light meson spectroscopy from lattice QCD – resonances

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hadspec
hadron spectrum collaboration
hadspec.org

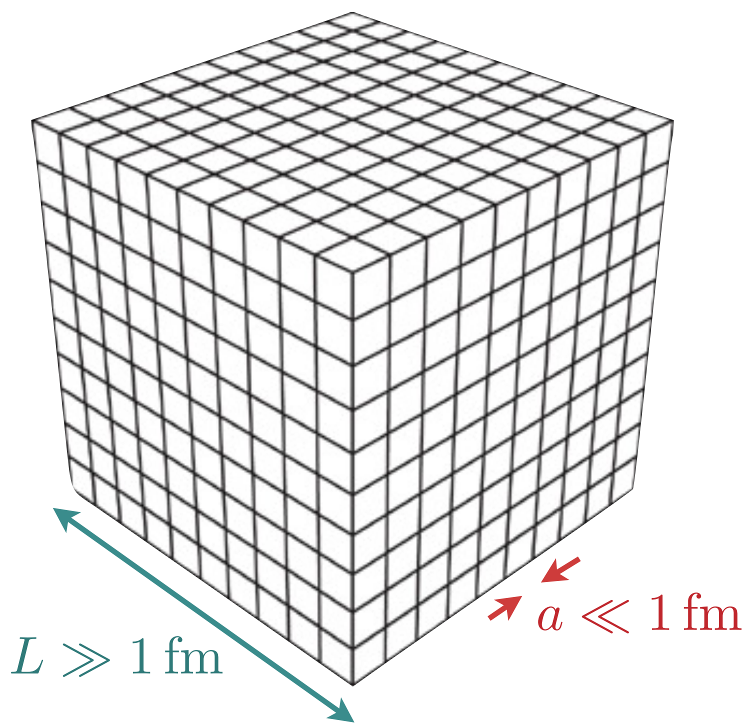
WILLIAM & MARY
CHARTERED 1693

QCD on a spacetime lattice

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, f(\psi, \bar{\psi}, A_\mu) \, e^{i\int d^4x \, \mathcal{L}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$$

sum over quark/gluon field configurations

in Euclidean spacetime, probability for a field configuration



generate field configurations → compute **correlation functions**

lattice QCD **is QCD** under controlled approximations

discretisation choice / finite lattice spacing ⇝ *relatively unimportant here*

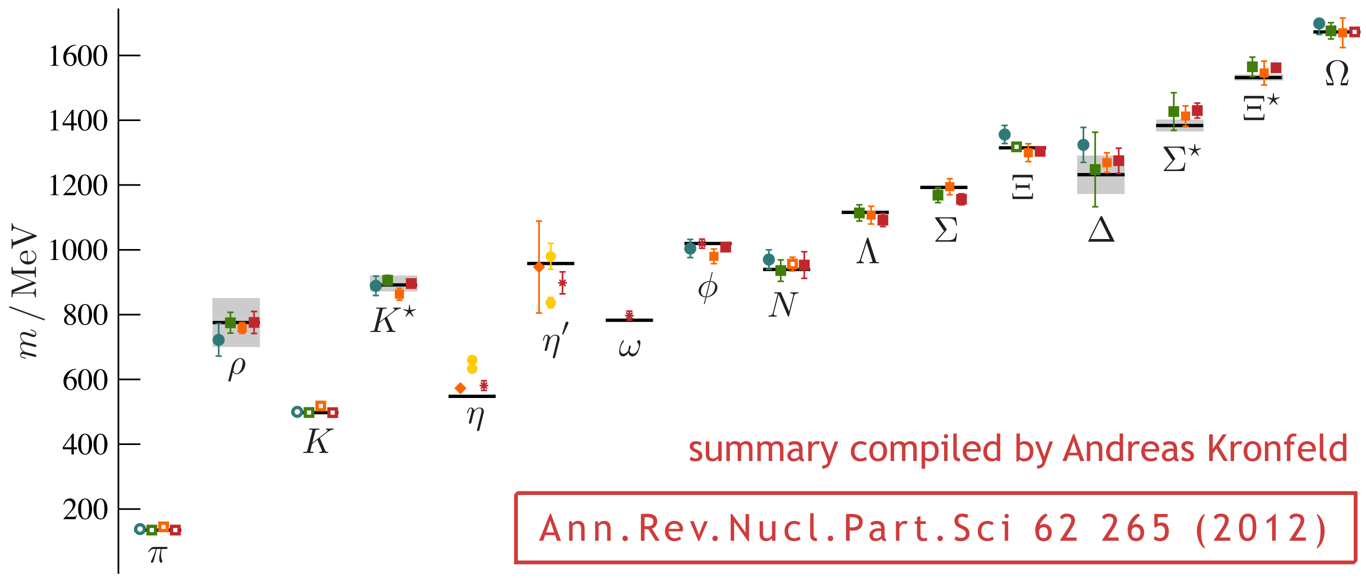
choice of quark mass value ⇝ *a tool to explore QCD*

finite spacetime volume ⇝ *how we access scattering*

computing a spectrum

spectrum in two-point correlation functions $$\langle 0|\mathcal{O}_i(t)\,\mathcal{O}_j(0)|0\rangle = \sum_n e^{-E_n t}\,\langle 0|\mathcal{O}_i|n\rangle\,\langle n|\mathcal{O}_j|0\rangle$$

lattice qcd light hadron spectrum

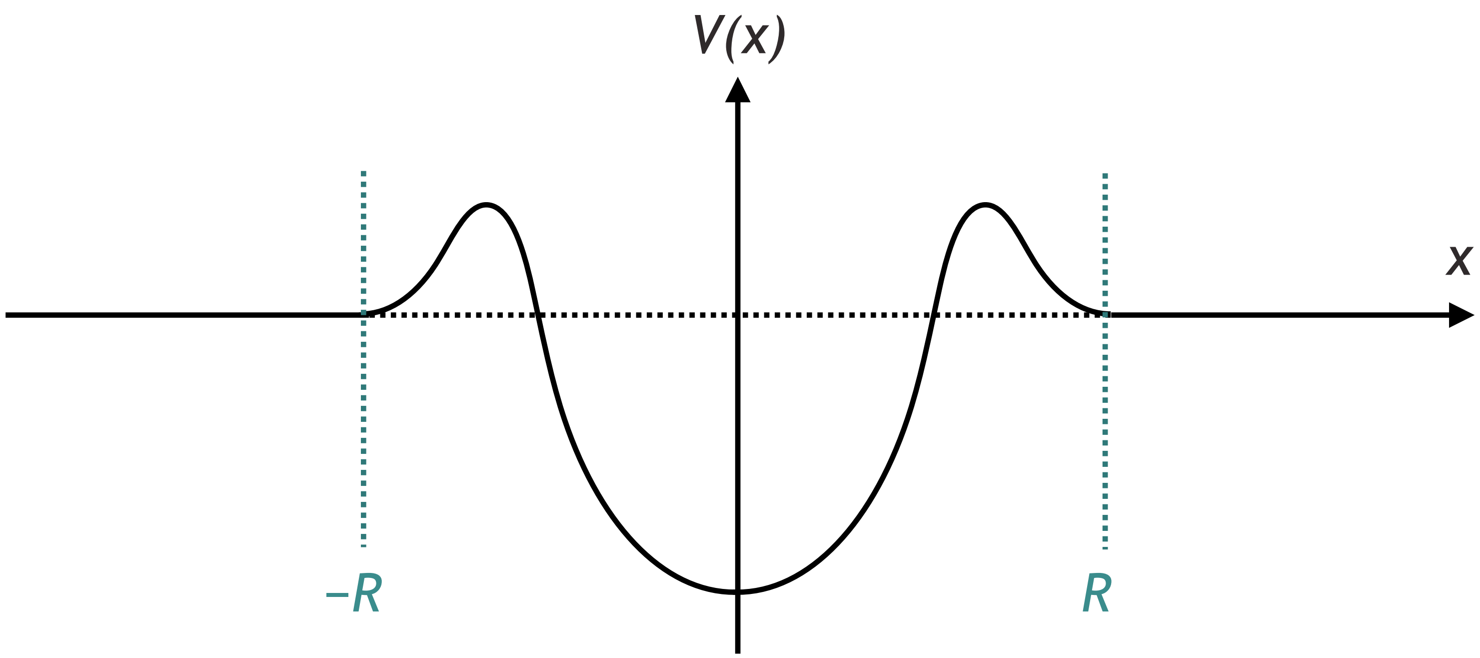


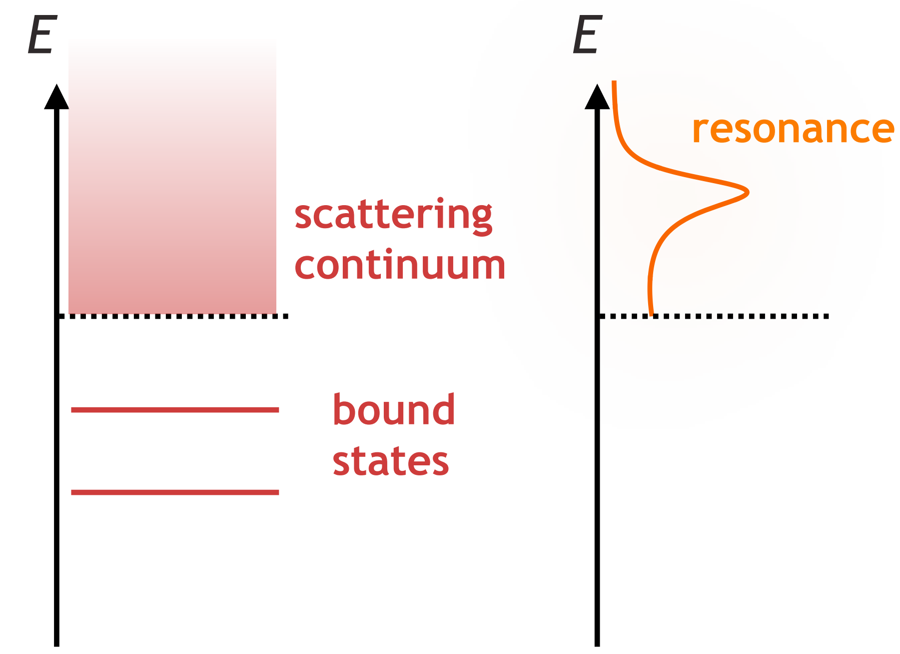
summary compiled by Andreas Kronfeld

Ann.Rev.Nucl.Part.Sci 62 265 (2012)

but what about excited states, the **resonances** of QCD … ?

resonances illustrated in quantum mechanics



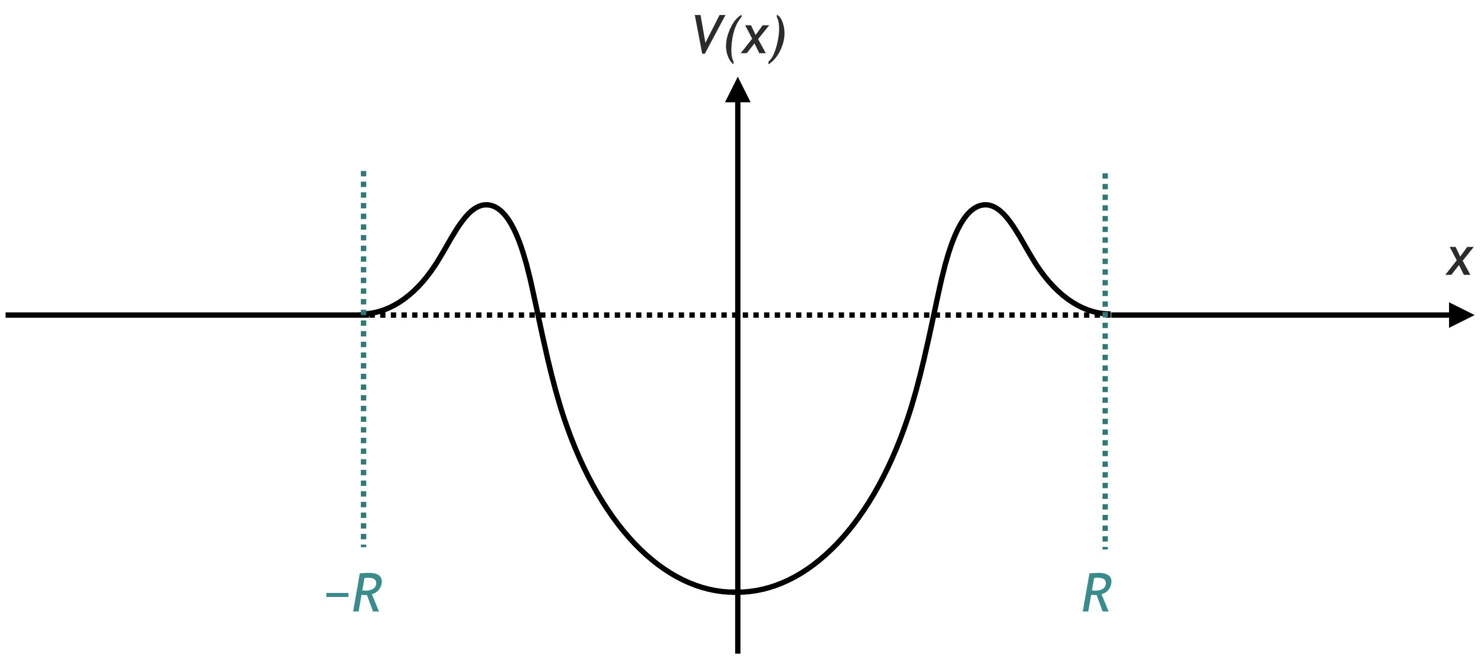


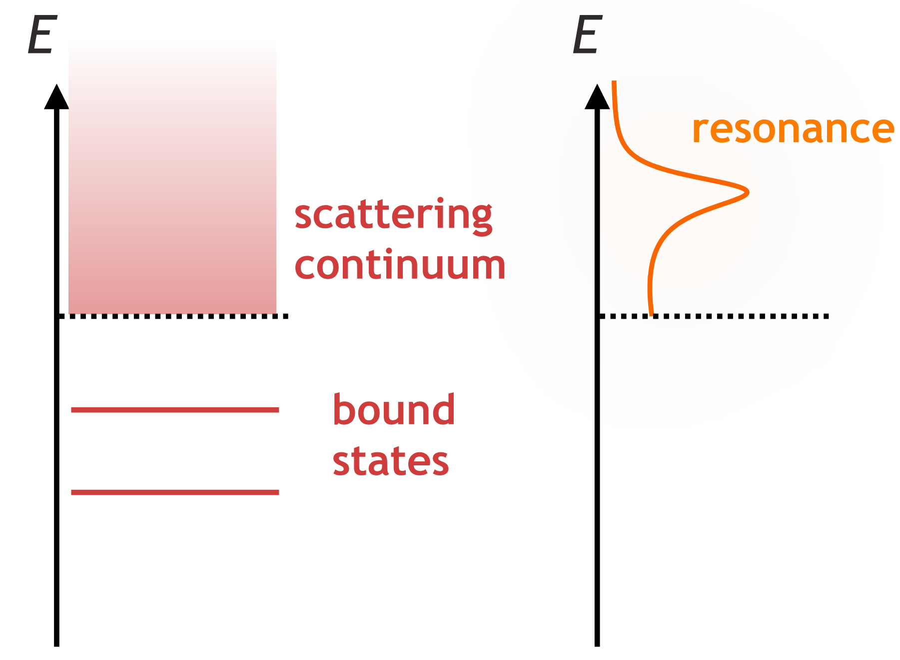
$$\psi(|x| > R) \sim \cos\left(p|x| + \delta(p)\right)$$

phase shift

resonances lie in the **continuous spectrum of scattering states**

resonances illustrated in quantum mechanics





$$\psi(|x| > R) \sim \cos\left(p|x| + \delta(p)\right)$$

phase shift

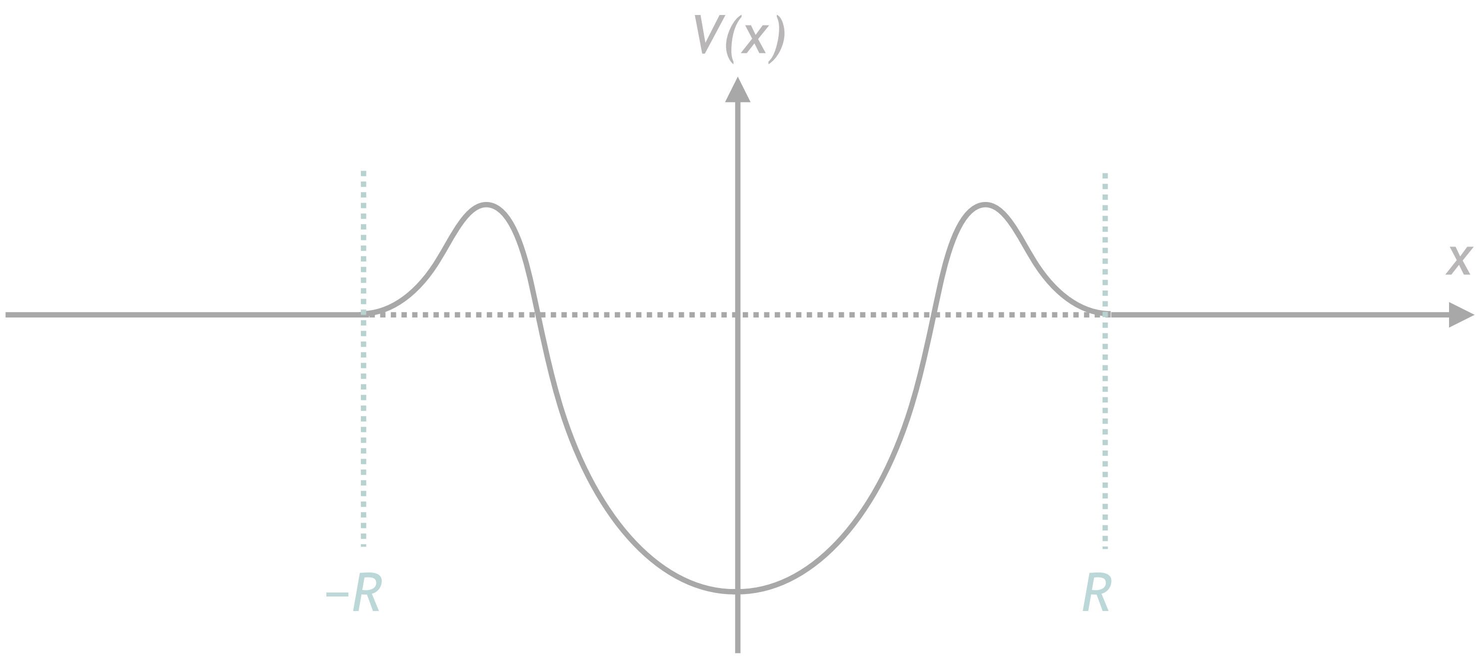
resonances lie in the **continuous spectrum of scattering states**

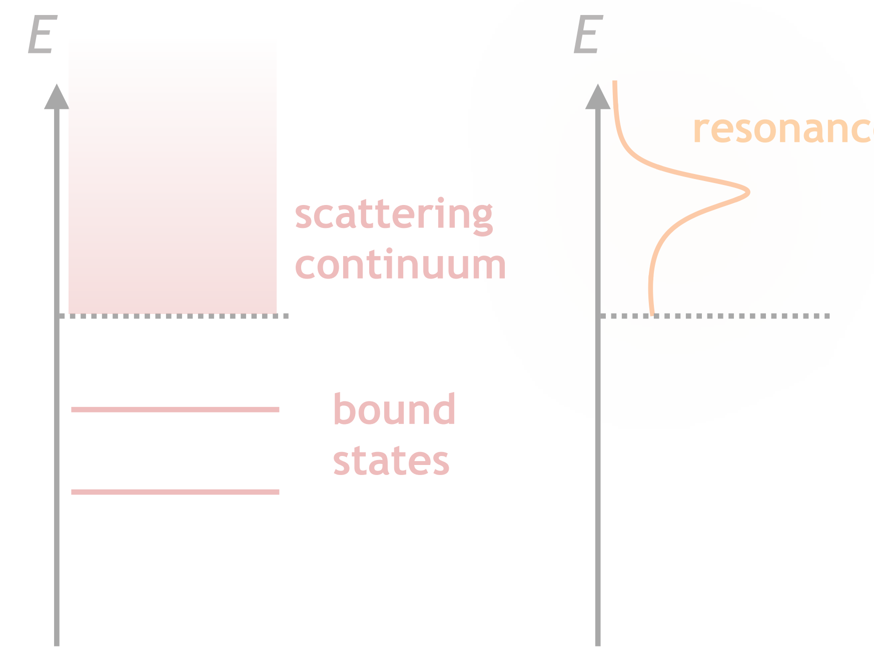
but ...

$$\langle 0|\mathcal{O}_i(t)\,\mathcal{O}_j(0)|0\rangle = \sum_n e^{-E_n t}\,\langle 0|\mathcal{O}_i|n\rangle\,\langle n|\mathcal{O}_j|0\rangle$$

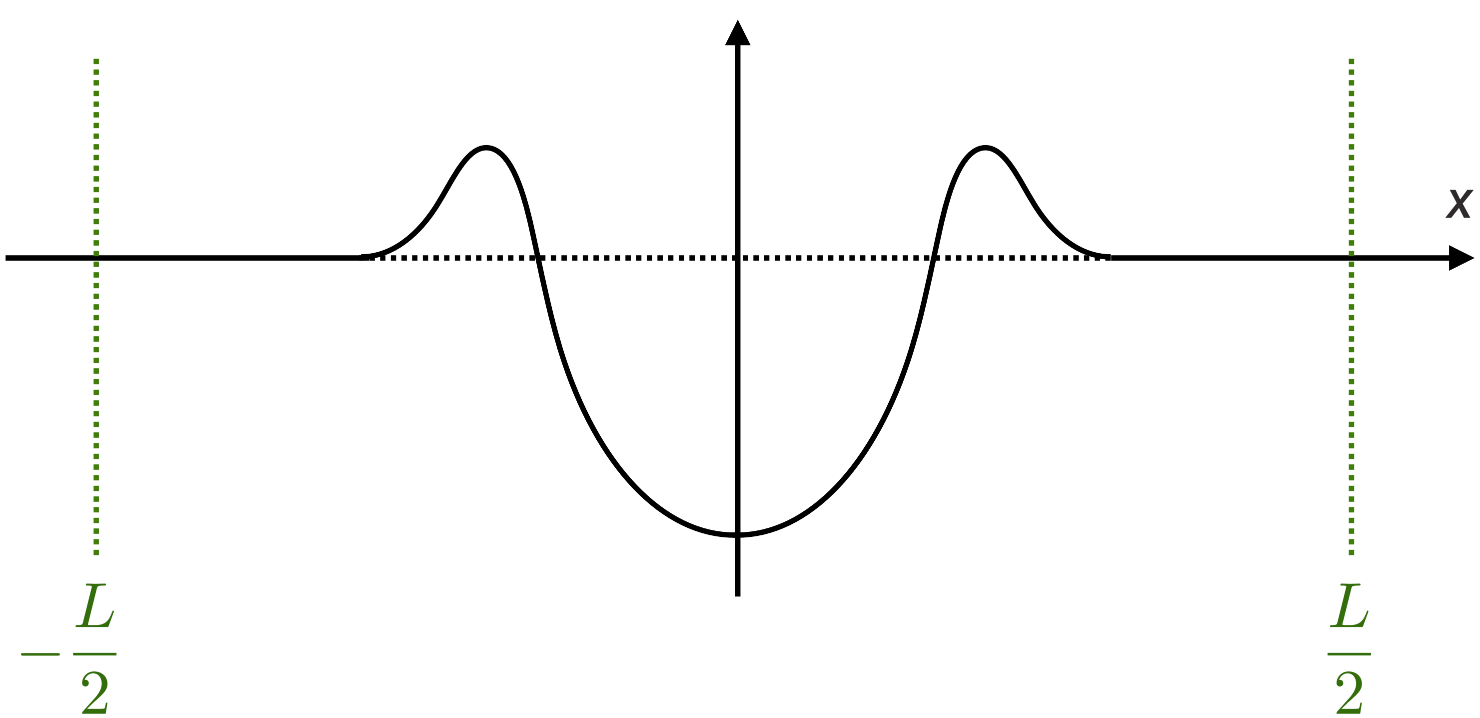
a **discrete** spectrum in finite-volume

resonances in a finite volume ?

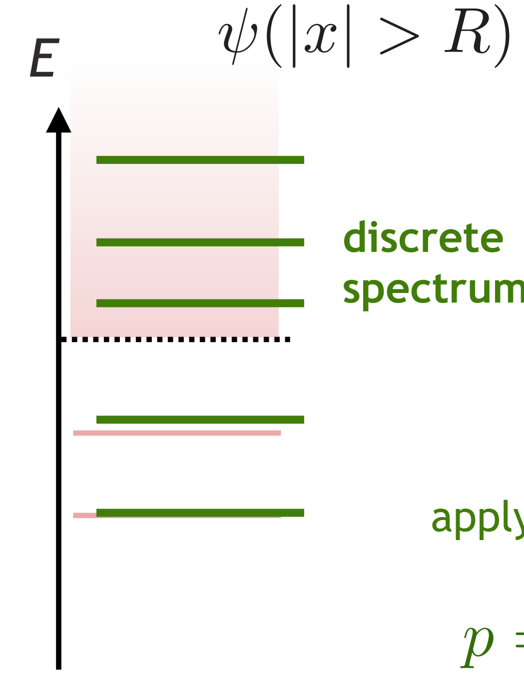




but in a periodic volume ...



$$\psi(|x| > R) \sim \cos\big(p|x| + \delta(p)\big)$$



applying the boundary conditions

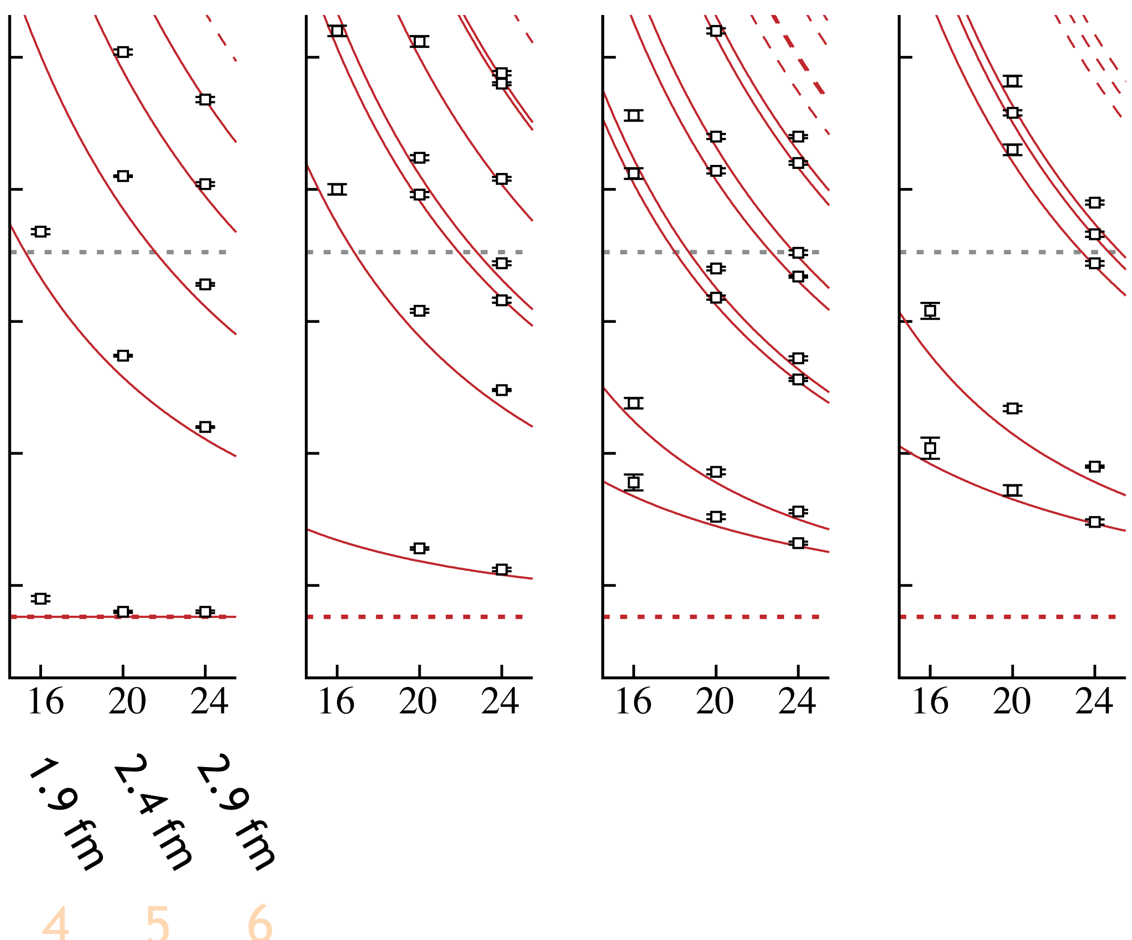
$$p = \frac{2\pi}{L} n - \frac{2}{L} \delta(p)$$

solved by **discrete** $p_n(L)$

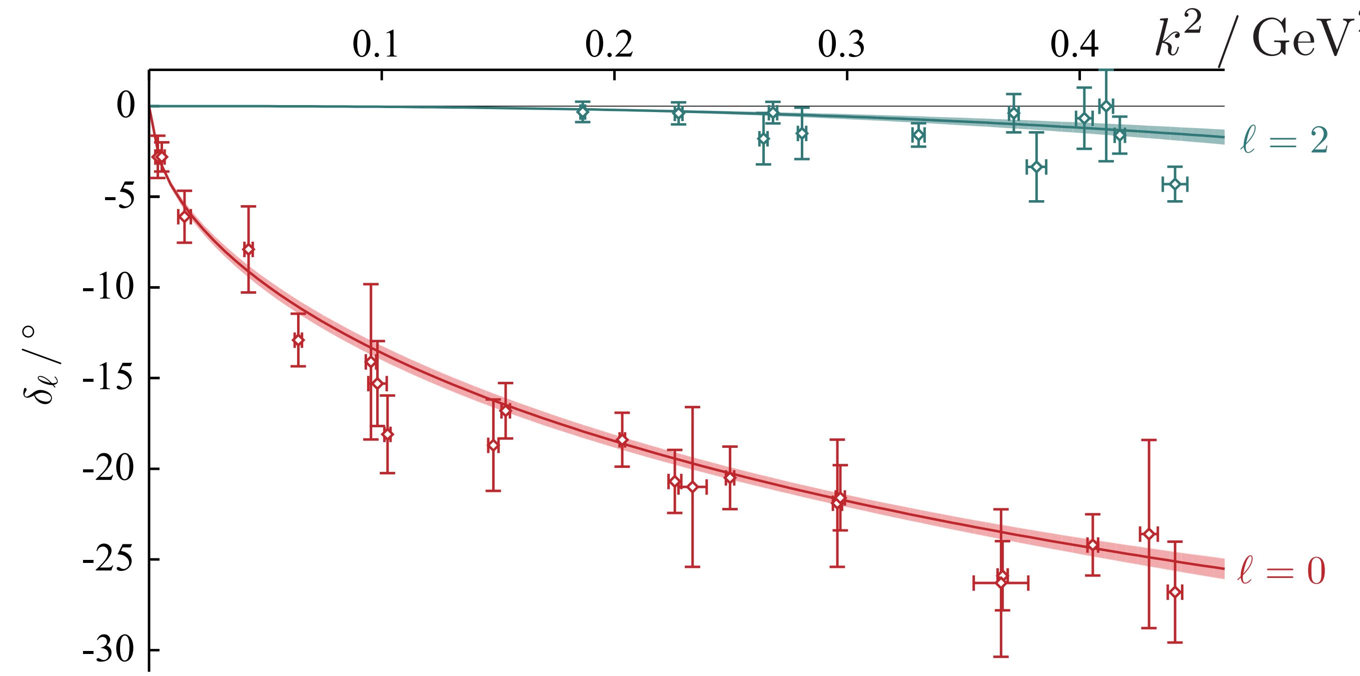
non-resonant elastic scattering — $\pi^+\pi^+$ (isospin=2)

PRD86 034031 (2012)

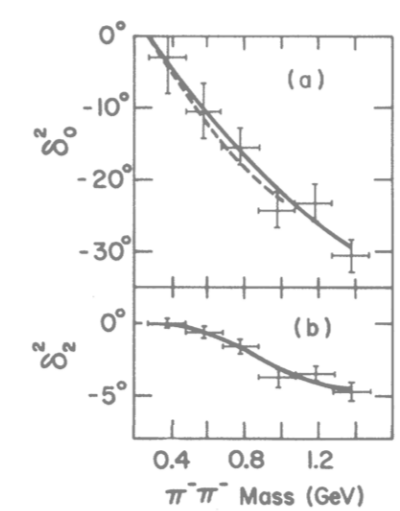
m_π ~ 391 MeV



scattering phase-shifts



repulsive scattering, no resonances

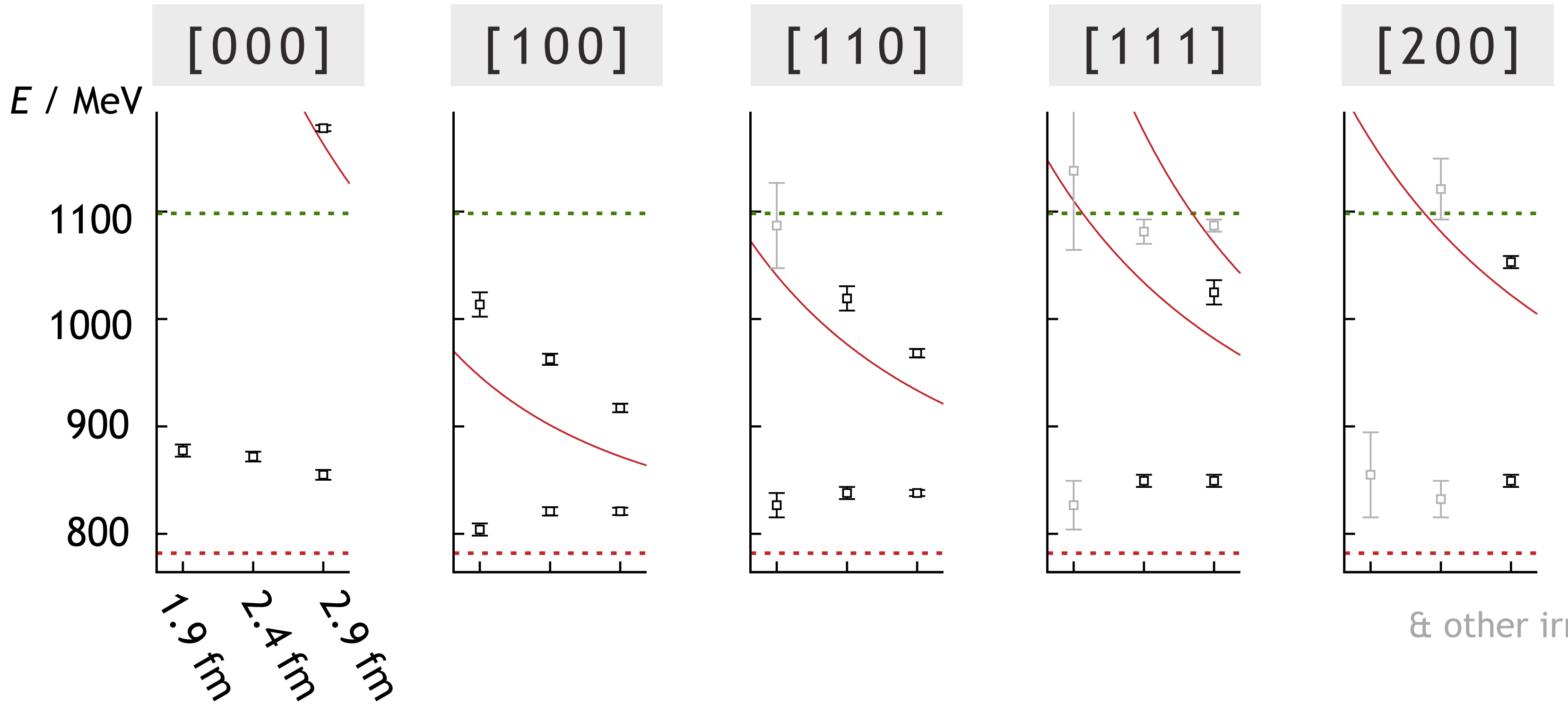


Cohen 1972

an elastic resonance — the ρ in $\pi\pi$ (isospin=1)

PRD87 034505 (2013)

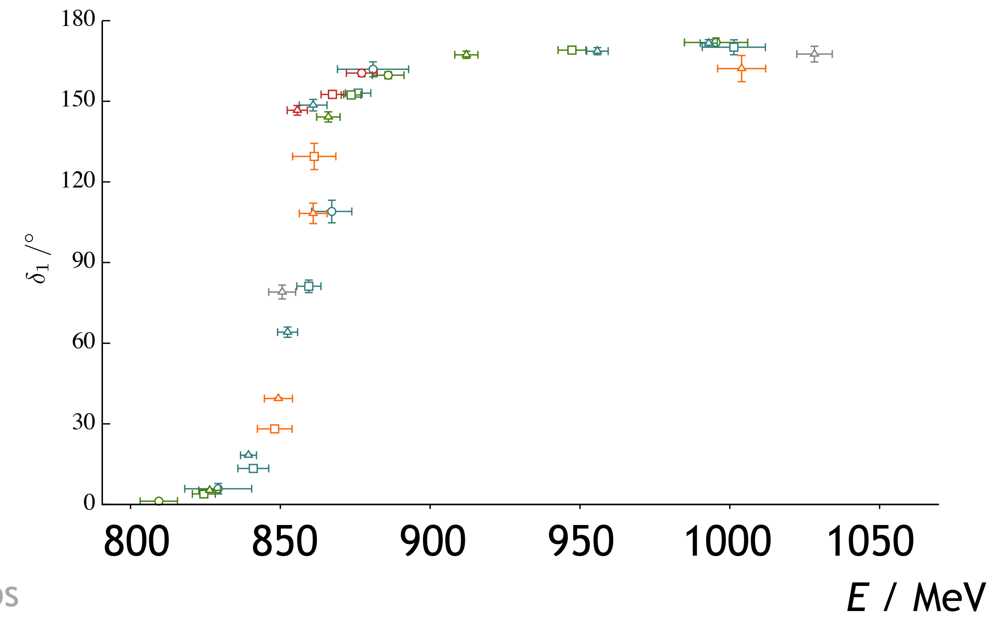
m_π ~ 391 MeV



$m_\pi L$ 4 5 6

& other irreps

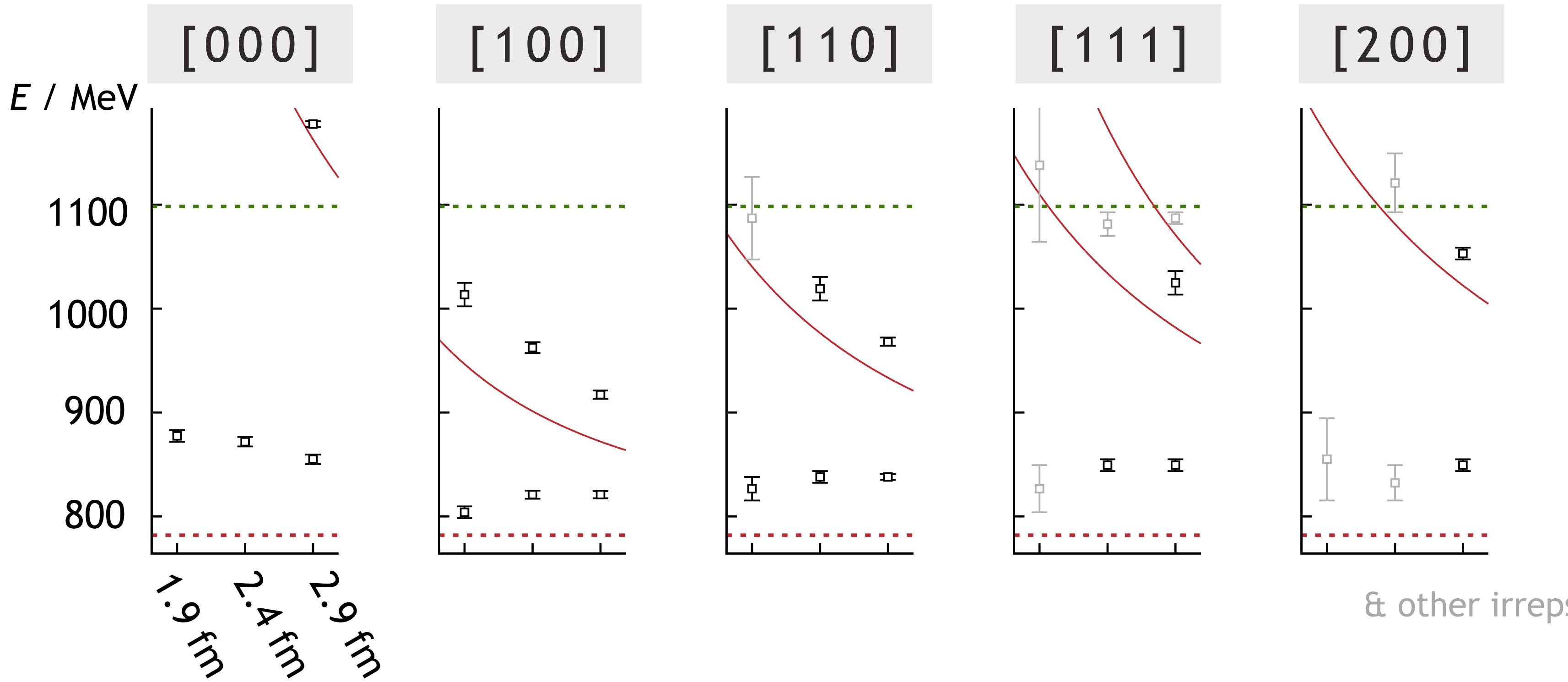
scattering phase-shift



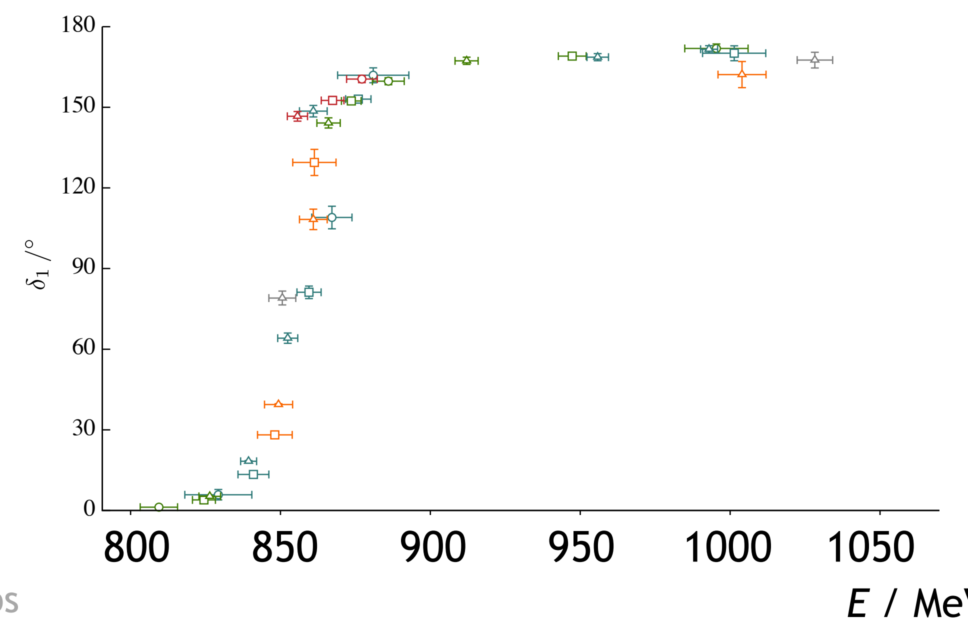
an elastic resonance — the ρ in $\pi\pi$ (isospin=1)

PRD87 034505 (2013)

$m_\pi \sim 391$ MeV

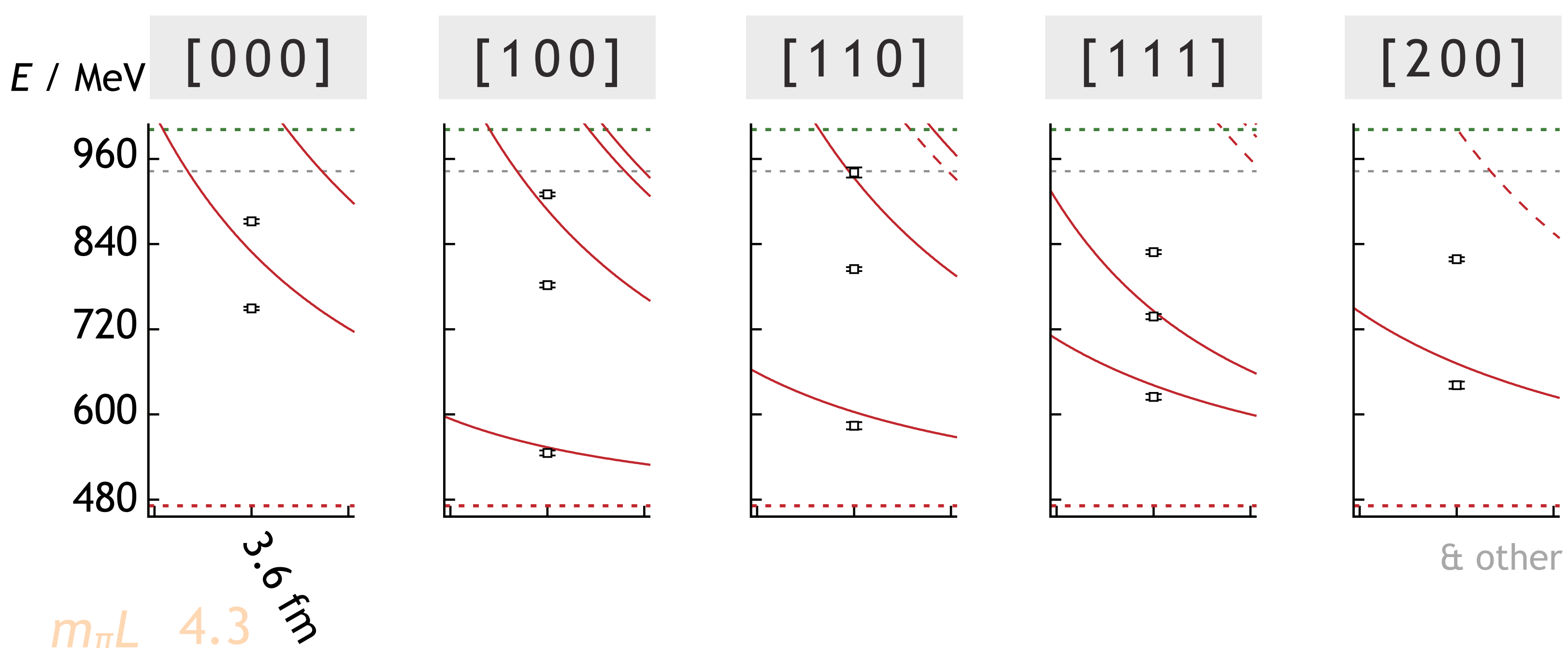


& other irreps



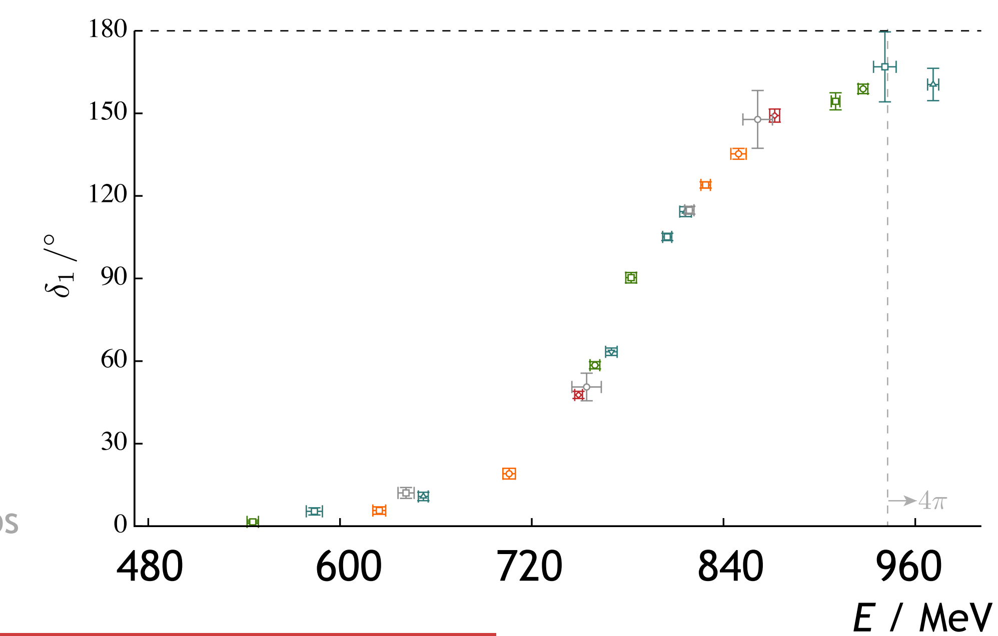
PRD92 094502 (2015)

$m_\pi \sim 236$ MeV



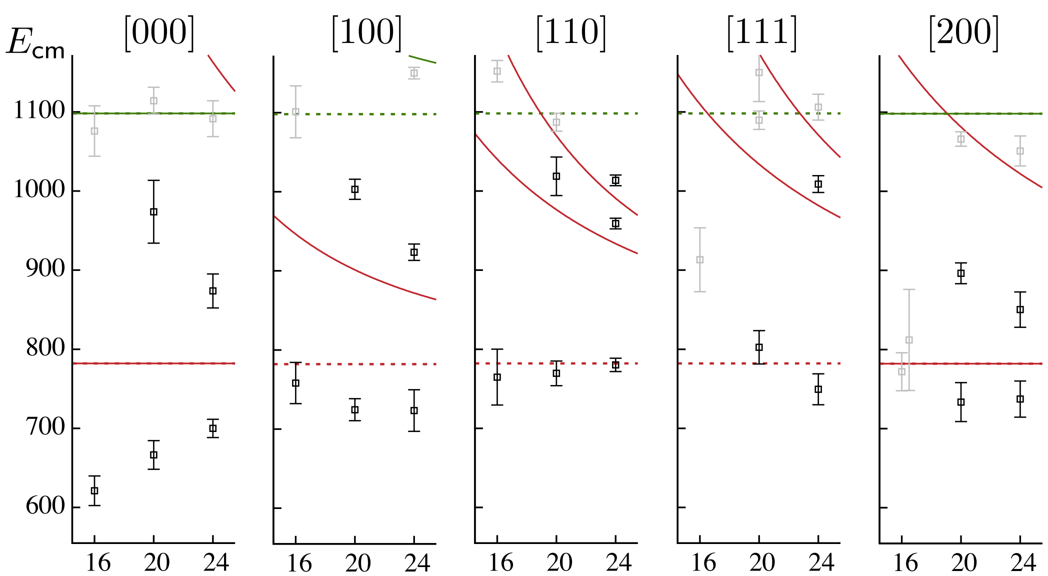
& other irreps

$m_\pi L$ 4.3



$\pi\pi$ isospin=0

$m_\pi \sim 391$ MeV



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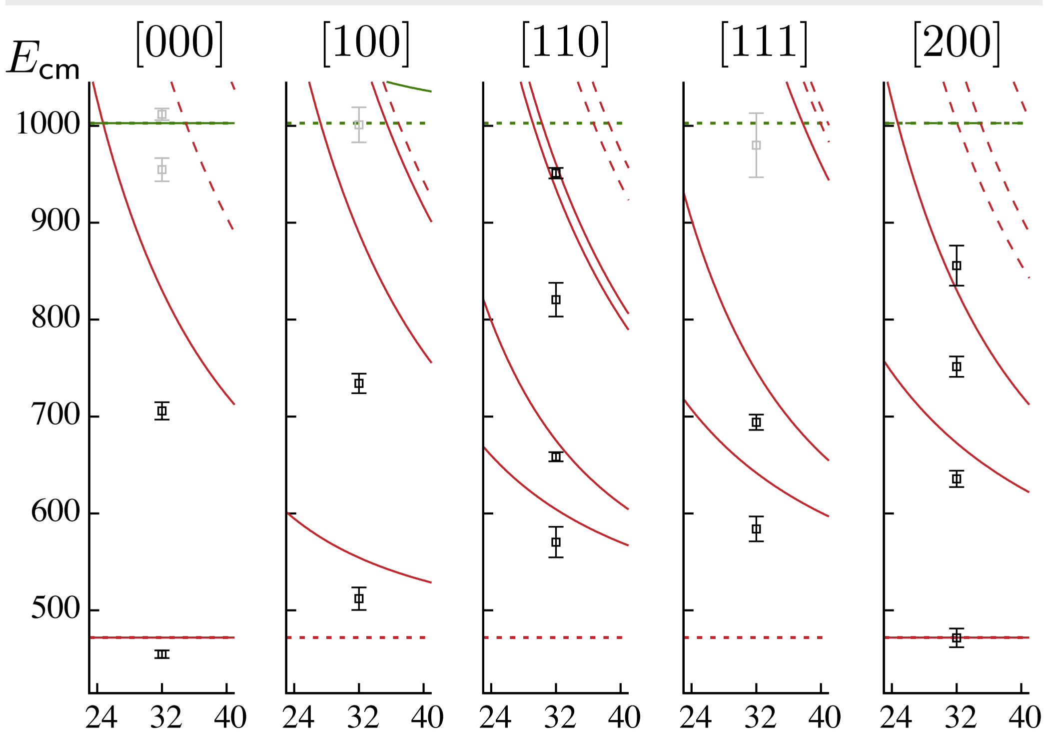
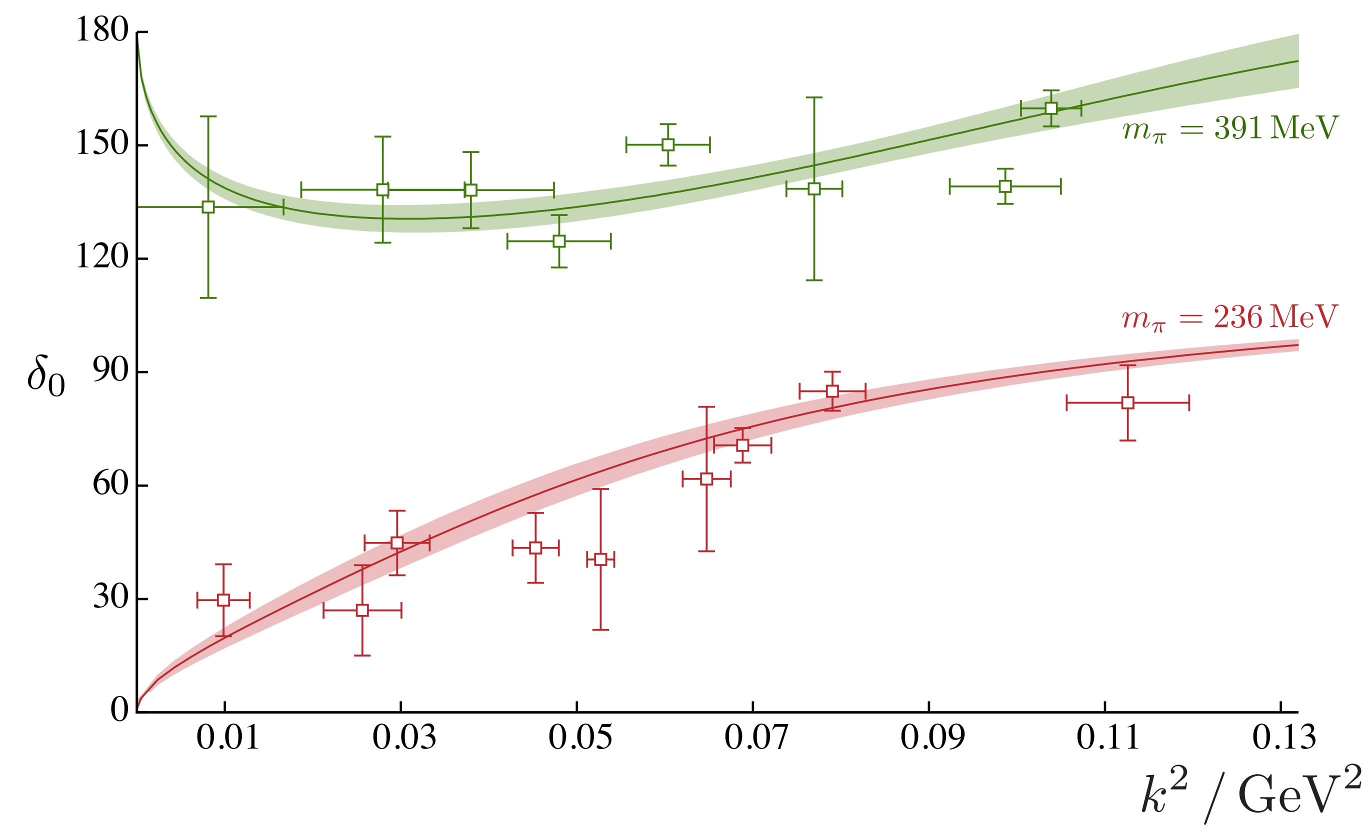




heavier quark mass — **a bound-state**

lighter quark mass — attraction, maybe a **broad resonance** ?

c.f. the experimental σ resonance …

one possible approach – mimic 'ideal' experiment: determine $t_{ab}^{(\ell)}(E)$ for real energies

maybe can then be analytically continued to complex energies to find poles ?

scattering matrix determines the finite-volume spectrum:

$$0 = \det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\big(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\big)\right]$$

for a given scattering matrix, has a discrete set of solutions $E_n(L)$

the finite-volume spectrum

one approach:

– **parameterize** the energy dependence of $\boldsymbol{t}(E)$

– solve $0 = \det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\big(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\big)\right]$

– compare 'model' spectrum to lattice spectrum ...

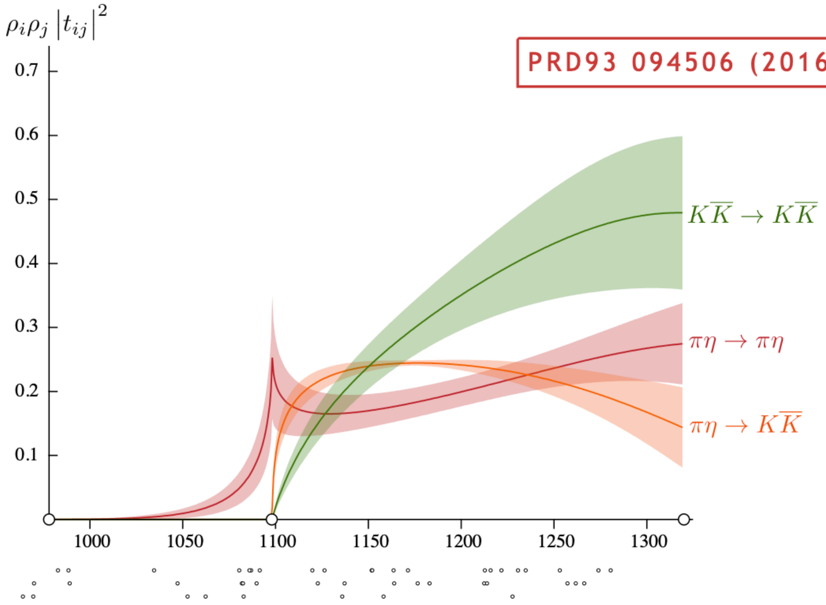
ensure important features are independent of parameterization details by varying parameterization ...

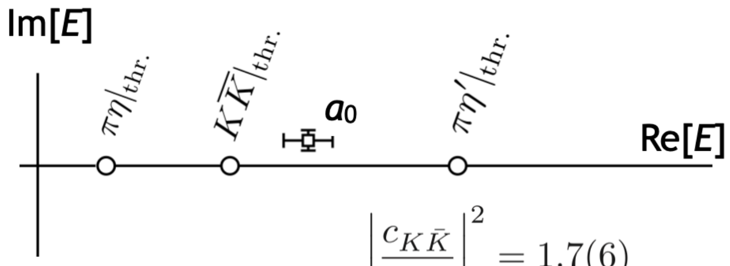
coupled-channel scattering from lattice QCD

m_π ~ 391 MeV

$a_0 \to \pi\eta, K\bar{K}$

PRD93 094506 (2016)



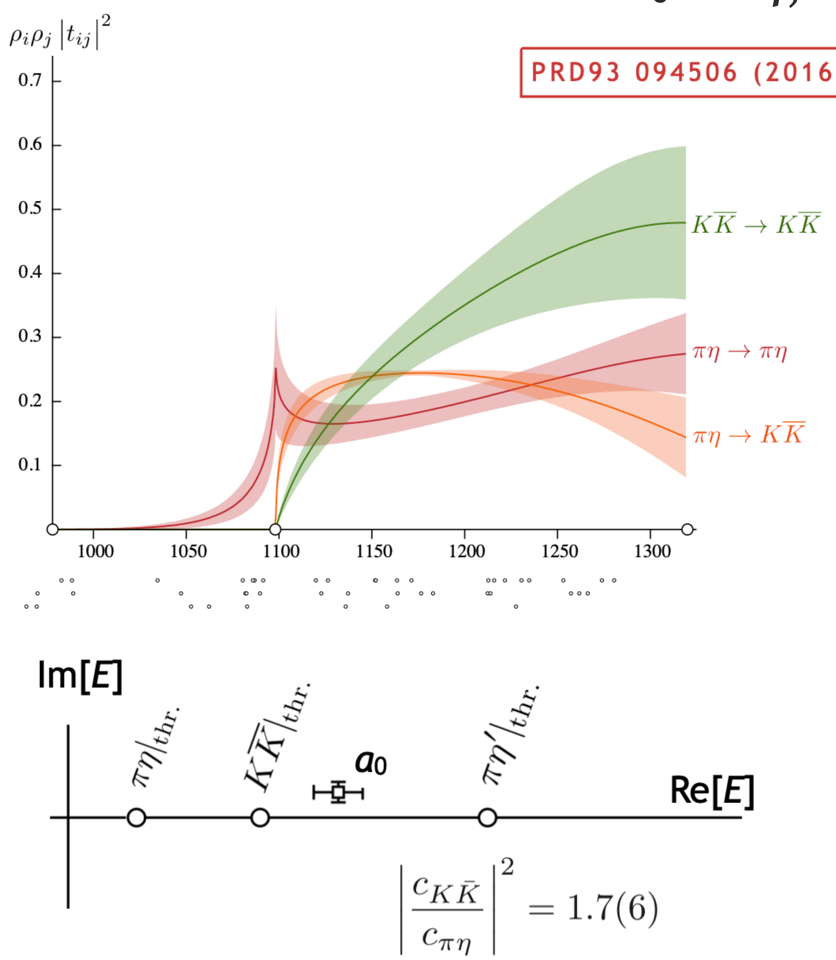


$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right|^2 = 1.7(6)$$

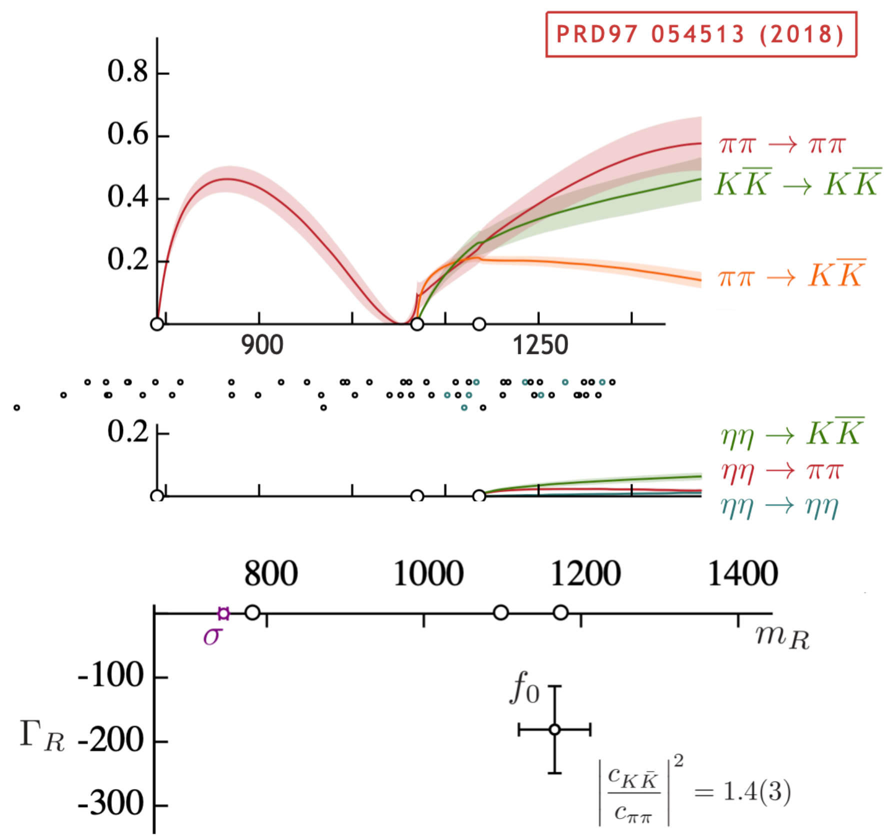
coupled-channel scattering from lattice QCD

m_π ~ 391 MeV

$a_0 \rightarrow \pi\eta, K\overline{K}$



$f_0 \rightarrow \pi\pi, K\overline{K}, \eta\eta$



coupled-channel scattering from lattice QCD

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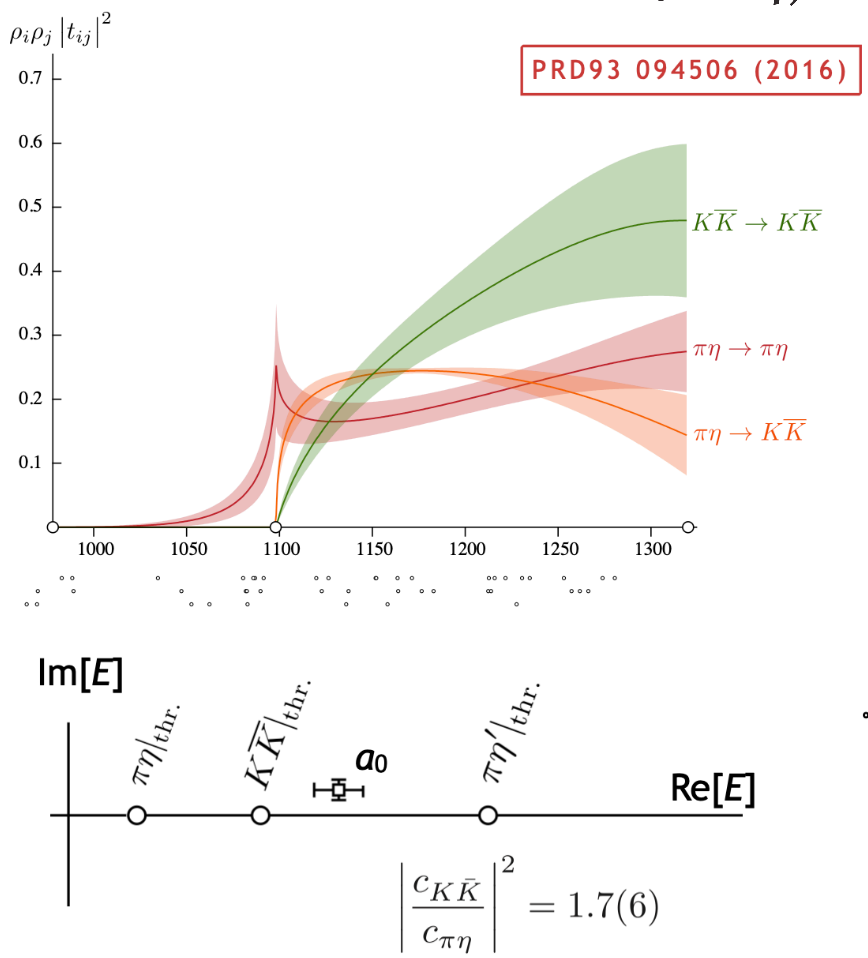
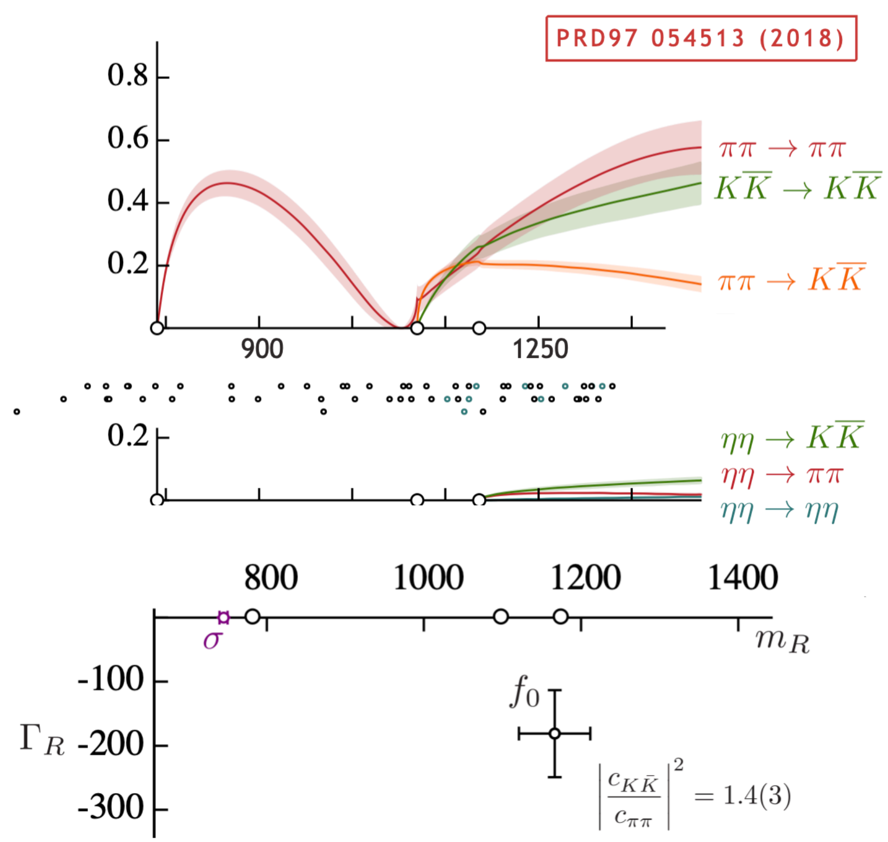


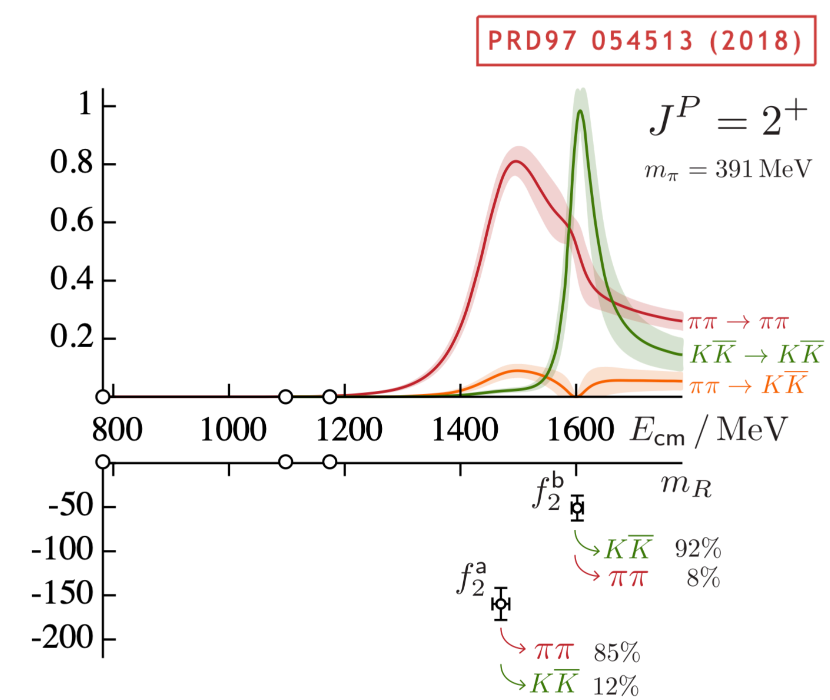
fourth sheet pole

$f_0 \to \pi\pi, K\bar{K}, \eta\eta$



second sheet pole

$f_2 \to \pi\pi, K\bar{K}$

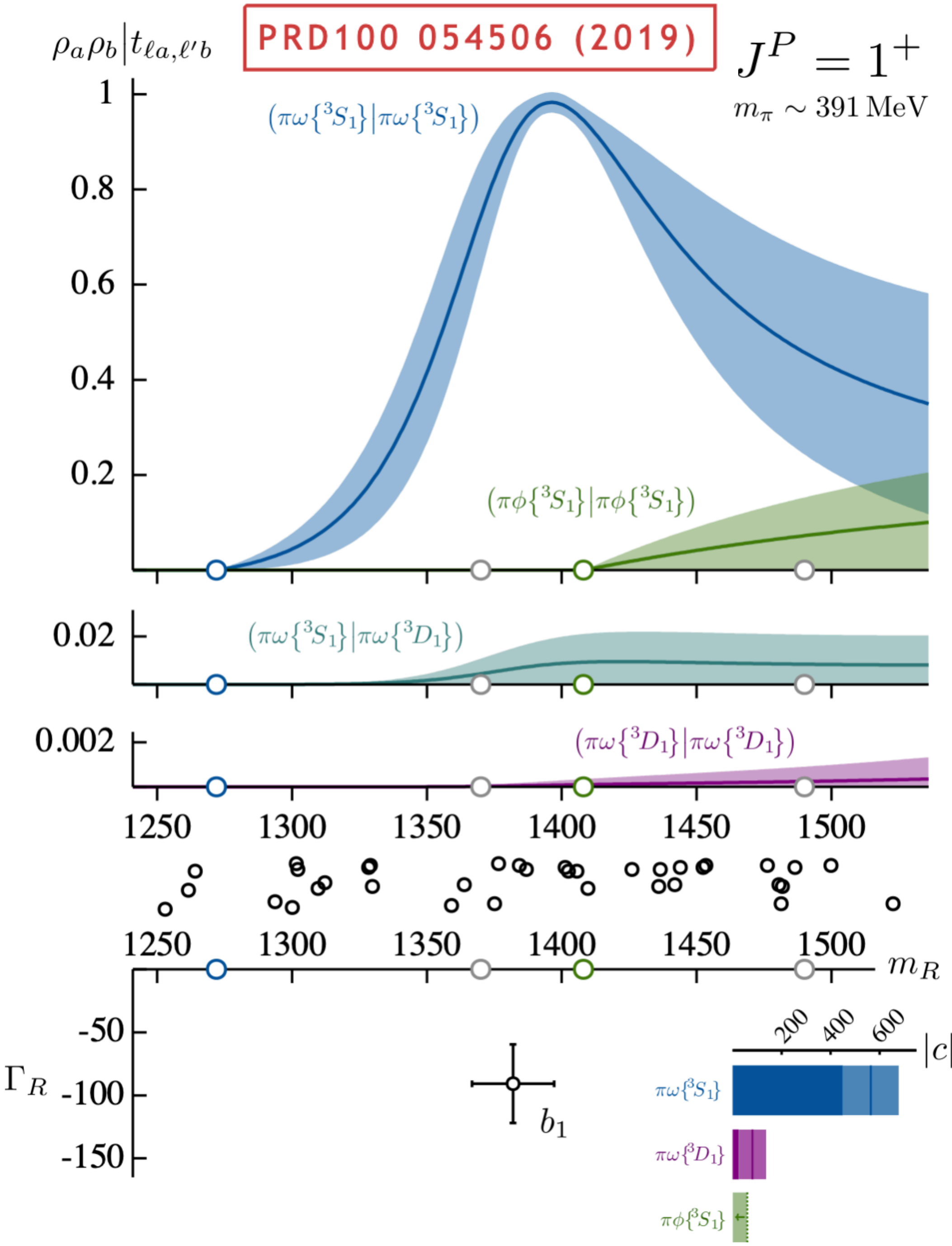


track pole movement with changing quark mass in future calculations

… recently scattering of hadrons with non-zero spin …

$b_1 \to \pi\omega, \pi\phi$

PRD100 054506 (2019)

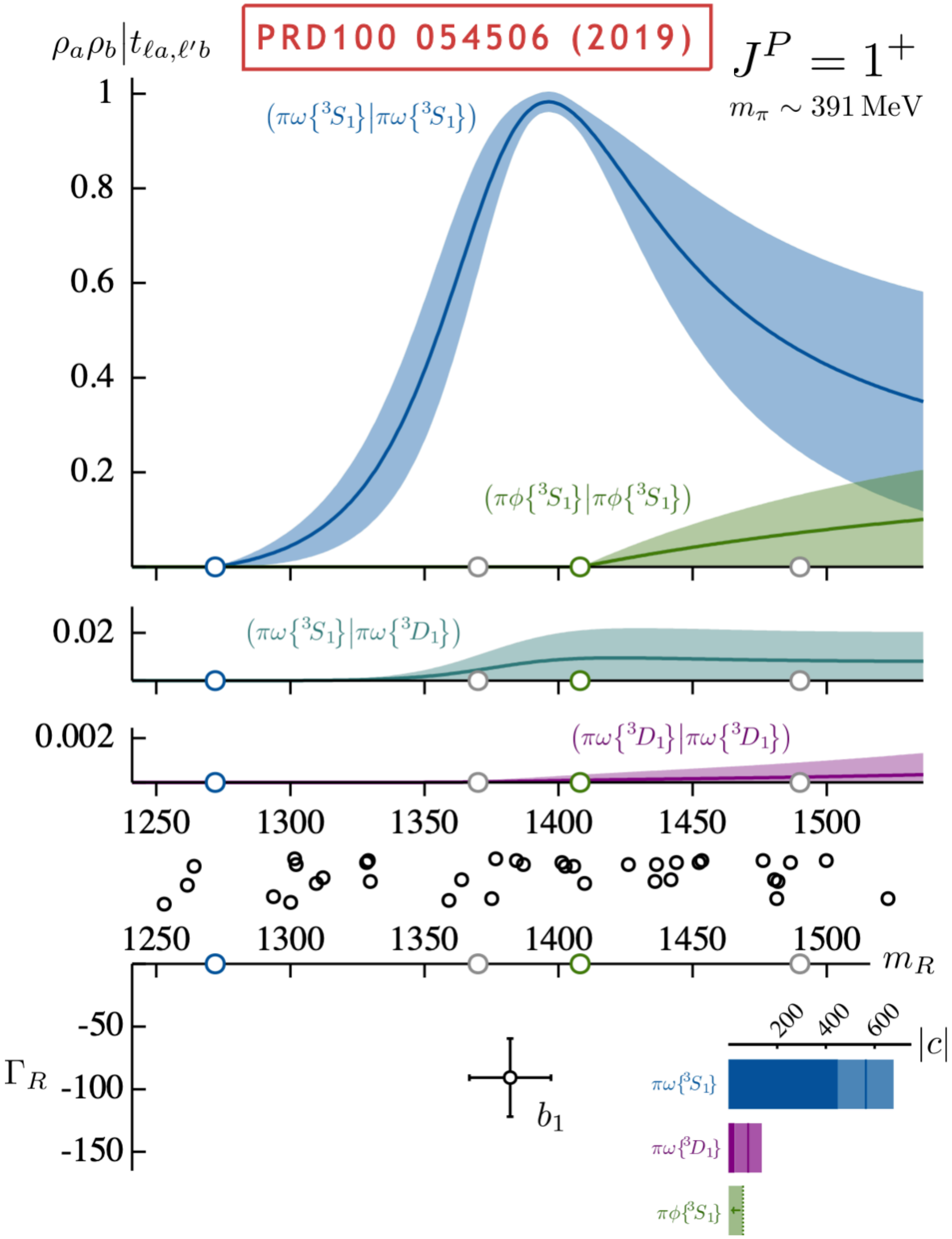


coupled-channel scattering from lattice QCD

… recently scattering of hadrons with non-zero spin – *coupled-partial waves* …

$b_1 \to \pi\omega, \pi\phi$

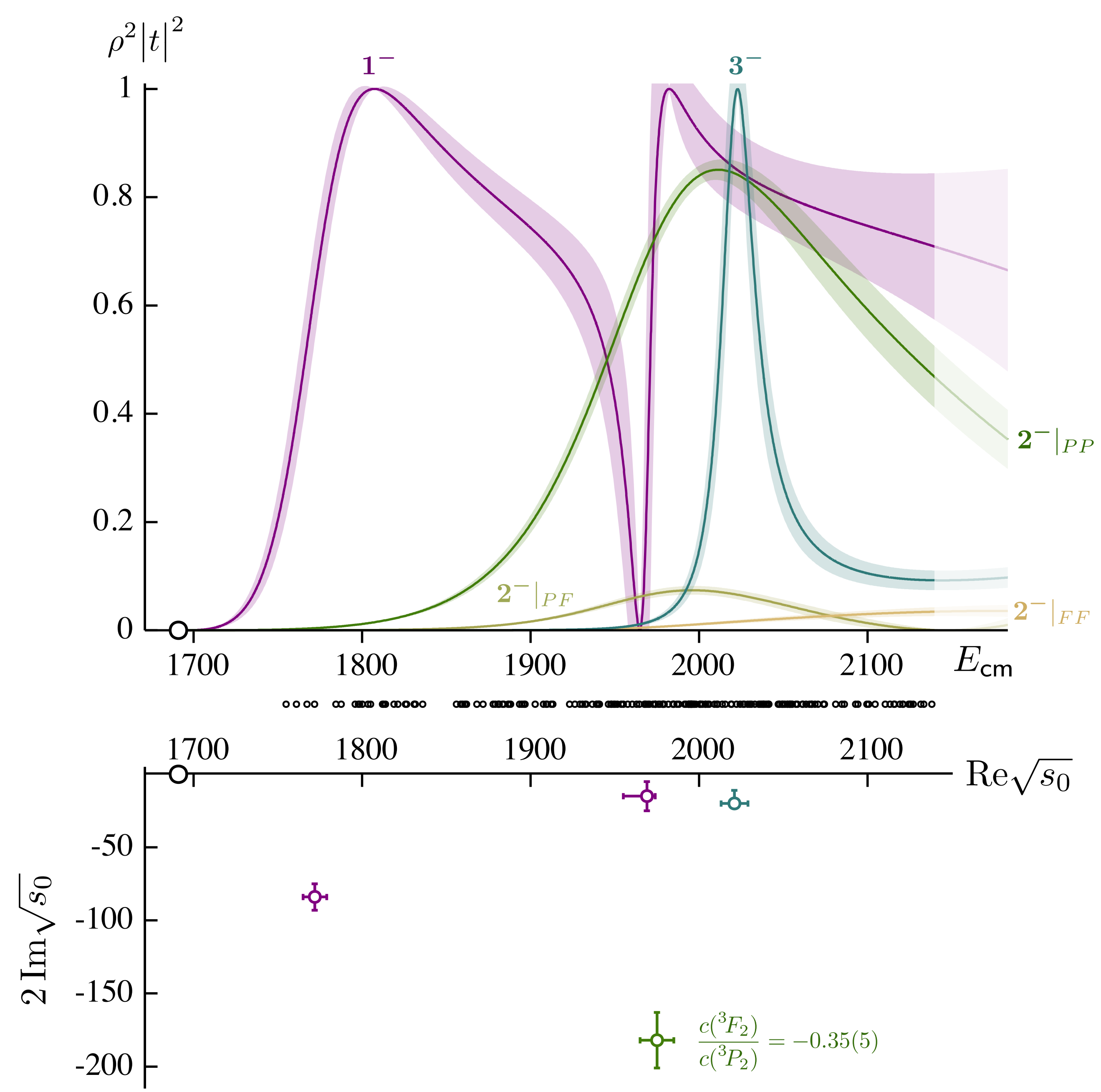
PRD100 054506 (2019)



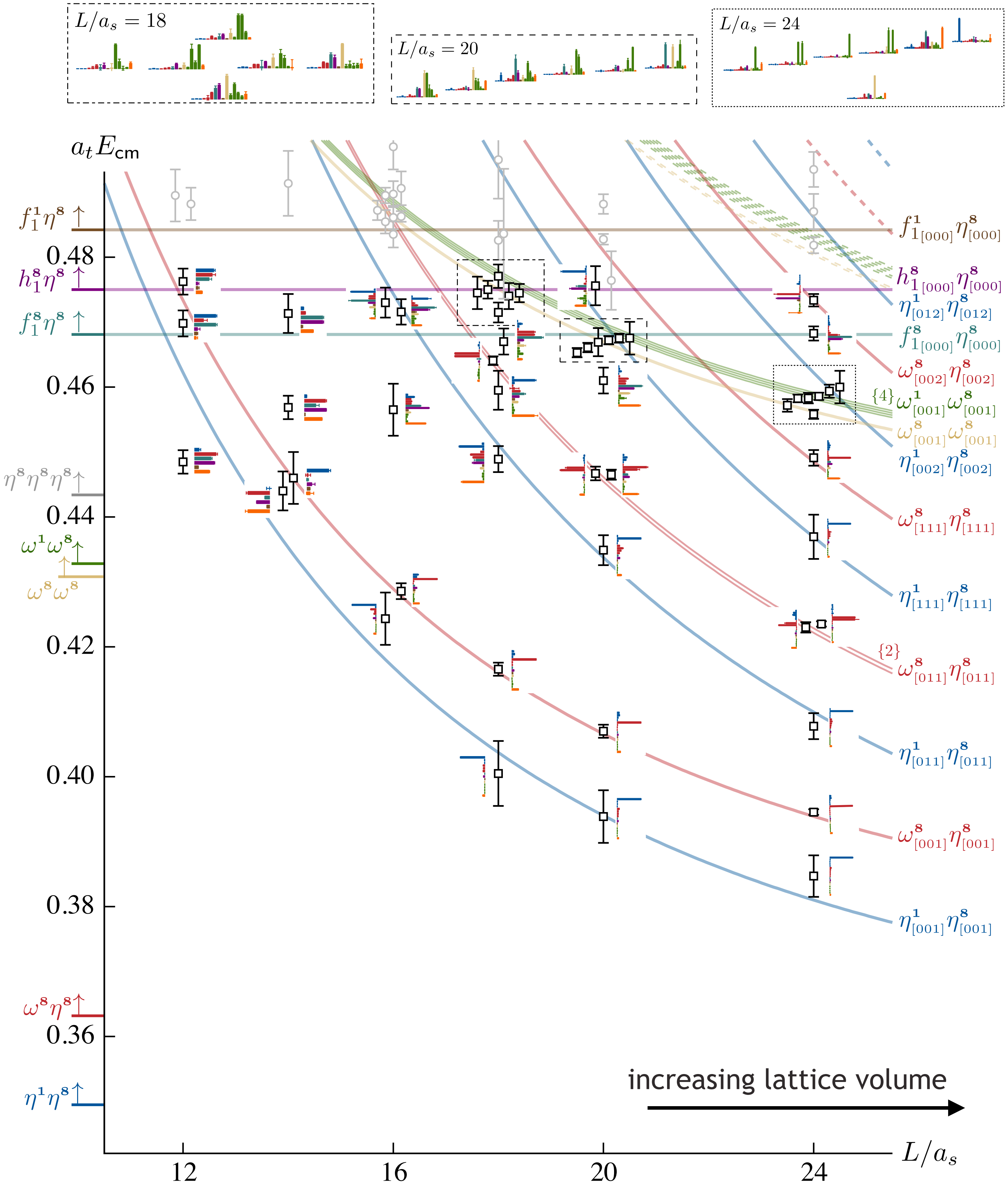
$J^{--} \to 0^{-+} 1^{--}$

with exact SU(3) flavor

PRD103 074502 (2021)



exotic 1−+ resonance at SU(3) flavor point



$\eta^8 \rightsquigarrow \pi, K, \eta$

$\eta^1 \rightsquigarrow \eta'$

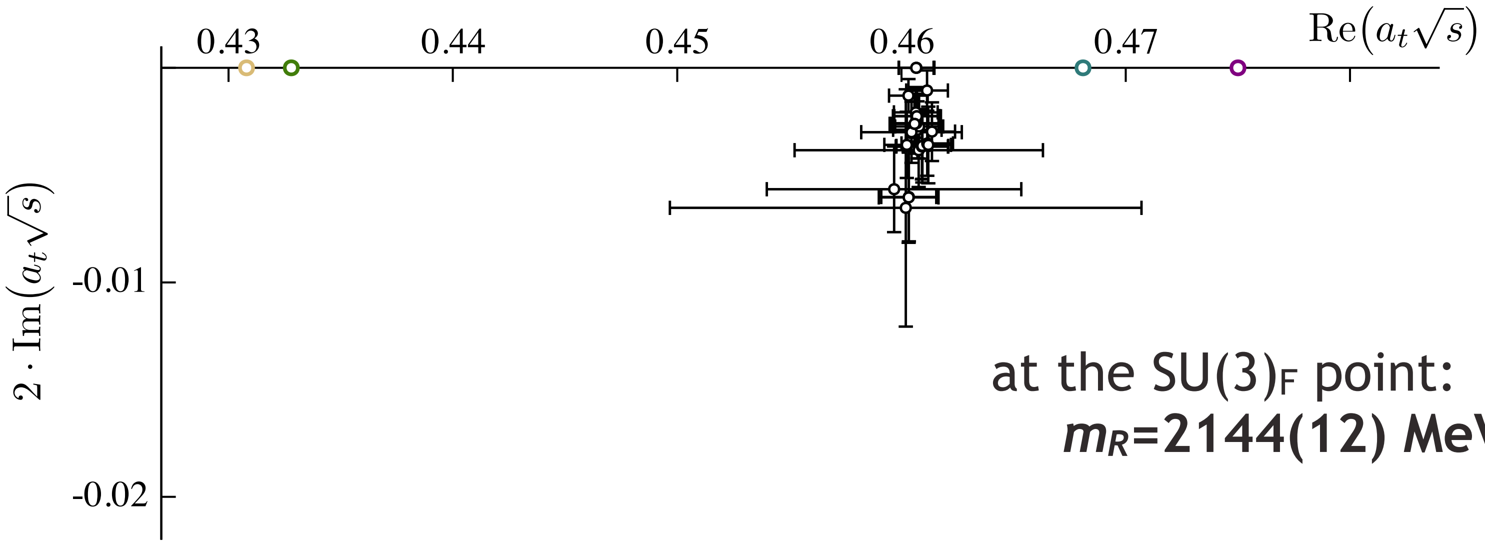
$\omega^8, \omega^1 \rightsquigarrow \rho, K^*, (\omega, \varphi)$

$h_1^8, h_1^1 \rightsquigarrow b_1, K_1, (h_1, h_1')$

$f_1^8, f_1^1 \rightsquigarrow a_1, K_1, (f_1, f_1')$

53 energy levels to constrain ‘eight’ channel scattering

octet 1^{-+} resonance pole & couplings

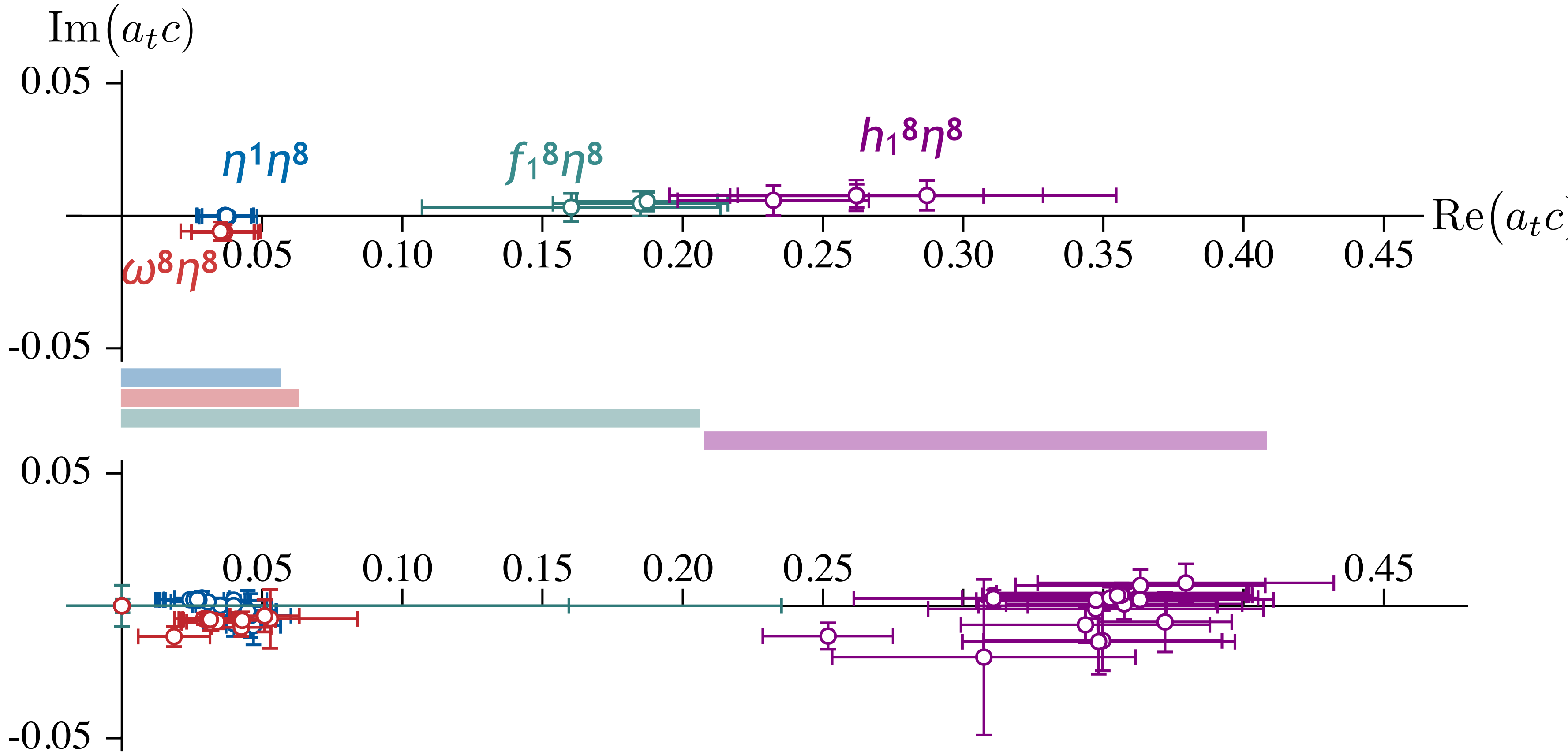


$$t_{ab}(s) \sim \frac{c_a c_b}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\tfrac{1}{2}\Gamma_R$$

at the SU(3)$_F$ point:

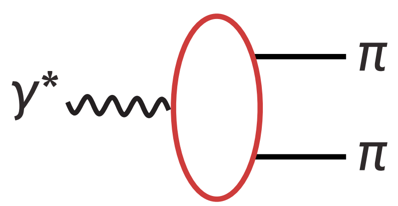
***m_R*=2144(12) MeV, *Γ_R*=21(21) MeV** (a narrow resonance)



resonance <u>below</u> $h_1{}^8\eta^8$ threshold, but with a large coupling

coupling in an external current — resonance form-factors

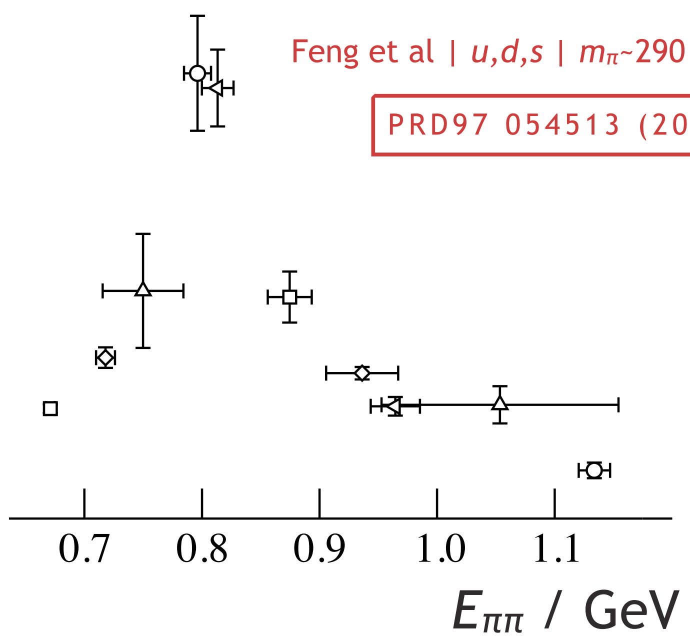
γ* → π π



$F_\pi(E_{\pi\pi})$

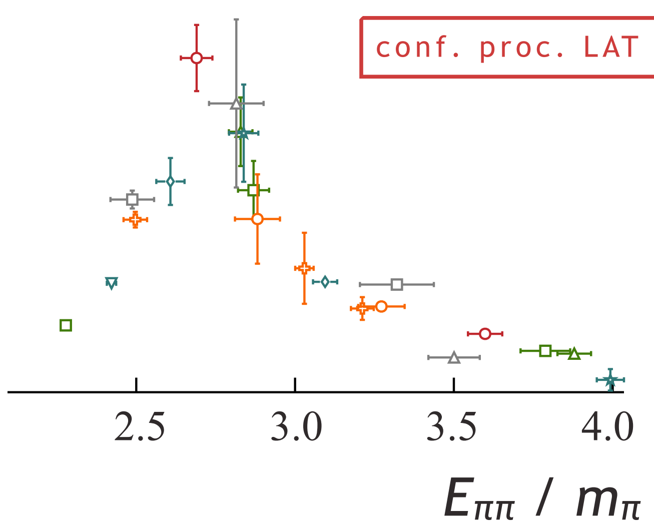
Feng et al | *u,d,s* | m_π~290 MeV

PRD97 054513 (2018)

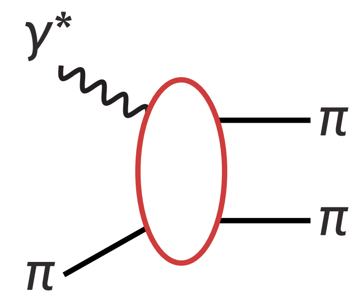


Bulava et al | *u,d,s* | m_π~280 MeV

conf. proc. LAT '15



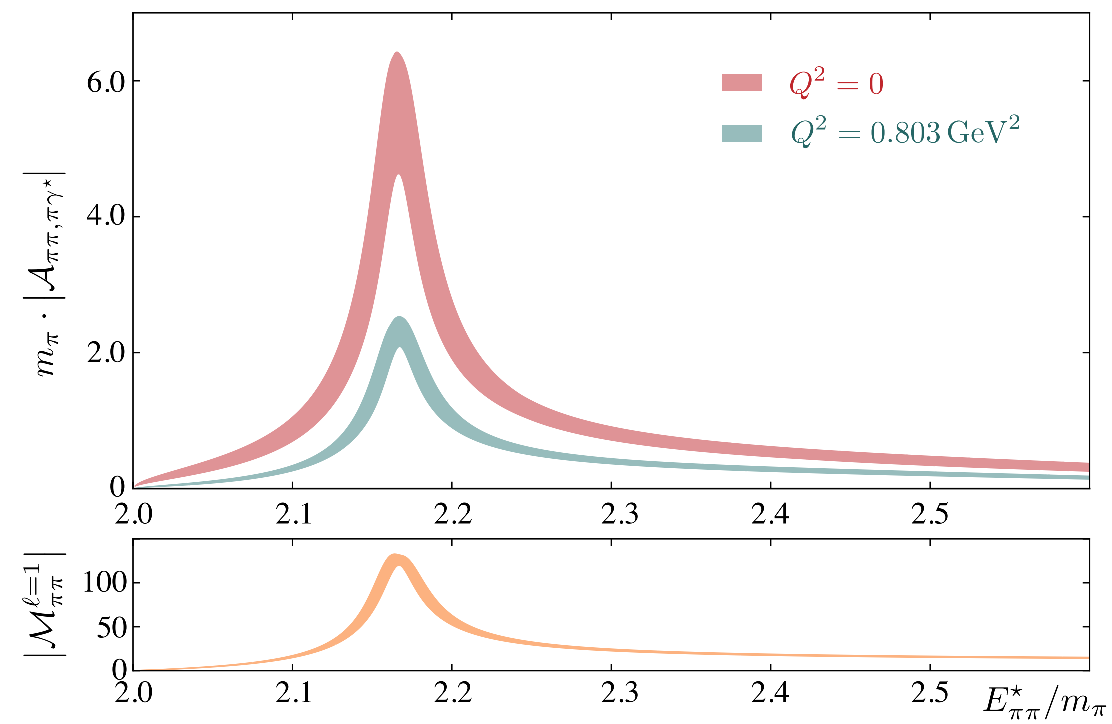
γ* π → π π

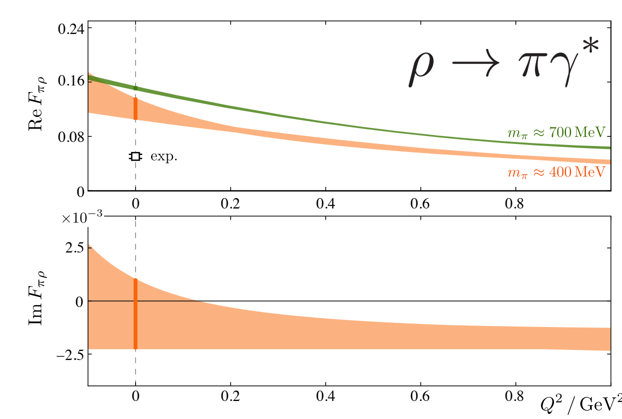


$\mathcal{A}(E_{\pi\pi}, Q^2)$

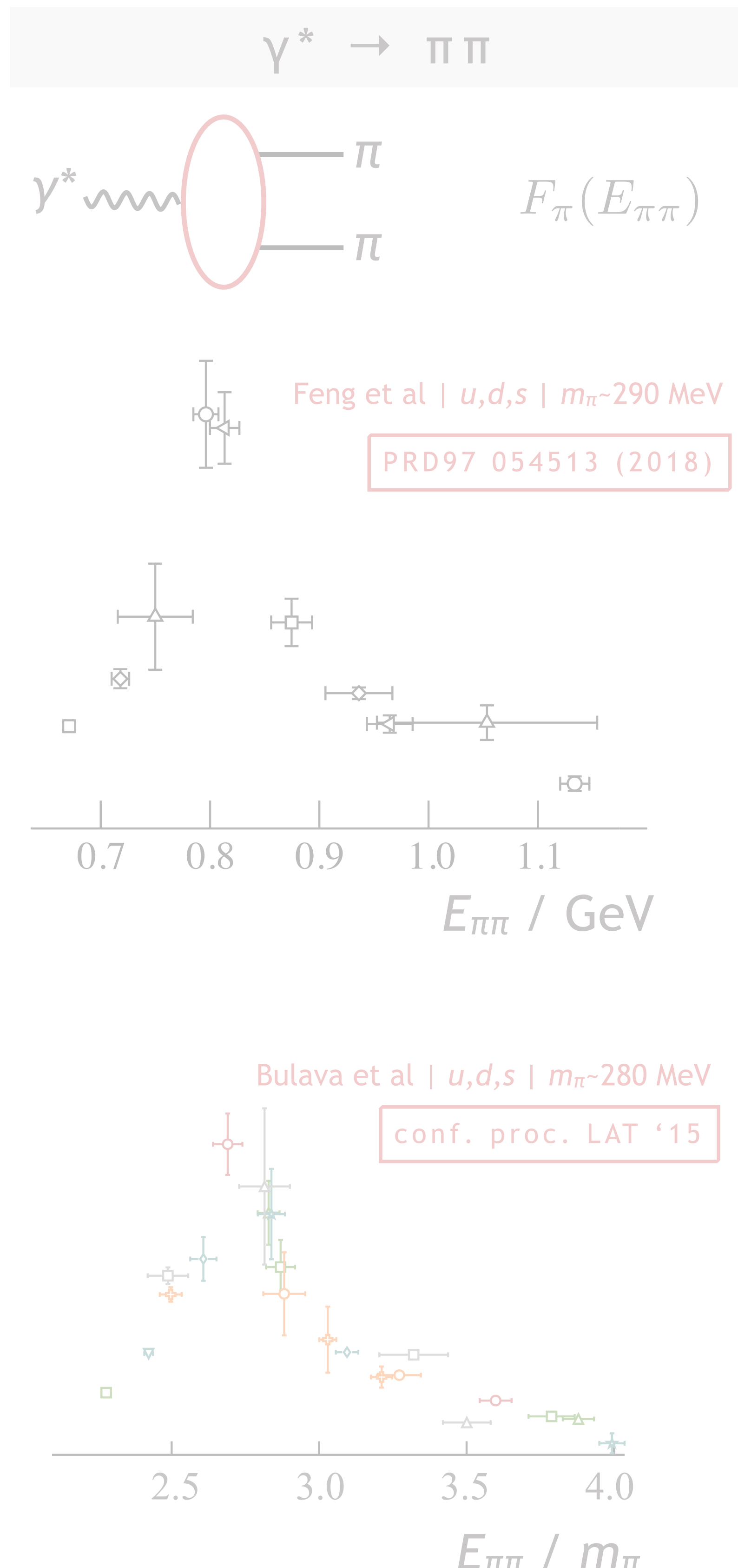
Briceno et al | *u,d,s* | m_π~391 MeV

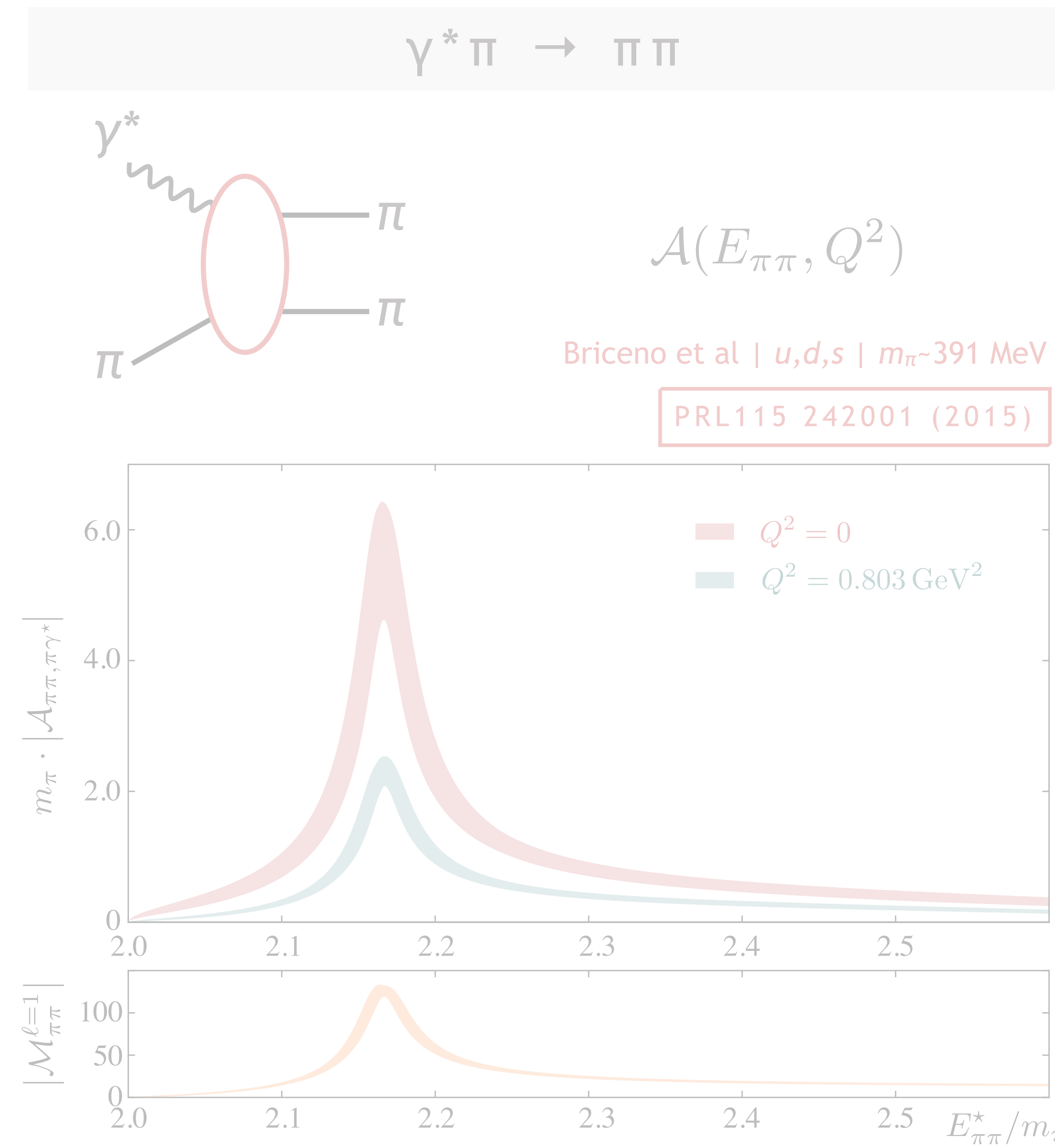
PRL115 242001 (2015)





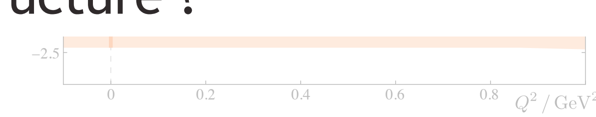
coupling in an external current — resonance form-factors





this technology can be used to study e.g. scalar meson resonance form-factors

determine their 'size' and 'flavor content' ?
inform models of their structure ?



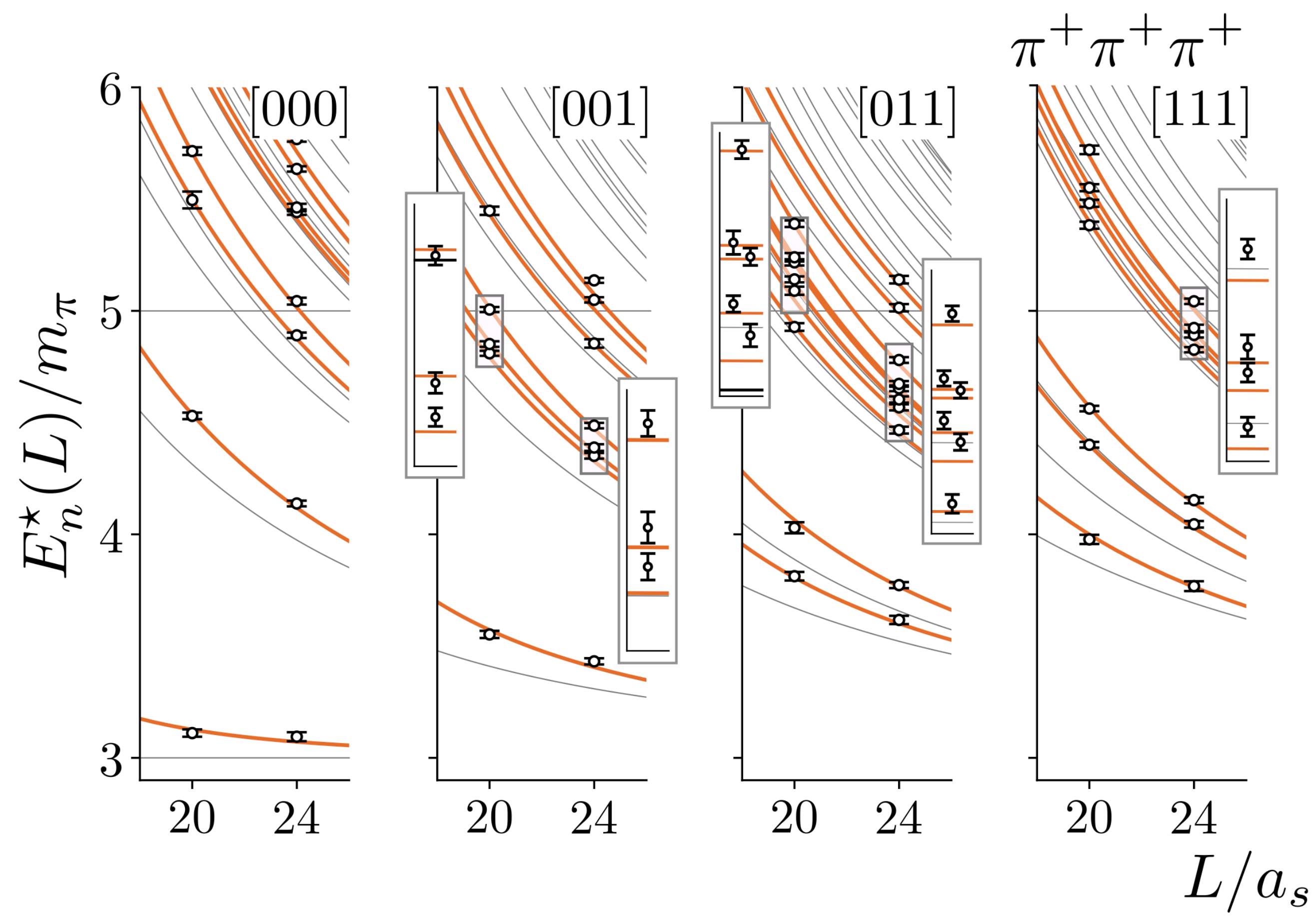
beyond 2→2 scattering

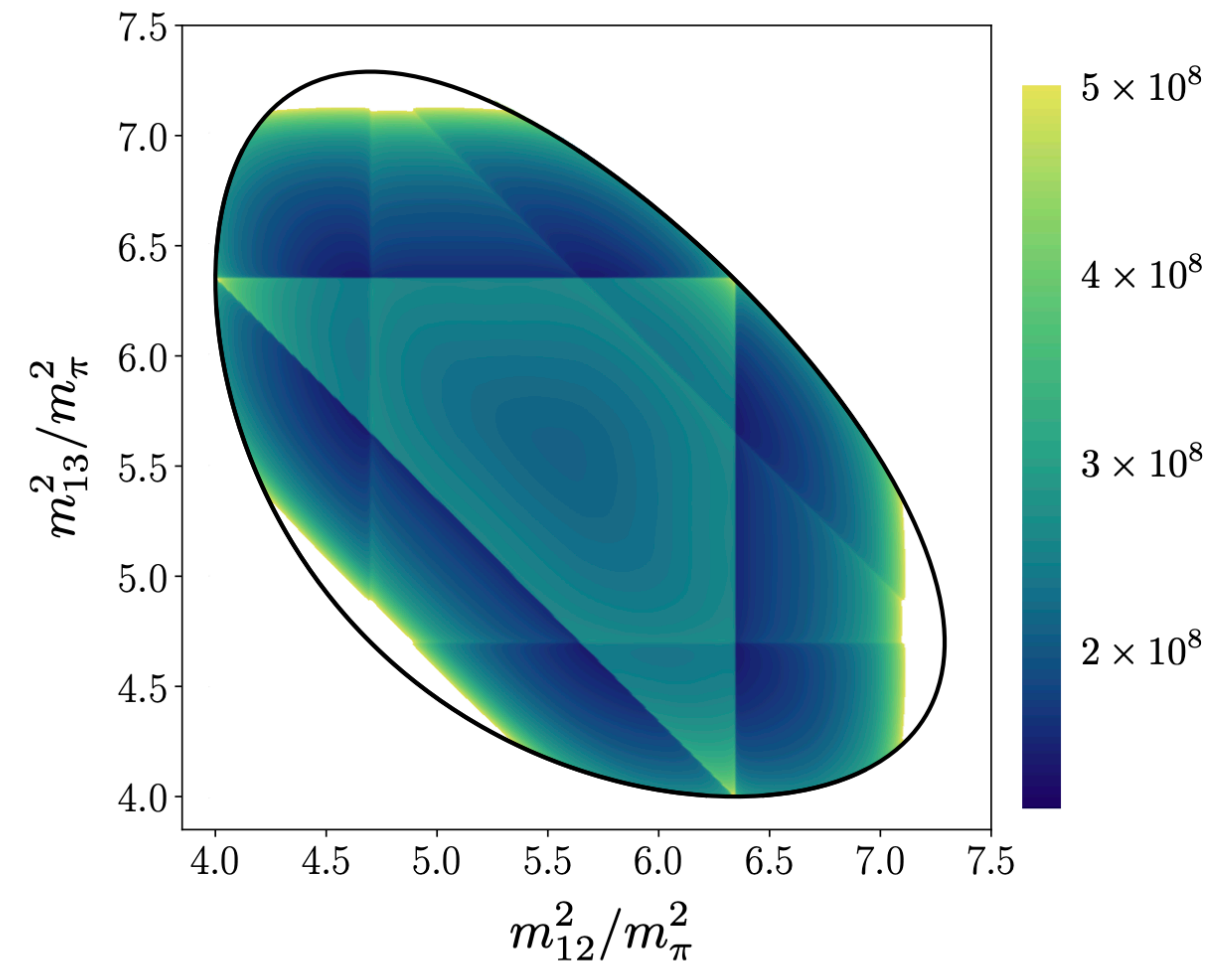
formalism used so far only valid for 2→2 scattering ⇒ required to stay below three-body thresholds …

three-body formalism now developed and applied in test cases

The energy-dependent $\pi^+\pi^+\pi^+$ scattering amplitude from QCD

Maxwell T. Hansen,[1,*] Raul A. Briceño,[2,3,†] Robert G. Edwards,[2,‡] Christopher E. Thomas,[4,§] and David J. Wilson[4,¶]
(for the Hadron Spectrum Collaboration)





summary

lattice QCD is increasingly becoming a practical tool to study **excited hadron resonances**

currently somewhat limited to heavier quark masses to avoid many-body decays

but can still learn a lot about strong QCD where previously we relied upon models

a new era of QCD-based hadron spectroscopy ?