MUonE simultaneous fit of $\Delta\alpha_{had}$ and systematic effects

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The Evaluation of the Leading Hadronic Contribution to the Muon g-2: Toward the MUonE Experiment
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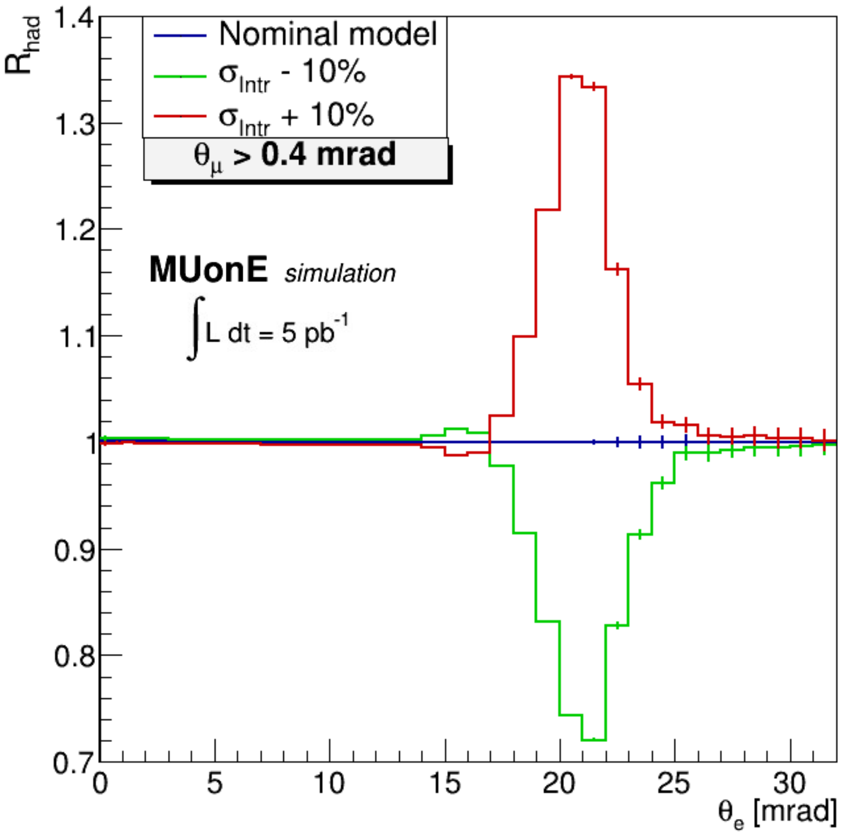
Framework used for the analysis

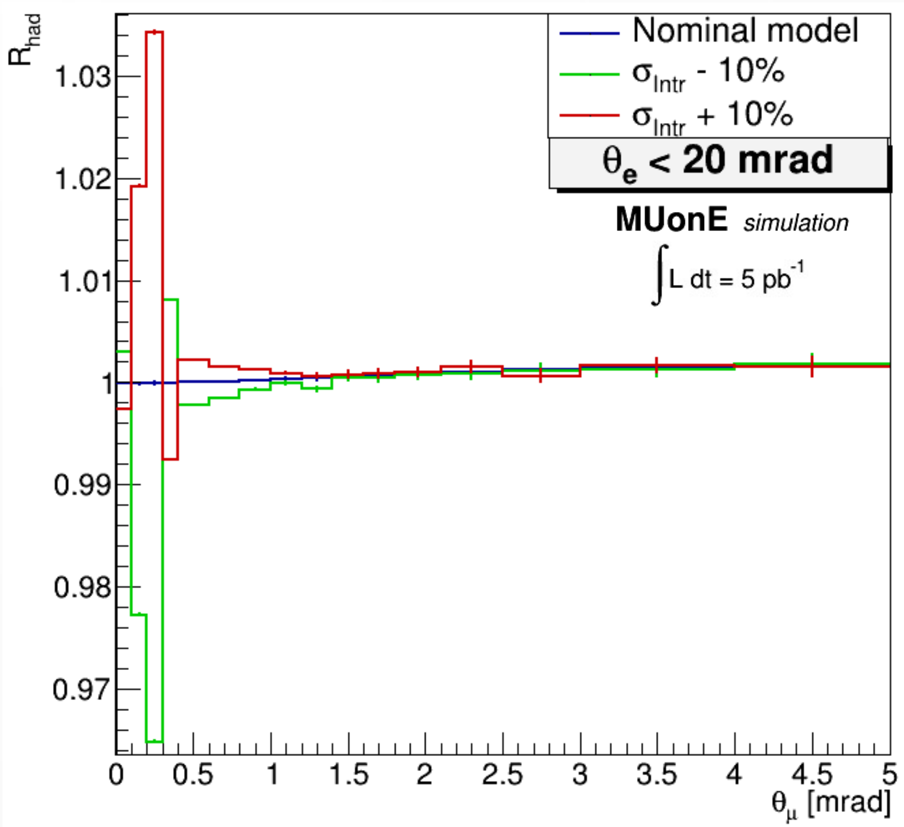
- NLO MonteCarlo generator: MESMER
  - Allows to change the muon beam energy and simulate the beam energy spread.
- C++ fast simulation to include detector effects:
  - Multiple scattering effects in the target.
  - Angular intrinsic resolution.
  - Effects applied to (θ_e, θ_μ) taken from the NLO generator: track reconstruction effects are currently neglected.
  - Further effects to be included: MS non-Gaussian tails, background effects, MS in the silicon sensors.

The need of including systematic effects in the analysis

Some systematic effects can produce huge distortions in the shape of the elastic scattering cross section.

Example: ±10% error on the angular intrinsic resolution.

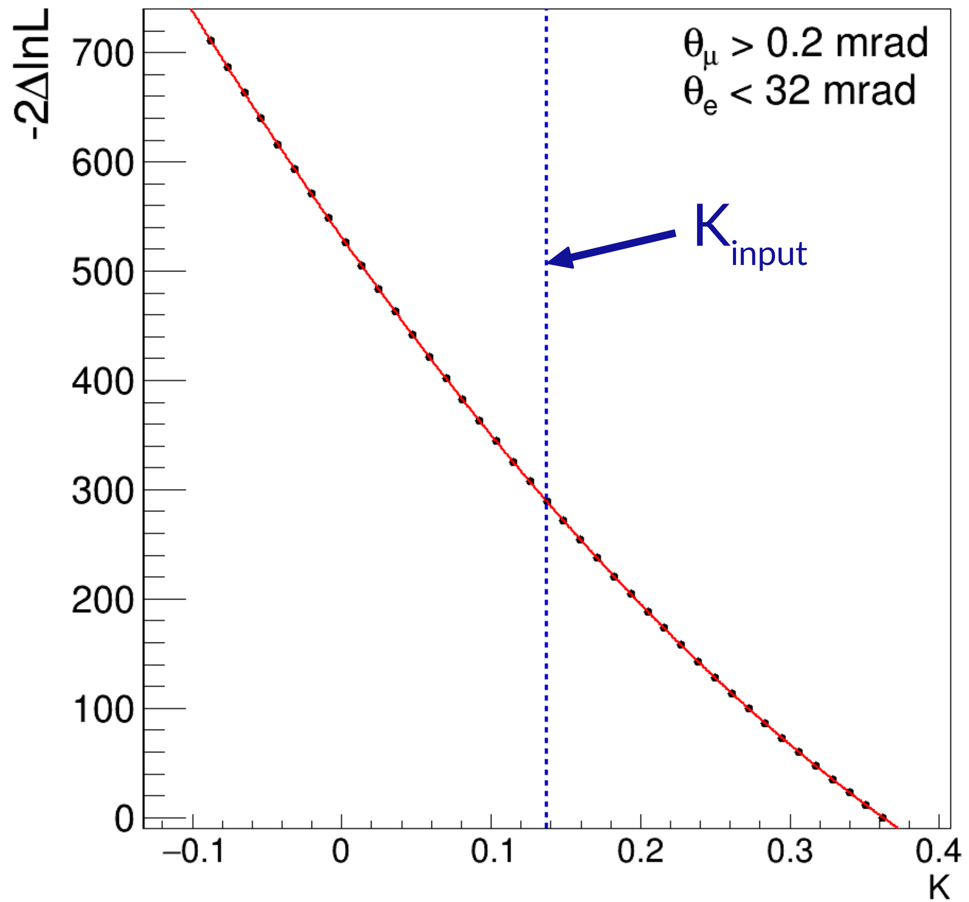




The need of including systematic effects in the analysis

Example: simulate a data sample with a shift on the angular intrinsic resolution *wrt* expectations.

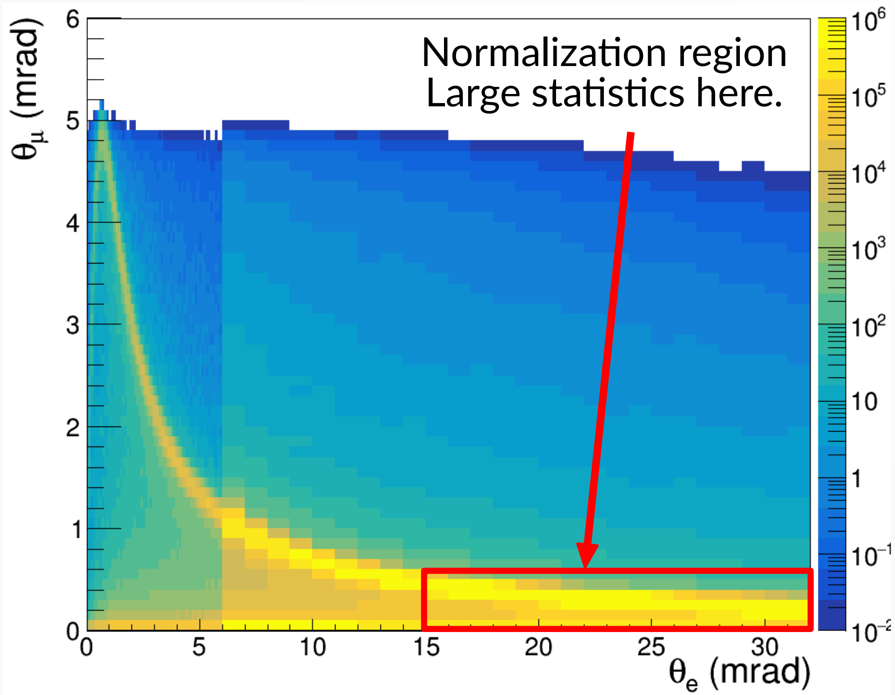
- Test Run statistics: $L_{TR} = 5\ pb^{-1}$.
- Expected angular intrinsic resolution: $\sigma_{Intr} = 0.02$ mrad.
- Shift in the pseudo-data sample: $\sigma_{Intr} \rightarrow \sigma_{Intr} + 5\%$.
- Template fit without accounting for this shift: the minimum is >5σ from K_{input}.

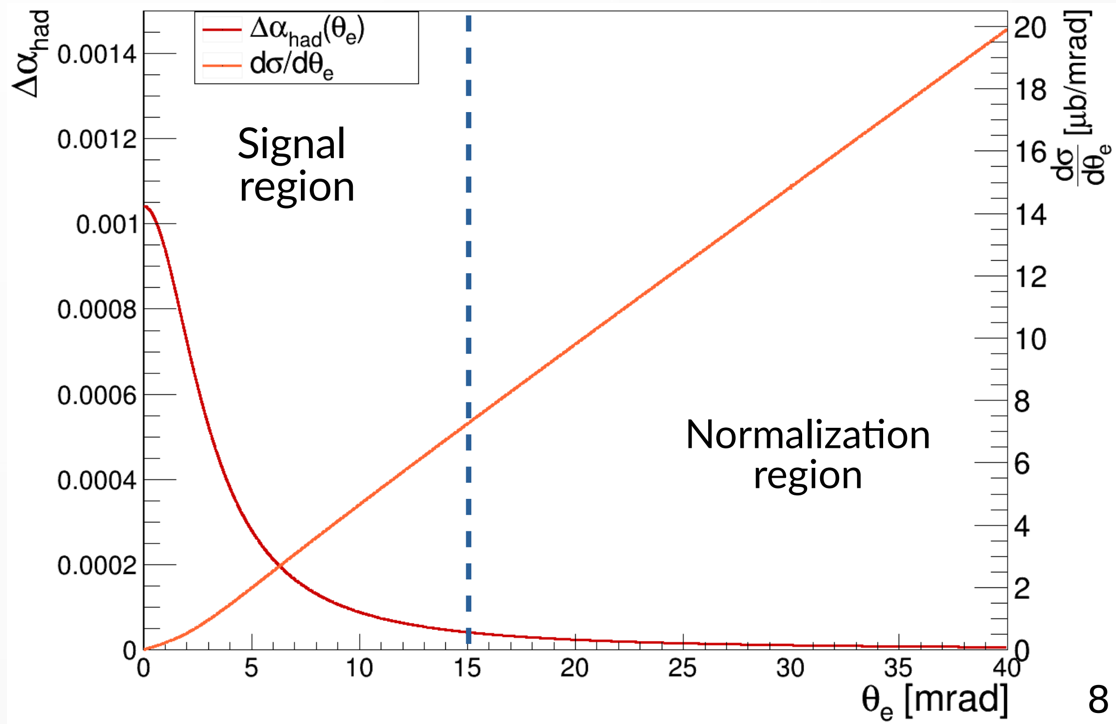


Strategy for the systematic effects

Introduce additional nuisance parameters in the analysis to include the systematic effects.

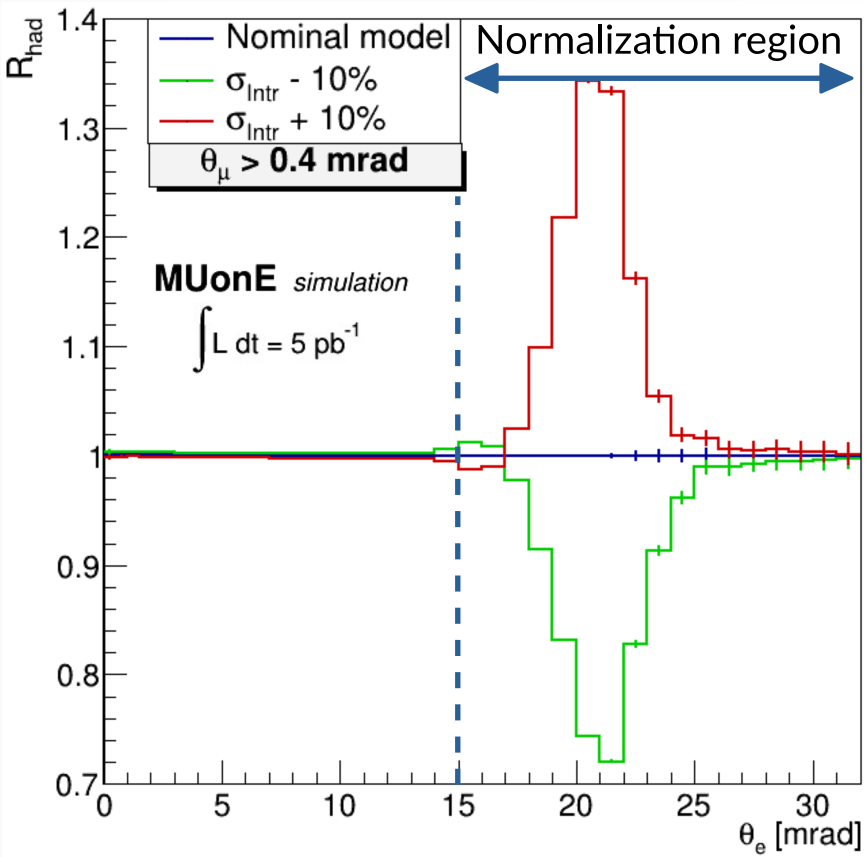
Main systematics have large effects in the normalization region. (no sensitivity to $\Delta\alpha_{had}$ here)

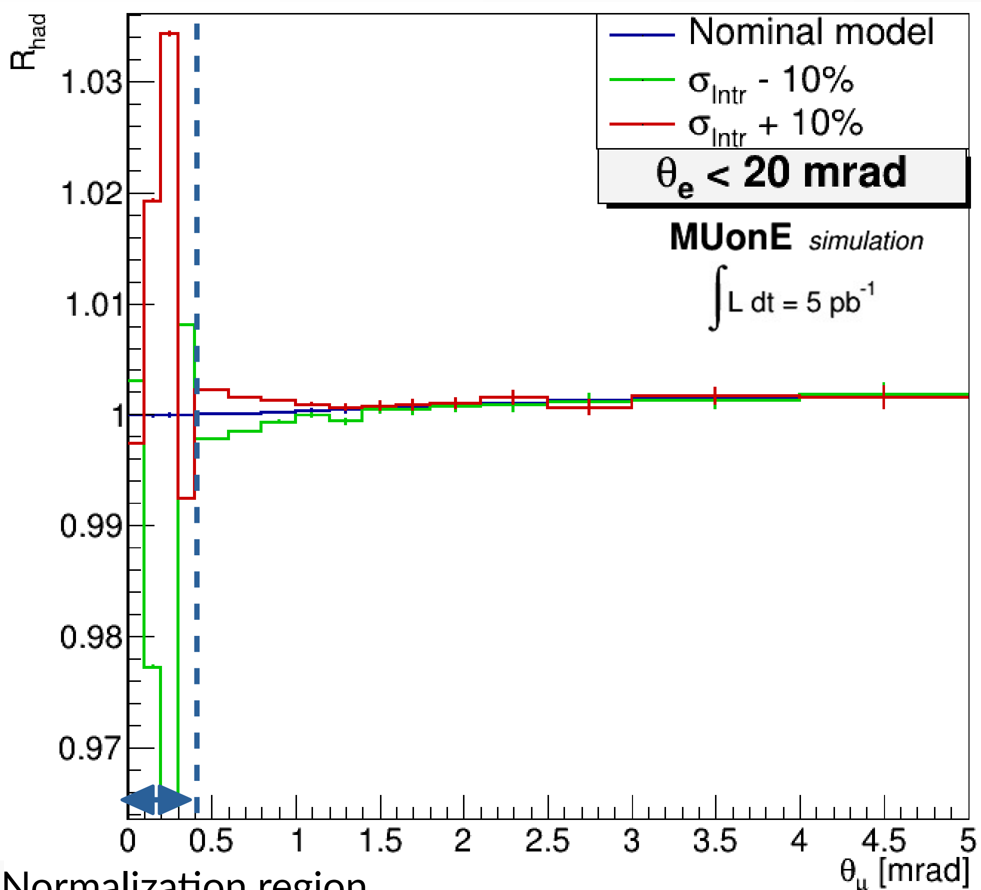




Systematic error on the angular intrinsic resolution

±10% error on the angular intrinsic resolution.





Normalization region

Systematic error on the multiple scattering

Expected precision on the multiple scattering model: ± 1%

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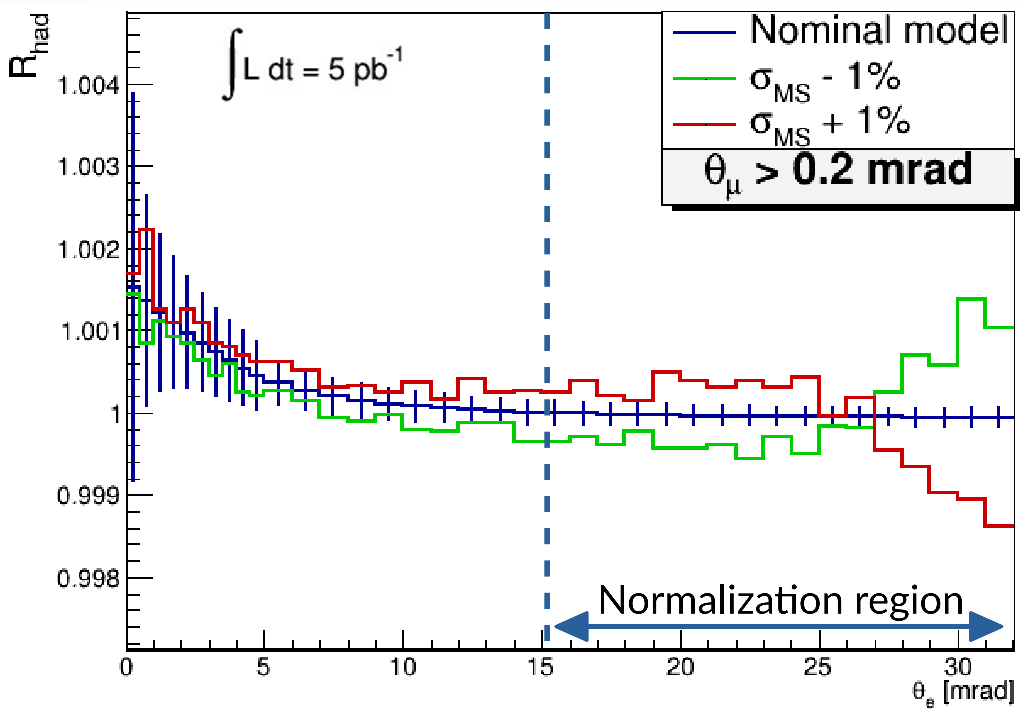
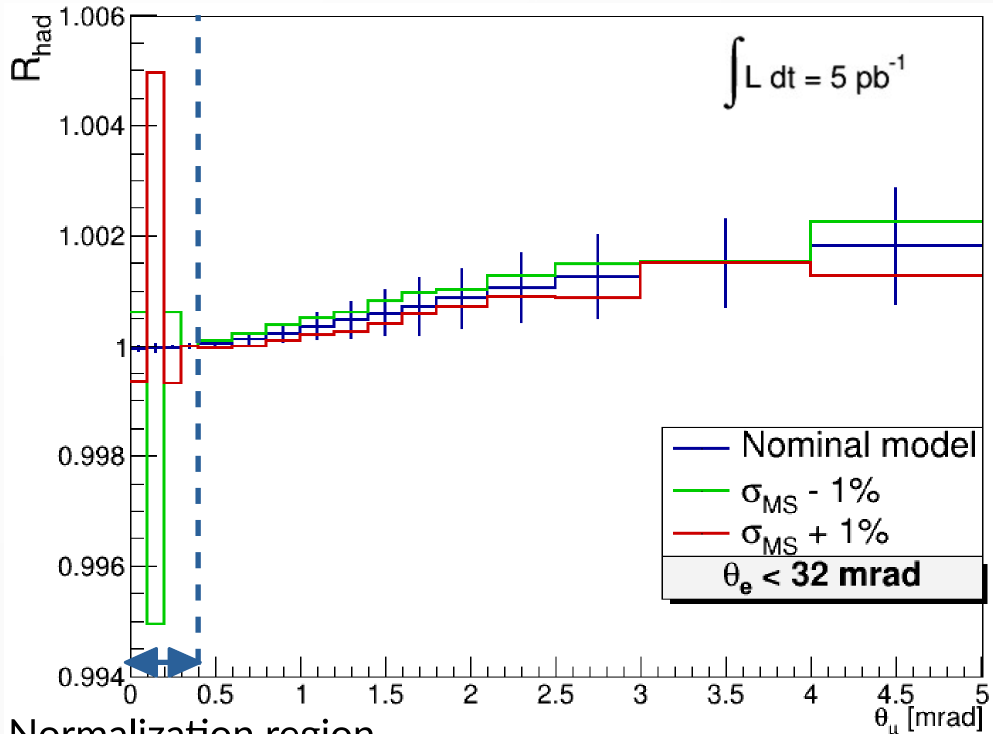


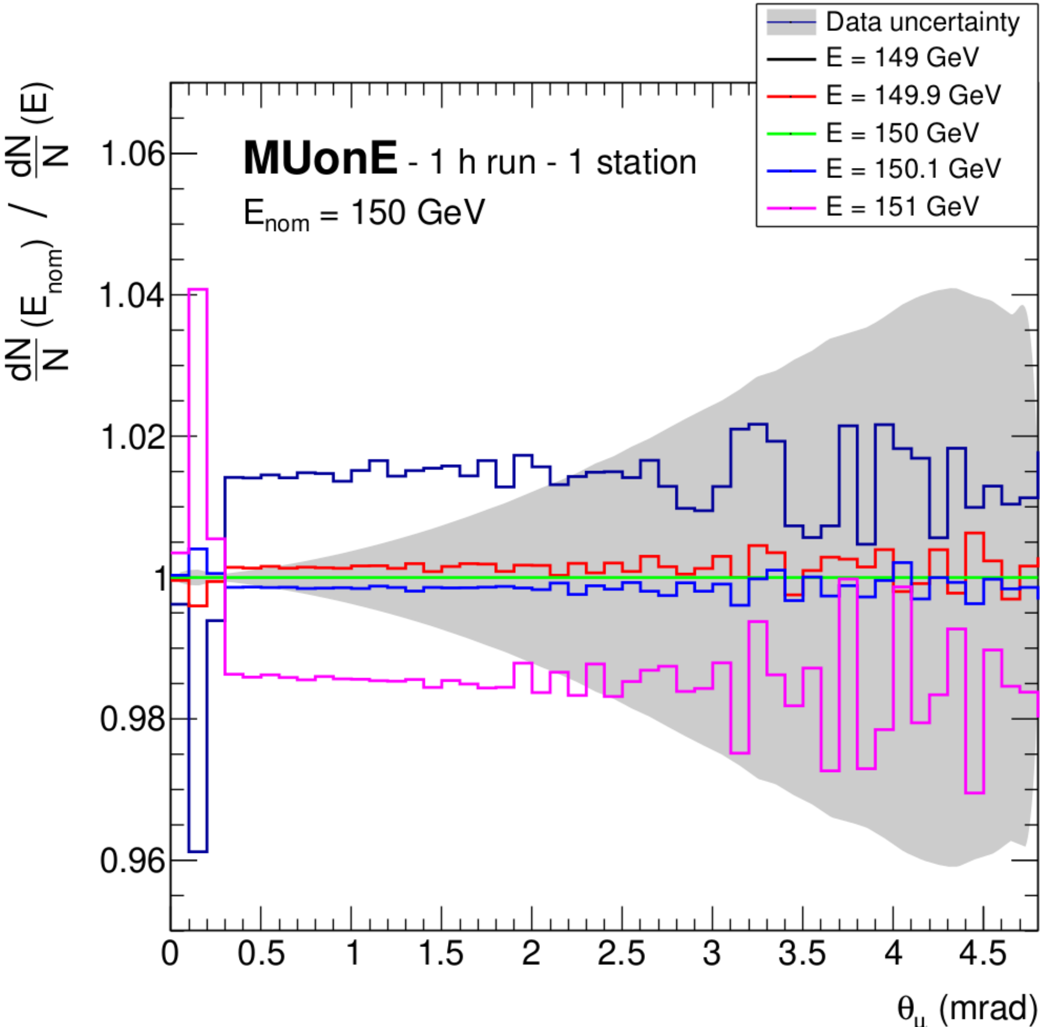


Normalization region

Systematic error on the muon beam energy

Accelerator division provides E_{beam} with O(1%) precision (~ 1 GeV).
It must be controlled by a physical process.

Effects of such shift on E_{beam} can be seen in our data in 1h of data taking per station.



G Abbiendi Phys. Scr. (2022) 97 054007

Strategy for the systematic effects

The Combine analysis tool is used to include the nuisance parameters in the fit procedure.

Binned likelihood fit:

$$\mathcal{L} = \prod_{i=1}^{N} \frac{n_i^{k_i}}{k_i!} e^{-n_i}$$

k_i = events in the *i-th* bin of data
n_i = events in the *i-th* bin of a given template
N = total number of bins

2 classes of nuisance parameters currently included:

- Normalization nuisance parameters, ν
- Shape nuisance parameters, μ

Nuisance parameters are used to adjust n_i and make it fit to k_i.

$$n_i \rightarrow n_i(\vec{\nu}, \vec{\mu})$$

Normalization nuisance parameters

Used to account for residual shifts in the normalization of template distributions with respect to data.

The expected number of events is modified as follows:

$$n_i \to n_i(\nu) = n_i(1+\varepsilon)^\nu$$

Nuisance parameter (ν)

Relative uncertainty on the systematic effect (ε)

Example: systematic error due to a limited knowledge of the luminosity → ε ~ O(1%)

Shape nuisance parameters

Used to control effects that change the *shape* of the differential cross section.

The expected number of events in each bin is modified as:

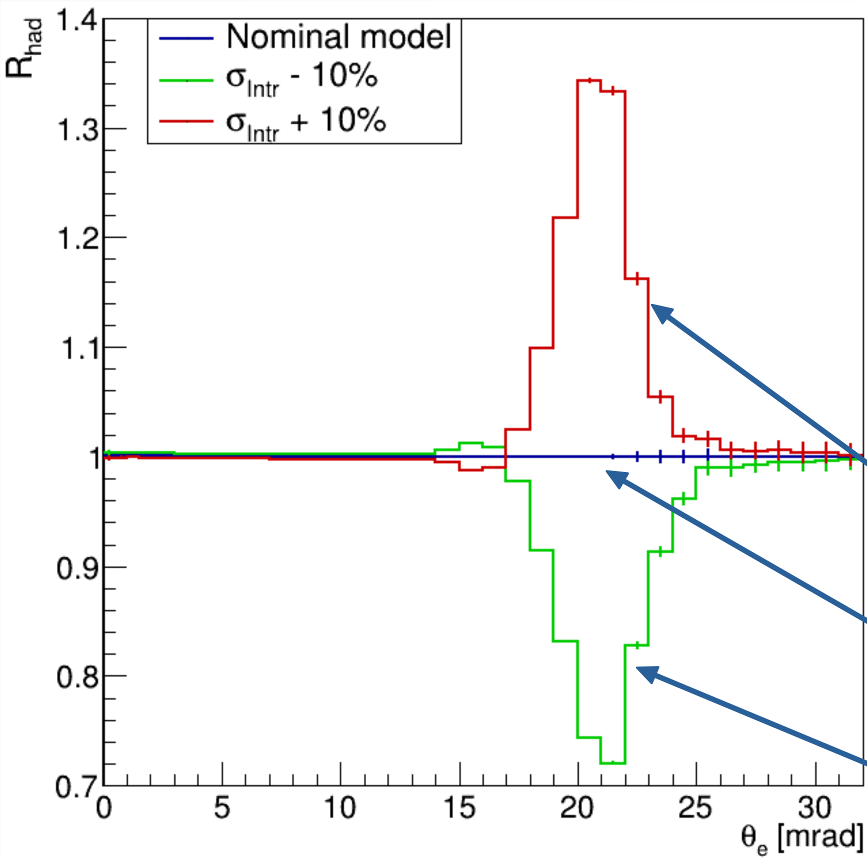
$$n_i \to n_i(\mu) = n_i[1 + s_i(\mu)]$$

Spline ensuring continuity and differentiability of 1st and 2nd derivatives.
Each bin has its own spline.

$$s_i(\mu) = \begin{cases} \frac{1}{2}\left[(\delta_i^+ - \delta_i^-)\mu + \frac{1}{8}(\delta_i^+ + \delta_i^-)(3\mu^6 - 10\mu^4 + 15\mu^2)\right] & |\mu| \le 1 \\ \delta_i^+ \mu & \mu > 1 \\ -\delta_i^- \mu & \mu < -1 \end{cases}$$

Shape nuisance parameters

$s_i(\mu)$ depends on $\delta_i^{\pm} = \frac{n_i^{\pm} - n_i^0}{n_i^0}$



Shape nuisance parameters are determined by vertical interpolation of the template histograms.

3 inputs are needed for the interpolation:

n_i^+ Template with systematic effect shifted by +1σ

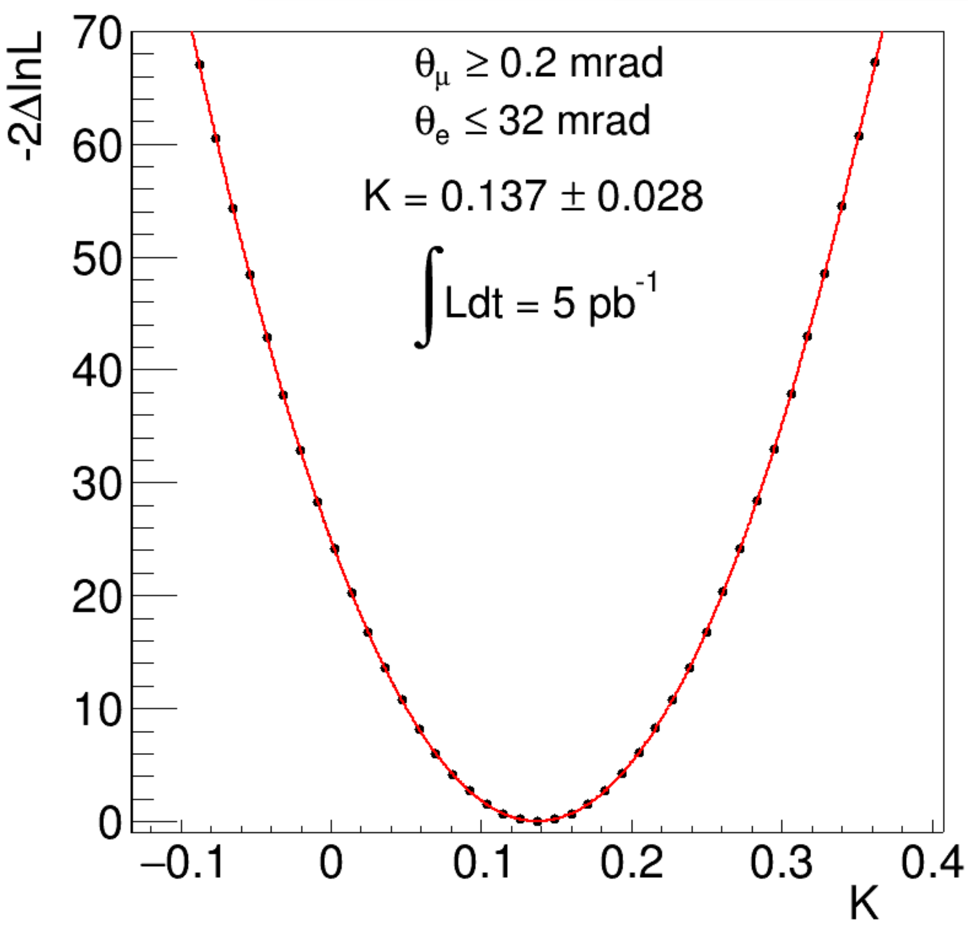
n_i^0 Template with the expected modelization

n_i^- Template with systematic effect shifted by -1σ

Analysis workflow

- Combine performs a likelihood fit to the nuisance parameters for each template.
- Obtain the profile likelihood as a function of K.
- Best fit value of K is determined by parabolic interpolation among the template points.
- Nuisance parameters values for $K = K_{best\ fit}$ are obtained by interpolation among the values obtained in the first step.

Statistical fit @L_{TR} = 5 pb^{-1}



Analysis workflow

Promising strategy: staged approach.

1. Use a small fraction of data to refine the knowledge of the main sources of systematic error with respect to the initial modelization.

2. Include the residual systematics as nuisance parameters in a combined fit with the signal parameter on the entire dataset.

Currently tested on the Test Run statistics including the main systematic errors.

Testing the procedure

Generate a pseudo-data sample introducing shifts in the main sources of systematic error with respect to the expectations.

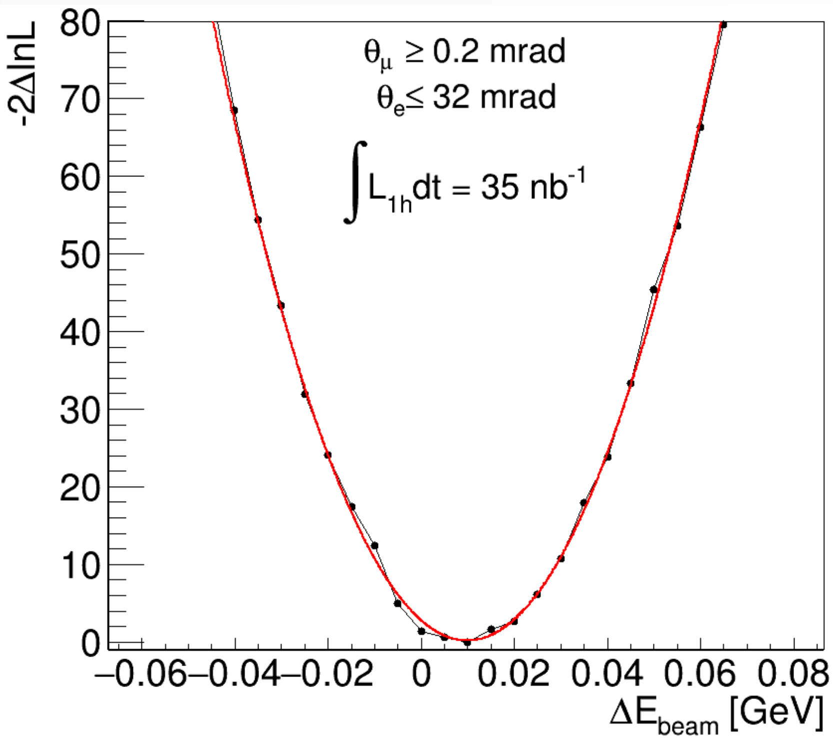
Source of systematics	Shift in the pseudo-data	Expected uncertainty
Beam energy scale	$E_{beam} \rightarrow E_{beam} + 6\,\mathrm{MeV}$	$\Delta E_{beam} = \pm 1\,\mathrm{GeV}$
Multiple scattering	$\sigma_{MS} \rightarrow \sigma_{MS} + 0.5\%$	$\Delta\sigma_{MS} = \pm 1\%$
Angular intrinsic resolution	$\sigma_{Intr} \rightarrow \sigma_{Intr} + 5\%$	$\Delta\sigma_{Intr} = \pm 10\%$
Luminosity		$\varepsilon = 1\%$

Are we able to determine precisely K and the nuisance parameters using this analysis strategy?

Step 1: identify the main systematic effects

1h of data taking per single station.

Allows to assume a fixed model for $\Delta\alpha_{had}$.



- Template fit as a function of E_{beam}.
- μ_{MS}: nuisance parameter for systematics on the multiple scattering.
- μ_{Intr}: nuisance parameter for systematics on the angular intrinsic resolution.
- ν: nuisance parameter for systematics on the normalization.

Selection cuts	Fit results
$\theta_e \leq 32\,\text{mrad}$ $\theta_\mu \geq 0.2\,\text{mrad}$	$\Delta E_{beam} = (0.006 \pm 0.006)$ GeV $\mu_{Intr} = (4.9 \pm 0.1)\%$ $\mu_{MS} = (0.6 \pm 0.1)\%$ $\nu = 0.01 \pm 0.03$

Similar results also for different selection cuts.

Update the knowledge on the sources of systematic error

Exploit results obtained in step 1 to refine the knowledge on the sources of systematic error.

Source of systematics	Expected uncertainty
Beam energy scale	$\Delta E_{beam} = \pm 1\,\mathrm{GeV}$
Multiple scattering	$\Delta\sigma_{MS} = \pm 1\%$
Angular intrinsic resolution	$\Delta\sigma_{Intr} = \pm 10\%$

→

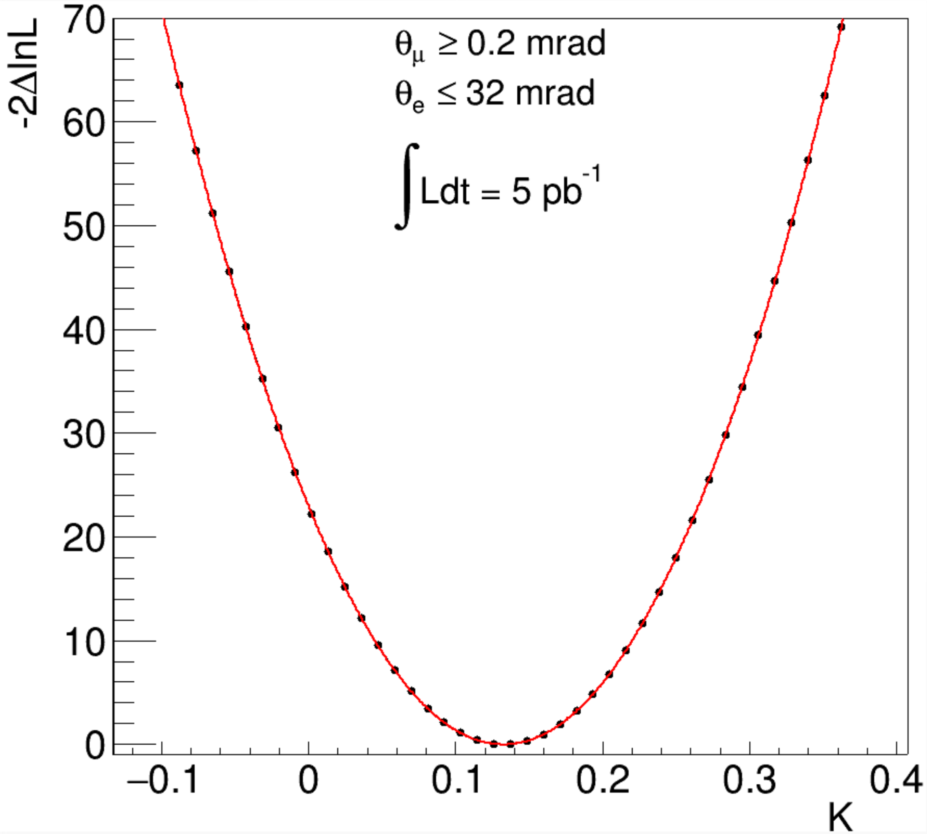
Updated model
$\Delta E_{beam} = \pm 20\,\mathrm{MeV}$
$\sigma_{MS} \to \sigma_{MS} + 0.6\%$ $\Delta\sigma_{MS} = \pm 0.5\%$
$\sigma_{Intr} \to \sigma_{Intr} + 5\%$ $\Delta\sigma_{Intr} = \pm 0.6\%$

Use this improved modelization to perform the combined fit to K and the residual systematics.

Step 2: combined fit signal + systematics

- Template fit as a function of K.
- Add a nuisance parameter for systematics on the beam energy: μ_{Ebeam}.

- K_{ref} = 0.137
- shift MS: +0.5%
- shift intr. res: +5%
- shift E_{beam}: +6 MeV



Selection cuts	Fit results
$\theta_e \leq 32\,\mathrm{mrad}$ $\theta_\mu \geq 0.2\,\mathrm{mrad}$	$K = 0.133 \pm 0.028$ $\mu_{\mathrm{MS}} = (0.47 \pm 0.03)\%$ $\mu_{\mathrm{Intr}} = (5.02 \pm 0.02)\%$ $\mu_{\mathrm{E_{Beam}}} = (6.5 \pm 0.5)\ \mathrm{MeV}$ $\nu = -0.001 \pm 0.003$

Similar results also for different selection cuts.

Input shifts identified correctly.
No degradation on the signal parameter.

Conclusions

- Proposed strategy to control the systematic effects: use the elastic scattering events to determine the main systematics, then perform a combined fit to the signal and the residual effects.
- Promising results for the Test Run.
- Next steps:
  - Include the track reconstruction algorithms in the simulation.
  - Add background processes.
  - Add further sources of systematic errors.
  - Verify the procedure with the full statistics (2 signal parameters).

Further systematic effects: theory

- Data @NNLO, templates @NLO: quantify the effect of the NNLO corrections.

- Residual systematic effects for the N^3LO?

- Quantify the effect of m_e:
  $m_e = 0$ vs m_e exact vs m_e series expansion?

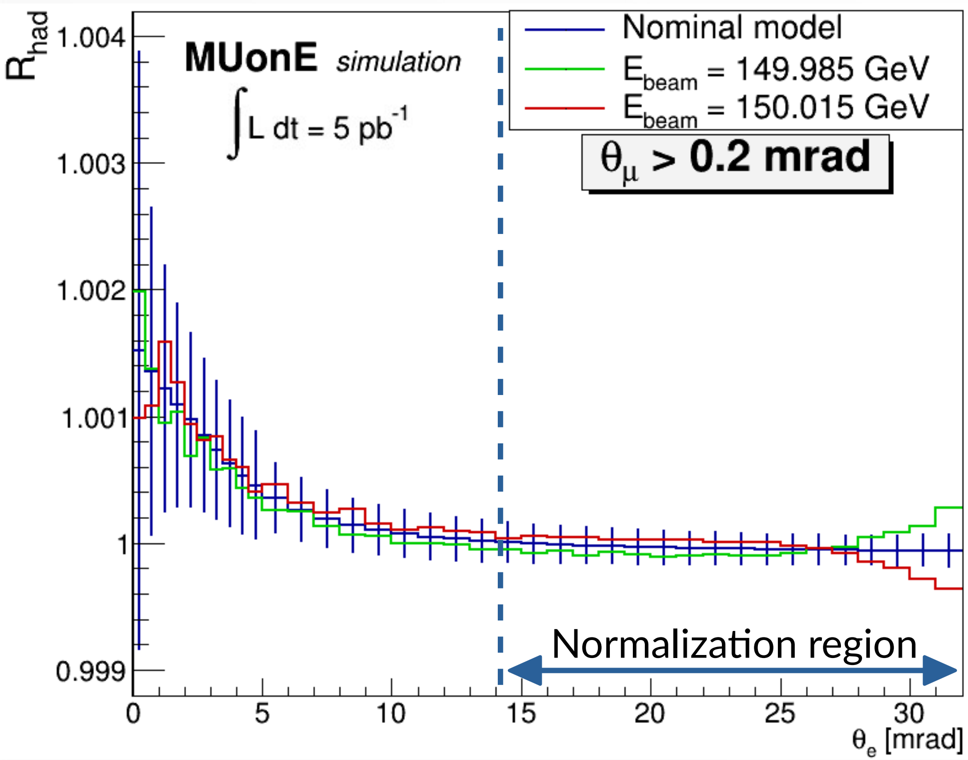
- Other?

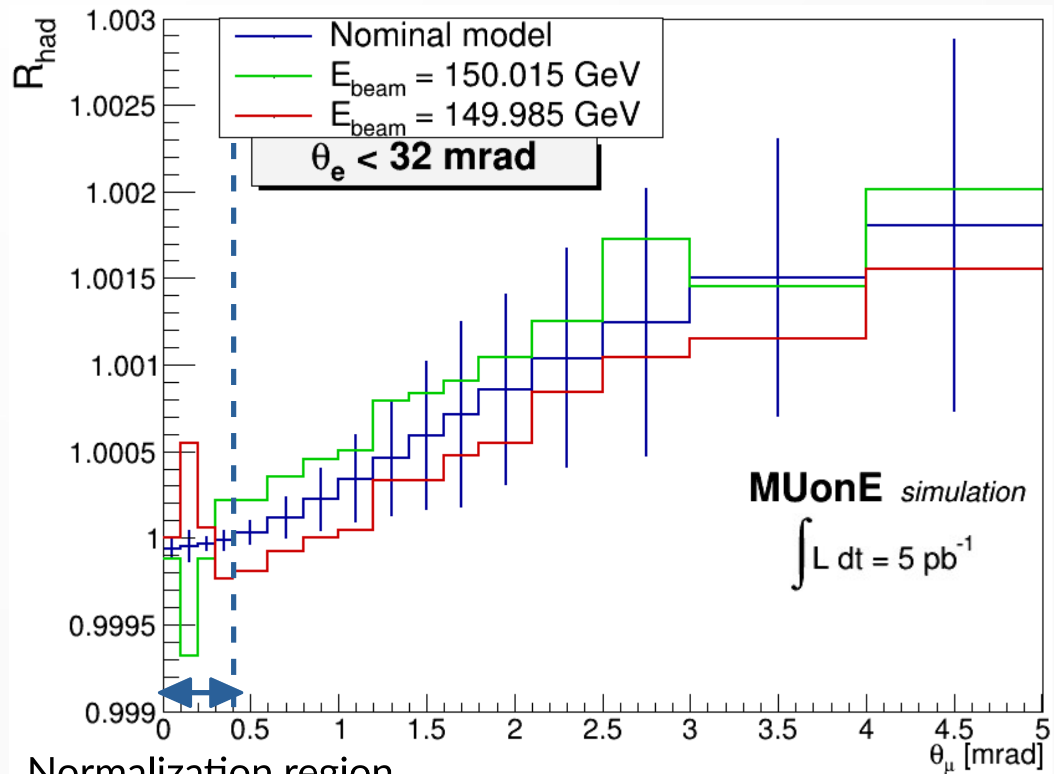
- What is needed for these tests:
  distributions with the nominal model +
  $\pm 1\sigma$ distributions.
  A parameterization of the expected distortion on the shape of the differential cross section due to the systematic effect is needed.

BACKUP

Systematic error on the beam energy scale

Effect of a ± 15 MeV shift

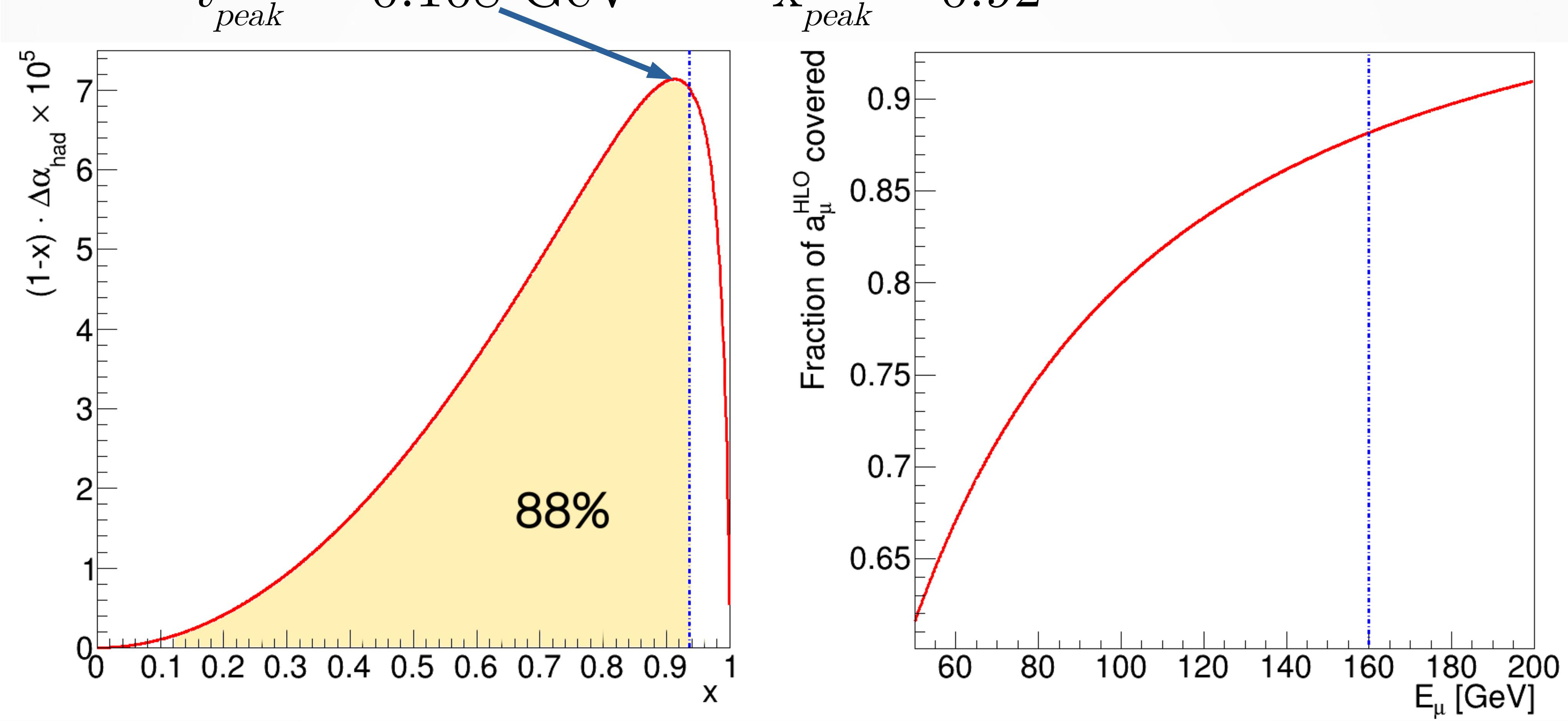




Normalization region

$$x < 0.936$$

$$t_{peak} \sim -0.108\ \mathrm{GeV}^2 \qquad x_{peak} \sim 0.92$$

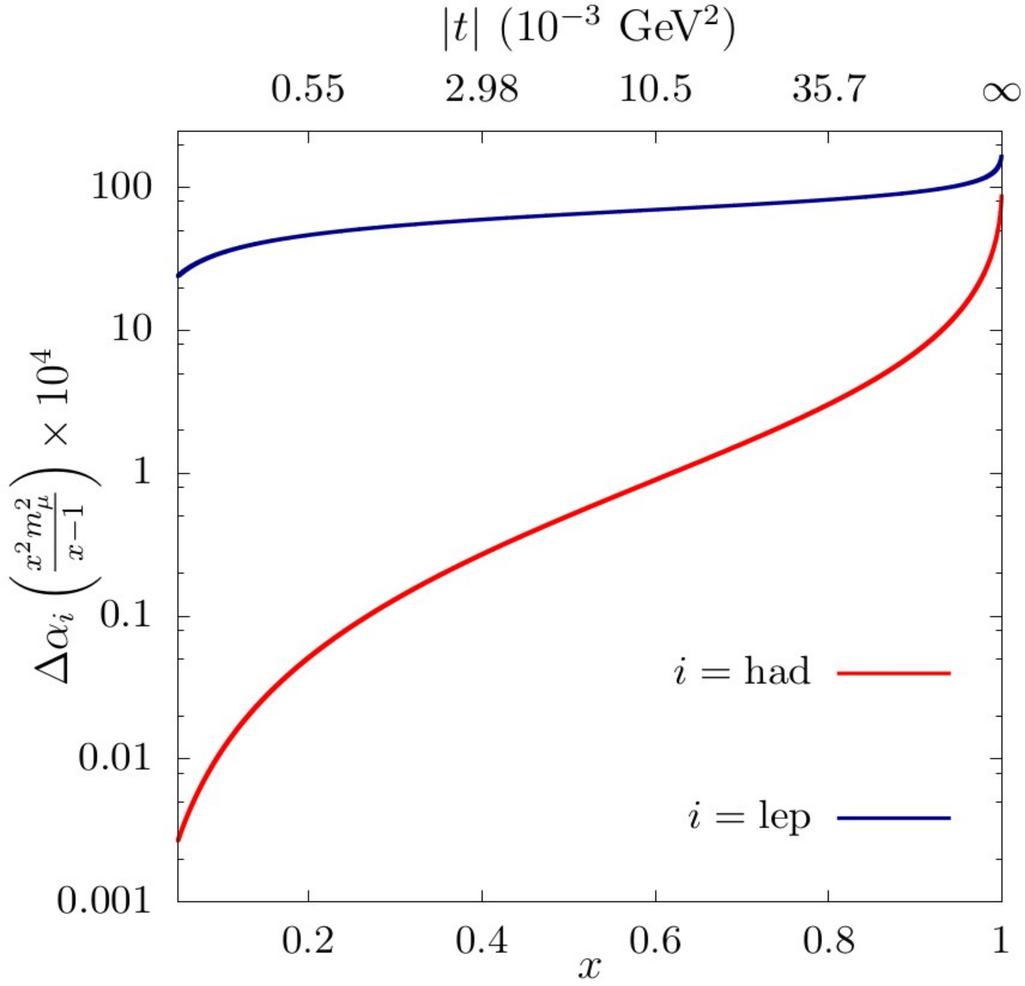


- 160 GeV muon beam on atomic electrons.

$$\sqrt{s} \sim 420\,\mathrm{MeV}$$

$$-0.153\,\mathrm{GeV}^2 < t < 0\,\mathrm{GeV}^2$$

$$\Delta\alpha_{had}(t) \lesssim 10^{-3}$$



Achievable accuracy

40 stations (60 cm Be) + 3 years of data taking ($\sim 4 \times 10^7$ s) ($I_\mu \sim 10^7\ \mu^+/s$) = ~0.3% statistical accuracy on a_μ^{HLO}

$\sim 4 \times 10^{12}$ events with $E_e > 1$ GeV

→ Competitive with the latest theoretical predictions.

Main challenge:
keep systematic accuracy at the same level of the statistical one

↓

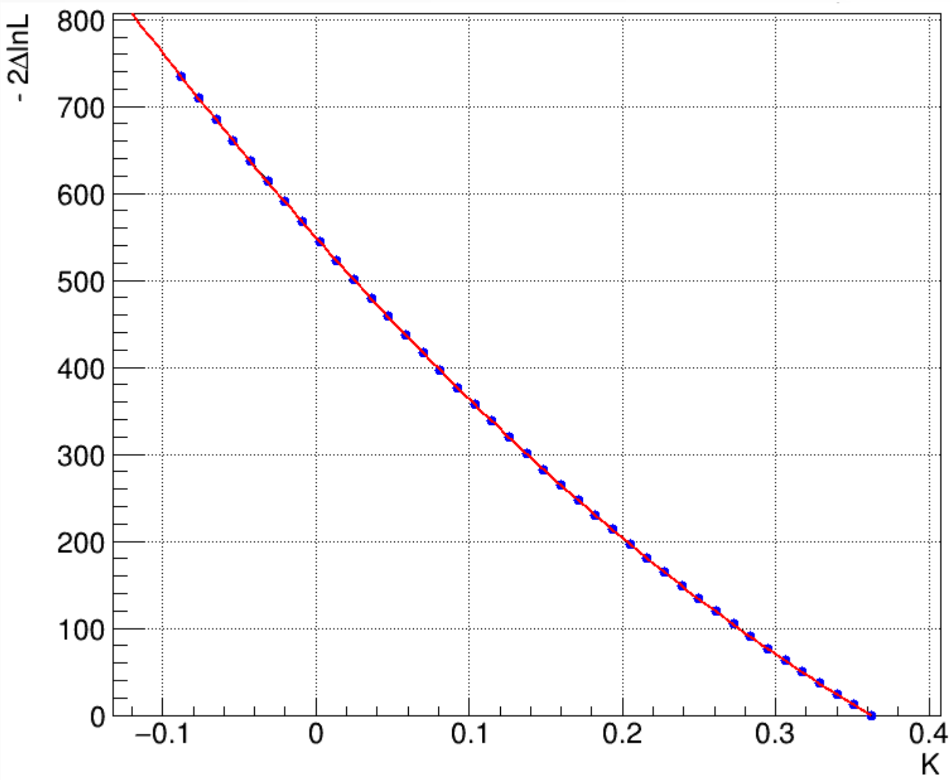
Systematic uncertainty of 10 ppm at the peak of the integrand function (low θ_e, large θ_μ)

Main systematic effects:

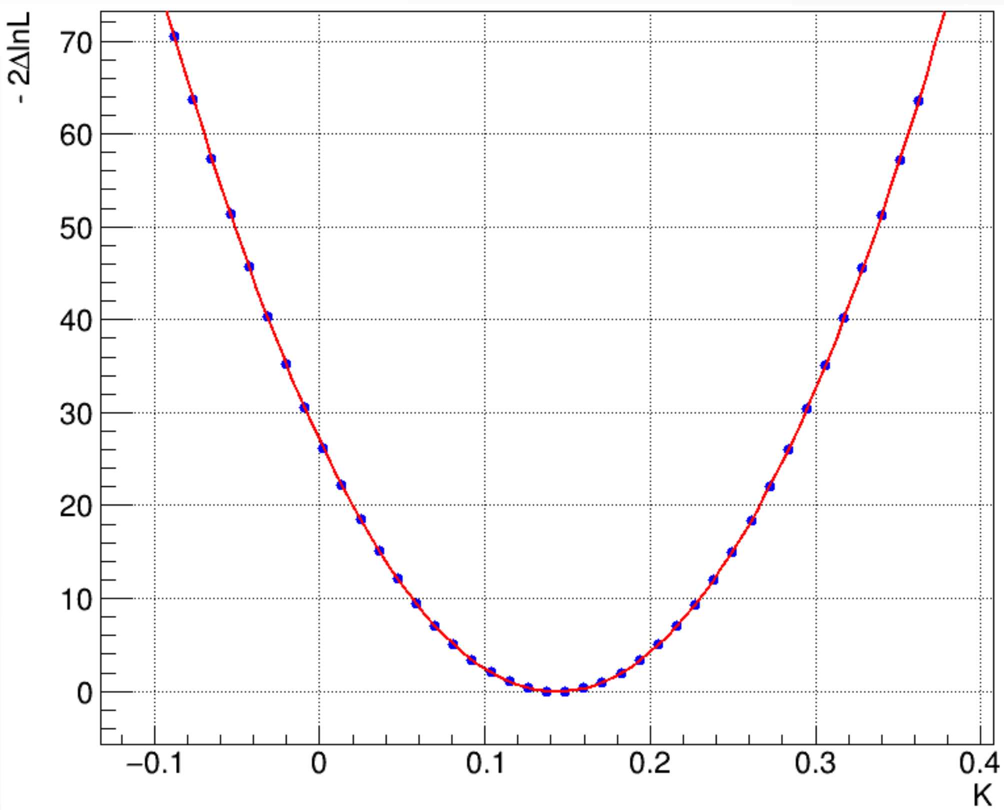
- Longitudinal alignment (~10 µm)
- Knowledge of the beam energy (few MeV)
- Multiple scattering (~1%)
- Angular intrinsic resolution (few %)

Simultaneous fit signal + nuisance parameters @L_{TR}

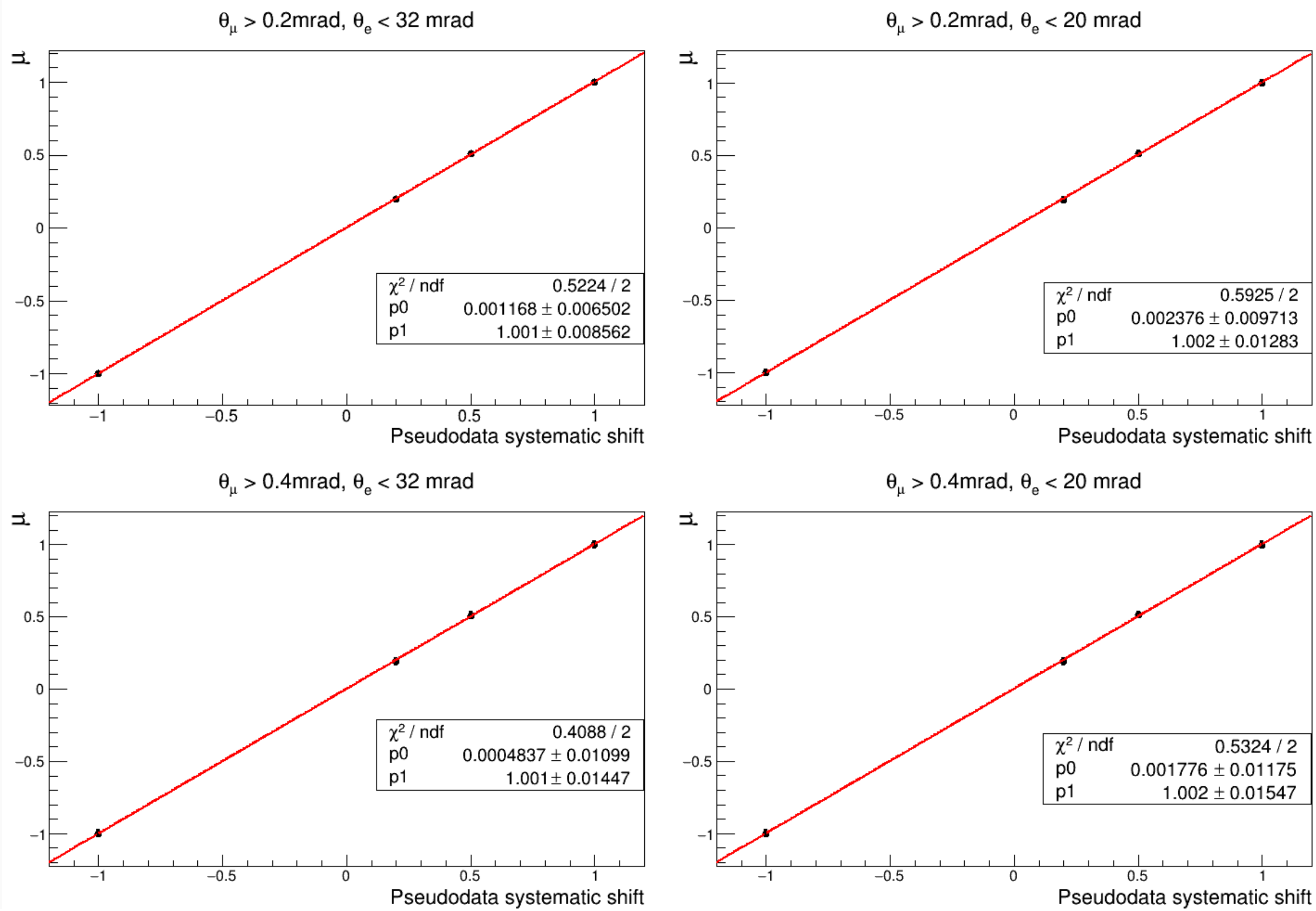
If the systematics are not taken into account in the fit...



If the nuisance parameters are introduced in the fit procedure...



Fit of MS nuisance using different pseudodata shifts



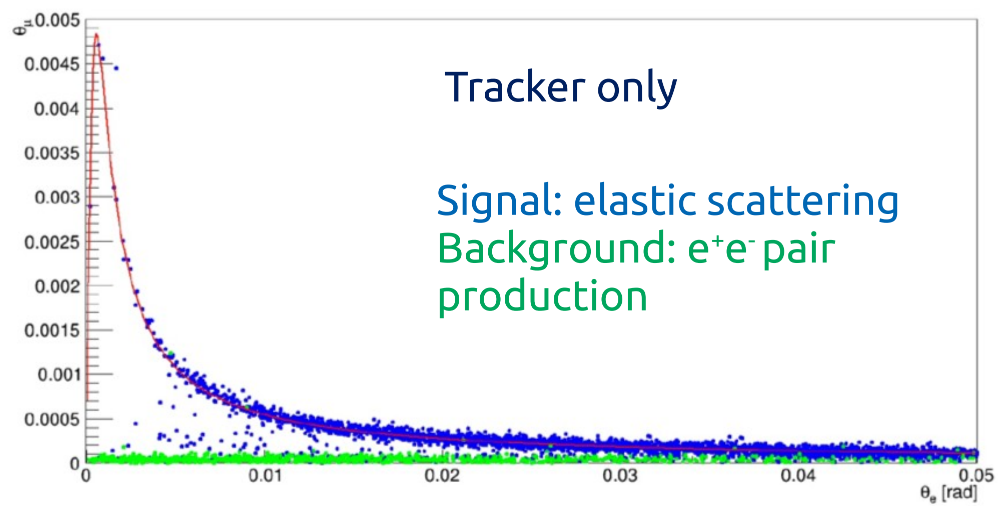
μ = {-1%, 0.2%, 0.5%, 1%}

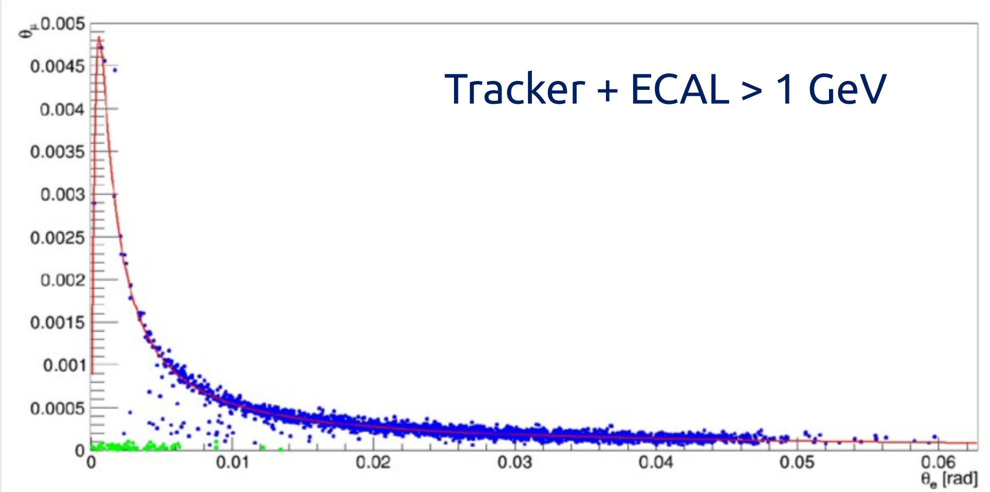
Linear relation between fitted value of μ and pseudodata shift

OK!

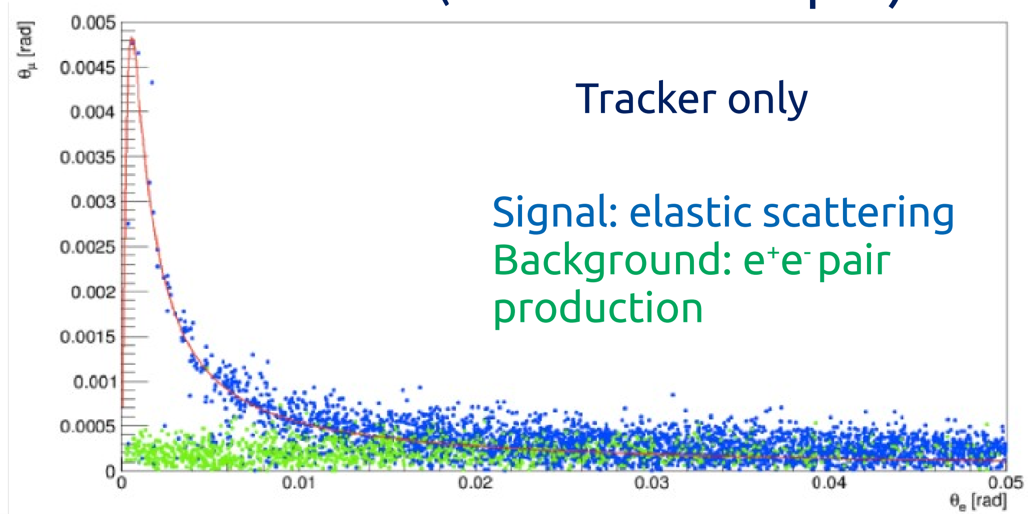
GEANT4 simulations

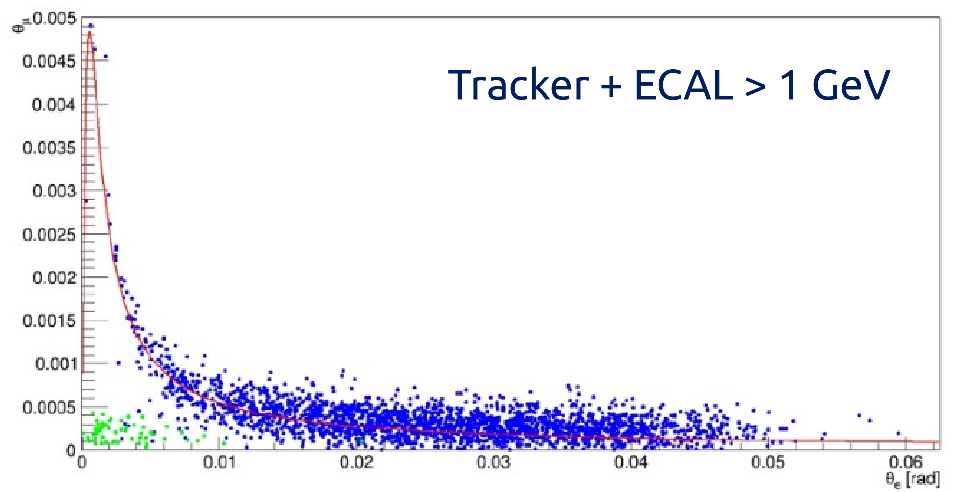
TB2017 (resolution ~7μm)





TB2018 (resolution ~40μm)



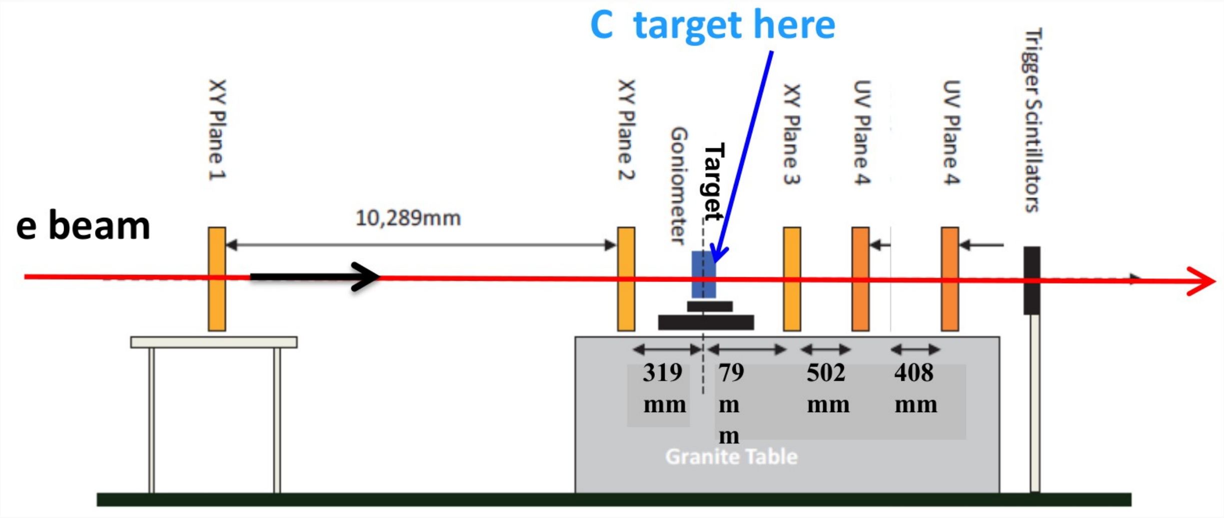


Multiple scattering: results from TB2017

Multiple scattering effects of electrons with 12 and 20 GeV on Carbon targets (8 and 20 mm)

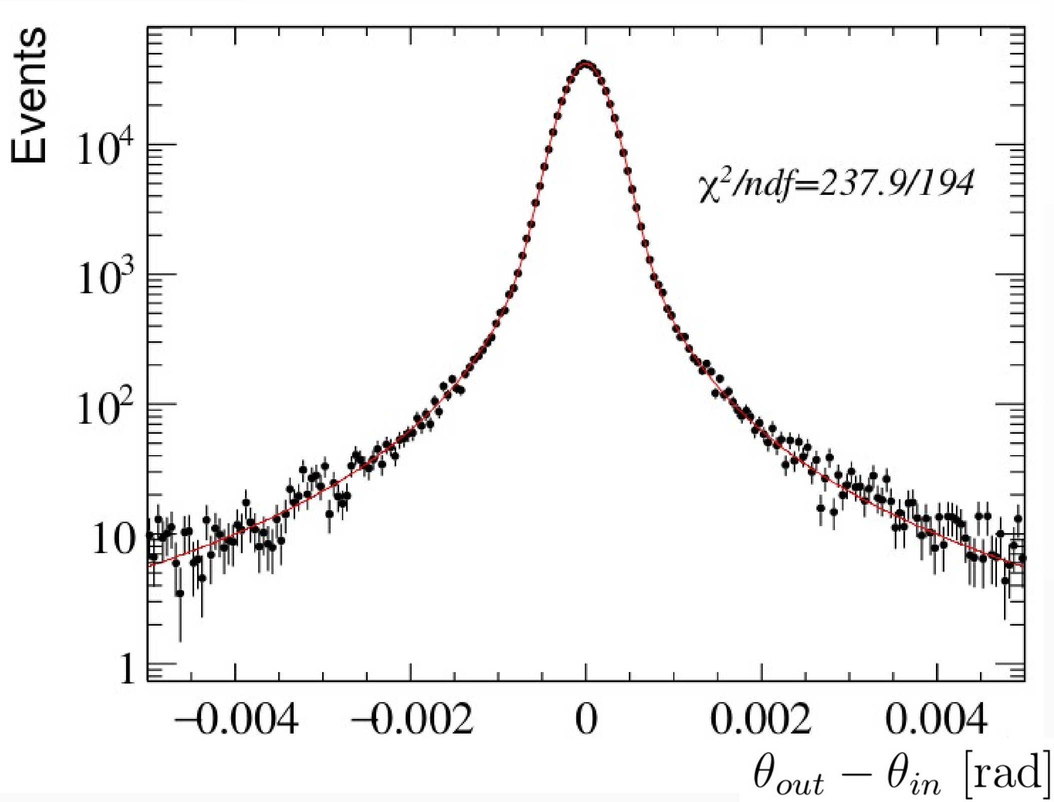
Main goals:

- to determine a parameterization able to describe also non Gaussian tails
- to compare data with a GEANT4 simulation of the apparatus



Multiple scattering: results from TB2017

$$f_e(\delta\theta_e^x) = N\left[(1-a)\frac{1}{\sqrt{2\pi}\sigma_G}e^{-\frac{(\delta\theta_e^x-\mu)^2}{2\sigma_G^2}} + a\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\sigma_T\Gamma(\frac{\nu}{2})}\left(1+\frac{(\delta\theta_e^x-\mu)^2}{\nu\sigma_T^2}\right)^{-\frac{\nu+1}{2}}\right]$$



$$\vec{p} = [N,\ a,\ \mu,\ \sigma_G,\ \nu,\ \sigma_T]$$

Results show a ~1% agreement between data and MC for the Gaussian core