MUonE analysis strategy

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MITP Topical Workshop *The Evaluation of the Leading Hadronic Contribution to the Muon g-2: toward the MUonE experiment*, Mainz, 14-18 Nov 2022

https://indico.mitp.uni-mainz.de/event/248

Outline

- Event selection
  - Backgrounds
  - Angular variables
  - Use of calorimeter and muon detector
- Signal fit strategy
- Systematic errors
  - Specific discussion in R.Pilato's talk
- Conclusions

EVENT SELECTION: BACKGROUNDS

Muon Interactions in the material

- MUonE is not a collider experiment.
- The μe scattering occurs in the detector material, not in vacuum
- There are physics backgrounds that have to be studied
  - $\mu N \rightarrow \mu N e^+ e^-$
  - $\mu N \rightarrow \mu N \gamma$
  - $\mu N \rightarrow \mu X$

  Scattering on Nuclei

  - $\mu\ e^- \rightarrow \mu\ e^- \gamma$ M. Alacevich et al., JHEP 02 (2019) 155
  - $\mu\ e^- \rightarrow \mu\ e^- e^+ e^-$ E.Budassi et al., JHEP 11 (2021) 098

- In the moment the only tool including all the physics backgrounds is GEANT4, which we are using for the Full Simulation of the detector
  - We have integrated the MESMER MC with the GEANT4 FullSim for simulating μe interactions
- We have no standalone tool for a Fast Simulation of physics backgrounds, which is crucial for the physics analysis

MUonE Detector Layout

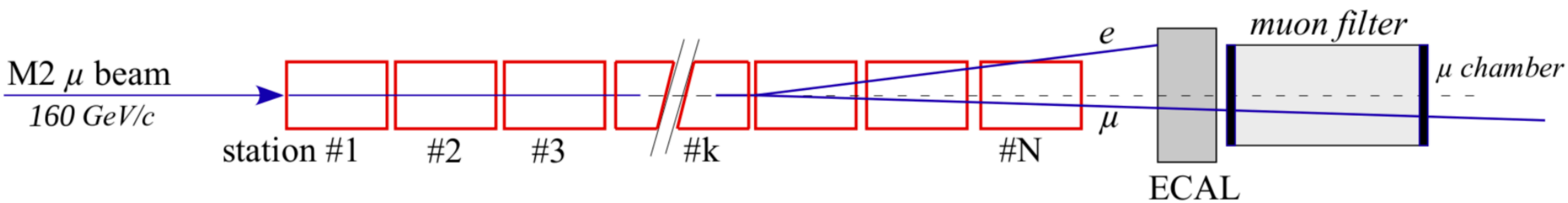
Letter-Of-Intent SPSC-I-252

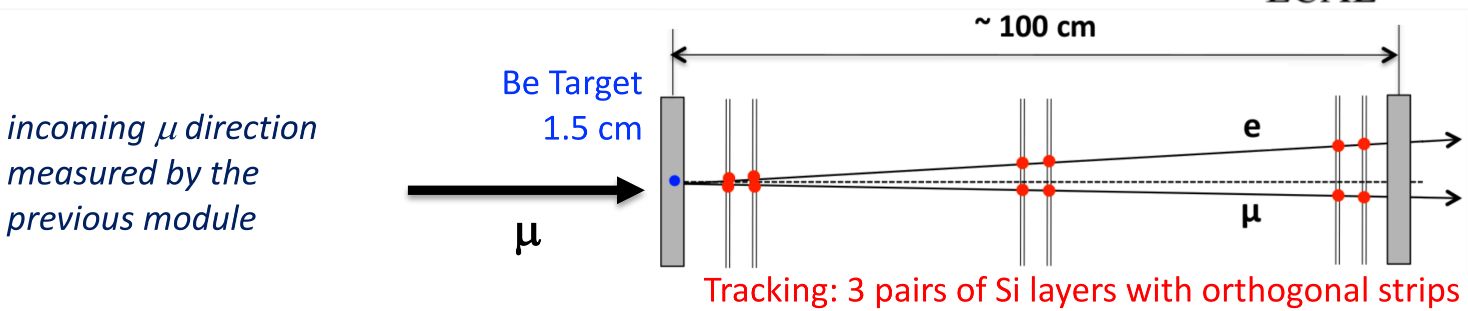
The detector concept is simple, the challenge is to keep the systematics at the same level as the statistical error .

- Large statistics to reach the necessary sensitivity
- Minimal distortions of the outgoing e/μ trajectories within the target material and small rate of radiative events

➢ Modular structure of 40 independent and precise tracking stations, with split light targets equivalent to 60cm Be

➢ ECAL and Muon filter after the last station, to help the ID and background rejection





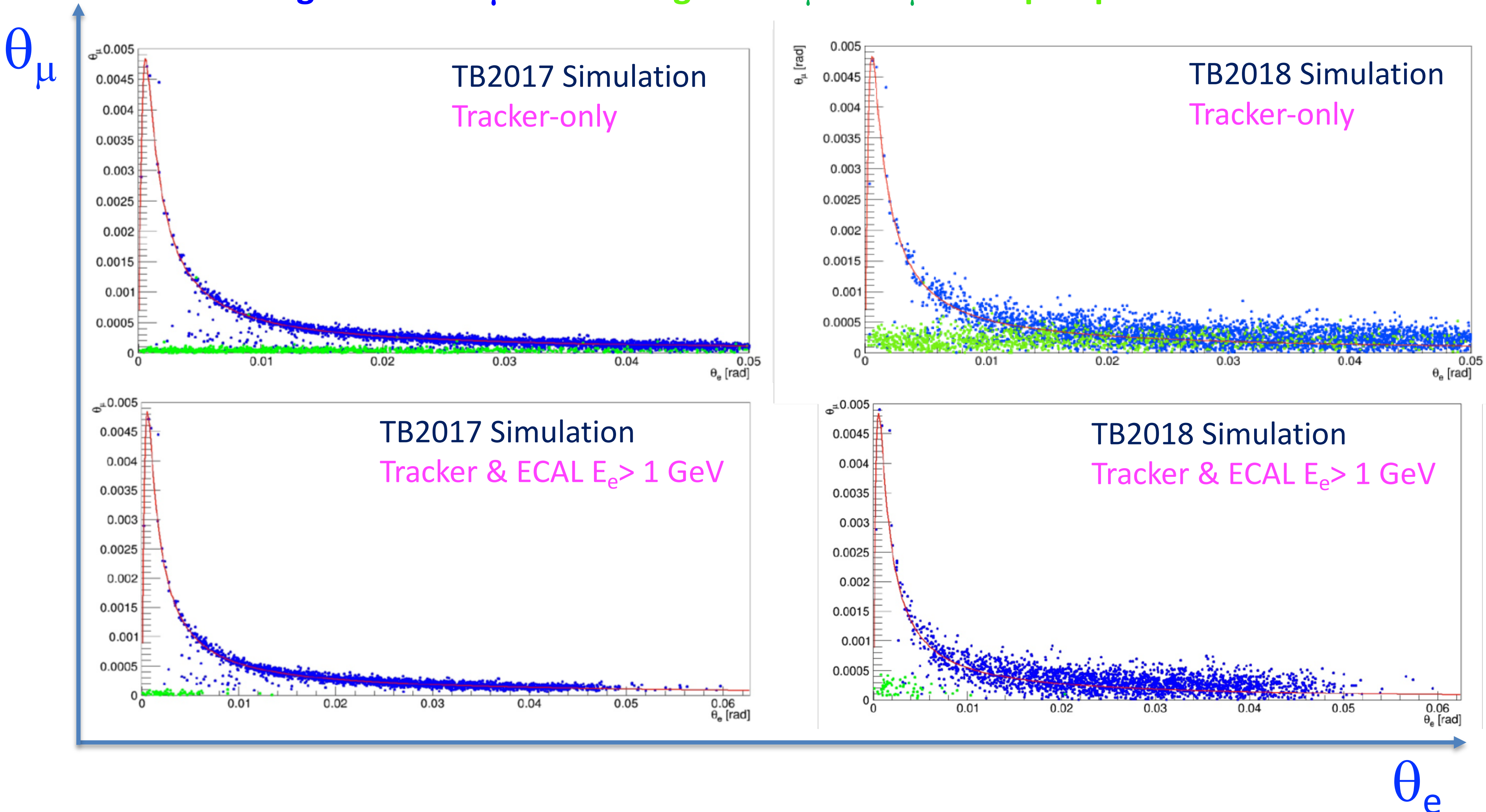
Boosted kinematics: θ_e<32mrad (for E_e>1 GeV), θ_μ<5mrad: the whole acceptance can be covered with a 10x10cm^2 silicon sensor at 1m distance from the target, reducing many systematic errors

GEANT4 simulations

Effect of the position resolution on θ_μ vs θ_e distribution:

(Left) TB2017: UA9 resolution 7μm ; (Right) TB2018: resolution ~35-40μm

Signal: elastic μe **Background: $\mu N \rightarrow \mu N e^+ e^-$ pair production**



GEANT4: e⁺e⁻ pair production

[GEANT4 Physics Reference Manual](#)

R.P. Kokoulin and A.A. Petrukhin, 1970-71
S.R. Kelner, 1998

$\mu N \rightarrow \mu N e^+ e^-$ $\quad Z^2 \quad\quad Z \quad$ $\mu e^- \rightarrow \mu e^- e^+ e^-$

$\zeta \simeq 1$

E = muon beam energy
ε = muon energy loss
v = ε/E

$$\sigma(Z, A, E, \epsilon) = \frac{4}{3\pi} \frac{Z(Z+\zeta)}{A} N_A (\alpha r_0)^2 \frac{1-v}{\epsilon} \int_0^{\rho_{\max}} G(Z, E, v, \rho)\, d\rho$$

$$G(Z, E, v, \rho) = \Phi_e + (m/\mu)^2 \Phi_\mu \quad \Phi_{e,\mu} = B_{e,\mu} L'_{e,\mu} \qquad \Phi_{e,\mu} = 0 \quad \text{whenever} \quad \Phi_{e,\mu} < 0$$

$$B_e = [(2+\rho^2)(1+\beta) + \xi(3+\rho^2)] \ln\left(1 + \frac{1}{\xi}\right) + \frac{1-\rho^2-\beta}{1+\xi} - (3+\rho^2);$$

$$\approx \frac{1}{2\xi}[(3-\rho^2) + 2\beta(1+\rho^2)] \quad \text{for} \quad \xi \geq 10^3;$$

$$B_\mu = \left[(1+\rho^2)\left(1+\frac{3\beta}{2}\right) - \frac{1}{\xi}(1+2\beta)(1-\rho^2)\right] \ln(1+\xi) + \frac{\xi(1-\rho^2-\beta)}{1+\xi} + (1+2\beta)(1-\rho^2);$$

$$B_\mu \approx \frac{\xi}{2}[(5-\rho^2) + \beta(3+\rho^2)] \quad \text{for} \quad \xi \leq 10^{-3};$$

$$\xi = \frac{\mu^2 v^2}{4m^2} \frac{(1-\rho^2)}{(1-v)}; \quad \beta = \frac{v^2}{2(1-v)};$$

$$L'_e = \ln \frac{A^* Z^{-1/3} \sqrt{(1+\xi)(1+Y_e)}}{1 + \frac{2m\sqrt{e} A^* Z^{-1/3}(1+\xi)(1+Y_e)}{Ev(1-\rho^2)}} - \frac{1}{2} \ln\left[1 + \left(\frac{3mZ^{1/3}}{2\mu}\right)^2 (1+\xi)(1+Y_e)\right];$$

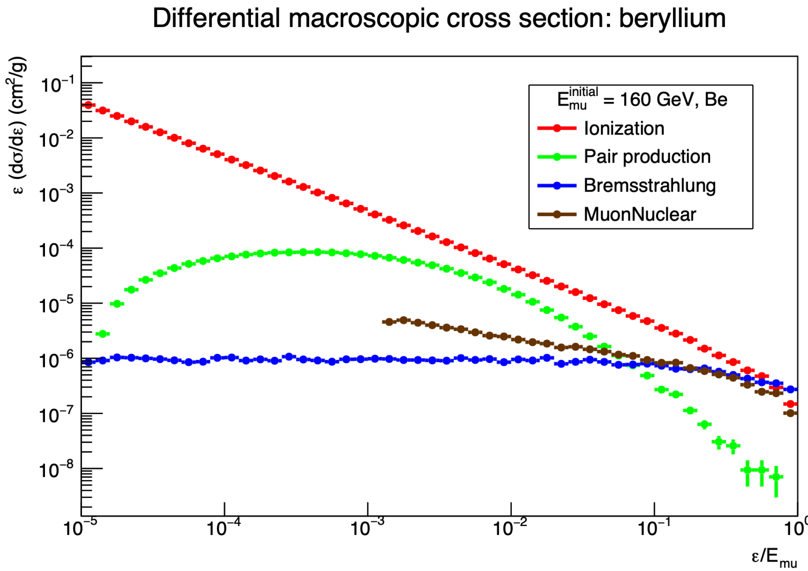
$$L'_\mu = \ln \frac{(\mu/m) A^* Z^{-1/3} \sqrt{(1+1/\xi)(1+Y_\mu)}}{1 + \frac{2m\sqrt{e} A^* Z^{-1/3}(1+\xi)(1+Y_\mu)}{Ev(1-\rho^2)}} - \ln\left[\frac{3}{2} Z^{1/3} \sqrt{(1+1/\xi)(1+Y_\mu)}\right].$$

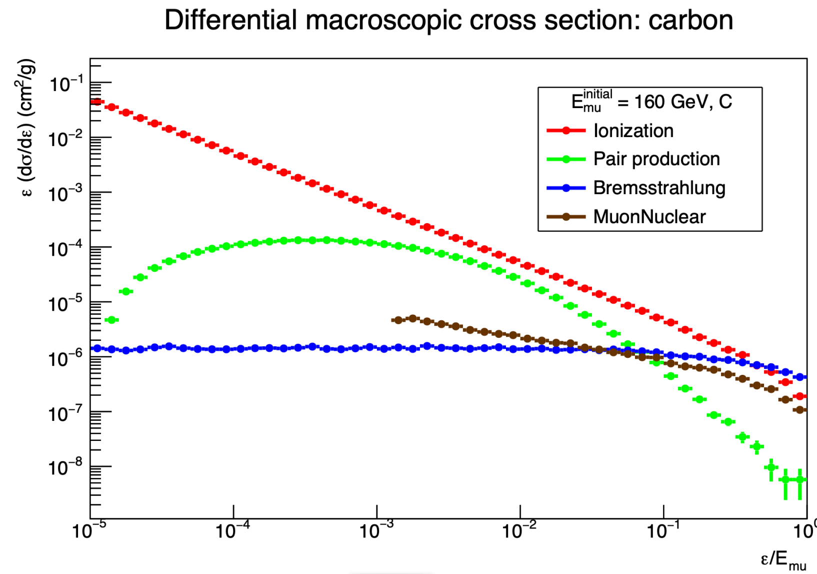
$$Y_e = \frac{5 - \rho^2 + 4\beta(1+\rho^2)}{2(1+3\beta)\ln(3+1/\xi) - \rho^2 - 2\beta(2-\rho^2)};$$

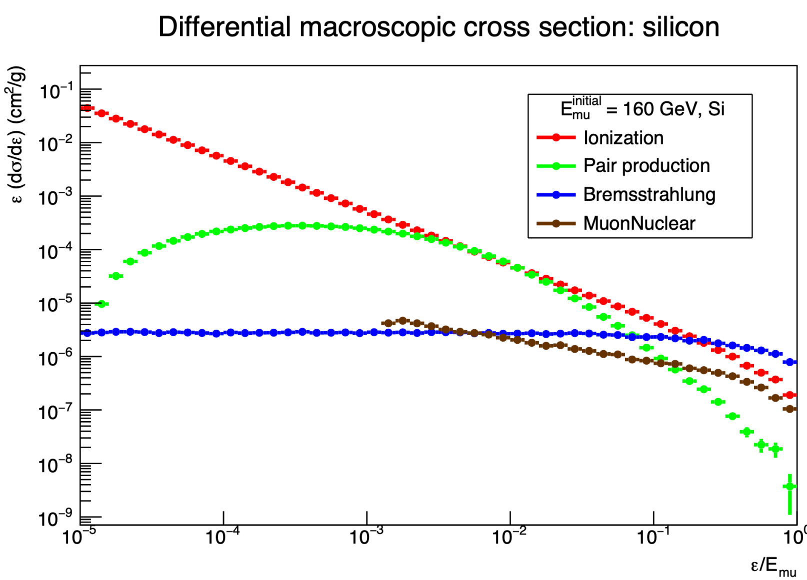
$$Y_\mu = \frac{4 + \rho^2 + 3\beta(1+\rho^2)}{(1+\rho^2)(\frac{3}{2}+2\beta)\ln(3+\xi) + 1 - \frac{3}{2}\rho^2};$$

$$\rho_{\max} = [1 - 6\mu^2/E^2(1-v)]\sqrt{1 - 4m/Ev}.$$

GEANT4: μ interaction cross sections







GEANT4 simulation

ε Muon Energy loss fraction

σ Macroscopic cross section

$\sigma = \sigma_A \, n_A/\rho_A$

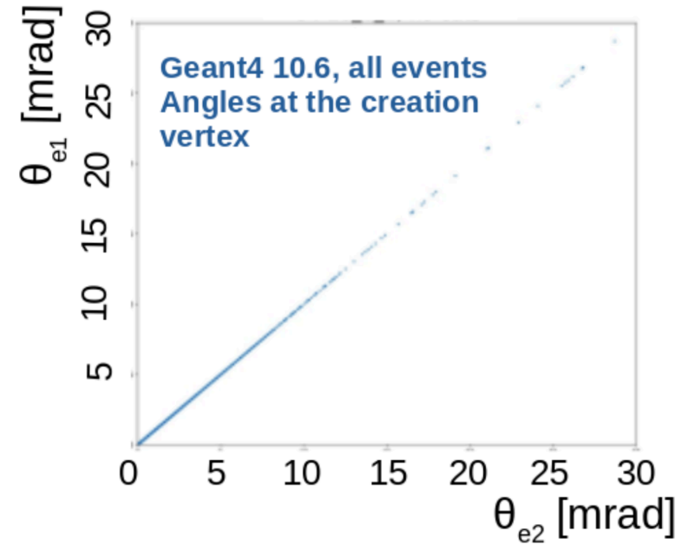
σ_A Atomic cross section

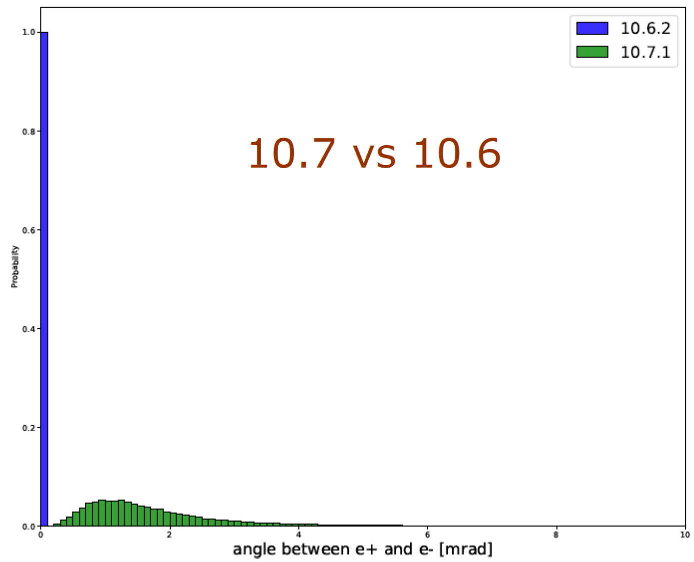
n_A density of atoms per unit volume

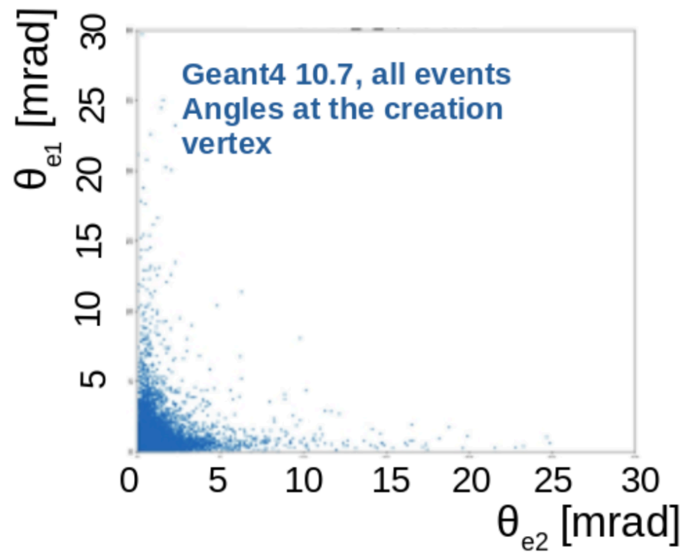
ρ_A material density in g/cm^3

Improving GEANT4

- The simulation of $\mu N \rightarrow \mu N e^+ e^-$ (angular distribution of e^+, e^-) has been made more realistic in v.10.7
- Still it is an approximation that could be further improved
  - The Bethe-Heitler process $\gamma N \rightarrow N' e^+ e^-$ is already described exactly in GEANT4
  - The vertex $\mu \rightarrow \mu \gamma^*$ could be described by the Weiszacker-Williams approximation







EVENT SELECTION: ANGULAR VARIABLES

Angular variables

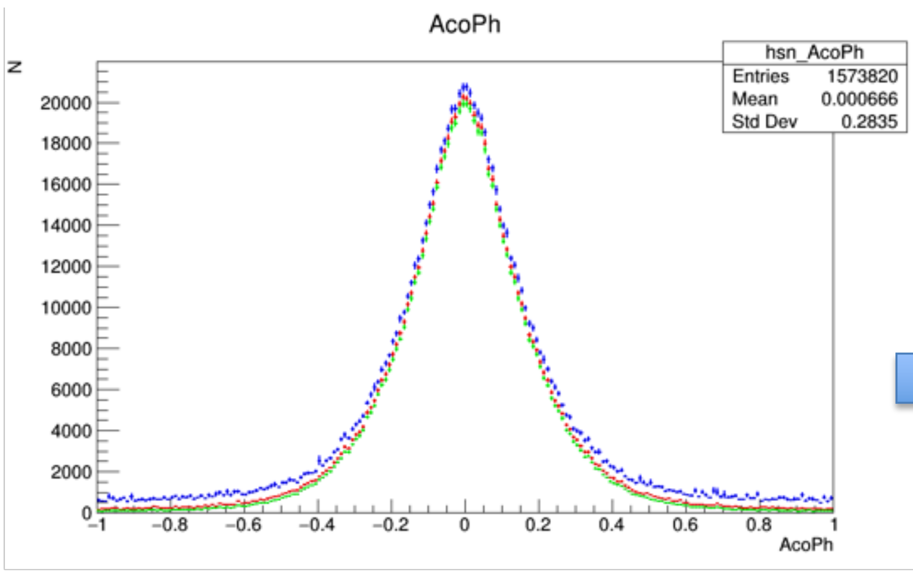
- Study using NLO MC
- Fast detector simulation, example close to the TB2018:
    - intrinsic resolution 80μrad
    - multiple scattering from a target of ~1cm C
    - NOTE: the design resolution of MUonE should be about a factor 4 better, ~ 20μrad
- The angular variables are sensitive to both NLO radiative effects and to the detector angular smearing
    - They are more useful with good detector resolution
- A calorimeter cut on the electron energy can show the effect of the suppression of both radiative and detector smearing effects
    - Tested E > 1, 2 GeV

Acoplanarity

Let $\mathbf{i}, \mathbf{m}, \mathbf{e}$ be unit vectors respectively along the directions of the incoming muon, the outgoing muon and the outgoing electron.

Angle between the scattering planes formed by the outgoing particles with the incoming muon

$$A_\Phi = \pm\left[\pi - \cos^{-1}\left(\frac{(\mathbf{i}\times\mathbf{m})\cdot(\mathbf{i}\times\mathbf{e})}{|\mathbf{i}\times\mathbf{m}|\,|\mathbf{i}\times\mathbf{e}|}\right)\right] \text{ for } \begin{bmatrix} T > 0 \\ T < 0 \end{bmatrix}$$



Blue: All Events θ_e < 32 mrad
Red: Electron E>1 GeV
Green: Electron E> 2 GeV

RMS of the distribution is ~200 mrad even after a cut E_e > 1-2 GeV

Elasticity

- The angular distance of a given event defined by the two angles $P=(\theta_e,\theta_\mu)$ from the nearest point C on the curve corresponding to the elastic scattering (at a given c.m.s. energy) can be taken as a measurement of the elasticity of the event
  - Calculable as a numerical minimisation
  - ideally for perfect elasticity D=0

- An approximation is the distance D_θ of P from the tangent line at $T=(\theta_e, C(\theta_e))$ where $\theta_\mu^C = C(\theta_e)$ is the equation of the elastic curve
  - Calculable analytically as: $D_\theta = \dfrac{\theta_\mu - C(\theta_e)}{\sqrt{1+\left(\frac{dC}{d\theta_e}\right)^2}}$
  - This is biased, particularly in the signal region. Defining the bias δ as:

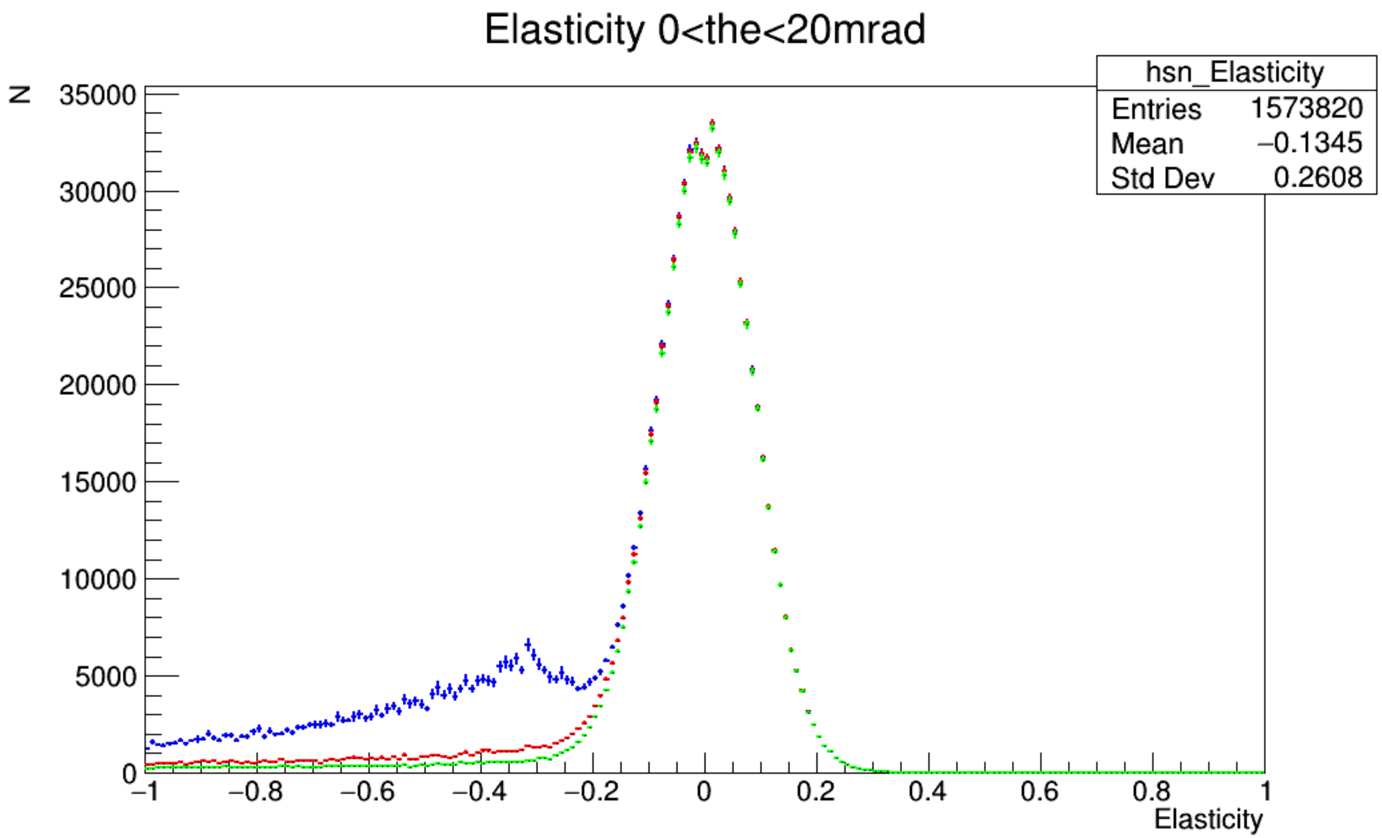
    $\delta = |D_\theta| - |D|$,

    it is positive or negative according to the position of P

    (above or below the curve) and the sign of the second derivative of the curve $c(\theta_e)$

Elasticity - inclusive

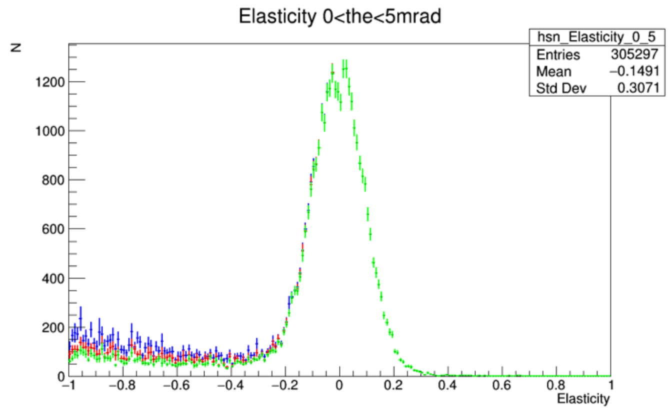
Blue: All Events
Red: Electron E>1 GeV
Green: Electron E> 2 GeV

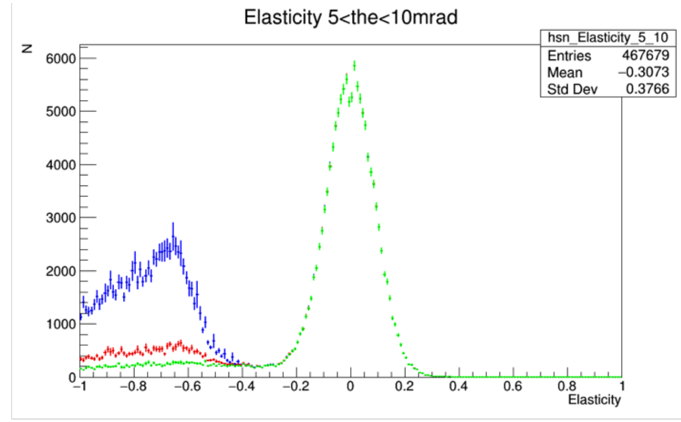


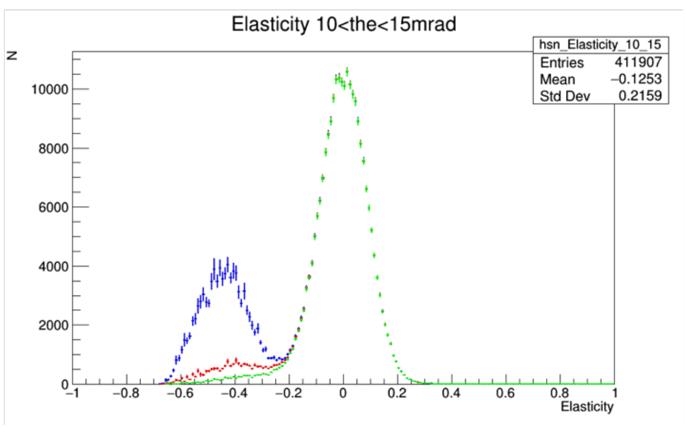
The left tail from radiative and detector smearing effects is effectively removed by energy cuts

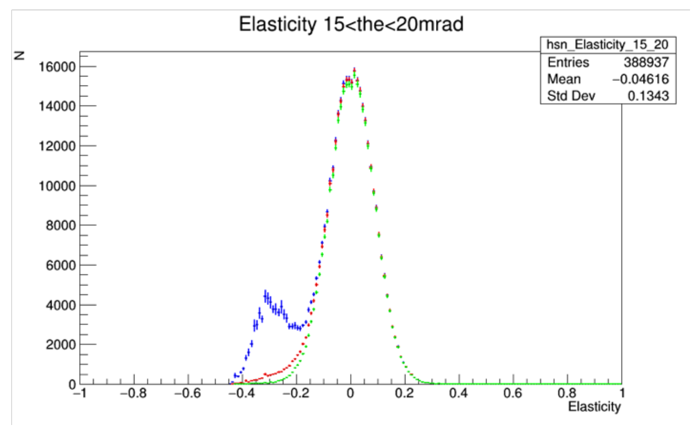
Elasticity - angular regions

Blue: All Events
Red: Electron E>1 GeV
Green: Electron E> 2 GeV









The left tail appears differently in different angular regions but is always suppressed effectively by energy cuts

EVENT SELECTION: USE OF CALORIMETER

Event Selection: use of ECAL and Muon detector

- In principle the analysis of μe elastic scattering events does not need the identification of the outgoing tracks
- However μ-e ID will be very useful to study systematics and determining detector performance
- ECAL measurement of electrons will be possible only for high-energy (low angle) electrons from events occurring at any station (although with reduced resolution for initial stations in the array)
- Instead muon identification will be possible with good performance for all interesting events from any station
- Nevertheless the last tracking station will be close to the ECAL, allowing to identify both μ and *e* in all events produced in the last station
  - It is important to study alternative event selections using the ECAL measurement of the electron energy which will be applicable at least to the last station.

Fast Simulation of ECAL

Master thesis of Eugenia Spedicato (Bologna, 2021): https://amslaurea.unibo.it/23207/

- Development of a fast simulation of the TestRun geometry (Tracking stations and ECAL) including the beam profile.
- Calorimeter response parametrised with the GFLASH model as used in CMS for the ECAL FastSim.
- NLO MESMER MC used to simulate events with real photon radiation

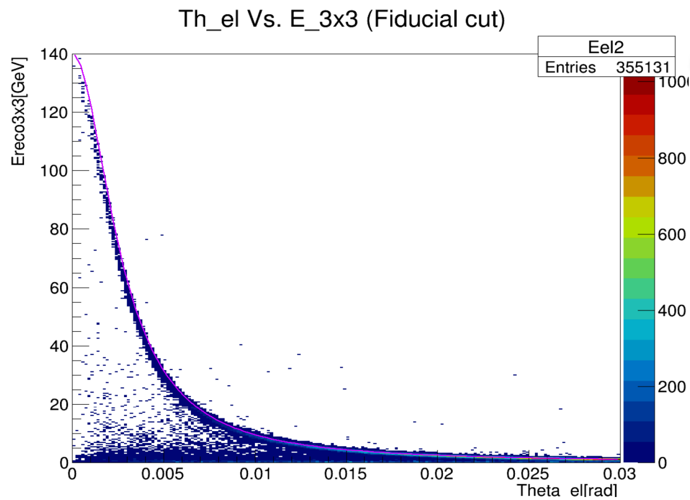
Definition of a clean calorimetric selection of Elastic events using:

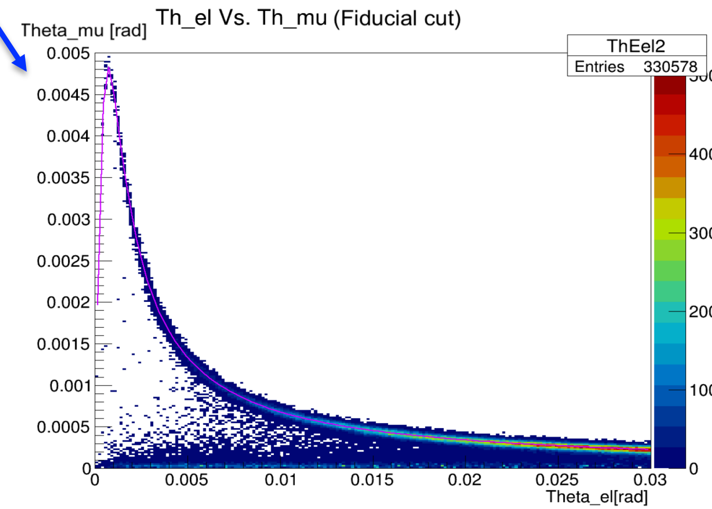
- ECAL Cluster energy
- $\Delta E(\theta_e)$ Difference w.r.t. expected energy for the given electron angle
- Distance of ECAL cluster centroid from the extrapolated track

Fast Simulation of ECAL (2)

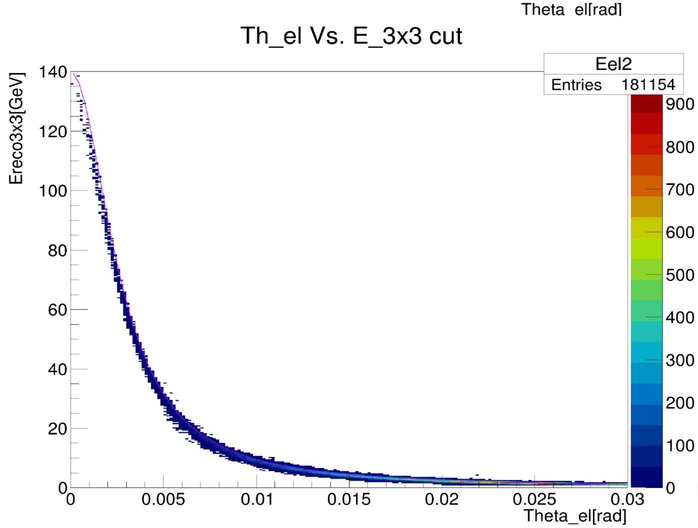
Before selection, only fiducial cuts

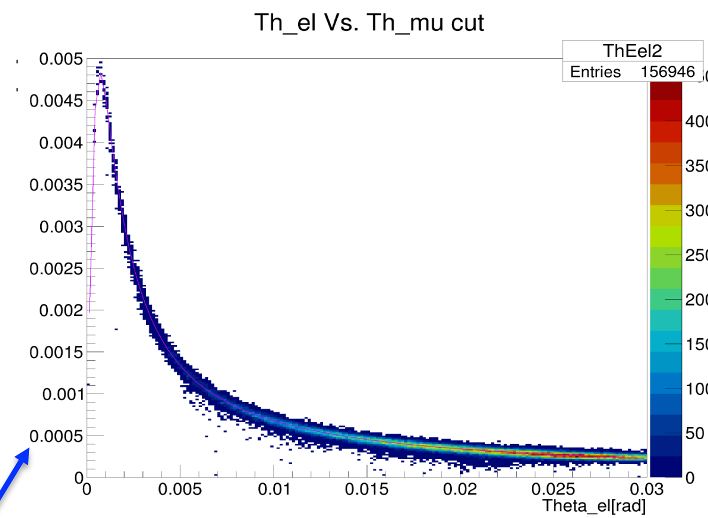
E_{3x3}





θ_μ



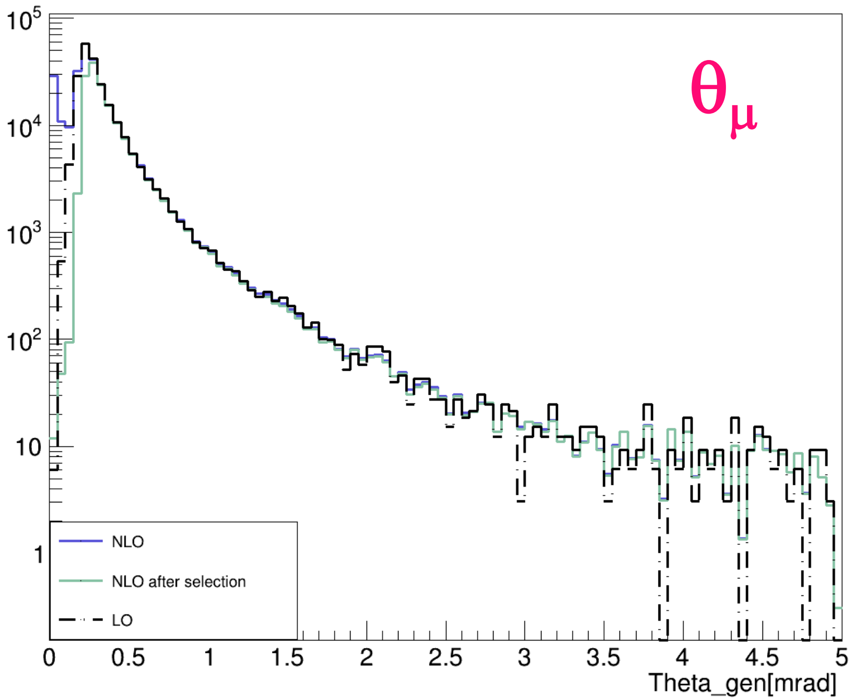


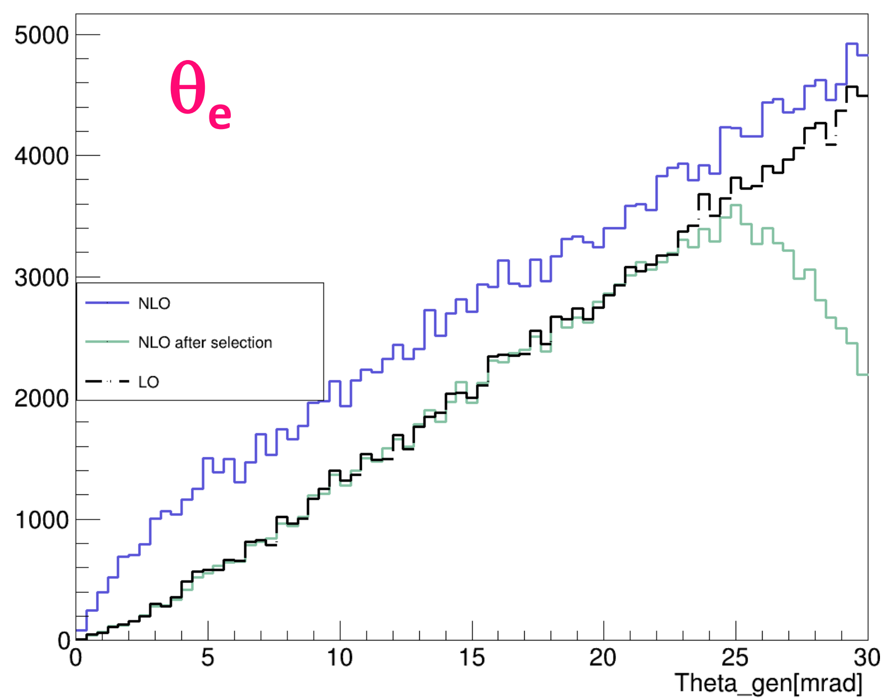
θ_e

After full calorimetric selection

θ_e

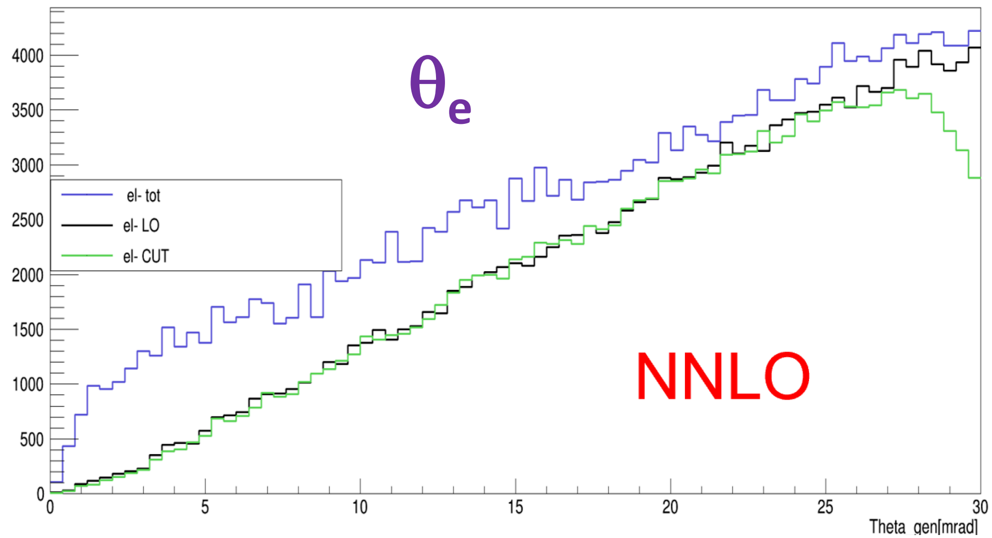
Fast Simulation of ECAL (3)

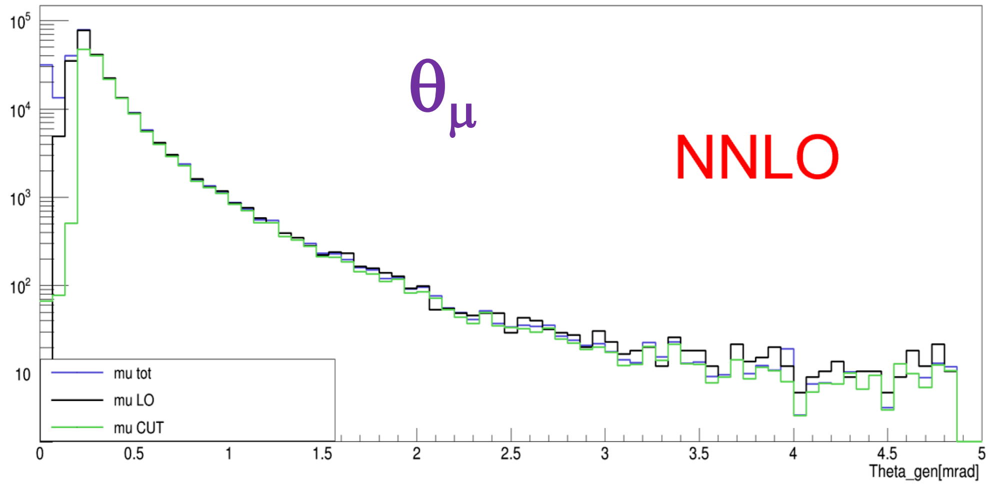




- Calorimetric selection is able to isolate a clean elastic sample:
- Selected NLO angular distributions are close to the LO
- Results confirming previous observations:
  - Electron distribution is substantially affected by radiative events
  - Muon distribution is robust

Fast Simulation of ECAL with NNLO





ECAL-base selection also tested with new NNLO MESMER code Sara Cesare, master thesis (Padova, 2022) http://hdl.handle.net/20.500.12608/34647

SIGNAL FIT STRATEGY

Analysis workflow: FastSim - FullSim

A competitive determination of a_μ^{HVP} requires a precision of $O(10^{-2})$ in the measurement of the hadronic running, which translates into an unprecedented precision of $O(10^{-5})$ in the shape of the differential cross section.

Reaching this accuracy requires a huge statistics of data, in the order of few times 10^{12} events.

Therefore even preliminary simulation studies would present a computational challenge.

The Fast Simulation is an absolute need for MUonE: the final analysis will likely use Fast Simulation, with detector effects parameterized from (smaller and dedicated) Full Sim samples.
Therefore the FastSim development has to proceed in parallel with the Full Simulation.

Given the above, the MC Data Formats representing MUonE MC events should be common to Fast and Full Simulation, such that analyses could be carried out in a similar way, with the same interfaces.

FastSim analysis strategy

- NLO MESMER MC
- $\Delta\alpha_{had}(t)$ from F.Jegerlehner's code(hadr5n12.f)$\rightarrow$ $a_\mu^{HLO} = 688.6 \times 10^{-10}$
- Detector resolution effects parametrized in a simplified way (including only: multiple scattering on 1.5cm Be target and intrinsic resolution σ_θ=0.02 mrad)
  - Neglecting: scattering on the Si planes, non-Gaussian tails, residual backgrounds
  - Neglecting: detailed track simulation and reconstruction
- Fit is done directly on the angular distributions of scattered μ and e
  - No attempt to estimate t (or x) event by event
  - $\theta_e < 32$ mrad (geometric acceptance)
  - $\theta_\mu > 0.2$ mrad (remove most of the background)
  - Both 1D and 2D distributions fitted. 2D is the most robust.
  - Ideally there is no need to identify the outgoing muon and electron, provided the event is a signal one. In this case we simply label the two angles as θ_L, θ_R ("Left" and "Right" w.r.t. an arbitrary axis)
- Shape-only fit: the absolute normalization shall not count.

MuE: FastSim – Analysis code

Public code: https://gitlab.cern.ch/muesli/nlo-mc/mue

- Package have been existing for quite some time, used in:
  - LoI analysis and updates
  - Studies of systematics (G.A., R.Pilato, M.Mantovano)
  - Fast Simulation of the TestRun setup, the Beam Profile and the ECAL (E.Spedicato, S.Cesare)
  - Starting point for the study of the simultaneous fit of signal+nuisances with Combine (R.Pilato)

- Recent updates:
  - use NNLO MESMER MC generator for μe scattering
  - embedded mode for MESMER MC generator, with (Fortran) generator functions called from a C++ steering program.

Hadronic running of α

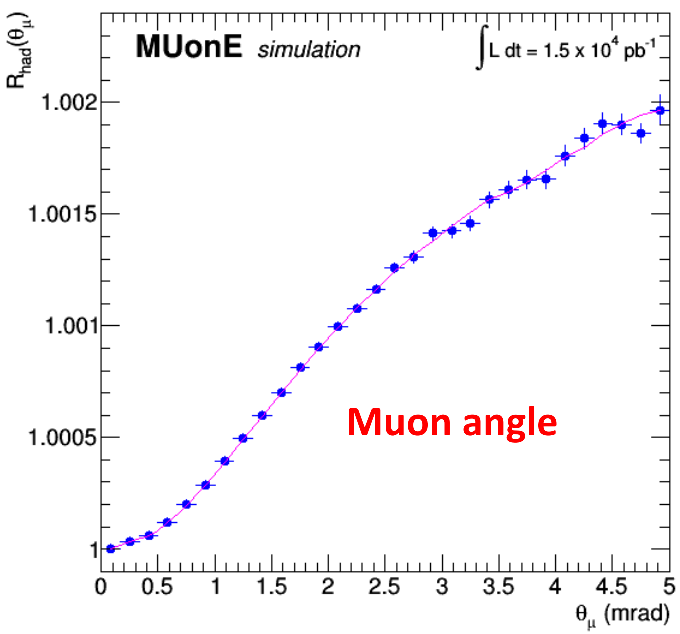
Most easily displayed by taking ratios of the MC predicted angular distributions (pseudodata) and the predictions obtained from the same MC sample reweighting $\alpha(t)$ to correspond to only the leptonic running.

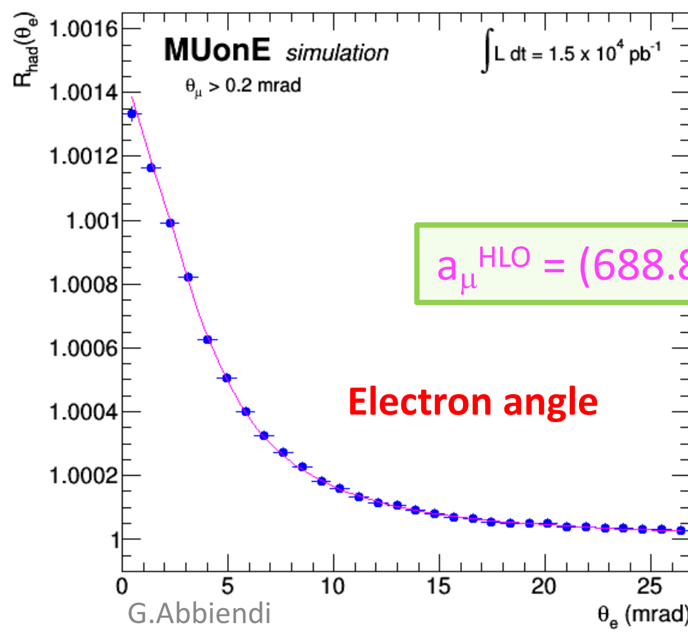
$$R_{had}(\theta) = \frac{d\sigma(\theta, \Delta\alpha_{had})}{d\sigma(\theta, \Delta\alpha_{had}=0)}$$

-- In this way most of the pure MC statistical fluctuations are cancelled.
-- (of course, this trick is applicable only to pseudodata analysis. With real data we will need to match the MC statistics to the data size)

Observable effect ~ 10^{-3} / wanted precision ~10^{-2} → required precision ~10^{-5}

The expected distributions are obtained from the nominal integrated luminosity
L = 1.5 x 10^{7} nb^{-1} (corresponding to 3-year run)





Example toy experiment

a_μ^{HLO} = (688.8 ± 2.4) x 10^{-10}

Stat.err.
0.35%

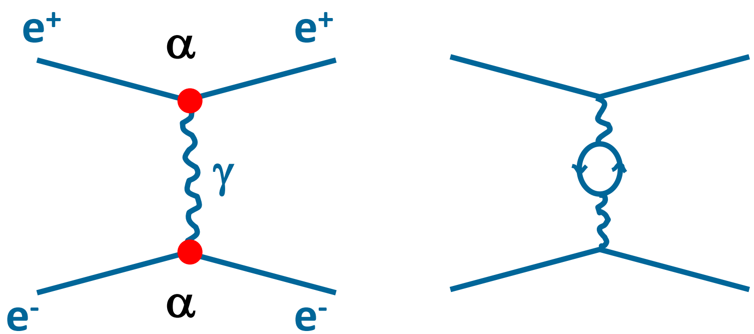
Measurement of $\Delta\alpha_{had}(t)$ spacelike at LEP

OPAL measurement: Bhabha scattering at small angle, with $1.8 < -t < 6.1$ GeV^2

Eur.Phys.J.C45(2006)1

about 10^7 events

precision at the per mille level



$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt}\left[\frac{\alpha(t)}{\alpha_0}\right]^2 (1+\varepsilon)(1+\delta_\gamma)+\delta_Z$$

Born term for t-channel single γ exchange

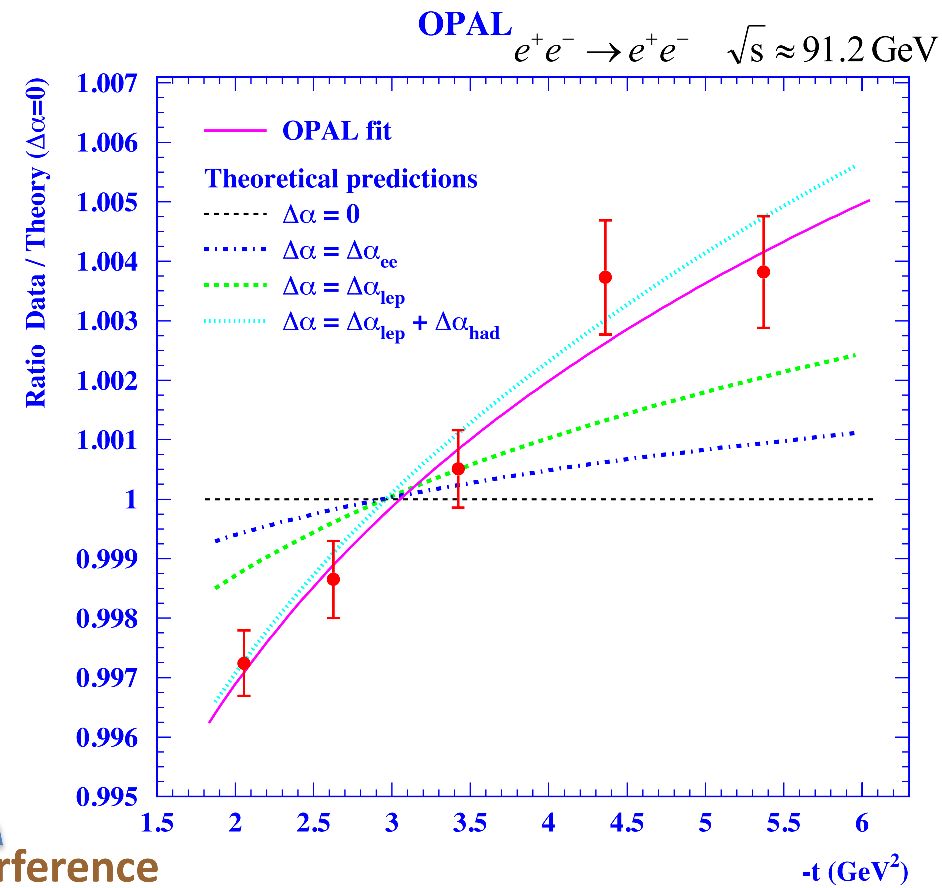
$$\boxed{\left(\frac{1}{1-\Delta\alpha(t)}\right)^2}$$

Effective coupling factorized

Photonic radiative corrections

s-channel γ exchange correction

Z interference correction



Other measurements in the space-like region by L3, VENUS

$\Delta\alpha_{had}$ parameterisation

Physics-inspired from the calculable contribution of lepton-pairs and top quarks at t<0

$q^2 = t < 0$

$$\Delta\alpha_{had}(t) = k\left\{-\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

M with dimension of mass squared, related to the mass of the fermion in the vacuum polarization loop

k depending on the coupling $\alpha(0)$, the electric charge and the colour charge of the fermion

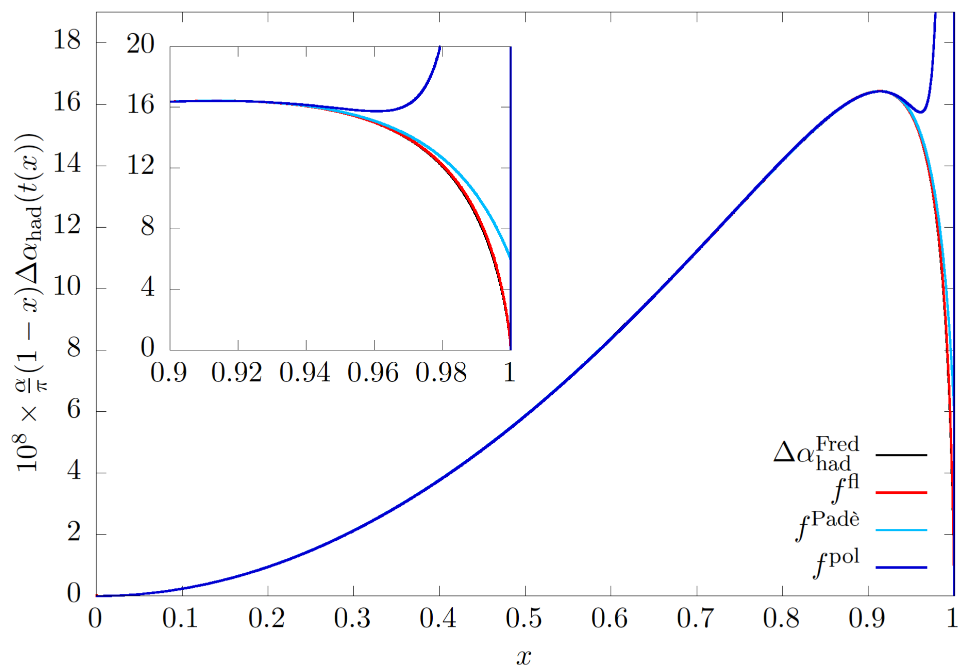
Low-|t| behavior dominant in the MUonE kinematical range:

$$\Delta\alpha_{had}(t) = -\frac{1}{15}\frac{k}{M}t$$

for $t \to -0$

$$\Delta\alpha_{had}(t) = \frac{k}{3}\left(\log\frac{|t|}{M} - \frac{5}{3}\right)$$

for $t \to -\infty$



Template Fit technique

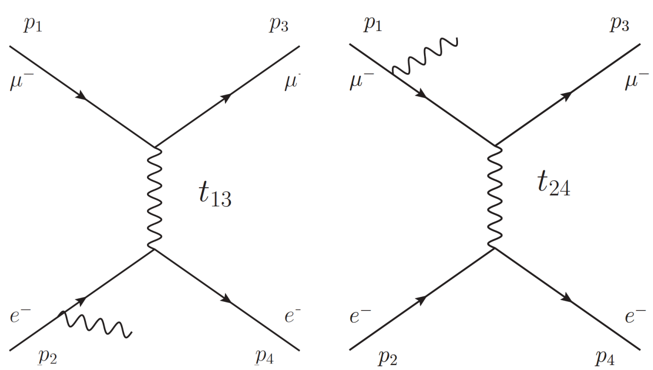
- MC templates for any useful distribution are built by reweighting the events to correspond to a given functional form of $\Delta\alpha_{had}(t)$

- $\Delta\alpha_{had}(t)$ is conveniently parameterised with the "Lepton-Like" form, one-loop QED calculation.

The 2->3 matrix element for one-photon emission at NLO can be split in 3 parts (radiation from mu or e leg and their interference), each one with a different running coupling factor → **3 coefficients**

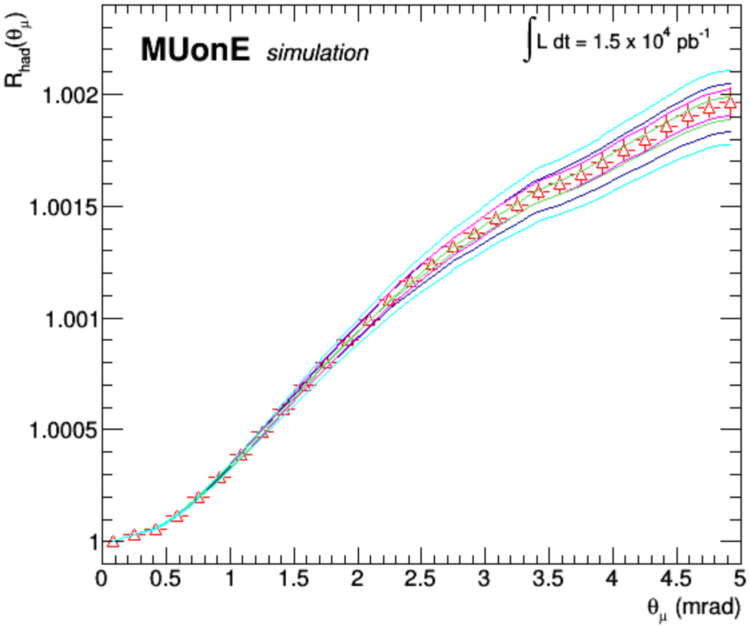
NOTE: at NNLO one needs **11 coefficients**

By saving the relevant coefficients at generation time we can easily reweight the events according to the chosen parameters in the $\Delta\alpha_{had}(t)$



Template fit

- Define a grid of points (K,M) in the parameter space covering a region of ±5σ around the expected values (with σ the expected uncertainty). Step size taken to be 0.5σ. This defines 21x21 = 441 templates for the relevant distributions.



- For every template in the grid calculate the χ^2 obtained with the pseudodata distribution:

$$\chi^2(K,M) = \sum_i^{bins} \frac{R_i^{data} - R_i^{(K,M)}}{\sigma_i^{data}}$$

  – Neglect the statistical errors of the templates as in the ratios they are vanishingly small.

- Minimise the χ^2 interpolating across the grid by parabolic approximation. Final errors correspond to $\Delta\chi^2=1$.

Determination of a_μ^{HLO} by the Master Integral

- From the fitted (K,M) values the hadronic contribution to $\Delta\alpha_{had}(t)$ is determined from the Lepton-Like parameterisation:

$$\Delta\alpha_{had}(t) = k\left\{-\frac{5}{9}-\frac{4M}{3t}+\left(\frac{4M^2}{3t^2}+\frac{M}{3t}-\frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

- Then, by using the master integral, we have the result in the full phase space:

$$\boxed{a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]}$$

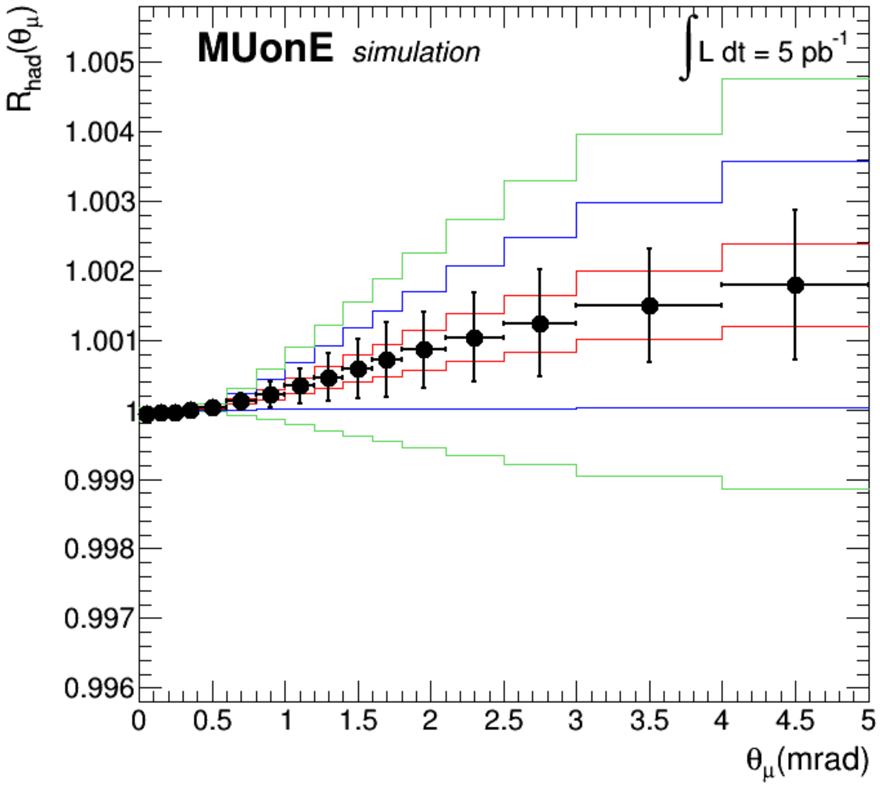
- The result for the nominal luminosity is a_μ^{HLO} = (688.8 ± 2.4) x 10^{-10}
  - statistical uncertainty of 0.35%

- The expectation from the used Jegerlehner's parameterization is: a_μ^{HLO} = 688.6 x 10^{-10}
  - difference from our fit is 0.2 x 10^{-10}, negligible w.r.t. the statistical uncertainty

Expected sensitivity of a First Physics Run

Expected integrated Luminosity with the Test Run setup with full beam intensity & detector efficiency ~ $1 pb^{-1}$/day

In one week ~$5pb^{-1}$ → ~10^9 μe scattering events with $E_e > 1$ GeV ($\theta_e < 30$ mrad)



Initial sensitivity to the hadronic running of α.

Pure statistical level: 5.2σ
2D (θ_μ,θ_e) K=0.136 ± 0.026

Definitely we will have sensitivity to the leptonic running (ten times larger)

Template fit with just one fit parameter K= k/M in the $\Delta\alpha_{had}$ parameterization.
The other parameter fixed at its expected value: M = 0.0525 GeV^2

SYSTEMATIC ERRORS

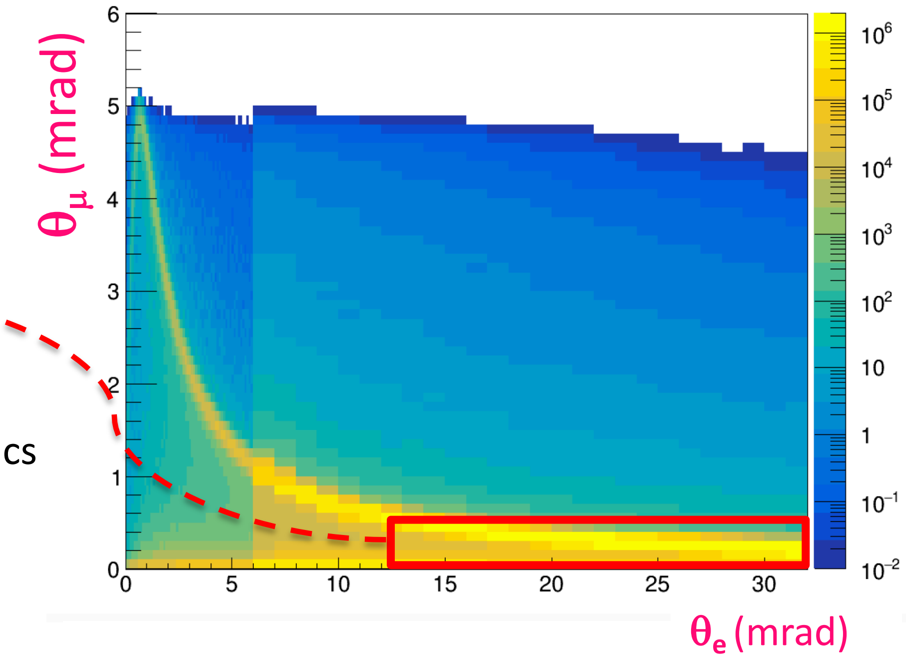
Event kinematics

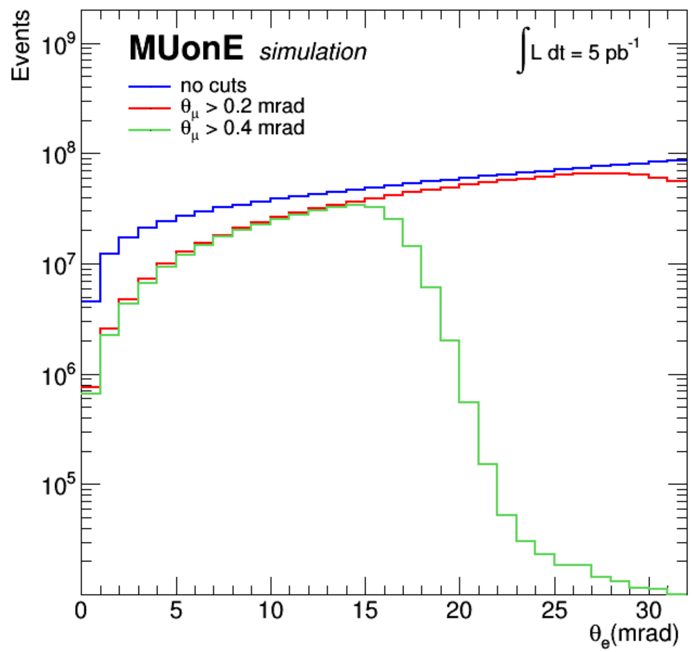
Normalisation region

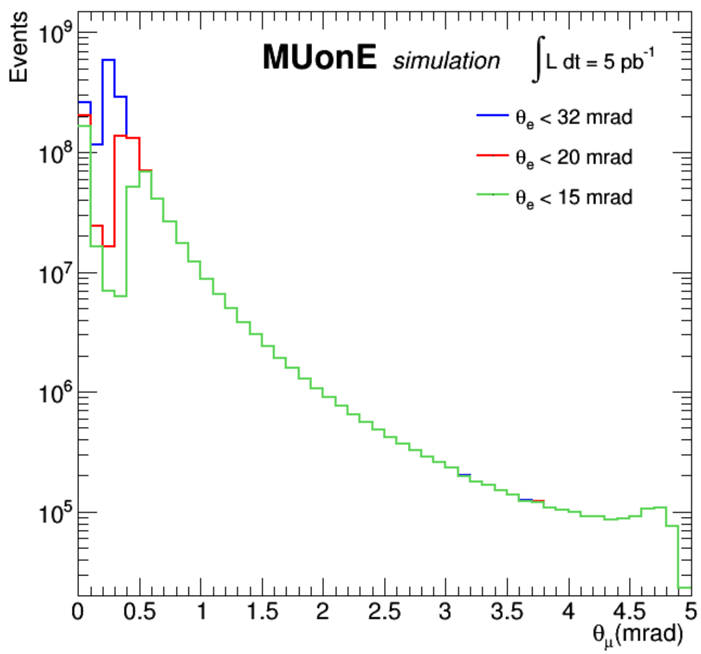
huge statistics

vanishing signal ($\Delta\alpha_{had}$)

convenient for studies of systematics







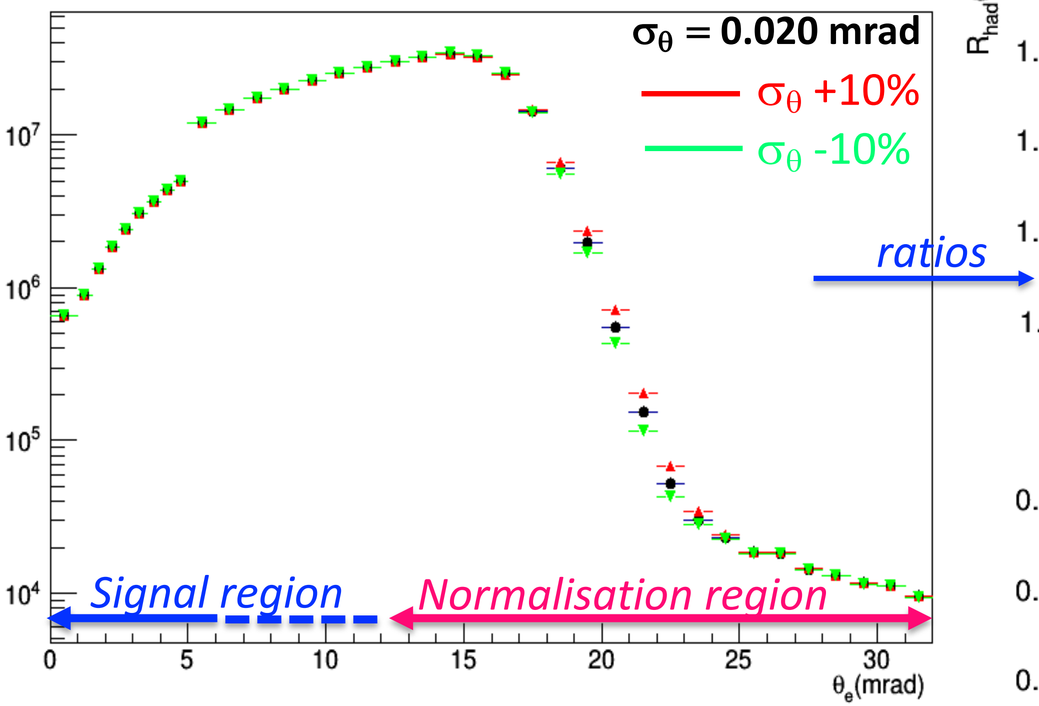
Probing systematics in the normalisation region

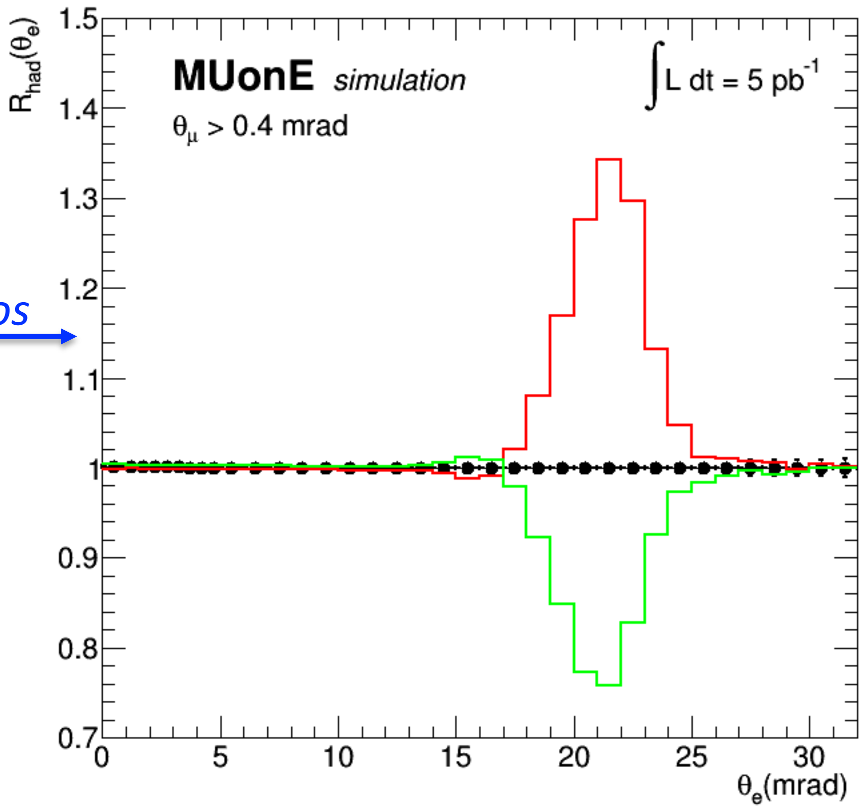
The **intrinsic angular resolution** can be probed by looking at the θ_e distribution after a cut on θ_μ distribution, e.g. cutting at $\theta_\mu > 0.4$ mrad

→ Effect of a ±10% error w.r.t. the nominal σ_θ = **0.020 mrad**

Huge distortion of 20-30% around electron angles of 20 mrad

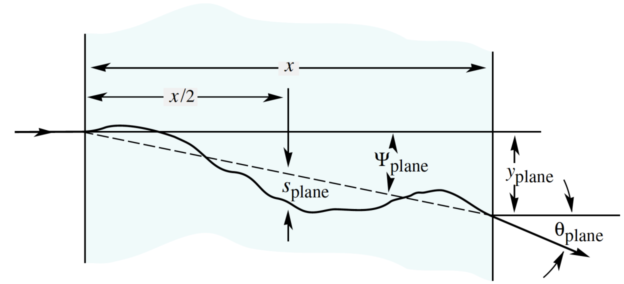
No effect in the signal region





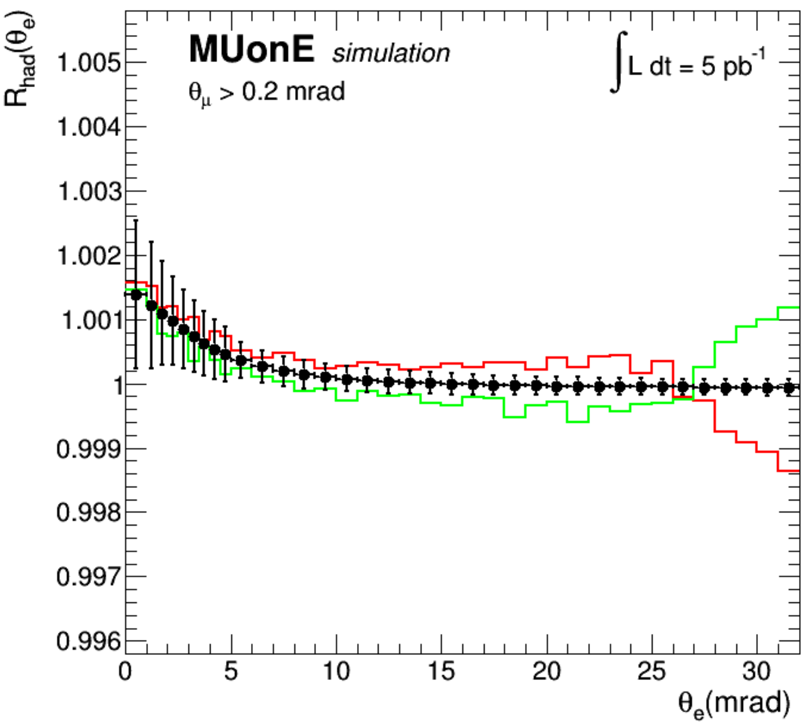
Systematics: Multiple Coulomb Scattering

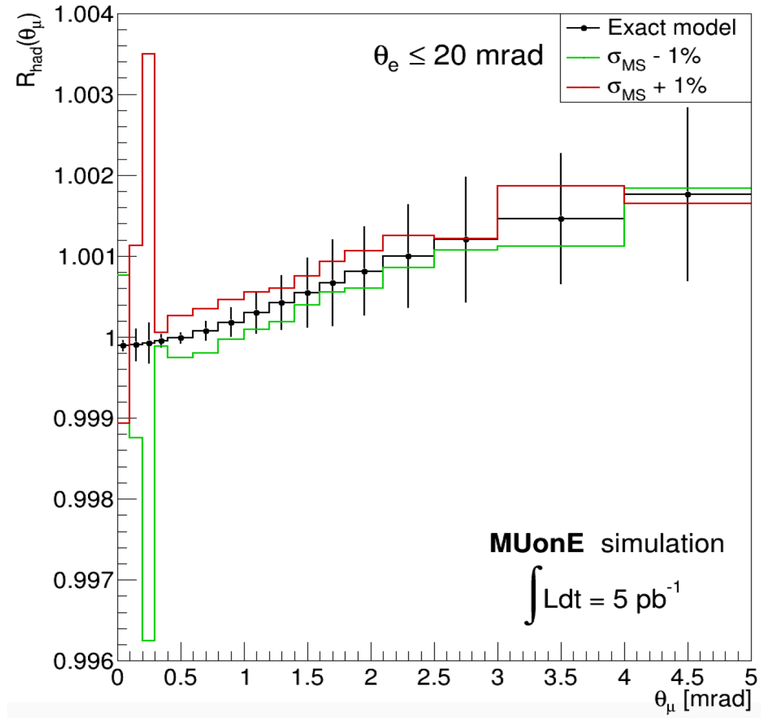
Effect of a flat error of ±1% on the core width of multiple scattering



+1%

-1%





Multiple scattering previously studied in a Beam Test in 2017: [JINST 15 (2020) P01017]
with 12–20 GeV electrons on 8-20 mm C targets

Systematics: Beam Energy scale

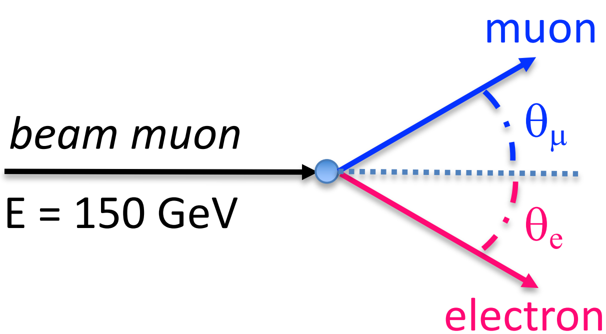
Time dependency of the beam energy profile has to be continuously monitored during the run:

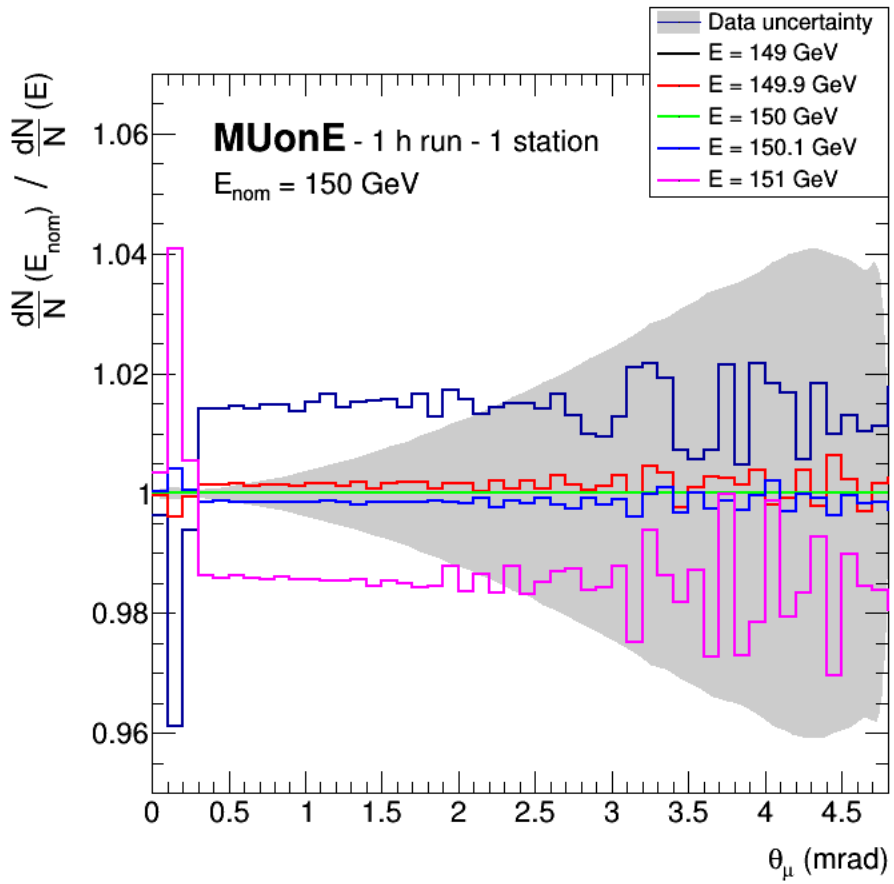
- SPS monitor
- COMPASS BMS

needed external infos

Effect of a syst shift of the average beam energy on the θ_μ distribution: 1h run / 1 station

However, the absolute beam energy scale has to be calibrated by a physics process: kinematical method on elastic μe events





For equal angles:

$\theta_\mu = \theta_e \equiv \theta$

$$\theta \simeq \sqrt{\frac{2m_e}{E}}$$

Can reach <3 MeV uncertainty in a single station in less than one week

From SPS E scale ~1% : 1.5 GeV

Combined Fit of Signal and Systematics

→ discussed in Riccardo Pilato's talk

Conclusions

- Study of the event selection has to be continued:
  - Backgrounds (in part. pair-production on material nuclei)
  - Cuts on angular variables must take into account the detector resolution (due to extended target and detector performance)
  - Calorimeter and Muon detector will be useful, probably necessary for studies of systematics
- Analysis strategy based on a template fit using
  - the best MC calculations for the μe scattering
  - a convenient parameterisation of the $\Delta\alpha_{had}(t)$
    - ❖ the so-called lepton-like parameterisation, with 2 free parameters seems optimal
  - Extrapolation to the full phase space of the fitted parameterisation
- Many systematics can be determined in the normalisation region where the signal is vanishing
- Residual systematics can be fitted simultaneously with the signal as nuisance parameters (with the Combine statistical tool)

Recent References

MUonE analysis first described in the LoI
Letter-Of-Intent SPSC-I-252 (June 2019)

Updated description in a recent paper:
Phys. Scripta 97 (2022) 054007 [arXiv:2201.13177]

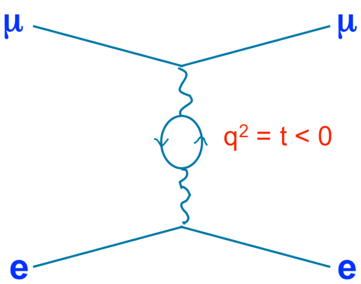
Latest advancements: Riccardo Pilato's PhD thesis

Web pages with links to documents (papers, conferences, theses)

- https://web.infn.it/MUonE/

BACKUP

μ-e elastic scattering



At LO

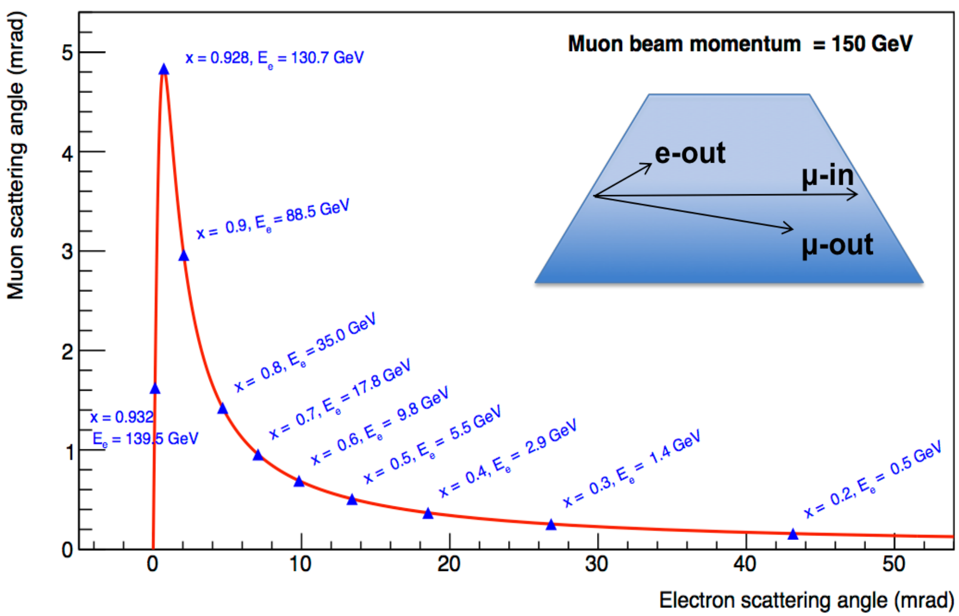
$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_e^2, m_\mu^2)}\left[\frac{(s - m_e^2 - m_\mu^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2}\right]$$

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha(0)}\right|^2 \qquad \alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)} \qquad \Delta\alpha(t) = \Delta\alpha_{lep}(t) + \Delta\alpha_{had}(t)$$

- **Elastic scattering: simple kinematics**
  - $t \cong -2\, m_e E_e$ E_e can be determined from the scattering angle θ_e and the beam energy

- Scattering angles θ_e and θ_μ correlated (helps selection: rejection of radiative/inelastic events)

- Elastic events are planar



Acoplanarity: definitions

There are several possible quantities related to the deviation of a given event from perfect coplanarity. They have different properties and numerically are very different.

Let $\mathbf{i}$, $\mathbf{m}$, $\mathbf{e}$ be unit vectors respectively along the directions of the incoming muon, the outgoing muon and the outgoing electron.

1) Triple product $\mathrm{T} = \mathbf{i} \cdot \mathbf{m} \times \mathbf{e}$

(used by NA7; geometrically the volume of the parallelepiped defined by the three vectors)

2) Angle between the incoming muon and the plane of the outgoing particles $(\mathbf{m}, \mathbf{e})$

$$\mathrm{A} = \frac{\pi}{2} - \cos^{-1}\left(\frac{\mathbf{i} \cdot \mathbf{m} \times \mathbf{e}}{|\mathbf{m} \times \mathbf{e}|}\right)$$

3) Angle between the scattering planes formed by the outgoing particles with the incoming muon

$$\mathrm{A}_{\Phi} = \pm\left[\pi - \cos^{-1}\left(\frac{(\mathbf{i} \times \mathbf{m}) \cdot (\mathbf{i} \times \mathbf{e})}{|\mathbf{i} \times \mathbf{m}|\,|\mathbf{i} \times \mathbf{e}|}\right)\right] \text{ for } \begin{bmatrix} \mathrm{T} > 0 \\ \mathrm{T} < 0 \end{bmatrix}$$

Notice: A_{Φ} tests also that the outgoing electron and muon are directed on opposite sides in the transverse plane, while T and A do not depend on this.

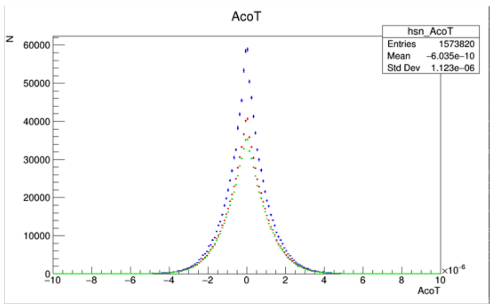
➢ this can provide significantly different power in suppressing the backgrounds, in particular also the pair production

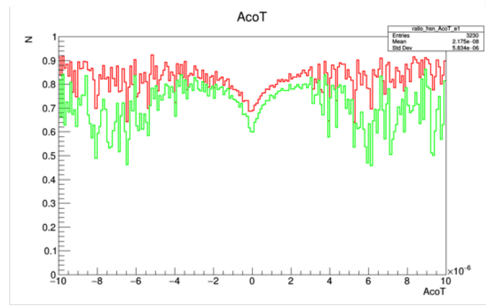
Acoplanarity variants

Blue: All Events / Red: Electron E>1 GeV / Green: Electron E> 2 GeV

Ratios: **E>X / All**

T

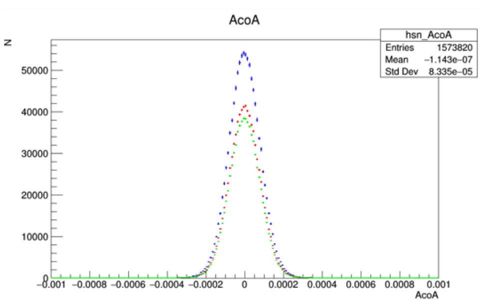


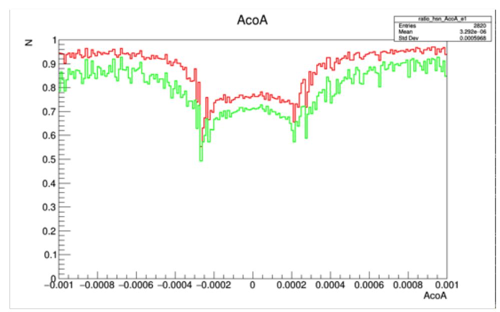


BAD!

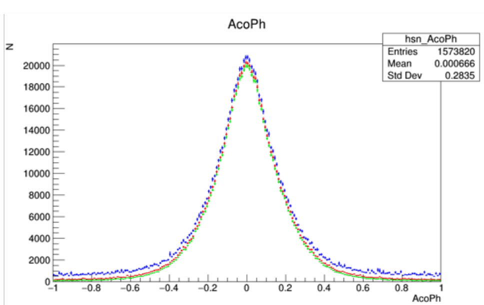
Energy cuts give unwanted pathological effects on T and A

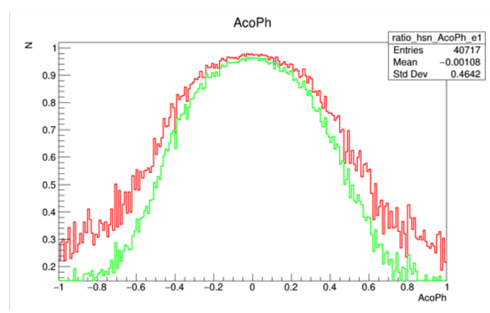
A





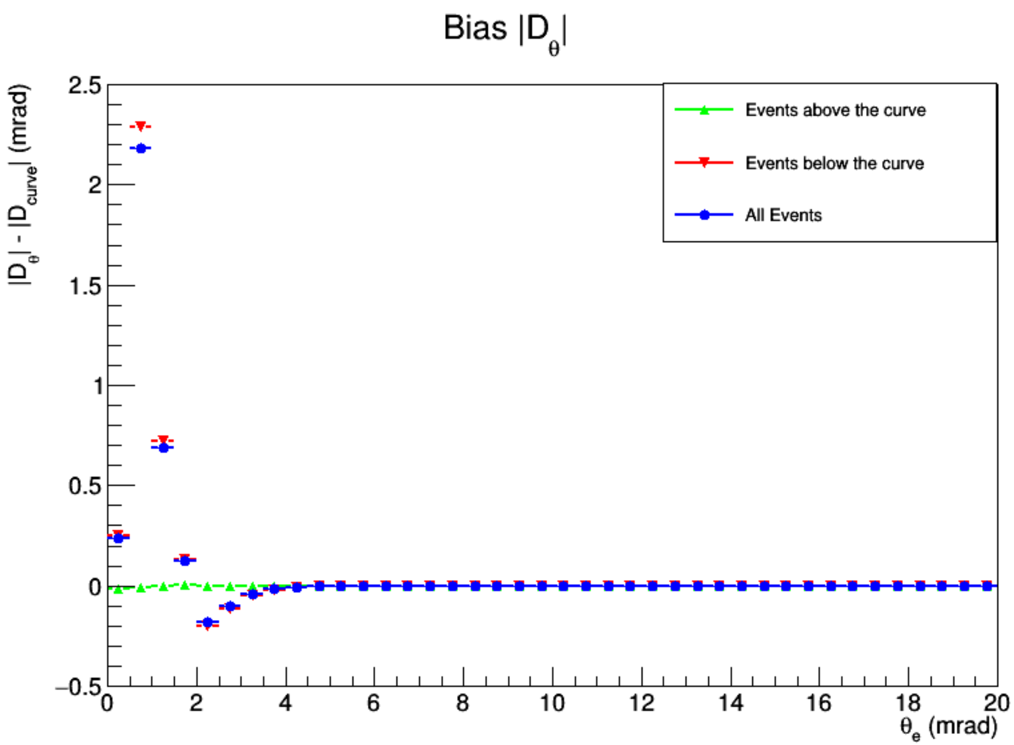
A_{Φ}





Good behaviour

Elasticity Bias of approximated D_θ



Bias of the approximated D_θ is very small for $\theta > 5$ mrad, but becomes large in the signal region. The observed sign depends on the second derivative of the curve