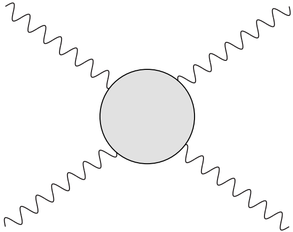
A theoretical perspective on two-photon physics

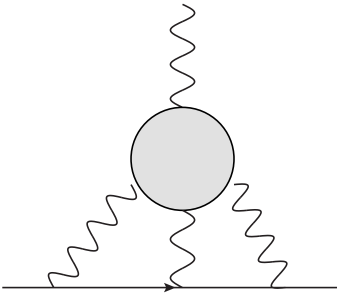
Christoph Lehner
(Uni Regensburg)

March 18, 2022 – Hadron Spectroscopy: The Next Big Steps

Focus of this talk: theory of Hadronic Light-by-Light (HLbL) scattering



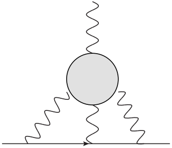
Focus of this talk: ... and mostly application to muon g-2

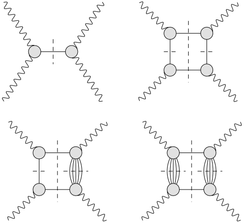


Two new avenues for a model-independent value for the HLbL

Dispersive analysis +
Experimental/lattice input

Direct lattice calculation





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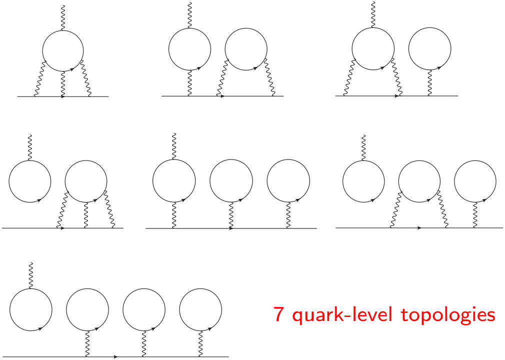
How to estimate uncertainty of truncation of cuts/states?



7 quark-level topologies

Dispersive analysis - recent results

- JHEP1704(2017)161 (Colangelo et al.): Pion-box plus S-wave rescattering $a_\mu^{\pi-box} + a_\mu^{\pi\pi,\pi-pole\ LHC,J=0} = -2.4(1) \times 10^{-10}$
- PRL121(2018)112002 (Hoferichter et al.); 1808.04823: Pion-pole contribution $a_\mu^{\pi-pole} = 6.26(30) \times 10^{-10}$ reconstructing $\pi \to \gamma^*\gamma^*$ form factor from $e^+e^- \to 3\pi, e^+e^-\pi^0$ and $\pi^0 \to \gamma\gamma$ width
- PRD100(2019)034520 (Mainz): Pion-pole contribution $a_\mu^{\pi-pole} = 6.23(23) \times 10^{-10}$ (Lattice+Dispersive FF normalization by PrimEx)
- Recent results for kaon-box: $a_\mu^{K-box} = -0.048(4) \times 10^{-10}$ (2112.11106 (Miramontes et al.)), $a_\mu^{K-box} = -0.048(1) \times 10^{-10}$ (2202.11106 (Stamen et al.))

Combining these results one finds: $a_\mu^{\pi-pole} + a_\mu^{\pi-box} + a_\mu^{\pi\pi} + a_\mu^{K-box} = 3.9(3) \times 10^{-10}$

Further estimates: $a_\mu^{\eta,\eta'} \approx 3 \times 10^{-10}$, $a_\mu^{\text{axial vector}} \approx 1 \times 10^{-10}$, $a_\mu^{\text{short distance}} \approx 1 \times 10^{-10}$ (see also lattice evaluations on subsequent slides)

Control of truncation error/short-distance constraints very important. For a complete list of recent improvements, see TI snowmass contribution to appear very soon.

Need improvements, e.g., for coupling of axial-vector resonances to two-photons in 1-2 GeV region. (BESIII/LQCD?)

g-2 Theory Initiative – Seattle consensus

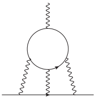
Contribution	PdRV(09) [6]	N/JN(09) [7, 138]	J(17) [38]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	$\left.\begin{matrix} \\ \end{matrix}\right\} -1(3)$
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

$a_\mu^{\mathrm{HLbL}} \times 10^{11}$

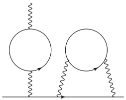
See also our whitepaper Phys. Rept. 887 (2020) 1-166

7 quark-level topologies of direct lattice calculation

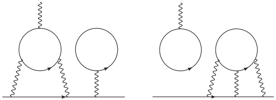
Hierarchy imposed by QED charges of dominant up- and down-quark contribution



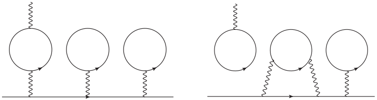
$Q_u^4 + Q_d^4 = 17/81$



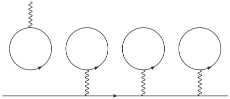
$(Q_u^2 + Q_d^2)^2 = 25/81$



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$

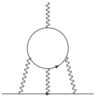


$(Q_u + Q_d)^4 = 1/81$

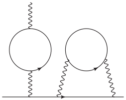
Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

7 quark-level topologies of direct lattice calculation

Hierarchy imposed by QED charges of dominant up- and down-quark contribution

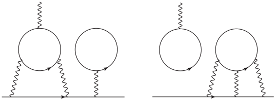


$Q_u^4 + Q_d^4 = 17/81$

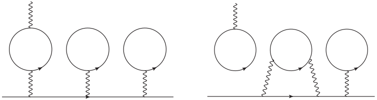


$(Q_u^2 + Q_d^2)^2 = 25/81$

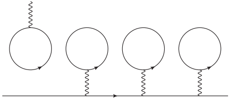
Dominant diagrams in top row: connected and leading disconnected diagram



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$



$(Q_u + Q_d)^4 = 1/81$

Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

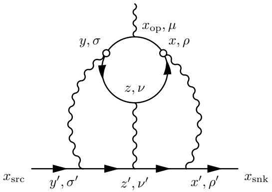
Development of HLbL lattice methodology (I)

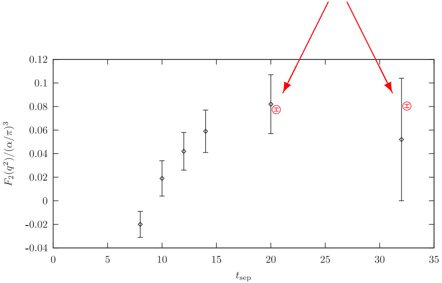
- QED non-perturbatively and momentum-space (PRL114(2015)012001)
- QED perturbatively and position-space
  - QED_L (PRD93(2016)014503, PRL118(2017)022005): $1/L^2$ finite-volume errors (with linear extent L); noise reduction through importance sampling
  - QED_∞: exponential finite-volume errors (PRL115(2015)222003, EPJ Web Conf. 175(2018)06023), subtraction prescriptions to reduce systematic errors (PRD96(2017)034515, arXiv:1811.08320)

Color code: Mainz, RBC/UKQCD

PRD93(2015)014503 (Blum, Christ, Hayakawa, Izubuchi, Jin, and Lehner):

New sampling strategy with 10x reduced noise for same cost (red versus black):





Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

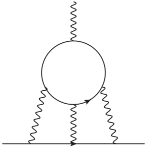
Development of HLbL lattice methodology (II)

- PRL118(2016)022005: Physical-pion mass for leading connected+disconnected diagrams at finite volume and lattice spacing $a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$
- PRD98(2018)074501: Forward scattering amplitude $(\gamma^*\gamma^* \to \gamma^*\gamma^*)$
- Phys. Rev. D 100, 034520 (2019): Pion-pole contribution $a_\mu^{\mathrm{HLbL},\pi^0} = 5.97(36) \times 10^{-10}$

Color code: Mainz, RBC/UKQCD

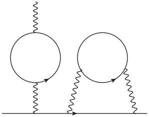
PRL118(2016)022005 (Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, and Lehner):

- Calculation at physical pion mass with finite-volume QED prescription ($\mathrm{QED_L}$) at single lattice cutoff of $a^{-1} = 1.73$ GeV and lattice size $L = 5.5$ fm.
- Connected diagram:



$$a_\mu^{\mathrm{cHLbL}} = 11.6(0.96) \times 10^{-10}$$

- Leading disconnected diagram:



$$a_\mu^{\mathrm{dHLbL}} = -6.25(0.80) \times 10^{-10}$$

- Large cancellation expected from pion-pole-dominance considerations is realized: $a_\mu^{\mathrm{HLbL}} = a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35(1.35) \times 10^{-10}$

Potentially large systematics due to finite-volume QED!

Development of HLbL lattice methodology (III)

- PRL124(2020)132002: $a \to 0$, $V \to \infty$, $m_\pi \approx 139$ MeV: $a_\mu^{\mathrm{HLbL}} = 7.87(3.06)(1.77) \times 10^{-10}$
- Eur. Phys. J. C 80, 869 (2020): Only SU(3) symmetric point ($m_\pi = m_K = 420$ MeV), connected plus leading disconnected diagrams, $a \to 0$ and $V \to \infty$
- Eur. Phys. J. C 81, 651 (2021): Also sub-leading diagrams and down to $m_\pi \approx 200$ MeV: $a_\mu^{\mathrm{HLbL}} = 10.68(1.47) \times 10^{-10}$
- Burri @ Lattice 2021 pion pole: $m_\pi \approx 139$ MeV, $a_\mu^{\mathrm{HLbL},\pi^0} \in [5.86, 6.57] \times 10^{-10}$ (statistical errors only)
- Chao @ Lattice 2021 pseudoscalar TFF: established control over η, η' in staggered quark formalism, promising statistical signal

Color code: BMWc, ETMC, Mainz, RBC/UKQCD

PHYSICAL REVIEW LETTERS **124,** 132002 (2020)

Editors' Suggestion Featured in Physics

Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment from Lattice QCD

Thomas Blum,[1,2] Norman Christ,[3] Masashi Hayakawa,[4,5] Taku Izubuchi,[6,2]
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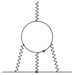
(Received 18 December 2019; accepted 27 February 2020; published 1 April 2020)

We report the first result for the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment with all errors systematically controlled. Several ensembles using $2+1$ flavors of physical mass Möbius domain-wall fermions, generated by the RBC and UKQCD collaborations, are employed to take the continuum and infinite volume limits of finite volume lattice QED + QCD. We find $a_\mu^{\mathrm{HLbL}} = 7.87(3.06)_{\mathrm{stat}}(1.77)_{\mathrm{sys}} \times 10^{-10}$. Our value is consistent with previous model results and leaves little room for this notoriously difficult hadronic contribution to explain the difference between the standard model and the BNL experiment.

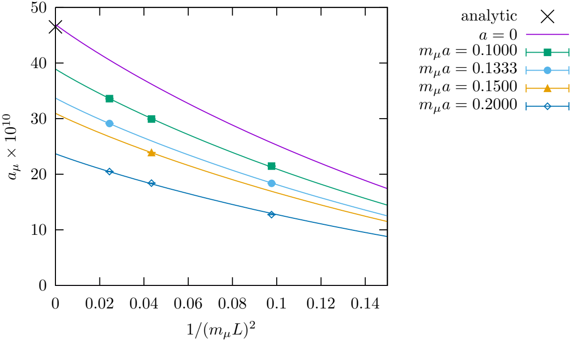
Lattice QCD ensembles at physical pion mass:

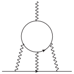
	48I	64I	24D	32D	48D	32Dfine
a^{-1} (GeV)	1.730	2.359	1.015	1.015	1.015	1.378
a (fm)	0.114	0.084	0.194	0.194	0.194	0.143
L (fm)	5.47	5.38	4.67	6.22	9.33	4.58
L_s	48	64	24	24	24	32
m_π (MeV)	139	135	142	142	142	144
m_μ (MeV)	106	106	106	106	106	106
# meas con	65	43	157	70	8	55
# meas discon	104	44	156	69	0	55

(multiple volumes and lattice spacings)

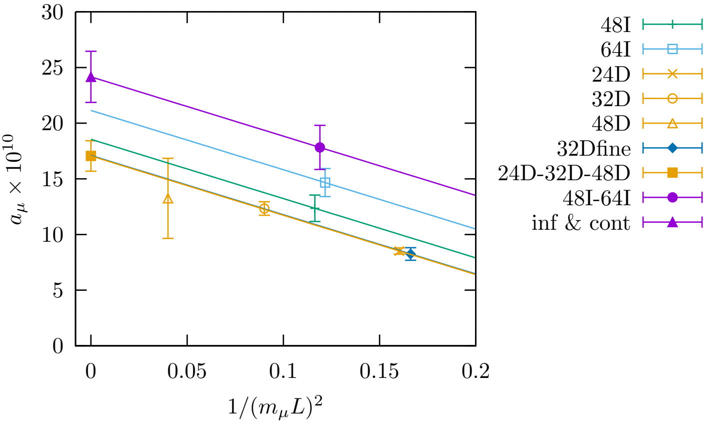
QED test (replace quark loop by lepton loop): 

$$a_\mu(L,a) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} + \frac{b_3}{(m_\mu L)^3}\Big) \times\Big(1 - c_1(m_\mu a)^2 + c_2(m_\mu a)^4\Big)$$

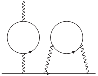


Connected diagram (QCD+QED): 

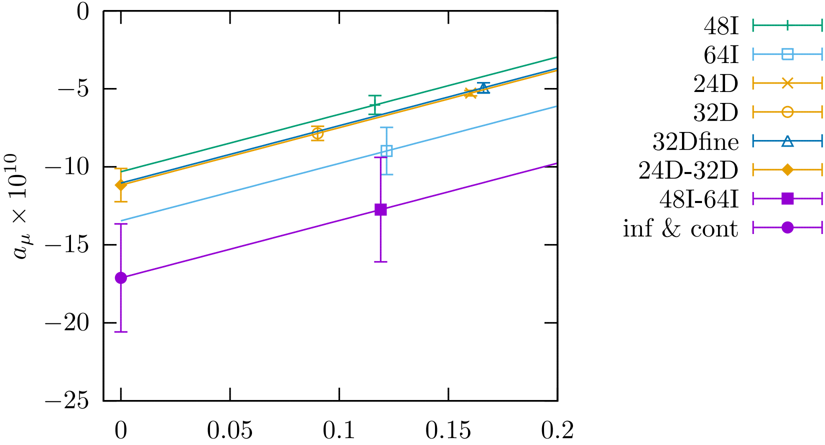
$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2}$$
$$- c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

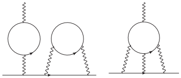


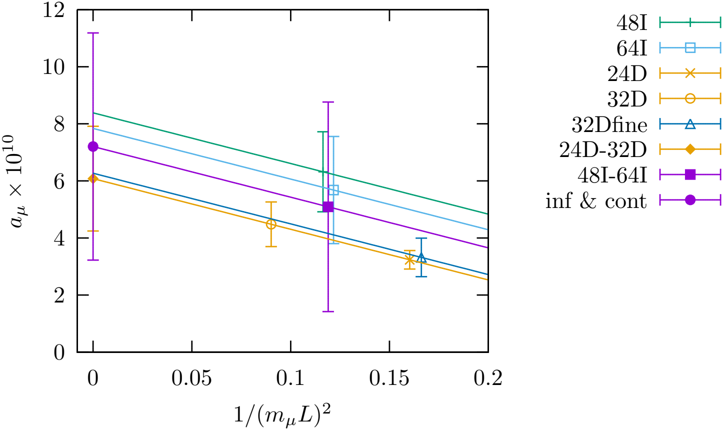
Hybrid method: Already used for 2018 HVP, for very noisy long-distance contribution in connected diagram fit to constant instead of $c_0 + a^2 c_1$ for continuum limit

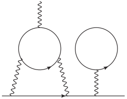
Leading disconnected diagram (QCD+QED): 

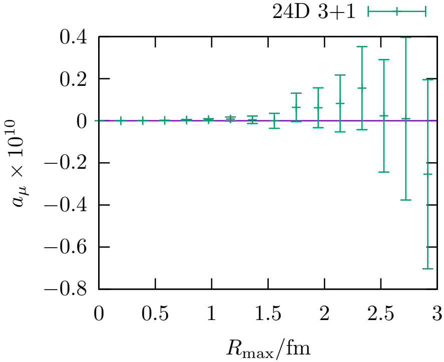
$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$



Connected plus leading disconnected (QCD+QED): 



Subleading diagrams: 



Summary of results:

	con	discon	tot
a_μ	24.16(2.30)	−16.45(2.13)	7.87(3.06)
sys hybrid $\mathcal{O}(a^2)$	0.20(0.45)	0	0.20(0.45)
sys $\mathcal{O}(1/L^3)$	2.34(0.41)	1.72(0.32)	0.83(0.56)
sys $\mathcal{O}(a^4)$	0.88(0.31)	0.71(0.28)	0.95(0.92)
sys $\mathcal{O}(a^2 \log(a^2))$	0.23(0.08)	0.25(0.09)	0.02(0.11)
sys $\mathcal{O}(a^2/L)$	4.43(1.38)	3.49(1.37)	1.08(1.57)
sys strange con	0.30	0	0.30
sys subdiscon	0	0.50	0.50
sys all	5.11(1.32)	3.99(1.29)	1.77(1.13)

$$\boxed{a_\mu^{\mathrm{tot}} = 7.87(3.06)_{\mathrm{stat}}(1.77)_{\mathrm{sys}} \times 10^{-10}}$$

Combined with data-driven result in Theory Initiative whitepaper result (Phys. Rept. 887 (2020) 1-166): $a_\mu^{\mathrm{HLbL}} = 9.2(1.8) \times 10^{-10}$

Eur. Phys. J. C 81, 651 (2021)

Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD: a complete calculation

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[2] Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France
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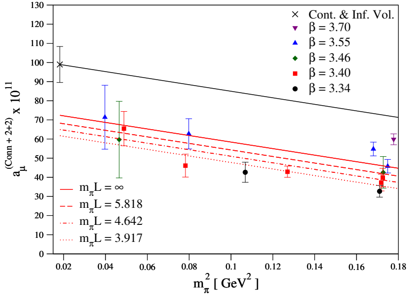
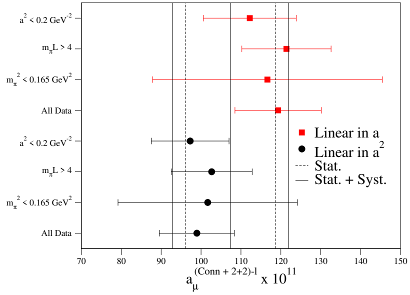
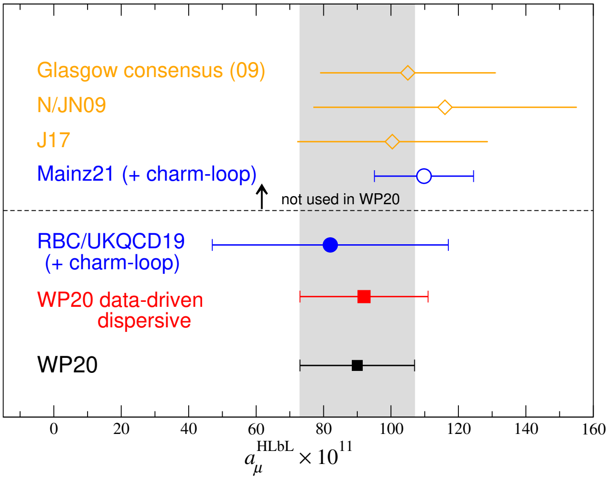
Contribution	Value $\times 10^{11}$
Light-quark fully-conn. and $(2+2)$	$107.4(11.3)_{\mathrm{stat.}}(9.2)_{\mathrm{syst.}}(6.0)_{\mathrm{chiral}}$
Strange-quark fully-conn. and $(2+2)$	$-0.6(2.0)$
$(3+1)$	$0.0(0.6)$
$(2+1+1)$	$0.0(0.3)$
$(1+1+1+1)$	$0.0(0.1)$
Total	$106.8(15.9)$

A breakdown of our result for a_μ^{hlbl}.





Current status



Summary of lattice efforts and outlook

- Two groups (RBC/UKQCD and Mainz) now have complete first-principles QCD+QED results for the HLbL; the two independent results agree with each other
- Additional groups have joined HLbL efforts with progress reports for the pion TFF (ETMC) and the η, η' TFF (BMW) at recent Lattice 2021
- Further improvements expected in following years (such as reduced statistical error for RBC/UKQCD, physical pion mass for Mainz)