On the possible molecular nature of the $X_0(2866)$ and the $Z_{cs}(3985)$ states

R. Molina, T. Branz, N. Ikeno, L.R. Dai and E. Oset

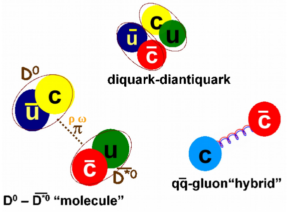
Table of contents

Intro

Introduction

Since the X(3872) many exotics discovered ...

- $Z_c(3900)$, BESIII, 2013
  close to $D\bar{D}^*$, $c\bar{q}q\bar{c}$ $(q = u, d)$
- $Z_{cs}(3985)$, BESIII, 2021
  close to $\bar{D}_s^* D/\bar{D}_s D^*$, $c\bar{q}s\bar{c}$
- $X_0(2866), X_1(2900)$, LHCb, 2020
  close to $D^*\bar{K}^*$, $c\bar{q}s\bar{q}$
- $T_{cc}(3875)$, LHCb, 2021
  close to DD^*, $c\bar{q}c\bar{q}$



$\Longrightarrow$ Do not fit into $q\bar{q}$ basic mesons of the quark model predictions

... Are the meson-meson molecules? tetraquarks?

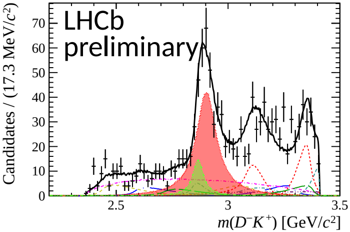
... Dynamics of the interaction?

New flavor exotic tetraquark ($C = -1; S = 1$)

LHCb (2020)

Two states $J^P = 0^+, 1^-$ decaying to $\bar{D}K$. First clear example of an heavy-flavor exotic tetraquark, $\sim \bar{c}\bar{s}ud$.

$$X_0(2866) : M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9\,\text{MeV},$$
$$X_1(2900) : M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5\,\text{MeV}.$$



R. Aaij et al. (LHCb Collaboration), PRL125(2020), PRD102(2020)

Flavour exotic states

- 2010. Prediction of several flavour exotic states

PHYSICAL REVIEW D **82,** 014010 (2010)

New interpretation for the $D^*_{s2}(2573)$ and the prediction of novel exotic charmed mesons

R. Molina,[1] T. Branz,[2] and E. Oset[1]

[1]*Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Apartado 22085, 46071 Valencia, Spain*

[2]*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

(Received 4 May 2010; published 21 July 2010)

In this manuscript we study the vector-vector interaction within the hidden-gauge formalism in a coupled channel unitary approach. In the sector $C = 1$, $S = 1$, $J = 2$ we get a pole in the T matrix around 2572 MeV that we identify with the $D^*_{s2}(2573)$, coupling strongly to the $D^*K^*(D^*_s\phi(\omega))$ channels. In addition we obtain resonances in other exotic sectors which have not been studied before such as $C = 1$, $S = -1$, $C = 2$, $S = 0$ and $C = 2$, $S = 1$. These "flavor-exotic" states are interpreted as $D^*\bar{K}^*$, D^*D^*, and $D^*_sD^*$ molecular states but have not been observed yet. In total we obtain nine states with different spin, isospin, charm, and strangeness of non-$C = 0$, $S = 0$ and $C = 1$, $S = 0$ character, which have been reported before.

DOI: 10.1103/PhysRevD.82.014010 PACS numbers: 14.40.Rt, 12.40.Vv, 13.75.Lb, 14.40.Lb

- Free parameter fixed with $D_{s2}(2573)$; couples to D^*K^*, $c\bar{q}q\bar{s}$
- Flavour exotic states with $I = 0$, $J^P = \{0, 1, 2\}^+$ coupling to $D^*\bar{K}^*$ are predicted, $c\bar{q}s\bar{q}$
- Doubly charm states, $I = 0$; $J^P = 1^+$, close to D^*D^* are predicted, $c\bar{q}c\bar{q}$, and $I = 1/2$; $J^P = 1^+$, close to $D^*D^*_s$ $c\bar{q}c\bar{s}$

Flavour exotic states

Molina,Branz,Oset, PRD82(2010)

C, S	Channels	$I[J^P]$	$\sqrt{s}$	$\Gamma_A(\Lambda = 1400)$	$\Gamma_B(\Lambda = 1200)$	State	$\sqrt{s}_{exp}$	Γ_{exp}
$1, -1$	$D^* \bar{K}^*$	$0[0^+]$	2848	23	59	$X_0(2866)$	2866	57
		$0[1^+]$	2839	3	3			
		$0[2^+]$	2733	11	36			
$1, 1$	$D^* K^*, D_s^* \omega$	$0[0^+]$	2683	20	71			
	$D_s^* \phi$	$0[1^+]$	2707	4×10^{-3}	4×10^{-3}			
		$0[2^+]$	2572	7	23	$D_{s2}(2573)$	2572.6 ± 0.9	20 ± 5
$1, 1$	$D^* K^*, D_s^* \rho$	$1[2^+]$	2786	8	11			
$2, 0$	$D^* D^*$	$0[1^+]$	3969	0	0			
$2, 1$	$D^* D_s^*$	$1/2[1^+]$	4101	0	0			

Table 1: Summary of the nine states obtained. The width is given for the model A, Γ_A, and B, Γ_B. All the quantities here are in MeV.

Form factors in the $D^* D\pi$ vertex; Model A: $F_1(q^2) = \frac{\Lambda_b^2 - m_\pi^2}{\Lambda_b^2 - q^2}$, Titov, Kampfer EPJA7, PRC65 with $\Lambda_b = 1.4, 1.5$ GeV and $g = M_\rho / 2 f_\pi$. Model B: $F_2(q^2) = e^{q^2/\Lambda^2}$ Navarra, Nielsen, Bracco PRD65 (2002), $\Lambda = 1, 1.2$ GeV and $g_D = g_{D^* D\pi}^{\rm exp} = 8.95$ (experimental value). Subtraction constant $\alpha = -1.6$.

Many studies appeared after these discoveries ...

- He, Wang, Zhu, EPJC80, 1026 (2020), Karliner, Rosner, PRD102(2020), $X_0(2866)$, compact tetraquark
- X. H. Liu, Yan et al., EPJC80(2020), $X_0(2866)$, Triangle Singularity
- M. Z. Liu, Xie, Geng, PRD102(2020), $X_0(2866)$, $D^*\bar{K}^*$ molecule (one-boson ex.), $X_1(2900)$ cannot be, Qi, Wang et al. EPJC81(2021), X_1 is a $\bar{D}_1 K$ molecule (ρ, ω ex.)
- Ge, X. H. Liu, EPJC81(2021), $Z_{cs}(4000)$, $X(4700)$, threshold effects
- Du, Albaladejo, F. K. Guo, Nieves, 2201.08253, energy dep. interaction, $Z_{cs}(3985)$ and $Z_c(3900)$ are SU(3) partners, $D^*\bar{D}_{(s)}/D\bar{D}^*_{(s)}$
- Du, Baru, Dong, Filin, Nieves, F. K. Guo, T_{cc}, PRD105, (2022), 3-body dynamics, $D^0D^0\pi^+$, contact+OPE, DD^* molecule
- Albaladejo, T_{cc} from DD^*, can have $I = 0$ or 1
- Feijoo, Liang, Oset, PRD104(2021), T_{cc} as DD^*, has $I = 0$, decay width to $D^0D^0\pi^+ \sim 43\ \mathrm{MeV}$
- Padmanath, Prelovsek, virtual s-wave bound state for $m_\pi = 280$ MeV of DD^* in LatticeQCD ...

The Local Hidden Gauge Approach

The hidden gauge formalism Bando,Kugo,Yamawaki

Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \tag{1}$$

$$\begin{aligned} \mathcal{L}^{(2)} &= \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \\ \mathcal{L}_{III} &= -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle + \frac{1}{2} M_V^2 \langle [V_\mu - \frac{i}{g} \Gamma_\mu]^2 \rangle \end{aligned} \tag{2}$$

$$D_\mu U = \partial_\mu U - ieQA_\mu U + ieUQA_\mu, \qquad U = e^{i\sqrt{2}P/f} \tag{3}$$

$$\begin{aligned} \mathcal{L}_{V\gamma} &= -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q \rangle \\ \mathcal{L}_{VPP} &= -ig \langle V^\mu [P, \partial_\mu P] \rangle; \quad g = M_V / 2f \\ \widetilde{\mathcal{L}}^{(2)} &= \frac{1}{12 f^2} \langle [P, \partial_\mu P]^2 + M P^4 \rangle . \end{aligned} \tag{4}$$

Local Hidden Gauge Approach
Vector-vector scattering

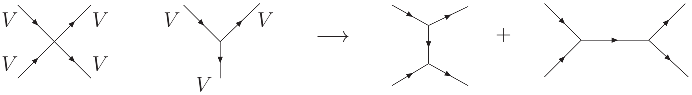
$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle \longrightarrow \mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle$$

$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu\rangle$$

$$\boldsymbol{V_{\mu\nu}} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]$$

$$g = \frac{M_V}{2f}$$

$$\boldsymbol{V_\mu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$



a) b) c) d)

Local Hidden Gauge Approach

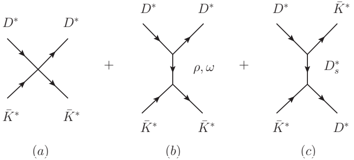


Figure 1: The $D^*\bar{K}^* \to D^*\bar{K}^*$ interaction at the tree level; (a) contact term; (b) exchange of light vectors; (c) exchange of a heavy vector.

Approximation

$$\epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (|\vec{k}|,0,0,k^0)/m$$

$$k^\mu = (k^0,0,0,|\vec{k}|)$$

$$\vec{k}/m \simeq 0,\ k_j^\mu \epsilon_\mu^{(l)} \simeq 0$$

$$\epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (0,0,0,1)$$

$$\mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle = ig\langle[V_\mu, \partial_\nu V_\mu]V^\nu\rangle$$

Spin projectors

$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu; \quad \mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu - \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu)$$

$$\mathcal{P}^{(2)} = \{\frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu + \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu) - \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu\} \ .$$

The $X_0(2866)$

Local Hidden Gauge Approach

Potential V: contact + vector-meson exchange (ρ, ω)

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-9.9g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-10.2g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-15.9g^2$

Table 2: Tree level amplitudes for $D^*\bar{K}^*$ in $I = 0$. Last column: $(V_{\rm th.})$.

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.7g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.9g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$15.7g^2$

Table 3: Tree level amplitudes for $D^*\bar{K}^*$ in $I = 1$. Last column: $(V_{\rm th.})$.

The interaction is attractive for $I = 0$ and repulsive for $I = 1$.

New flavor exotic tetraquark ($C=1, S=-1$)

Two-meson loop function

$$G_i(s) = \frac{1}{16\pi^2}\left(\alpha + \mathrm{Log}\frac{M_1^2}{\mu^2} + \frac{M_2^2 - M_1^2 + s}{2s}\mathrm{Log}\frac{M_2^2}{M_1^2} + \frac{p}{\sqrt{s}}\left(\mathrm{Log}\frac{s - M_2^2 + M_1^2 + 2p\sqrt{s}}{-s + M_2^2 - M_1^2 + 2p\sqrt{s}} + \mathrm{Log}\frac{s + M_2^2 - M_1^2 + 2p\sqrt{s}}{-s - M_2^2 + M_1^2 + 2p\sqrt{s}}\right)\right),$$

Bethe-Salpeter

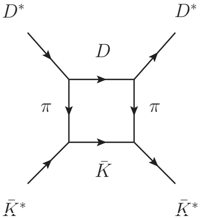
$$\boxed{T = [\hat{1} - VG]^{-1} V}$$

The states with $J^P = \{0, 2\}^+$ decay into $D\bar{K}$

$$\mathcal{L}_{VPP} = -ig\langle [P, \partial_\mu P] V^\mu \rangle$$

$F(q) = e^{((p_1^0 - q^0)^2 - \vec{q}^{\,2})/\Lambda^2}$ Navarra, PRD65(2002)

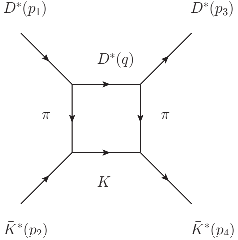
with $q_0 = (s + m_D^2 - m_K^2)/2\sqrt{s}$.



New flavor exotic tetraquark ($C = 1, S = -1$)

Recent work: Molina, Oset PLB811 2020, $\alpha = -1.474$, $\Lambda = 1300$.
Evaluation of the decay width of the $J^P = 1^+$ state

$$\mathcal{L} = \frac{iG'}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle\delta_\mu V_\nu \delta_\alpha V_\beta P\rangle$$
$$\mathcal{L}_{VPP} = -ig\langle[P, \partial_\mu P]V^\mu\rangle$$



Amplitude:

$$-it = \frac{9}{2}(G' g m_{D^*})^2 \int \frac{d^4q}{(2\pi)^4}\epsilon^{ijk}\epsilon^{i'j'k'}\left(\frac{1}{(p_1-q)^2 - m_\pi^2 + i\epsilon}\right)^2$$
$$\times \frac{1}{q^2 - m_{D^*}^2 + i\epsilon}\frac{1}{(p_1+p_2-q)^2 - m_K^2 + i\epsilon}$$
$$\times \epsilon^{j(1)}\epsilon^{m(2)}\epsilon^{k(3')}q^i q^m \epsilon^{j'(1)}\epsilon^{m'(4)}\epsilon^{k'(3')}q^{i'}q^{m'}F^4(q) \qquad (5)$$

Decay of the $X_0(2900)$ to $D^*\bar{K}$

Taking now into account that,

$$\int \frac{d^3q}{(2\pi)^3} f(\vec{q}^{\,2}) q^i q^m q^{i'} q^{m'} = \frac{1}{15} \int \frac{d^3q}{(2\pi)^3} f(\vec{q}^{\,2}) \vec{q}^{\,4} (\delta_{im}\delta_{i'm'} + \delta_{ii'}\delta_{mm'} + \delta_{im'}\delta_{m'i}) \;,$$

one obtains,

$$4\epsilon^{j(1)}\epsilon^{m(2)}\epsilon^{j(3)}\epsilon^{m(4)} - \epsilon^{j(1)}\epsilon^{j(2)}\epsilon^{m(3)}\epsilon^{m(4)} - \epsilon^{j(1)}\epsilon^{m(2)}\epsilon^{m(3)}\epsilon^{j(4)} \;,$$

which is a combination of the spin projectors, $5\mathcal{P}^{(1)} + 3\mathcal{P}^{(2)}$, zero component for $J = 0$ (violates parity). Taking q on-shell,

$$\boxed{\mathrm{Im}\,t = -\frac{3}{2}\frac{1}{8\pi}(G' g m_{D^*})^2 q^5 \left(\frac{1}{(m_D^* - \omega^*(q))^2 - \omega^2(q)}\right)^2 \frac{1}{\sqrt{s}} F^4(q)}$$

$$\omega(q) = \sqrt{m_K^2 + \vec{q}^{\,2}};\, \omega^*(q) = \sqrt{m_{D^*}^2 + \vec{q}^{\,2}};\, q = \frac{\lambda^{1/2}(s, m_{D^*}^2, m_K^2)}{2\sqrt{2}}$$

Decay of the $X_0(2900)$ to $D^*\bar{K}$

$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$X_0(2866)$

Table 4: New results including the width of the D^*K channel.

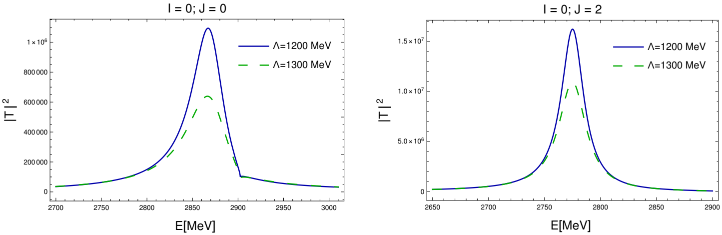


Figure 2: $|T|^2$ for $C = 1, S = -1, I = 0, J = 0$ and $J = 2$.

Decay of the $X_0(2900)$ to $D^*\bar{K}$

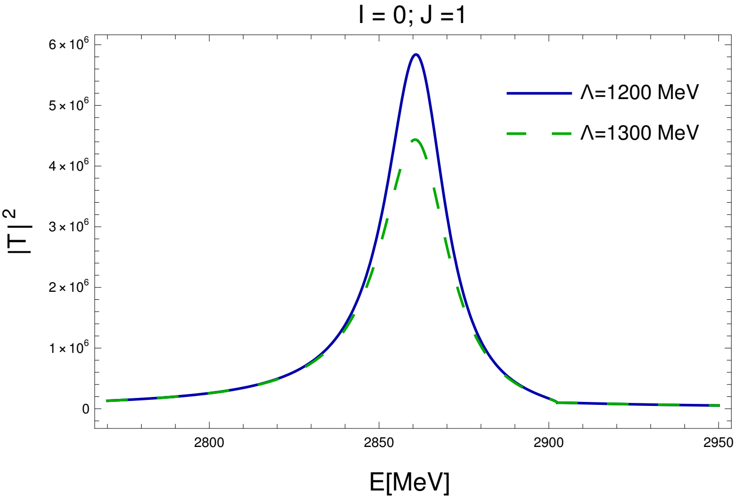


Figure 3: $|T|^2$ for $C = 1, S = -1, I = 0, J = 0$ and $J = 1$.

How can we observe the $J^P = 1^+$ state?

Amo Sanchez et al. (BABAR), PRD83(2011).

The $\bar{B}^0 \to D^{*+}\bar{D}^{*0}K^-$ reaction:

- It proceeds via external emission (favoring the decay)
- It has the largest branching fraction (1.06%)
- It can produce the $D^{*+}K^-$ in $I = 0$ (decay mode of the 1^+ state).

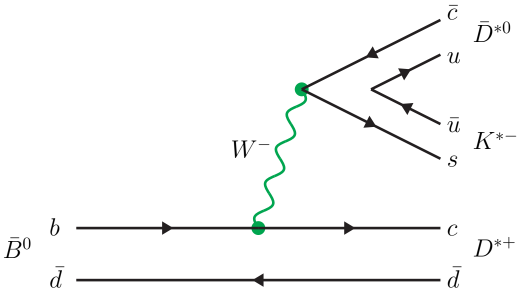


Figure 4: Diagrammatic decay of the $\bar{B}^0 \to \bar{D}^{*0}D^{*+}K^{*-}$ at the quark level.

How can we observe the $J^P = 1^+$ state?

Hadronization + decay

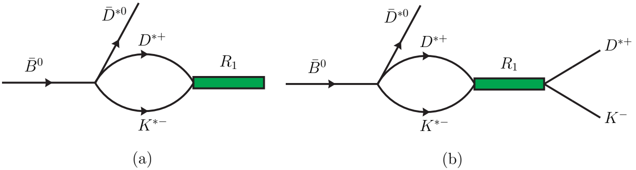


Figure 5: (a) Rescattering of $D^{*+}K^{*-}$ to give the resonance R_1 of $I = 0, J^P = 1^+$; (b) Further decay of R_1 to $D^{*+}K^-$.

$$|D^*\bar{K}^*; I = 0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}K^{*-} + D^{*0}\bar{K}^{*0})\,. \tag{6}$$

$\bar{B}^0 \to \bar{D}^{*0}D^{*+}K^{*-}$ vertex: $(1)\bar{D}^{*0}$, $(2)D^{*+}$, $(3)K^{*-}$

$$(s - wave) \quad -i\,t = -i\,C\,\boldsymbol{\epsilon}^{(1)} \cdot (\boldsymbol{\epsilon}^{(2)} \times \boldsymbol{\epsilon}^{(3)}) = -i\,C\,\epsilon_{ijk}\epsilon_i^{(1)}\epsilon_j^{(2)}\epsilon_k^{(3)}$$

$R_I \to VV$ vertex: $-\frac{1}{\sqrt{2}}g_{R,D^*\bar{K}^*}\mathcal{P}^{(J=1)}$; $\quad \mathcal{P}^{(J=1)} = \frac{1}{2}(\epsilon_i^{(2)}\epsilon_j^{(3)} - \epsilon_j^{(2)}\epsilon_i^{(3)})$

How can we observe the $J^P = 1^+$ state?

$$\sum_{pol} \epsilon_i^{(1)} \epsilon_{ii'j'} \epsilon_m^{(1)} \epsilon_{mi'j'} = \epsilon_{ii'j'} \epsilon_{ii'j'} = \delta_{ii'} \delta_{jj'} - \delta_{i'j'} \delta_{j'i'} = 9 - 3 = 6$$

$$\sum_{pol} |t'|^2 = \frac{6}{4} C^2 |g_{R_1, D^* \bar{K}^*}|^2 |G_{D^* \bar{K}^*}(M_{\rm inv})|^2 |g_{R_1, D^* \bar{K}}|^2 \left| \frac{1}{M_{\rm inv}^2(R_1) - M_{R_1}^2 + i M_{R_1} \Gamma_{R_1}} \right|^2$$

with $M_{\rm inv}^2 = (P_{D^{*+}} + P_{K^-})^2$. The effective $|g_{R_1, D^* \bar{K}}|^2$ coupling is obtained from the $R_1 \to D^* \bar{K}$ width.

$$\boxed{\frac{d\Gamma}{dM_{\rm inv}(D^{*+}K^-)} = \frac{1}{(2\pi)^3} \frac{1}{4 M_{\bar{B}^0}^2} p_{\bar{D}^{*0}} \tilde{p}_{K^-} \sum |t'|^2} \quad (7)$$

where $\tilde{p}_{K^-} = \frac{\lambda^{1/2}(M_{\rm inv}^2(D^{*+}K^-), m_{D^*}^2, m_{\bar{K}}^2)}{2 M_{\rm inv}(D^{*+}K^-)}$, $p_{\bar{D}^{*0}} = \frac{\lambda^{1/2}(M_{\bar{B}^0}^2, m_{\bar{D}^{*0}}^2, M_{\rm inv}^2(D^{*+}K^-))}{2 M_{\bar{B}^0}}$.

Background for $\bar{B}^0 \to \bar{D}^{*0} D^{*+} K^-$ $\quad -it = -iC \boldsymbol{\epsilon}(D^{*0}) \cdot \boldsymbol{\epsilon}(D^{*+})$

$$\frac{d\Gamma_{\rm bac}}{dM_{\rm inv}(D^{*+}K^-)} = \frac{1}{(2\pi)^3} \frac{1}{4 M_{\bar{B}^0}^2} p_{\bar{D}^{*0}} \tilde{p}_{\bar{K}} \, 3 \, C^2 \quad (8)$$

How can we observe the $J^P = 1^+$ state?

Dai,Molina and Oset, arXiv:2202.00508 (2022)

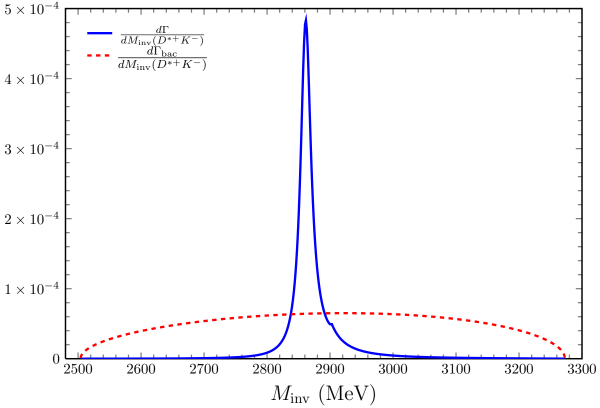


Figure 6: $\frac{d\Gamma}{dM_{\rm inv}}$ for the R_1 production versus the background, $\frac{d\Gamma_{\rm bac}}{dM_{\rm inv}}$, in the $\bar{B}^0 \to \bar{D}^{*0} D^{*+} K^{*-}$ reaction in a global arbitrary normalization. $M_{\rm inv}$ is the invariant mass of $D^{*+}K^-$. $\mathcal{B}_R(R_1; R_1 \to D^{*+}K^-) = 4.24 \times 10^{-3}$.

How can we observe the $J^P = 1^+$ state?

Similar results for the $\bar{B}^0 \to D^{*+}K^{*-}K^{*0} \to R_1 K^{*0} \to D^{*+}K^- K^{*0}$ process. Dai, Molina and Oset, arXiv:2202.11973(2022)

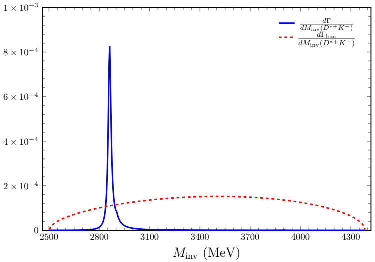


Figure 7: $\frac{d\Gamma}{dM_{\rm inv}}$ for R_1 production and $\frac{d\Gamma_{\rm bac}}{dM_{\rm inv}}$ for background in the $\bar{B}^0 \to K^{*0}D^{*+}K^-$ reaction in global arbitrary units versus the $D^{*+}K^-$ invariant mass.

The $X_0(2866)$ can also be seen in:

$\bar{B}^0 \to K^0 D^{*+}K^{*-} \to K^0 X_0 \to K^0 D^+ K^-$ arXiv:2202.11973

$B^- \to D^- D^{*+}K^{*-} \to D^- X_0 \to D^- D^+ K^-$ arXiv: 2202.00508

The $Z_{cs}(3985)$

Hidden-charm strange state

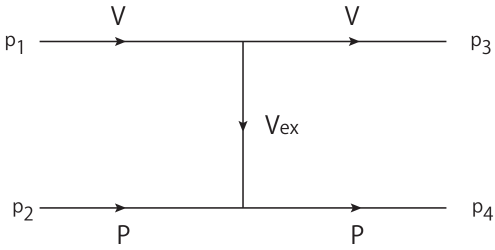
BESIII (2020) has reported a state, $Z_{cs}(3985)$, from the $D_s^{*-}D^0$, $D_s^- D^{*0}$ invariant mass distribution of $e^+e^- \to K^+(D_s^{*-}D^0 + D_s^- D^{*0})$.

$$M = 3982.5^{+1.8}_{-2.6} \pm 2.1 \text{ MeV}, \quad \Gamma = 12.8^{+5.3}_{-4.4} \pm 3.0 \text{ MeV}. \qquad (9)$$

7 MeV above the $D_s^{*-}D^0$, $D_s^- D^{*0}$ threshold. SU(3) partner of the $Z_c(3900)$ state?, changing u or d by an s quark.

Hidden-Gauge Formalism Ikeno,Molina,Oset, PRL814(2021)

$$\text{Channels} : J/\psi K^- \,(1), \quad K^{*-}\eta_c\,(2), \quad D_s^{*-}D^0\,(3), \quad D_s^- D^{*0}\,(4)$$



Hidden-charm strange state

$$V_{ij} = C_{ij} g^2 (p_2 + p_4)(p_1 + p_3); \quad \bar{m}_D = 1916 \text{ MeV}, \bar{m}_{D^*} = 2060 \text{ MeV}$$

$$C_{ij} = \begin{pmatrix} 0 & 0 & \frac{1}{m^2_{D^*}} & \frac{1}{m^2_{D^*_s}} \\ & 0 & \frac{1}{m^2_{D^*_s}} & \frac{1}{m^2_{D^*}} \\ & & -\frac{1}{m^2_{J/\psi}} & 0 \\ & & & -\frac{1}{m^2_{J/\psi}} \end{pmatrix},$$

and the product $\vec{\epsilon} \cdot \vec{\epsilon}\,'$ has been omitted. We build the combinations:

$$A = \frac{1}{\sqrt{2}}(D_s^- D^{*0} + D_s^{*-} D^0),$$

$$B = \frac{1}{\sqrt{2}}(D_s^- D^{*0} - D_s^{*-} D^0).$$

Combination A couples to $J/\psi K^-$, $K^{*-}\eta_c$, while B not.

$$J/\psi K^- \ (1), \quad K^{*-}\eta_c \ (2), \quad \frac{1}{\sqrt{2}}(D_s^- D^{*0} + D_s^{*-} D^0) \ (3)$$

Hidden-charm strange state: $e^+e^- \to K^+(D_s^{*-} + D_s^- D^{*0})$

$$V_{ij} = g^2 C_{ij} \frac{1}{2}\left[3M_{12}^2 - (m_1^2 + m_2^2 + m_3^2 + m_4^2) - \frac{1}{M_{12}^2}(m_1^2 - m_2^2)(m_3^2 - m_4^2)\right],$$

where $M_{12}^2 = (p_1 + p_2)^2$. q^2 correction in the propagators: $q^2 = 0$ for V_{ii} and $q^2 = M^2 + M_{D^*}^2 - 2EM_{D^*}^2$, for V_{13}, V_{23}, with $E = \frac{M_{12}^2 + M^2 - m^2}{2M_{12}}$.

$$C_{ij} = \begin{pmatrix} 0 & 0 & \frac{\sqrt{2}}{\bar{m}_{D^*}^2} \\ & 0 & \frac{\sqrt{2}}{\bar{m}_{D^*}^2} \\ & & -\frac{1}{m_{J/\psi}^2} \end{pmatrix}.$$

$$G_l = \int \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{1}{(P^0)^2 - (\omega_1 + \omega_2)^2 + i\epsilon}$$

$\omega_1 = \sqrt{m^2 + \vec{q}^{\,2}}$, $\omega_2 = \sqrt{M^2 + \vec{q}^{\,2}}$, and $|\vec{q}| < q_{\max}$. $q_{\max} \sim 700$–850 MeV in Aceti,Oset, PRD90(2014) BS factor, $\psi = -\frac{1}{3} + \frac{4}{3}\left(\frac{m_L}{m_H}\right)^2$ (2.8)

$$T = [1 - VG]^{-1} V,$$

$$p = \frac{\lambda^{1/2}(s, m_K^2, M_{\bar{D}_s D^*}^2)}{2\sqrt{s}},$$

$$\tilde{q} = \frac{\lambda^{1/2}(M_{\bar{D}_s D^*}^2, m_{D_s}^2, m_{D^*}^2)}{2M_{\bar{D}_s D^*}},$$

$$\frac{d\sigma}{dM_{\bar{D}_s D^*}} = \frac{1}{s\sqrt{s}}\, p\tilde{q}\, N\, |T_{33}|^2$$

Hidden-charm strange state: $e^+e^- \to K^+(D_s^{*-} + D_s^- D^{*0})$

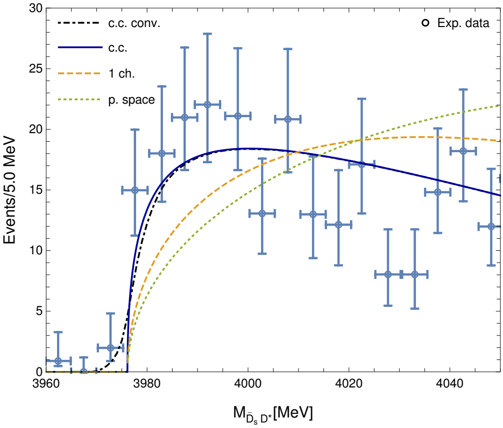


Figure 8: Results of $d\sigma/dM_{\bar{D}_s D^*}$. Solid line: Result for the $\bar{D}_s D^* + \bar{D}_s^* D$ combination with its coupled channels (c.c.). Dashed line: Result for the single channel $\bar{D}_s D^* - \bar{D}_s^* D$ combination (1 ch.). Dotted line: phase space. Dashed-dotted line: result folded with the experimental resolution (c.c. conv.).

Are there Z_{cs} states from $D_s^*\bar{D}^*$?

Ikeno, Molina and Oset, PRD105(2022) Channels: $D_s^{*+}\bar{D}^{*0}$, $J/\psi K^{*+}$

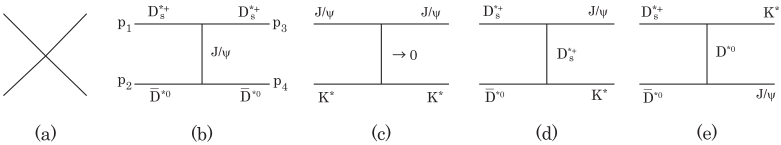
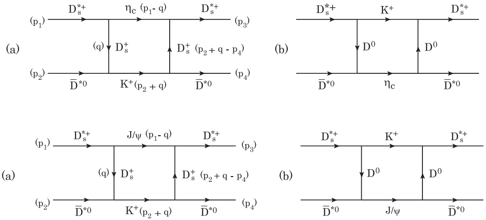


Figure 9: Contact term (a) and vector exchange terms (b, c, d, e) involved in the interaction of the coupled channels.

$\eta_c K^+$, $J/\psi K^+$ included via box diagrams:



Are there Z_{cs} states from $D_s^* \bar{D}^*$?

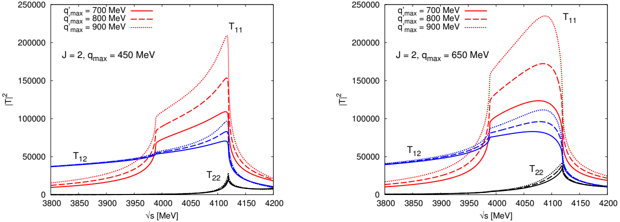


Figure 10: $|T|^2$ for the each channel are shown for $J = 2$ and the different $q'_{\rm max}$ value of the $J/\psi K^*$ channel. The value of $q_{\rm max} = 450, 650$ MeV is fixed for the $D_s^* \bar{D}^*$ channel.

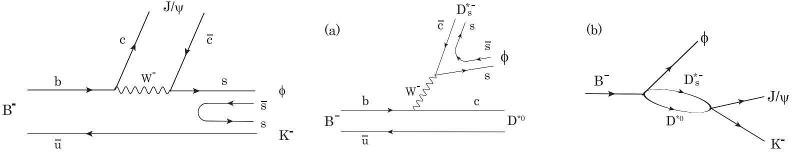


Figure 11: Left: Mechanism for $B^- \to J/\psi \phi K^-$ decay based on internal emission. Center. External emission and right, decay into $J/\psi K^-$.

Are there Z_{cs} states from $D_s^* \bar{D}^*$?

Invariant mass distribution for $B^- \to J/\psi \phi K^-$

$$\frac{d\Gamma}{dM_{\rm inv}(J/\psi K)} = \frac{1}{(2\pi)^3} \frac{1}{4M_B^2} \, p_\phi \, \tilde{p}_K \, |t|^2 \qquad (10)$$

with $p_\phi = \frac{\lambda^{1/2}(M_B^2, M_\phi^2, M_{\rm inv}^2(J/\psi K))}{2M_B}$, $\qquad \tilde{p}_K = \frac{\lambda^{1/2}(M_{\rm inv}^2(J/\psi K), M_{J/\psi}^2, M_K^2)}{2M_{\rm inv}(J/\psi K)}$

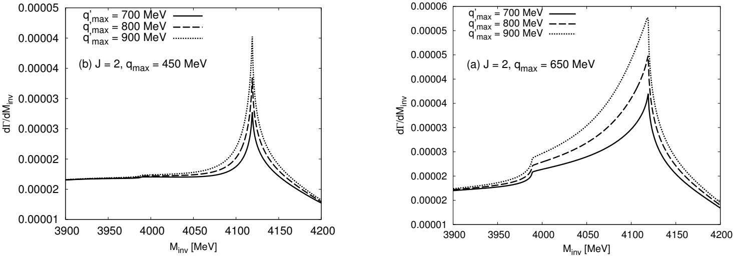
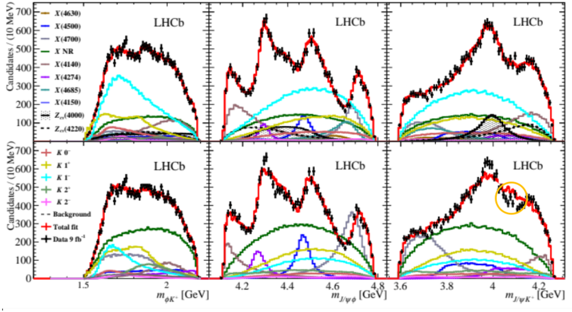
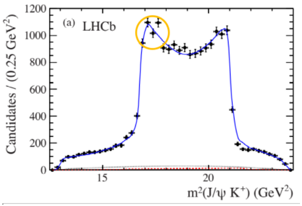


Figure 12: The mass distribution for $B^- \to J/\psi \phi K^-$ is show as a function of $M_{\rm inv}(J/\psi K)$. (a) $q_{\rm max} = 450$ MeV and (b) $q_{\rm max} = 650$ MeV are fixed for the $D_s^* \bar{D}^*$ channel and the different values of $q'_{\rm max}$ for the $J/\psi K^*$ channel are used as shown in the figure. The result corresponds to the case of $J = 2$ for $D_s^* D^{*0}$.

Are there Z_{cs} states from $D_s^* \bar{D}^*$?

$B^+ \to J/\psi \phi K^+$, LHCb, PRL127(2021)





$\bar{B}^0 \to J/\psi K^- K^+$, LHCb, PRD87(2013)

Conclusions

Conclusions

- The $X_0(2866)$ is a good candidate to be a $D^*\bar{K}^*$ molecular state
- We have proposed several reactions where the $J = 1^+$ state (partner of the X_0) can be observed
- The $Z_{cs}(3985)$ can be explained as a threshold effect from the coupled-channel interaction
- There can be also another cusp corresponding to the $D_s^*\bar{D}^*$ interaction with $J/\psi K^*$, and we suggest to look around 4120 MeV in the data of the B decays