MUonE: theory status

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The Evaluation of the Leading Hadronic Contribution To the Muon $g-2$: Towards the MUonE Experiment

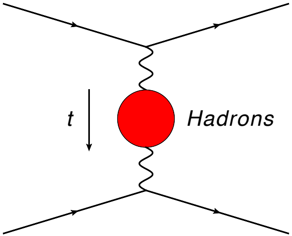
MITP, Mainz, 14-18 November 2022

- ⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  Measuring the leading hadronic contribution to the muon g-2 via μe *scattering*
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$
$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



⇝ The hadronic VP correction to the running of α enters

- ⋆ $\Delta\alpha_{\rm had}(t)$ can be directly measured in a (single) experiment involving a space-like scattering process and $a_\mu^{\rm HLO}$ obtained through numerical integration

  Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- ⋆ **A data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

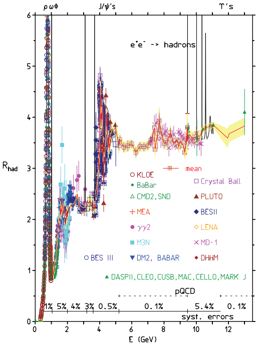
- By modifying the kernel function $\frac{\alpha}{\pi}(1-x)$, also $a_\mu^{\rm HNLO}$ and $a_\mu^{\rm HNNLO}$ can be provided

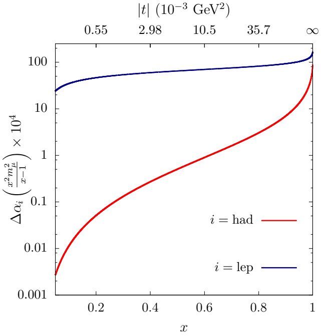
  Balzani, Laporta, Passera, Phys.Lett.B834 (2022) 137462

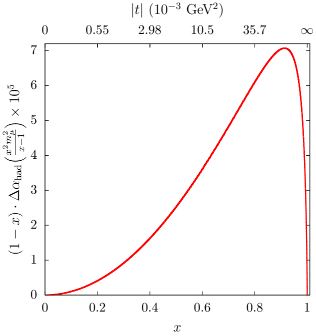
  talks by E. Balzani and S. Laporta

From time-like to space-like evaluation of $a_\mu^{\rm HLO}$

Time-like $\mapsto$ **Space-like**







Smooth function

- $\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required
- $\mapsto$ **Space-like:** in principle, one single experiment, *it's a one-loop effect, very high accuracy needed*

MUonE: $\mu e \to \mu e$ @CERN

Abbiendi et al., EPJC 77 (2017) 3, 139

Abbiendi et al., *Letter of Intent: the MUonE project, CERN-SPSC-2019-026, SPSC-I-252 (2019)*

- Scattering μ's on e's in a low Z target looks like an ideal process (fixed target experiment)
- **It is a pure t-channel process at tree level**
- **The M2 muon beam ($E_\mu \simeq 160$ GeV) is available at CERN**
- $\sqrt{s} \simeq 0.4$ GeV and $-0.143 < t < 0$ GeV^2 (and extrapolate to $x \to 1$)
- With ~ 3 years of data taking, a statistical accuracy of 0.35% on a_μ^{HLO} can be achieved

talk by G. Abbiendi

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\text{had}}$$

$\Delta\alpha_{\text{had}}$ is a 0.1% effect in this region $\to$ to measure it at 1%, σ must be controlled at the 10^{-5} level

Systematics

- experimental $\Longrightarrow$ talks by G. Abbiendi, R.N. Pilato and U. Marconi

- theoretical: higher order radiative corrections modify the shapes

$$\sigma \quad = \quad \sigma^{(0)} + \left(\frac{\alpha}{\pi}\right)\sigma^{(1)} + \left(\frac{\alpha}{\pi}\right)^2\sigma^{(2)} + \ldots + \left(\frac{\alpha}{\pi}\right)^n\sigma^{(n)}$$

  - order of magnitude estimate, barring infrared logs and setting $c_{i,j} \sim 10$
  - $c_{1,1}\left(\frac{\alpha}{\pi}\right) L \sim 0.2$ $\quad c_{1,0}\left(\frac{\alpha}{\pi}\right) \sim 2.5 \cdot 10^{-2}$
  - $c_{2,2}\left(\frac{\alpha}{\pi}\right)^2 L^2 \sim 5 \cdot 10^{-3}$ $\quad c_{2,1}\left(\frac{\alpha}{\pi}\right)^2 L \sim 5 \cdot 10^{-4}$ $\quad c_{2,0}\left(\frac{\alpha}{\pi}\right)^2 \sim 5 \cdot 10^{-5}$
  - $c_{3,3}\left(\frac{\alpha}{\pi}\right)^3 L^3 \sim 1.5 \cdot 10^{-4}$ $\quad c_{3,1}\left(\frac{\alpha}{\pi}\right)^3 L^2 \sim 1.5 \cdot 10^{-5}$ $\quad c_{3,0}\left(\frac{\alpha}{\pi}\right)^3 L \sim 1.5 \cdot 10^{-6}$

  - the most advanced technologies for NNLO calculations and higher order resummation and matching are needed

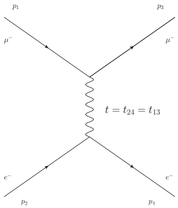
Theoretical progress since 2017

thanks also to past

MUonE theory workshops

- Theory Kickoff Workshop, Padova, 4-5 September 2017
- MITP Workshop, Mainz 19-23 February 2018
- 2^{nd} Workstop/ThinkStart, Zürich, 4-7 February 2019

First step towards precision: QED NLO (2018)



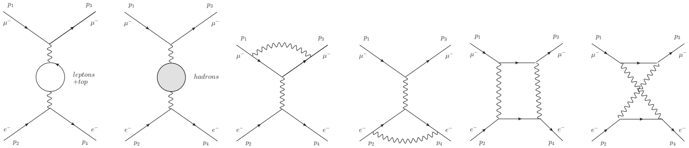
- analytical expression for tree level

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

(Van Nieuwenhuizen 1971, D'Ambrosio 1983, Kukhto et al. 1987, Bardin, Kalinovskaya 1997)

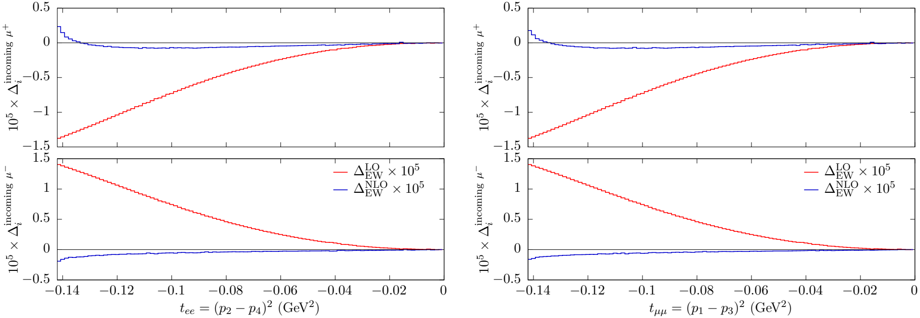
- NLO virtual diagrams



- and corresponding real emission diagrams
- **NLO matrix elements** calculated with finite m_μ and m_e mass effects and a **Monte Carlo** program, MESMER, has been taylored to the fixed target kinematics of MUonE

Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini, JHEP 02 (2019) 155

Weak interaction effects (LO and NLO) (2018)

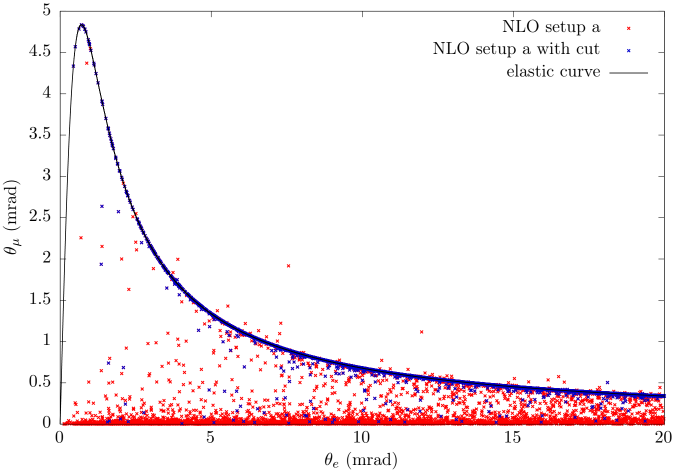


Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini, JHEP 02 (2019) 155

- **tree-level Z-exchange important** at the 10^{-5} level ($\sim tG_\mu/4\pi\alpha\sqrt{2}$ in the Fermi theory)
- purely weak RCs (in QED NLO units) at a few 10^{-6} level

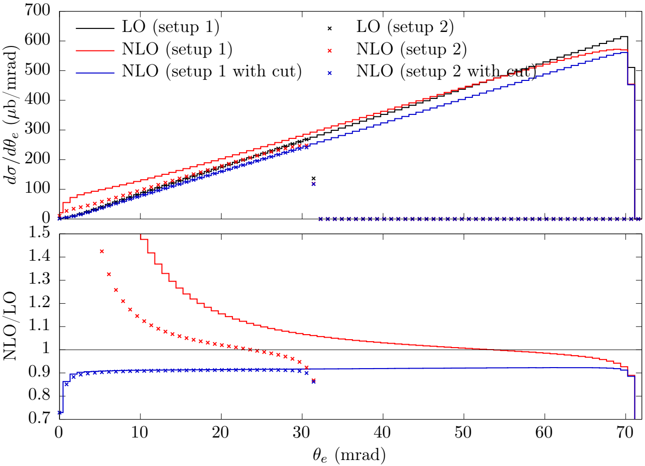
What can we learn from “simple” NLO?

μ-e angle correlation in the lab



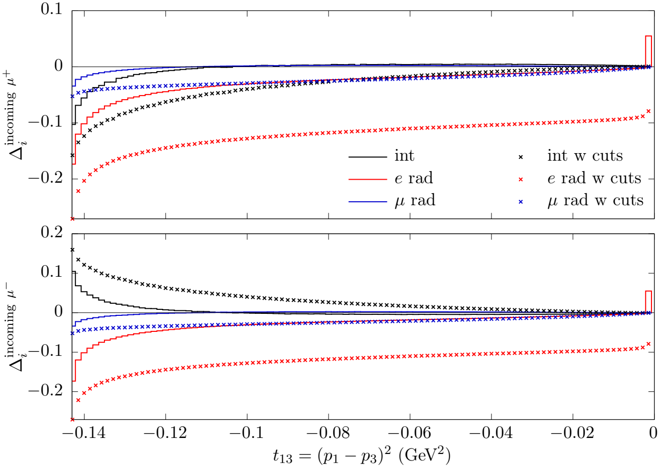
- many points fall out of the $2 \to 2$ correlation curve $\theta_\mu - \theta_e$ because of the radiative events

Giant $K-$factor for $d\sigma/d\theta_e$



- due to the almost vanishing LO cross cross section for $\theta_e \to 0$

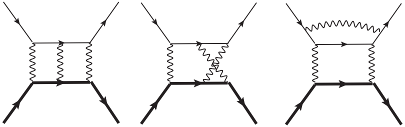
Radiation from electron leg not always dominant, in particular for “almost” elastic events



- interference between electron and muon leg can be of the same order of magnitude as the contribution from (virtual and real) radiation along the electron leg

Second step, towards *photonic* radiative corrections at **NNLO** (2020)

- **exact calculation of corrections along one lepton line**
  - two independent calculations, with different subtraction procedures — Carloni Calame et al., JHEP 11 (2020) 028, P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027
  - implemented in **Mesmer** and **McMule** — talks by E. Budassi, T. Engel, M. Rocco, Y. Ulrich
- **complete calculation (including up-down interference) approximated in Mesmer** — talk by E. Budassi
  - interference of LO $\mu e \to \mu e$ amplitude with



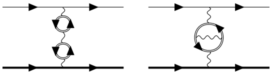
⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the infra-red divergent structure

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** — R. Bonciani *et al.*, PRL 128 (2022)

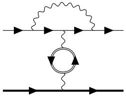
  talks by W.T. Bobadilla and J. Ronca
- **complete calculation with massification in McMule** — T. Engel, C. Gnendinger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118
  - **complete calculation including leading log $\propto \ln(m_e^2/Q^2)$ and m_e^0 terms** — talks by T. Engel, M. Rocco, Y. Ulrich

talk by C.L. Del Pio

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes

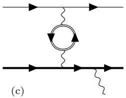


(a) (b)



$+ \quad \cdots$

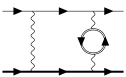
(a)



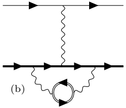
$+ \quad \cdots$

(c)

- interfered with $\mu e \to \mu e$ amplitude



(a)



(b)

$+ \quad \cdots$

Here the 2-loop integral is evaluated with dispersion relation techniques

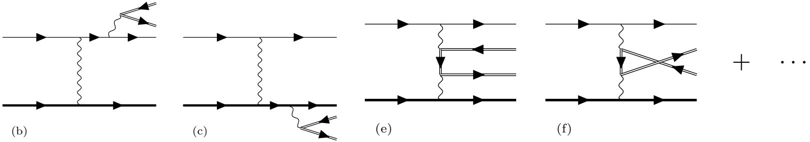
used e.g. in the past for Bhabha: Actis et al., Phys. Rev. Lett. 100 (2008) 131602; Carloni Calame et al., JHEP 07 (2011) 126

$$\frac{g_{\mu\nu}}{q^2+i\epsilon} \to g_{\mu\nu}\frac{\alpha}{3\pi}\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{R_\ell(z)}{q^2-z+i\epsilon} = g_{\mu\nu}\frac{\alpha}{3\pi}\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{1}{q^2-z+i\epsilon}\left(1+\frac{4m_\ell^2}{2z}\right)\sqrt{1-\frac{4m_\ell^2}{z}}$$

E. Budassi et al., JHEP 11 (2021) 098

talk by C.L. Del Pio

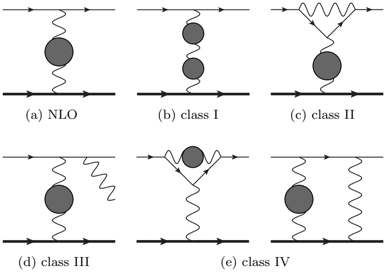
- they also contribute at NNLO accuracy
- squared absolute vaule of

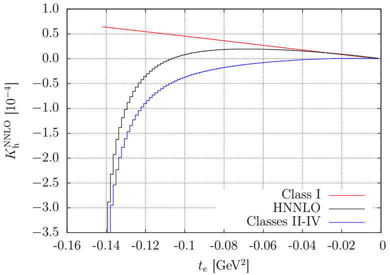


- the emission of an extra electron pair $\mu e \to \mu e\ e^+ e^-$ is potentially a dramatically large (reducible) background, because of the presence of “peripheral” diagrams
- NNLO virtual and real pairs implemented in **`Mesmer`**

NNLO hadronic contributions (2019)

- using the dispersion relation approach





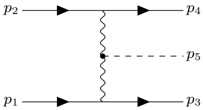
Fael, Passera, Phys. Rev. Lett. 122 (2019) 192001

- relevant on the target precision scale

- space-like method to calculate hadronic NNLO corrections

M. Fael, JHEP02 (2019) 027

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**



- **π^0 production** recently calculated and shown to be well below 10^{-5} w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

talk by C.L. Del Pio

Possible New Physics issues

- **Possible contamination of New Physics in the $\Delta\alpha(t)$ measurement?**

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** NP mediators investigated through EFT by considering dim-6 operators
  - excluded (at the 10^{-5} level) by existing data
- Effects of **light** NP mediators investigated through EFT by considering dim-6 operators
  - spin-0 NP mediators (ALPs)
  - spin-1 NP mediators (Dark Photons, light Z')
  - excluded (at the 10^{-5} level) by existing data

Possible New Physics studies in complementary regions

- **interesting proposals for NP searches at MUonE (new light mediators) in $2 \to 3$ processes**
  - invisibly decaying light Z' in $\mu e \to \mu e Z'$

    Asai et al., Phys. Rev. D106 (2022) 5
    - a relevant background can be $\mu e \to \mu e \pi^0$, in addition to $\mu e \to \mu e \gamma$
  - long-lived mediators with displaced vertex signatures $\mu e \to \mu e A' \to \mu e e^+ e^-$

    Galon et al., arXiv:2202.08843
  - through scattering off the target nuclei $\mu N \to \mu N X \to \mu N e^+ e^-$

    Grilli di Cortona and E. Nardi, Phys. Rev. D105 (2022) L111701

Summary

- Given its precision requirements, MUonE represents a challenge for present theoretical calculations and simulation tools

- at present we have two Monte Carlo tools, **`Mesmer`** and **`McMule`**
  - NLO QED corrections cross-checked with perfect agreement
  - NNLO QED corections from single lepton legs in perfect agreement
  - YFS inspired approximation to the full NNLO QED in **`Mesmer`**
  - full NNLO QED with electron "massification" in **`McMule`**
  - efforts for N^3LO started
    - very recent result on three-loop QCD form factors for massive quarks

      M. Fael, F. Lange, K. Schönwald and M. Steinhauser, Phys, Rev. D106 (2022) 034029

      talk by M. Fael
    - N^3LO kick-off workshop/thinkstart, Durham, 3-5 August 2022 (organized by Y. Ulrich)