Recent *BABAR* results on measurement of exclusive hadronic cross sections

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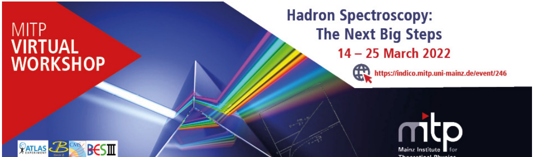
Hadron Specttroscopy: The Next Big Step
MITP virtual workshop



Outline

- New precision measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section
  Phys.Rev.D 104, 112003 (2021)

- First measurements of $e^+e^- \to \pi^+\pi^-4\pi^0$ and $e^+e^- \to \pi^+\pi^-3\pi^0\eta$ cross sections
  Phys.Rev.D 104, 112004 (2021)

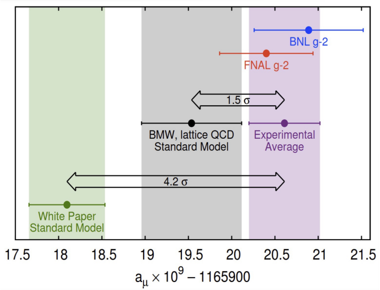
- First measurements of $e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ and $e^+e^- \to 2(\pi^+\pi^-)2\pi^0\eta$ cross sections
  Phys.Rev.D 103, 092001 (2021)

$a_\mu = \frac{1}{2}(g-2)_\mu$ precision test of SM

SM prediction for muons: $a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr}$

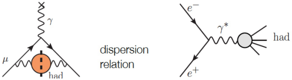
absolute value dominated by $a_\mu^{QED} + a_\mu^{EW}$

ERROR dominated by a_μ^{hadr}: not calculable perturbatively



$4.2\ \sigma$ (WP/SM) or $1.5\ \sigma$ (LQCD/SM)

a_μ^{hadr}: LQCD or data-driven dispersive approach



$K(s)$: analytically known kernel function

$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$ experimental input

relies on hadronic cross section measurements

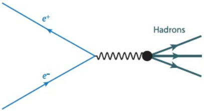
Largest contribution to the integral from hadronic cross section at low energies

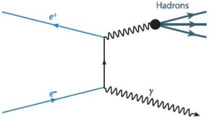
For $\sqrt{s} \lesssim 2\ GeV$ finite number of final states contribute:

$\sigma(e^+e^- \to$ hadrons) can be obtained as sum of all exclusive cross sections

BABAR hadronic cross section measurements using ISR

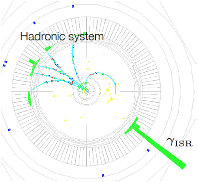
Initial State Radiation from e^+e^- allows to measure cross sections at all center-of-mass energies $\sqrt{s'}$ below the nominal $\sqrt{s}$ of the beams:





$$\frac{\mathrm{d}\sigma(s;s';\theta_\gamma)}{\mathrm{d}s'\mathrm{d}\theta_\gamma} = W(s;s';\theta_\gamma)\cdot\sigma_x(s')$$

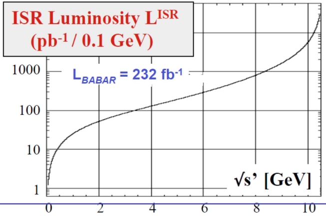
tag photon to identify ISR events



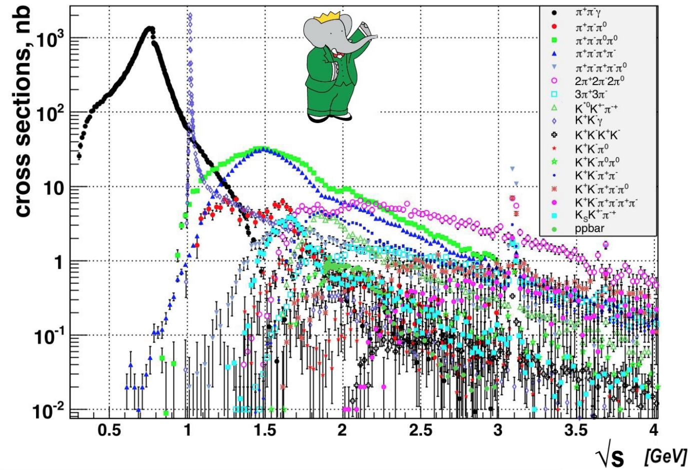
- hadrons in fiducial detector region
- fully reconstruct the final state
- kinematic fit: energy resolution

boost $\Longrightarrow$ harder momentum spectrum for daughter particles

- cross sections down to threshold
- measure σ at all $\sqrt{s}$ simultaneously
- large "effective" luminosity



Light hadrons cross sections measured by *BABAR*



Light hadrons cross sections measured in ISR by *BABAR*

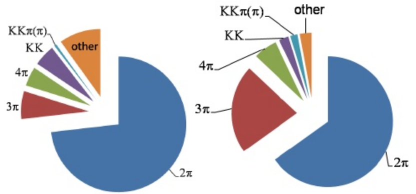
Many **first measurements**: (superseded results omitted)

$\pi^+\pi^-\pi^0$	469 fb^{-1}	Phys. Rev. D 104, 112003 (2021)
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^0\pi^0\eta$	469 fb^{-1}	Phys. Rev. D 104, 112004 (2021)
$2(\pi^+\pi^-)\pi^0\pi^0\pi^0$ and $2(\pi^+\pi^-)\pi^0\pi^0\eta$	469 fb^{-1}	Phys. Rev. D 103, 092001 (2021)
$\pi^+\pi^-\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^0\eta$	469 fb^{-1}	Phys. Rev. D 98, 112015 (2018)
$\pi^+\pi^-\eta$	469 fb^{-1}	Phys. Rev. D 97, 052007 (2018)
$\pi^+\pi^-\pi^0\pi^0$	454 fb^{-1}	Phys. Rev. D 96, 092009 (2017)
$K^0_S K^\pm\pi^\mp\pi^0$ and $K^0_S K^\pm\pi^\mp\eta$	454 fb^{-1}	Phys. Rev. D 95, 092005 (2017)
$K^0_S K^0_L\pi^0$, $K^0_S K^0_L\eta$, and $K^0_S K^0_L\pi^0\pi^0$	469 fb^{-1}	Phys. Rev. D 95, 052001 (2017)
K^+K^- (γ undetected)	469 fb^{-1}	Phys. Rev. D 92, 072008 (2015)
$K^0_S K^0_L$, $K^0_S K^0_L\pi^+\pi^-$, $K^0_S K^0_S\pi^+\pi^-$, and $K^0_S K^0_S K^+K^-$	469 fb^{-1}	Phys. Rev. D 89, 092002 (2014)
K^+K^-	232 fb^{-1}	Phys. Rev. D 88, 032013 (2013)
$p\bar{p}$	469 fb^{-1}	Phys. Rev. D 87, 092005 (2013)
$p\bar{p}$ (E_{cm} : $3.0 \div 6.5$ GeV)	469 fb^{-1}	Phys. Rev. D 88, 072009 (2013)
$\pi^+\pi^-\pi^+\pi^-$	454 fb^{-1}	Phys. Rev. D 85, 112009 (2012)
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, and $K^+K^-K^+K^-$	454 fb^{-1}	Phys. Rev. D 86, 012008 (2012)
$\pi^+\pi^-$	232 fb^{-1}	Phys.Rev.Lett. 103, 231801 (2009)
$K^+K^-\eta$, $K^+K^-\pi^0$ and $K^0_s K^\pm\pi^\mp$	232 fb^{-1}	Phys. Rev. D 77, 092002 (2008)
$\Lambda\bar{\Lambda}$, $\Lambda\bar{\Sigma}^0$, and $\Sigma^0\bar{\Sigma}^0$	230, fb^{-1}	Phys. Rev. D 76, 092006 (2007)
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$ and $K^+K^-\pi^+\pi^-\eta$	232 fb^{-1}	Phys. Rev. D 76, 092005 (2007)
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$ and $K^+K^-2(\pi^+\pi^-)$	232 fb^{-1}	Phys. Rev. D 73, 052003 (2006)

$e^+e^- \to \pi^+\pi^-\pi^0$

It gives the second largest contribution to a_μ^{HLO} and its error

Abs. contr. to a_μ^{HLO}



Error (squared) contr. a_μ^{HLO}

Previous *BABAR* measurement based on 20% of dataset available now

Cross section had been measured in $1.05 \div 3$ GeV

PRD 70, 072004 (2004)

New measurement using the whole dataset extends cross section below 1.05 GeV, in the region of ρ, ω and ϕ resonances

accuracy on a_μ^{HLO} contribution due to $e^+e^- \to \pi^+\pi^-\pi^0$ currently $\approx 3\%$

new measurement will improve accuracy to $\approx 1.5\%$

$e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$

Detect all final state particles

Select events using kinematic fit (cut on χ^2)

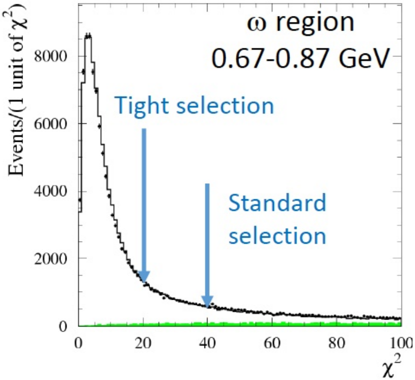
Several additional cuts reduce background by factor 2

Remaining ISR and $q\bar{q}$ background subtracted using simulation normalized to data.

Above 1.1 GeV sizeable FSR background from $e^+e^- \to a_1\gamma$, $a_2\gamma$ processes.

Estimated by pQCD with 100% uncertainty.

up to 8% contribution near 1.3 GeV

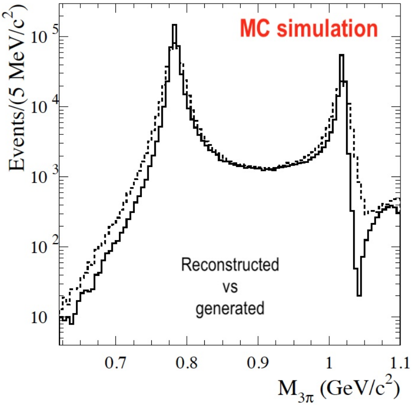


$\pi^+\pi^-\pi^0$ mass spectrum below 1.1 GeV

Below 1.1 GeV the mass spectrum has a sharp structure

unfolding required to determine true spectrum

cross section result depends on the assumed mass resolution



The ω and ϕ widths are well known

$\Longrightarrow$ use data to correct the simulated resolution function

Tails of the resolution depend on the χ^2 cut applied in selecting events:

$\Longrightarrow$ try more than one cut value

Fit to the $\pi^+\pi^-\pi^0$ mass spectrum

The mass spectrum fitted with VDM model including

$\omega(782) + \omega(1420) + \omega(1680) + \phi(1020)$ resonances

$\omega(782)$ and ϕ widths fixed to PDG average

$+$ the rare $\rho(770) \to 3\pi$ decay

For $\chi^2 < 20$ (nominal fit) the mass spectrum is well described by introducing an additional Gaussian smearing to the MC resolution function

$\sigma_s = 1.5 \pm 0.2$ MeV

$m_\omega - m_{PDG} = 0.042 \pm 0.055$ MeV

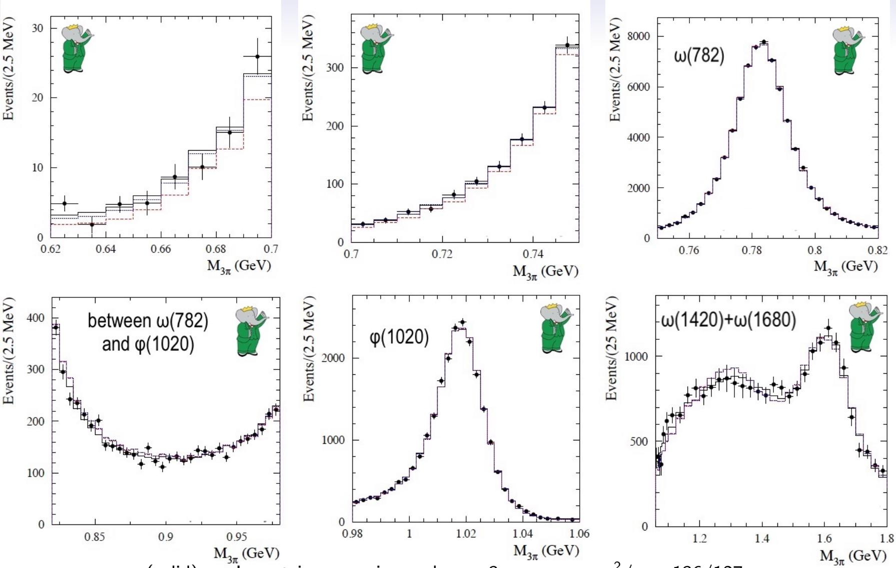
$m_\phi - m_{PDG} = 0.095 \pm 0.084$ MeV

For $\chi^2 < 40$ (cross check): additional Lorentzian smearing required to describe tails $fraction = 0.7 \pm 0.2\,\%; \quad \gamma = 63 \pm 35$ GeV

consistent results for all other parameters

The data spectrum CANNOT be adequately described with $\mathcal{B}(\rho \to 3\pi) \equiv 0$

Fits to the $\pi^+\pi^-\pi^0$ mass spectrum ($\chi^2 < 40$)



(solid): Lorentzian smearing and $\rho \to 3\pi$ $\chi^2/\nu = 136/127$
(dash): No Lorentzian smearing and no $\rho \to 3\pi$ $\chi^2/\nu = 201/131$
(dot): Lorentzian smearing and no $\rho \to 3\pi$ $\chi^2/\nu = 180/129$

Fit results on resonance parameters

For $\omega(782)$ and $\phi(1020)$ the products $\Gamma_{ee} \times \mathcal{B}_{3\pi}$ are in reasonable agreement with world average values:

$$\Gamma(\omega \to e^+e^-) \cdot \mathcal{B}(\omega \to \pi^+\pi^-\pi^0) = (0.5698 \pm 0.0031 \pm 0.0082)\ \mathrm{keV}$$

world average: $(0.557 \pm 0.011)\ \mathrm{keV}$

$$\Gamma(\phi \to e^+e^-) \cdot \mathcal{B}(\phi \to \pi^+\pi^-\pi^0) = (0.1841 \pm 0.0021 \pm 0.0080)\ \mathrm{keV}$$

world average: $(0.1925 \pm 0.0043)\ \mathrm{keV}$

The rare decay $\rho \to \pi^+\pi^-\pi^0$ is observed with significance greater than 6σ

the value and the relative phase wrt to the $\omega(782)$ amplitude are in agreement with the only previous measurement by SND

SND: Phys.Rev.D 63,07002 (2001)

$$\mathcal{B}(\rho \to \pi^+\pi^-\pi^0) = (0.88 \pm 0.23 \pm 0.30) \times 10^{-4}$$

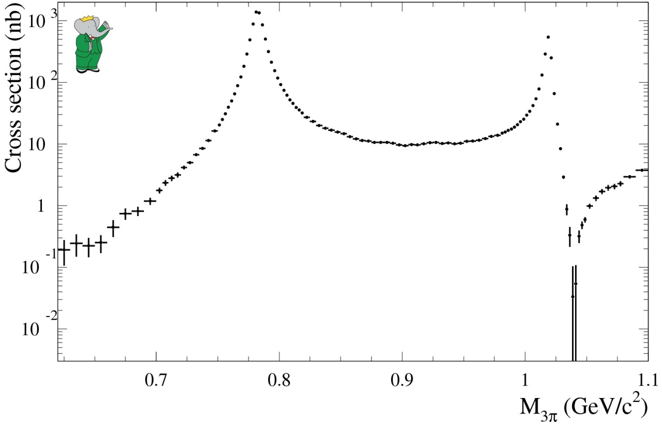
SND: $(1.01^{+0.54}_{-0.34} \pm 0.34) \times 10^{-4}$

$$\phi_\rho - \phi_\omega = -(99 \pm 9 \pm 15)^o$$

SND: $-(135^{+17}_{-13} \pm 9)^o$

$e^+e^- \to \pi^+\pi^-\pi^0$ cross section below 1.1 GeV

The parameters of the smearing function determined in the VDM fit are used to correct the simulated resolution function



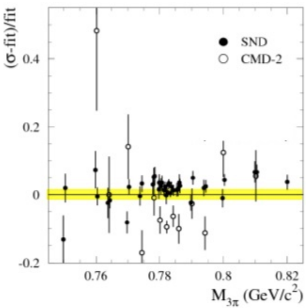
The unfolding is performed using the IDS (iterative dynamically stabilized) method (B. Malaescu, arXiv:0907.3791)

Systematic uncertainty at $\omega(782)$ and $\phi(1020)$ peak $\approx$1.3%

$e^+e^- \to \pi^+\pi^-\pi^0$ cross section below 1.1 GeV: comparison with previous measurements

SND: Phys.Rev.D 68,052006 (2003)

CMD-2: Phys.Lett.B 578,285 (2004)

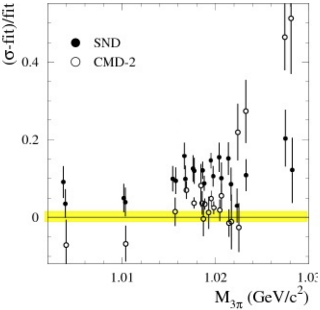


SND-*BABAR* difference $\simeq 2\%$
below syst. (3.4% SND, 1.4% *BABAR*)

CMD-2 (1.8% stat and 1.3% syst) is $\simeq 7\%$ smaller than *BABAR*
$\approx 2.7\sigma$ difference

SND: Phys.Rev.D 63,072002 (2001)

CMD-2: Phys.Lett.B 642,203 (2006)

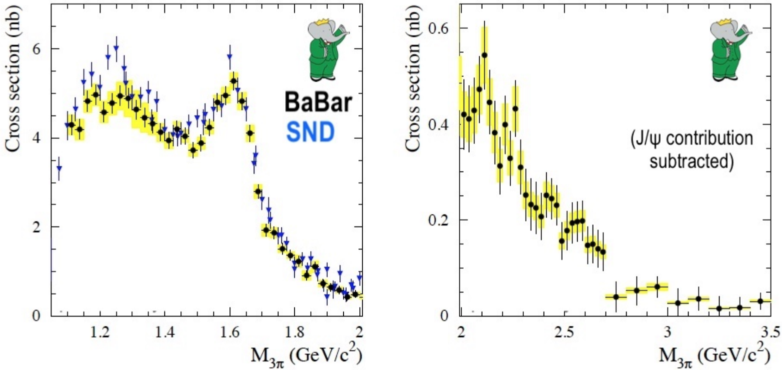


SND-*BABAR* difference $\simeq 11\%$
syst: 5% (SND); 1.4% (*BABAR*)

CMD-2-*BABAR* difference $\simeq 3\%$
syst: 2.5% (CMD-2); 1.4% (*BABAR*)

$e^+e^- \to \pi^+\pi^-\pi^0$ cross section above 1.1 GeV

No smearing is needed above 1.1 GeV. Syst. uncertainty: $4 \div 15\%$
dominated by background subtraction



Significant localized differences around 1.25 GeV and 1.5 GeV between *BABAR* and SND (Eur.Phys.J. C 80, 993 (2020))

Impact on $a_\mu^{3\pi}$

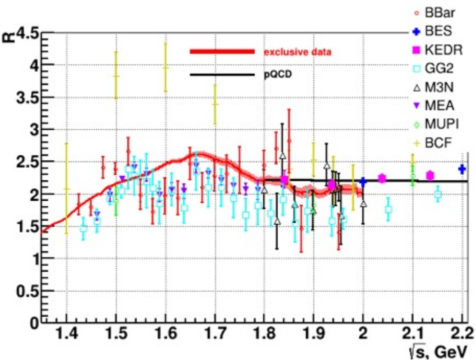
$M_{3\pi}$ GeV/c^2	$a_\mu^{3\pi} \times 10^{10}$
0.62–1.10	$42.91 \pm 0.14 \pm 0.55 \pm 0.09$
1.10–2.00	$2.95 \pm 0.03 \pm 0.16$
< 2.00	$45.86 \pm 0.14 \pm 0.58$
< 1.80[A]	$46.21 \pm 0.40 \pm 1.40$
< 1.97[B]	46.74 ± 0.94
< 2[C]	44.32 ± 1.48

[A] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur.Phys.J. C 80, 241 (2020)
[B] A. Keshavarzi, D. Nomura and T. Teubner, Phys.Rev.D 101, 014029 (2020)
[C] F. Jegerlehner, Springer Tracts Mod. Phys. 274, 1 (2017)

The value of $a_\mu^{3\pi}$ calculated using the $e^+e^- \to \pi^+\pi^-\pi^0$ cross-section is in reasonable agreement with earlier calculations

the error on this contribution is reduced by a factor ≈ 2

Is anything still missing?



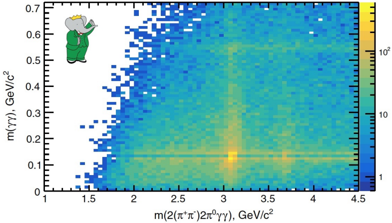
Because of large, highly segmented calorimeter BaBar has advantage for study of the multi-photon reactions over detectors at VEPP2000 (SND and CMD-3) even effective integrated luminosity is already lower. It was demonstrated for the study of the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0\pi^0\eta$ reactions Phys.Rev.D 98, 112015 (2018).

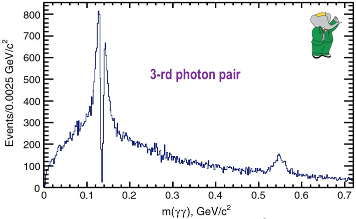
Currently, the sum of exclusive cross sections near 2.0 GeV shows a systematic deviation from the QCD predictions. BABAR, SND and CMD-3 measurements of previously unmeasured processes, e.g.
$e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0, \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\pi^0, K_S K^\pm\pi^\mp\pi^0\pi^0, K^-K^+\pi^0\pi^0\pi^0$, ...
may help to understand if this deviatiopn is real.

Phys.Rev.D 103, 092001 (2021)

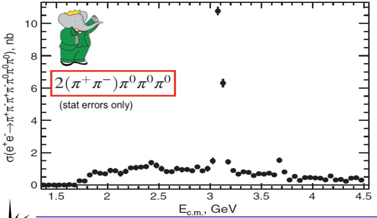
$e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ and $2(\pi^+\pi^-)2\pi^0\eta$ (first measurement)

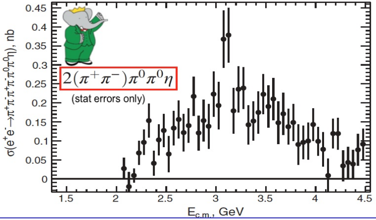
All candidate events with 4 oppositely charged tracks, one γ_{ISR} photon candidate, 2 photon pairs with $m_{\gamma\gamma}$ compatible with π^0 and a third photon pair + kinematic fit. Signal events selected based on $\chi^2 < 50$; background from χ^2 sidebands. + some additional cuts to reduce background.





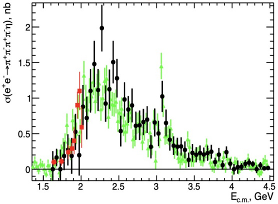
Fit the third photon pair mass distribution in bins of $m(2(\pi^+\pi^-)2\pi^0\gamma\gamma)$ to determine the $2(\pi^+\pi^-)3\pi^0$ and $2(\pi^+\pi^-)2\pi^0\eta$ yields to determine cross sections





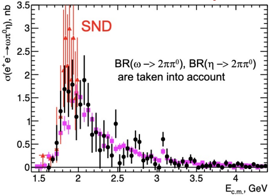
Intermediate states in $e^+e^- \to 2(\pi^+\pi^-)3\pi^0$

$e^+e^- \to \eta\pi^+\pi^-\pi^+\pi^-$



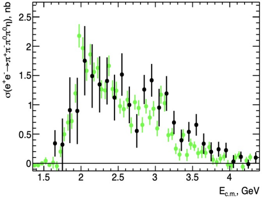
BaBar $\eta \to \gamma\gamma$ (Phys.Rev. D76 (2007) 092005)
CMD-3 $\eta \to \pi^+\pi^-\pi^0$ Phys.Lett. B792 (2019) 419-423

$e^+e^- \to \omega\pi^0\eta$



BaBar data for the $e^+e^- \to \omega\pi^0\eta(\eta \to \gamma\gamma)$ reaction
Phys.Rev. D98 (2018) no.11, 112015, and still are lower than SND measurement.

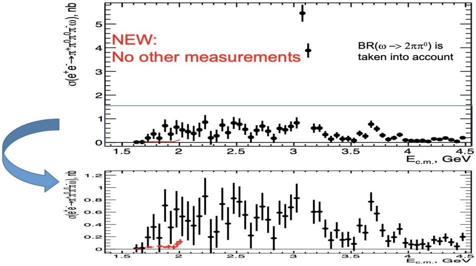
$e^+e^- \to \eta\pi^+\pi^-\pi^0\pi^0$



BaBar result for $\eta \to \gamma\gamma$ mode
Phys.Rev. D98 (2018) no.11, 112015

Good agreement with previously measured cross sections in different decay modes of ω, η

$e^+e^- \to \omega\,\pi^+\pi^-\pi^0\pi^0$



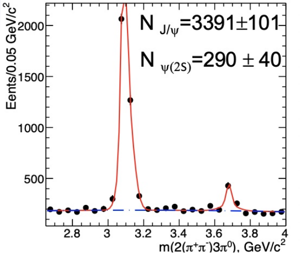
CMD-3 results are shown for the $e^+e^- \to \omega\,\pi^+\pi^-\pi^+\pi^-$ reaction
Phys.Lett. B792 (2019) 419-423

Phys.Rev.D 103, 092001 (2021)

(not shown: $\rho^\pm\pi^\mp\pi^+\pi^-2\pi^0$)

No significant evidence for other intermediate states

$e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ and $2(\pi^+\pi^-)2\pi^0\eta$: charmonium



Phys.Rev.D 103, 092001 (2021)

Measured quantity	Measured value (eV)	J/ψ or $\psi(2S)$ branching fraction (10^{-3})	
		Calculated, this work	PDG [22]
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\pi^0}$	$345.0 \pm 10.0 \pm 50.0$	$62.0 \pm 2.0 \pm 9.0$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\omega\pi^+\pi^-\pi^0\pi^0} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0}$	$165.0 \pm 9.0 \pm 25.0$	$33.0 \pm 2.0 \pm 5.0$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\eta\pi^+\pi^-\pi^0\pi^0} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0}$	$6.0 \pm 4.0 \pm 1.0$	$4.8 \pm 3.2 \pm 0.8$	2.3 ± 0.5
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\pi^+\pi^-\pi^+\pi^-\eta} \cdot \mathcal{B}_{\eta\to\pi^0\pi^0\pi^0}$	$5.6 \pm 2.6 \pm 0.8$	$2.6 \pm 1.2 \pm 0.5$	2.26 ± 0.28
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\rho^\pm\pi^\mp\pi^+\pi^-\pi^0\pi^0}$	$155.0 \pm 26.0 \pm 36.0$	$28.0 \pm 4.7 \pm 6.6$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\rho^+\rho^-\pi^+\pi^-\pi^0}$	$32.0 \pm 13.0 \pm 15.0$	$5.7 \pm 2.4 \pm 2.7$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi\to\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\eta} \cdot \mathcal{B}_{\eta\to\gamma\gamma}$	$9.1 \pm 2.6 \pm 1.4$	$4.2 \pm 1.2 \pm 0.6$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S)\to\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\pi^0}$	$33.0 \pm 5.0 \pm 5.0$	$14.0 \pm 2.0 \pm 2.0$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S)\to J/\psi\pi^0\pi^0} \cdot \mathcal{B}_{J/\psi\to\pi^+\pi^-\pi^+\pi^-\pi^0}$	$14.8 \pm 2.6 \pm 2.2$	$34.7 \pm 6.1 \pm 5.2$	33.7 ± 2.6
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S)\to J/\psi\pi^+\pi^-} \cdot \mathcal{B}_{J/\psi\to\pi^+\pi^-\pi^0\pi^0\pi^0}$	$19.2 \pm 4.5 \pm 3.2$	$23.8 \pm 5.6 \pm 3.6$	27.1 ± 2.9
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S)\to\omega\pi^+\pi^-\pi^0\pi^0} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0}$	$18.0 \pm 4.0 \pm 3.0$	$8.7 \pm 1.9 \pm 1.5$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S)\to\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\eta} \cdot \mathcal{B}_{\eta\to\gamma\gamma}$	<1.9 at 90% C.L.	<2.0 at 90% C.L.	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S)\to\pi^+\pi^-\pi^+\pi^-\eta} \cdot \mathcal{B}_{\eta\to\pi^0\pi^0\pi^0}$	<2.3 at 90% C.L.	<2.4 at 90% C.L.	1.2 ± 0.6

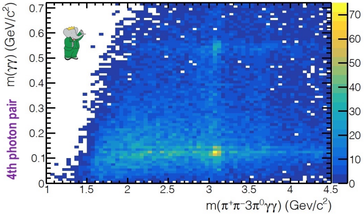
Phys.Rev.D 104, 112004 (2021)

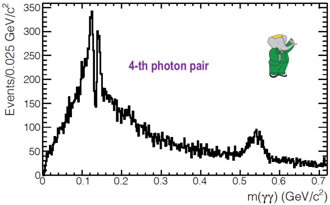
$e^+e^- \to \pi^+\pi^-4\pi^0$ and $\pi^+\pi^-3\pi^0\eta$ (first measurement)

Events with 2 oppositely charged tracks, one γ_{ISR} photon candidate, 3 photon pairs with $m_{\gamma\gamma}$ compatible with π^0 and a fourth photon pair

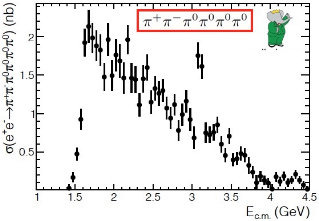
Signal events selected based on $\chi^2 < 70$; background from χ^2 sidebands

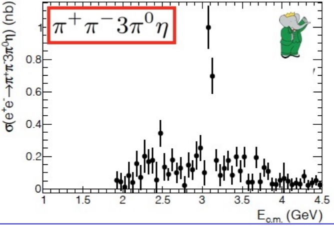
some additional cuts to reduce background





Fit the fourth photon pair invariant mass distribution in bins of $m(\pi^+\pi^-3\pi^0\gamma\gamma)$ to determine the $\pi^+\pi^-4\pi^0$ and $\pi^+\pi^-3\pi^0\eta$ yields to determine cross sections





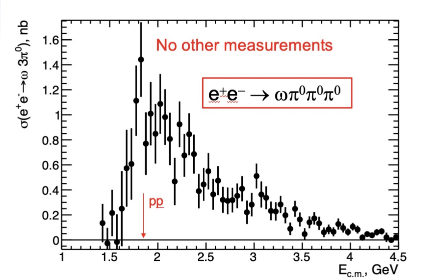
Phys.Rev.D 104, 112004 (2021)

Intermediate states in $e^+e^- \to \pi^+\pi^-4\pi^0$

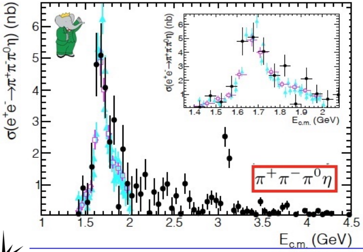
Intermediate states: $\pi^+\pi^-\pi^0\eta$ $\omega\eta$ $\omega 3\pi^0$ $(\rho\pi)3\pi^0$

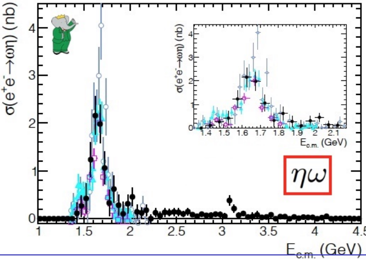
and possibly $\rho^+\rho^-2\pi^0$ above 2.9 GeV

Sum of intermediate states seem to saturate the observed cross section



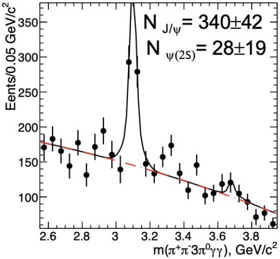
Below 2 GeV agreement with SND and CMD-2 measurements of $\pi^+\pi^-\pi^0\eta$ and $\omega\eta$:





$e^+e^- \to \pi^+\pi^-4\pi^0$ and $\pi^+\pi^-3\pi^0\eta$: charmonium

Phys.Rev.D 104, 112003 (2021)



Measured Quantity	Measured Value (eV)	J/ψ or $\psi(2S)$ Branching Fraction (10^{-3}) Derived, this work	PDG [29]
$\Gamma_{ee}^{J/\psi}\cdot B_{J/\psi\to\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0}$	35.8±4.4±5.4	6.5±0.8±1.0	no entry
$\Gamma_{ee}^{J/\psi}\cdot B_{J/\psi\to\eta\pi^+\pi^-\pi^0}\cdot B_{\eta\to\pi^0\pi^0\pi^0}$	21.1±1.7±3.2	11.9±0.9±2.3	no entry
$\Gamma_{ee}^{J/\psi}\cdot B_{J/\psi\to\omega\eta}\cdot B_{\omega\to\pi^+\pi^-\pi^0}\cdot B_{\eta\to\pi^0\pi^0\pi^0}$	4.9±2.1±0.7	3.0±1.3±0.5	1.74±0.20
$\Gamma_{ee}^{J/\psi}\cdot B_{J/\psi\to\omega\pi^0\pi^0\pi^0}\cdot B_{\omega\to\pi^+\pi^-\pi^0}$	9.4±2.3±1.5	1.9±0.5±0.3	no entry
$\Gamma_{ee}^{J/\psi}\cdot B_{J/\psi\to\pi^+\pi^-\pi^0\pi^0\pi^0\eta}\cdot B_{\eta\to\gamma\gamma}$	10.6±1.6±1.6	4.9±0.8±0.8	no entry
$\Gamma_{ee}^{\psi(2S)}\cdot B_{\psi(2S)\to\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0}$	3.3±2.3±0.5	1.4±1.0±0.2	no entry
$\Gamma_{ee}^{\psi(2S)}\cdot B_{\psi(2S)\to\eta\pi^+\pi^-\pi^0}\cdot B_{\eta\to\pi^0\pi^0\pi^0}$	<3. 0 at 90% C.L.	<3. 5 at 90% C.L.	no entry
$\Gamma_{ee}^{\psi(2S)}\cdot B_{\psi(2S)\to\omega\eta}\cdot B_{\omega\to\pi^+\pi^-\pi^0}\cdot B_{\eta\to\pi^0\pi^0\pi^0}$	<1. 1 at 90% C.L.	<1. 4 at 90% C.L.	<0. 11 at 90% C.L.
$\Gamma_{ee}^{\psi(2S)}\cdot B_{\psi(2S)\to\omega\pi^0\pi^0\pi^0}\cdot B_{\omega\to\pi^+\pi^-\pi^0}$	<1. 6 at 90% C.L.	<0. 8 at 90% C.L.	no entry
$\Gamma_{ee}^{\psi(2S)}\cdot B_{\psi(2S)\to\pi^+\pi^-\pi^0\pi^0\pi^0\eta}\cdot B_{\eta\to\gamma\gamma}$	<1. 9 at 90% C.L.	<2. 0 at 90% C.L.	no entry

Conclusions

New measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section Phys.Rev.D 104, 112003 (2021)

- based on the entire BABAR dataset
- measured in the range $0.62 \div 3.5$ GeV
- 1.3% systematic uncertainty near the maxima of $\omega(782)$ and $\phi(1020)$
- the error on the leading order contribution to muon magnetic anomaly from $e^+e^- \to \pi^+\pi^-\pi^0$ ($E < 2$ GeV) reduced by a factor ≈ 2

First measurements of $e^+e^- \to \pi^+\pi^-4\pi^0$ and $e^+e^- \to \pi^+\pi^-3\pi^0\eta$ cross sections

Phys.Rev.D 104, 112004 (2021)

- The $e^+e^- \to \pi^+\pi^-4\pi^0$ cross section seems to be saturated by intermediate states: $\pi^+\pi^-\pi^0\eta$, $\omega 3\pi^0$, $(\rho\pi)3\pi^0$ and possibly $\rho^+\rho^-2\pi^0$ intermediate states
- All possible combinations for the $e^+e^- \to 6\pi$ cross section have been measured by BABAR- iso-spin relations are not needed for the HVP calculation.
- new J/ψ and $\psi(2S)$ decay modes

First measurements of $e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ and $e^+e^- \to 2(\pi^+\pi^-)2\pi^0\eta$ cross sections

Phys.Rev.D 103, 092001 (2021)

- The $e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ cross section seems to be saturated by intermediate states: $2(\pi^+\pi^-)\eta$, $\omega\pi^0\eta$, $\rho^\pm\pi^\mp\pi^+\pi^-2\pi^0$, $\eta\pi^+\pi^-2\pi^0$, $\omega\pi^+\pi^-2\pi^0$
- new J/ψ and $\psi(2S)$ decay modes

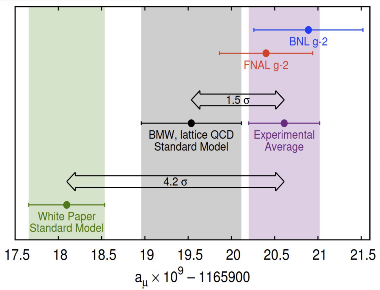
BACKUP

$a_\mu = \frac{1}{2}(g-2)_\mu$ precision test of SM

SM prediction for muons: $a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr}$

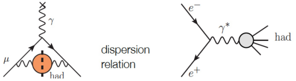
absolute value dominated by $a_\mu^{QED} + a_\mu^{EW}$

ERROR dominated by a_μ^{hadr}: not calculable perturbatively



$4.2\ \sigma$ (WP/SM) or $1.5\ \sigma$ (LQCD/SM)

a_μ^{hadr}: LQCD or data-driven dispersive approach



$K(s)$: analytically known kernel function

$$a_\mu^{\rm had,LO} = \frac{\alpha^2(0)}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$ experimental input

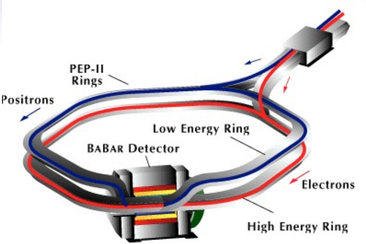
relies on hadronic cross section measurements

Largest contribution to the integral from hadronic cross section at low energies

For $\sqrt{s} \lesssim 2\ GeV$ finite number of final states contribute:

$\sigma(e^+e^- \to \text{hadrons})$ can be obtained as sum of all exclusive cross sections

The *BABAR* experiment



PEP-II asymmetric e^+e^- collider operating at center of mass energies near the $\Upsilon(4S)$ (for most of the time)

$$\sqrt{s} = 10.58\,\mathrm{GeV}/c^2$$

Asymmetric detector:

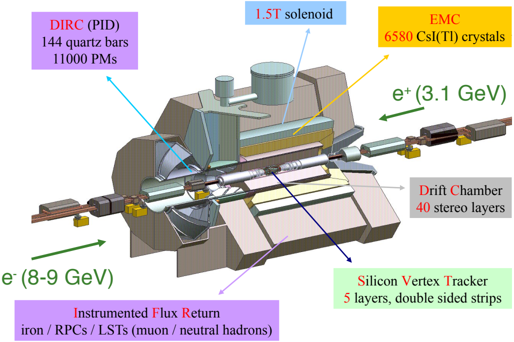
$$-0.9 < \cos\theta^* < 0.85$$

wrt electron beam

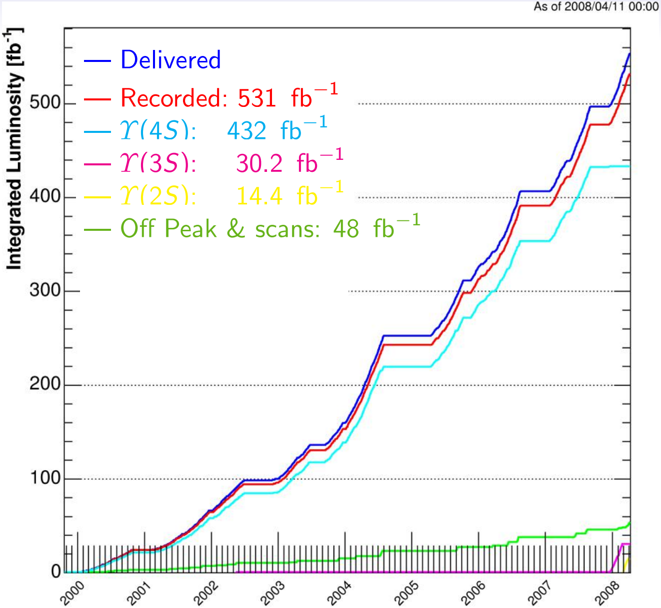
excellent performance:

- vertexing
- tracking
- PID
- calorimeter

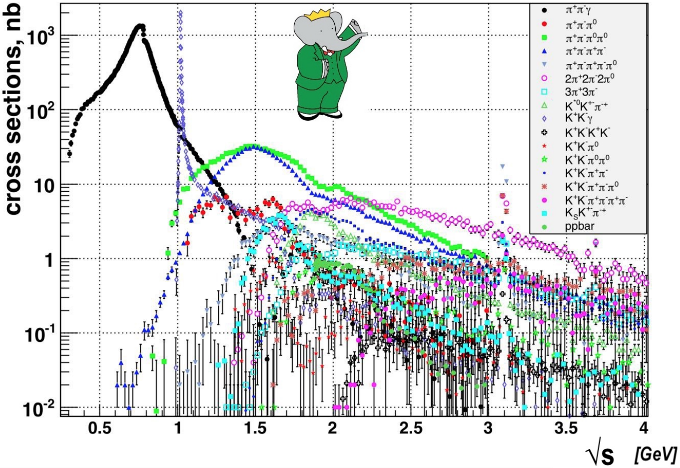
General-purpose detector



Data samples



Light hadrons cross sections measured by *BABAR*



Substructures in $e^+e^- \to \pi^+\pi^-4\pi^0$