Glueball Molecules

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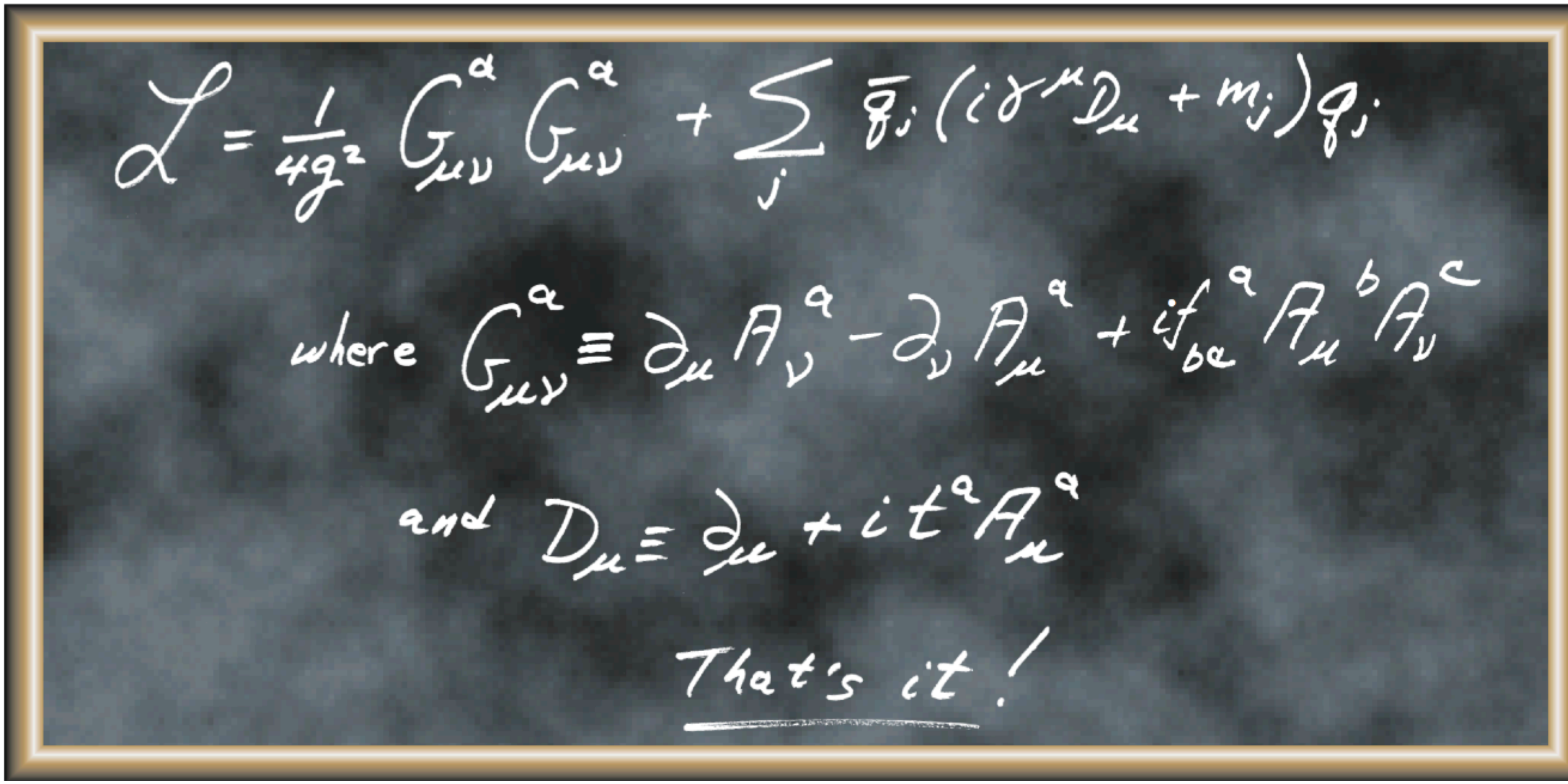
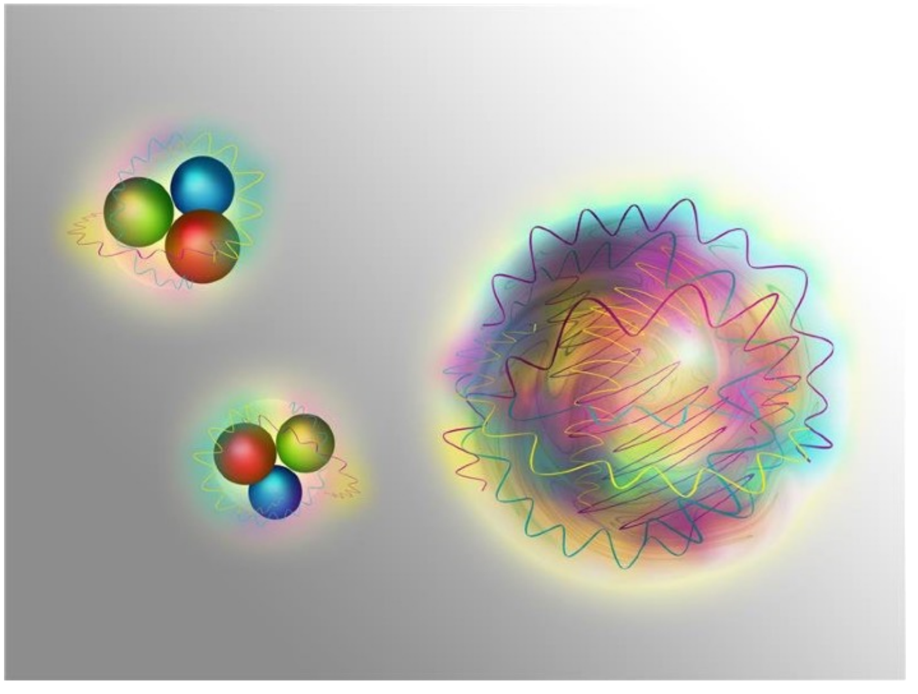
Quantum Chromodynamics is simple!



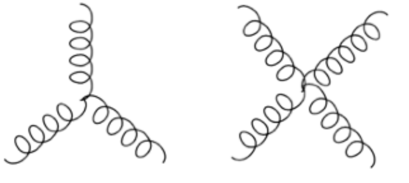
F. Wilczek, "QCD made simple", Physics Today, August 2000

Ok, maybe Quantum Chromodynamics is not so simple…



- QCD Lagrangian is written in terms of ``wrong" degrees of freedom: we see mesons and baryons, not quarks/gluons!
- Since gluons carry color charge, they can self-interact

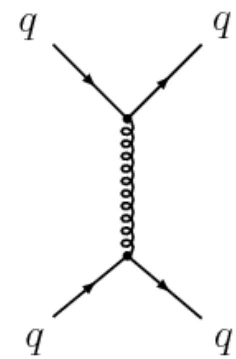
Can there be bound states of pure glue?



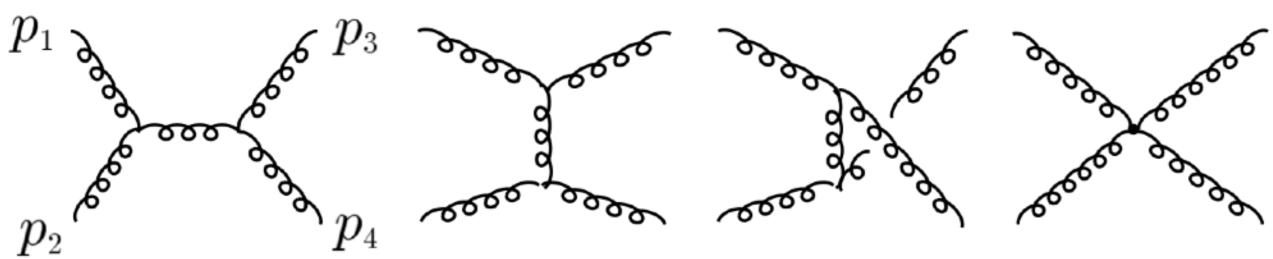
Curious: Higgs field has nothing to do with mass!

Glueball spectrum

- Can we predict glueball spectrum?
  - quark models: quark-antiquark potential
  - not so easy for gluons: gauge invariance



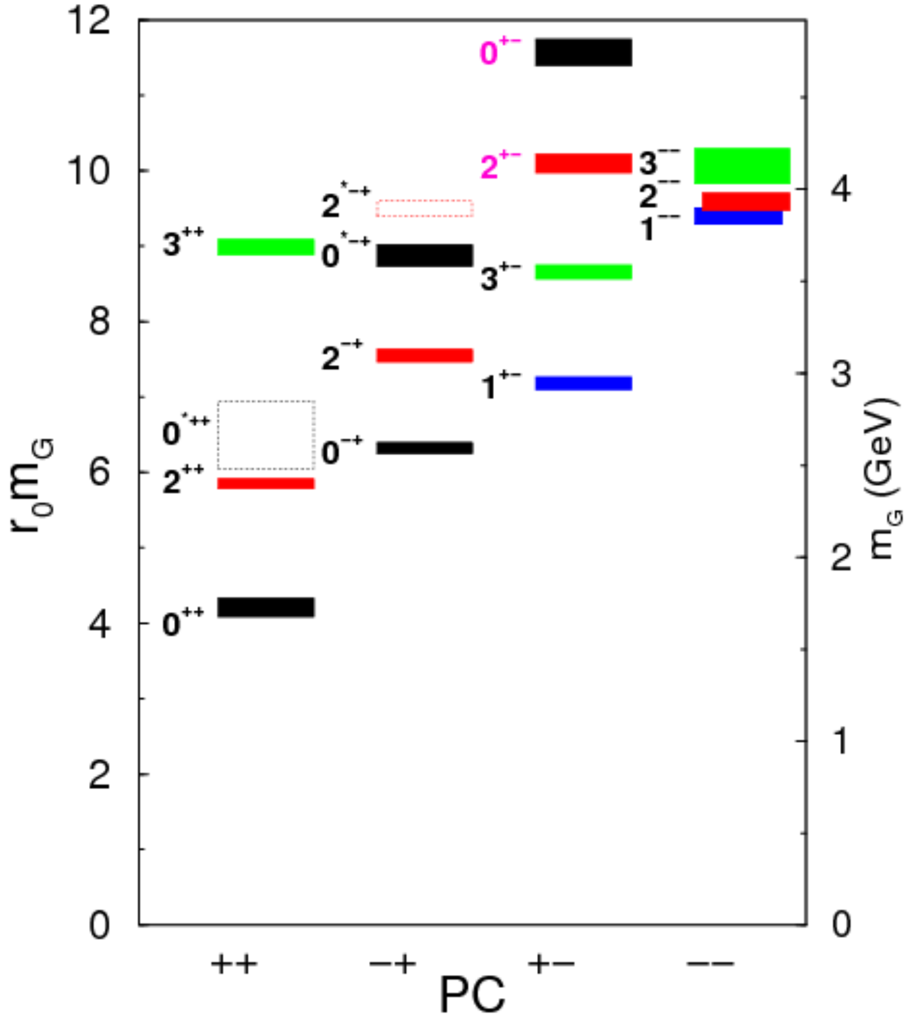
  - quark models (constituent, flux tube, bag, etc.)



Cornwall and Soni, PLB120 (1983) 431
Hou and Wong, PRD67, 034003 (2003)

  - since gluons have spin-one, all glueballs are bosons
- Lattice QCD, QCD Sum Rules, …

Old: lattice and model-dependent

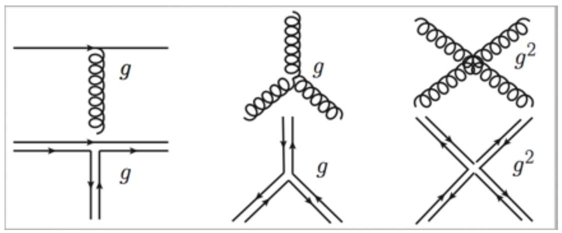


J^{PC}	Constituent	Lattice	Experiment
0^{++}	{1730} (0.72)	1730 (0.72)	1500^b (0.76)
	2685 (1.12)	2670 (1.11)	2105^b (1.06)
	2710 (1.13)		2320^c (1.17)
	2790 (1.16)		
2^{++}	{2400} (1.00)	2400 (1.00)	1980^b (1.00)
	2693 (1.12)	3290 (1.37)	2020^d (1.02)
	2700 (1.13)		2240^d (1.13)
	2730 (1.14)		2370^d (1.20)
	2810 (1.17)		
0^{-+}	2570 (1.07)	2590 (1.08)	2140^d (1.08)
	2765 (1.15)		2190^b (1.11)
1^{-+}	2605 (1.09)		
	2770 (1.15)		
2^{-+}	2615 (1.09)	3100 (1.29)	2040^d (1.03)
	2775 (1.16)	3890 (1.62)	2300^d (1.16)
1^{++}	2690 (1.12)		2340^d (1.18)
3^{++}	2694 (1.12)	3690 (1.54)	2000^d (1.01)
			2280^d (1.15)
4^{++}	2695 (1.12)	3650^a (1.52)	2044^d (1.03)
			2320^d (1.17)
$3g(0^{-+})$	3780 (1.58)	3640 (1.52)	
$3g(1^{--})$	3680 (1.53)	3850 (1.60)	
$3g(3^{--})$	3690 (1.54)	4130 (1.72)	

Morningstar and Peardon, PRD60, 034509 (1999)

Hou and Wong, PRD67, 034003 (2003)

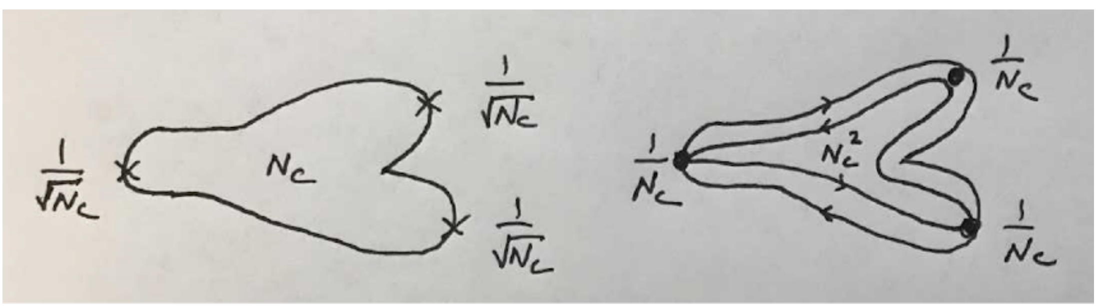
- Should we expect wide or narrow glueball states?
  - difficult to say model-independently; lots of model-dependent results
  - large N_c counting rules can provide guidance ('t Hooft limit)



Each coupling: $g \sim \frac{1}{\sqrt{N_c}}$

Each quark loop: N_c

  - meson and glueball decay amplitudes



$\sim N_c^{-\frac{1}{2}}$ $\qquad$ $\sim N_c^{-1}$

$$A_{n(q\bar{q})} \sim N_c^{-\frac{n-2}{2}}$$

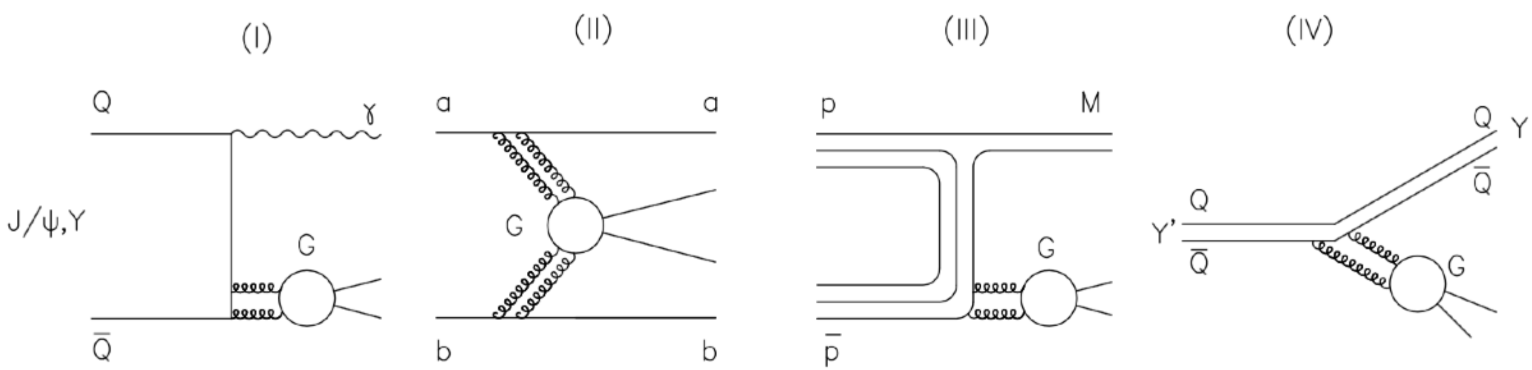
$$A_{n(G)} \sim N_c^{-(n-2)}$$

$$A_{n(G),m(q\bar{q})} \sim N_c^{-\frac{n}{2}+m-1}$$

- Glueballs are narrow in the large N_c limit, expect smaller widths

Experimental searches for glueballs

- It appears that 0^{++} glueball is the lightest glueball state
  - it must be produced copiously in the glue-rich environment and couples strongly to the color-singlet di-gluon (radiative J/ψ decays)
  - its production in gamma-gamma collisions must be suppressed
  - the decay/production amplitude for the glueballs is flavor



  - it must be narrow (at least in the large N_c limit; also chiral)

Chanowitz, PRL95, 172001 (2005)

- All of this is generally true for other glueball states as well

“Experimental” searches for glueballs

automatic sweet dumpling **machine/rice glue balls making machine**

$852.00-$2,738.00/ Set

1 Set (Min. Order)

Verified 9 YRS CN Supplier >

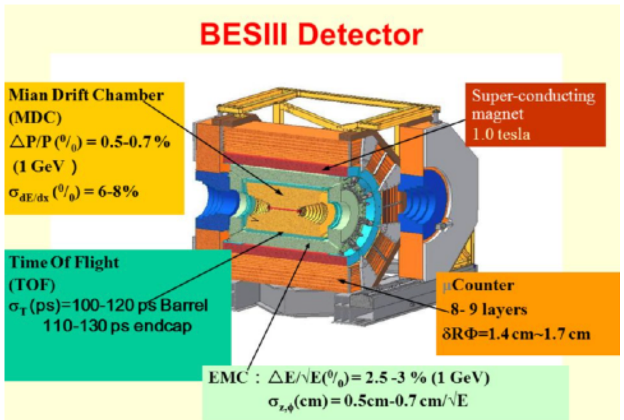
3.9 ★ (5)

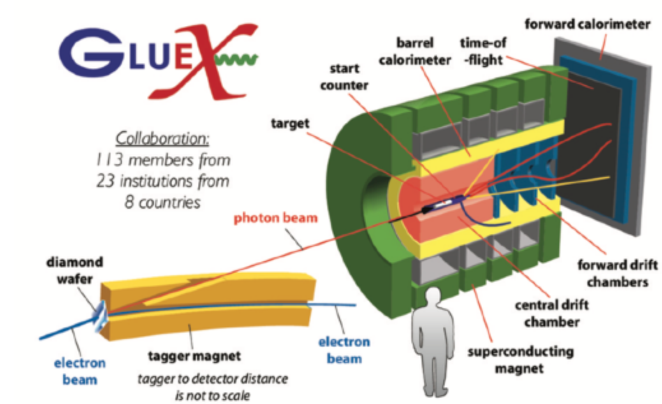
Contact Supplier

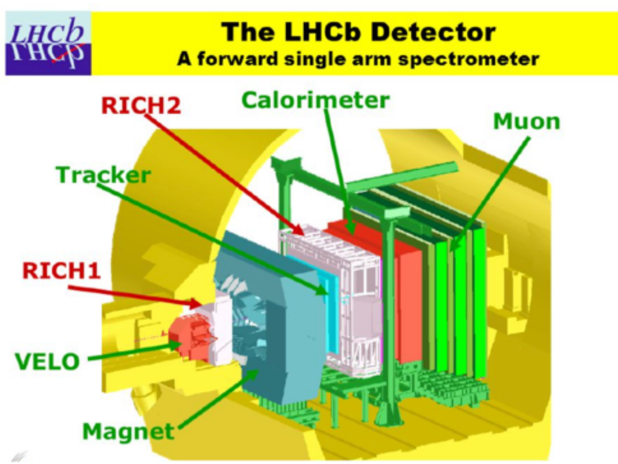
Compare

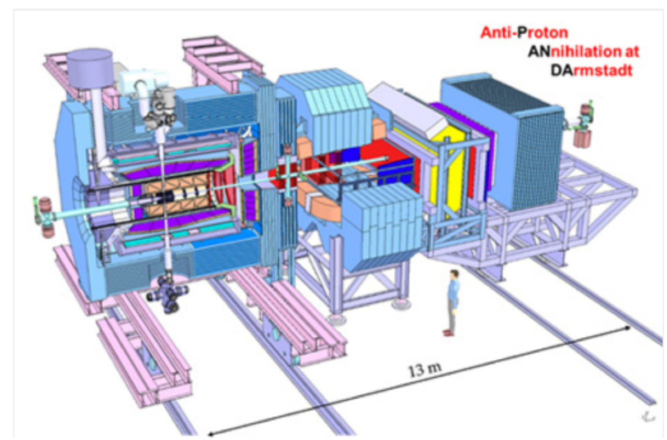
Experimental searches for glueballs

- Searches at dedicated and general-purpose detectors









- No convincing observation of a glueball state yet. Why?

Problems with finding glueballs?

- Glueballs and $q\bar{q}$ states might have the same quantum numbers
  - quantum mechanics requires mixing of those states…
  - … which means that "pure glueballs" do not exist!
  - let us still concentrate on scalar 0^{++} states

$$f_0(500),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

  - these states are admixtures $|f_{0i}\rangle = \alpha_i|N\rangle + \beta_i|S\rangle + \gamma_i|G\rangle$

$$N \equiv n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$$

$$S \equiv s\bar{s},$$

  - fit to experiment (see next)
  - various fits exist for the relative coefficients, here is a recent one

$$\begin{pmatrix} |f_0(1370)\rangle \\ |f_0(1500)\rangle \\ |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.819(89) & 0.290(91) & -0.495(118) \\ -0.399(113) & 0.908(37) & -0.128(52) \\ 0.413(87) & 0.302(52) & 0.859(54) \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}$$

Cheng, Chua, and Liu, PRD92, 094006 (2015)

Molecular states with glueballs

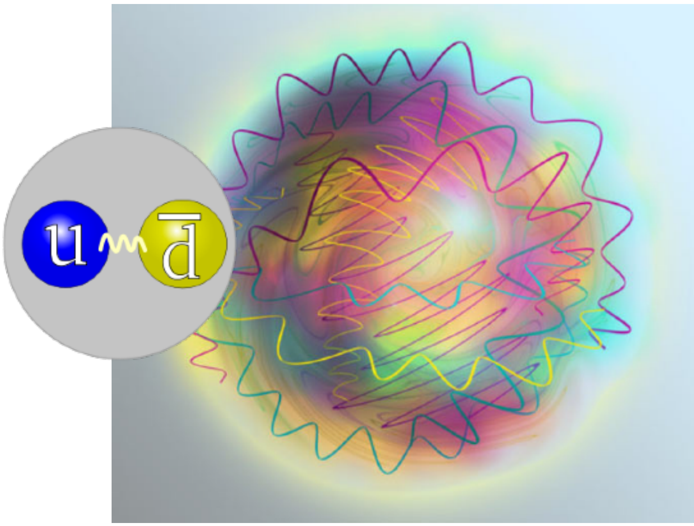
- Are there any other mechanisms for "glueball hadronization"?
  - meson-meson and meson-baryon molecular states:
    - why not glueball-meson or glueball-baryon molecular states?
    - glueballs have smaller widths than mesons in the large Nc, which might have implications for some observed highly excited states
  - some hints from Nature from observations of a few unusual states?
    - for small binding energy:

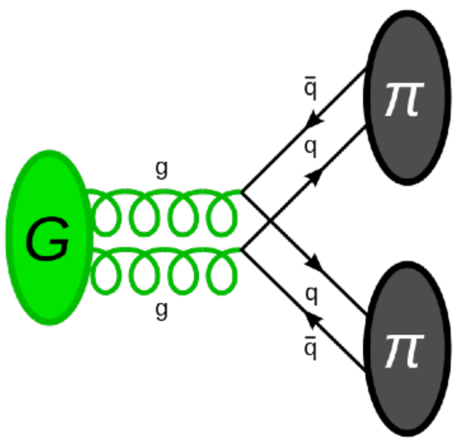
$$m_{G(0^{++})} + m_\pi \approx m_{\pi(1800)}$$

$$m_{G(1^{--})} + m_\pi \approx m_{X(3872)}$$

  - need non-relativistic description of components to build molecular states (consider lightest glueball and lightest octet of pseudoscalars)

Molecular states with glueballs





- Lifetime of the state is expected to be governed by a lifetime of the glueball component
  - smaller widths, at least from the large N_c arguments
  - possible large mixing with highly excited q-qbar states
  - expect unusually long-lived "highly excited states"
- Alternatively can be viewed as a "glueball excitation of a state"
- The lightest state: 0^{-+} or a "pseudo-glueball" $\mathscr{P}$

Scalar glueball in EFT

- It is sufficient to have an effective description of a 0^{++} glueball
  - consider massless QCD $\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + i\bar{q} \not{D} q$
  - use the fact that QCD is classically invariant under dilatations

$$x^\mu \to \lambda x^\mu \ , \quad \psi_q(x) \to \lambda^{3/2} \psi_q(\lambda x) \ , \quad A^a_\mu(x) \to \lambda A^a_\mu(\lambda x)$$

  - this symmetry is broken at quantum level

$$(T_{\rm YM})^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0$$

  - can introduce a scalar dilaton field G describing the trace anomaly

$$\mathcal{L}_{\rm dilaton} = \frac{1}{2} \left(\partial_\mu \tilde{G} \right)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left[\tilde{G}^4 \log \left| \frac{\tilde{G}}{G_0} \right| - \frac{1}{4} \tilde{G}^4 \right]$$

Migdal and Shifman

Glueball molecules

- To calculate the binding energy need to couple pions and glueballs
  - use linear sigma model $\mathcal{L} = \mathcal{L}_{\rm LSM} + \mathcal{L}_{\rm dilaton} + \mathcal{L}_{\rm int}$

$$\begin{aligned}\mathcal{L}_{\rm LSM} = & \ {\rm Tr}\left[(\partial^\mu\Phi)^\dagger(\partial_\mu\Phi)\right] - \lambda_1\left({\rm Tr}\left[\Phi^\dagger\Phi\right]\right)^2 \\ & - \lambda_2 \ {\rm Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right] + {\rm Tr}\left[H\left(\Phi^\dagger+\Phi\right)\right] \\ & + c\left(\det(\Phi^\dagger)+\det(\Phi)\right),\end{aligned}$$

$$\Phi = \frac{1}{2}\left(\sigma + i\eta_N\right)\sigma^0 + \frac{1}{2}\left(\vec{a}_0 + i\vec{\pi}\right)\cdot\vec{\sigma}$$

Jankowski et al, PRD84, 054007 (2011)

  - … with the interaction term

$$\mathcal{L}_{\rm int} = -m_0^2 \ {\rm Tr}\left[\left(\frac{\tilde{G}}{\Lambda}\right)^2 \Phi^\dagger\Phi\right]$$

- Small momentum transfer: match to determine πG coupling

Glueball molecules

- Matching to NR EFT for pions and glueballs
  - expand G and σ about the minimum ($G \rightarrow \Lambda + G$, $\sigma \to \sigma + \langle\sigma\rangle$)...

$$\mathcal{L}_{\sigma G} = -\frac{m_0^2 \langle\sigma\rangle}{\Lambda^2} G^2 \sigma + \ldots$$

$$\mathcal{L}_{\pi\pi\sigma} = -\lambda_1 \left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 - \lambda_2 \; \mathrm{Tr}\left[\left(\Phi^\dagger \Phi\right)^2\right] \quad \longrightarrow \quad \mathcal{L}_{\pi \mathrm{G}} = \lambda \pi^2 G^2$$

$$\mathcal{L}_{\pi \mathrm{G}} = \lambda \pi^2 G^2$$

  - ... resulting in

$$\mathcal{L}_{\pi \mathrm{G}} = \lambda \pi^2 G^2 \quad \text{with} \quad \lambda = \frac{m_0^2}{2\Lambda^2}\left[1 - \frac{\langle\sigma\rangle^2}{m_\sigma^2}\left(2\lambda_1 + \lambda_2\right)\right]$$

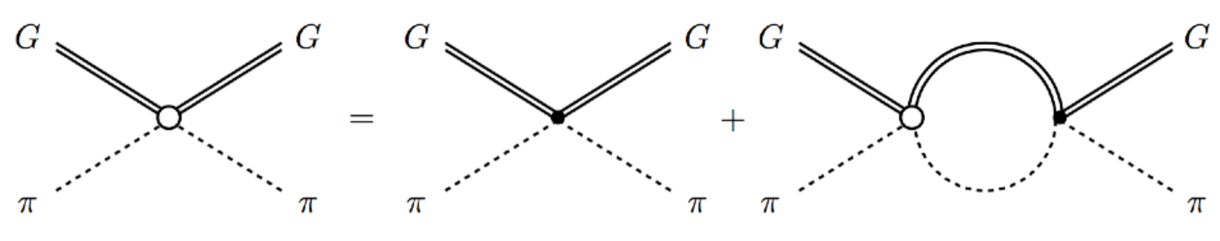
- Now we can calculate the low energy π-G scattering amplitude

Glueball molecules: binding energy

- Calculate binding energy from the pole of transition amplitude
  - in quantum mechanics

$$T_{\pi G} = \frac{4\pi}{\mu_{\pi G}} \frac{1}{p \cot \delta_s - ip} = -\frac{4\pi}{\mu_{\pi G}} \frac{a}{1 + ipa}$$

  - QFT: solve Lippmann-Schwinger equation to find the transition amplitude



$$iT_{\pi \mathrm{G}} = -i\lambda + \int \frac{d^4 q}{(2\pi)^4} \left(iT_{\pi \mathrm{G}}\right) G_{\pi \mathrm{G}} \left(-i\lambda\right)$$

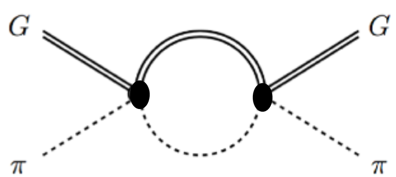
- Need to evaluate one loop integral: divergence?

Glueball molecules: binding energy

- Calculate binding energy from the pole of transition amplitude
  - resuming the "bubbles"…

$$T_{\pi \mathrm{G}} = \frac{\lambda}{1 + i\lambda \widetilde{A}}$$

  - …need to calculate (expect a divergence, move to d-1 dim), $\lambda \to \lambda_R$



$$\widetilde{A} = -\frac{i}{2}\frac{\mu_{\pi G}}{m_G m_\pi}\int \frac{d^3 q}{(2\pi)^3}\frac{1}{\vec{q}^2 - 2\mu_{\pi G}E - i\epsilon}$$

  - … which is very similar to Weinberg's treatment of a deuteron!

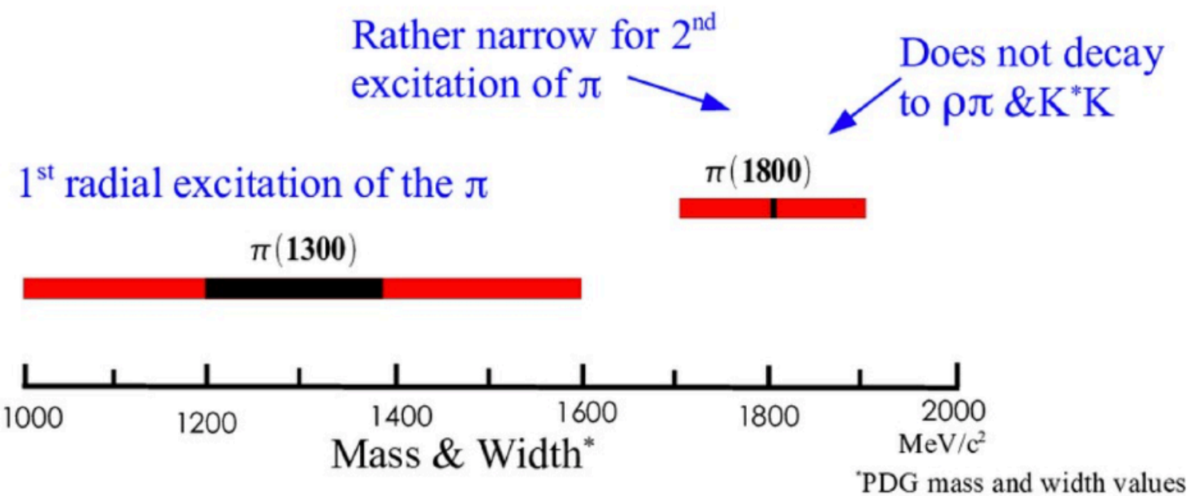
$$E_{\mathrm{bound}} = E_{pole} = \frac{32\pi^2}{\lambda_R^2}\frac{m_\pi^2 m_G^2}{\mu_{\pi G}^3}$$

  - …and a bound state energy of about several MeV

The $\pi(1800)$ puzzle?

$\pi(1800)$ 0^{-+} Hybrid

$$\pi(1800) \to f_0(980)\pi,\ f_0(500)\pi,\ a_0(980)\eta,\ \omega\rho,\ \eta\eta'\pi,\ K_0^*(1430)K$$



Many† have suggested that the $\pi(1800)$ is a 0^{-+} hybrid meson

†See for example T. Barnes, F. E. Close, P. R. Page, & E. S. Swanson Phys. Rev. D55 4157 (1997)

P. Eugenio, talk at 2016 APS April meeting

$\pi(1800)$ as a glueball molecule

- It appears that most issues with understanding of $\pi(1800)$ would go away if a dominant part of the $\pi(1800)$ wave function is built up from a glueball-π molecule
  - lifetime of a glueball-pi molecule is driven by a glueball lifetime
    - expect smaller width than usual qq-bar excitations
  - $\pi(1800)$ mass is tantalizingly close to that of a G(0^{++})-π molecule
    - for small binding energy, as considered before,

$$m_{G(0^{++})} + m_\pi \approx m_{\pi(1800)}$$

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

Wikipedia's definitions of a "Duck test"

Things to take home

- Glueballs are expected to be there from QCD
  - smaller widths, at least from the large N_c arguments
  - possible large mixing with highly excited q-qbar states
  - expect unusually long-lived highly excited states
- Proposed a new mechanism for "glueball hadronization"
- Alternatively can be viewed as a "glueball excitation of a qq-bar or a qqq state"
  - has direct implications for the N* program at JLab
  - opens up new opportunities in identifying gluon degrees of freedom of ordinary hadrons
- How do you know that X(3872) and other molecules/tetraquarks contains charmed quarks? What about new pentaquark states?


The Evening
EXTRA!

Non-relativistic pions

- For the weakly-bound system need non-relativistic pions
  - not an unusual situation for pionic atoms!

Kong and Ravndal, PRD61, 077506 (2000)

  - kinetic part

$$\mathcal{L}_0(\pi_i)=\pi_i^*\left(i\frac{\partial}{\partial t}+\frac{1}{2m_i}\nabla^2\right)\pi_i$$

  - interaction part

$$\mathcal{L}_{int}(\boldsymbol{\pi})=\frac{1}{4}A_0(\pi_0^*\pi_0^*\pi_0\pi_0)+B_0(\pi_+^*\pi_-^*\pi_+\pi_-)$$

$$+\frac{1}{2}C_0(\pi_+^*\pi_-^*\pi_0\pi_0+\pi_0^*\pi_0^*\pi_+\pi_-)$$

$$+\frac{1}{4}D_0(\pi_+^*\pi_+^*\pi_+\pi_++\pi_-^*\pi_-^*\pi_-\pi_-$$

$$+2\pi_+^*\pi_0^*\pi_+\pi_0+2\pi_-^*\pi_0^*\pi_-\pi_0)$$

  - NR pion propagator

$$G(E,\mathbf{k})=\frac{1}{E-\mathbf{k}^2/2m_\pi+i\epsilon}$$