What lineshapes of resonances teach us about their nature

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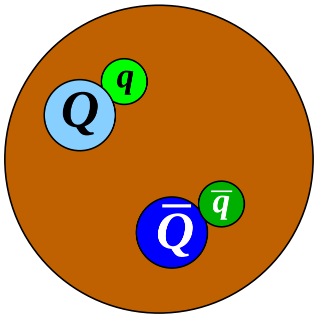
Based on

- F.-K. Guo et al. “Hadronic molecules,” Rev. Mod. Phys. 90(2018)015004
- I. Matuschek et al., “On the nature of near-threshold bound and virtual states,” Eur. Phys. J. A 57 (2021) no.3, 101
- V. Baru et al., “Effective range expansion for narrow near-threshold resonances,” arXiv:2110.07484
- C. Hanhart et al., “Lineshapes for composite particles with unstable constituents,” PRD81 (2010), 094028

Overview

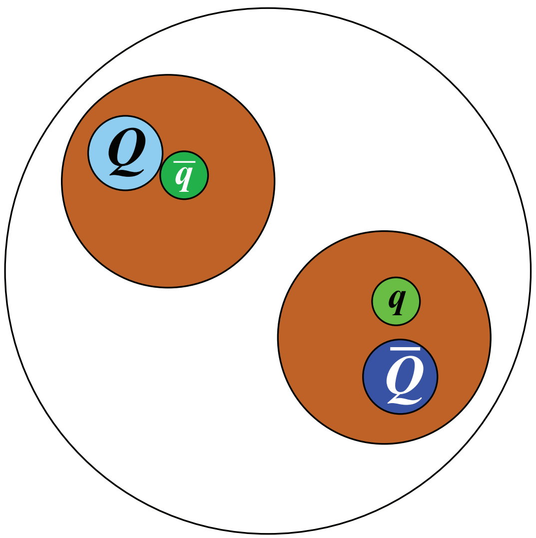
We want employ line shapes to distinguish



e.g. Quarkonia or Tetraquarks

→ Compact object formed from $\bar{Q}Q$ or (Qq) and $(\bar{Q}\bar{q})$

and



Hadronic-Molecules

→ Extended object made of $(\bar{Q}q)$ and $(Q\bar{q})$

Bohr radius = $1/\gamma = 1/\sqrt{2\mu E_b}$

$\gg 1$ fm $\gtrsim$ confinement radius

for near threshold states

Tool: The Weinberg compositeness criterion

Definition using non–rel. QM

Weinberg Phys.Rev.137(1965)B672, Baru et al. (2004)

Expand in terms of non–interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|h_1h_2\rangle \end{pmatrix},$$

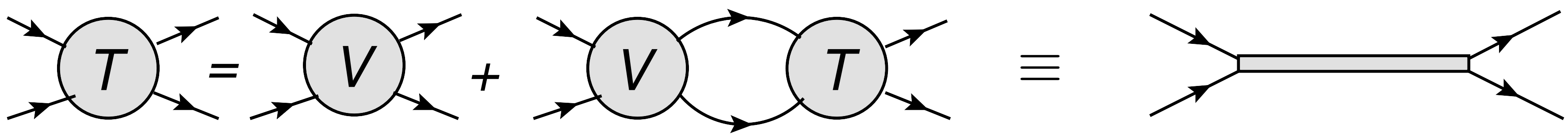
here $|\psi_0\rangle$ = elementary state and $|h_1h_2\rangle$ = two–hadron cont., then

$\lambda^2 = |\langle\psi_0|\Psi\rangle|^2$ = probability to find bare state in physical state

$\rightarrow$ λ^2 is the quantity of interest!

Crucial observation: S. Weinberg, Phys. Rev. 130(1963)776; 131, 440 (1963)

Non-pert. hadron-hadron interactions equivalent to pole term + perturbative interaction



$\Longrightarrow$ Dynamical information transferred into coupling

Derivation

Therefore: $\hat{H}_{hh} = \hat{H}^0_{hh} = \vec{p}^{\,2}/(2\mu)$ contains only kinetic terms!

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}^0_{hh} \end{pmatrix} \longrightarrow \chi(p) = \lambda \frac{f(p^2)}{E - p^2/(2\mu)}$$

introducing the transition form factor $\langle\psi_0|\hat{V}|hh\rangle = f(p^2)$

Therefore

$$|\Psi\rangle = \lambda \begin{pmatrix} |\psi_0\rangle \\ -\frac{f(p^2)}{E_B + p^2/(2\mu)}|h_1 h_2\rangle \end{pmatrix},$$

For the normalization of the physical state we get

$$1 = \langle\Psi|\Psi\rangle = \lambda^2 \left(1 + \int \frac{d^3p}{(2\pi)^3} \frac{f^2(p^2)}{(E_B + p^2/(2\mu))^2}\right)$$

provides connection between λ^2 and hadronic properties

$\Longrightarrow$ Need to understand the integral

Effective Coupling

using $$\int \frac{f^2(p^2)d^3p}{(p^2/(2\mu)+E_B)^2} = \frac{4\pi^2\mu^2 f(0)^2}{\sqrt{2\mu E_B}} + \mathcal{O}\left(\frac{\sqrt{E_B\mu}}{\beta}\right)$$

for s–waves; $1/\beta$= range of forces; $\mu f(0)^2/(2\pi) = g^2$; $\gamma = \sqrt{2\mu E_B}$

$$1 = \lambda^2\left(1 + \frac{\mu g^2}{\gamma} + \mathcal{O}\left(\frac{\gamma}{\beta}\right)\right) \Longrightarrow \boxed{g^2 = \frac{\gamma}{\mu}\left(\frac{1-\lambda^2}{\lambda^2}\right)}$$

This gives for the residue, $g^2_{\mathrm{eff(NR)}} = (2\pi/\mu)\lambda^2 g^2$:

$$g^2_{\mathrm{eff(NR)}} = 2\pi(1-\lambda^2)\gamma/\mu^2 \leq 2\pi\gamma/\mu^2$$

$(1-\lambda^2)$ = Quantifies molecular component in physical state

> The structure information is hidden in the effective coupling, extracted from experiment, independent of the phenomenology used to introduce the pole(s)

Connecting to effective range expansion

The scattering amplitude is in terms of the previous parameters

$$T_{\rm NR}(E) = \frac{2\pi}{\mu} \frac{g^2}{E + E_B + g^2(ik+\gamma)}$$

where $k^2 = 2\mu E$ & $g^2 = \infty$ for molecule / $g^2 = 0$ for compact state

The effective range expansion reads:

$$T_{\rm NR}(E) = -\frac{2\pi}{\mu} \frac{1}{1/a + (r/2)k^2 - ik}$$

and we get from matching coefficients

$$\frac{1}{a} = -\frac{E_B}{g^2} + \gamma \qquad \Longrightarrow a = -2\frac{1-\lambda^2}{2-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right)$$

$$r = -\frac{1}{g^2\mu} \qquad \Longrightarrow r = -\frac{\lambda^2}{1-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right)$$

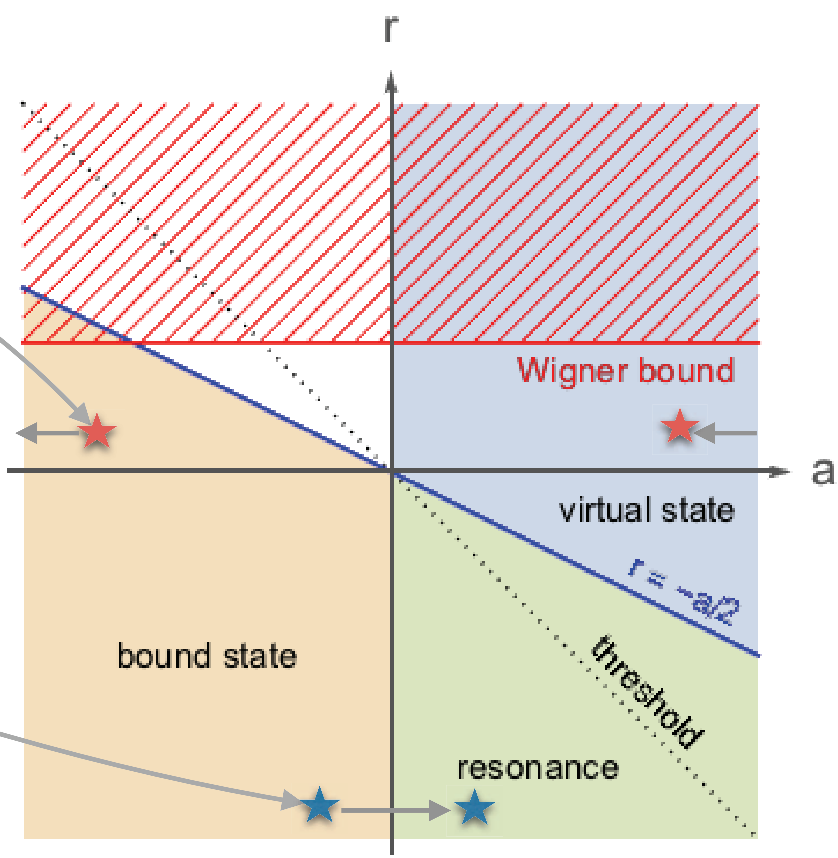
Weinbergs analysis and a generalisation

I. Matuschek et al., EPJA57(2021)3

Assume attractive interaction
(bound state $a<0$, all others $a>0$)

Molecules: $|a| \gg |r|$ and $|r| \simeq$ range

Compact states: $|a| \ll |r|$ and
$r < 0$ with $|r| \gg$ range

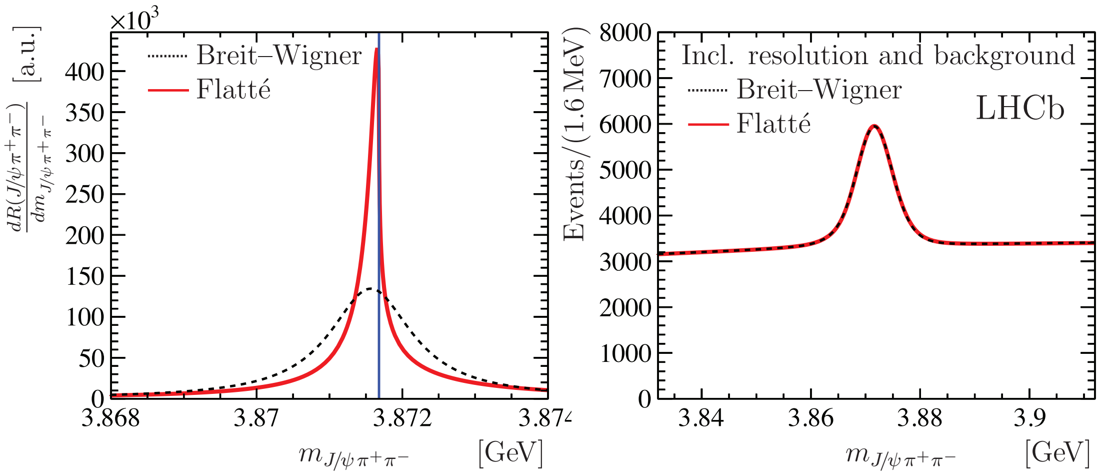


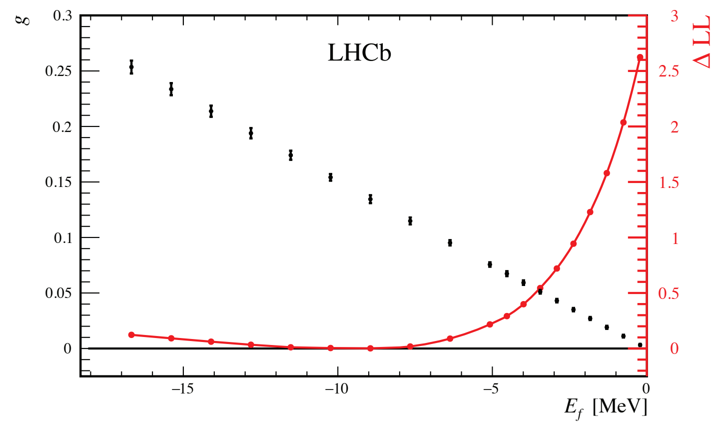
When a changes sign (r fixed): Molecule $\rightarrow$ virtual state
Compact state $\rightarrow$ resonance

Subsummed in compositness $\bar{X} = 1/\sqrt{1+|2r/a|}$

other approaches: Sekihara, Hyodo, Oset, Oller, Nieves, Jido ...
mostly relying on on-shell factorisation of the potential; little about virtual states

$\chi_{c1}(3872)$ also known as $X(3872)$





LHCb, PRD102(2020)092005

C.H. at al., PRD76(2007)034007

Data analysed employing for the rate

$$\frac{\Gamma_\rho(E)}{\left|E-E_f+\frac{i}{2}[g_1^2\sqrt{2\mu_1 E}+g_2^2\sqrt{2\mu_2(E-\delta)}+\Gamma_\rho(E)+\Gamma_\omega(E)+\Gamma_0]\right|^2}$$

with E_f fixed to -7.18 MeV: $g_1^2=g_2^2=g^2=0.108\pm 0.003$ such that

$$-r=2/(\mu_1 g^2)+\sqrt{\mu_2/(2\mu_1^2\delta)}\simeq(3.8+1.4)\ \text{fm}\gg 1/M_\pi$$

Does this mean $\chi_{c1}(3872)$ is a compact state?

A. Esposito et al., PRD105(2022)L031503 & L. Maiani's talk

The second term in

$$-r = 2/(\mu_1 g_1^2) + \frac{g_2^2}{g_1^2}\sqrt{\mu_2/(2\mu_1^2\delta)} \simeq (3.8 + 1.4)\text{ fm} \gg 1/M_\pi$$

comes from isospin-symmetry, $g_1^2 = g_2^2 = g^2$ and the expansion

$$ik_2 = \sqrt{2\mu_2(\delta - k_1^2/(2\mu_1))} = \sqrt{2\mu_2\delta} - \frac{1}{2}\sqrt{\frac{\mu_2}{2\mu_1^2\delta}}\, k_1^2 + \mathcal{O}\left(\left(\frac{k_1^2}{\mu_1\delta}\right)^2\right)$$

which “measures” the contribution from the charged channel and does not have a proper isospin limit ($\delta \to 0$). However,

- it scales with g^2 ($\to \infty$ for molecule)
- we thus see that this contribution is sizable

$\to$ needs to be removed to understand structure

Thus the quantity relevant for the Weinberg analysis is thus

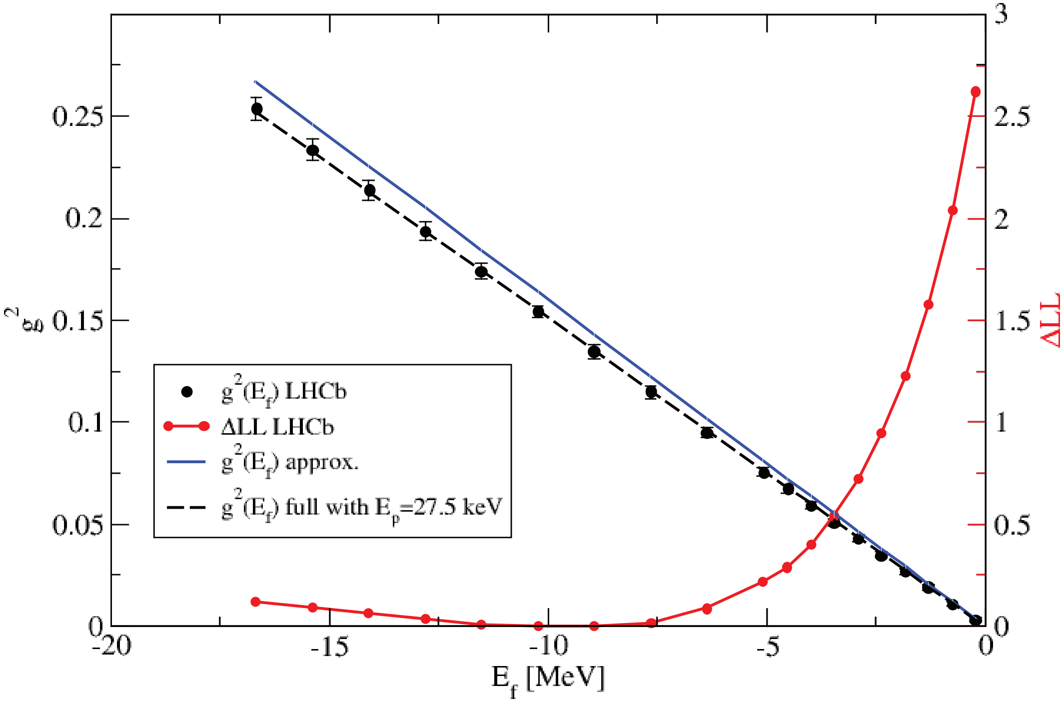
$$-r_{\text{eff.}} = 2/(\mu_1 g^2) \leq 3.8 \text{ fm}$$

Regarding the correlations

$$\frac{\Gamma_\rho(E)}{\left|E - E_f + \frac{i}{2}\left[g^2\left(\sqrt{2\mu_1 E} + \sqrt{2\mu_2(E-\delta)}\right) + \Gamma_\rho(E) + \Gamma_\omega(E) + \Gamma_0\right]\right|^2}$$

For $E < 0$ the parts in red define E_p, the real part of pole location:

$$E_p = E_f + \frac{g^2}{2}\left(\sqrt{2\mu_1|E_p|} + \sqrt{2\mu_2(\delta + |E_p|)}\right) \implies g^2(E_f, E_p)$$



Since $E_p \ll \delta$ one may approximate correlation parameter free

$$g^2(E_f, 0) = -\sqrt{\frac{2}{\mu_2 \delta}} E_f$$

To remove correlation:

Express E_f by E_p

Consequence:

The formula that should be used in the analysis:

$$\frac{\Gamma_\rho(E)}{\left|E-E_p+\frac{i}{2}\left[g^2\left(\sqrt{2\mu_1 E}\mp i\gamma_1+\sqrt{2\mu_2(E-\delta)}-i\gamma_2\right)+\Gamma_{\rm inel.}(E)\right]\right|^2}$$

for pole on the physical (unphysical) $D^0\bar{D}^{*0}$ sheet and

where $\gamma_1=\sqrt{2\mu_1|E_p|}$ and $\gamma_2=\sqrt{2\mu_2(\delta+|E_p|)}$

The LHCb data only provides lower bound for g

If one allows for $\Delta LL=1$, one finds $g^2>0.1$ and accordingly

$$-r_{\rm eff.}<4\ \text{fm} \quad \text{and} \quad \bar{X}=\frac{1}{\sqrt{1+2|r_{\rm eff.}/\Re(a)|}}>0.94\ ,$$

fully consistent with a molecular interpretation

Similar numbers emerge for the T_{cc} state ...

How to get positive r?

$\rightarrow$ The formulas were derived neglecting finite range corrections

$\rightarrow$ The Wigner bound (causality!) requires $r < R \sim 1/\beta$

E.P. Wigner, Phys.Rev 98(1955)145

$\Longrightarrow$ Zero range interactions call for neg. effective ranges

The longest range interaction is the one π exchange, however

in the charm system $\pi D\bar{D}$ can go on-shell

$\Longrightarrow$ no fixed sign of potential

We need hadronic EFT to quantify the effects!

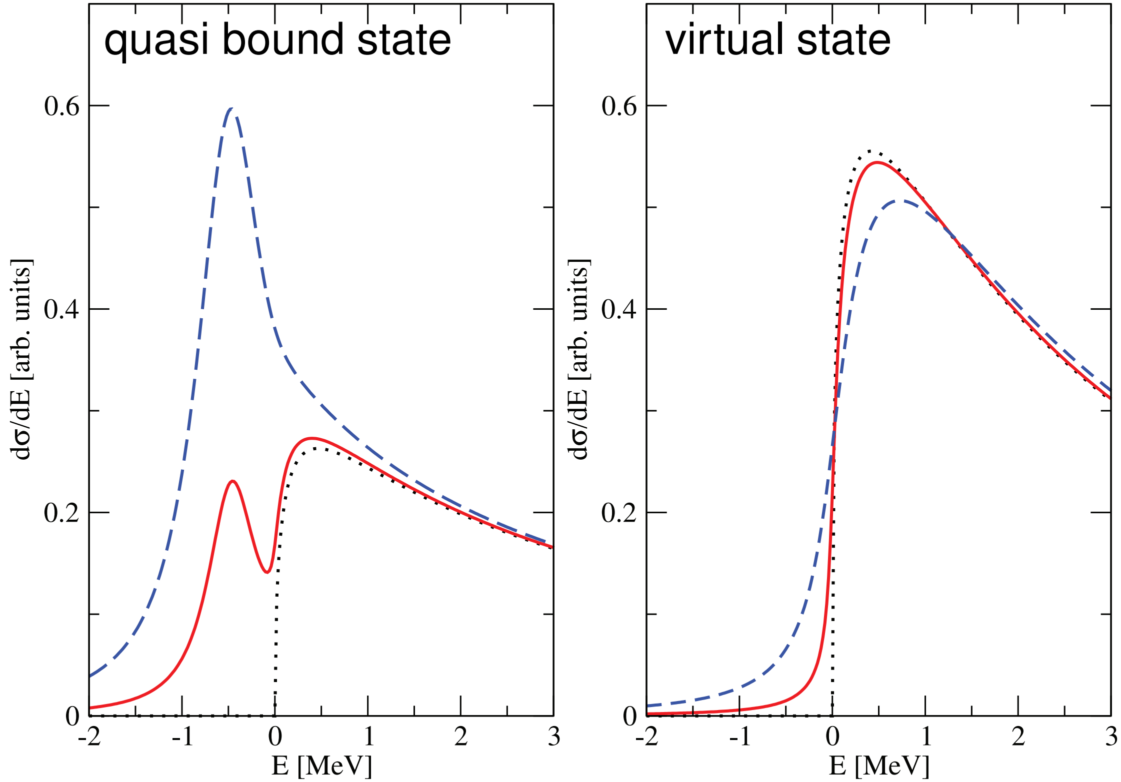
pert. pions: Mehen, Valderrama, Mikhasenko, ...; non-pert. pions: Baru, Filin, Du, Guo, C.H., ...

$\Longrightarrow$ three-body calculation for T_{cc}: $r_{\mathrm{OPE}} = +0.4$ fm

M. L. Du et al., PRD105 (2022)014024.

Molecules with unstable constituents

E. Braaten and M. Lu, PRD76(2007)094028, C. H. et al., PRD81(2010)094028



→ $E_b = 0.5$ MeV below nominal threshold,

→ additional width: $\Gamma_{\rm inel.} = 1.5$ MeV

→ constituent width

$0,\ 0.1,\ 1$ MeV

Molecules with unstable const. can show peculiar line shapes

Strong rise above nominal threshold, because of

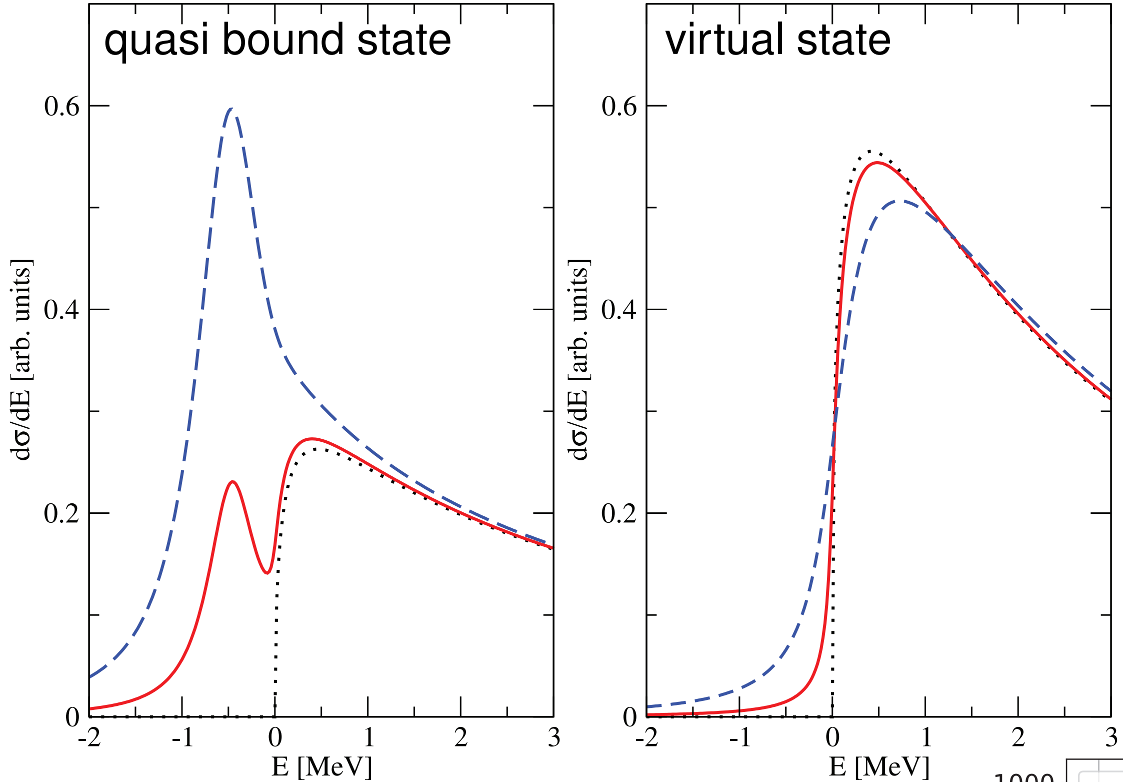
→ nearby pole

→ with large residue

Form depends on interplay of scales

Molecules with unstable constituents

E. Braaten and M. Lu, PRD76(2007)094028, C. H. et al., PRD81(2010)094028



→ $E_b = 0.5$ MeV below nominal threshold,

→ additional width: $\Gamma_{\text{inel.}} = 1.5$ MeV

→ constituent width

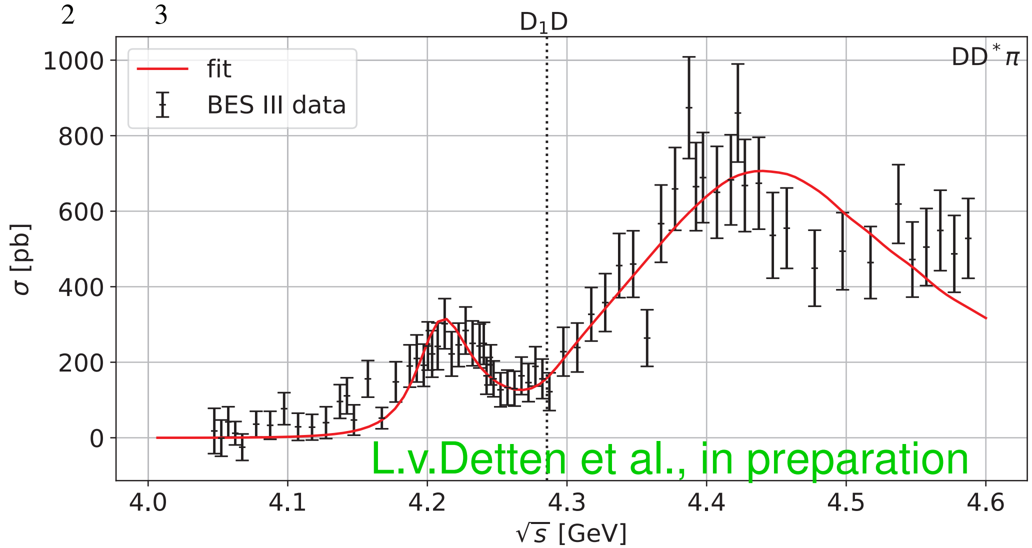
$0,\ 0.1,\ 1$ MeV

$e^+e^- \to D^*\bar{D}\pi$ data show strong evidence for $Y(4230)$ as $D_1(2420)\bar{D}$ bound state with

$E_b = 55$ MeV

$D_1(2420)$ ($\Gamma = 30$ MeV)

mixing into $D_1(2430)$ ($\Gamma = 300$ MeV)



Conclusion

At present the data on $\chi_{c1}(3872)$ aka $X(3872)$ and T_{cc}^{+} are consistent with a molecular interpretation, but so far a sizeable compact component cannot be excluded.

For more definite statements we need

- $\rightarrow$ Reanalysis of LHCb data with correlations removed
- $\rightarrow$ Combined analysis of inelastic and elastic channels
- $\rightarrow$ Direct measurement of line shape (PANDA?)
- $\rightarrow$ Information on (iso)spin partner states

Line shapes carry important structure information

... thank you very much for your attention