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Inter-experiment Partial Wave Analysis

Hadron Spectroscopy: The Next Big Steps

MITP Virtual Workshop, 14-25 March 2022

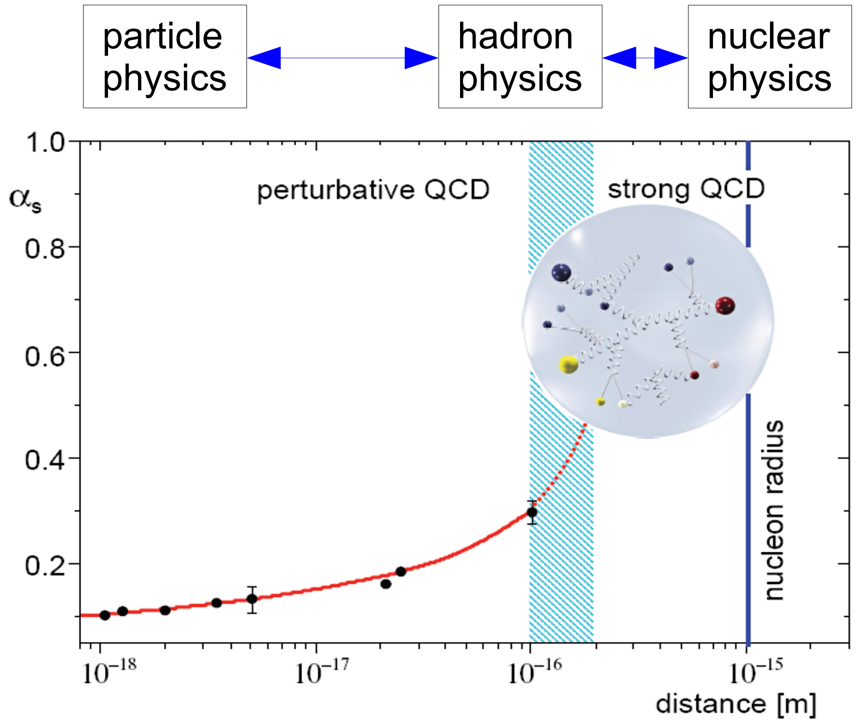
Outline

- Introduction
  - why coupled channel analyses with data from different experiments?
- Proper descriptions for the dynamics
  - fulfillment of unitarity and analyticity
- Coupled Channel analysis of $\bar{p}p \rightarrow K^+K^-\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$ and $\pi\pi$-scattering data
- Investigation of the spin-exotic $I^G(J^{PC})= 1^-(1^{-+})$ wave with $\bar{p}p$-, π^-p- and $\pi\pi$-data
- Use of the obtained parametrization for the dynamics on $\gamma\gamma$ reactions
- Summary and conclusion

Introduction

- Light mesons are bound states consisting of u-, d- and s-quarks
- Cover the non-perturbative QCD regime
- Description very challenging
  - lattice QCD
  - phenomenological models
- Observation and measurements of the resonance properties very challenging
  - many overlapping resonances with same quantum numbers
  - decays in different channels
  - distinction between conventional $q\bar{q}$-mesons and exotics difficult

energy dependence of α_s



Access to the Inner Structure of a Resonance

- Characteristics of the production
  - γ induced processes include QED effects
  - exotics with gluonic content should
    - ➔ be mainly produced in gluon rich environments like radiative J/ψ decays, central production, or $\bar{p}p$ annihilation
    - ➔ couple weakly to γ induced processes like $\gamma\gamma$ fusion

- Characteristics of the decay pattern
  - glueballs: flavour blind decay with a rather narrow width
  - molecules: decay into a meson pair close to threshold

Coupled Channel Analysis

- Determination of pole parameters and coupling strength to the production and decay systems are essential
- Analysis of only one single channel is not sufficient
- Coupled channel analyses of data from different production mechanisms and decay systems are needed and the most elegant way
- Advantages compared to single channel fits
  - ➢ common and unique description of the dynamics
  - ➢ better description of threshold effects
  - ➢ better fulfillment of the conservation of unitarity
  - ➢ more constraints due to common amplitudes

Dynamical Functions

- Breit-Wigner functions widely used
  - good approximation for isolated resonances appearing in a single channel
  - violate the unitarity
  - extracted resonance parameters are not unique and depend on the production and decay process
- More sophisticated descriptions needed for
  - resonances decaying into multiple channels
  - several resonances with the same quantum numbers appearing in the same channel
  - resonances located at thresholds → distortion of the line shape

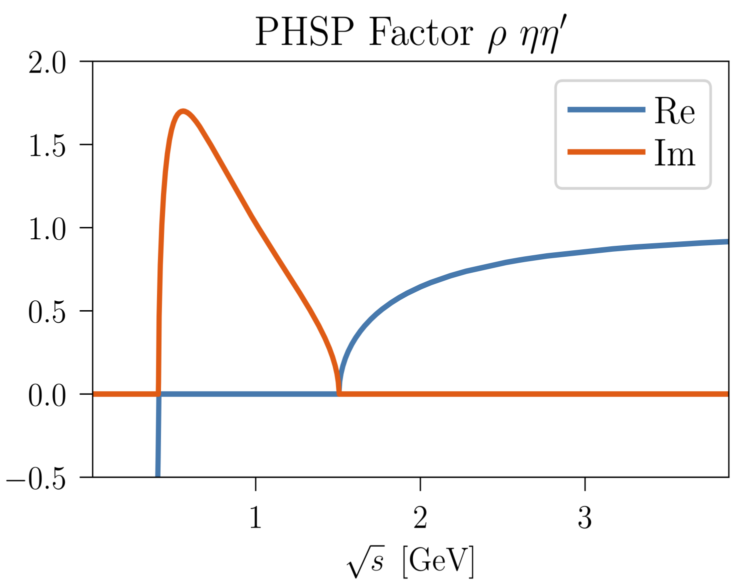
Approaches with an adequate consideration

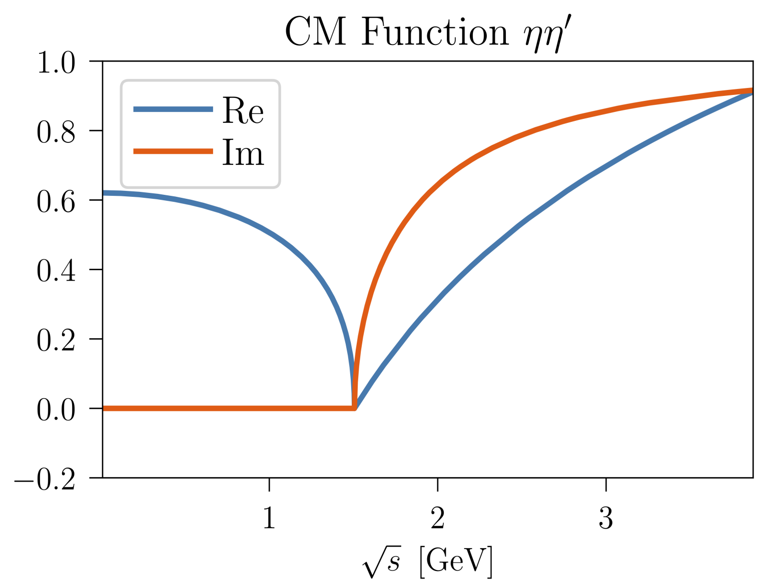
of unitarity and analyticity needed

(K-matrix, N/D-method, Two-potential decomposition)

Analyticity in K-Matrix Approach

- Scattering process described by T-matrix: $T = (I - i\,K\,\rho)^{-1}\,K$
- Production process described via P- and F-vector: $F = (I - i\,K\,\rho)^{-1}\,P$
- K-matrix with standard phase space factors: $\rho = \sqrt{\left[1 - \left(\frac{m_a + m_b}{m}\right)^2\right]\left[1 - \left(\frac{m_a - m_b}{m}\right)^2\right]}$
  - violates constraints from analyticity: unphysical cuts for unequal masses
- Proper description with Chew-Mandelstam function from *Phys Rev D19, 054008 (2015)*
  - ρ must be replaced by CM(s): $T = (I + K\,CM(s))^{-1}\,K$





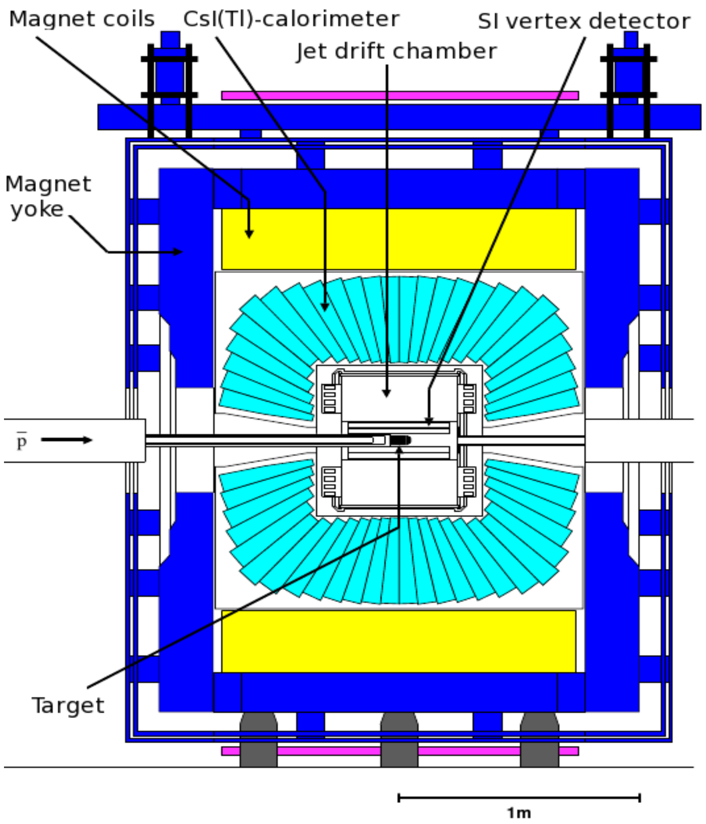
PAWIAN

- **PA**rtial **W**ave **I**nteractive **AN**alysis software package
- Supports $\bar{p}p$- and e^+e^--annihilation, $\gamma\gamma$-fusion and πp- and $\pi\pi$- scattering
- Hypothesis and other settings defined via configuration files
  - spin formalisms: e.g. canonical, helicity, ...
  - dynamics: Breit-Wigner, K-matrix, ...
- Channels with arbitrary number of final state particles
- Event based maximum likelihood fit using MINUIT2
- Support for parallelization
- Analysis tools: extraction of pole positions, branching fractions, ...
- Event generator, histogramming, efficiency correction, ...

Analyses of Crystal Barrel at LEAR data are an excellent opportunity for the investigation of physics aspects relevant for P̄ANDA

See talk about P̄ANDA by M. Küßner

- In operation between 1989 and 1996
- Fixed target experiment
- p̄p annihilation at rest and in flight
  - highest beam momentum 1.94 GeV/c
  - overlap with P̄ANDA
- Physics program
  - spectroscopy of light mesons and search for exotic states



$\bar{p}p \rightarrow K^+ K^- \pi^0, \pi^0 \pi^0 \eta, \pi^0 \eta \eta$

Eur. Phys. J. C (2020) 80:453 *Crystal Barrel Collaboration*

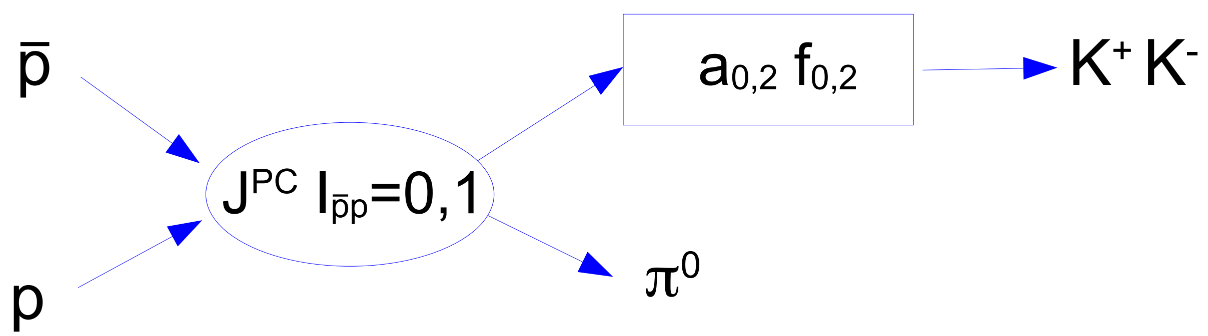
Coupled channel analysis of $\bar{p}p \rightarrow \pi^0\pi^0\eta, \pi^0\eta\eta$ and $K^+K^-\pi^0$ at 900 MeV/c and of $\pi\pi$-scattering data

M. Albrecht[1], C. Amsler[4,5], W. Dünnweber[3], M. A. Faessler[3], F. H. Heinsius[1], H. Koch[1], B. Kopf[1,a], U. Kurilla[1,6], C. A. Meyer[2], K. Peters[1,6], J. Pychy[1], X. Qin[1], M. Steinke[1], U. Wiedner[1]

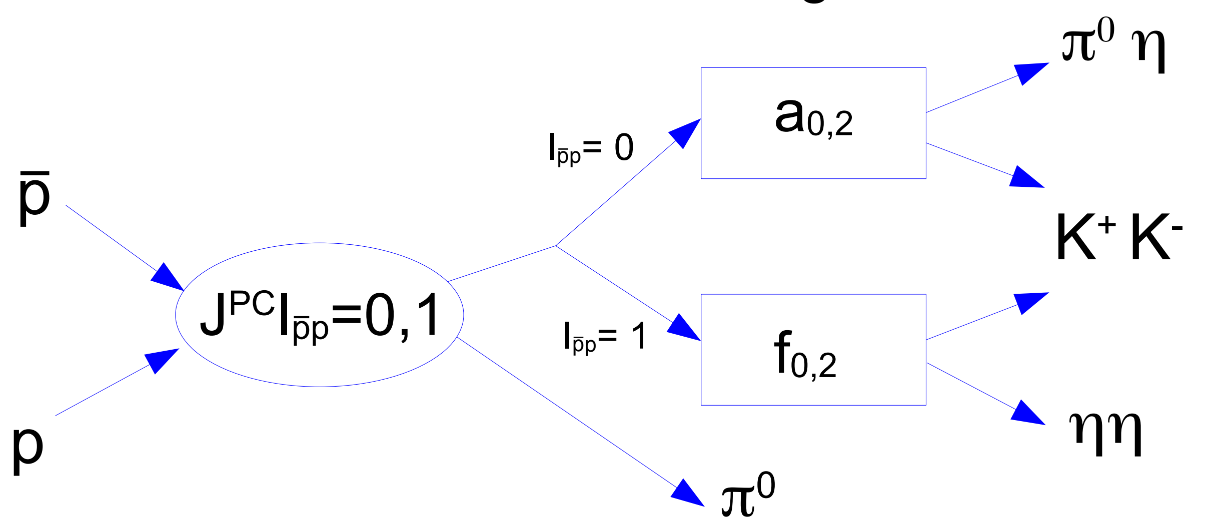
- Many a_0, a_2, f_0 and f_2 resonances appear in two or all three channels
  - constraints due to common production amplitudes
- Exotic spin wave π_1 so far only seen in $\bar{p}p$ data at rest
  - also visible in $\bar{p}p$ in flight data ?
- Event based fits take into account the complete phase-space with amplitudes from the initial down to the final state

$\bar{p}p \rightarrow K^+ K^- \pi^0, \pi^0 \pi^0 \eta, \pi^0 \eta \eta$

Single channel fit for $K^+K^-\pi^0$:
a and f resonances are not distinguishable



Coupled channel fit for $K^+K^-\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$:
a and f resonances are distinguishable



Why scattering data?

- Processes only characterized by elasticity and phase motion
  - good and easy access to resonance properties
- Phase shifts and elasticity is properly taken into account also for $\bar{p}p$ data
- Available from measurements and theory for I=0 S- and D-wave and I=1 P-wave
- Good constraints for f_0, f_2 and ρ resonances

$\bar{p}p \to K^+ K^- \pi^0,\ \pi^0 \pi^0 \eta,\ \pi^0 \eta\, \eta$

Best Fit Result achieved for

- K-matrix description for
  - f_0 with 5 poles and 5 channels
  - f_2 with 4 poles and 4 channels
  - ρ with 2 poles and 3 channels
  - a_0 and a_2 with 2 poles and 2 channels, each
  - $\pi_1^0 \to \pi^0\eta$ in $\pi^0\pi^0\eta$ with 1 pole and 2 channels
  - $(K\pi)_S$-wave: fixed parameterization from FOCUS-experiment

all pole positions and coupling strengths are free parameters

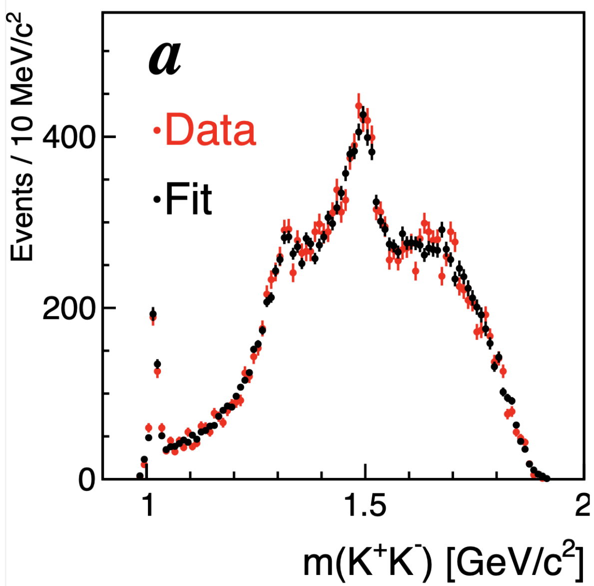
Phys. Lett. B653 (2007) 1-11

- Breit-Wigner description for
  - $\Phi(1020) \to K^+ K^-$
  - $K^{*\pm}(892) \to K^\pm \pi^0$

$\bar{p}p \rightarrow K^+ K^- \pi^0,\ \pi^0 \pi^0 \eta,\ \pi^0 \eta\ \eta$

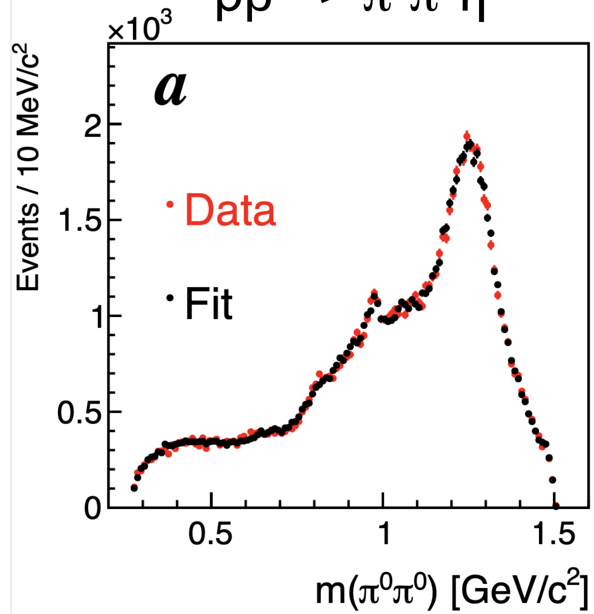
Eur. Phys.J. C (2020) 80, 453

$\bar{p}p \rightarrow K^+K^-\pi^0$



channel	contribution %
K*(892) K	45.0 ±1.3 ±11.0
$\rho\ \pi^0$	17.2 ±1.0 ±4.0
$f_2\,\pi^0$	17.1 ±0.7 ±10.0
$f_0\,\pi^0$	7.4 ±0.3 ±4.1
$(K\pi)_S$ K	6.1 ±0.4 ±4.9
$a_2\,\pi^0$	6.4 ±0.2 ±2.9
$\phi\ \pi^0$	2.5 ±0.3 ±0.3
$a_0\,\pi^0$	6.1 ±0.2 ±2.8
Σ	107.8 ±1.9 ±12.5

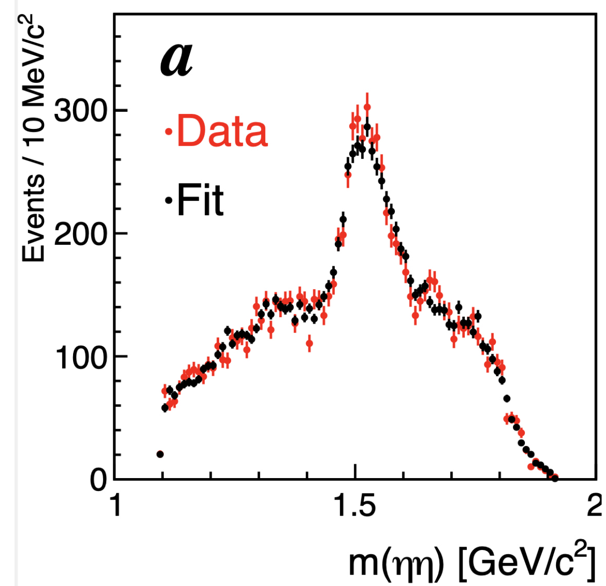
$\bar{p}p \rightarrow \pi^0\pi^0\eta$



channel	contribution %
$f_2\,\eta$	52.3 ±0.8 ±5.0
$a_2\,\pi^0$	33.0 ±0.6 ±2.9
$f_0\,\eta$	10.7 ±0.4 ±1.8
$a_0\,\pi^0$	22.4 ±0.4 ±1.0
$\pi_1\ \pi^0$	16.7± 0.5± 3.0
Σ	135.0 ±1.2 ±8.7

spin-exotic 1^{-+} contribution

$\bar{p}p \rightarrow \pi^0\eta\eta$



channel	contribution %
$f_0\,\pi^0$	23.7 ±1.2 ±2.3
$a_2\,\eta$	18.8 ±1.1 ±5.6
$a_0\,\eta$	28.6 ±1.1 ±7.5
$f_2\,\pi^0$	30.1 ±1.3 ±2.7
Σ	101.2 ±2.4 ±11.7

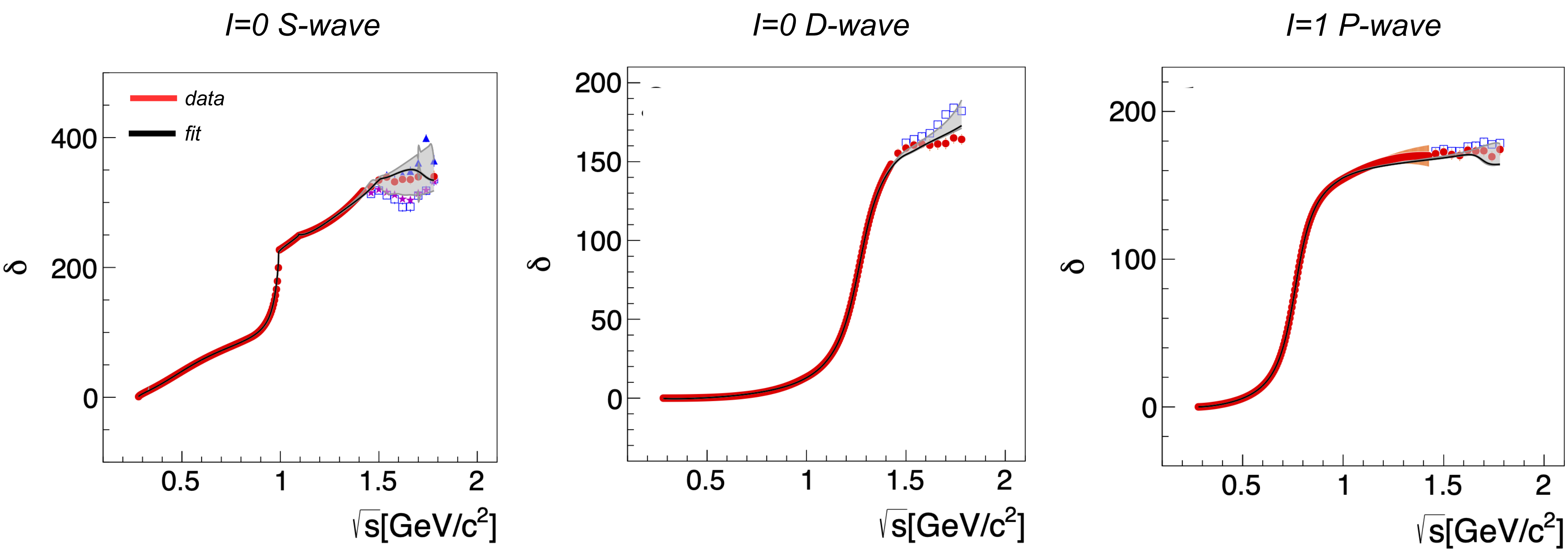
ππ Scattering Data

- All scattering data are well described
- $\pi\pi \to \pi\pi$ elasticity and
  $\pi\pi \to K\bar{K}$, $\eta\eta$, $\eta\eta'$ are not shown here

used data

Phys. Rev. D83(2011) 074004
Nucl. Phys B64 (1973) 134-162
Nucl. Phys B100 (1975) 205-224
J. Phys G40 (2013) 043001
Nucl. Phys B64 (1973) 134-162
Nucl. Phys B269 (1986) 485
Nouvo Cimento A80 (1984) 363

Phases for $\pi\pi \to \pi\pi$

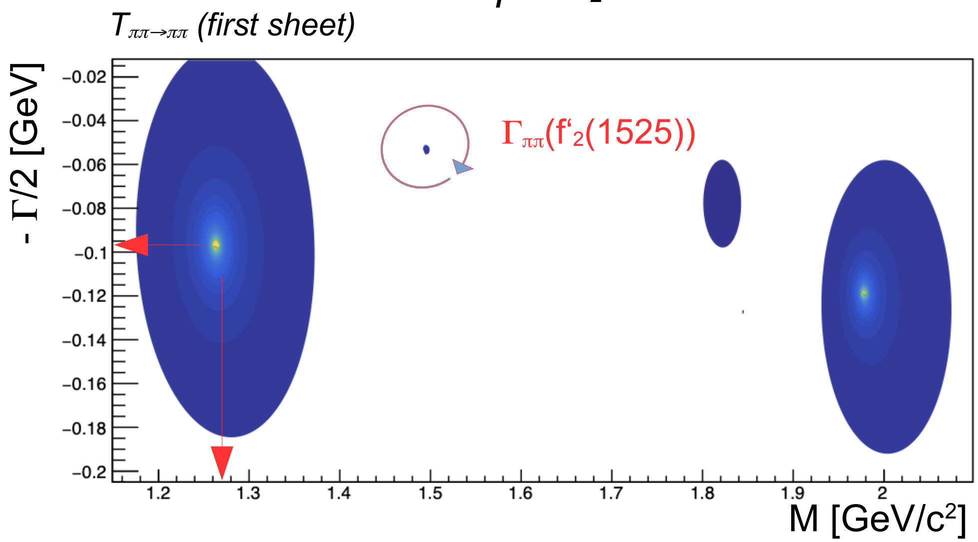


Extraction of Resonance Properties

- K-matrix contains all resonance parameters
- Masses and widths defined by the pole position in the complex energy plane of the T-matrix sheet closest to the physical sheet
- Related partial decay width can be extracted via the residues:

$$Res^{\alpha}_{k\to k} = \frac{1}{2\pi i} \oint_{C_{z\alpha}} \sqrt{\rho_k} \cdot T_{k\to k}(z) \cdot \sqrt{\rho_k}\, dz$$

Example: f_2-wave



More than 50 different resonance properties extracted
on the relevant Riemann-sheets for f_0, f_2, a_0, a_2 and ρ resonances

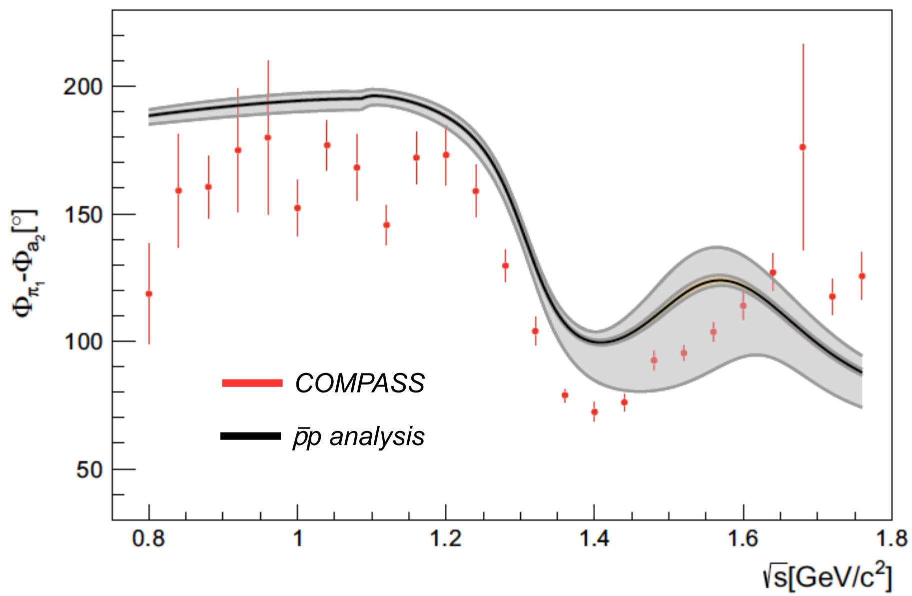
1^{-+} Wave in $\bar{p}p \to \pi^0 \pi^0 \eta$

- 1^{-+} wave seen in the decay $\pi^0\eta$
- K-matrix description with 1 pole and two channels $\pi\eta$ and $\pi\eta'$
  - motivated by JPAC analysis with COMPASS data
  - no data for $\pi\eta'$
  - $\pi\eta'$ channel is only used for unitarity
- Phase difference between the π_1 and a_2 wave from $T_{\pi\eta\to\pi\eta}$ in good agreement with COMPASS measurement

Phys. Lett. B740 (2015) 303-311
Phys. Lett. B811 (2020) 135913 (erratum)

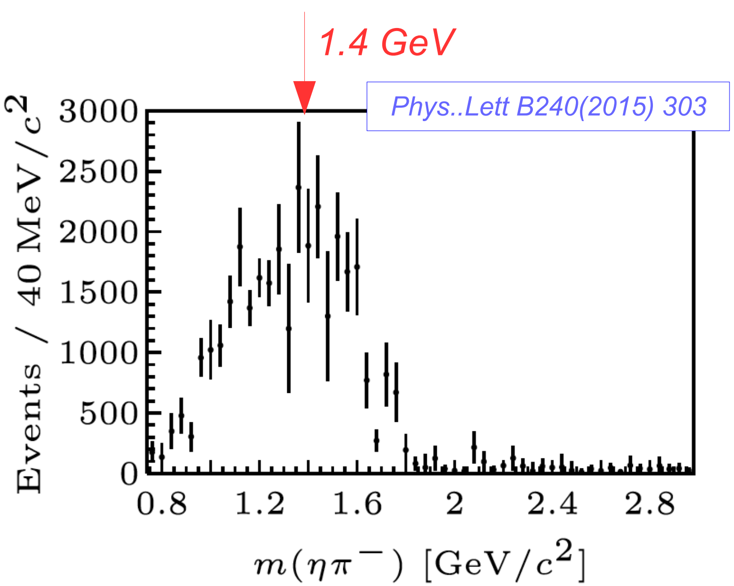
- Pole position of the π_1: $(1404.7 \pm 3.5\ (\text{stat.})\ ^{+9.0}_{-17.3}\ (\text{sys.}))\ \text{MeV}/c^2$

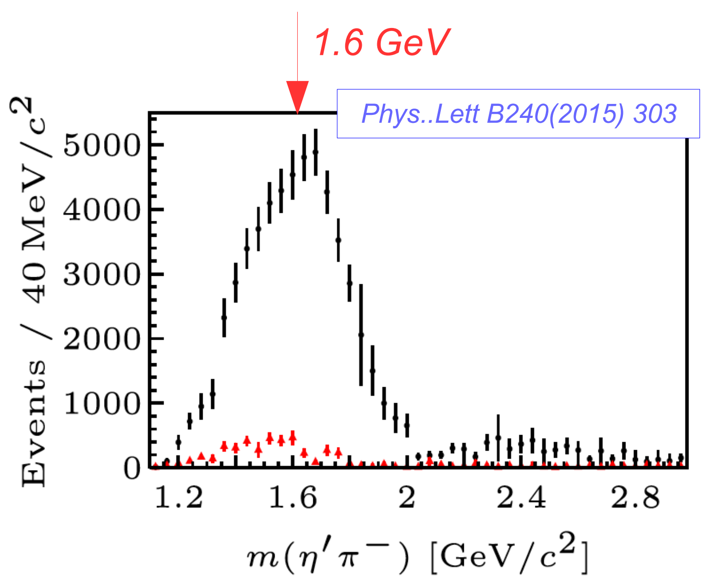
$$(628.3 \pm 27.1\ (\text{stat.})\ ^{+35.8}_{-138.2}\ (\text{sys.}))\ \text{MeV}$$

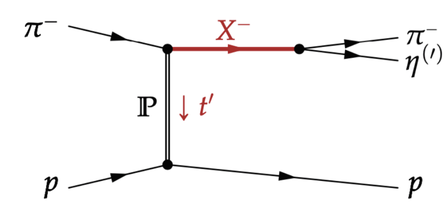


JPAC Analysis of COMPASS Data

- Coupled channel analysis of the 1^{-+} and 2^{++} wave in $\pi^- p \to \pi^- \eta^{(\prime)} p$
- 2 π_1 hybrid candidates below 2 GeV are listed in PDG
  - at around 1.4 GeV only seen in $\pi\,\eta$
  - at around 1.6 GeV seen in $\pi\,\eta'$ but not in $\pi\,\eta$
  - parameters obtained by Breit-Wigner fits
- Only one π_1 hybrid state predicted slightly below 2 GeV
- Enforcing analyticity and unitarity utilizing N/D method
- Mass shapes and phase shifts between 1^{-+} and 2^{++} are considered
- Decay channels to $\rho\,\pi$, $b_1\,\pi$ or $f_1\,\pi$ not considered



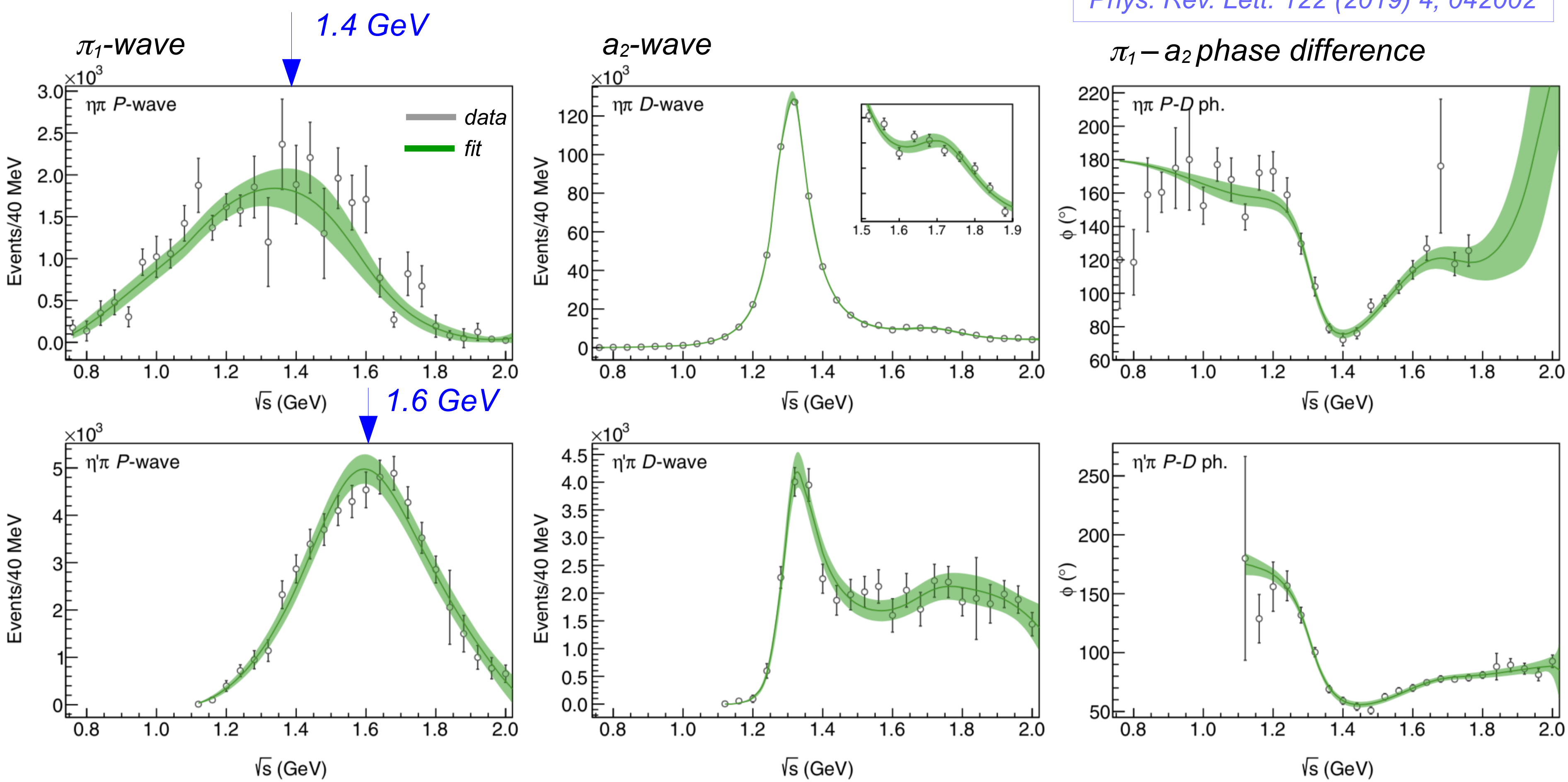




JPAC Analysis of COMPASS Data

- JPAC analysis: peak at 1.4 GeV in $\pi\eta$ and 1.6 GeV in $\pi\eta'$ are described by one pole at $(1564 \pm 24 \pm 86) - i(246 \pm 27 \pm 51)$ MeV
- Cannot be described by only one resonance with Breit-Wigner description

Phys. Rev. Lett. 122 (2019) 4, 042002



Coupled Channel Analysis with p̄p & COMPASS Data

- Extension: simultaneous fit of ππ-scattering data, p̄p → $K^+ K^- \pi^0$, $\pi^0 \pi^0 \eta$, $\pi^0 \eta\, \eta$ and $\pi^- p \rightarrow \pi^- \eta^{(\prime)} p$
- Good description with one pole scenario for the 1^{-+} wave using K-matrix → confirmation of the JPAC analysis based on N/D-method

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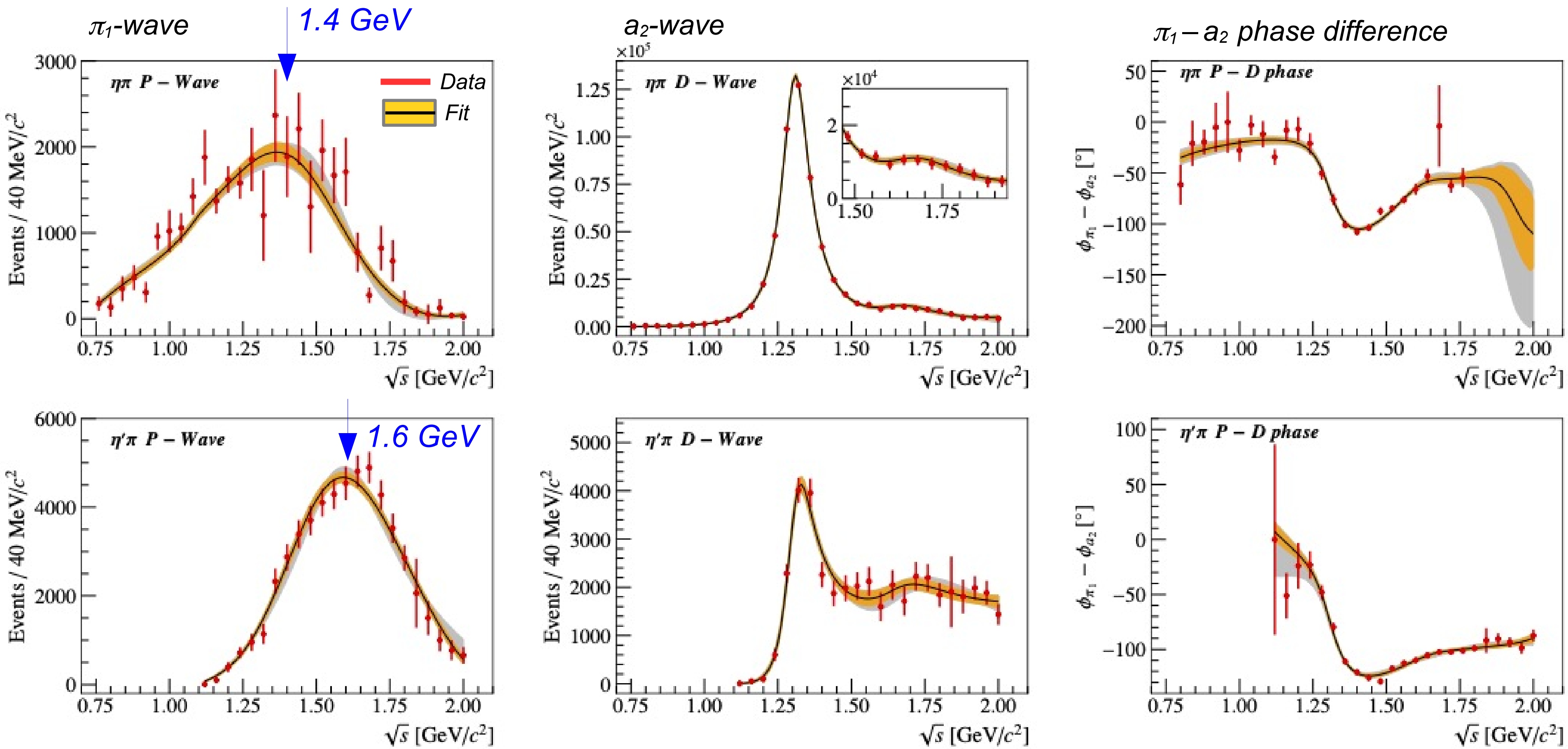
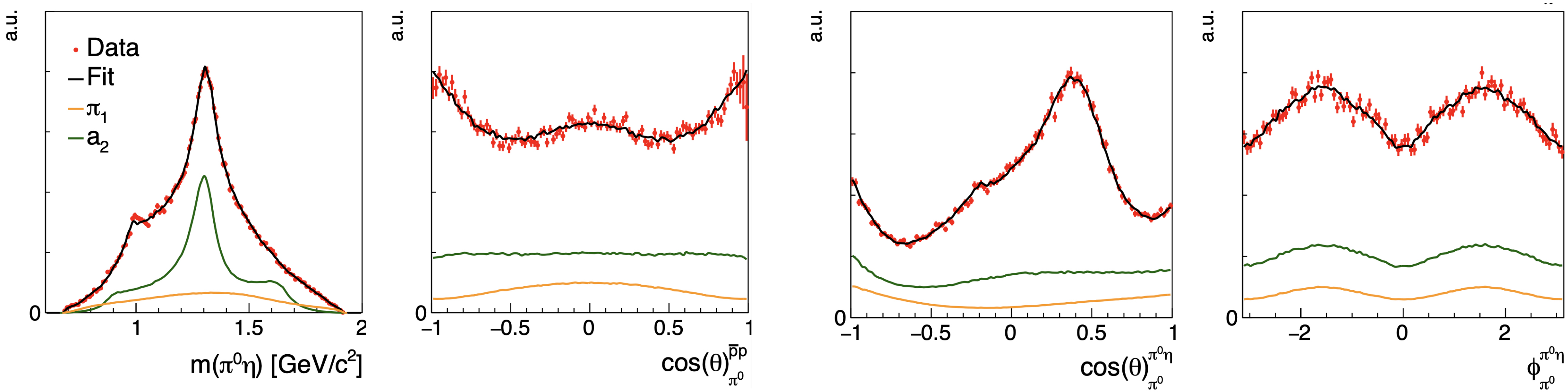


Coupled Channel Analysis with p̄p & COMPASS Data

$\bar{p}p \rightarrow \pi^0 \pi^0 \eta$

Eur. Phys.J. C (2021) 81, 1056



- Masses, widths and coupling strengths of the decays extracted from the poles in the complex energy plane
- π_1 mass is moving from 1.4 GeV/c^2 to 1.6 GeV/c^2 with $\pi\eta'$ data

Table 1 Obtained masses, total widths and ratios of partial widths for the pole of the spin-exotic π_1-wave and for the two poles in the a_2-wave, the $a_2(1320)$ and the $a_2(1700)$. The first uncertainty is the statistical and the second the systematic one

Name	Pole mass (MeV/c^2)	Pole width (MeV)	$\Gamma_{\pi\eta'}/\Gamma_{\pi\eta}$ (%)	$\Gamma_{K\bar{K}}/\Gamma_{\pi\eta}$ (%)
$a_2(1320)$	$1318.7 \pm 1.9^{+1.3}_{-1.3}$	$107.5 \pm 4.6^{+3.3}_{-1.8}$	$4.6 \pm 1.5^{+7.0}_{-0.6}$	$31 \pm 22^{+9}_{-11}$
$a_2(1700)$	$1686 \pm 22^{+19}_{-7}$	$412 \pm 75^{+64}_{-57}$	$3.5 \pm 4.4^{+6.9}_{-1.2}$	$2.9 \pm 4.0^{+1.1}_{-1.2}$
π_1	$1623 \pm 47^{+24}_{-75}$	$455 \pm 88^{+144}_{-175}$	$554 \pm 110^{+180}_{-27}$	–

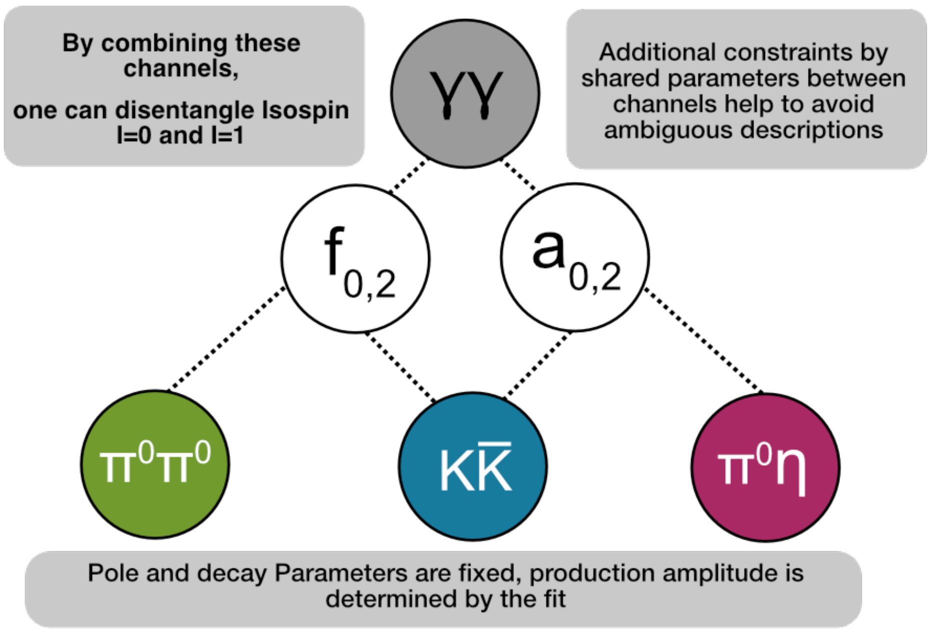
Obtained K-Matrix Parameters for other Use Cases

Coupled Channel Analysis of $\gamma\gamma$ Reactions @ BESIII

- Isospin 0 and 1 can be well separated in the K^+K^- channel
- f_0, f_2, a_0 and a_2 poles and decay constants fixed to obtained K-matrix parametrization from

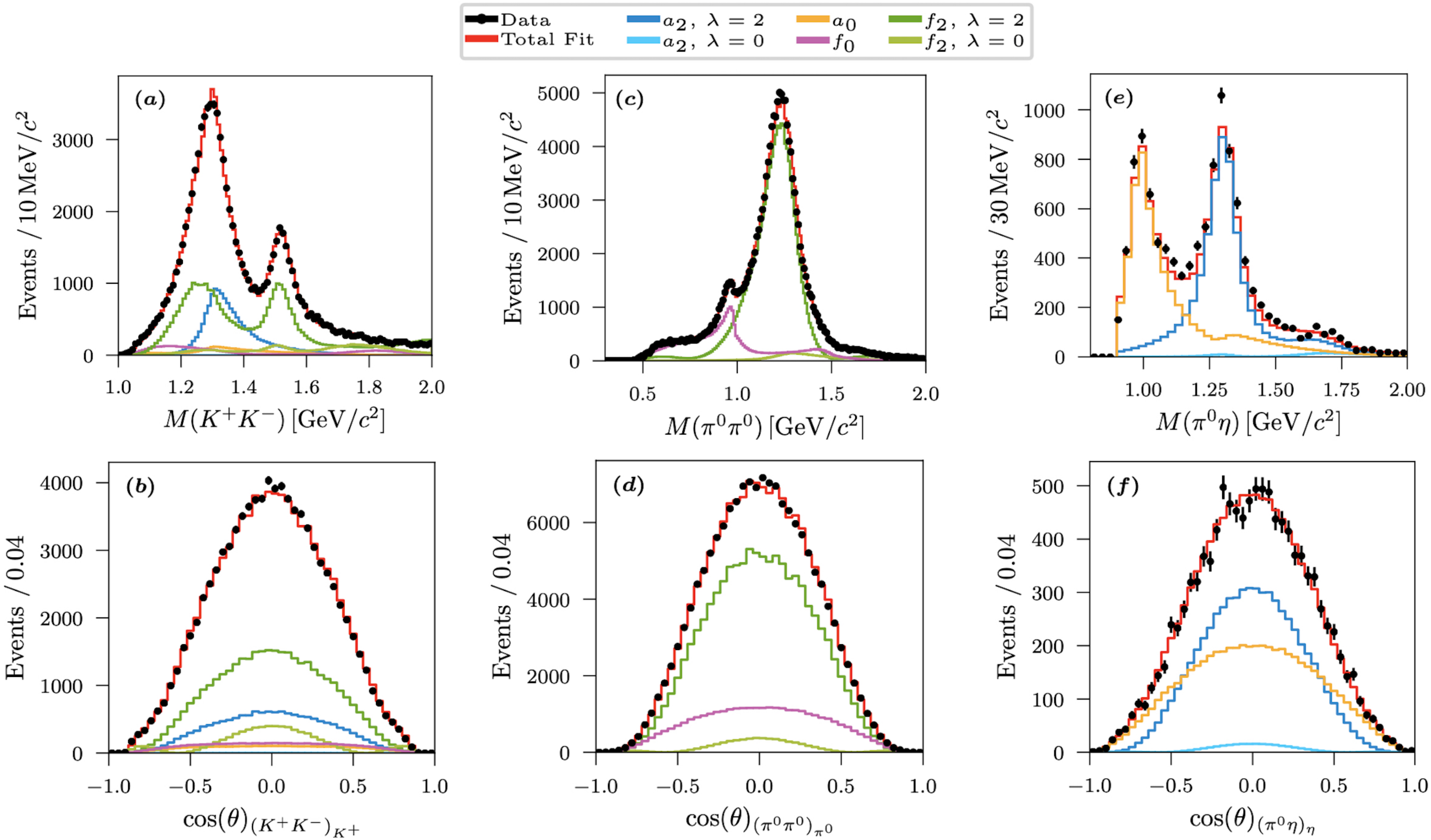
  Eur. Phys.J. C (2021) 81, 1056

- Proper access to $\gamma\gamma$-widths



$\gamma\gamma \rightarrow K^+K^-$, $\pi^0\pi^0$ and $\pi^0\eta$ @ BESIII

M. Küßner, PhD 2022, RUB

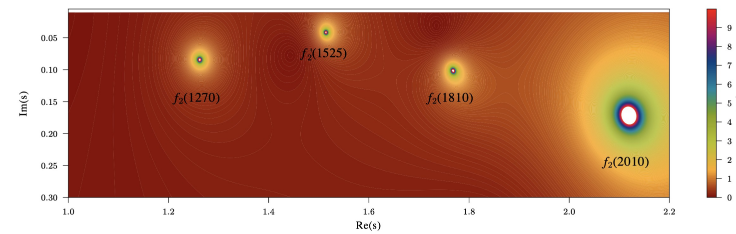


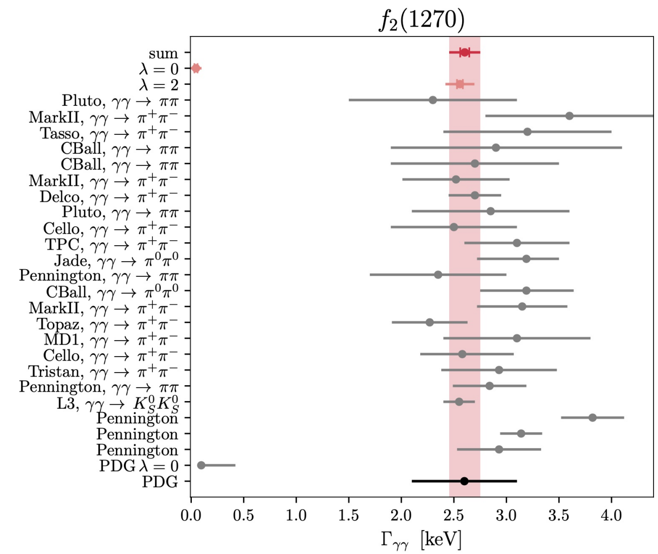
$\gamma\gamma \to K^+K^-$, $\pi^0\pi^0$ and $\pi^0\eta$ @ BESIII

Obtained $\gamma\gamma$-Widths

M. Küßner, PhD 2022, RUB

- Extracted via pole residues of the F-vector on the relevant Riemann-sheet
- Very accurate measurements for several f_0, f_2, a_0 and a_2 resonances with masses below 1.8 GeV/c^2
- First determination of the $f_2'(1525)$-widths for the helicities 0 and 2 separately





Future plans:

PWA with additional data from radiative J/ψ decays

with comparison of the production strengths

Summary and Conclusion

- Determination of the resonance parameters and coupling strengths to different production and decay processes important for classifying states
- Coupled channel analyses of data from different production mechanisms and decay systems are needed
- Sophisticated descriptions of the dynamics needed by taking into account analyticity and unitarity
- Examples of analyses in the light mesons sector are shown with data from $\bar{p}p$-annihilation, $\gamma\gamma$-fusion, πp-reactions and $\pi\pi$-scattering processes
  - peaks at 1.4 GeV in $\pi\eta$ and at 1.6 GeV in $\pi\eta'$ of the spin exotic 1^{-+} wave can be described by only one pole using K-matrix formalism

Back-up Slides

K-Matrix

Aitchison: „Nucl Phys A189 (1972) 417

S.U. Chung, E.Klempt „A Primer on K-matrix Formalism“, BNL Preprint (1995)

- A two body scattering process can be fully described by the S-matrix

$$S = I + 2i\sqrt{\rho}\,T\,\sqrt{\rho}$$

- T-matrix can be expressed by K-matrix: $T = (I - i\,K\,\rho)^{-1}\,K$

- Elements of the K-matrix:

bare coupling to channel i an j of resonance α

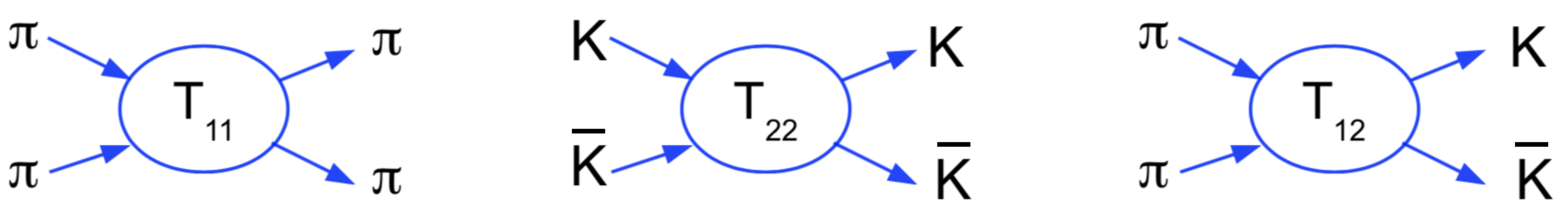
$$K_{ij} = \sum_{\alpha} \frac{g_{\alpha_i} g_{\alpha_j}}{m_\alpha^2 - s} + \sum_k c_{kij}\, s^k$$

K-matrix element for channel i (initial) and j (final)

bare mass of resonance α

energy dependent background term

- Example: channel 1: $\pi\pi$, channel 2: K $\overline{\text{K}}$



K $\qquad (I - i\,K\,\rho)^{-1}$

K-Matrix with P-Vector Approach

Aitchison: „The K-Matrix formalism for overlapping resonances", Nucl Phys A189 (1972) 417

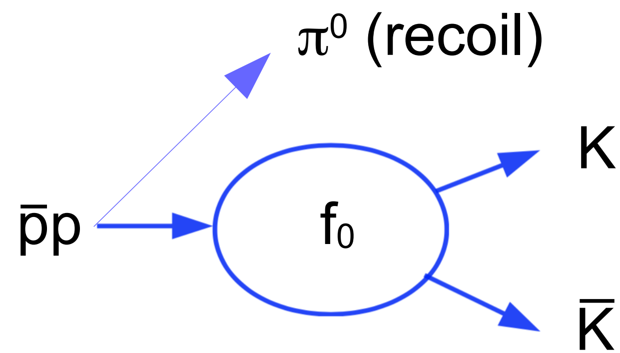
- Generalization of the K-matrix formalism to the case of production of resonances in more complex reactions

- Dynamical function for P-vector approach: $F = (I - i\,K\,\rho)^{-1}\,P$

with: $$P_i = \sum_{\alpha} \frac{\beta_\alpha g_{\alpha_i}}{m_\alpha^2 - s} + \sum_k c_{ki} s^k$$

same pole structure as for K-matrix

- Example: $\bar{p}p \rightarrow f_0\,\pi^0 \rightarrow (K\bar{K})\,\pi^0$

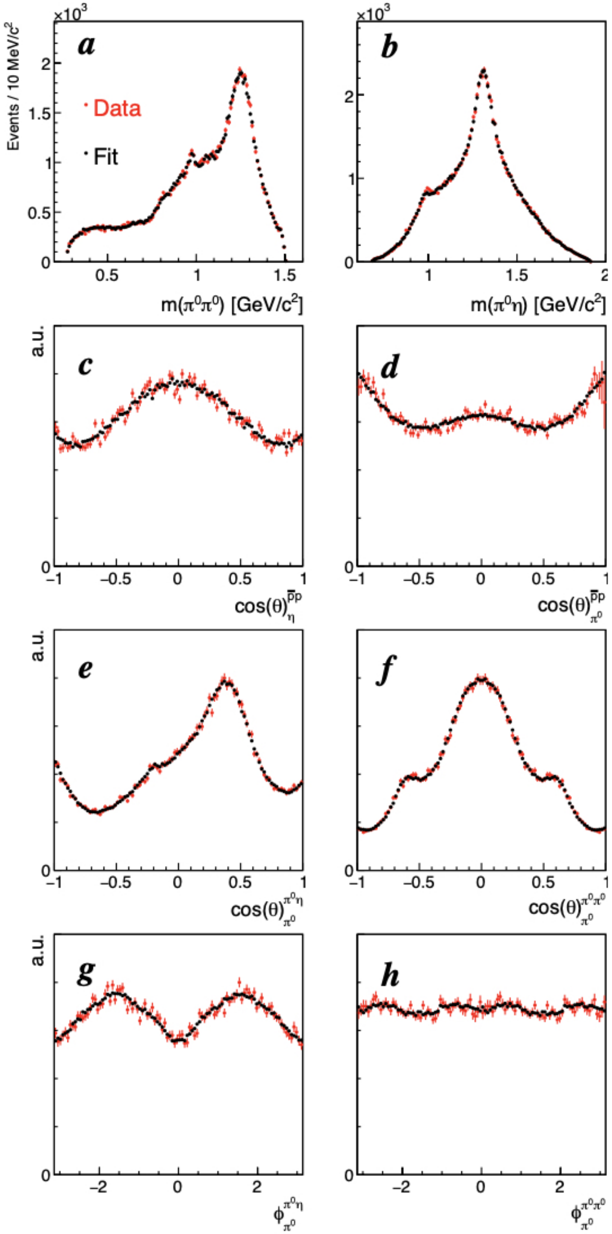


P $\qquad$ $(I - i\,K\,\rho)^{-1}$

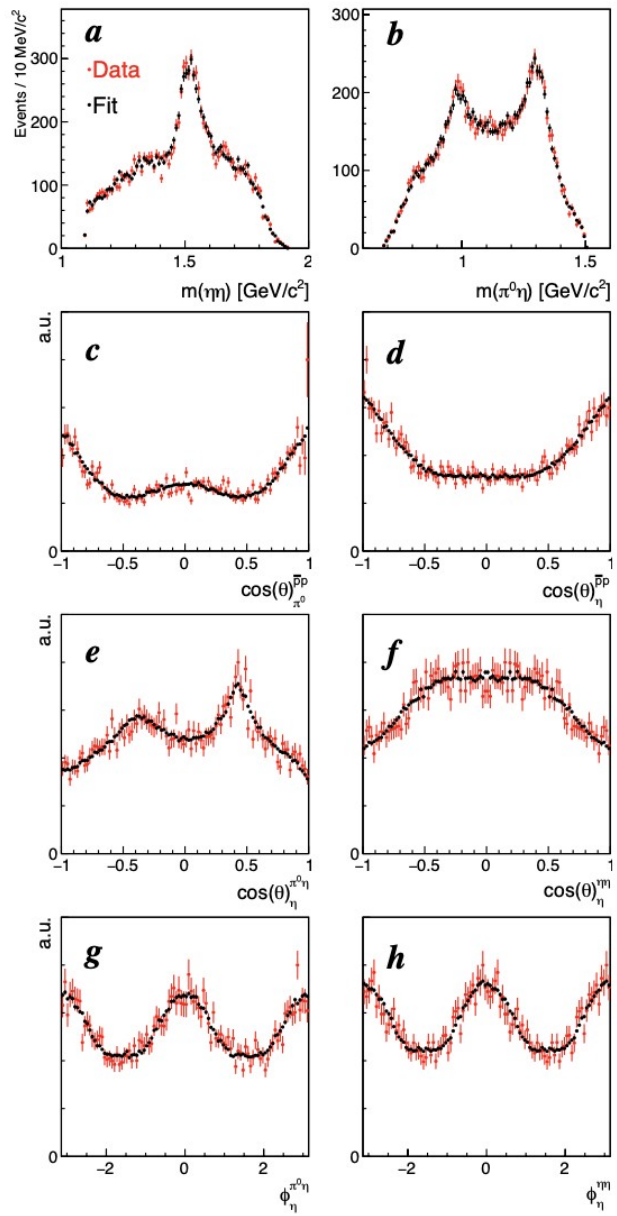
Fit Results

Eur. Phys.J. C (2021) 81, 1056

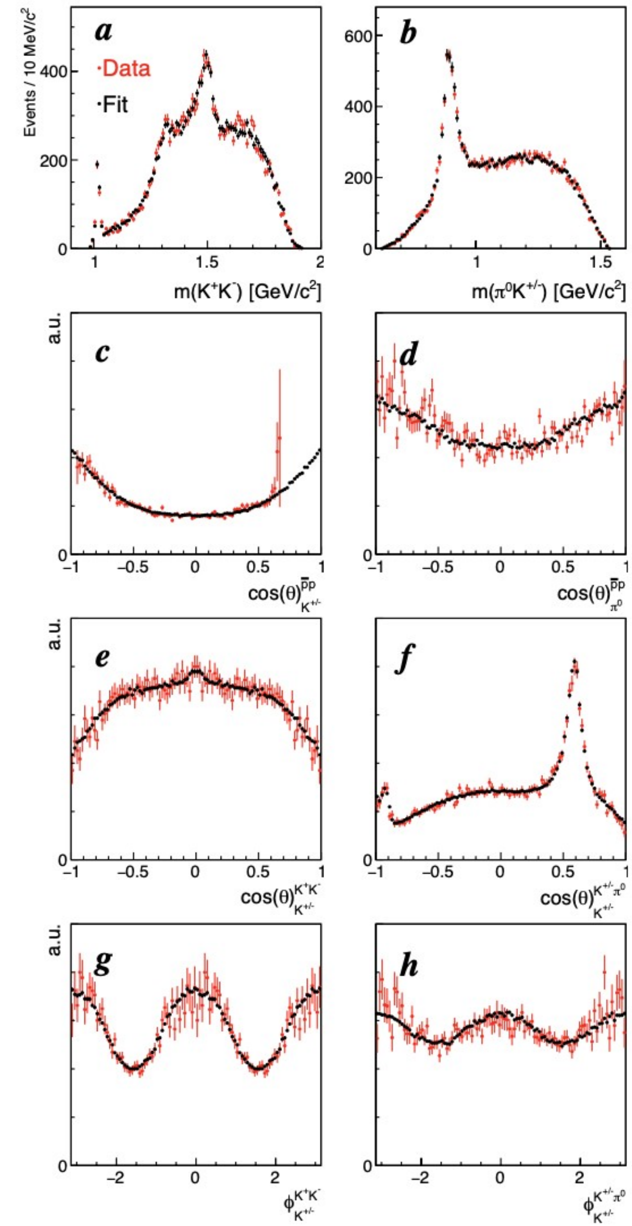
$\bar{p}p \rightarrow \pi^0\pi^0\eta$



$\bar{p}p \rightarrow \pi^0\eta\eta$



$\bar{p}p \rightarrow K^+K^-\pi^0$



Fit Results

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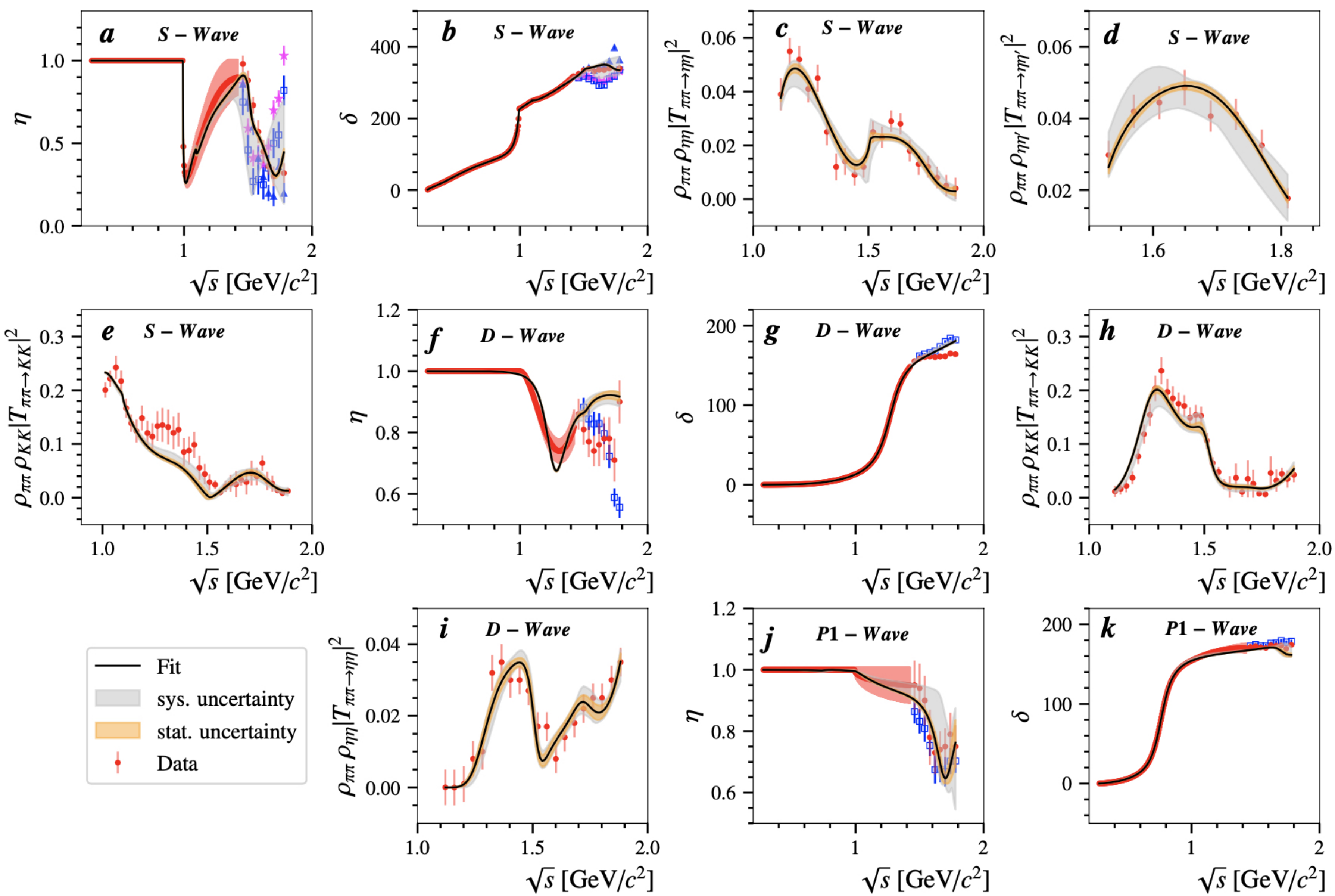

Obtained Resonance Properties

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name	relevant data	Breit-Wigner mass [MeV/c^2]	Breit-Wigner width Γ [MeV]
$K^*(892)^\pm$	$\bar{p}p$	$893.8 \pm 1.0 \pm 0.8$	$56.3 \pm 2.0 \pm 1.0$
$\phi(1020)$	$\bar{p}p$	$1018.4 \pm 0.5 \pm 0.2$	4.2 (fixed)

name	relevant data	pole mass [MeV/c^2]	pole width Γ [MeV]
$f_0(980)^{---++}$	scat	$977.8 \pm 0.6 \pm 1.4$	$98.8 \pm 6.6 \pm 11.2$
$f_0(980)^{--+++}$	scat	$992.6 \pm 0.3 \pm 0.5$	$61.2 \pm 1.2 \pm 1.7$
$f_0(1370)$	scat	$1281 \pm 11 \pm 26$	$410 \pm 12 \pm 50$
$f_0(1500)$	$\bar{p}p$ + scat	$1511.0 \pm 8.5\, ^{+3.5}_{-14.0}$	$81.1 \pm 4.5\, ^{+26.9}_{-0.5}$
$f_0(1710)$	$\bar{p}p$ + scat	$1794.3 \pm 6.1\, ^{+47.0}_{-61.2}$	$281 \pm 32\, ^{+12}_{-80}$
$f_2(1810)$	scat	$1769 \pm 26\, ^{+3}_{-26}$	$201 \pm 57\, ^{+13}_{-87}$
$f_2(X)$	scat	$2119.9 \pm 6.4\, ^{+25.7}_{-1.1}$	$343 \pm 11\, ^{+32}_{-11}$

name	relevant data	pole mass [MeV/c^2]	pole width Γ [MeV]	$\Gamma_{\pi\eta'}/\Gamma_{\pi\eta}$ [%]
π_1	$\bar{p}p + \pi p$	$1623 \pm 47\, ^{+24}_{-75}$	$455 \pm 88\, ^{+144}_{-175}$	$554 \pm 110\, ^{+180}_{-27}$

name	relevant data	pole mass [MeV/c^2]	pole width Γ [MeV]	$\Gamma_{KK}/\Gamma_{\pi\eta}$ [%]
$a_0(980)^{--}$	$\bar{p}p$	$1002.7 \pm 8.8 \pm 4.2$	$132 \pm 11 \pm 8$	$14.8 \pm 7.1 \pm 3.6$
$a_0(980)^{-+}$	$\bar{p}p$	$1003.3 \pm 8.0 \pm 3.7$	$101.1 \pm 7.2 \pm 3.0$	$13.5 \pm 6.2 \pm 3.1$
$a_0(1450)$	$\bar{p}p$	$1303.0 \pm 3.8 \pm 1.9$	$109.0 \pm 5.0 \pm 2.9$	$396 \pm 72 \pm 72$

name	relevant data	pole mass [MeV/c^2]	pole width Γ [MeV]	$\Gamma_{KK}/\Gamma_{\pi\eta}$ [%]	$\Gamma_{\pi\eta'}/\Gamma_{\pi\eta}$ [%]
$a_2(1320)$	$\bar{p}p + \pi p$	$1318.7 \pm 1.9\, ^{+1.3}_{-1.3}$	$107.5 \pm 4.6\, ^{+3.3}_{-1.8}$	$31 \pm 22\, ^{+9}_{-11}$	$4.6 \pm 1.5\, ^{+7.0}_{-0.6}$
$a_2(1700)$	$\bar{p}p + \pi p$	$1686 \pm 22\, ^{+19}_{-7}$	$412 \pm 75\, ^{+64}_{-57}$	$2.9 \pm 4.0\, ^{+1.1}_{-1.2}$	$3.5 \pm 4.4\, ^{+6.9}_{-1.2}$

name	relevant data	pole mass [MeV/c^2]	pole width Γ [MeV]	$\Gamma_{\pi\pi}/\Gamma$ [%]	Γ_{KK}/Γ [%]	$\Gamma_{\eta\eta}/\Gamma$ [%]
$f_2(1270)$	$\bar{p}p$ + scat	$1262.4 \pm 0.2\, ^{+0.2}_{-0.3}$	$168.0 \pm 0.7\, ^{+1.7}_{-0.1}$	$87.7 \pm 0.3\, ^{+4.8}_{-4.4}$	$2.6 \pm 0.1\, ^{+0.1}_{-0.2}$	$0.3 \pm 0.1\, ^{+0.0}_{-0.1}$
$f_2'(1525)$	$\bar{p}p$ + scat	$1514.7 \pm 5.2\, ^{+0.3}_{-7.4}$	$82.3 \pm 5.2\, ^{+11.6}_{-4.5}$	$2.1 \pm 0.3\, ^{+0.8}_{-0.0}$	$67.2 \pm 4.2\, ^{+5.0}_{-3.8}$	$9.8 \pm 3.8\, ^{+1.7}_{-3.3}$
$\rho(1700)$	$\bar{p}p$ + scat	$1700 \pm 27\, ^{+13}_{-16}$	$181 \pm 25\, ^{+0.0}_{-16}$	$13.6 \pm 1.2\, ^{+0.9}_{-0.5}$	$0.8 \pm 0.1\, ^{+0.0}_{-0.0}$	-