T_{cc}^{+} coupled channel analysis and predictions

[Based on M. Albaladejo, arXiv:2110.02944]

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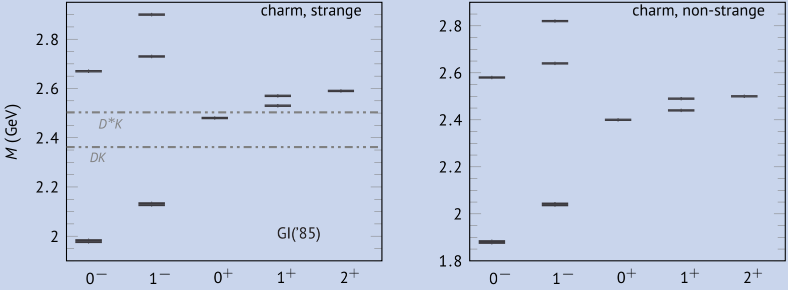
Hadron Spectroscopy: The Next Big Steps. March 16, 2022

Generalitat Valenciana
Gobierno de España
Ministerio de Ciencia e Innovación

Outline

Quark model in the singly heavy sector

- Quark model $c\bar{n}$ is still our baseline: *"In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework."* [Godfrey, Isgur, PR,D32,189('85)]



- The discovery of $D^*_{s0}(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is "equivalent" to the discovery of $X(3872)$ in charmonium-like system.

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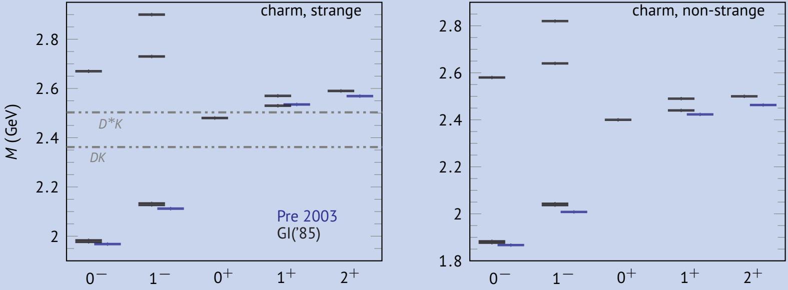

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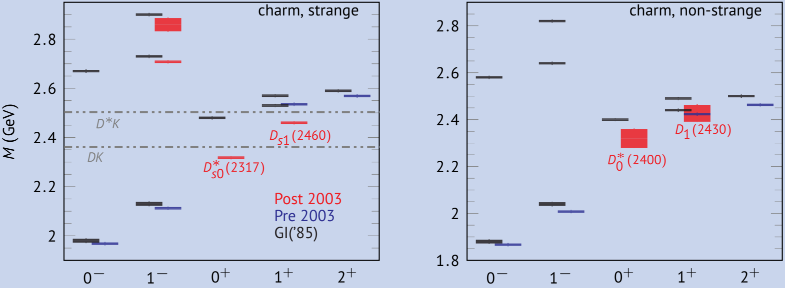

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T_{cc}^{+} and previous predictions

- T_{cc}^{+} is a **tetraquark** with constituent $cc\bar{u}\bar{d}$
- Models give broad range of predictions.
- Not observed until now (only Ξ_{cc}^{++} [LHCb])

  [PRL,119,112001('17)]

- LQCD: not conclusive in the charm sector; more agreement in the bottom sector.

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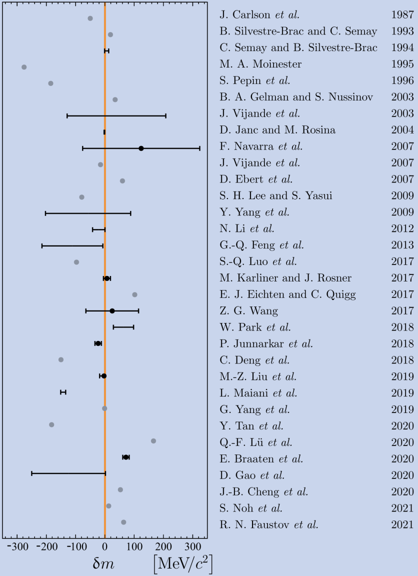
  [Bicudo *et al.*,PR,D103,114506('21)]



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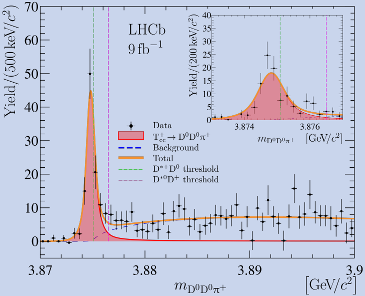
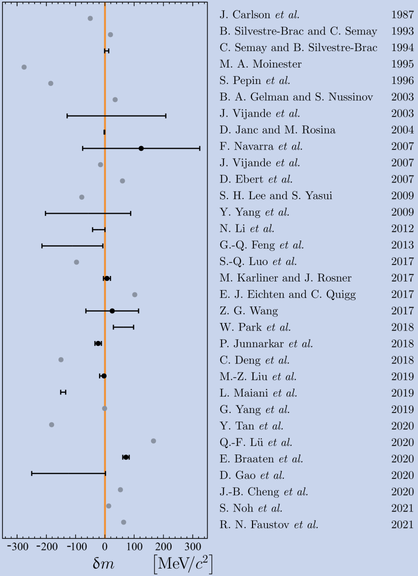
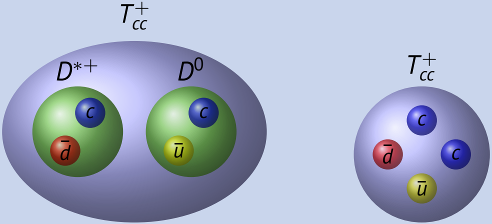
- Then comes LHCb… [2109.01038;2109.01056]





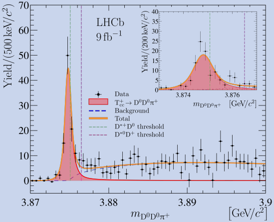
Tetraquark? Molecule *vs* compact tetraquarks

- [PRL,115('15),072001] "Observation of J/ψ resonances consistent with **pentaquark** states in $\Lambda_b^0 \to J/\psi K^- p$ decays"
- [2109.01038] "Observation of an exotic narrow doubly charmed **tetraquark**"
- [2109.01056] "Study of the doubly charmed **tetraquark** T_{cc}^+"
- Misleading nomenclature (not LHCb fault!)

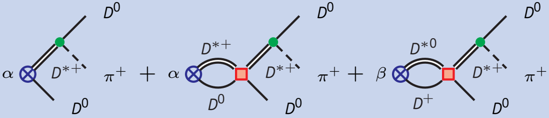


- Nomenclature A: A tetraquark is anything with constituent $4q$. **Compact tetraquarks** *vs* **molecular tetraquarks**.
- Nomenclature B: Tetraquarks *vs* molecules

Production Model



- LHCb spectrum is essentially T_{cc}^+ signal and a D^*D phase space background
- Reasonable to assume that all $DD\pi$ events are produced through D^*D
- Small range ($\sim$ 30 MeV) $DD\pi$ invariant mass: assume D^*D in S-wave



$$\mathcal{N}_{\rm ev}(Q^2) = \mathcal{N}_0 \left(\frac{Q^2_{\rm th}}{Q^2}\right)^{\frac{3}{2}} \int_{s_{\rm th}}^{s_{\rm max}(Q^2)} {\rm d}s \int_{t_-(s,Q^2)}^{t_+(s,Q^2)} {\rm d}t \sum_\lambda \left|\mathcal{M}_\lambda(Q^2,s,t,u)\right|^2 \,,$$

$$\mathcal{M}_\lambda(Q^2,s,t,u) = g_{D^*D\pi}\, p_\pi^\nu \epsilon_S^\mu(\lambda) \left[\frac{K_t(Q^2)}{t-m^2_{D^*_{(t)}}}\left(-g_{\mu\nu}+\frac{k_\mu^{(t)}k_\nu^{(t)}}{t}\right) + \frac{K_u(Q^2)}{u-m^2_{D^*_{(u)}}}\left(-g_{\mu\nu}+\frac{k_\mu^{(u)}k_\nu^{(u)}}{u}\right)\right]$$

$$K_t(Q^2) = \alpha\left(1+G_1(Q^2)T_{11}(Q^2)\right) C_{D^{*+}\to D^0\pi^+} + \beta G_2(Q^2)T_{12}(Q^2)\, C_{D^{*+}\to D^0\pi^+} \,.$$

D^*D scattering amplitude

- Coupled T-matrix for the $D^{*+}D^0$, $D^{*0}D^+$ channels:

$$T^{-1}(E) = V^{-1}(E) - \mathcal{G}(E) \, ,$$

- $I_z = 0$: the isospin decomposition reads:

$$\left|D^{*+}D^0\right\rangle = -\frac{1}{\sqrt{2}}\left(|D^*D, I=1\rangle + |D^*D, I=0\rangle\right) \, ,$$

$$\left|D^{*0}D^+\right\rangle = -\frac{1}{\sqrt{2}}\left(|D^*D, I=1\rangle - |D^*D, I=0\rangle\right) \, ,$$

$V(E)$: **interaction** kernels written in terms of $C_{I=0,1}$ (constants):

$$V(E) = \frac{1}{2}\begin{pmatrix} C_0 + C_1 & C_1 - C_0 \\ C_1 - C_0 & C_0 + C_1 \end{pmatrix}$$

$\mathcal{G}(E)$: **loop functions** of the $D^{*+}D^0$, $D^{*0}D^+$ channels:

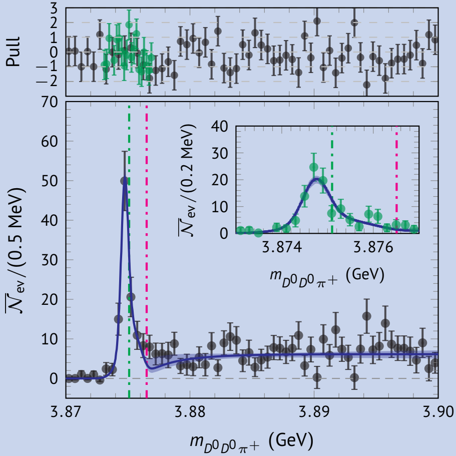
$$G_i(E) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{e^{-\frac{2\vec{k}^2}{\Lambda^2}}}{E - E^i_{\rm th} - \frac{\vec{k}^2}{2\mu_i}}$$

- Width of the D^*: the loop functions are analytically continued to complex values of the D^* mass, $m_{D^*} \to m_{D^*} - i\Gamma_{D^*}/2$.
- Two values for the cutoff, $\Lambda = 0.5$ GeV and $\Lambda = 1.0$ GeV.
- The V-matrix elements depend now on the cutoff, $C_I(\Lambda)$.

Results: Fit

- Exp. resolution taken from LHCb ($\delta \simeq 400$ keV):

$$\overline{\mathcal{N}}_{\rm ev}(E) = \int \mathrm{d}E' \, R_{\rm LHCb} \, (E, E') \, \mathcal{N}_{\rm ev}(E')$$



Parameter	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$C_0(\Lambda)$ [fm^2]	$-0.7008(22)$	$-1.5417(121)$
$C_1(\Lambda)$ [fm^2]	$-0.440(79)$	$-0.71(27)$
β/α	0.228(108)	0.093(79)
χ^2/dof	0.95	0.92

- Good agreement ($\chi^2/\mathrm{dof} = \{0.92, 0.95\}$)
- Check: pull of the data seems randomly distributed.
- Statistical uncertainties obtained by MC bootstrap of the data

Spectroscopy

- Bound state pole in T-matrix, $\det(\mathbb{1} - V\,G) = 0$:

$$T_{ij}(E) = \frac{\tilde{g}_i \tilde{g}_j}{E^2 - \left(M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2\right)^2} + \cdots$$

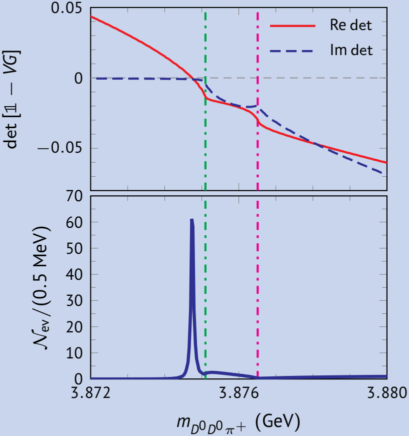
- Width: $m_{D^*} - i\,\Gamma_{D^*}/2 \Rightarrow M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2$
- Pole position (wrt $D^{*+}D^0$ threshold):

Λ (GeV)	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
1.0	$-357(29)$	77(1)
0.5	$-356(29)$	78(1)

- Good agreement with LHCb determination:

	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
[2109.01038]	$-273(61)$	410(165)
[2109.01056]	$-360(40)$	48(2)

- Our width is somewhat larger than the ~ 50 keV obtained by LHCb and [Feijoo *et al.*, 2108.02730], [Ling *et al.*, 2108.00947].
- [Du *et al.*, 2110.13765]: $\Gamma_{T_{cc}^+}$ depending on the model used.



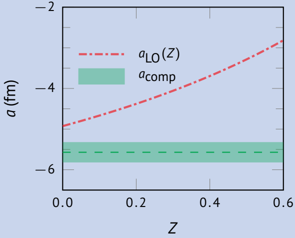
- Results similar to [LHCb, 2109.0156] (top) and [Feijoo *et al.*, 2108.02730; Du *et al.*, 2110.13765] (bottom).

Molecular state?

- Weinberg compositeness [Weinberg, PR,137,B672('65)] : $P = 1 - Z \simeq \frac{\mu^2 g^2}{2\pi\gamma_B} = -g^2 G'(E_B)$
- We get $P_{D^{*+}D^0} = 0.78(5)(2)$, $P_{D^{*0}D^+} = 0.22(5)(2)$, $\sum_i P_i = 1$
- Isospin limit, $P_I = 1$ (for $I = 0$ or 1): **purely molecular state (model built-in!)**

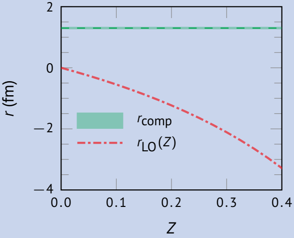
- Relation to ERE parameters a, r [Weinberg('65)]

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} + \cdots\ ,$$
$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \cdots\ .$$



- This result must be applied to a **single channel** case: **isospin limit**

Λ (GeV)	0.5	1.0
E_B (keV)	833(67)	856(53)
$a_{I=0}$ (fm)	−5.57(25)	−5.18(16)
$r_{I=0}$ (fm)	0.63	1.26



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- Relation to ERE parameters a, r
  [Weinberg('65)]+[Albaladejo, Nieves: 2203.04864]

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} - 2\delta r\left(\frac{1-Z}{2-Z}\right)^2 + \cdots ,$$

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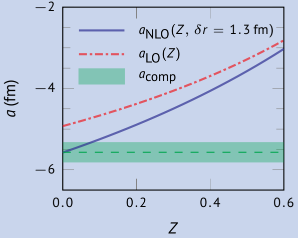
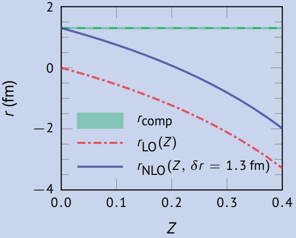




HQSS partner

- Heavy-Quark Spin Symmetry (HQSS) predicts that heavy-meson interactions are independent of the heavy-quark spin in the limit $m_Q \to \infty$.
- Relation between $D^*D^* \to D^*D^*$ and $D^*D \to D^*D$ amplitudes.
- The interaction kernels of the $I(J^P)$ D^*D^* systems are related to those of the D^*D ones as:

$$\left\langle D^*D^*,\, 0(1^+) \left| \hat{V} \right| D^*D^*,\, 0(1^+) \right\rangle = \left\langle D^*D,\, 0(1^+) \left| \hat{V} \right| D^*D,\, 0(1^+) \right\rangle = V_0 \,,$$
$$\left\langle D^*D^*,\, 1(2^+) \left| \hat{V} \right| D^*D^*,\, 1(2^+) \right\rangle = \left\langle D^*D,\, 1(1^+) \left| \hat{V} \right| D^*D,\, 1(1^+) \right\rangle = V_1 \,.$$

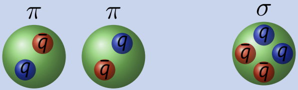
- We predict the existence of T_{cc}^{*+}, a D^*D^* molecular state, HQSS partner of T_{cc}^+, with a binding energy (wrt the different D^*D^* thresholds) of **1.1–1.5 MeV**.

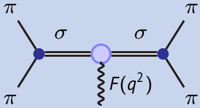
	$\delta M_{T^*_{cc}}$ (keV)			
	Isoscalar solution		Isovector solution	
	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$D^{*+}D^{*+}$			$-1580(71)$	$-1156(79)$
$D^{*+}D^{*0}$	$-1561(71)$	$-1148(79)$	$-1561(71)$	$-1148(79)$
$D^{*0}D^{*0}$			$-1543(71)$	$-1140(79)$

- Similar predictions are obtained in a later work [Dai, Molina, and Oset, 2110.15270]
- Previous works predicting D^*D^* states: [Molina *et al.*, PR,D82,014010('10); Liu *et al.*, PR,D99,094018('19)].

Size

- Can we address the question of $4q$, $q\bar{q}$, molecule based on the size of the object?





- For $\pi\pi$ scattering, σ meson: MA, Oller, PR,D86,034003('12)
  - $\sqrt{\langle r^2\rangle^S_\sigma} \simeq 0.44$ fm vs $\sqrt{\langle r^2\rangle^S_\pi} \simeq 0.81$ fm
- Perhaps only theoretical? Future lattice QCD calculations?

Briceño *et al.*, PR,D103,114512('21) [and refs. therein]

Summary

- Hadron spectroscopy keeps living exciting times, as shown by the discovery of the T_{cc}^{+} state: a tetraquark with double charm.
- We have analyzed a coupled channel ($D^{*+}D^0$, $D^{*0}D^+$) T-matrix, where one would expect the T_{cc}^{+} to show up.
- A simple production model allows a good description of the data with few parameters
- A bound state originating from the $I = 0$ interaction appears in the T-matrix, identified with the T_{cc}^{+} state.
- This state is found to be largely molecular.
- A D^*D^* molecular state (T_{cc}^{*}) is predicted with a binding energy of $\sim$ 1–1.5 MeV (w.r.t. D^*D^* threshold), and with $I(J^P) = 0(1^+)$ or $1(2^+)$ depending on T_{cc}^{+} isospin.
- Our results are similar to several earlier and later theoretical works [Feijoo, Liang, and Oset, 2108.02730; Du *et al.*, 2110.13765; Dai, Molina, and Oset, 2110.15270]

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[Based on M. Albaladejo, arXiv:2110.02944]

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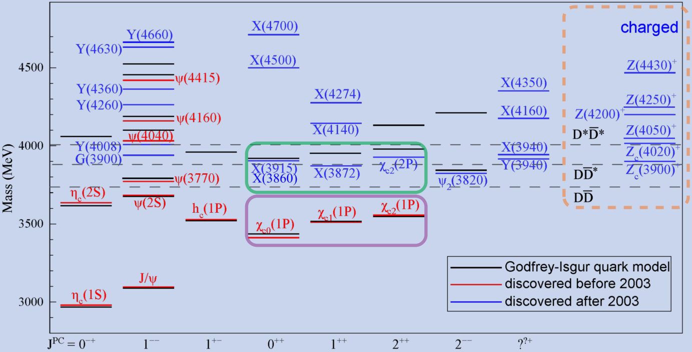
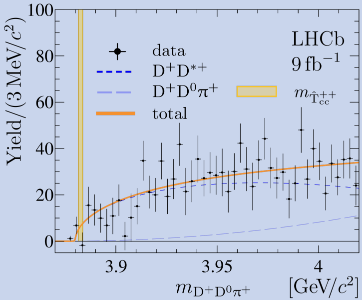


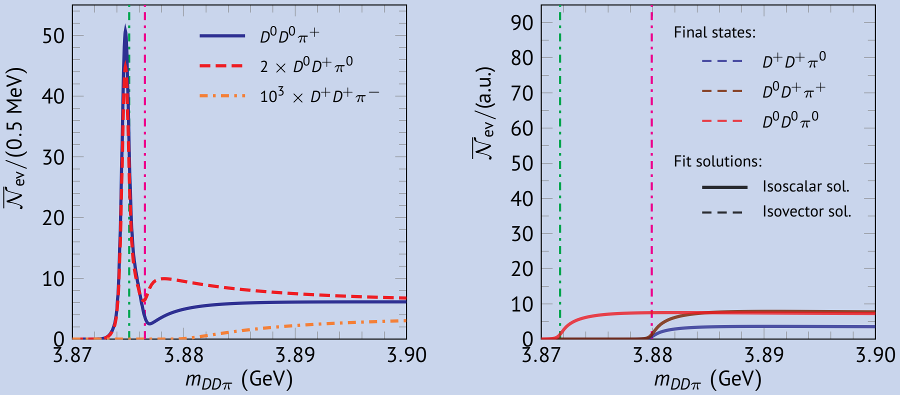
- $\chi_{cJ}(1P)$ well established, "very CQM model" state.
- $X(3872)$ discovered by Belle [PRL,91,262001('03)] (also 2003!)
  $J^{PC} = 1^{++}$ and $\Gamma \sim 1$ MeV established by LHCb [PR,D92,011102('15);PR,D102,092005('20); JHEP,08(2020),123].
- $\chi_{cJ}(2P)$ Not established. Influence of open thresholds? Is $X(3872)$ a molecular states?
- Z_c states have $I = 1$, clearly "tetraquarks" ($c\bar{c}u\bar{d}$, ...)

Isospin $I = 0$ or $I = 1$, and fit degeneracy

- **Q**: Can I know the T_{cc}^{+} isospin from **this** analysis? **A**: **No**
- There's a degeneracy in the solutions: the model is "invariant" under a simultaneous exchange: $C_0 \leftrightarrow C_1$ and $\beta \leftrightarrow -\beta$
- Physically, this is due to the fact that $D^0 D^0 \pi^+$ has $I_z = 0$, so you cannot know whether T_{cc}^{+} is $|I\, I_z\rangle = |1\, 0\rangle$ or $|0\, 0\rangle$.
- We will keep both solutions $I = 0$ or $I = 1$ for T_{cc}^{+} and discuss their differences (whenever they exist!)
- LHCb [2109.01056] has shown an additional spectrum, $D^+ D^0 \pi^+$ ($I_z = 1$), in which no sign of a T_{cc}^{++} is observed, but with much less statistics:



Other distributions



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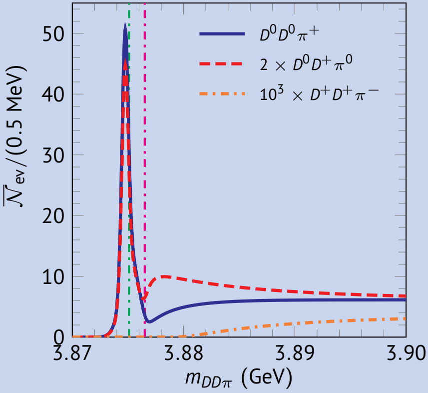

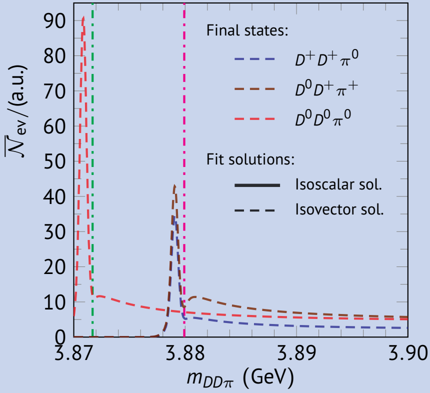

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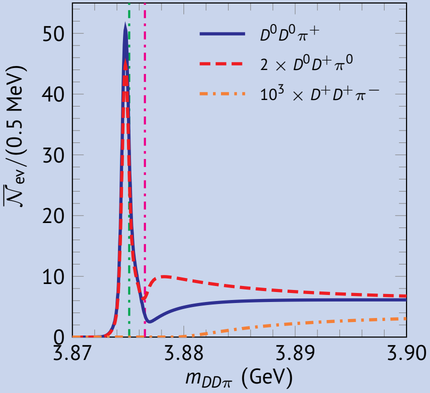
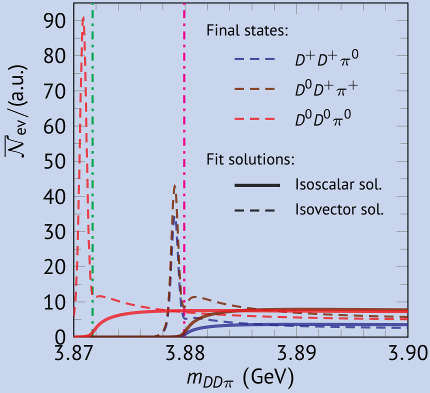




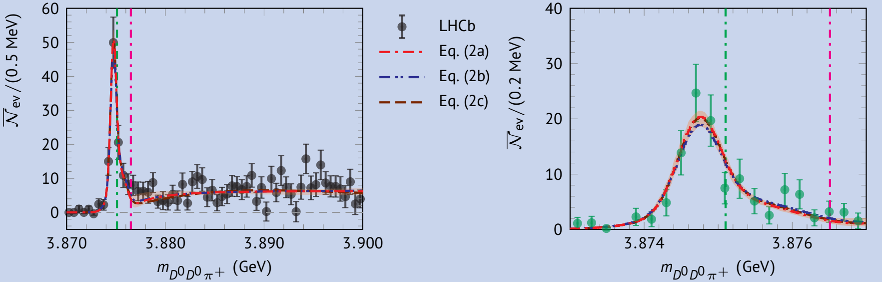
Other parameterizations

- Other paramterizations could lead to different line shapes and/or properties: pole position, scattering length, molecular probability,...
- No large variations are observed when the most inmediate generalizations are employed
- In particular, always large molecular probability

$$V_0(s) = C_0(\Lambda) \quad \longrightarrow \quad V_0(s) = C_0(\Lambda) + b_0(\Lambda)k^2 \; , \tag{2a}$$

$$V_0(s) = C_0(\Lambda) \quad \longrightarrow \quad V_0(s) = \frac{d_0(\Lambda)}{E - M_0(\Lambda)} \; , \tag{2b}$$

$$V_1(s) = C_1(\Lambda) \quad \longrightarrow \quad V_1(s) = C_1(\Lambda) + b_1(\Lambda)k^2 \; . \tag{2c}$$



Nomenclatures about Z

Reasonable

- $Z \simeq 0$ is a molecule
- (mostly molecular, mixed with some small compact component)
- $Z \simeq 1$ is a compact state
- (mostly compact, mixed with some small molecular component)

Purist

- Only $Z = 0$ is a molecule
- Only $Z = 1$ is a compact state

Extremist (biased)

- Only $Z = 0$ is a molecule!
- $Z \simeq 1$ is a compact state

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