Light hadron spectroscopy: Superconformal quantum mechanics and its holographic embedding

Guy F. de Téramond

UCR

Hadron Spectroscopy:

The Next Big Steps

MITP Workshop

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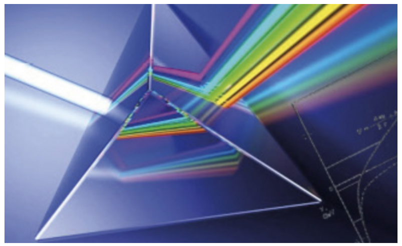

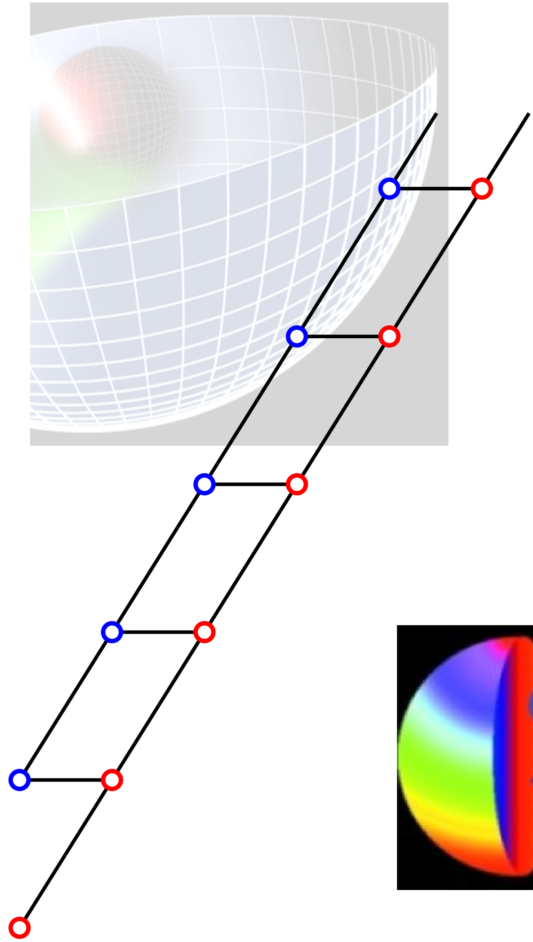

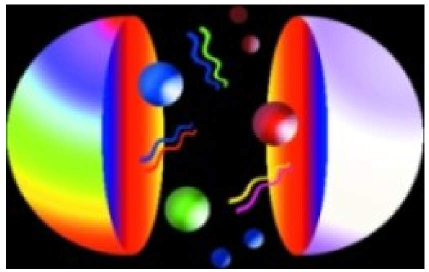

Image credit: N. Evans

With Stan Brodsky, Hans G. Dosch, Alexandre Deur, Tianbo Liu, Raza Sabbir Sufian, Marina Nielsen and Liping Zou

I. Recent insights into the nonperturbative structure of QCD based on

   (a) Light-front quantization

   (b) Gauge/gravity correspondence

   (c) Superconformal algebra

   lead to semiclassical wave equations which incorporate essential elements not obvious from the QCD Lagrangian, such as the introduction of a mass scale, confinement, a massless pion in the chiral limit, and the connection between mesons, baryons and tetraquarks

II. Extensions of the holographic LF QCD approach (HLFQCD) incorporate the exclusive-inclusive connection and provide nontrivial interrelations between the dynamics of form factors and polarized and unpolarized quark and gluon distributions with pre-QCD nonperturbative approaches such as Regge theory and the Veneziano model

- In this talk I give an overview of (I) in the ultrarelativistic limit of zero quark masses

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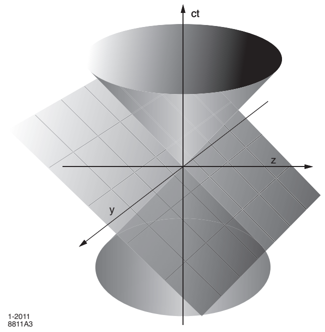
1 Semiclassical approximation to QCD in the light front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]



- Light-front quantization uses the null plane $x^+ = x^0 + x^3 = 0$ tangential to the light cone as the initial surface (Dirac 1949)

- Schrödinger-like equation in the light-front (LF)

$$i \frac{\partial}{\partial x^+} |\psi\rangle = P^- |\psi\rangle, \qquad P^- |\psi\rangle = \frac{\mathbf{P}_\perp^2 + M^2}{P^+} |\psi\rangle,$$

for hadron with 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, where P^- is a dynamical generator and P^+ and $\mathbf{P}_\perp$ kinematical

- LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^+ P^- - \mathbf{P}_\perp^2$

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \qquad |\psi\rangle = \sum_n \psi_n |n\rangle$$

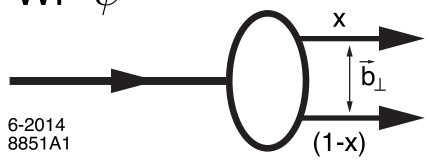
- Simple structure of LF vacuum allows a quantum-mechanical probabilistic interpretation of hadronic states in terms of wave functions, $\psi_n = \langle n|\psi\rangle$, similar to usual Schrödinger equation

- The mass spectrum for a two-parton bound state is computed from the hadron matrix element

$$\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = M^2\langle\psi(P')|\psi(P)\rangle$$

- We factor out the longitudinal $X(x)$ and orbital $e^{iL\varphi}$ dependence from the LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\varphi}X(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



with invariant impact "radial" LF variable $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$ and $L = \max|L^z|$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple

$$M^2 = \int d\zeta\,\phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\,\phi^*(\zeta)\,U(\zeta)\,\phi(\zeta)$$

where the effective potential U includes all interactions, including those from higher Fock states

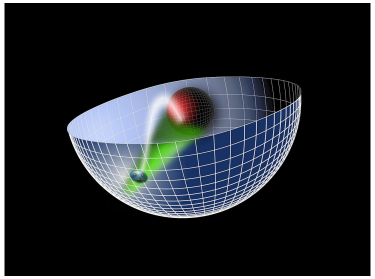
- The Lorentz invariant equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ becomes a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- Critical valuel $L=0$ corresponds to the lowest possible stable solution
- Relativistic and frame-independent semiclassical WE: It has identical structure of AdS WE

2 Higher-spin wave equations in AdS

[GdT, H. G. Dosch and S. J. Brodsky, PRD **87**, 075005 (2013)]



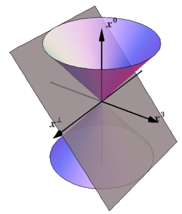
Integer spin

- Start with AdS action for a tensor-J field $\Phi_{N_1 \dots N_J}$ with a dilaton field φ

$$\left[- \frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \frac{(\mu R)^2}{z^2} \right] \Phi_J(z) = M^2 \Phi_J(z)$$

plus kinematical constraints to eliminate lower spin from the symmetric tensor $\Phi_{N_1 \dots N_J}$

- Upon the substitution $\Phi_J(z) = z^{(d-1)/2-J} e^{-\varphi(z)/2} \, \phi_J(z)$ we find for $d = 4$ the semiclassical QCD light-front wave equation (z is the holographic variable of AdS space)



$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

with $\zeta^2 = z^2 = x(1-x) b_\perp^2$ the LF invariant separation between two quarks and

$$U(\zeta, J) = \tfrac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta)$$

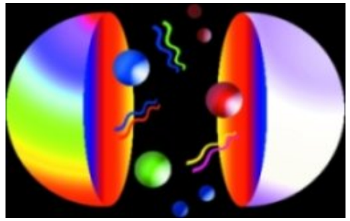
the effective LF confinement potential with AdS mass-radius $(\mu R)^2 = -(2-J)^2 + L^2$

- QM condition $L^2 \geq 0$ equivalent to the Breitenlohner-Freedman AdS bound $(\mu R)^2 \geq 4$ for $J = 0$

Half-integer spin

- Start with Rarita-Schwinger action in AdS for spinor-J field $\Psi_{N_1 \ldots N_{J-1/2}}$ with potential V
- Upon the substitution $\Psi_J^{\pm}(z) = z^{(d-1)/2-J} \psi_J^{\pm}(z) u^{\pm}$ we find for $d = 4$

$$-\frac{d}{dz}\psi_- - \frac{\nu + \frac{1}{2}}{z}\psi_- - V(z)\psi_- = M\psi_+$$
$$\frac{d}{dz}\psi_+ - \frac{\nu + \frac{1}{2}}{z}\psi_+ - V(z)\psi_+ = M\psi_-$$

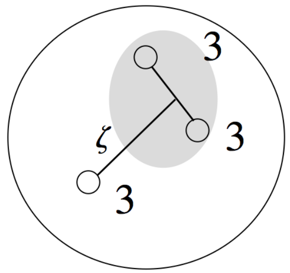


with $|\mu R| = \nu + 1/2$ and equal probability $\int dz\, \psi_+(z)^2 = \int dz\, \psi_-^2(z)$

Image credit: N. Evans

- System of linear Eqs in AdS is equivalent to second order Eqs. Mapping to the light front $z \to \zeta$:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U^+(\zeta)\right)\psi_+ = M^2\psi_+$$
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4(L+1)^2}{4\zeta^2} + U^-(\zeta)\right)\psi_- = M^2\psi_-$$



the semiclassical LF WE with ψ_+ and ψ_- corresponding to LF orbital L and $L+1$ with

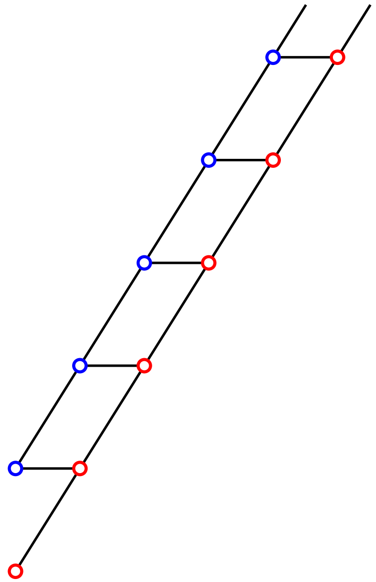
$$U^{\pm}(\zeta) = V^2(\zeta) \pm V'(\zeta) + \frac{1 + 2L}{\zeta} V(\zeta), \qquad L = \nu,$$

a J-independent potential in agreement with the observed degeneracy in the baryon spectrum

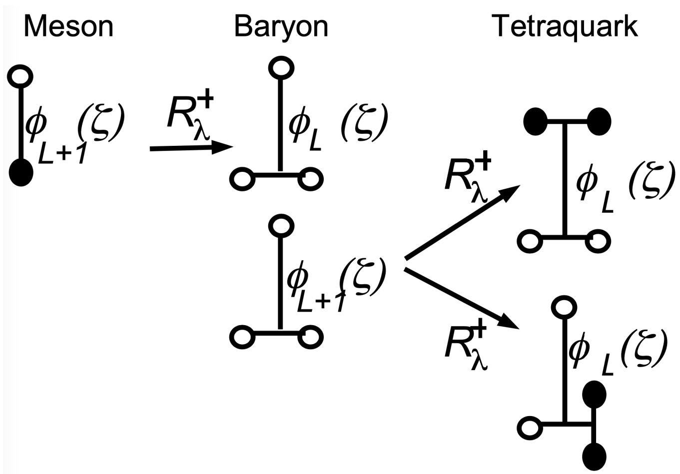
3 Superconformal algebraic structure and emergence of a mass scale

[V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]



- $V(\zeta)$ plays the role of a superpotential: Its form is fixed by superconformal QM
- Superconformal algebra underlies in HLFQCD the scale invariance of the QCD Lagrangian: Its breaking leads to the emergence of a scale in the Hamiltonian keeping the action conformal invariant
- It also incorporates the connection between mesons, baryons and tetraquarks underlying the $SU(N)_C$ representation properties: $\overline{N} \to N \times N$



$$\overline{\mathbf{3}} \to \mathbf{3} \times \mathbf{3} \qquad \mathbf{3} \to \overline{\mathbf{3}} \times \overline{\mathbf{3}}$$

Supersymmetric Quantum Mechanics

[E. Witten, NPB **188**, 513 (1981)]

- SUSY QM contains two fermionic generators Q and $Q^\dagger$ and a bosonic generator, the Hamiltonian H

$$\tfrac{1}{2}\{Q, Q^\dagger\} = H$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \quad [Q, H] = [Q^\dagger, H] = 0$$

which closes under the graded algebra $sl(1/1)$

- Since $[Q^\dagger, H] = 0$, the states $|E\rangle$ and $Q^\dagger|E\rangle$ for $E \neq 0$ are degenerate, but for $E = 0$ we can have the trivial solution $Q^\dagger|E = 0\rangle = 0$ (the pion)

- Matrix representation of SUSY generators $Q, Q^\dagger$ and H

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \qquad H = \frac{1}{2}\begin{pmatrix} q\, q^\dagger & 0 \\ 0 & q^\dagger q \end{pmatrix}$$

- For a conformal theory (f dimensionless)

$$q = -\frac{d}{dx} + \frac{f}{x}, \qquad q^\dagger = \frac{d}{dx} + \frac{f}{x}$$

- Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to the generator of conformal transformations $\frac{1}{2}\{S, S^\dagger\} = K$

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \qquad S^\dagger = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}$$

and leads to a conformal enlarged algebra [Haag, Lopuszanski and Sohnius (1974)]

- Following Fubini and Rabinovici we define the fermionic generator $R = Q + \lambda\, S$, $[\lambda]$ = GeV^2,

$$\{R_\lambda, R_\lambda^\dagger\} = G_\lambda$$

$$\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \quad [R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$$

which also closes under the graded algebra $sl(1/1)$:

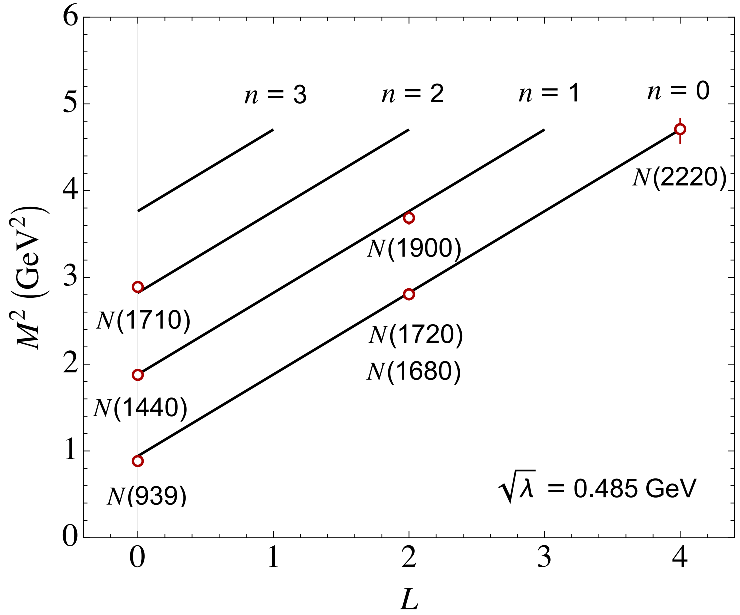
- In a 2×2 matrix representation the Hamiltonian equation $G|\phi\rangle = E|\phi\rangle$ leads to

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda\, f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2}\right) \phi_1 = E\, \phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda\, f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2}\right) \phi_2 = E\, \phi_2$$

4 Light-front mapping and baryons

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]



- Upon LF mapping to the superconformal eigenvalue equations

$$x \mapsto \zeta, \quad E \mapsto M^2, \quad f \mapsto L + \tfrac{1}{2}$$

$$\phi_1 \mapsto \psi_-, \quad \phi_2 \mapsto \psi_+$$

we recover the nucleon bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L+1)\right)\psi_+ = M^2\psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L\right)\psi_- = M^2\psi_-$$

with $U^+ = \lambda^2\zeta^2 + 2\lambda(L+1)$ and $U^- = \lambda^2\zeta^2 + 2\lambda L$

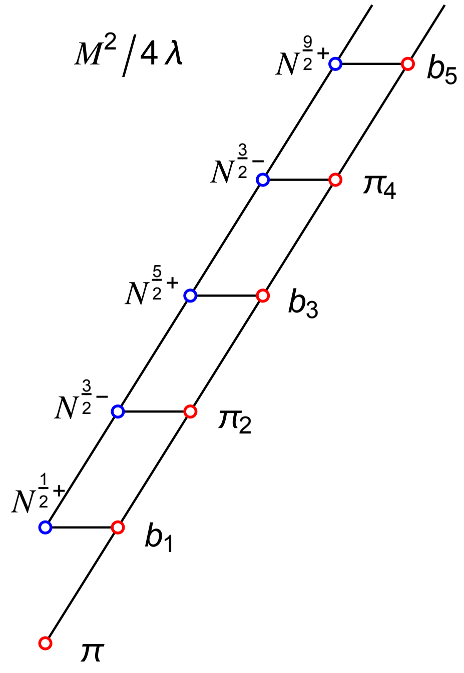
- Eigenvalues $M^2 = 4\lambda(n + L + 1)$

- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2), \quad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, PRD **85**, 076003 (2012)]

5 Superconformal meson-baryon-tetraquark symmetry



[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

- Upon substitution in the superconformal equations

$$
\begin{aligned}
x &\mapsto \zeta, \quad E \mapsto M^2, \\
\lambda &\mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \tfrac{1}{2} = L_B + \tfrac{1}{2} \\
\phi_1 &\mapsto \phi_M, \quad \phi_2 \mapsto \phi_B
\end{aligned}
$$

we find the LF meson (M) – baryon (B) bound-state equations

$$
\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M(L_M - 1)\right)\phi_M = M^2\,\phi_M
$$

$$
\left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_N + 1)\right)\phi_B = M^2\,\phi_B
$$

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $L_M = L_B + 1$
- L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator cluster in the baryon: effective quark-diquark approximation

- Special role of the pion as a unique state of zero energy

$$R^\dagger|M,L\rangle = |B, L-1\rangle, \quad R^\dagger|M, L=0\rangle = 0$$

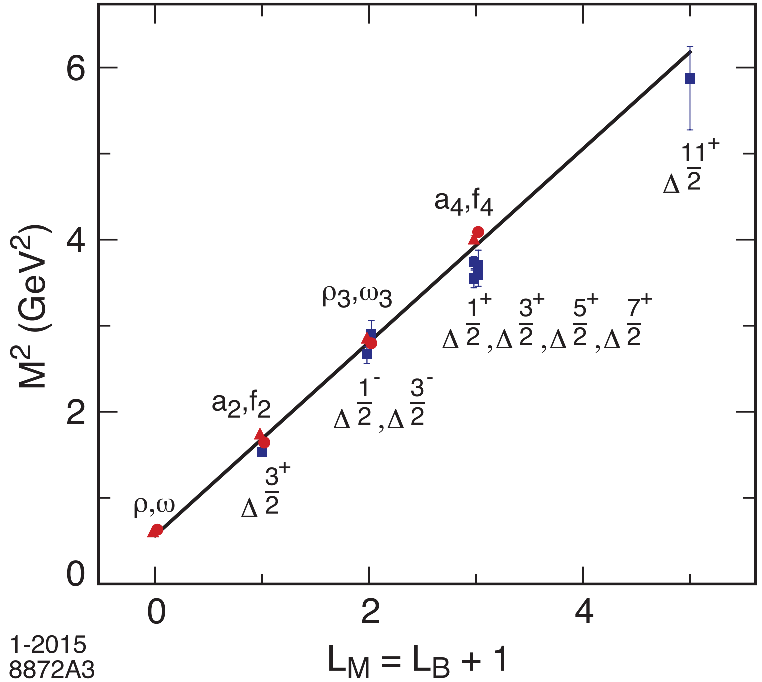
- Spin-dependent Hamiltonian for mesons and baryons with internal spin $S = 0, 1$

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé (2016)]

$$G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S$$

- Supersymmetric 4-plet

$$\begin{aligned} M_M^2 &= 4\lambda\,(n + L_M) + 2\lambda S \\ M_B^2 &= 4\lambda\,(n + L_B + 1) + 2\lambda S \\ M_T^2 &= 4\lambda\,(n + L_T + 1) + 2\lambda S \end{aligned}$$



- Expected accuracy $1/N_C^2 \sim 10\%$

- Quark masses and CSB from longitudinal dynamics

$$\left(-\sigma^2 \partial_x\left(x(1-x)\,\partial_x\right) + \frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)\chi(x) = \Delta M^2\,\chi(x)$$

- Y. Li and J. P. Vary, PLB **825**, 136860 (2022), GdT and S. J Brodsky, PRD **104**, 116009 (2021)

$$M_\pi^2 = \Delta M^2 = \sigma(m_q + m_{\overline{q}}) + \mathcal{O}\big((m_q + m_{\overline{q}})^2\big)$$

6 Nucleon SU(6) spin-parity gap

- Phenomenological extension to account for the SU(6) spin-parity gap

$$G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\,(S + P)$$

with $S, P = 0, 1$

- PDG Pole-position mass value