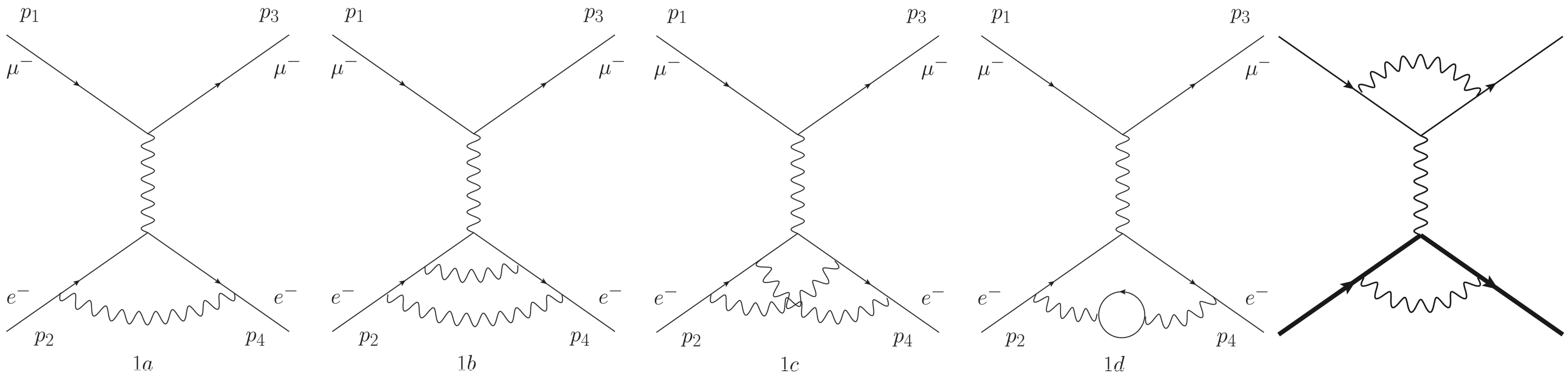
  - **Acoplanarity cut:** $\xi = \Big| \pi - |\phi_e - \phi_\mu| \Big| < \xi_c = 3.5$ mrad.

M. Alacevich, C. M. Carloni Calame *et al.*, JHEP 02 (2019) 155.

NNLO Photonic Corrections

NNLO Photonic Corrections: exact contributions

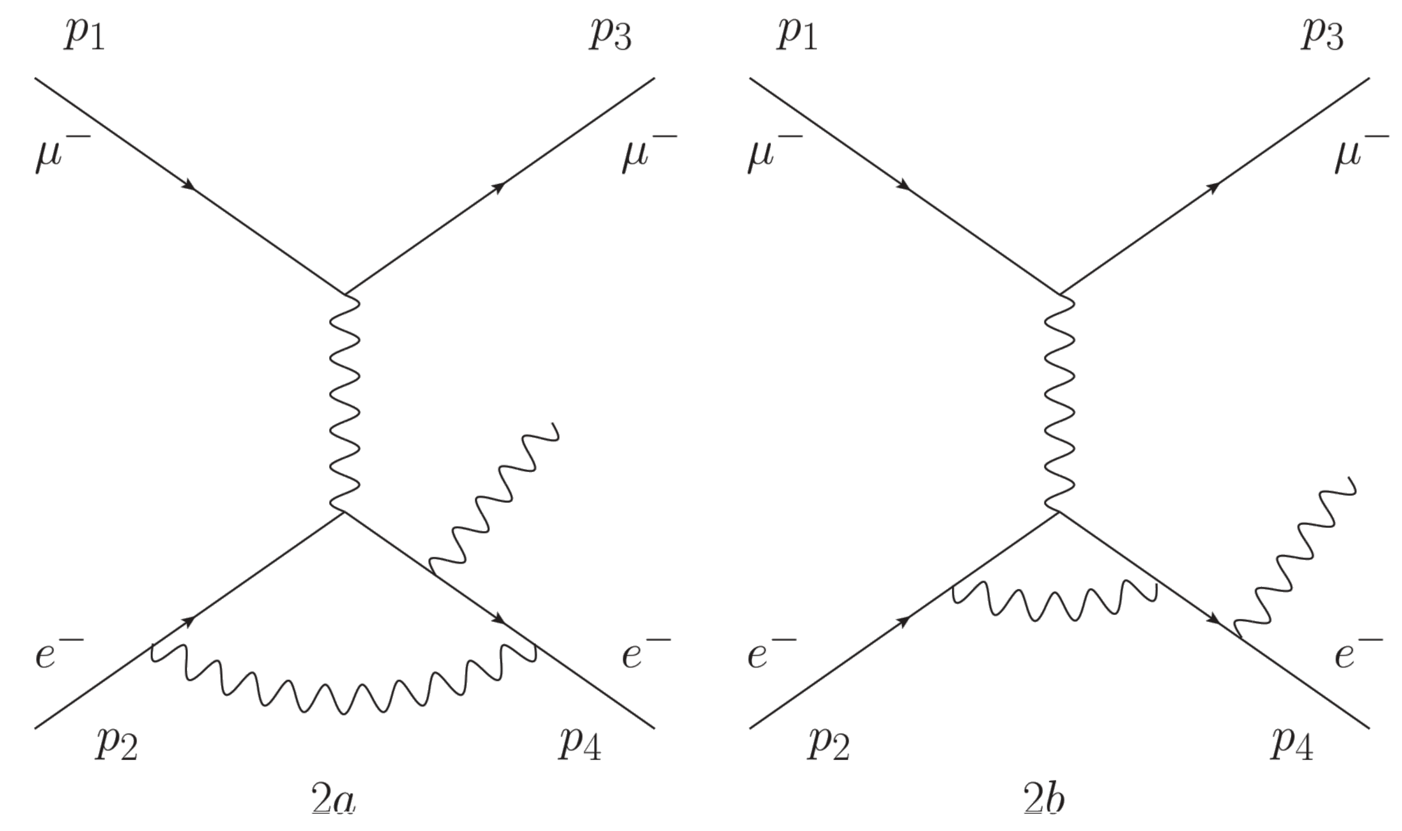
- |NLO virtual diagrams|^2
- Virtual NNLO photonic contributions are included exactly for electron or muon leg emission. 2-loop QED vertex from factors taken from Mastrolia and Remiddi.

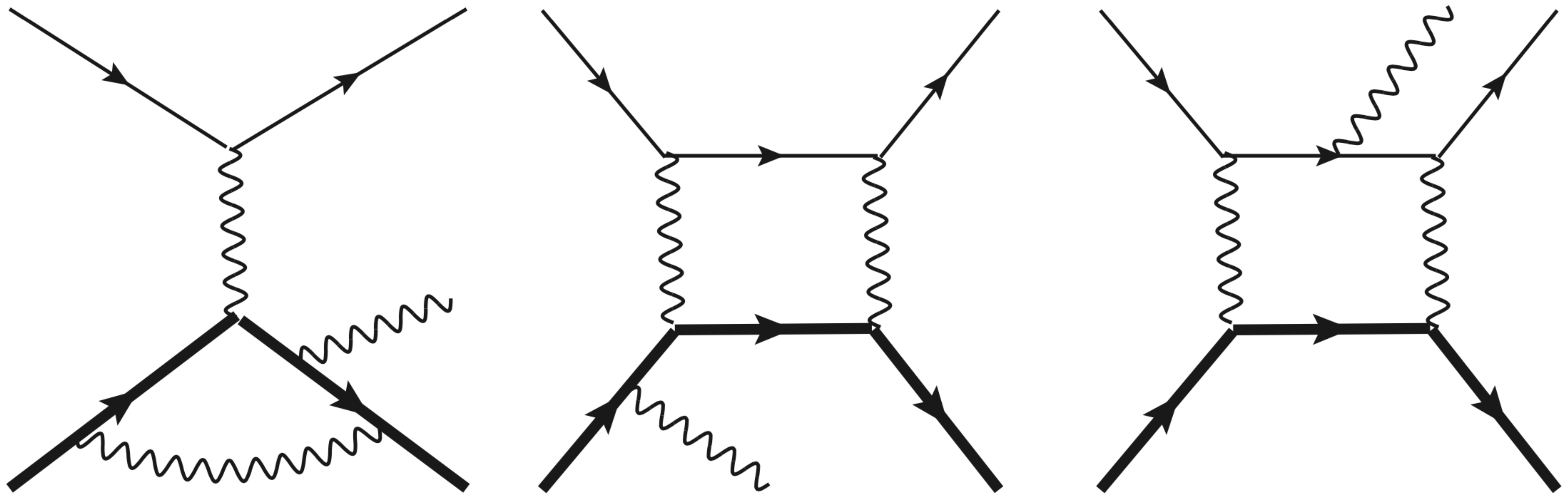


C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.
P. Mastrolia and E. Remiddi, Nucl.Phys.B 664 (2003), 341-356.

NNLO Photonic Corrections: exact contributions

- Interference of LO $\mu e \to \mu e \gamma$ amplitude with





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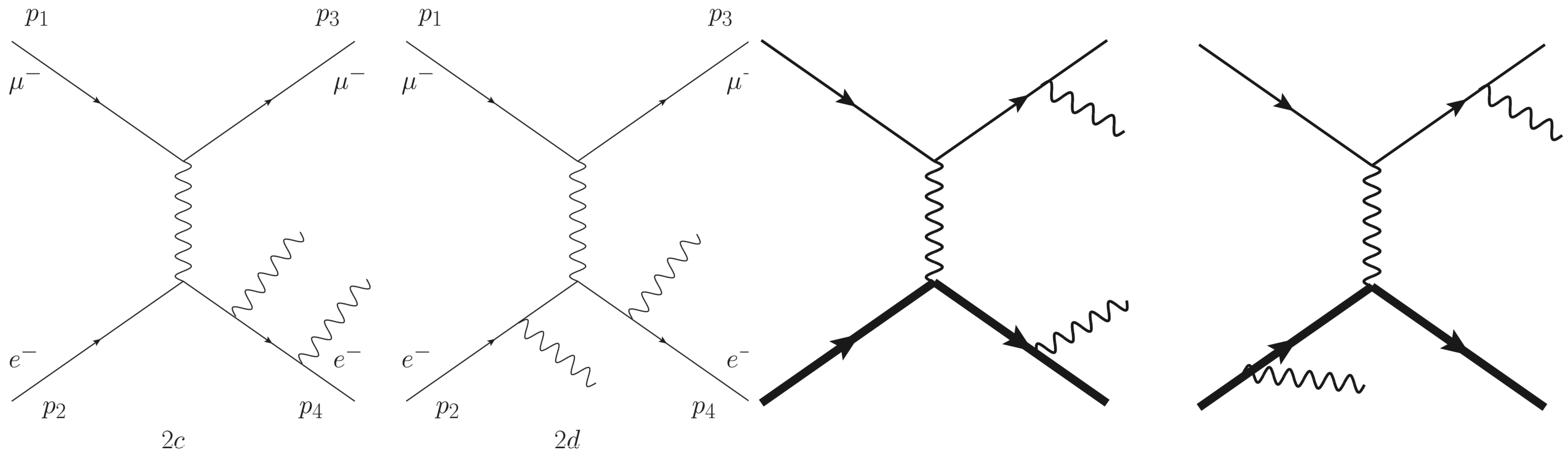
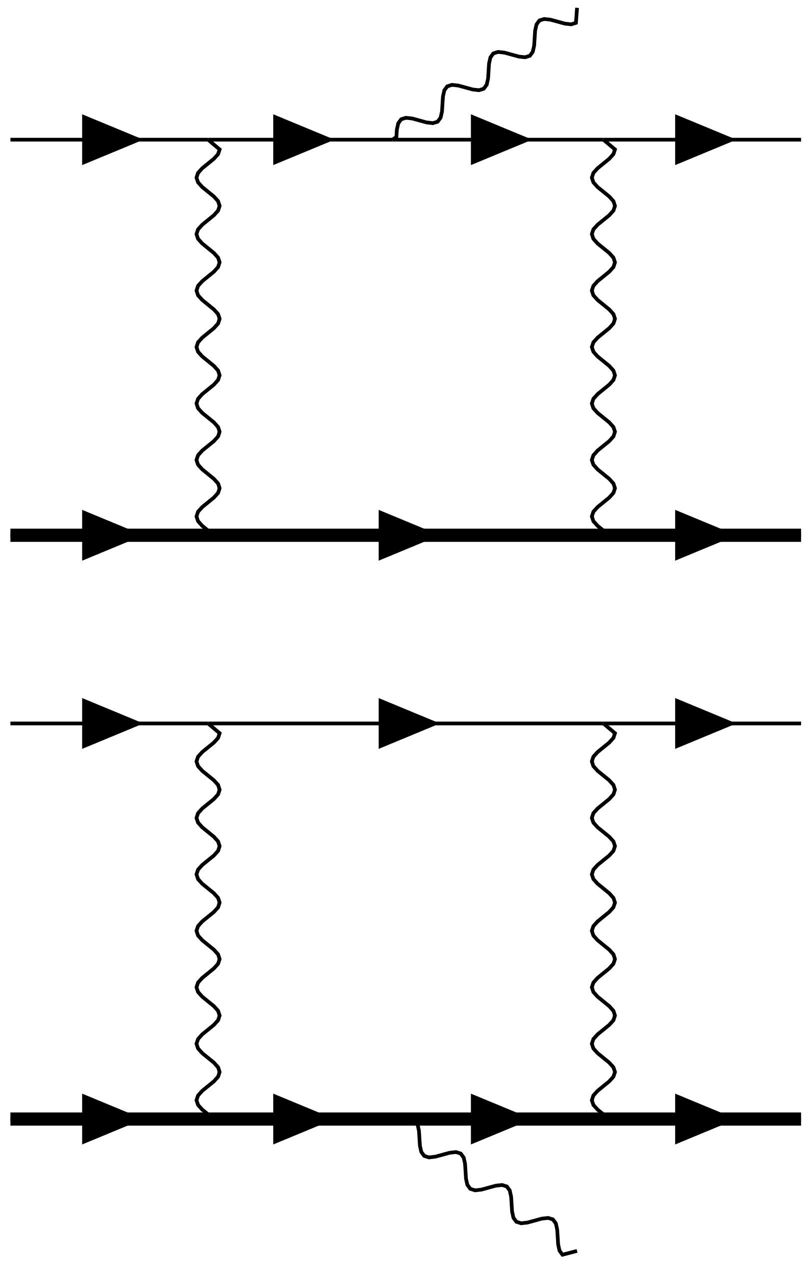
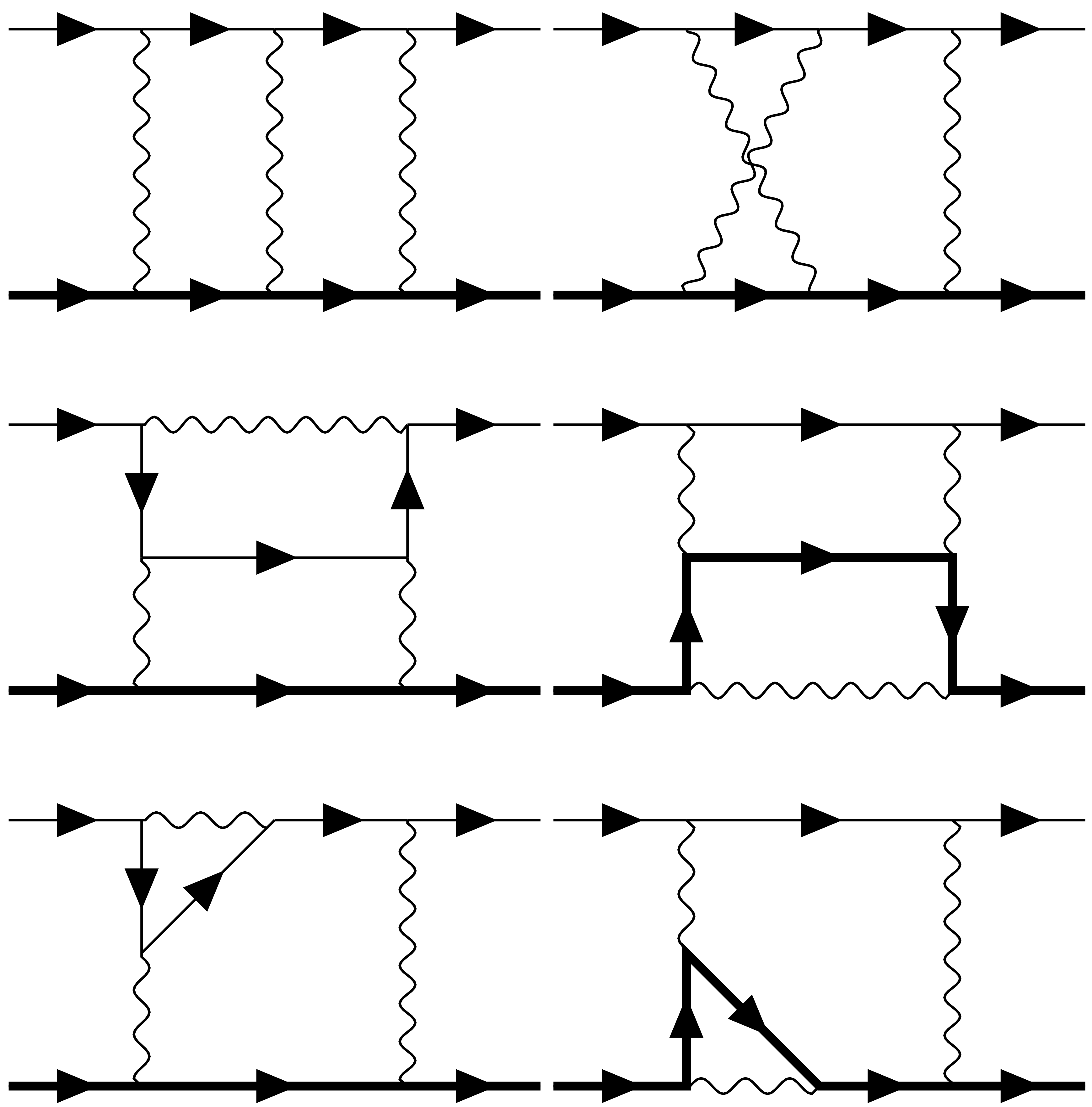




- 1-loop corrections to real photon emission exactly included: *e.g.* pentagon diagrams.
- Double real emission included exactly;
- We use a small $m_\gamma = \lambda$ and the slicing method;
- The gauge-invariant subset of NNLO corrections along the electron line are cross-checked against McMule.

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.
P. Banerjee, T. Engel, A. Signer, Y. Ulrich. SciPost Phys. 9 (2020), 027; P. Banerjee *et al.*, Eur.Phys.J.C 80 (2020) 6, 591.

NNLO Photonic Corrections: approximated contributions



- Of the two-loop virtual diagrams with a virtual photon insertion on top of NLO boxes, only the IR part is included exactly (YFS).
- The non-IR remnants are approximate.
- All photonic NNLO effects weigh at most some % at the Phase Space boundaries.
- Work is in progress for the full $\mu e \to \mu e$ at NNLO (Padova&PSI).

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.

NNLO Photonic Corrections: in formulas...

$$\widetilde{\mathcal{M}}^{\alpha^2} = \underbrace{\mathcal{M}_e^{\alpha^2} + \mathcal{M}_\mu^{\alpha^2} + \mathcal{M}_{e\mu,\,1\text{L}\times1\text{L}}^{\alpha^2}}_{\text{Exact}} + \underbrace{\frac{1}{2} Y_{e\mu}^2 \mathcal{T} + Y_{e\mu}\left(Y_e + Y_\mu\right)\mathcal{T} + (Y_e + Y_\mu)\mathcal{M}_{e\mu}^{\alpha^1,\text{R}} + Y_{e\mu} M^{\alpha^1,\text{R}}}_{\text{YFS approximated}}$$

- $\mathscr{M}^{\alpha^2}_{e(\mu)}$: two virtual γ are both attached to the electron (muon) leg;
- $\mathscr{M}^{\alpha^2}_{e\mu,\,1\text{L}\times1\text{L}}$: 1 virtual γ is attached to each leg.

- This contains all the IR part.
- It misses the non-IR remnant of the two-loop diagrams with two γs connecting the e and μ legs $\mathscr{M}^{\alpha^2,\text{R}}$.

$$Y_{ij} = \begin{cases} \frac{1}{8}\frac{\alpha}{\pi} Q_i^2 \left[B_0\left(0, m_i^2, m_i^2\right) - 4m_i^2 C_0\left(m_i^2, 0, m_i^2, \lambda^2, m_i^2, m_i^2\right)\right] & \text{for } i = j \\ \frac{\alpha}{\pi} Q_i Q_j \vartheta_i \vartheta_j \left[p_i \cdot p_j \; C_0\left(m_i^2, (\vartheta_i p_i + \vartheta_i p_j)^2, m_j^2, \lambda^2, m_i^2, m_j^2\right) + \right. & \\ \left. \qquad + \frac{1}{4} B_0\left((\vartheta_i p_i + \vartheta_j p_j)^2, m_i^2, m_j^2\right)\right] & \text{for } i \neq j \end{cases}$$

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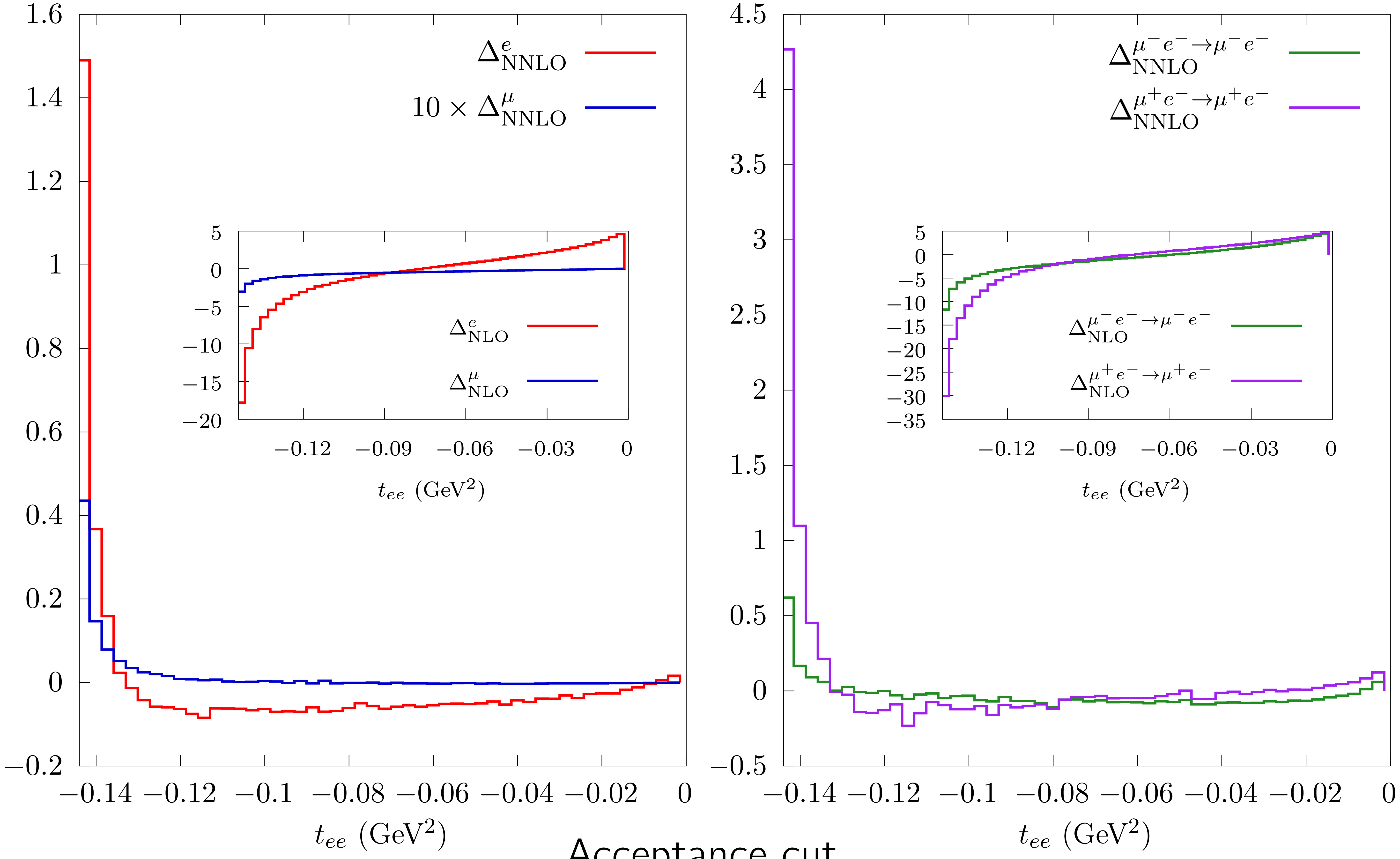
NNLO Photonic Corrections: results



Acceptance cut

$$\Delta_i^{\rm NNLO} = \frac{d\sigma_i^{\rm NNLO} - d\sigma_i^{\rm NLO}}{d\sigma_i^{\rm LO}} \times 100$$

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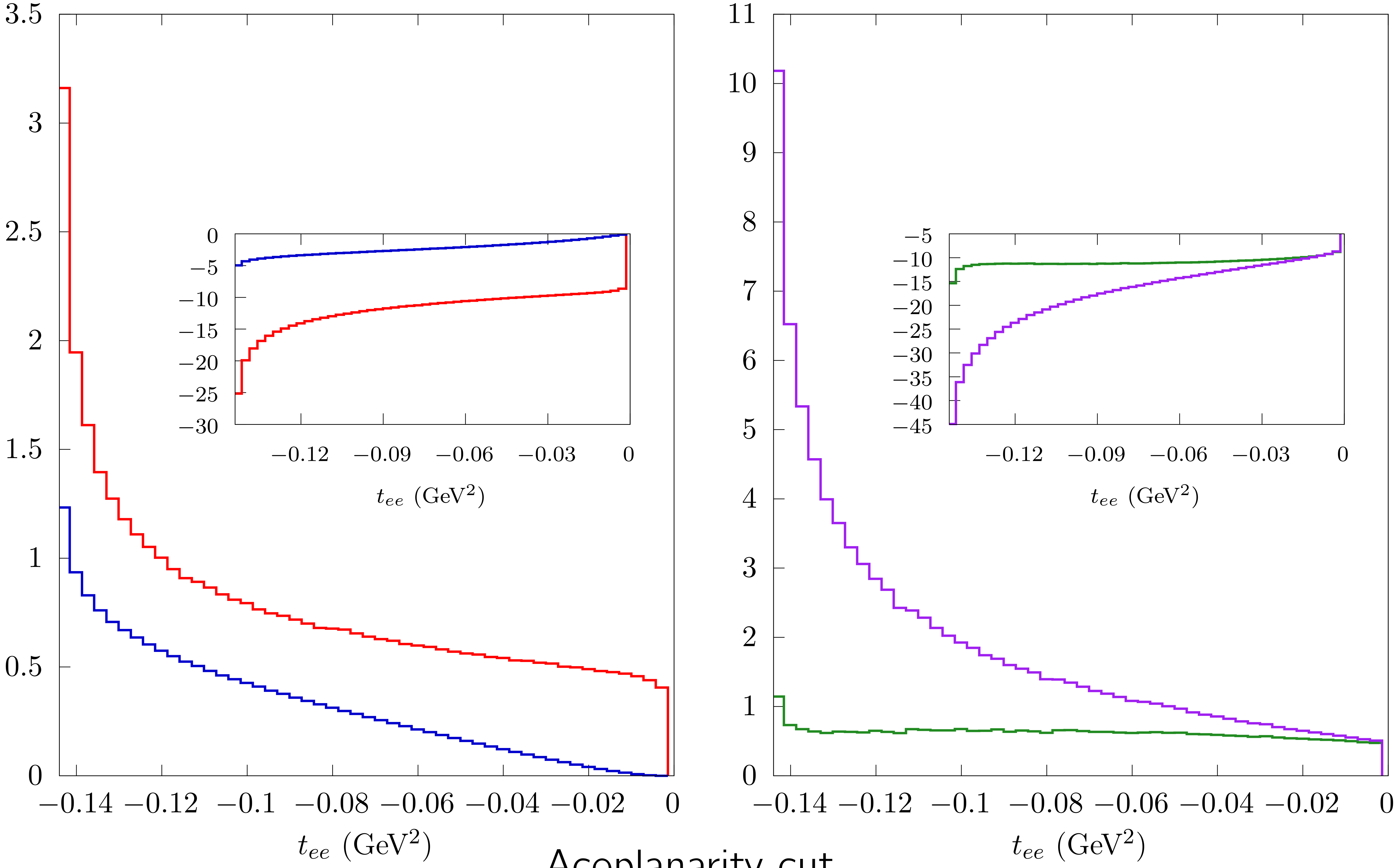

Acoplanarity cut

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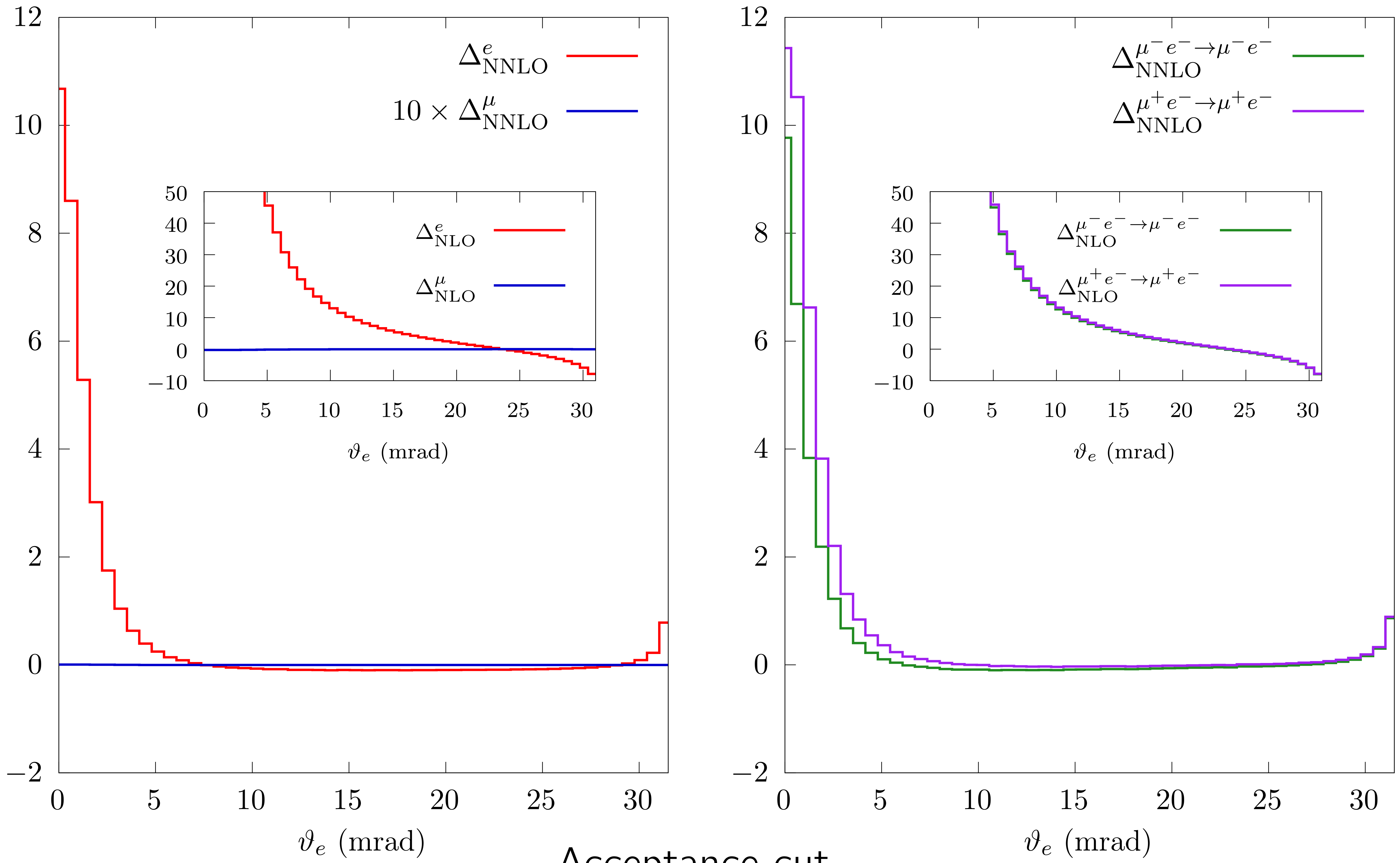

Acceptance cut

$$\Delta_i^{\mathrm{NNLO}} = \frac{d\sigma_i^{\mathrm{NNLO}} - d\sigma_i^{\mathrm{NLO}}}{d\sigma_i^{\mathrm{LO}}} \times 100$$

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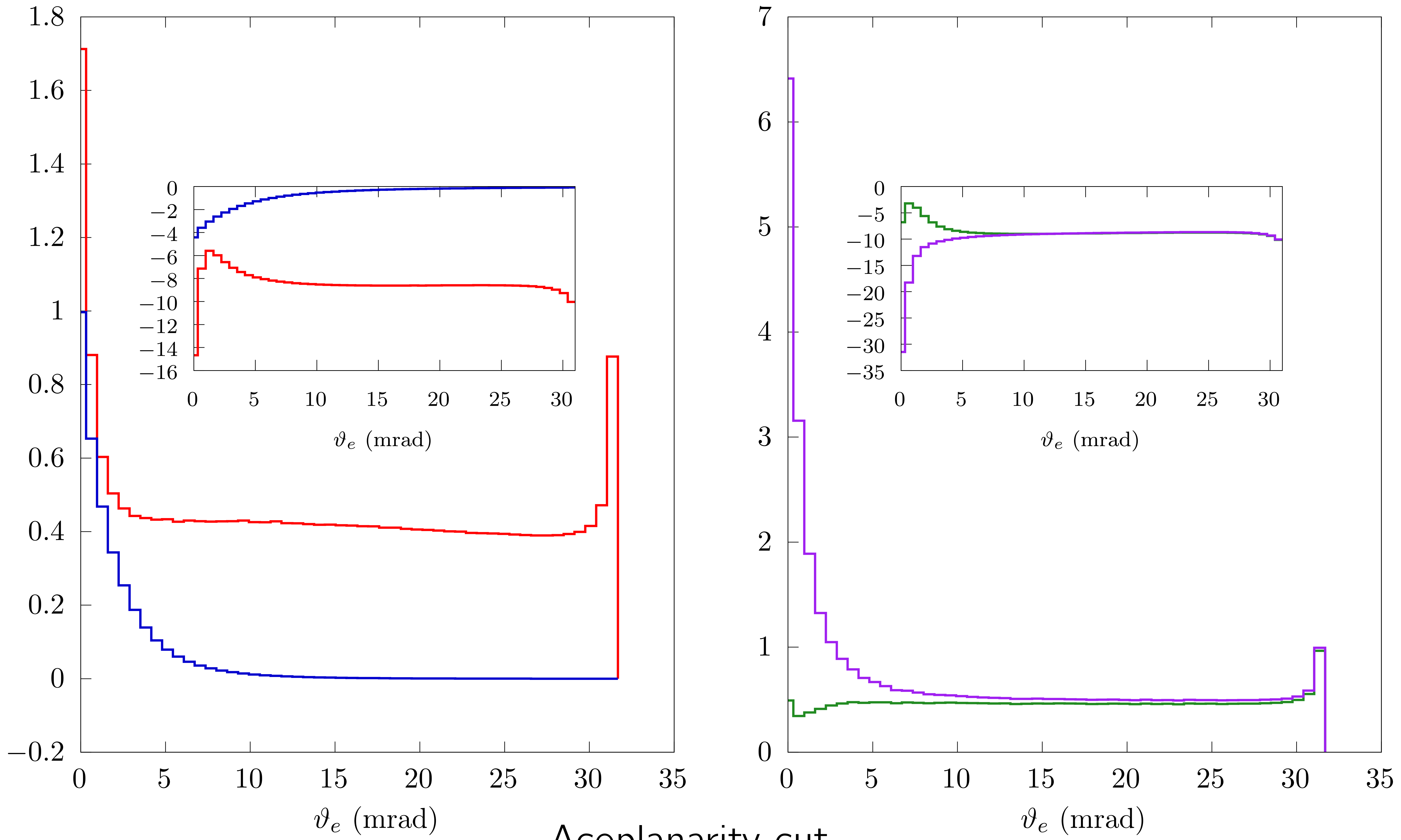

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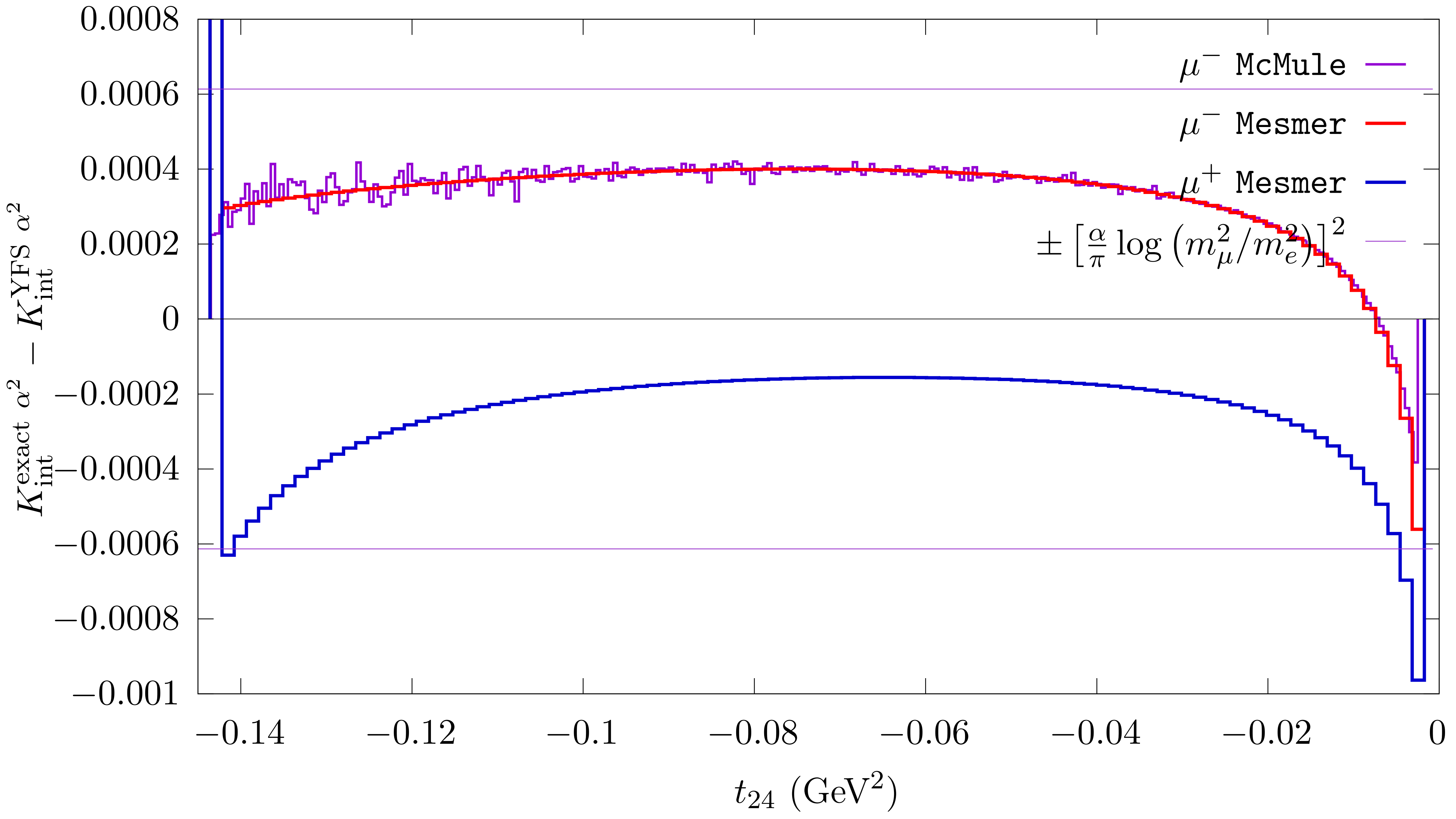
Exact NNLO photonic corrections

- The complete two-loop corrections to $f\bar{f} \to F\bar{F}$ have been calculated by Bonciani *et. al* for $m_f = 0$. Using **crossing symmetry** it can be used for $\mu e \to \mu e$;

- The amplitudes with $m_e = 0$ can undergo the **massification** procedure, to get the collinear divergences in terms of $\ln\left(Q^2/m_e\right)$;

- NNLO double boxes are CPU expensive (>1 s/event on a single core).



Difference between YFS-approximated and exact NNLO photonic K factor.

R. Bonciani *et. al*, PRL 128 (2022) 2.
T. Engel *et al.* JHEP 02 (2019) 118

PRELIMINARY!

MESMER

Relevant references

- Alacevich *et al.*, Muon-electron scattering at NLO, JHEP 02 (2019) 155
- Carloni Calame *et al.*, Towards muon-electron scattering at NNLO, JHEP 11 (2020) 028
- E. Budassi *et al.*, NNLO virtual and real leptonic corrections to muon-electron scattering, JHEP 11 (2021) 098
- E. Budassi *et al.*, Single π^0 production in μe scattering at MUonE, PLB 829 (2022) 137138

What can MESMER calculate?

- NLO QED corrections: single real or virtual γ;
- NNLO photonic corrections (approximate);
- Exact NNLO lepton pair contributions and $\mu e \to \mu e l^+ l^-$, with $l = e\,,\mu$ (not public yet);
- $\mu e \to \mu e \pi^0$, with $\pi^0 \to \gamma\gamma$ (not public yet).

MESMER

MESMER (**M**uon **E**lectron **S**cattering with **M**ultiple **E**lectromagnetic **R**adiation)

Fully differential Monte Carlo event generator for high-precision simulation of μe scattering at low energies, developed for the MUonE experiment.

The code

- The code can be found in this github repository.
- MESMER is mostly written in Fortran 77 language.
- Some external libraries are used for one-loop integrals and pseudo-random number generation;
- HVP is included using Jegerlehner's, Keshavarzi-Nomura-Teubner's and Ignatov's routines.

Parameters & output

- Some kinematical constraints are imposed at generation level. `cuts.F` is the routine that selects kinematical cuts and can be modified according to the user's needs;

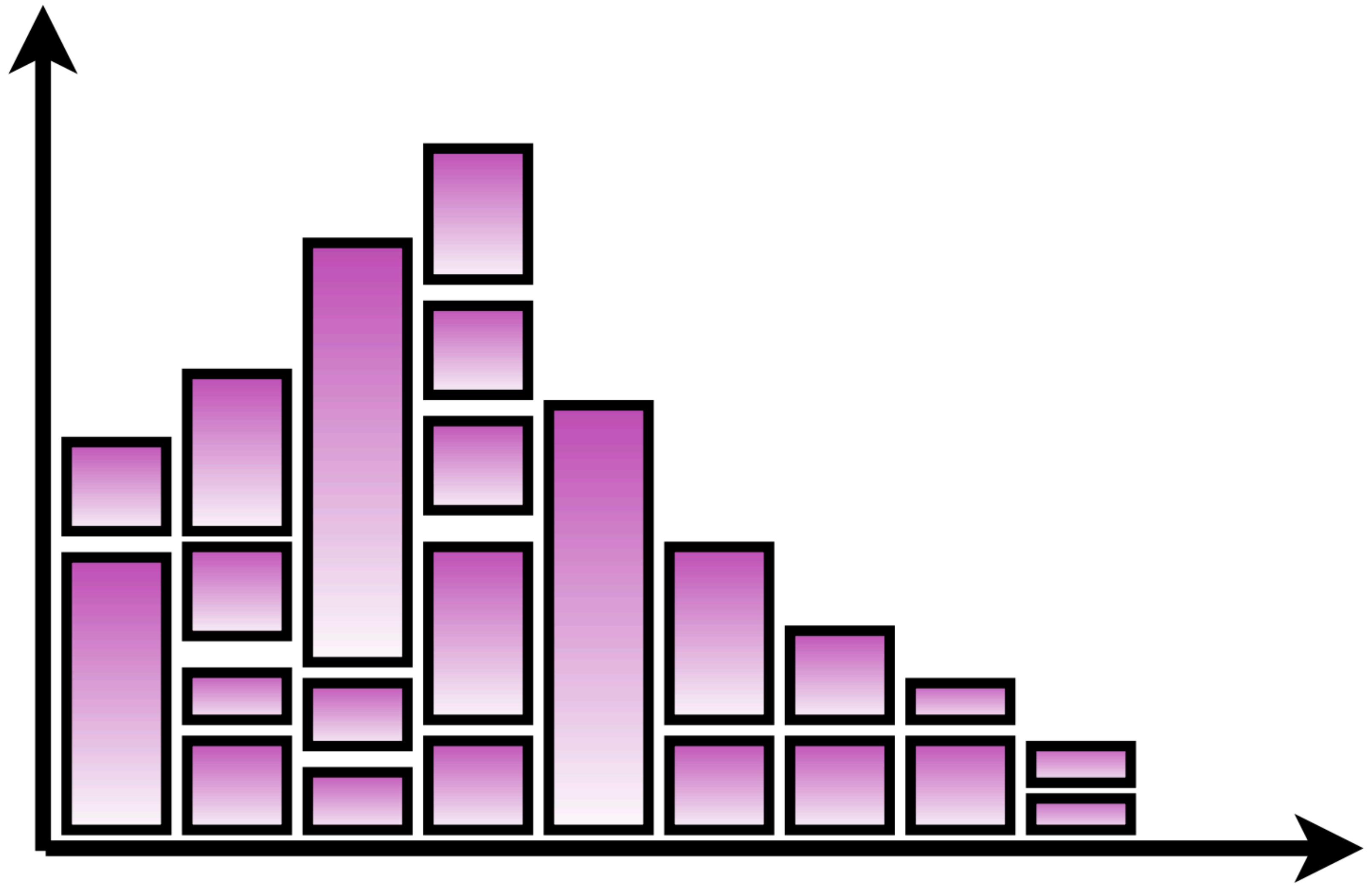
- A generic incoming muon momentum can be fed in input event by event, for realistic simulations (beam profile);

- Events are stored in a format that has been thought out with the experimental colleagues (weights, momenta *etc.*);

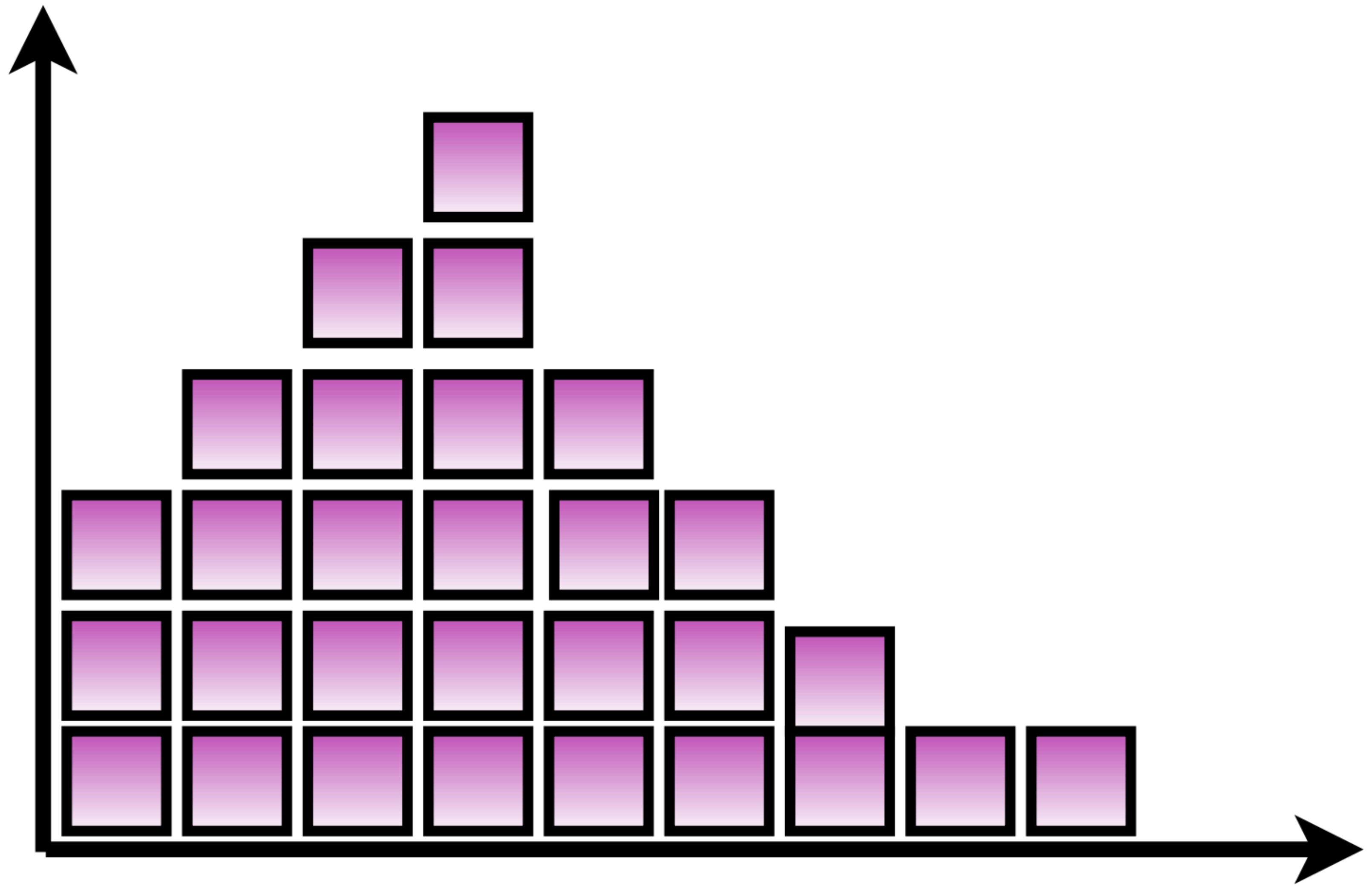
- Differential distributions are written in specific files, ready to be plotted (*e.g.* with gnuplot).

- A `C/C++` interface is provided to use the code as a library in a driver simulation program.

Weighting vs. Un-weighting



- Events have their weight;
- It needs to be carried throughout the whole detector simulation;
- Fast generation.



- Events are not weighted: they are distributed according to the cross section;
- Slow due to the un-weighting procedure.
- It is tricky to guess a correct maximiser for weights before the generation.

Reweighting

$$\sigma = \sum_{i=0}^{n} \frac{w_i}{n} \qquad \sigma_{reweighted} = \sum_{i=0}^{n} \frac{w_i}{n} \times r_i\,; \qquad r_i = \frac{w_i^{rew}}{w_i}$$

- In the same run we can calculate different re-weighting coefficients r_i to study the inclusion/exclusion of different contributions (*e.g.* VP, HVP parametrisations *etc.*);
- No need to re-generate another MC sample to account for different effects!
- Since the MC sample is the same, one can exloit the statistical correlation on the weights to reduce errors.

Now it’s Clara’s turn.