Properties of low-lying charmonia and bottomonia from lattice QCD+QED

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Hadron spectroscopy, March 2022

Lattice QCD enables precision tests of the Standard Model when accurate theory and experiment results can be brought together.

The ground-state heavyonium mesons (η_c, J/ψ, η_b, Υ) can provide such tests

D. Hatton, CD, J. Koponen, P.Lepage, A.Lytle, 2005.01845, 2101.08103

HPQCD DiRAC

To achieve high precision we have:

E. Follana et al, HPQCD, hep-ph/0610092

1) Used HPQCD's Highly Improved Staggered Quark (HISQ) action - designed as a very accurate discretisation of the Dirac equation.

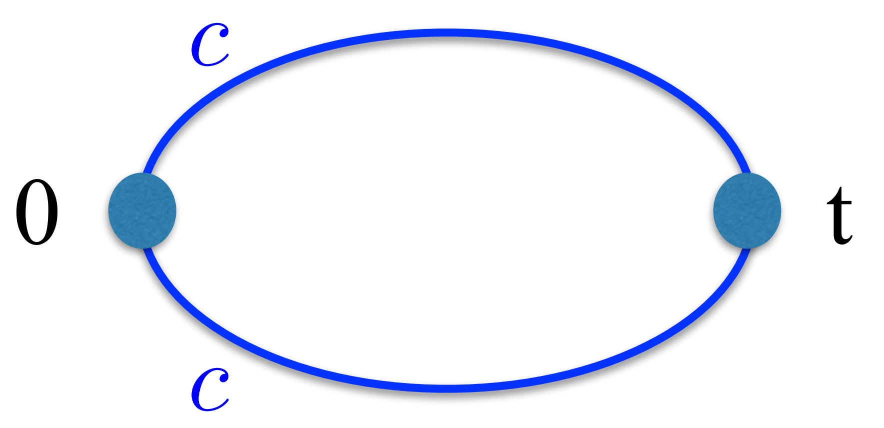
2) Used 17 gluon field configs that include u, d, s and c sea quarks with the HISQ action (generated by MILC). Lattice spacing (a) values range from 0.15fm to 0.03fm. u/d sea quark masses from ms/5 to physical.

3) Included 'quenched' QED to allow for effects from electric charge of c/b quark. (Random photon field incorporated with gluon when solving Dirac eq.)

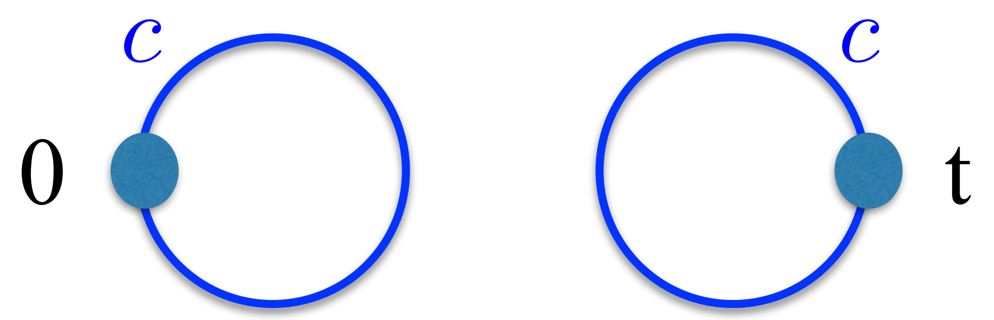
Calculational details for charmonium

D. Hatton et al, HPQCD, 2005.01845

Calculate ‘connected’ two-point correlation functions for 0^- and 1^- for both lattice QCD and QCD+QED. Statistical accuracy very high.
NOTE: annihilation to gluons NOT included. Key question: What impact does this have?



included for 0^- and 1^-



not included

Fit correlators to obtain ground-state masses (E_0) and amplitudes (A_0)

$$C(t) = \sum_i A_i (e^{-E_i t} + e^{-E_i (L_t - t)})$$

Convert A_0 into a ‘decay constant’ : $$f_{0^-} = 2m_q \sqrt{\frac{2A_0^{0^-}}{(E_0^{0^-})^3}} \qquad f_{1^-} = Z_V \sqrt{\frac{2A_0^{1^-}}{E_0^{1^-}}}$$

Z_V is renormalisation factor to match lattice vector current to that in continuum QCD - calculate accurately using intermediate symmetric MOM scheme

D. Hatton et al, HPQCD, 1909.00756

Tune m_c so J/ψ mass = experiment for every gluon ensemble (lattice spacing fixed from w_0 and f_π).
Do this for both QCD and QCD+QED (so ‘physical tuning’ same in both cases).

Impact of QED on charmonium

Meson mass shift with QED:

Net positive effect on J/ψ mass means m_c is lower in QCD+QED. Must also calculate Z_m in QCD+QED. Total effect = -0.2% at scale of 3 GeV.

$$\Delta M_{q_1\overline{q}_2} = Ae_{q_1}e_{\overline{q}_2} + Be_{q_1}^2 + Ce_{q_2}^2$$

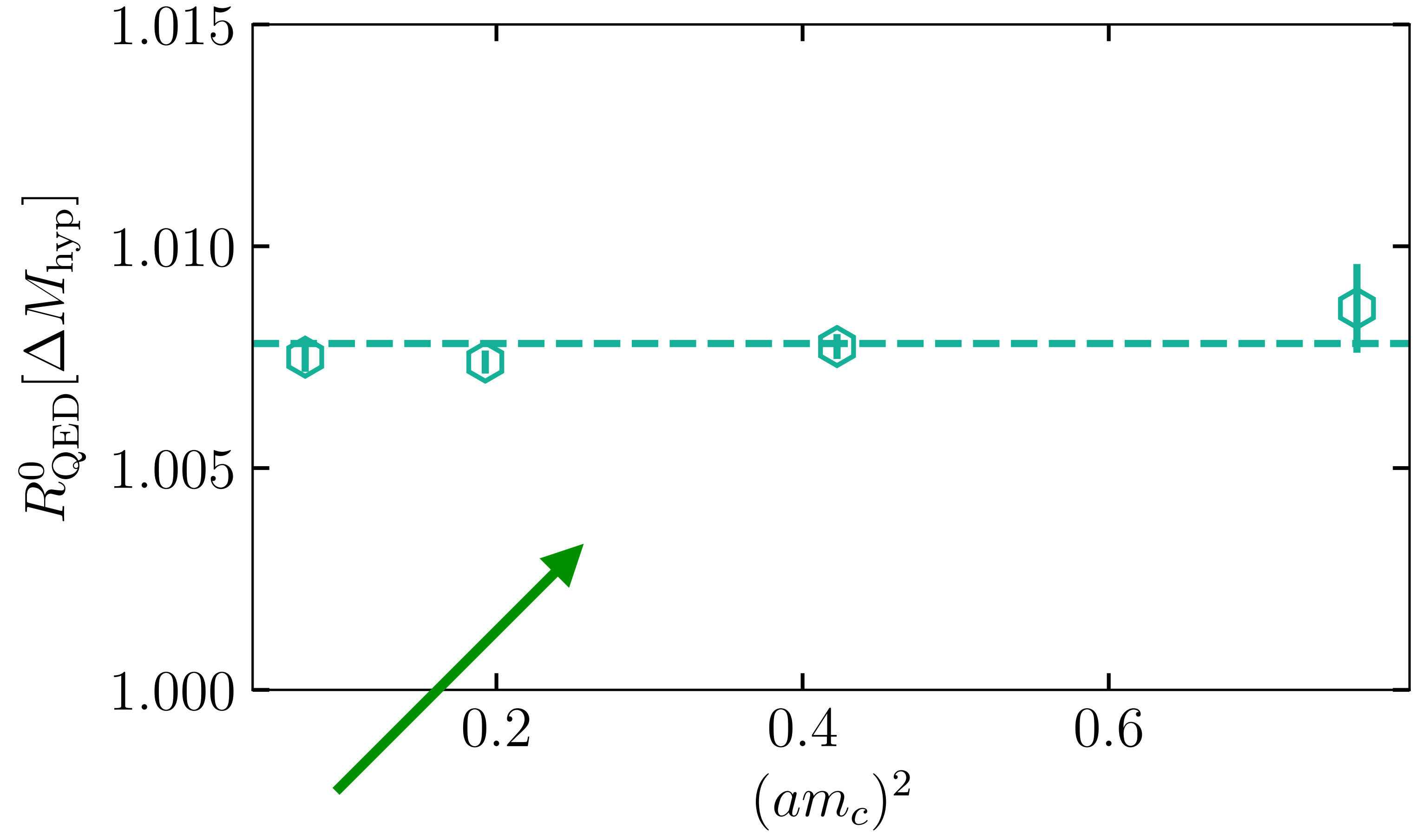
Interaction term - negative for neutral mesons

Self-energy terms - positive (and larger)

$$R^0 = \left.\frac{\mathrm{QCD}+\mathrm{QED}}{\mathrm{QCD}}\right|_{\text{fixed am}}$$

Ratio of QCD+QED to QCD at fixed bare quark mass is convenient to calculate, but does not allow for quark mass retuning.

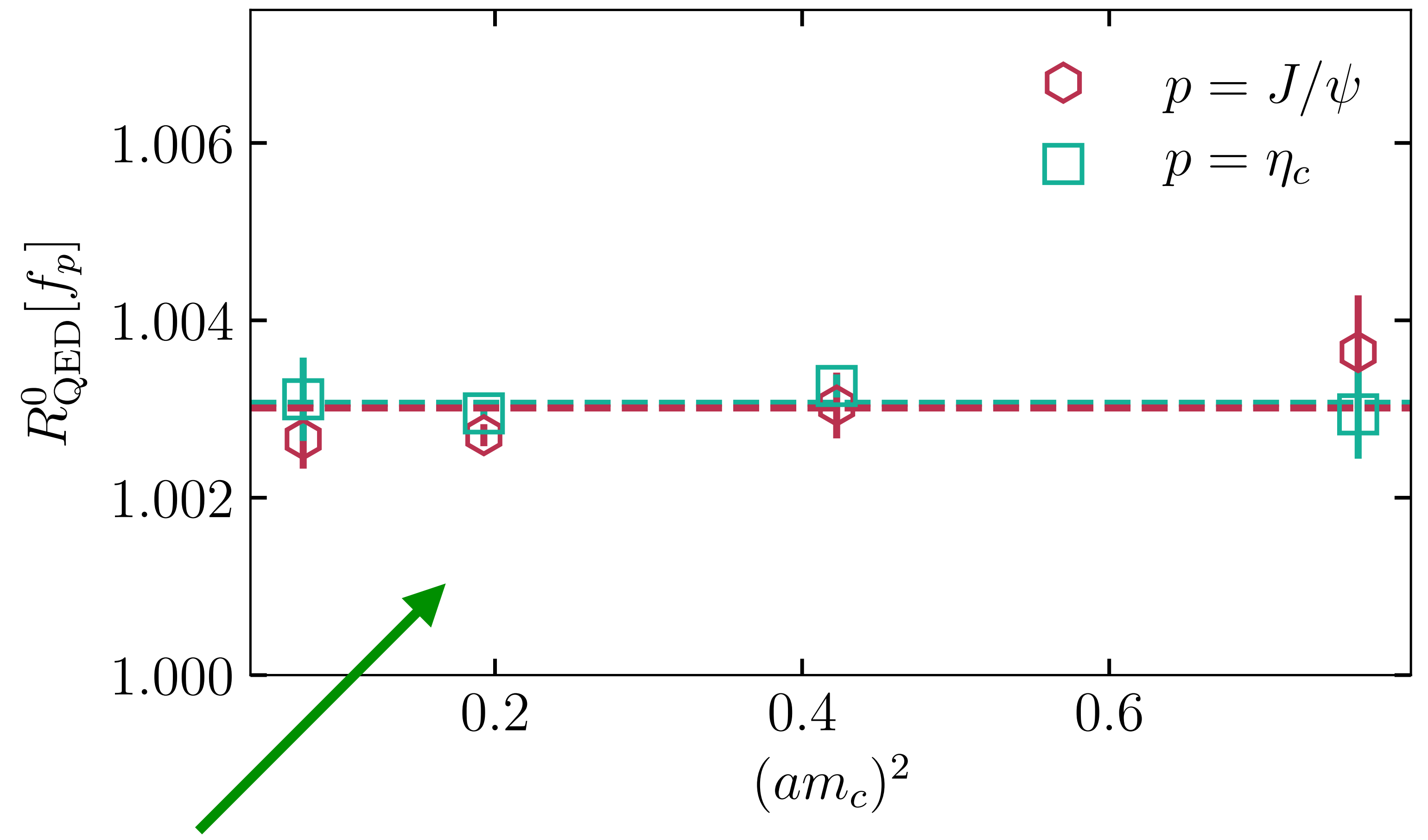
Impact of QED on hyperfine splitting



Including QED INCREASES the hyperfine splitting by 0.8% (0.7% direct and 0.1% from retuning m_c)

There is an additional QED contribution from J/ψ annihilation to a photon - estimate with pert. th. as +0.7 MeV.

Impact of QED on decay constants



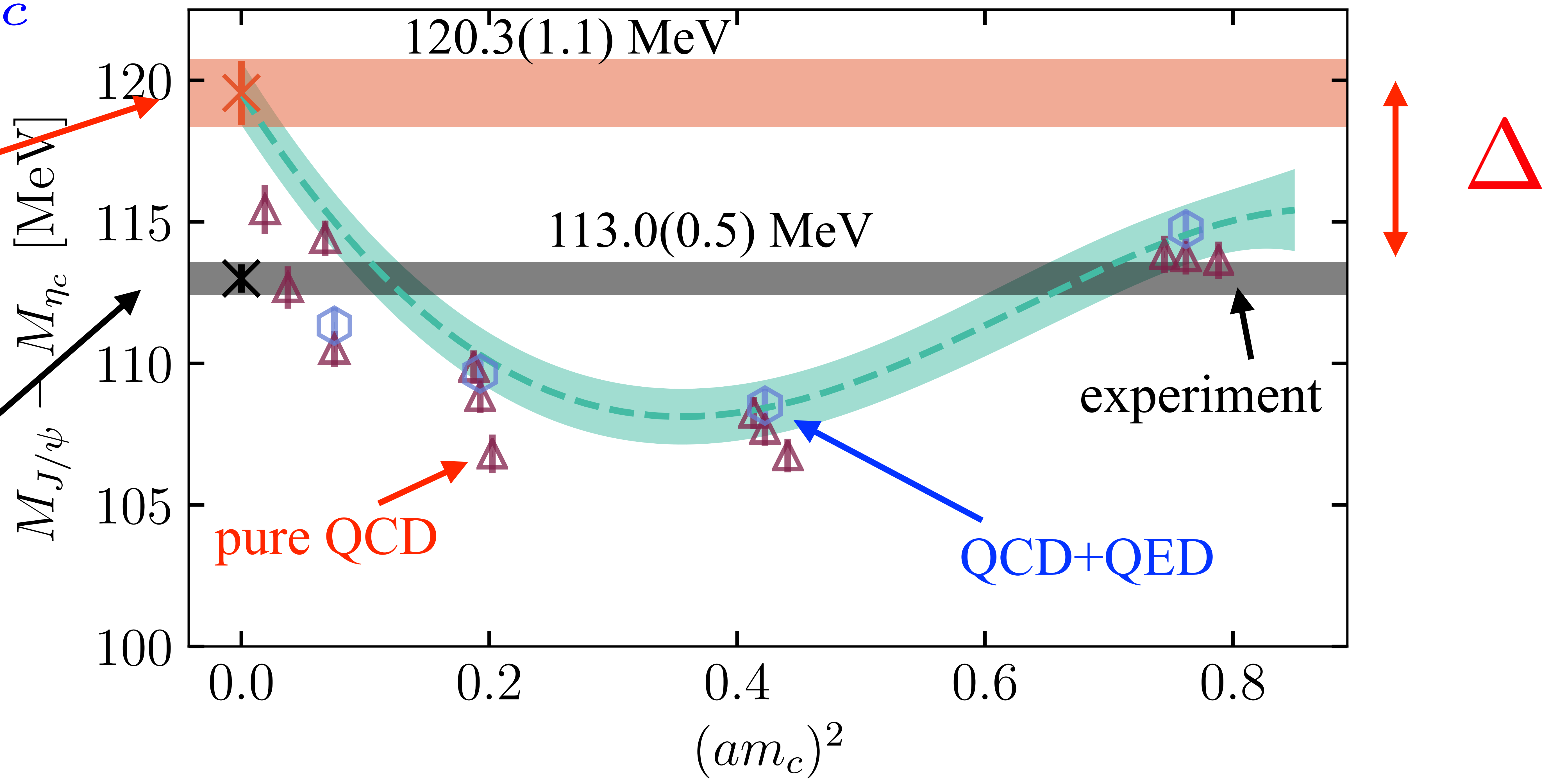
Including QED INCREASES the decay constants by 0.2% (0.3% direct and -0.1% from retuning m_c)

Hyperfine Splitting: $M_{J/\psi} - M_{\eta_c}$

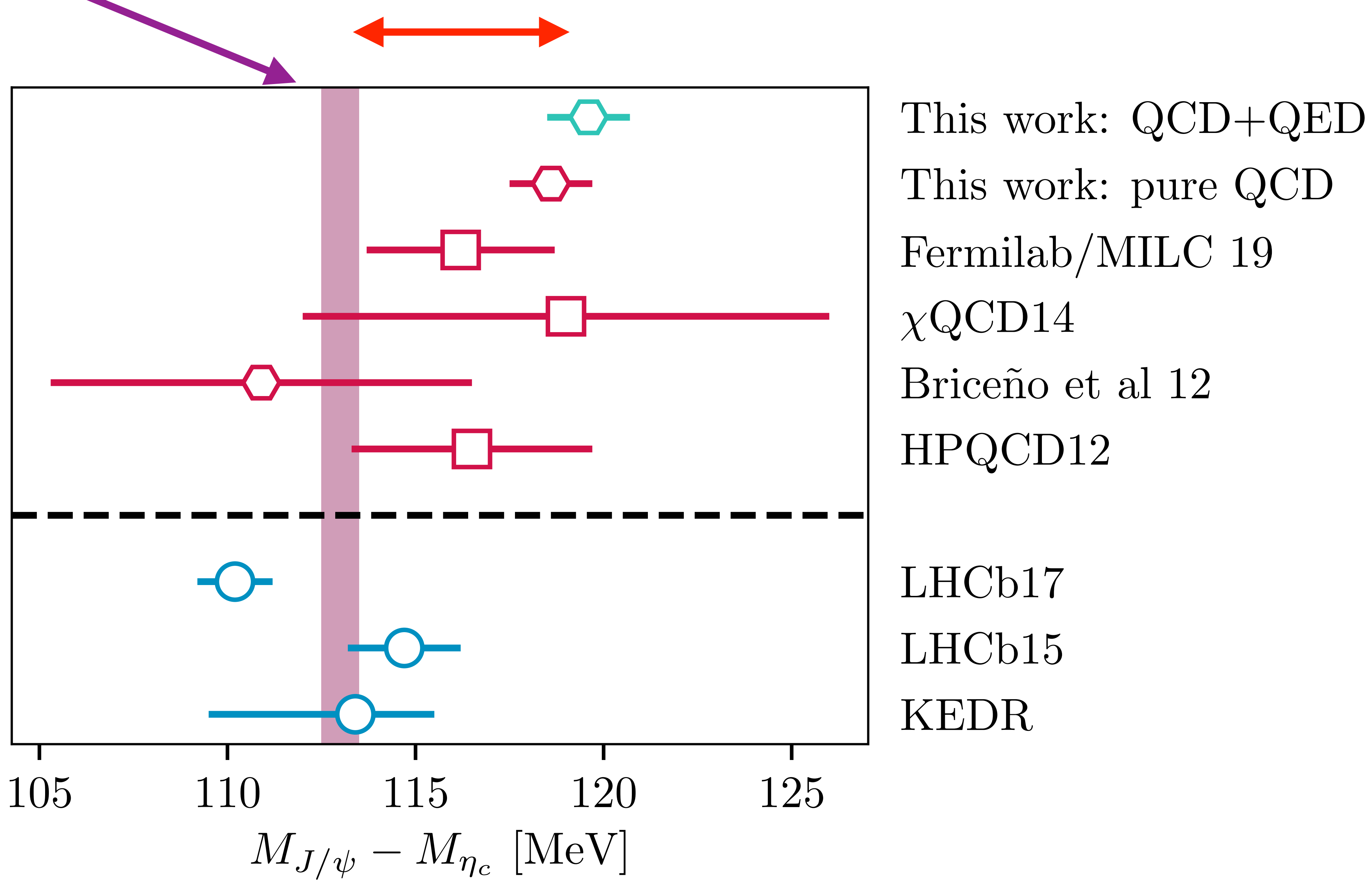
D. Hatton et al, HPQCD, 2005.01845

Hyperfine splitting in lattice QCD+QED - extrapolate to a=0 and physical u/d masses in sea. (Note: a^2 effects are small for HISQ; allow a^{2n} terms up to a^{10} in extrapolation.)

Experimental av.



Experimental av.



Conclude: lattice QCD+QED connected calculation of hyperfine splitting disagrees with experiment. If this is because of missing η_c annihilation, then

$$\Delta M_{\eta_c}^{\text{annihiln}} = +7.3(1.2)\ \text{MeV}$$

Leading order NRQCD pert. th. gives -3 MeV (related to total Γ=32(1) MeV), but subleading terms could easily change sign. Not (yet) calculable directly in lattice QCD.

Decay constants of η_c and J/ψ

$$\langle 0|\overline{\psi}\gamma_5\gamma_0\psi|\eta_c(\vec{p}=0)\rangle = f_{\eta_c} M_{\eta_c}$$

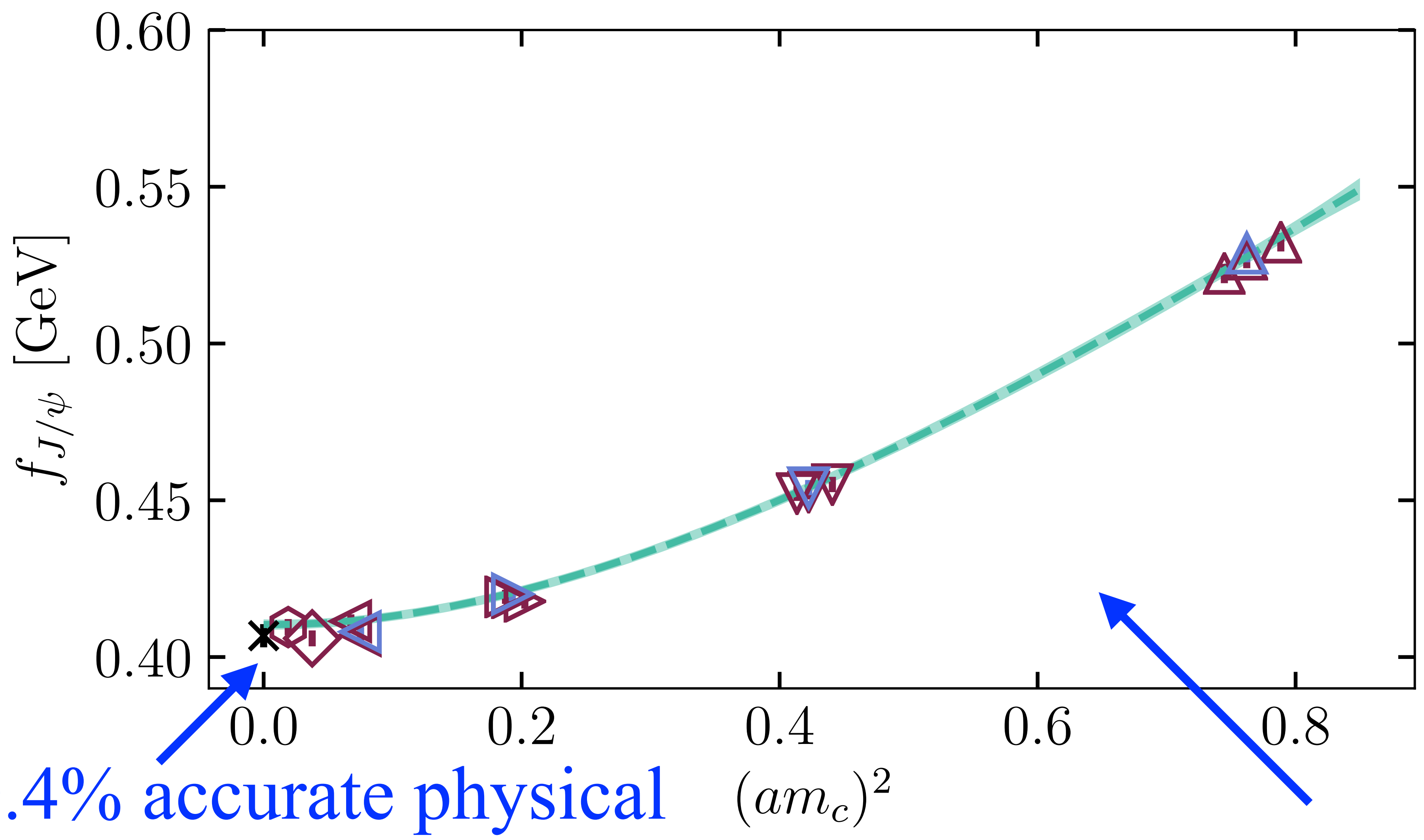
$$\langle 0|\overline{\psi}\gamma_\mu\psi|J/\psi(\vec{p}=0)\rangle = f_{J/\psi} M_{J/\psi}\varepsilon_\mu$$

Must normalise lattice vector current carefully - use RI-SMOM since respects Ward identity on lattice.

D. Hatton et al, HPQCD, 1909.00756

$$\Gamma(J/\psi \to \ell^+\ell^-) = \frac{4\pi}{3}\alpha^2_{\rm QED} Q_c^2 \frac{f^2_{J/\psi}}{M_{J/\psi}}$$

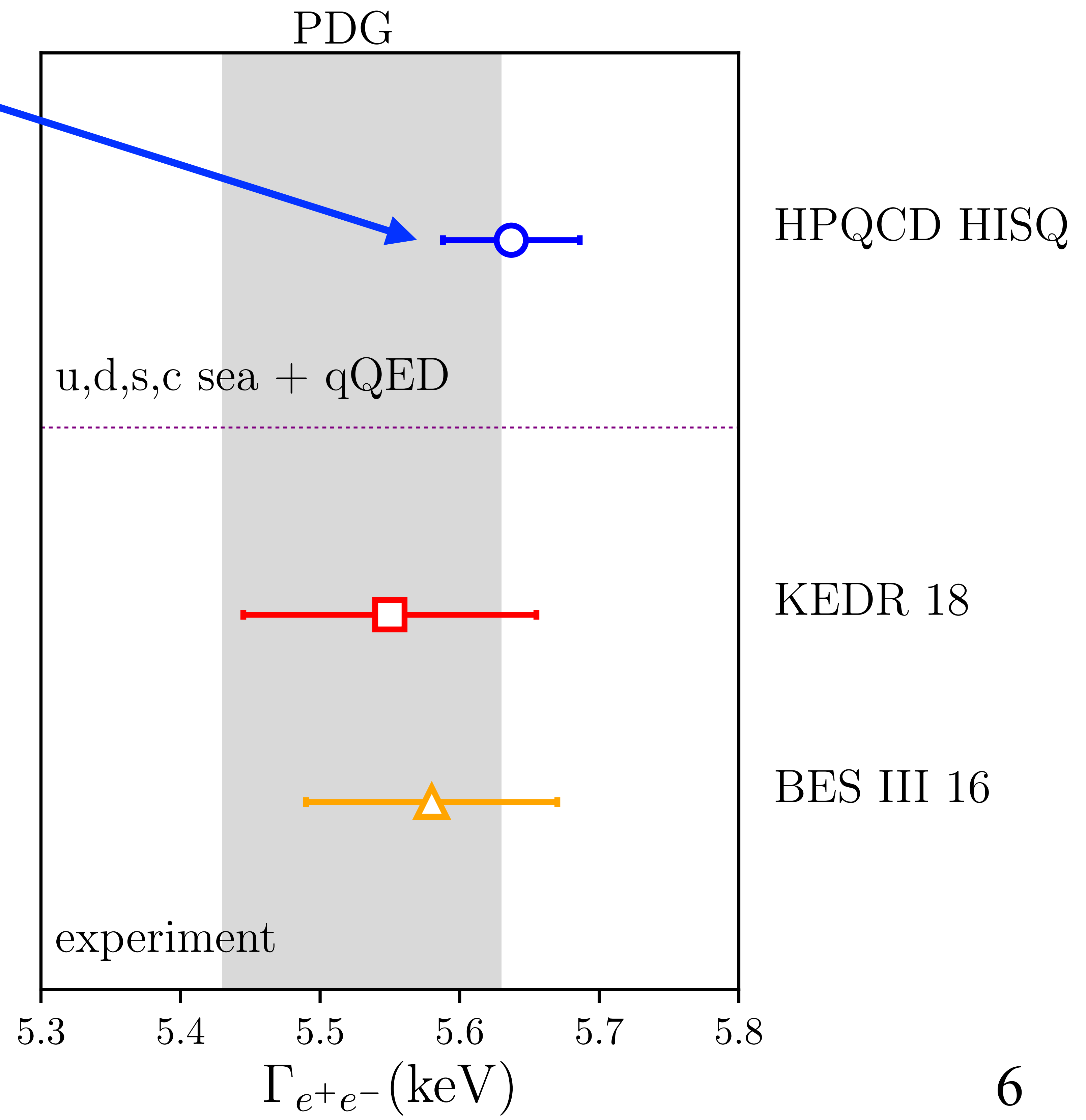
Hadronic parameter for annihilation to a photon



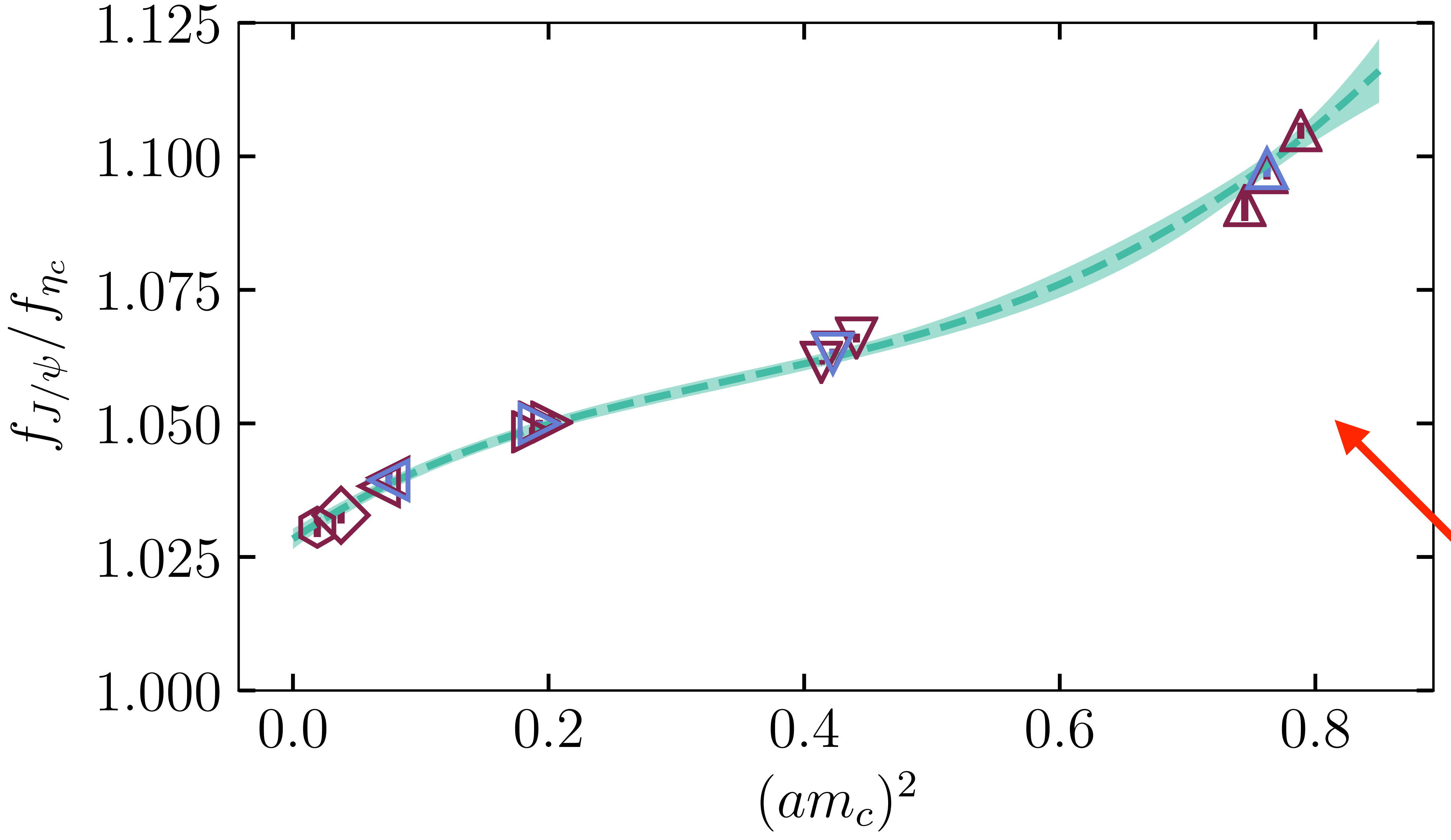
0.4% accurate physical result: 410.4(1.7) MeV

Extrapolation to a=0

$$\Gamma(J/\psi \to e^+e^-) = 5.637(49)\ \text{keV}$$



Agrees with, but more accurate than, experiment



Systematics cancel in decay constant ratio

$$\frac{f_{J/\psi}}{f_{\eta_c}} = 1.0284(19)$$

0.2% accurate! and > 1

D. Hatton et al, HPQCD, 2005.01845

Extend calculations to bottomonium

D. Hatton et al, HPQCD, 2101.08103

Use HPQCD's 'heavy-HISQ' approach, increasing quark masses above that of charm but keeping ma<1. Can reach the b quark mass for a=0.045 fm and a=0.03 fm. Systematic errors are smaller than for nonrelativistic approaches to the b.

By fitting results as a function of m_h and lattice spacing can obtain results at b in continuum limit.
Use cubic splines for m_h-dependence

The b mass is fixed as the point where the vector meson mass is that of the Υ.

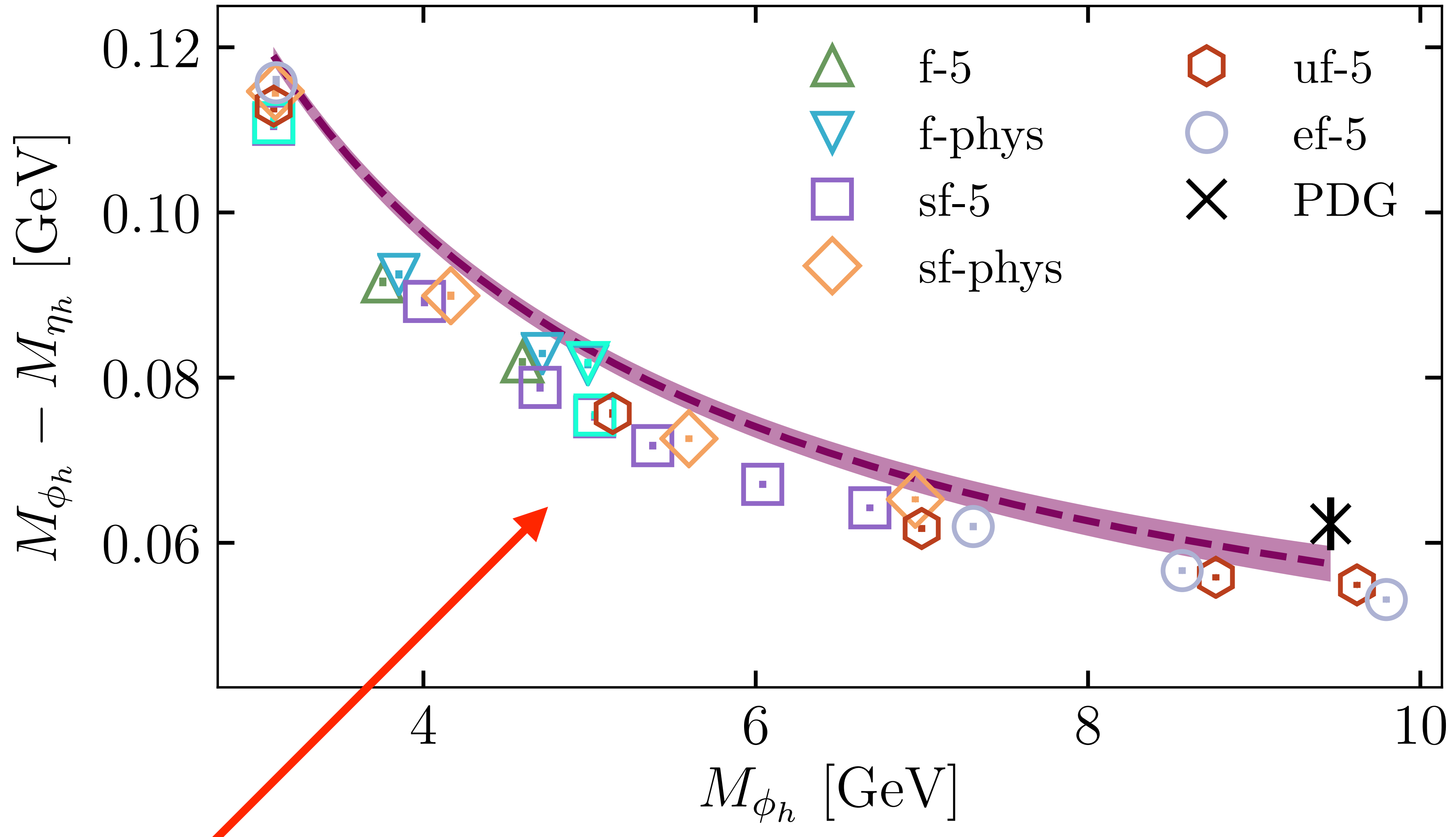
QED effects are much smaller here because $e_b = 1/3$.

Improved by factor 3 over earlier lattice results.

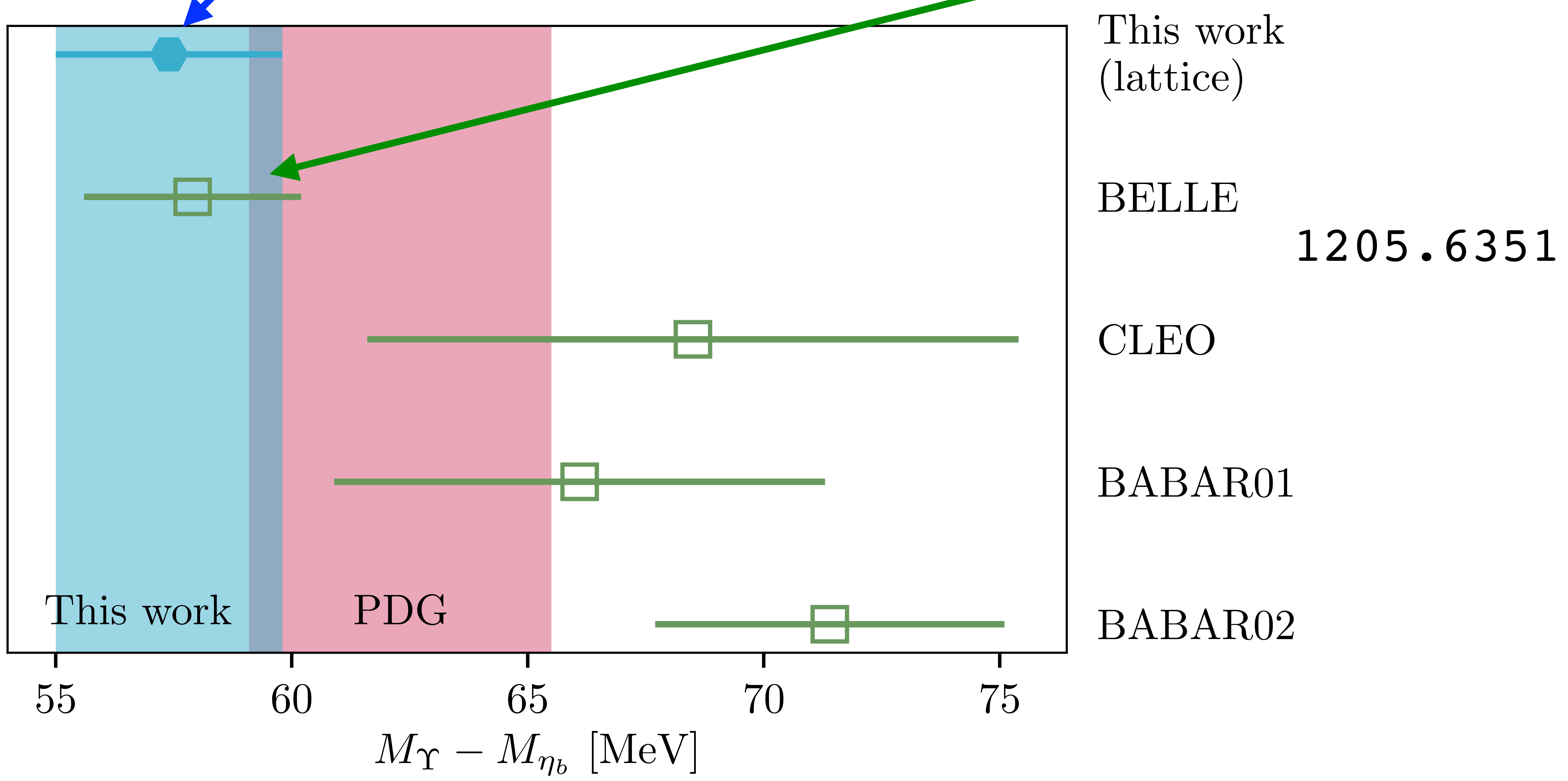
$$M_\Upsilon - M_{\eta_b} = 57.5(2.5)\ \mathrm{MeV}$$

Good agreement with Belle

Hyperfine splitting



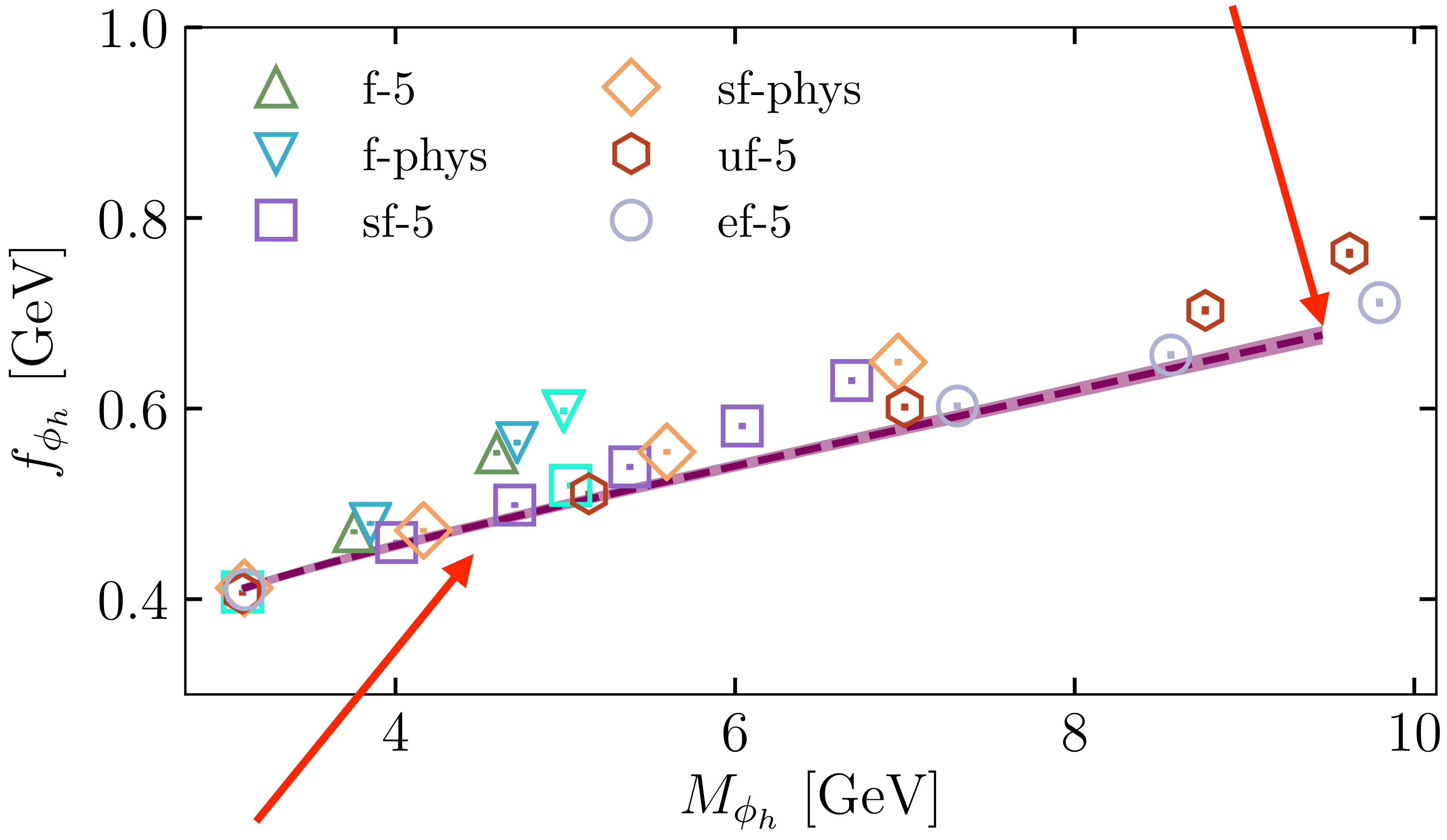
Curve gives physical (continuum) dependence of splitting on heavyonium vector mass.

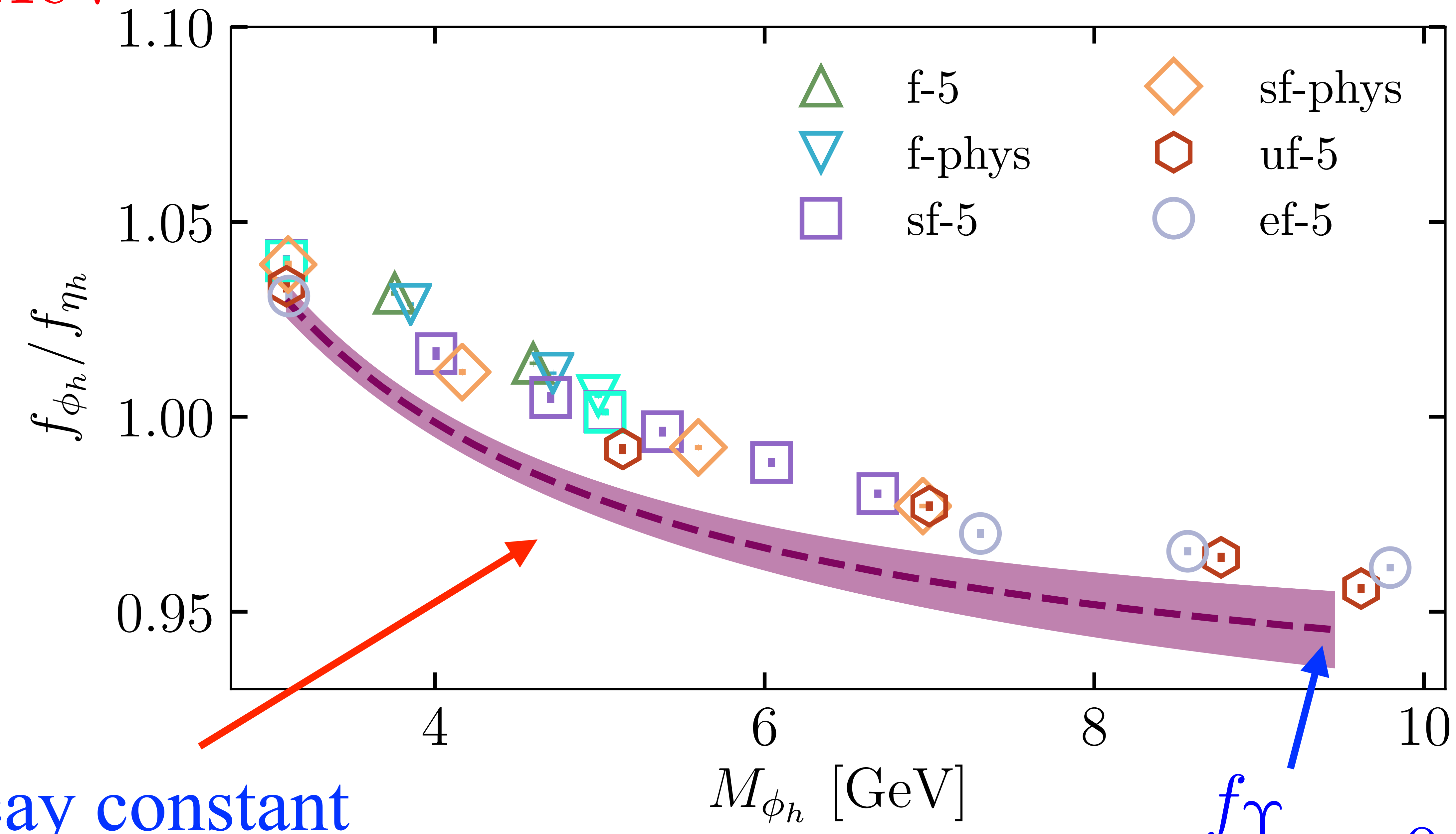


Result at b from quark-line connected correlators agrees with experiment. Pert. th. gives disc. contribution of -1 MeV ($\Gamma_\eta = 10$ MeV); expect this to be more reliable than for charmonium

Decay constants

$f_\Upsilon = 677.2(9.7)$ MeV





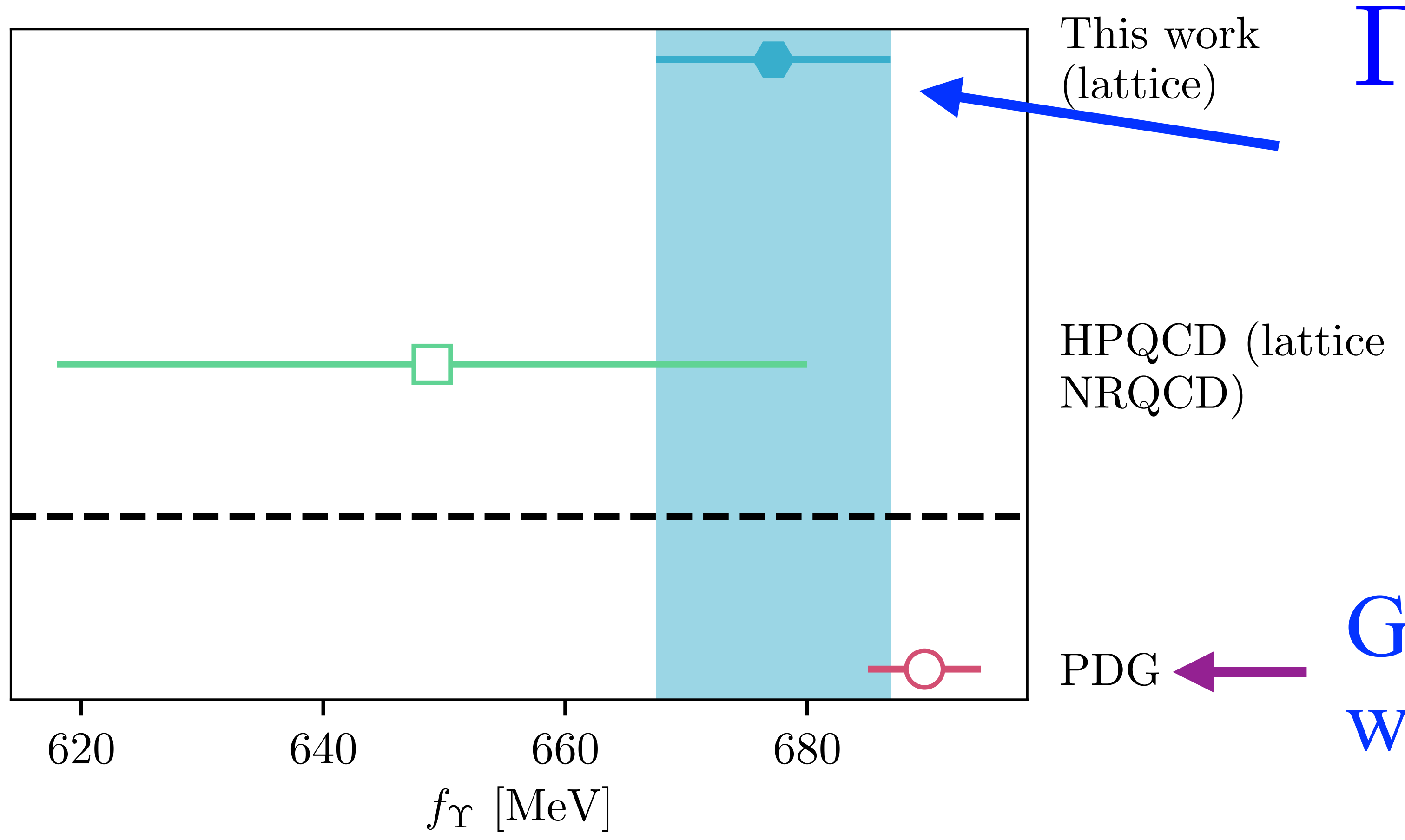
Curve gives physical (continuum) dependence of decay constant on heavyonium vector mass from c to b.

Decay constant ratio >1 for c; <1 for b

$$\frac{f_\Upsilon}{f_{\eta_b}} = 0.945(10)$$

$$\Gamma(\Upsilon \to \ell^+\ell^-) = \frac{4\pi}{3}\alpha_{\rm QED}^2 Q_b^2 \frac{f_\Upsilon^2}{M_\Upsilon}$$

Compare rate for annihilation to leptons predicted by lattice QCD to the experimental result - see good agreement



$$\Gamma(\Upsilon \to e^+e^-) = 1.292(37) \text{ keV}$$

Good agreement with experiment

Big improvement in accuracy over previous lattice QCD results.

Determination of quark masses from lattice QCD+QED

D. Hatton et al, HPQCD, 1805.06225, 2005.01845, 2102.09609

m_c

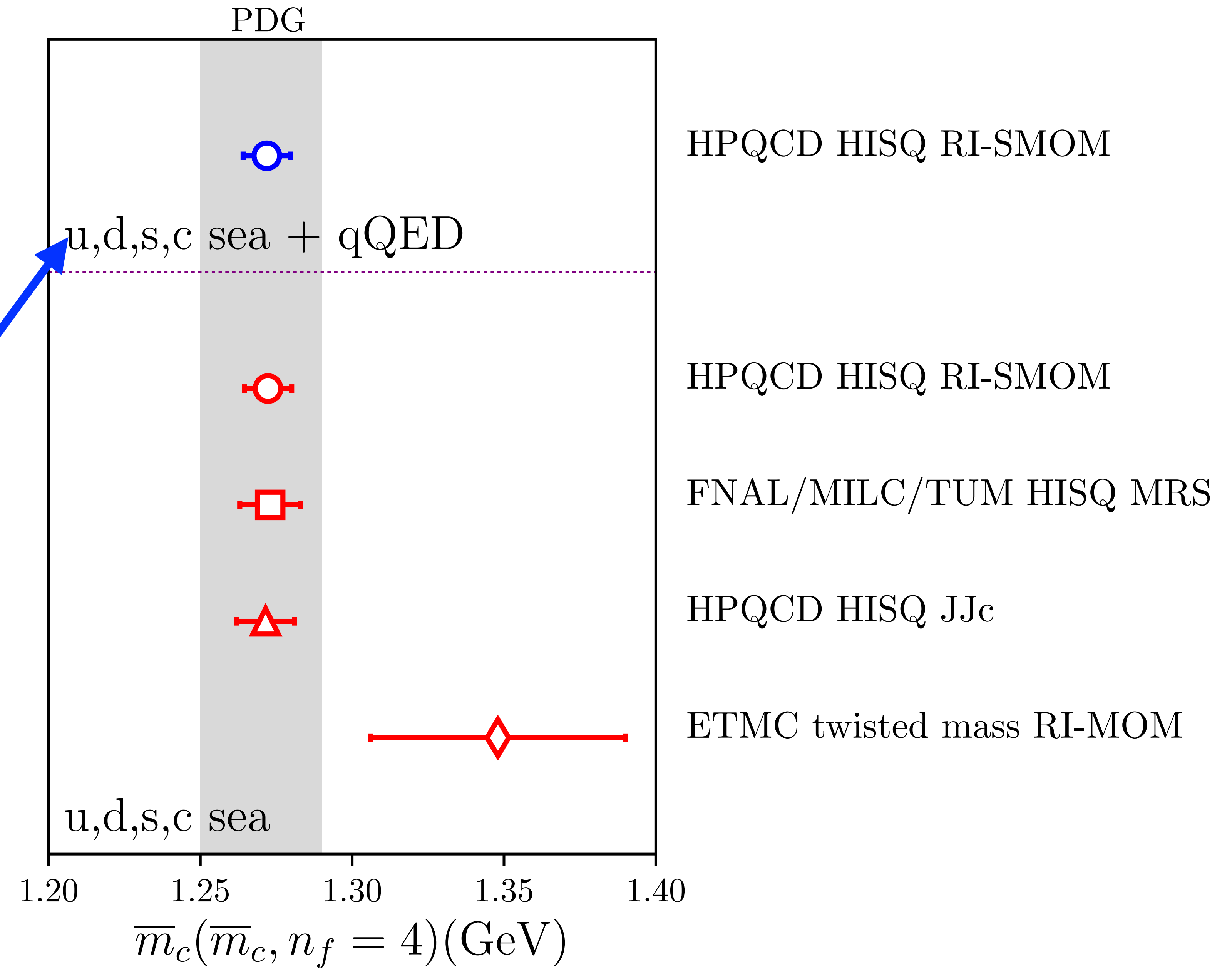
Tune lattice mass from J/ψ.

Determine Z_m via SMOM scheme on lattice allowing for nonperturbative artefacts.

Combined impact of QED: -0.2% at scale 3 GeV.

In $\overline{\text{MS}}$ scheme $\overline{m}_c(n_f = 4, 3\,\text{GeV}) = 0.9841(51)\,\text{GeV}$

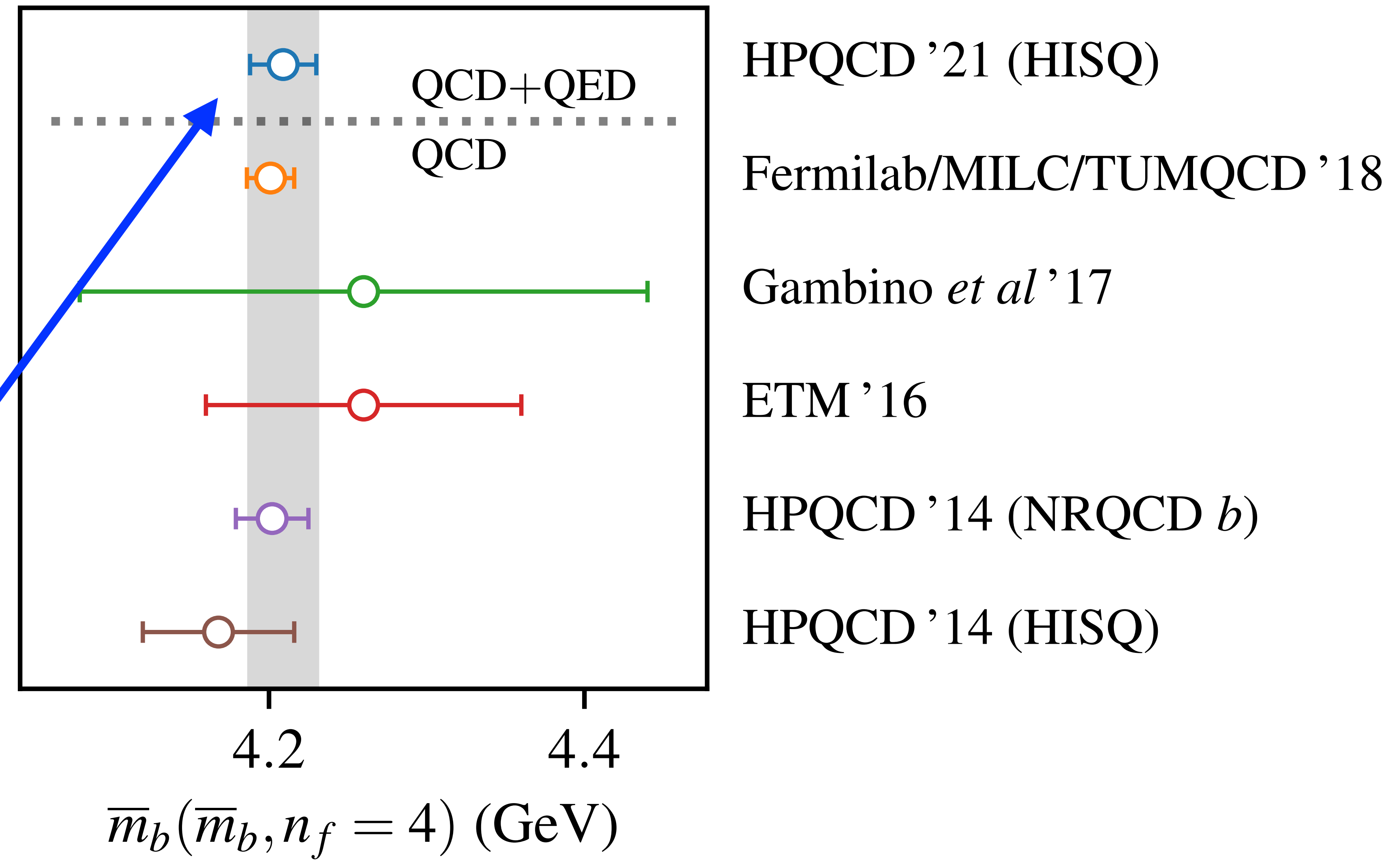
$\overline{m}_c(n_f = 4, \overline{m}_c) = 1.2719(78)\,\text{GeV}$



m_b

1) Calculate ratio of m_h to m_c as a function of heavyonium meson mass in pure QCD we can then read off mb/mc for lattice masses = ratio of MSbar masses in continuum limit. Key point: independent of scale, μ

2) Determine QED corrections to this (next slide) - ratio no longer scale-independent since Q not the same

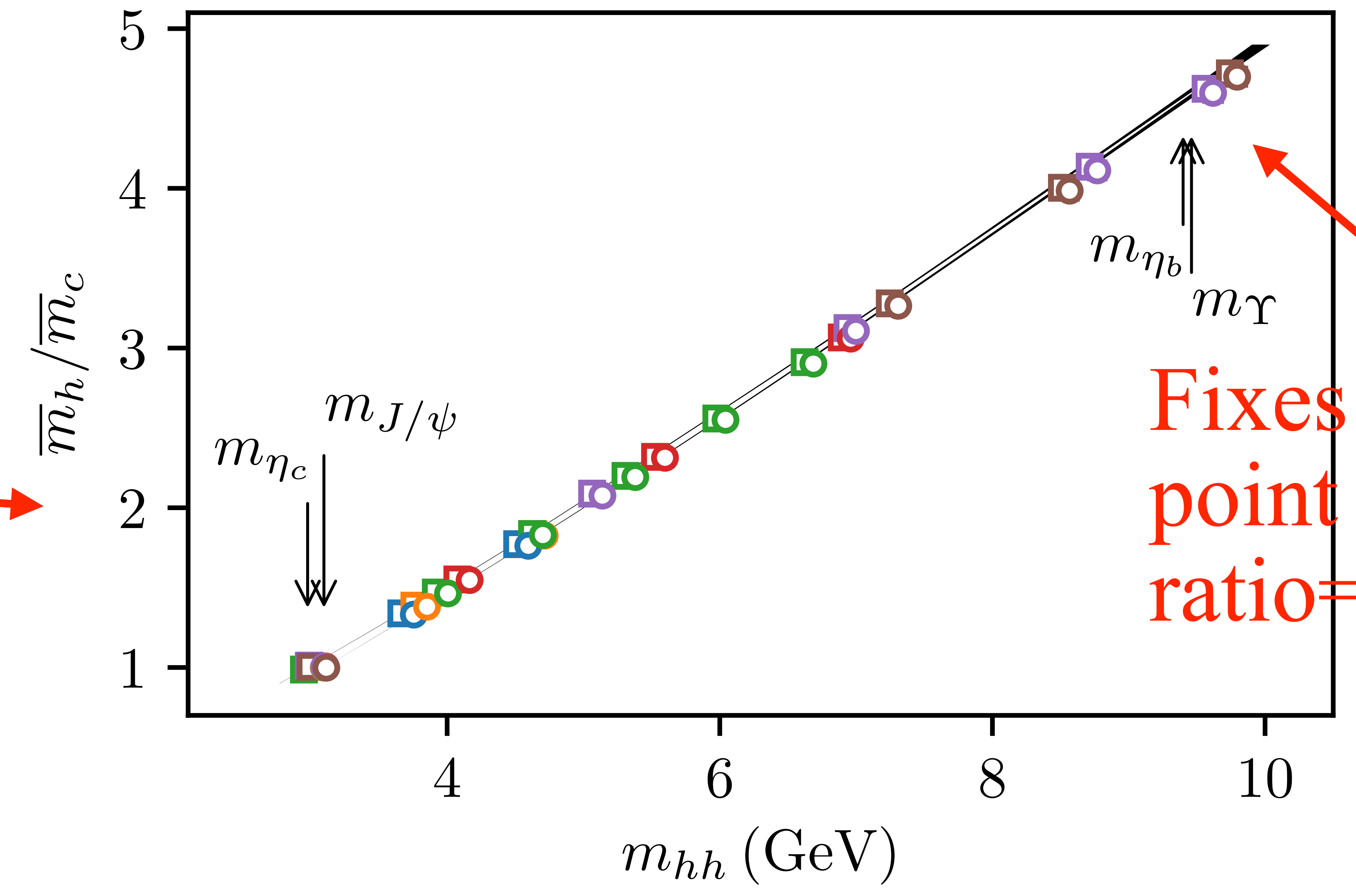


2) Use ratio and m_c above to give: $\overline{m}_b(n_f = 4, \overline{m}_b) = 4.209(21)\,\text{GeV}$

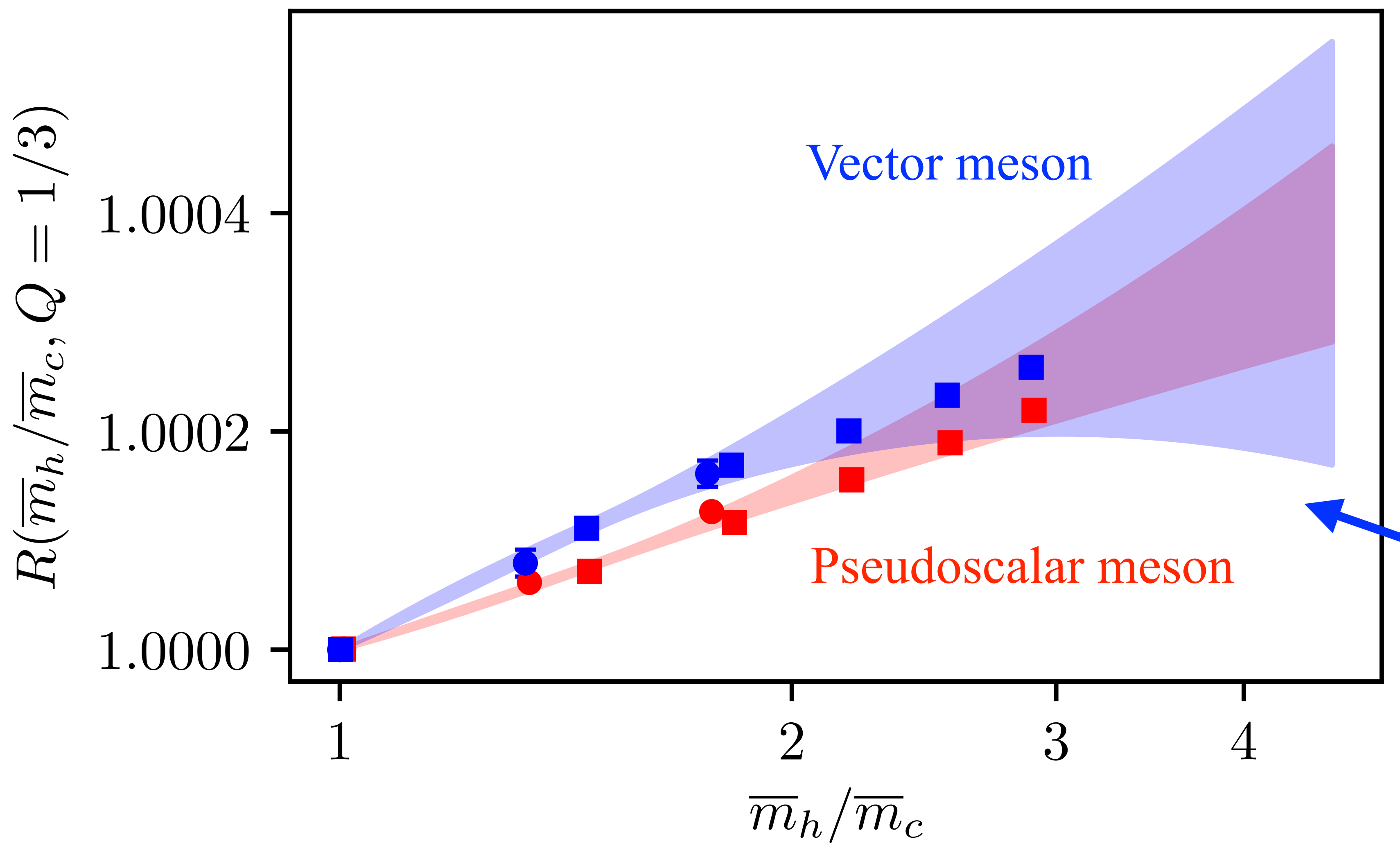
Determination of the b quark mass in lattice QCD+QED

D. Hatton et al, HPQCD, 2102.09609

1) Ratio of m_h to m_c as a function of heavyonium meson mass in pure QCD. This is equal to the mass ratio in the MSbar scheme in the continuum limit and is scale-independent.



Fixes b mass point ratio=4.578(12)



2) Ratio of m_h to m_c ratios in QCD+QED to that in QCD for the case where each quark has Q=1/3, so ratio still scale-independent. Effects are tiny.

3) Adjust m_c in ratio for Q=1/3 -> Q=2/3 - biggest QED effect = 3/4 of -0.2% at 3 GeV

Final result for ratio (now scale-dependent): $$\left.\frac{\overline{m}_b(3\,\mathrm{GeV})}{\overline{m}_c(3\,\mathrm{GeV})}\right|_{\mathrm{QCD+QED}} = 4.586(12)$$

Conclusions

HPQCD

• Lattice QCD results for η_c and J/ψ have reached high precision, and now include the effects of the c quark electric charge (QCD+quenched QED).

• The hyperfine splitting result shows that the impact of η_c annihilation on its mass is +7(1) MeV

• The calculation of the J/ψ leptonic width is now more accurate (0.9%) than experiment and agrees well with it.

• Extension of the calculation to bottomonium gives a hyperfine splitting that agrees well with recent experiment, and an Υ leptonic width that also agrees with experiment. The ratio of vector to pseudoscalar decay constants flips from >1 at c to <1 at b.

• Masses for c and b quarks now determined including the effect of QED.

Future

• Accurate determination underway of decay modes that test meson structure: need more accurate experiment

$$J/\psi \to \eta_c\gamma$$
$$\eta_c \to \gamma\gamma$$

• Extend to accurate (1 MeV level) determination of heavy-light meson masses, including effects of QED. Heavy-light vector meson vector and tensor decay constant calculations underway

SPARES

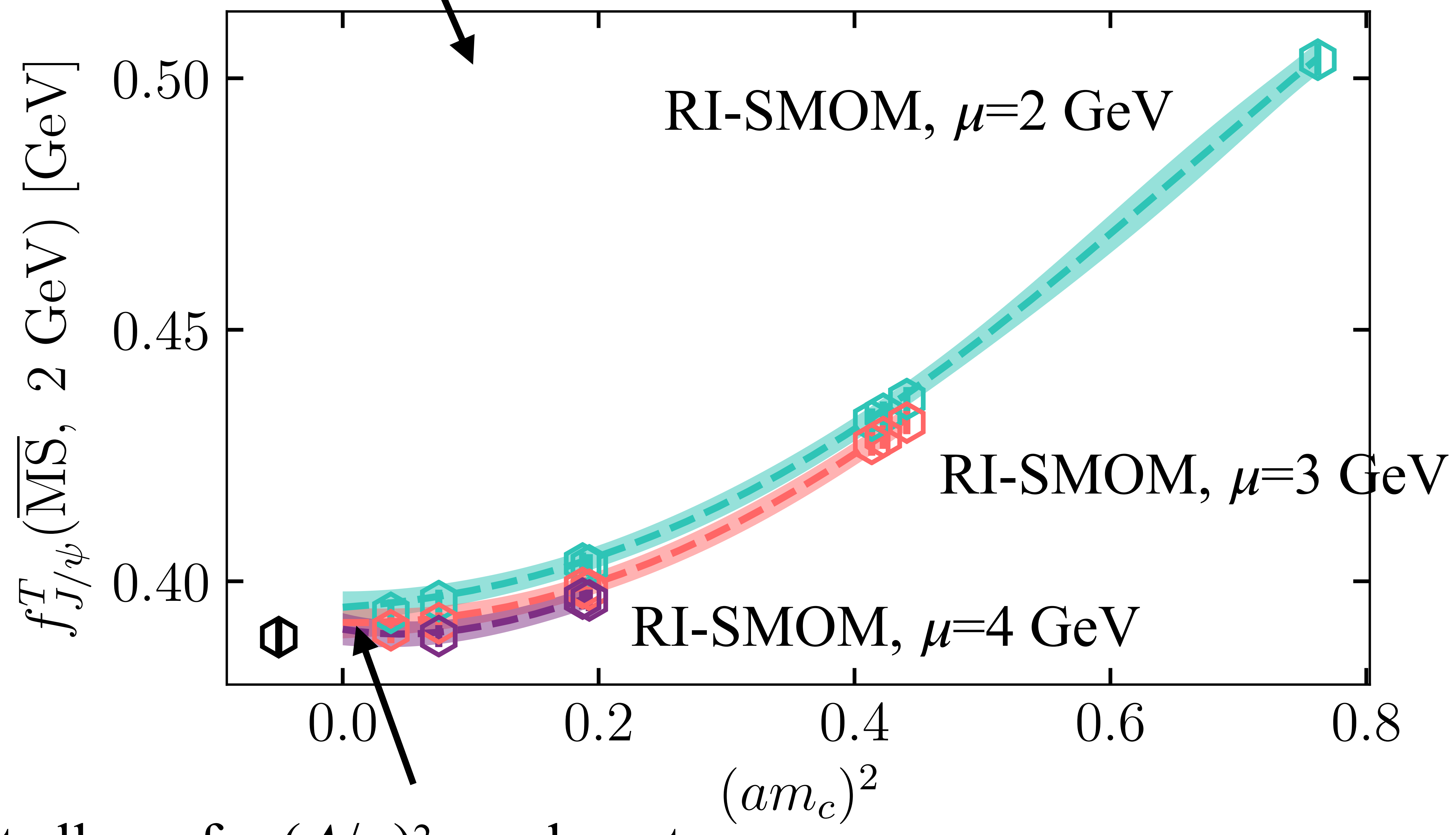
Tensor decay constant of the J/ψ

D. Hatton et al, HPQCD, 2008.02024

$$\langle 0|\overline{\psi}\sigma_{\alpha\beta}\psi|J/\psi\rangle = i f^T_{J/\psi}(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha)$$

scheme and scale-dependent

Must renormalise tensor current in lattice QCD - use intermediate momentum-subtraction scheme, RI-SMOM, on lattice and account for nonperturbative ‘condensate’ contributions

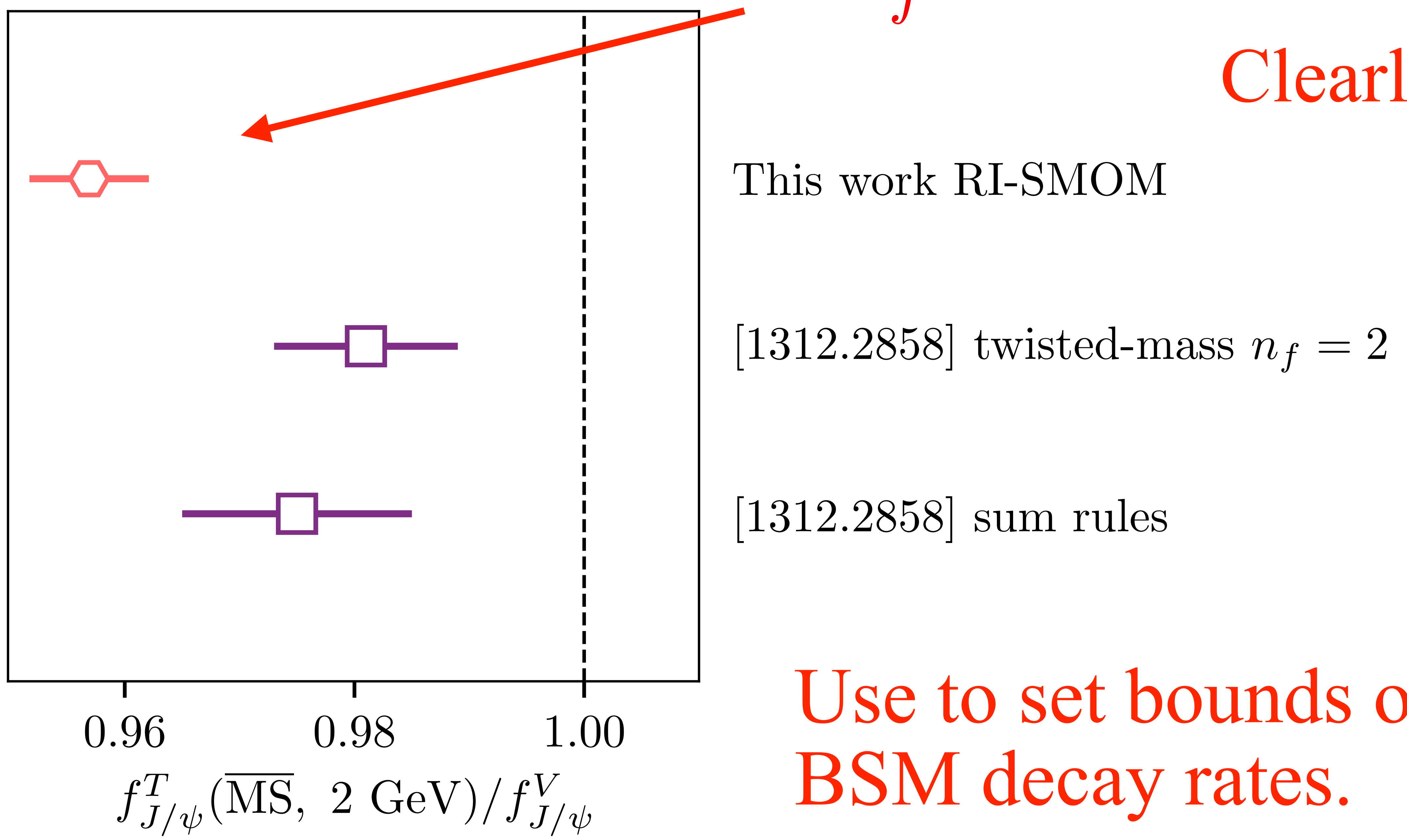


Fit allows for $(\Lambda/\mu)^2$ condensates

Ratio of tensor to vector decay constants

$$\frac{f^T(\overline{\mathrm{MS}}, 2\,\mathrm{GeV})}{f^V} = 0.957(5)$$

Clearly <1



Use to set bounds on BSM decay rates.

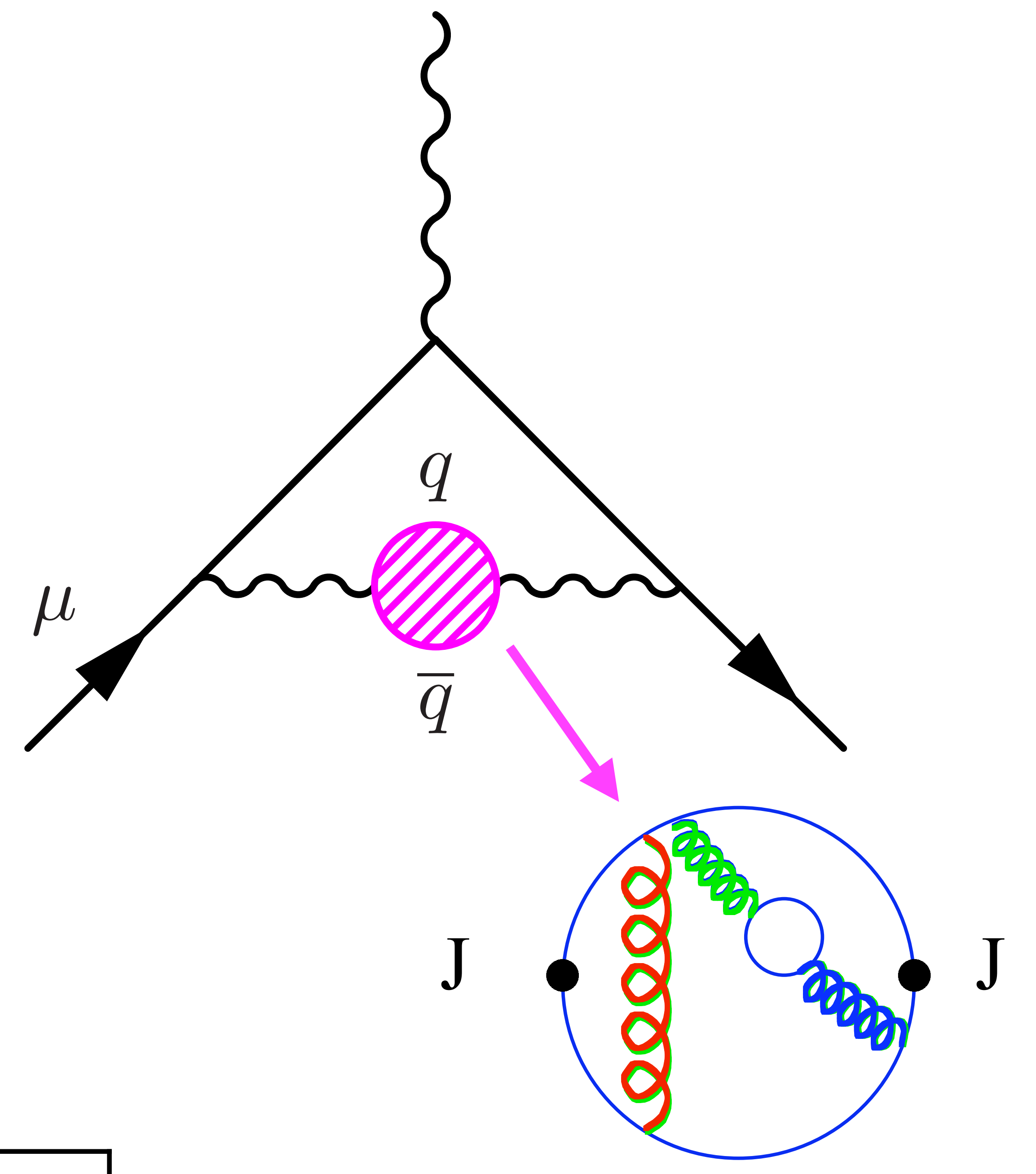
Charm quark contribution to anomalous magnetic moment of the muon, a^c_μ

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$$

$$a_\mu = \frac{g-2}{2}$$

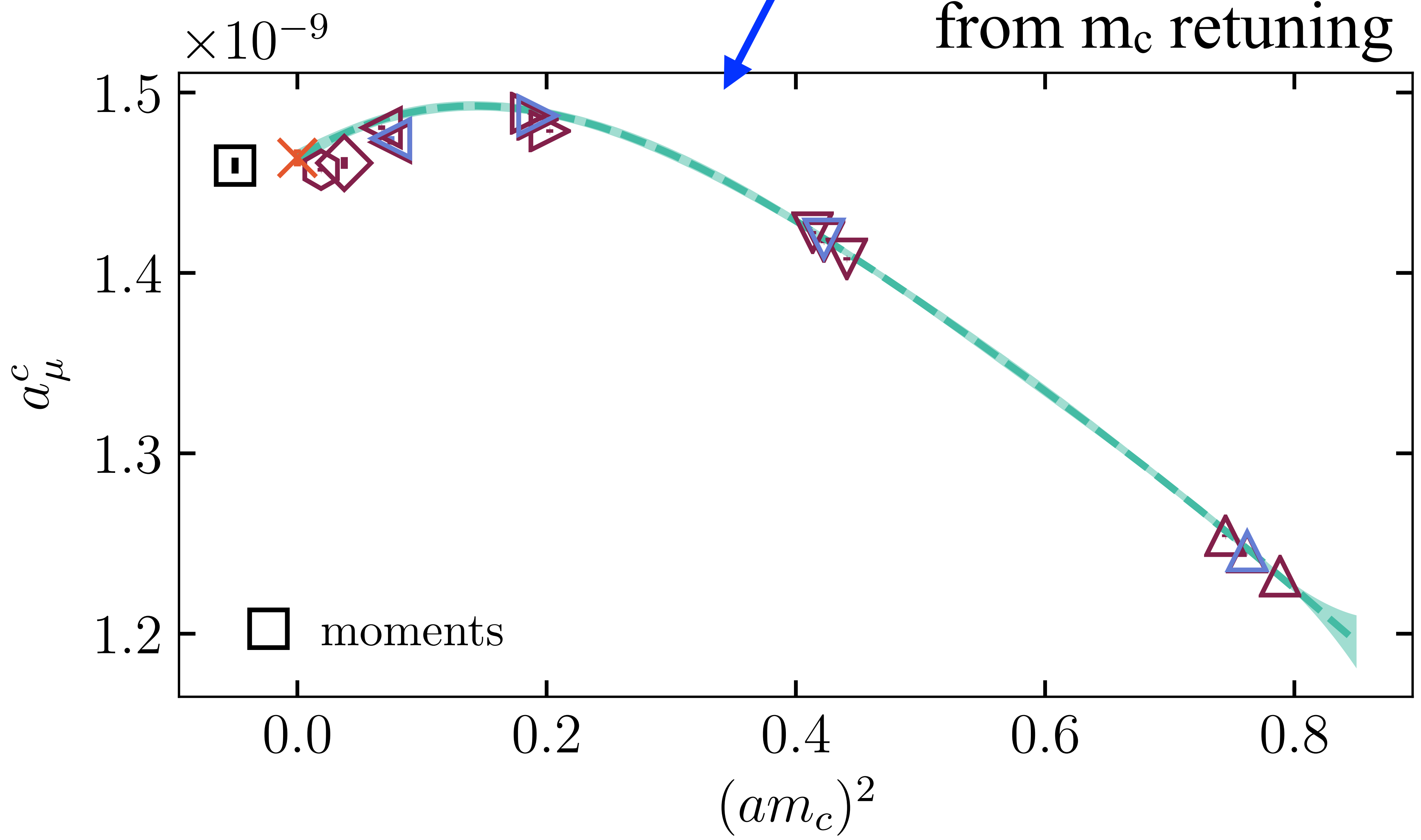
Least well-known contribution is that of 'hadronic vacuum polarisation' - quark bubble for each flavour attached to photon.

Calculate in lattice QCD+QED using vector correlators, either via time-moments or by direct integration



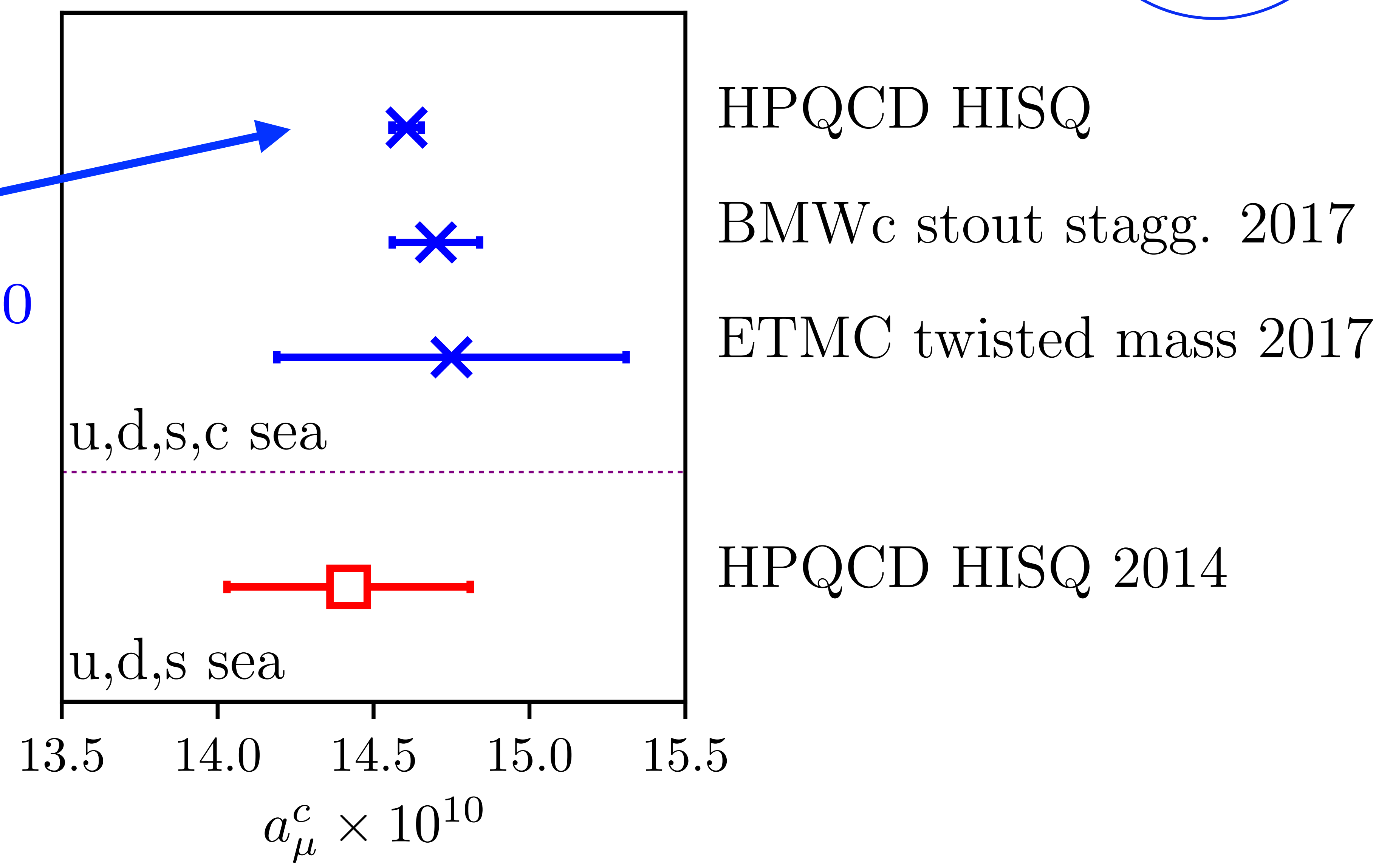
Extrapolation to a=0

Impact of QED = +0.2% from m_c retuning



$$a^c_\mu = 14.638(47) \times 10^{-10}$$

Small part of total HVP, but still needed for accurate SM a_μ



Error budgets

Error budget for our final result for the charmonium hyperfine splitting and decay constants including quenched QED corrections. The uncertainties shown are given as a percentage of the final result.

	ΔM_{hyp}	$f_{J/\psi}$	f_{η_c}
$a^2 \to 0$	0.13	0.09	0.03
Z_V	-	0.05	-
Pure QCD statistics	0.24	0.12	0.05
QCD+QED statistics	0.08	0.05	0.02
w_0/a	0.24	0.11	0.08
w_0	0.87	0.34	0.24
Valence mistuning	0.02	0.05	0.01
Sea mistuning	0.06	0.01	0.00
Total (%)	0.96	0.40	0.26

	$M_\Upsilon - M_{\eta_b}$	f_Υ	f_{η_b}
statistics	2.40	0.77	0.38
SVD cut	1.48	0.44	0.67
w_0	0.55	0.61	0.59
w_0/a	0.66	0.23	0.18
Z_V	-	0.29	-
M_{ϕ_h} dependence	0.03	0.01	0.00
$1/M_{\phi_h}$ dependence	0.05	0.02	0.01
$(am_h)^{2k}$ discretisation effects	1.14	0.17	0.18
$(a\Lambda)^{2k}$ discretisation effects	0.48	0.24	0.31
$(am_h)^2(a\Lambda)^2$ discretisation effects	0.42	0.28	0.45
light and strange sea quark mistuning	1.45	0.73	0.98
charm sea quark mistuning	1.08	0.29	0.27
QED M_{ϕ_h} dependence	0.29	0.07	0.08
QED $1/M_{\phi_h}$ dependence	0.19	0.01	0.00
Total (%)	3.99	1.43	1.59

	$\overline{m}_b/\overline{m}_c\,[m^P_{hh}]$	$\overline{m}_b/\overline{m}_c\,[m^V_{hh}]$	$\overline{m}_b/\overline{m}_c\,[\mathrm{avg}]$
$(am_h)^2 \to 0$	0.20	0.21	0.20
w_0, w_0/a	0.10	0.18	0.12
σ_u	0.12	0.12	0.09
g_m, ζ	0.05	0.05	0.05
m_{cc}	0.06	0.01	0.04
m_{bb}	0.03	0.00	0.02
$(am_h)^2\delta m^{\mathrm{sea}}_{uds} \to 0$	0.06	0.07	0.06
$\delta m^{\mathrm{sea}}_c \to 0$	0.03	0.03	0.03
$d\tilde{m}_c/dm_{cc}$	0.03	0.02	0.02
Total (%)	0.27	0.32	0.27

D. Hatton et al, HPQCD, 2005.01845, 2101.08103, 2102.09609