Muon g-2: Theory Status

Marc Knecht

Centre de Physique Théorique UMR7332,
CNRS Luminy Case 907, 13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

Present experimental situation

a$_\mu$: Unblinding

$a_\mu \times 10^9 - 1165900$

G. Venanzoni, CERN Seminar, 8 April 2021

69
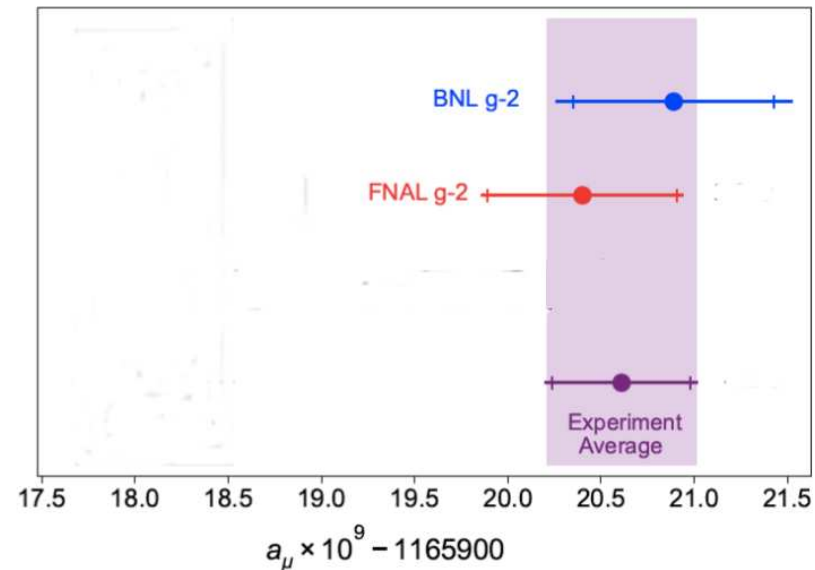
New measurement by FNAL-E989

$$a_{\mu^+}^{\text{E989}} = 116\,592\,040(54) \cdot 10^{-11} \;[0.46\text{ppm}]$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

In line with previous BNL-E821 measurement ~ 20 years ago

$$a_\mu^{\text{E821}} = 116\,592\,089(63) \cdot 10^{-11} \;[0.54\text{ppm}]$$

G. W. Bennett et al. [Muon g-2 Coll.], PRD 73, 072003 (2006)

and in line with decades of muon storage ring experiments

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72
FNAL E989	2021	μ^+	11 659 2040(54)	0.46
FNAL E989	2023?	μ^+	???	~ 0.14
[J-PARC E34	2027?	μ^+	???	~ 0.45]

Present experimental situation $\longrightarrow$ quite satisfactory
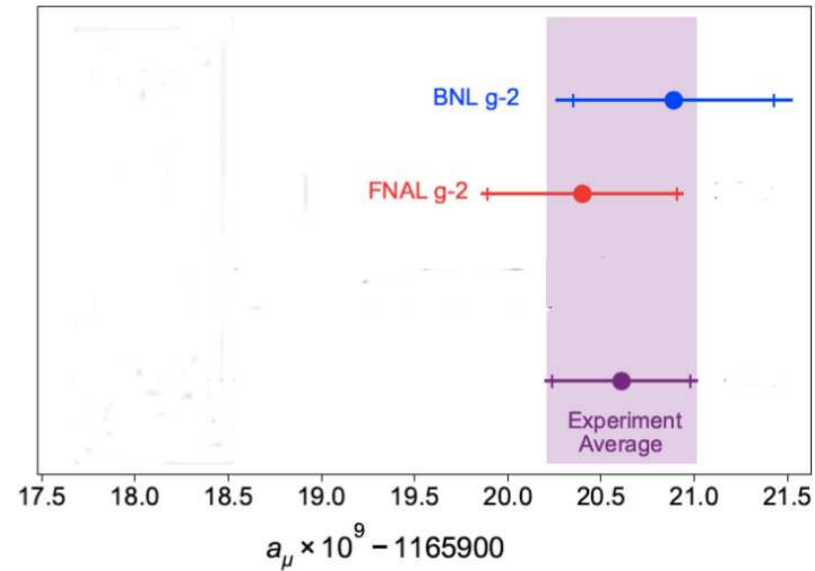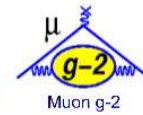
$$a_\mu^{\text{exp;WA}} \quad = \quad 116\,592\,061(41) \cdot 10^{-11} \ [0.35\text{ppm}]$$

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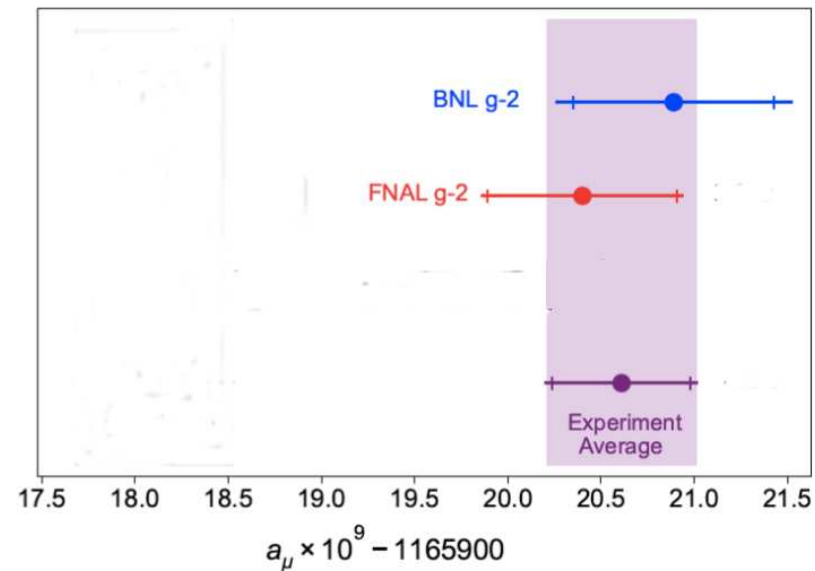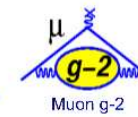
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G. Venanzoni, CERN Seminar, 8 April 2021

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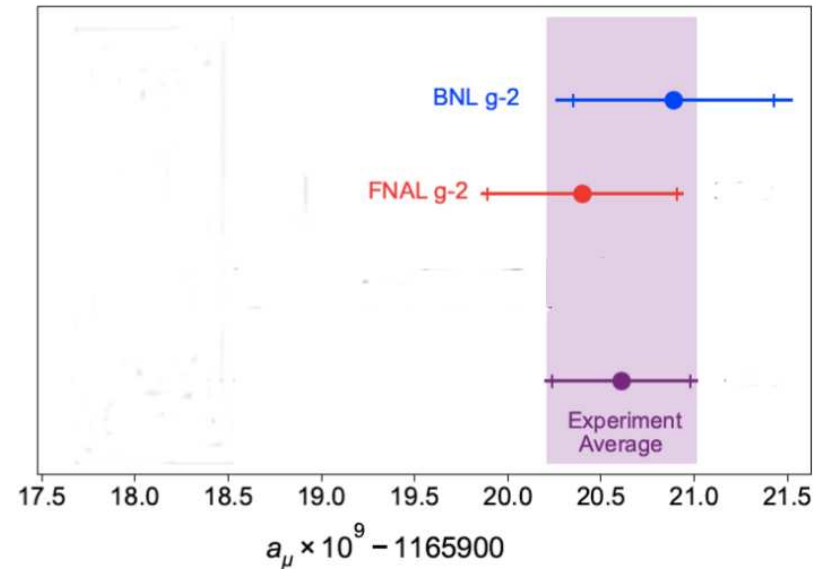
G. Venanzoni, CERN Seminar, 8 April 2021

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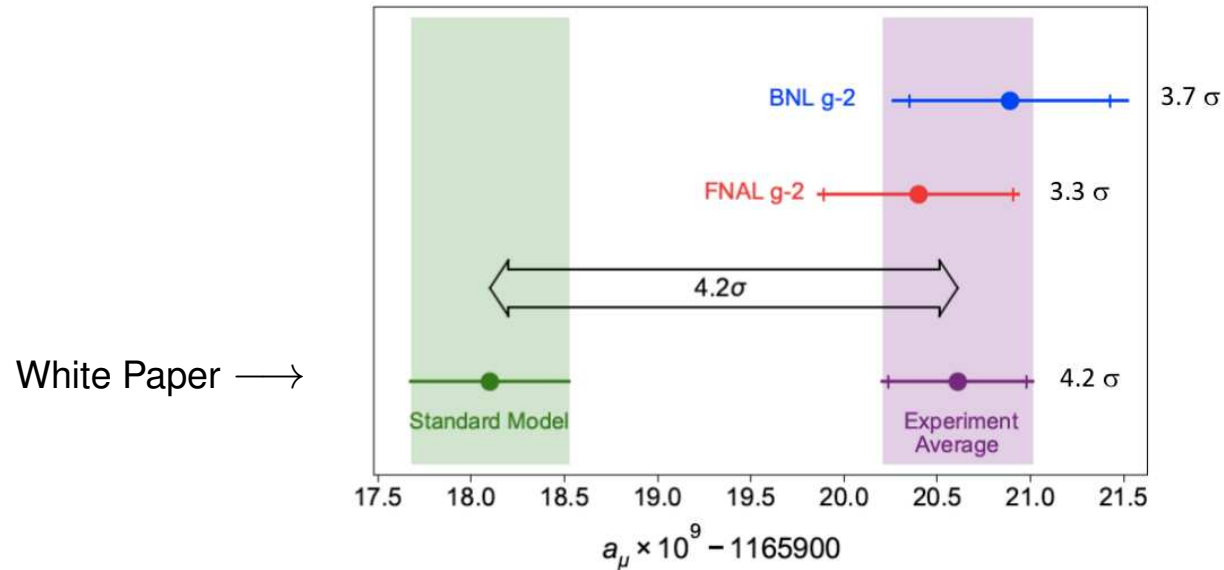
Theory situation?

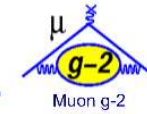Full and detailed account [up to June 15, 2020] given in the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

Present situation

G. Venanzoni, CERN Seminar, 8 April 2021

$$a_\mu^{\text{th;WP}} = 116\,591\,810(43) \cdot 10^{-11} \ [0.35\text{ppm}]$$

Leads to a discrepancy th. vs. exp. at the level of 4.2σ

$$a_\mu^{\text{exp;WA}} - a_\mu^{\text{th;WP}} = 251(59) \cdot 10^{-11} \ [4.2\sigma]$$

Present situation

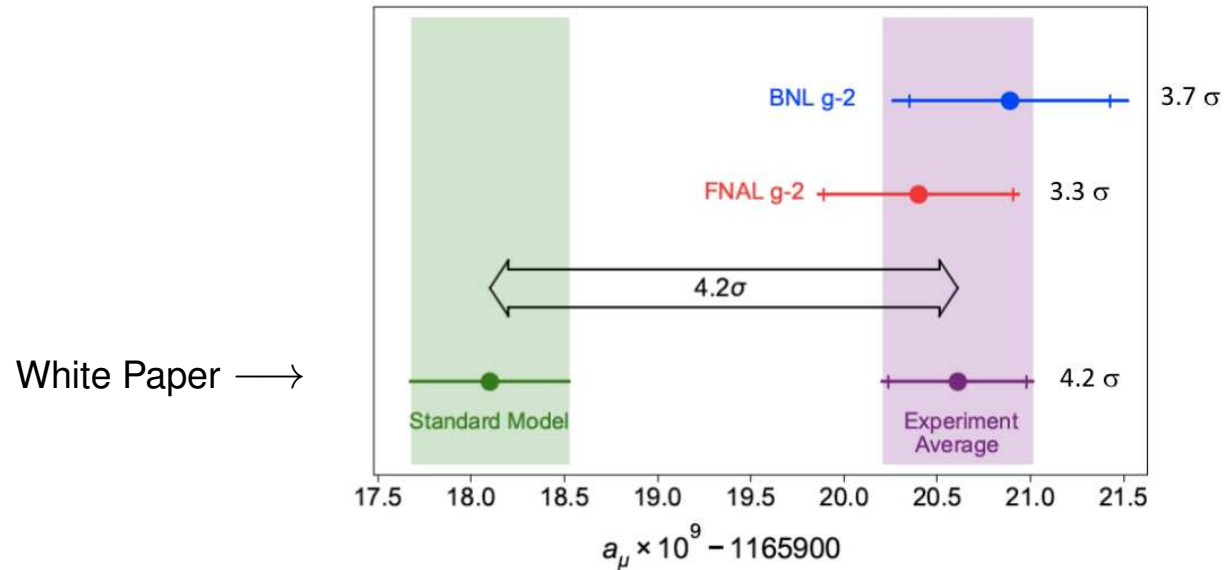$$a_\mu^{\text{th;WP}} \;=\; 116\,591\,810(43) \cdot 10^{-11} \; [0.35\text{ppm}]$$
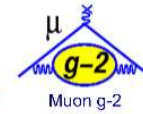
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$$a_\mu^{\text{exp;WA}} - a_\mu^{\text{th;WP}} \;=\; 251(59) \cdot 10^{-11} \; [4.2\sigma]$$

Does $a_\mu^{\text{th;WP}} = a_\mu^{\text{th;SM}}$ still hold today?

Contribution from QED

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$\longrightarrow$ loops with only photons and leptons

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$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

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$\longrightarrow$ becomes technically challenging $(1, 6, 72, 891, 12\,672, \ldots)$

$$a_\mu{}^{\text{QED}} \;=\; C_\mu^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\mu^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\mu^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\mu^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\mu^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

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$\longrightarrow$ multiflavour QED

$$C_\mu^{(2n)} \;=\; A_1^{(2n)} + A_2^{(2n)}(m_\mu/m_e) + A_2^{(2n)}(m_\mu/m_\tau) + A_3^{(2n)}(m_\mu/m_e, m_\mu/m_\tau)$$

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$$\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$$

A few comments about the QED contributions

- expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

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- $A_1^{(8)}$ has also been evaluated! ($\longrightarrow a_e$) S. Laporta, Phys. Lett. B 772, 232 (2017)

- Mass dependent parts $A_2^{(8)}(m_\mu/m_e)$, $A_2^{(8)}(m_\mu/m_\tau)$, $A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau)$
evaluated numerically
  T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

and crossed-checked by independent QFT methods   A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

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  and crossed-checked by independent QFT methods   A. Kataev, Phys. Rev. D 86, 013019 (2012)

  A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

- $(\alpha/\pi)^5$: 6 classes, 32 gauge invariant subsets

  Five of these subsets are known analytically   S. Laporta, Phys. Lett. B 328, 522 (1994)

  J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

  Complete numerical results available

  T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops ($\longrightarrow a_e$)
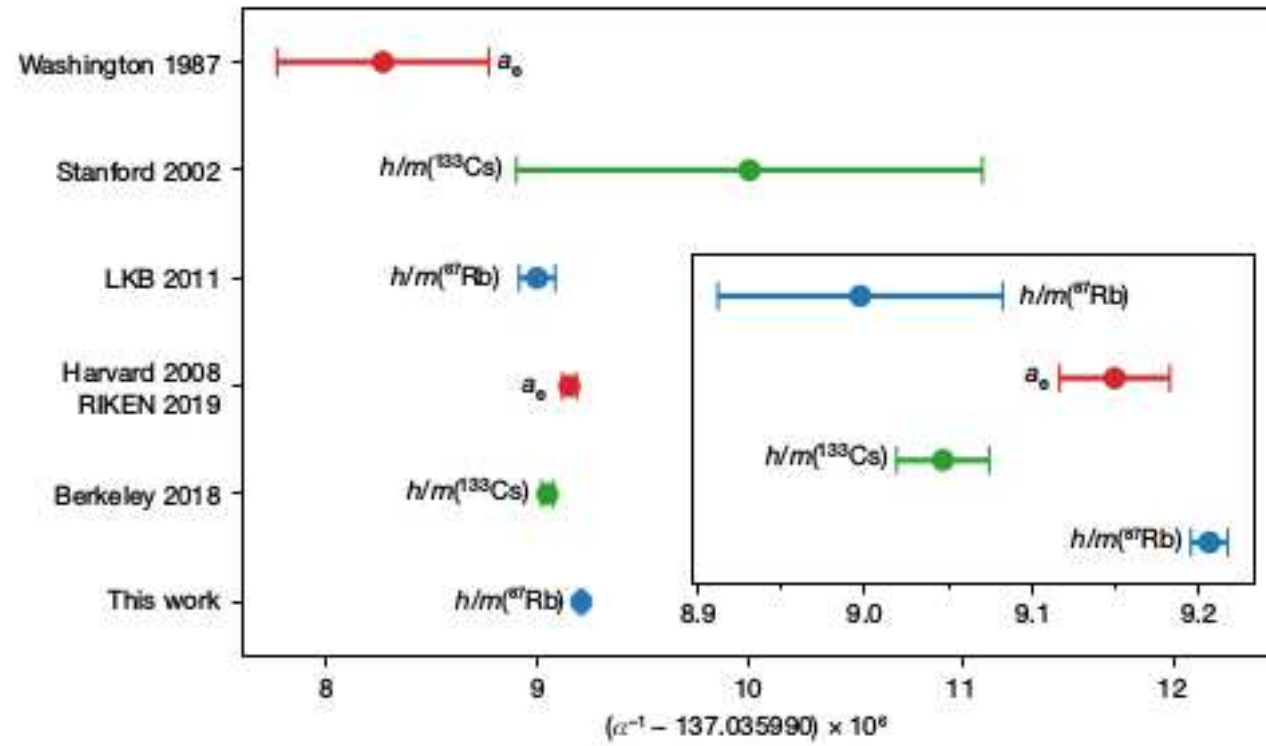
QED contribution :

$C_\mu^{(2)}$	0.5
$C_\mu^{(4)}$	$0.765\,857\,425(17)$
$C_\mu^{(6)}$	$24.050\,509\,96(32)$
$C_\mu^{(8)}$	$130.878\,0(61)$
$C_\mu^{(10)}$	$750.72(93)$

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Requires an experimental determination of α with

$$\frac{\Delta\alpha}{\alpha} \sim \frac{\Delta a_\mu}{a_\mu} \sim 0.14\text{ppm}$$

- Washington 1987 — a_e
- Stanford 2002 — $h/m(^{133}\text{Cs})$
- LKB 2011 — $h/m(^{87}\text{Rb})$
- Harvard 2008 / RIKEN 2019 — a_e
- Berkeley 2018 — $h/m(^{133}\text{Cs})$
- This work — $h/m(^{87}\text{Rb})$

Inset:
- $h/m(^{87}\text{Rb})$
- a_e
- $h/m(^{133}\text{Cs})$
- $h/m(^{87}\text{Rb})$

$(\alpha^{-1} - 137.035990) \times 10^{6}$
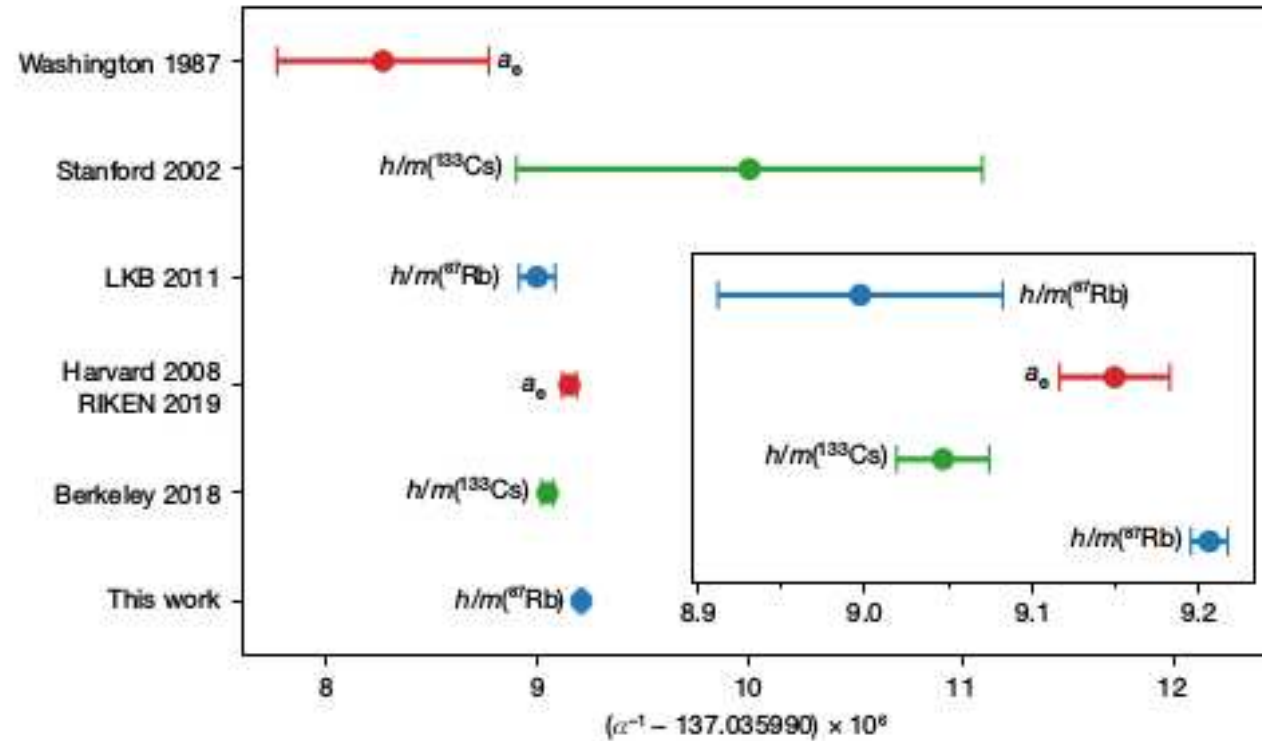
$\longrightarrow$ existing tension/discrepancy between $\alpha(\mathrm{Cs}18)$ and $\alpha(\mathrm{Rb}20)$ (but also between $\alpha(\mathrm{Rb}11)$ and $\alpha(\mathrm{Rb}20)$) of no concern for a_μ

$\longrightarrow$ for a_μ the value of α could be provided by the qH effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \quad [19.7\mathrm{ppb}]$$

P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)

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- $a_\mu^{\text{QED}}(\text{Cs}18) = 116\,584\,718.931(7)_{\text{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs19)} \cdot 10^{-11}$

- $$a_\mu^{\text{exp;WA}} - a_\mu^{\text{QED}}(\text{Cs}18) = 7342(41) \cdot 10^{-11}$$

● QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

● The missing part has to be provided by weak and strong interactions (or else, new physics...)

A few comments about the QED contributions

● Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13} \qquad \Delta a_\mu^{\mathsf{exp}} = 41 \cdot 10^{-11}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \longrightarrow \quad \sim 14 \cdot 10^{-11}$$

● Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 380 \cdot 10^{-11} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 5 \cdot 10^{-11}$$

● Drastic increase with n in the coefficients $C_\mu^{(2n)}$ $[\pi^2 \ln(m_\mu/m_e) \sim 50!]$

● Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3} \ln \frac{m_\mu}{m_e} - \frac{5}{9} \right]^3 \cdot 10 \left(\frac{\alpha}{\pi} \right)^6 \sim 5.4 \cdot 10^3 \cdot \left(\frac{\alpha}{\pi} \right)^6 \sim 0.08 \cdot 10^{-11}$$

● No sign of substantial contribution to a_μ from higher order QED

$$\left(\frac{\alpha}{\pi} \right)^6 = 1.56 \ldots \cdot 10^{-16} \quad ==> \quad \text{would require} \quad C_\mu^{(12)} \sim 10^6 ?!$$

Contribution from the electroweak sector

Weak contributions : W, Z,... loops

One-loop contributions

$$a_\mu^{\text{weak}(1)} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$

$$= 19.48\times10^{-10}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

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M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)

M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)

A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

Complete three-loop short-distance leading logarithms

G. Degrassi and G. F. Giudice, Phys. Rev. D 58, 053007 (1998)

Updated a few years ago: $a_\mu^{\text{weak}} = 15.36(10) \cdot 10^{-10}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

Recent numerical evaluation: $a_\mu^{\text{weak}} = (152.9 \pm 1.0) \cdot 10^{-11}$

T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

Contribution from the strong interactions

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- hadronic vacuum polarization
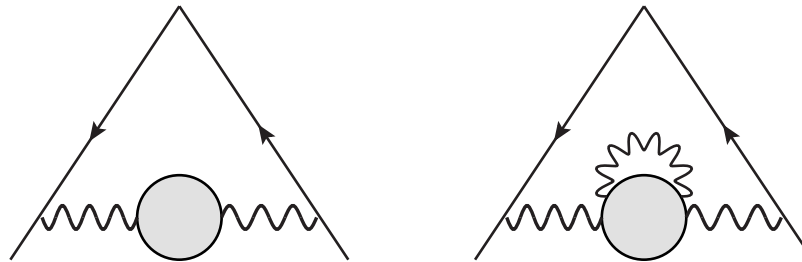
- (virtual) hadronic light-by-light (HLxL
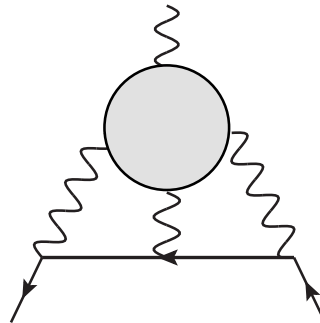
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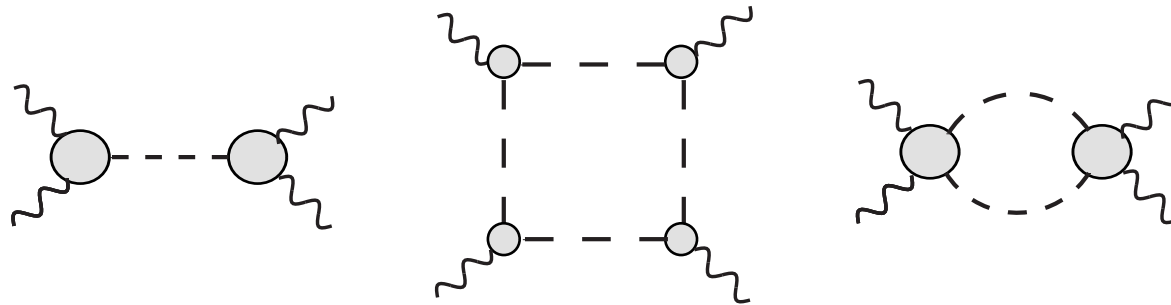
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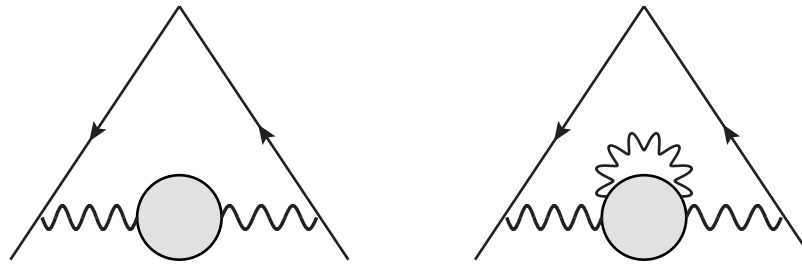
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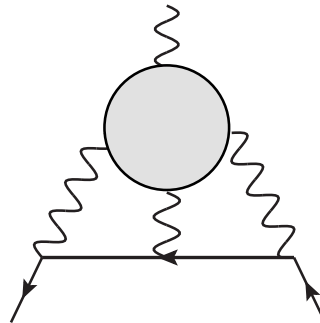
$\longrightarrow$ non-perturbative regime of QCD

Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$

- Involves the 4th-rank vacuum polarization tensor $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4), \sum q_i = 0$

- Used to be estimated via phenomenological models (pion pole,...)

- More recently: evaluation improved through dispersive approach

$$\Pi = \Pi^{\pi^0, \eta, \eta' \text{ poles}} + \Pi^{\pi^{\pm}, K^{\pm} \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Needs input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

A. Nyffeler, arXiv:1602.03398 [hep-ph]

- Short-distance constraints have been worked out...

K. Melnikov, A. Vainshtein, Phys.Rev.D 70, 113006 (2004)

J. Bijnens, N. Hermansson-Truedsson, A. Rodríguez-Sánchez, Phys.Lett. B 798, 134994 (2019)

J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, JHEP 10, 203 (2020); JHEP 04, 240 (2021)

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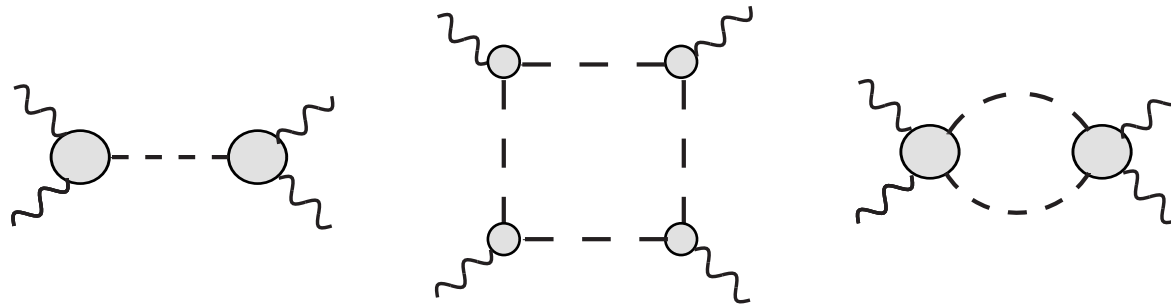
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G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub, P. Stoffer, JHEP03, 101 (2020)

J. Lüdtke, M. Procura, Eur. Phys. J. C 80, 1108 (2020)

J. G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, Eur. Phys. J. C 81, 702 (2021)

Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$

- Involves the 4th-rank vacuum polarization tensor $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$, $\sum q_i = 0$

- Used to be estimated via phenomenological models (pion pole,...)

- More recently: evaluation improved through dispersive approach

- More recently: lattice QCD evaluations

T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

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White Paper summary

$$a_\mu^{\text{HLxL}} = 92(19) \cdot 10^{-11}$$

Hadronic light-by-light

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- More recently: lattice QCD evaluations

- Post-WP: New lattice QCD result for HLxL at 15% accuracy

$$a_\mu^{\mathrm{HLxL}} = 109.6(15.9) \cdot 10^{-11}$$

E.-H. Chao et al., Eur. Phys. J. C 81, 651 (2021)

E.-H. Chao et al., Eur. Phys. J. C 82, 664 (2022)

Hadronic light-by-light

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- Post-WP: New lattice QCD result for HLxL at 15% accuracy
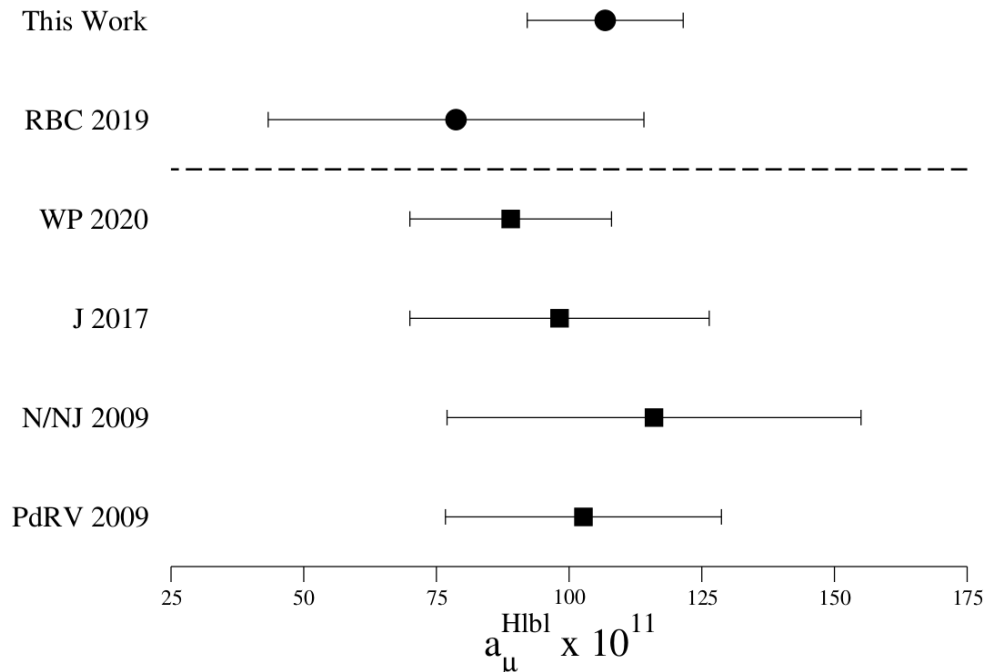
~10% accuracy goal seems within reach

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^\infty \frac{ds}{s} K(s) R^{\text{had}}(s) \quad K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)

L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)

M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization
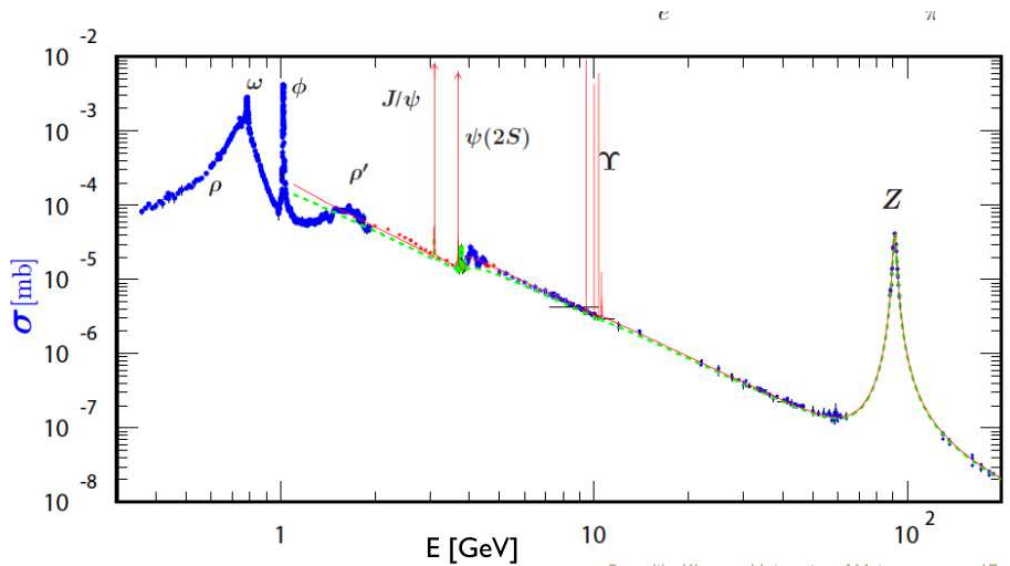
- Can be evaluated using available experimental data

- Combination of ~ 39 exclusive channels

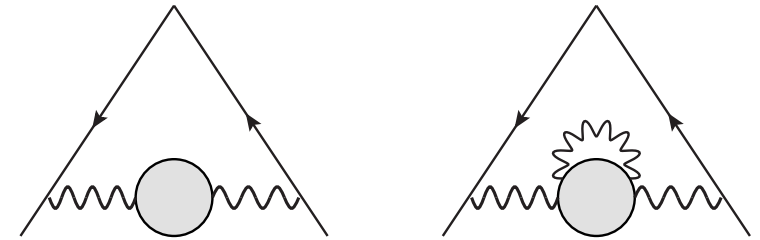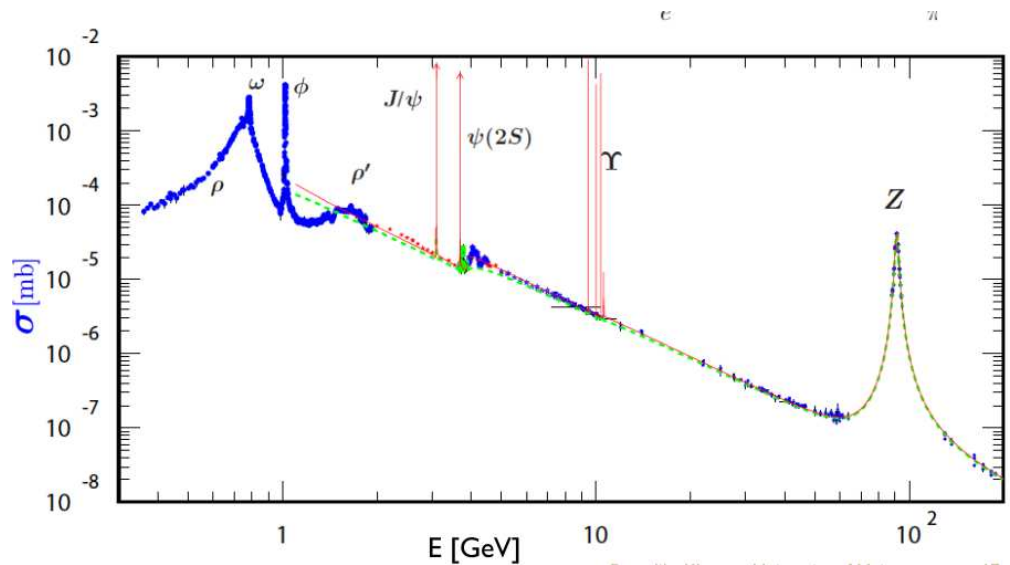$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Hadronic vacuum polarization

- Can be evaluated using available experimental data
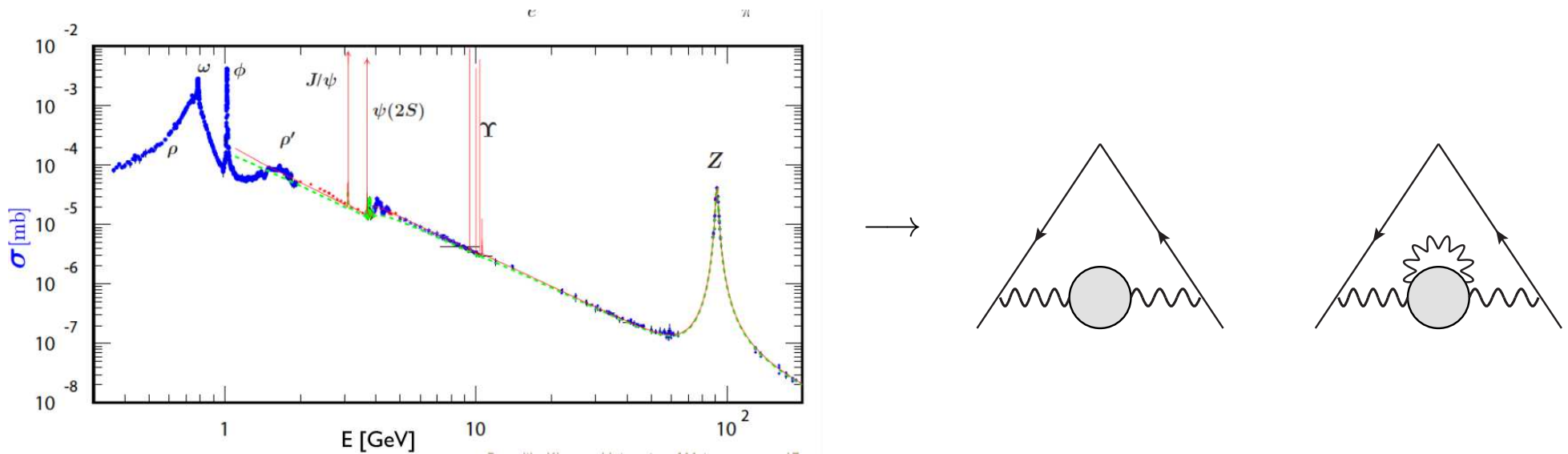
- Some tension, for instance, in the region of the ρ resonance

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600-900\,\mathrm{MeV})$
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)
KLOE comb.	366.9(2.1)
BaBar 09	376.7(2.0)(1.9)
BESIII 16	368.2(2.5)(3.3)
SND 04	371.7(5.0)
CMD-2 03,06	372.4 (3.0)

Hadronic vacuum polarization

- Can be evaluated using available experimental data
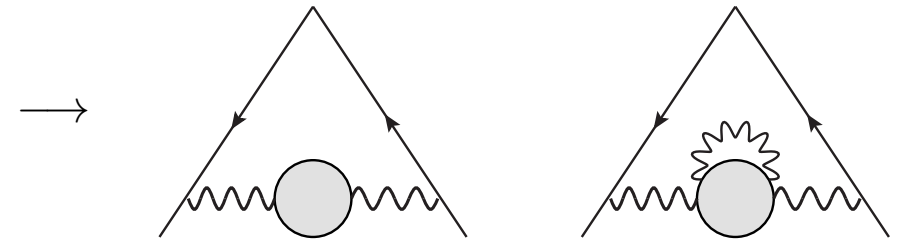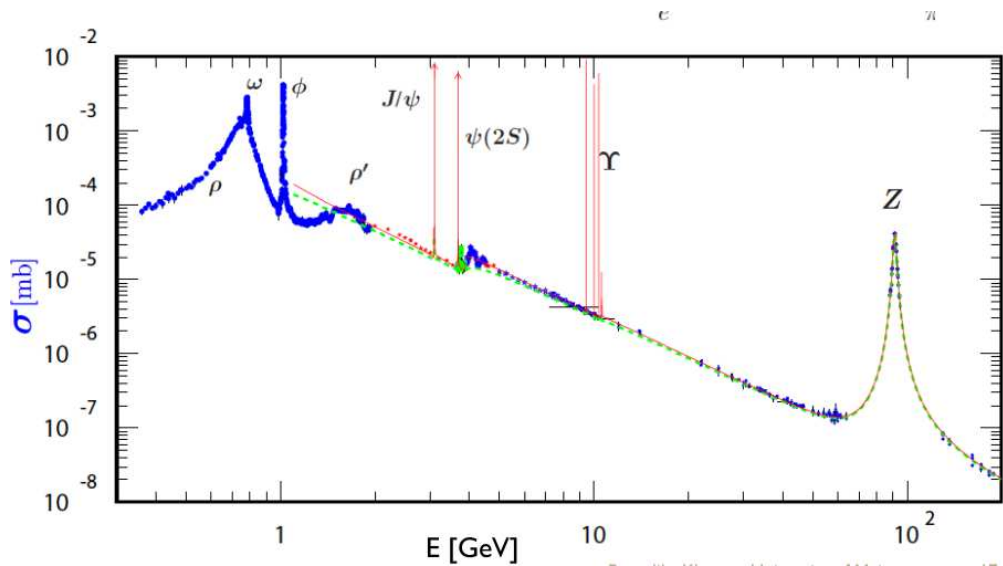
- Dispersive analyses of the $\pi\pi$ and $\pi\pi\pi$ channels provide cross-checks

$$a_\mu^{\pi\pi}|_{\leq 1GeV} = 495.0(1.5)(2.1) \cdot 10^{-10}$$

G. Colangelo, M. Hoferichter, P. Stoffer, JHEP 1902,006 (2019)

$$a_\mu^{\pi\pi\pi}|_{\leq 1.8GeV} = 46.2(6)(6) \cdot 10^{-10} \qquad a_\mu^{\text{HVP-LO}} = 692.3(3.3) \cdot 10^{-10}$$

M. Hoferichter, B.-L. Hoid, B. Kubis, JHEP 08, 137 (2019)

Hadronic vacuum polarization
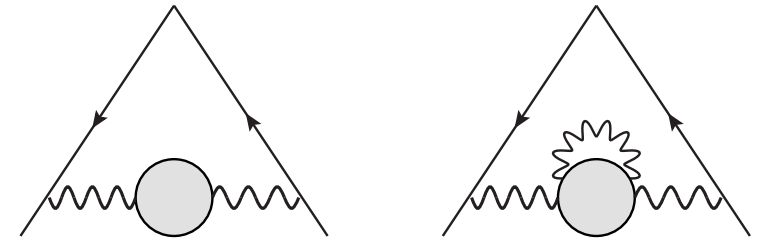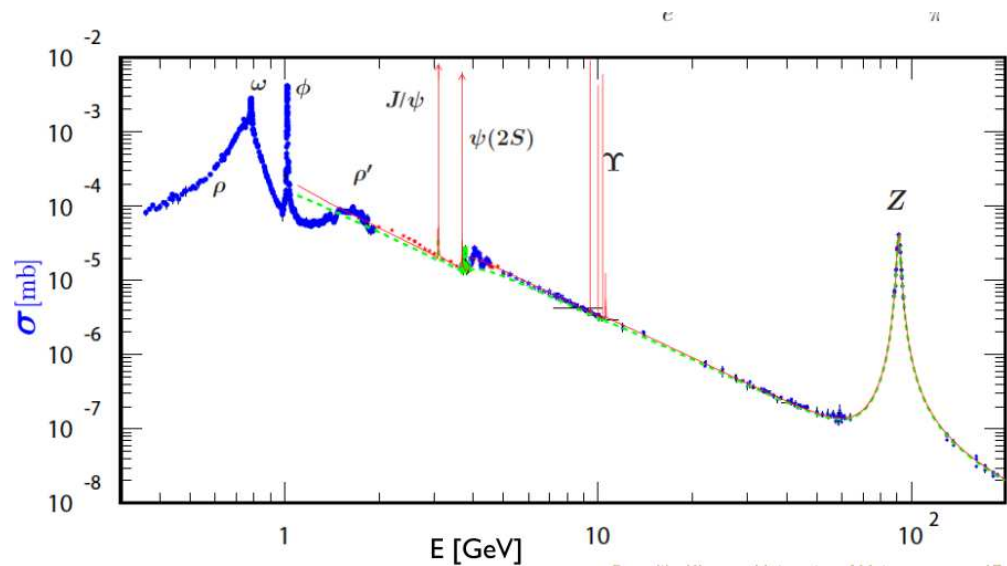
- Can be evaluated using available experimental data

- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- More is to come

$\longrightarrow$ SND, CMD-3 (ongoing analysis of the $\pi\pi$ channel)

$\longrightarrow$ BaBar, BES III, BELLE II

$\longrightarrow$ Possiblity to measure HVP in the space-like region from μe scattering

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

Hadronic vacuum polarization

- Can be evaluated using available experimental data

- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- Several lattice results (for the time being, stick to WP)

A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)

C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]

E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]

D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)

T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)

S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)

M. Della Morte *et al.*, JHEP 10, 020 (2017)

White Paper summary

- Data evaluation:

$$a_\mu^{\mathrm{HVP;LO}} = 6931(40) \cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NLO}} = -98.3(7) \cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NNLO}} = 12.4(1) \cdot 10^{-11}$$

- Lattice WA: $a_\mu^{\mathrm{HVP;LO}} = 7043(150) \cdot 10^{-11}$
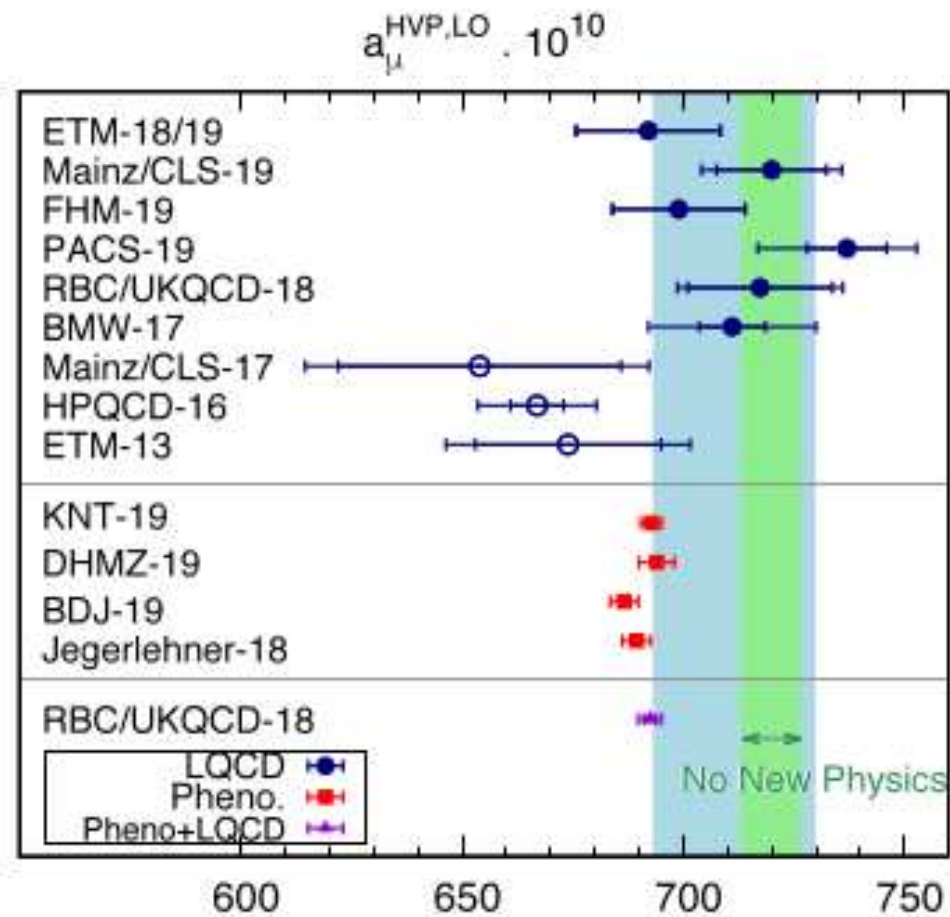
White Paper summary

$$a_\mu^{\text{th;WP}} = 116\,591\,810(43) \cdot 10^{-11}$$

The situation today

$a_\mu^{\mathrm{th;WP}}$ based on

$$a_\mu^{\mathrm{HVP-LO;WP}} = 6931(40) \cdot 10^{-11}$$
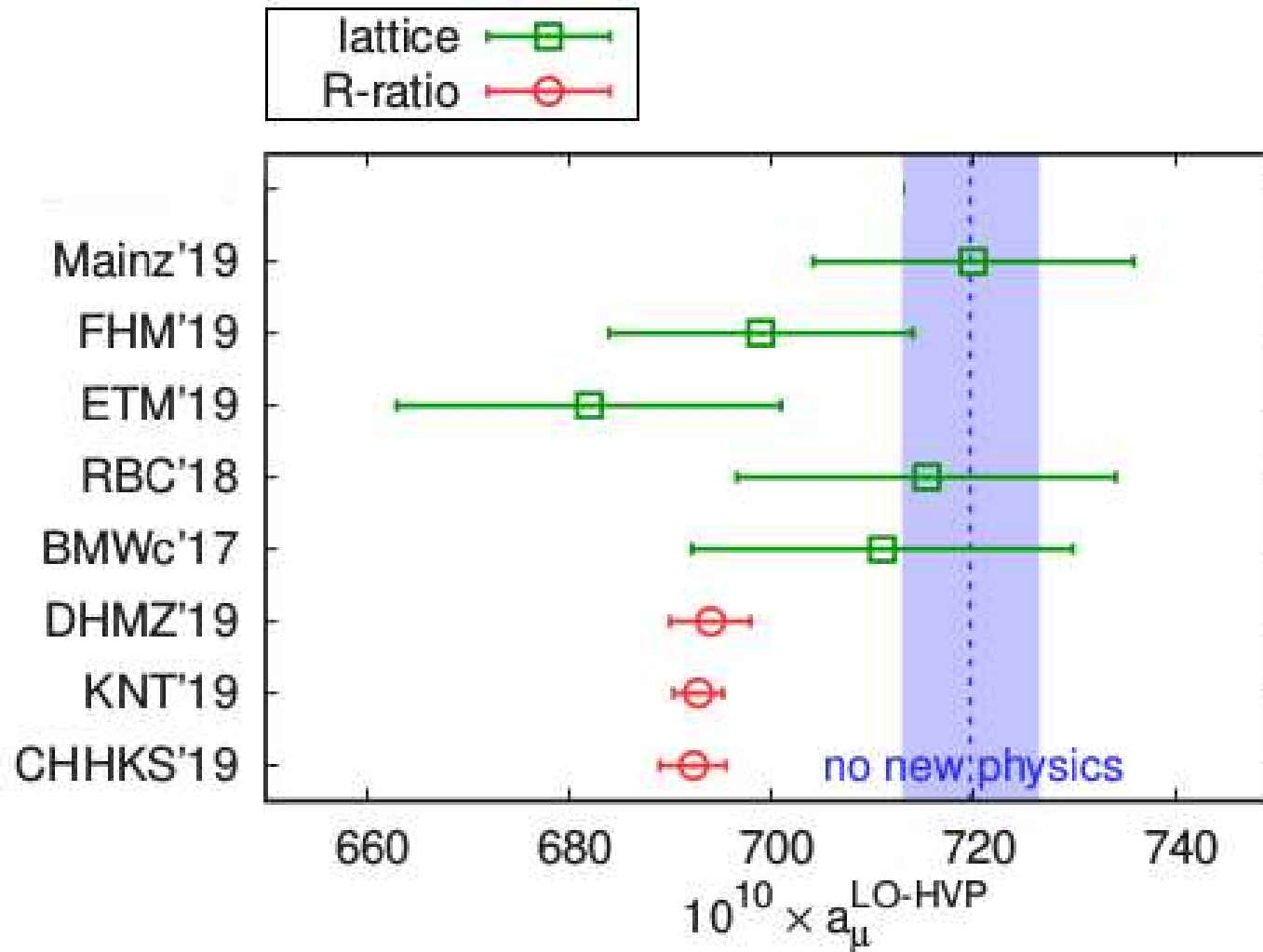
The situation today

New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021)
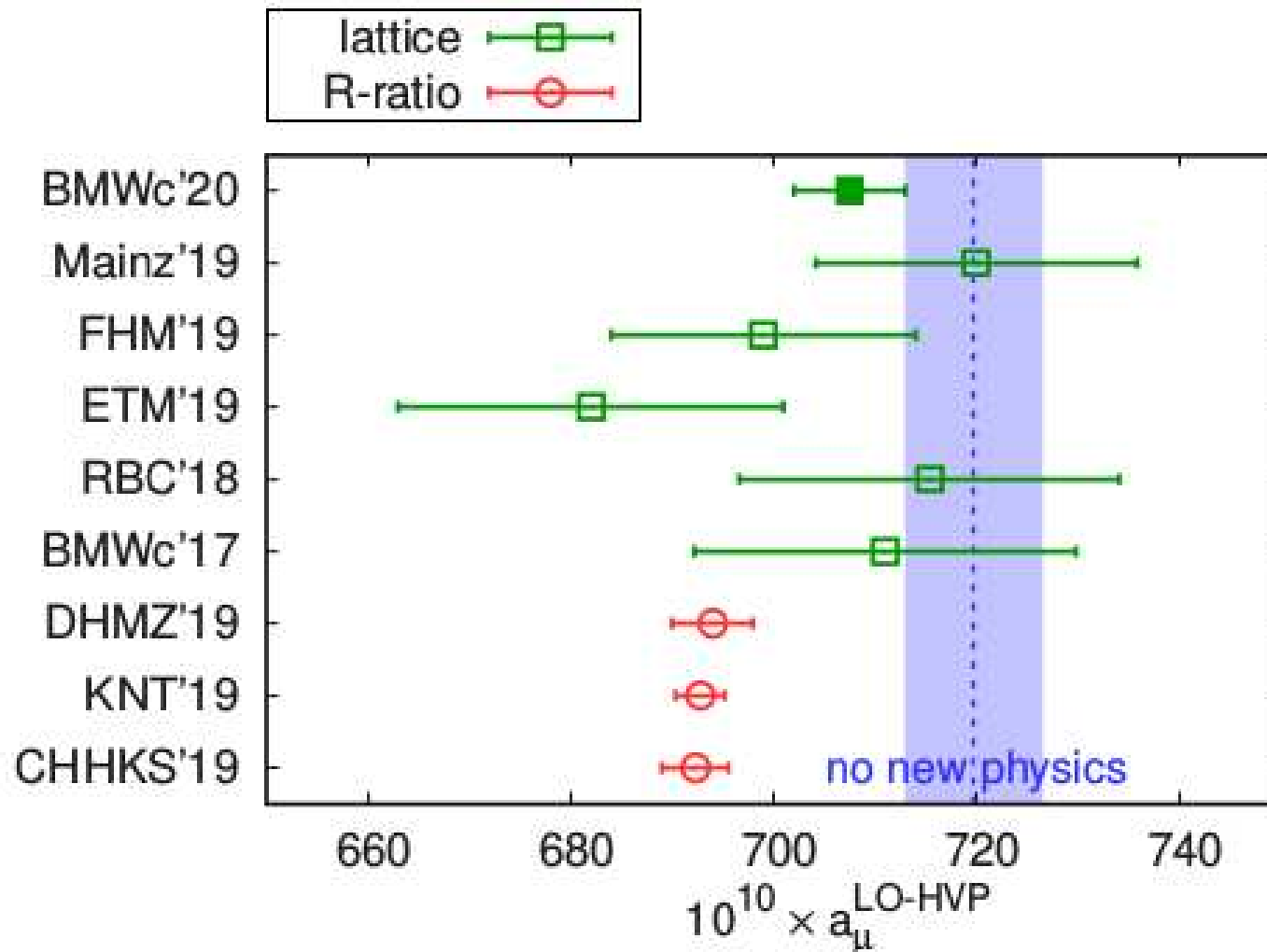
The situation today

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S. Borsanyi et al., Nature 593, 7857 (2021)

Systematic effets (finite size, discretization,...) need to be scrutinized

Requires independent confirmation

Post-WP results

New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\text{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021)

Systematic effets (finite size, discretization,...) need to be scrutinized
Requires independent confirmation

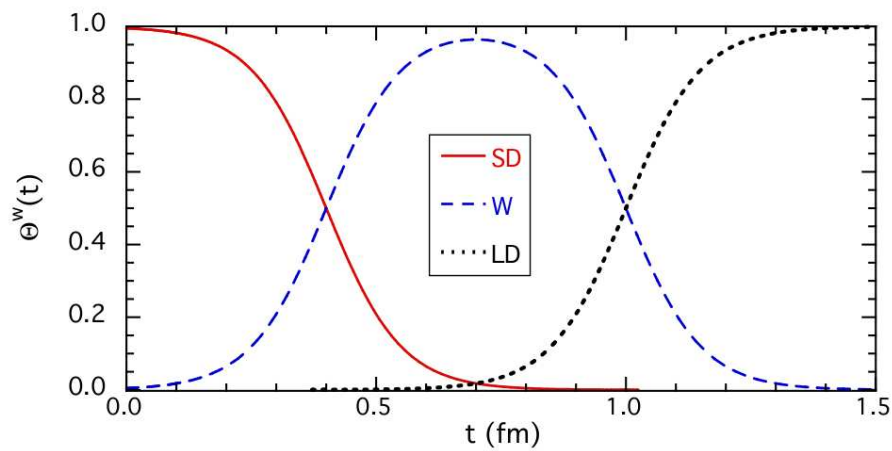S. Borsanyi et al., Nature 593, 7857 (2021)

M. Cè et al., arXiv:2206.06582 [hep-lat]

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

Several independent cross-checks for the intermediate window
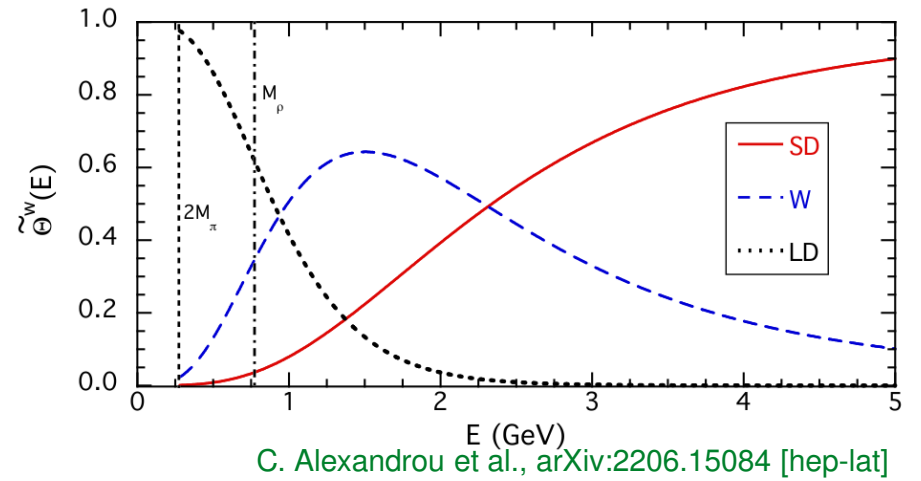a_μ^{IW}: $0.4\text{fm} \leq t_{\text{E}} \leq 1.0\text{fm}$
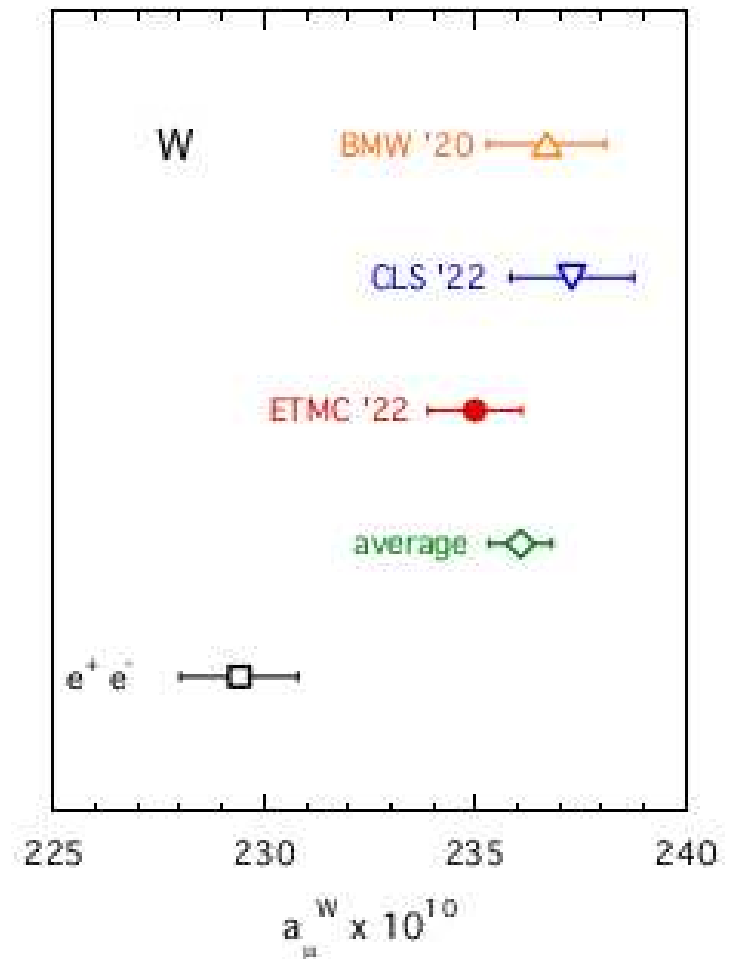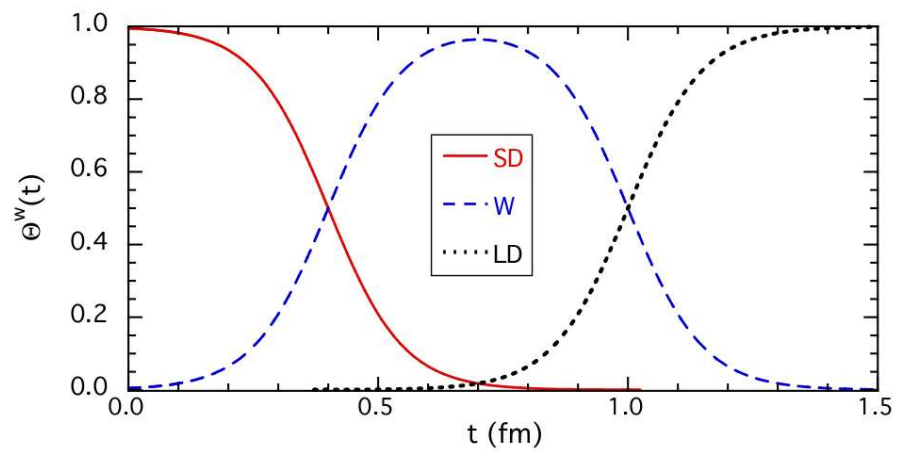
C. Alexandrou et al., arXiv:2206.15084 [hep-lat]
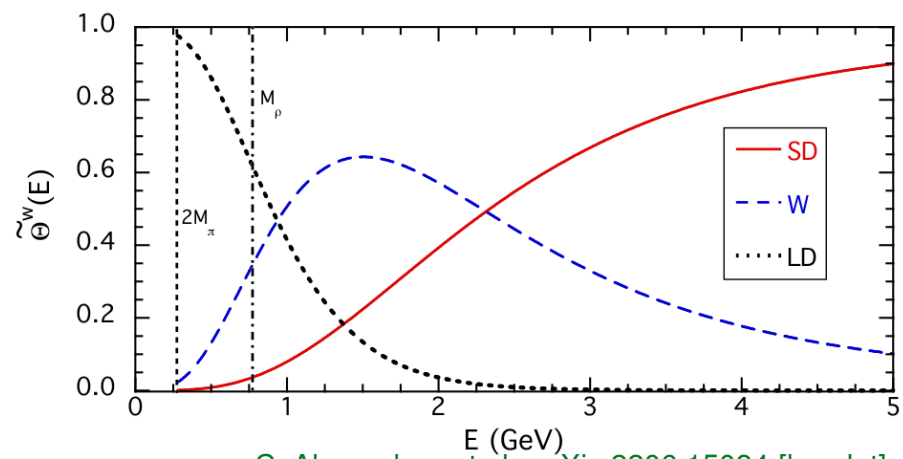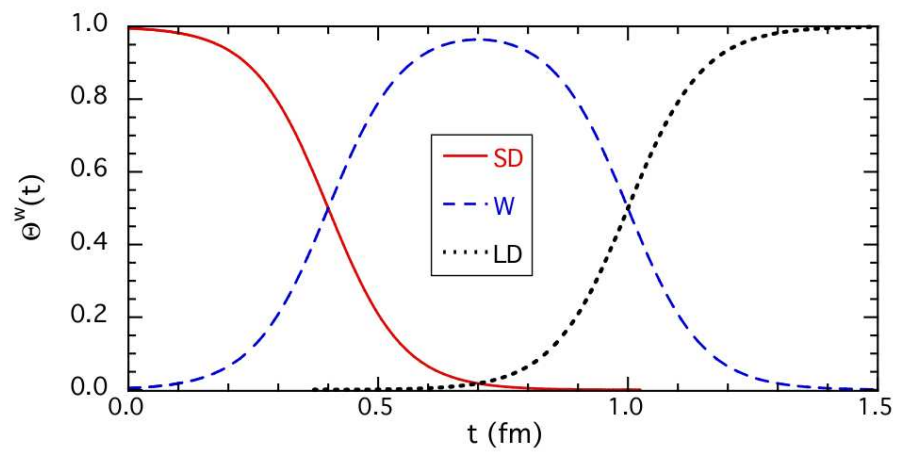
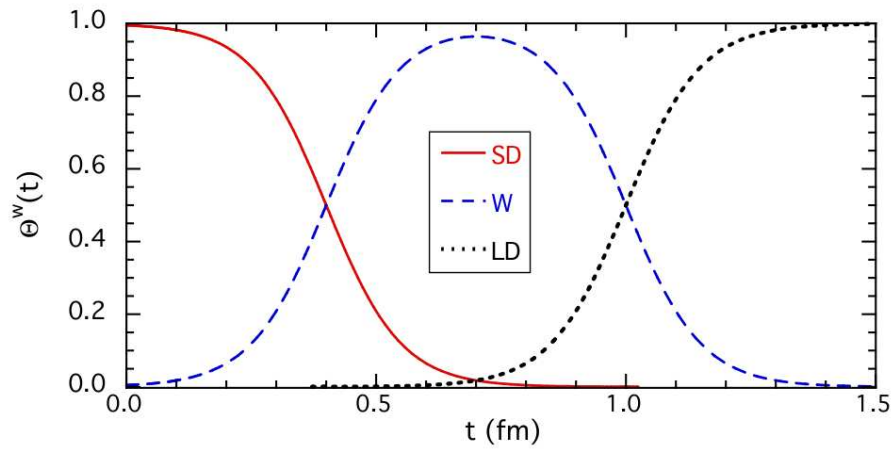C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

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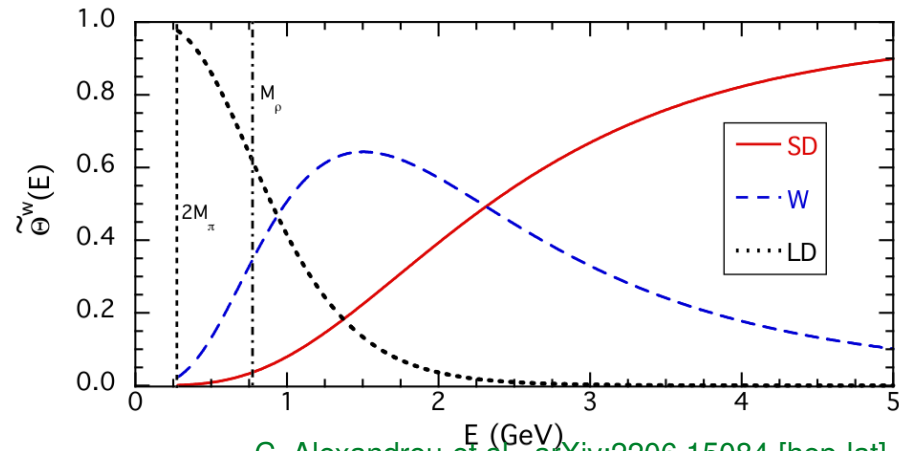
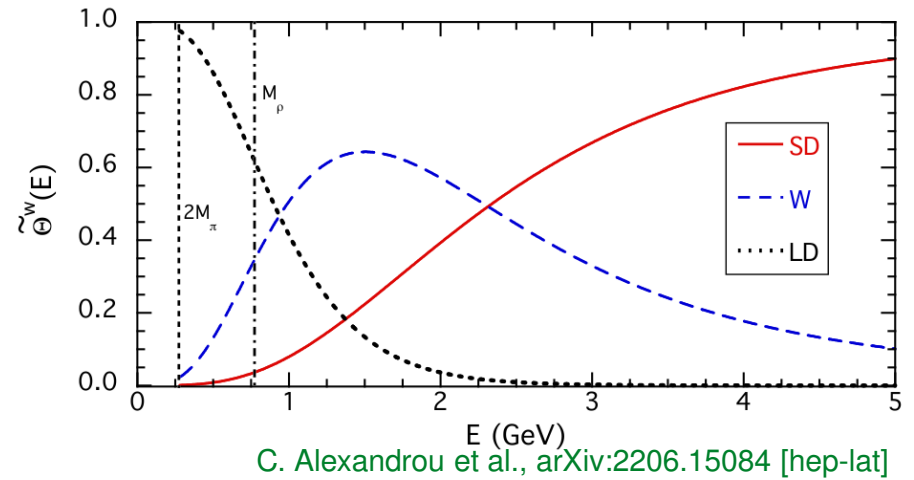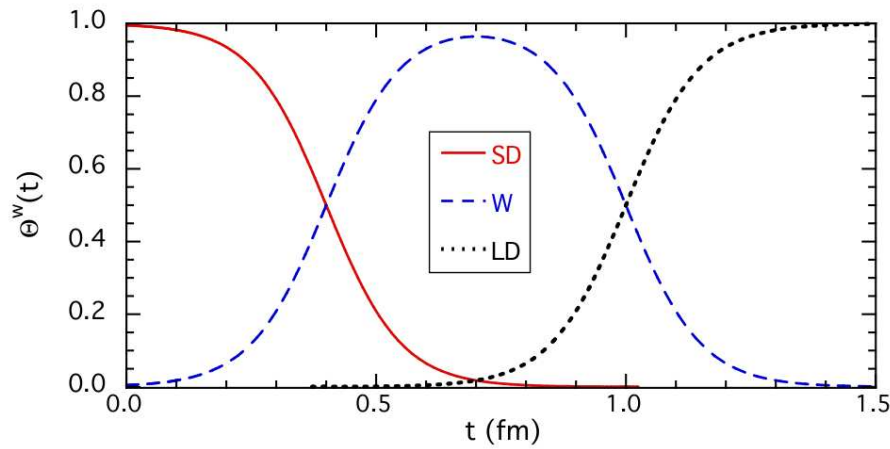C. Alexandrou et al., arXiv:2206.15084 [hep-lat]
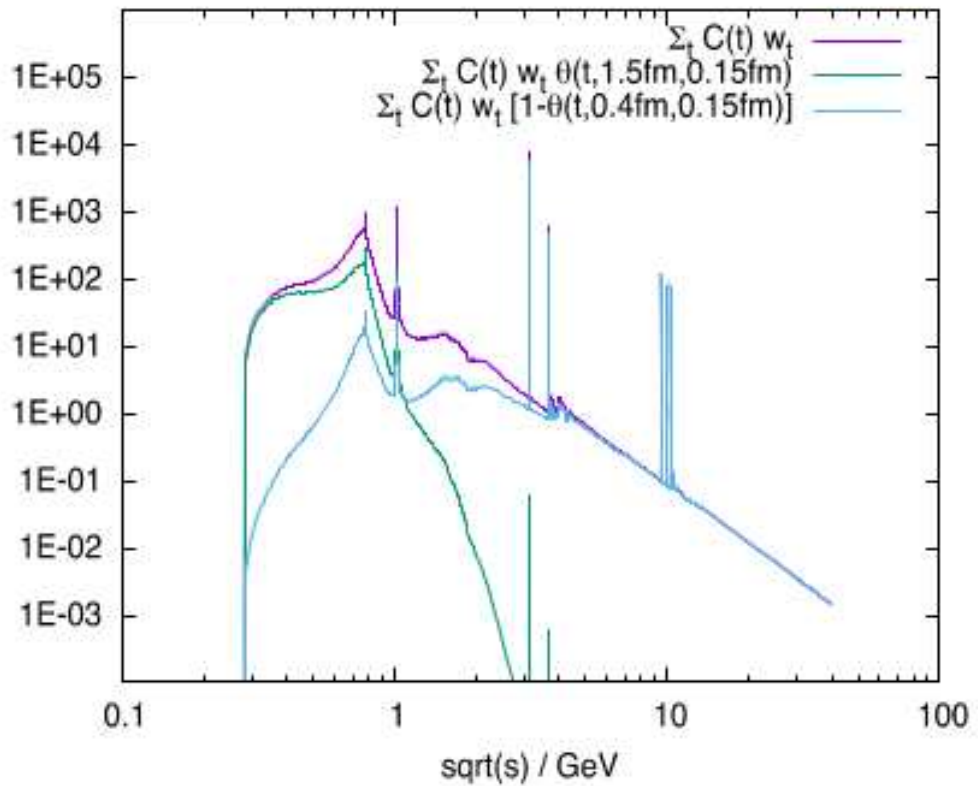
$$a_\mu^{\text{SDW}} \sim 10\% \text{ of } a_\mu^{\text{HVP;LO}}$$

$$a_\mu^{\text{IW}} \sim 30\% \text{ of } a_\mu^{\text{HVP;LO}}$$

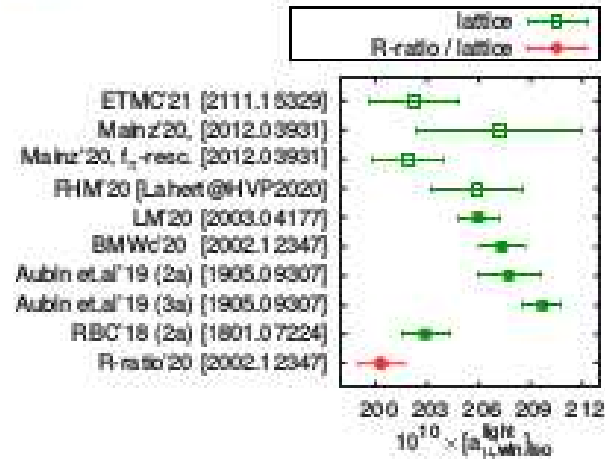$$a_\mu^{\text{LDW}} \sim 60\% \text{ of } a_\mu^{\text{HVP;LO}}$$

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

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$$a_\mu^{\mathrm{IW}} \sim 30\% \text{ of } a_\mu^{\mathrm{HVP;LO}} \quad \text{but } 50\% \text{ of the difference with data} - \text{based determination}$$
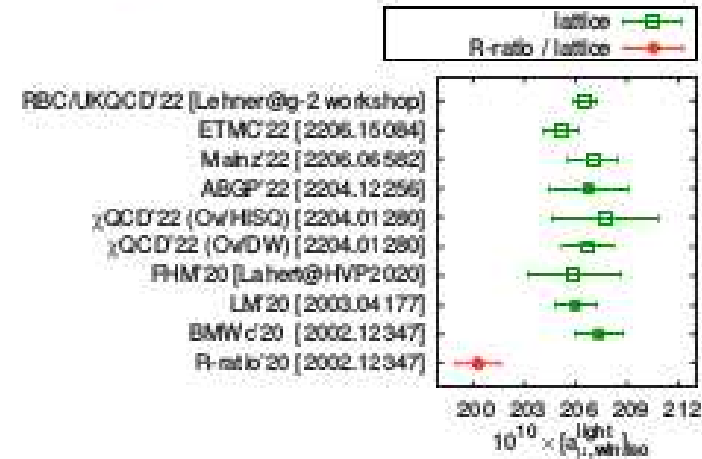
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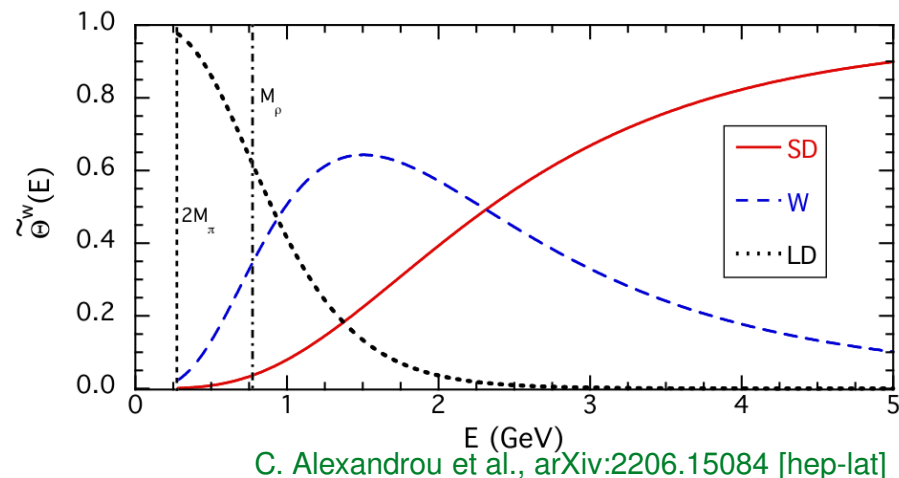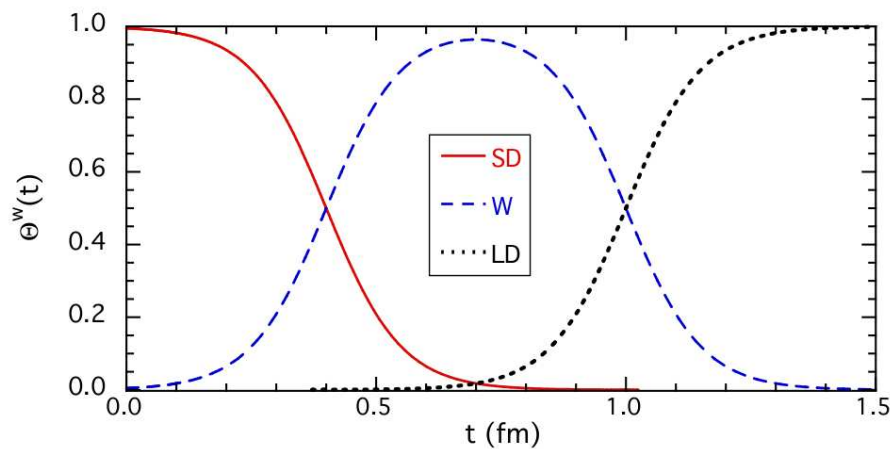
Window observable

Status in 2021:

Status now:

- Latest result from each group ⟶ consensus within lattice community
- R-ratio vs lattice discrepancy has to be understood

B. Thot, FCCP 2022

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

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More windows can allow for finer studies of the discrepancy between LQCD and data-based determination (correlations matter)

G. Colangelo, A. X. El-Khadra, M. Hoferichter, A. Keshavarzi, C. Lehner, P. Stoffer, T. Teubner, Phys. Lett. B 833, 137313 (2022)

Work in progress

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

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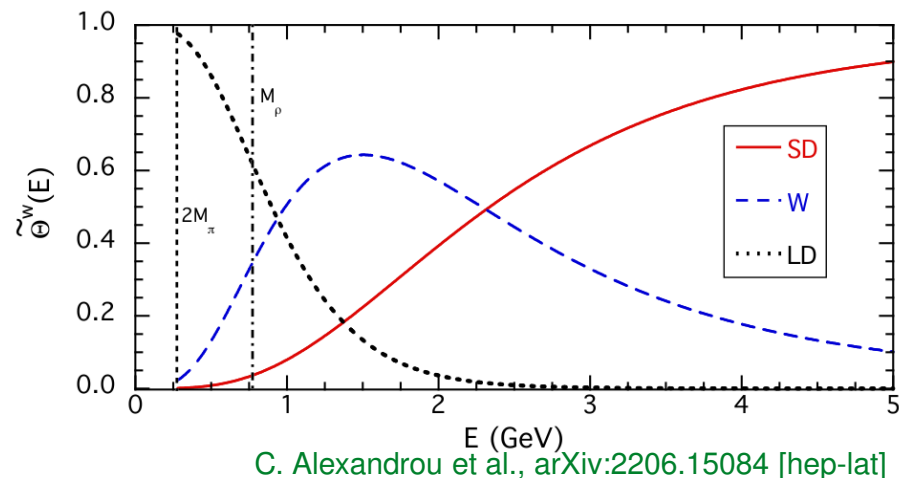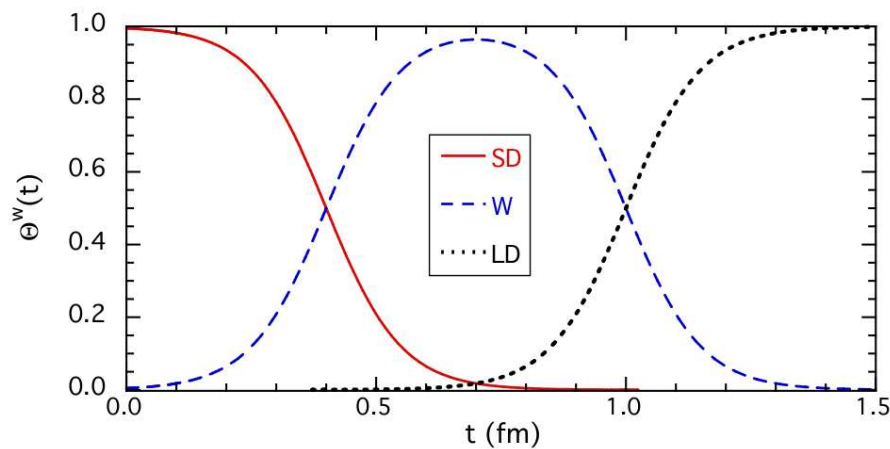
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Work in progress

So far, no complete LQCD cross-check of BMWc result available

Conclusion

FNAL-E989 running fine, next announcement eagerly awaited

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If $a_\mu^{\text{th;WP}} = a_\mu^{\text{th;SM}}$, discrepancy with $a_\mu^{\text{exp;WA}}$ will be difficult to explain within SM (many cross-checks)

$$a_\mu^{\text{exp}} - a_\mu^{\text{th;WP}} \sim \begin{cases} a_\mu^{\text{QED}}(\alpha^4) \\ 60 \cdot a_\mu^{\text{QED}}(\alpha^5) \\ 5 \cdot a_\mu^{\text{weak(2)}} \\ 3 \cdot a_\mu^{\text{HLxL}} \end{cases}$$

FNAL-E989 running fine, next announcement eagerly awaited

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Do we really understand $a_\mu^{\text{HVP}}\big|_{\text{data}}$ and $a_\mu^{\text{HVP}}\big|_{\text{LQCD}}$?

(at the level of precision that is claimed)

- Possibility to measure HVP in the space-like region from μe scattering?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $d\sigma/dt \propto |\alpha(t)|^2 \; t \leftrightarrow \theta_2 \, \theta_e \in$

- $a_\mu^{\rm HVP} - \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\rm had}\left(-\frac{x^2}{1-x} m_\mu^2\right)$

$t = \frac{x^2 m_\mu^2}{x-1}, \; 0 \leq -t < +\infty, \; 0 \leq x < 1$

- $a_\mu^{\rm HVP}$ given by the integral

- measurement of $\Delta\alpha_{\rm had}$ in the space-like region

- contribution at small t enhanced

- a 0.3% error can be achieved in 2y of data taking with $1.3 \times 10^7 \; \mu/{\rm s}$ at $E_\mu = 150$ GeV (CERN)

$\longrightarrow$ challenging (systematics at the level of 10ppm)

$\longrightarrow$ inclusive measurement (more like lattice QCD)

$\longrightarrow$ MUonE coll. LoI CERN-LHCC-2017-009/CMS-TDR-014

$\longrightarrow$ test run (proof of concept, assessment of systematics,...) scheduled for the end of 2021

$\longrightarrow$ Running time during LHC-Run3 (sufficient time left?)

$\longrightarrow$ postponed to 2023?

$x_{\rm peak} \simeq 0.914$
$t_{\rm peak} \simeq -0.108$ GeV^2
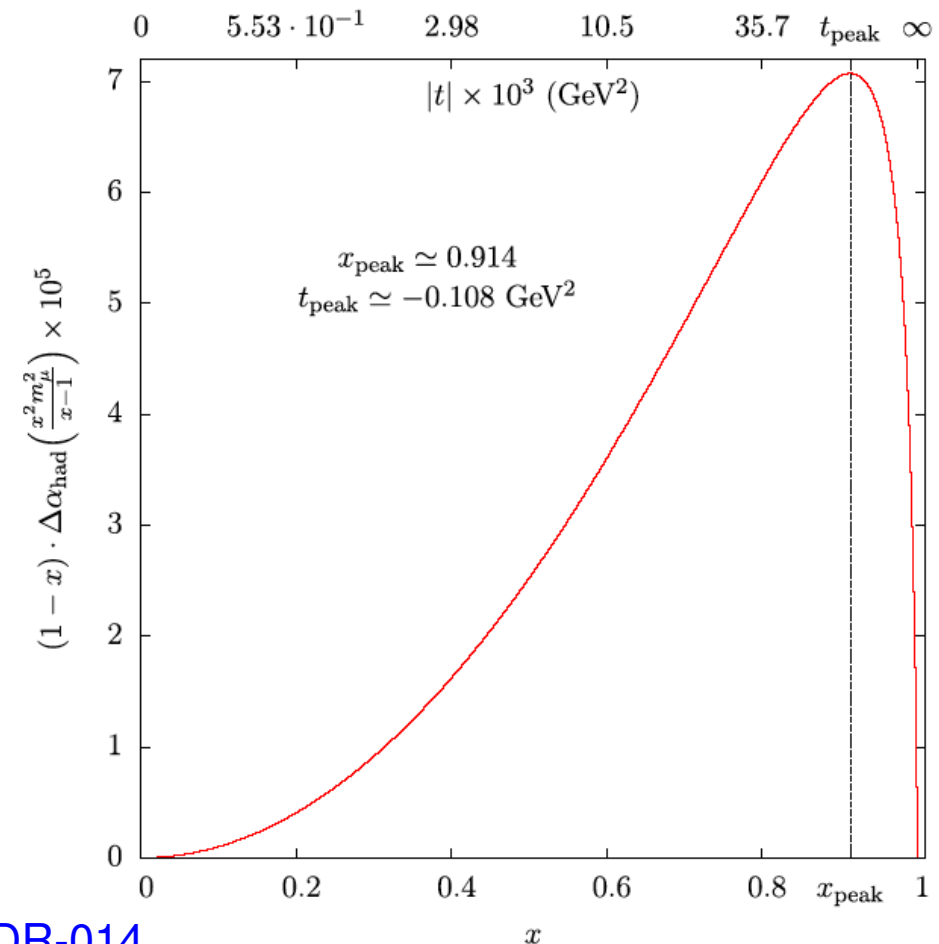
Thanks for your attention!