Hadron Spectroscopy: The Next Big Steps
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Mainz Institute for Theoretical Physics

"ccu baryon: Investigating quark-diquark model"

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OUTLINE:

- Introduction
- Quark-diquark model
- $\Xi_{cc}{}^{++}$ baryon
- Outlook and Remarks

INTRODUCTION

- theory of strong interactions: gluons, quarks …
- perturbation in α_s at small distances (large energies), asymptotic freedom
- non perturbative at large distances (small energies), confinement
- MAIN PROBLEM: we live in a confined world :)

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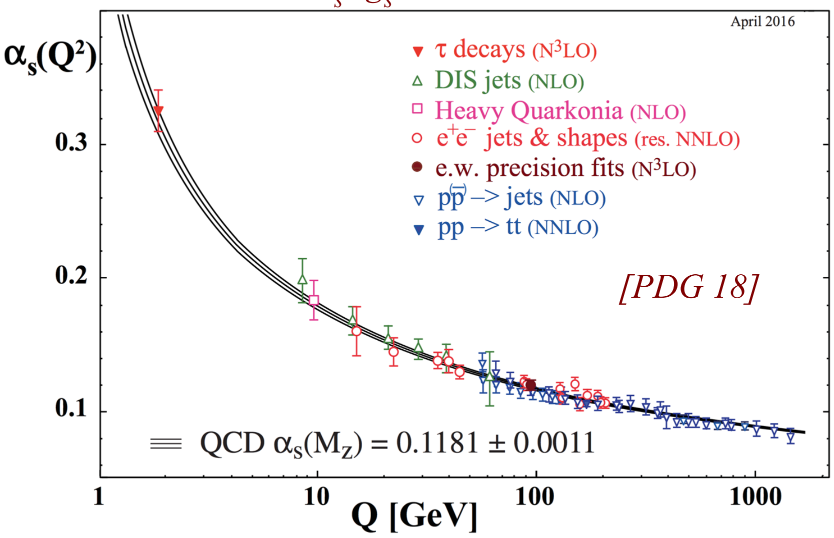
$$\mathcal{L} = \sum_q \bar{\psi}_{q,a}(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab})\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu}$$

$\alpha_s = g_s^2/4\pi$



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requires non perturbative techniques!

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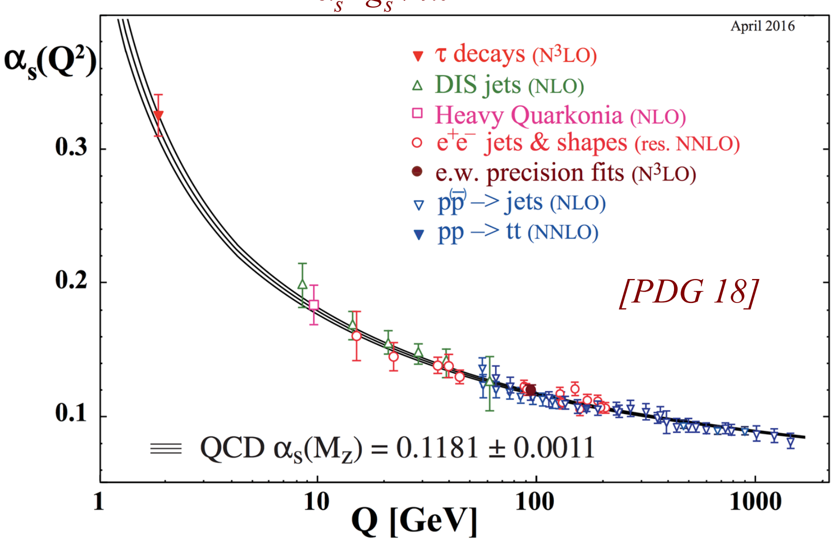

INTRODUCTION

QUARK MODEL

- Hadrons are bound states of quarks and gluons *(Gell-mann and Zweig, 64)*
- Mesons: *Baryon number 0,* quark-antiquark states
- Baryons: *Baryon number 1,* 3 quark states
- Only colour neutral combinations of quarks exist.

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- *Gell-Mann 64:*

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest

R. L. Jaffe, Phys. Rev. **D15**, 267 (1977); R. L. Jaffe, Phys. Rev. **D15**, 281 (1977).

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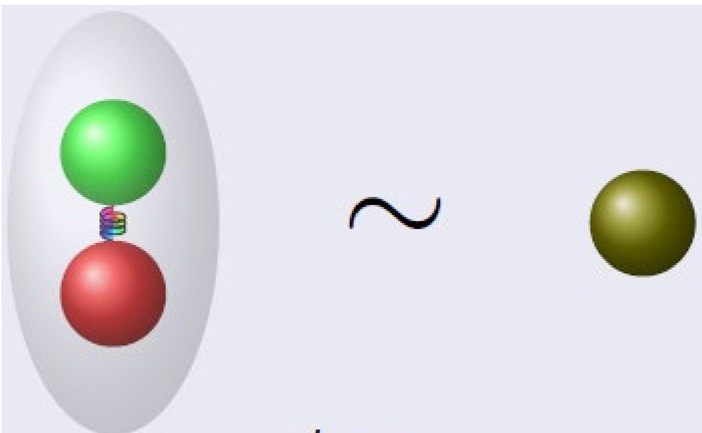
DIQUARK

- The idea of diquarks is almost as old as quarks
- A diquark is defined as a colored bound state of two quarks, although it has not been observed yet
- In terms of color, two quarks can behave like an antiquark:

$$3 \otimes 3 = \bar{3} \oplus 6$$

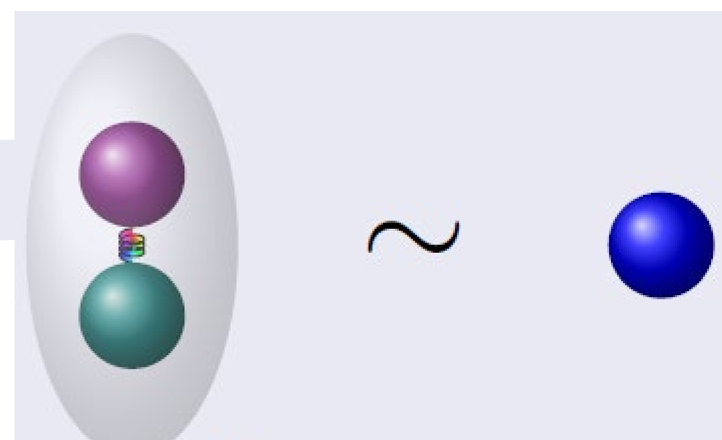


$$\epsilon_{abc} q^b q^c \sim \bar{q}_a$$

In a colorless group if one quark (antiquark) is replaced with antidiquark (diquark), the group remains colorless.

- Two antiquarks can behave in the same manner as quark:

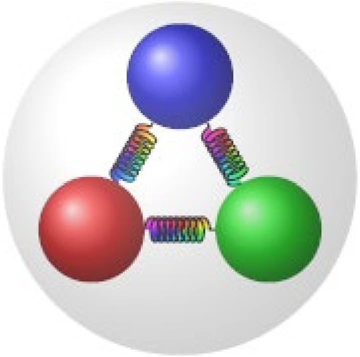
$$\bar{3} \otimes \bar{3} = 3 \oplus \bar{6}$$



$$\epsilon^{abc} \bar{q}_b \bar{q}_c \sim q^a$$

DIQUARK IN BARYON

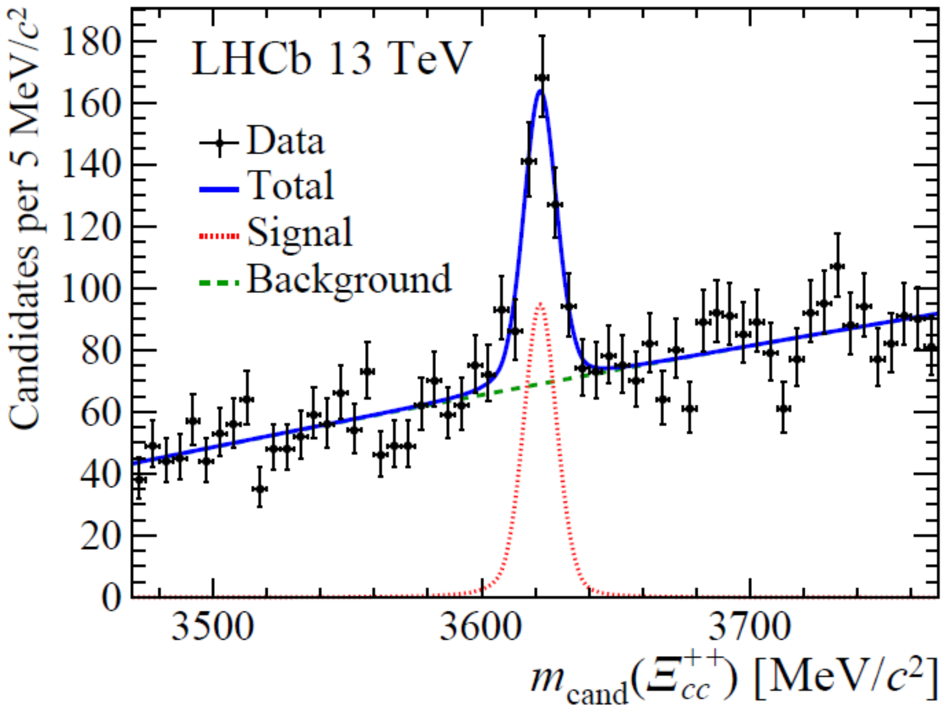
- Baryons are made of three valence quarks:



- Baryons can be studied as three-body problem
- The possibility of pairwise clustering of quarks in baryons has been studied in the early days of Quark Model (Prog. Theor. Phys. 36, 846 (1966); Phys. Rev. 155, 1601–1606 (1967); Phys. Rev. 178, 2197–2200 (1969))
- In addition to this, there are some indications such as mean separations, no need to antisymmetrizing the wave function, reducing the degrees of freedom (Rev. Mod. Phys. 65, 1199–1234 (1993); Prog. Part.Nucl. Phys. 116, 103835 (2021) arXiv:2008.07630 [hep-ph]) that the quark–diquark approximation is plausible for a doubly heavy baryon (qQQ)

DOUBLY CHARMED BARYON

- The Ξ_{cc} baryon (with ccu quark content) was observed in 2017 by LHCb with a mass around 3621 MeV



- This observation trigerred many phenonemological studies in the literature.
- Since this is a doubly charmed and doubly charged state, it is believed that the charm diquark (?) may play a role in the dynamics.

DOUBLY CHARMED BARYON

- Since cc diquark is heavy system, and relative slow motion of the tightly bound color antitriplet cc diquark in ccu baryon is similar to a quarkonium.
- Furthermore, it can be expected that two heavy quarks in the diquark are very close to each other so that they are seen as a whole structure by the light quark.
- In the quark–diquark model, two heavy quarks inside a baryon are thought to form a heavy diquark acting as a static color source for the third constituent light quark.
- We studied ccu baryon with a potential model as follows:

$$V(r) = \frac{\kappa \alpha_s}{r} + br,$$

$$V_S(r) = -\frac{2}{3(2\mu)^2}\nabla^2 V_V(r)\mathbf{S}_1\cdot\mathbf{S}_2 = -\frac{2\pi\kappa\alpha_S}{3\mu^2}\delta^3(r)\mathbf{S}_1\cdot\mathbf{S}_2\,.$$

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V_{\mathrm{eff}}(r)\right]\varphi(r) = 2\mu E\varphi(r)$$

$$V_{\mathrm{eff}}(r) \equiv 2\mu\left[V(r) + V_S(r)\right] + \frac{L(L+1)}{r^2}$$

DOUBLY CHARMED BARYON

- We have taken into account possible diquark clusterings:

$$\Xi_{cc}^{++} = [uc]\, c,$$
$$\Xi_{cc}^{++} = \{uc\}\, c,$$
$$\Xi_{cc}^{++} = \{cc\}\, u.$$

State	Spin	Mass
$[uc]$	$S=0$	2.150
$\{uc\}$	$S=1$	2.203
$\{cc\}$	$S=1$	3.133

State	$J^P = \frac{1}{2}^+$	$J^P = \frac{3}{2}^+$
$[uc]_{S=0}\, c_{S=\frac{1}{2}}$	3.687	-
$\{uc\}_{S=1}\, c_{S=\frac{1}{2}}$	3.647	3.773
$\{cc\}_{S=1}\, u_{S=\frac{1}{2}}$	3.621	4.019

DOUBLY CHARMED BARYON

- We also calculate wave function at the origin
- A large value of the wave function at the origin corresponds to a compact state

$$|\Psi(0)|^2 = |Y_0^0(\theta,\phi)R_{n,\ell}(0)|^2 = \frac{|R_{n,\ell}(0)|^2}{4\pi}$$

$$|\Psi(0)|^2 = \frac{\mu}{2\pi}\langle\frac{d}{dr}V(r)\rangle \Rightarrow |R(0)|^2 = 2\mu\langle\frac{d}{dr}V(r)\rangle.$$

State	$\lvert R(0)\rvert^2$
$[uc]_{S=0}\, c_{S=\frac{1}{2}}$	1.835
$\{uc\}_{S=1}\, c_{S=\frac{1}{2}}$	1.875
$\{cc\}_{S=1}\, u_{S=\frac{1}{2}}$	0.247

DOUBLY CHARMED BARYON

References	$J^P = (\frac{1}{2})^+$	$J^P = (\frac{3}{2})^+$	Method
$[uc]\,c$	3.687	-	Quark-Diquark Model
$\{uc\}\,c$	3.647	3.773	
$\{cc\}\,u$	3.621	4.019	
Ref. [25]	3.66	3.74	Feynman-Hellmann + Semiemperical
Ref. [26]	3.66	3.81	Relativistic Quark Model
Ref. [30]	3.478	3.610	Nonrelativistic Quark Model
Ref. [35]	3.620	3.727	Relativistic Quark Model
Ref. [44]	3.676	3.753	Nonrelativistic Quark Model
Refs. [49, 50]	3.72(0.20)	3.69-3.72	QCD Sum Rule
Ref. [58]	3.511	3.687	Hypercentral Constituent Quark Model
Ref. [87]	3.524	3.548	Nonrelativistic Quark Model + Potential Model
Ref. [88]	3.547	3719	Bethe-Salpeter Model
Refs. [110, 111]	3.570	3.610	QCD Sum Rule
Ref. [112]	3.610(09)(12)	3.694(07)(11)	Lattice QCD
Ref. [113]	3.685	3.754	Nonrelativistic Quark Model
Refs. [114, 115]	3.520	3.695	Regge Phenomenology
Ref. [116]	3.561(22)	3.642(26)	Lattice QCD
Ref. [117]	3.627	3.690	Nonrelativistic Quark Model
Ref. [118]	3.610	3.692	Latice QCD
Ref. [119]	3.606	3.675	Relativistic Quark Model
Ref. [120]	3.633	3.696	Chromagnetic Model
Ref. [121][a]	3.63± 0.02	3.63± 0.02	Bethe-Salpeter Model
Ref. [121][b]	3.62± 0.02	3.62± 0.02	
Ref. [121][c]	3.55± 0.01	3.62± 0.01	
Ref. [121][d]	3.54± 0.01	3.62± 0.01	
Ref. [122]	$3.63^{+0.08}_{-0.07}$	$3.75^{+0.07}_{-0.07}$	QCD Sum Rule
Ref. [123]	3.396	3.434	Nonrelativistic Quark Model + Potential Model
Ref. [124]	3.601^{-28}_{+28}	3.703^{-28}_{+28}	Bethe-Salpeter Model
Ref. [125]	3.519	3.555	Quark-Diquark Model
Ref. [126]	4.26 ± 0.19	3.90 ± 0.10	QCD Sum Rule
Ref. [127]	3.55	3.59	Bag Model

DOUBLY CHARMED BARYON

$$|\Xi_{ccu}; s=\frac{1}{2}\rangle = \frac{1}{3\sqrt{2}}[2c\uparrow c\uparrow u\downarrow - c\uparrow c\downarrow u\uparrow - c\downarrow c\uparrow u\uparrow$$
$$+ 2c\uparrow u\downarrow c\uparrow - c\downarrow u\uparrow c\uparrow - c\downarrow u\downarrow u\downarrow$$
$$+ 2u\downarrow c\uparrow c\uparrow - u\downarrow c\downarrow c\downarrow - u\uparrow c\downarrow c\uparrow],$$
$$|\Xi^*_{ccu}; s=\frac{3}{2}\rangle = \frac{1}{\sqrt{3}}[c\uparrow c\uparrow u\uparrow + c\uparrow c\uparrow u\uparrow$$
$$+ c\uparrow c\uparrow u\uparrow],$$

$$|\Xi_{cc}^{++}\rangle = \frac{4}{3}\mu_c - \frac{1}{3}\mu_u \text{ for } J^P = \frac{1}{2}^+,$$
$$|\Xi_{cc}^{++*}\rangle = 2\mu_c + \mu_u \text{ for } J^P = \frac{3}{2}^+.$$

DOUBLY CHARMED BARYON

TABLE VII. Comparison of the magnetic moments of $J^P = \frac{1}{2}^+$ Ξ_{cc}^{++} baryon with those predicted by other approaches (in unit of μ_N).

References	Magnetic Moment	Method
$[uc]\,c$	-0.044	Quark-Diquark Model
$\{uc\}\,c$	-0.044	
$\{cc\}\,u$	-0.045	
Ref. [37]	-0.10	Relativistic Quark Model
Ref. [41]	0.13	Relativistic Three-Quark Model
Ref. [42]	-0.208	Nonrelativistic Quark Model
Ref. [58]	0.031	Hypercentral Constituent Quark Model
Ref. [62]	-0.23 ± 0.05	QCD Sum Rule
Ref. [70]	0.35	Heavy Baryon Chiral Perturbation Theory
Ref. [91]	-0.25	Heavy Baryon Chiral Perturbation Theory
Ref. [125]	-0.054	Quark-Diquark Model
Ref. [128]	-0.133	Hypercentral Constituent Quark Model
Ref. [129]	0.114	MIT Bag Model
Ref. [130]	-0.12	Quark Model
Ref. [131]	-0.47	Skyrmion (Set 1)
Ref. [131]	-0.47	Skyrmion (Set 2)
Ref. [132]	0.006	Chiral Constituent Quark Model
Ref. [133]	-0.20	Nonrelativistic Quark Model
Ref. [134]	0.17	MIT Bag Model
Ref. [135]	-0.154	Logarithmic Confining Potential (Set 1)
Ref. [135]	-0.172	Logarithmic Confining Potential (Set 2)
Ref. [136]	-0.11	MIT Bag Model

DOUBLY CHARMED BARYON

TABLE VIII. Comparison of the magnetic moments of $J^P = \frac{3}{2}^+$ Ξ_{cc}^{++*} baryon with those predicted by other approaches (in unit of μ_N).

References	Magnetic Moment	Method
$\{uc\}\,c$	2.347	Quark-Diquark Model
$\{cc\}\,u$	2.203	
Ref. [42]	2.67	Nonrelativistic Quark Model
Ref. [58]	2.218	Hypercentral Constituent Quark Model
Ref. [71]	3.50	Covariant Baryon Chiral Perturbation Theory (EOMS) Case 1
Ref. [71]	3.51	Covariant Baryon Chiral Perturbation Theory (HB) Case 1
Ref. [71]	2.89	Covariant Baryon Chiral Perturbation Theory (EOMS) Case 2
Ref. [71]	2.80	Covariant Baryon Chiral Perturbation Theory (HB) Case 2
Ref. [90]	3.51	Heavy Baryon Chiral Perturbation Theory (Set 1)
Ref. [90]	3.63	Heavy Baryon Chiral Perturbation Theory (Set 2)
Ref. [94]	2.94 ± 0.95	QCD Sum Rule
Ref. [125]	2.513	Quark-Diquark Model
Ref. [128]	2.75	Hypercentral Constituent Quark Model
Ref. [131]	3.16	Skyrmion (Set 1)
Ref. [131]	3.18	Skyrmion (Set 2)
Ref. [132]	2.66	Chiral Constituent Quark Model
Ref. [134]	2.54	MIT Bag Model
Ref. [136]	2.35	MIT Bag Model
Ref. [137]	2.41	Effective Mass Scheme
Ref. [137]	2.52	Screening Effect Scheme

DOUBLY CHARMED BARYON

- For further details see

The status of Ξ_{cc}^{++} baryon: investigating quark–diquark model

Halil Mutuk (Ondokuz Mayis U.) (Dec 12, 2021)

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OUTLOOK AND REMARKS

- The discovery of doubly charmed baryon may pave the way for possible triply heavy baryon states. But more interestingly, it may open a perspective about our understanding of the exotic hadrons:

PRL **119,** 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

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Recently, the LHCb Collaboration discovered the first doubly charmed baryon $\Xi_{cc}^{++} = ccu$ at 3621.40 ± 0.78 MeV, very close to our theoretical prediction. We use the same methods to predict a doubly bottom tetraquark $T(bb\bar{u}\bar{d})$ with $J^P = 1^+$ at $10\,389 \pm 12$ MeV, 215 MeV below the $B^-\bar{B}^{*0}$ threshold and 170 MeV below the threshold for decay to $B^-\bar{B}^0\gamma$. The $T(bb\bar{u}\bar{d})$ is therefore stable under strong and electromagnetic interactions and can only decay weakly, the first exotic hadron with such a property. On the other hand, the mass of $T(cc\bar{u}\bar{d})$ with $J^P = 1^+$ is predicted to be 3882 ± 12 MeV, 7 MeV above the D^0D^{*+} threshold and 148 MeV above the $D^0D^+\gamma$ threshold. $T(bc\bar{u}\bar{d})$ with $J^P = 0^+$ is predicted at 7134 ± 13 MeV, 11 MeV below the $\bar{B}^0D^0$ threshold. Our precision is not sufficient to determine whether $bc\bar{u}\bar{d}$ is actually above or below the threshold. It could manifest itself as a narrow resonance just at threshold.

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THE END

Thank you for your attention!