# $T_{cc}$ and other exotic states with two open heavy quarks

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Local hidden gauge approach and chiral Lagrangians

Predictions for molecular D\* K\*bar and D\*D\* states in 2010

Discussion to the light of the  $T_{cc}$  state

Predictions for D(s)(\*)D(s)(\*) and B(s)(\*)B(s)(\*) states



VP INTERACTION IN THE LOCAL HIDDEN GAUGE APPROACH Bando et al Phys Rep. 164

$$\begin{array}{ll} \underbrace{V}{} & \underbrace{V}{} & \underbrace{V}{} & \underbrace{\mathcal{L}_{VVV}}{} = ig\langle (V_{\mu}\partial_{\nu}V^{\mu} - \partial_{\nu}V_{\mu}V^{\mu})V^{\nu} \rangle & \text{Neglecting the p/M}_{\nu} \\ g = M_{V}/2f \; (M_{V} \approx 800 \; \text{MeV}, \; f = 93 \; \text{MeV}) \\ \mathcal{L}_{VPP} = -ig\langle V^{\mu}[P, \partial_{\mu}P] \rangle \\ -it = -g(V^{\mu}\partial_{\nu}V_{\mu} - \partial_{\nu}V_{\mu}V^{\mu})_{ij}V_{ji}^{\nu}\frac{i}{q^{2} - M_{V}^{2}}V_{lm}^{\nu}[P, \partial_{\nu'}P]_{ml} \\ \sum_{pol} \epsilon_{ji}^{\nu}\epsilon_{lm}^{\nu'} = \left(-g^{\nu\nu'} + \frac{q^{\nu}q^{\nu'}}{M_{V}^{2}}\right)\delta_{jl}\delta_{im} \\ -it = -i\frac{g^{2}}{M_{V}^{2}}\langle (V^{\mu}\partial_{\nu}V_{\mu} - \partial_{\nu}V_{\mu}V^{\mu})[P, \partial^{\nu}P] \rangle \end{array}$$

 $\mathcal{L} = -\frac{1}{4f^2} \langle [V^{\mu}, \partial_{\nu} V^{\mu}] [P, \partial^{\nu} P] \rangle \quad \text{Chiral Lagrangian of M. C. Birse, Z. Phys. A 355, 231 (1996)}$ 

New interpretation for the Ds2\*(2573) and the prediction of novel exotic charmed mesons R. Molina, T. Branz, E. Oset, PHYSICAL REVIEW D 82, 014010 (2010)

 $V_{\mu}$ 

State predicted of D\* K\*bar nature. The local hidden gauge for VV interaction has an extra contact term

$$\begin{aligned} \mathcal{L}_{VVVV} &= \frac{1}{2}g^2 \langle [V_{\mu}, V_{\nu}] V^{\mu} V^{\nu} \rangle \\ & \text{Spin projection operators} \\ & = \begin{pmatrix} \frac{\omega + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega - \rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ R^{*-} & \bar{K}^{*0} & \phi & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D^{*-} \\ D^{*0} & D^{*+} & D^{*+} & J/\psi \end{pmatrix}_{\mu} \\ \mathcal{P}^{(2)} &= \{\frac{1}{2}(\epsilon_{\mu}\epsilon_{\nu}\epsilon^{\mu}\epsilon^{\nu} + \epsilon_{\mu}\epsilon_{\nu}\epsilon^{\nu}\epsilon^{\mu}) - \frac{1}{3}\epsilon_{\mu}\epsilon^{\mu}\epsilon_{\nu}\epsilon^{\nu}\epsilon^{\nu} \} \end{aligned}$$

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J	Amplitude	Contact	V exchange	~Tota
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^{2}$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_{\epsilon}^*}^2} + \frac{1}{2}g^2(\frac{1}{m_{\omega}^2} - \frac{3}{m_{\rho}^2})(p_1 + p_3).(p_2 + p_4)$	-9.98
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D_{\epsilon}}^2} + \frac{1}{2}g^2(\frac{1}{m_{\omega}^2} - \frac{3}{m_{\rho}^2})(p_1+p_3)(p_2+p_4)$	-10.2
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^{2}$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{1}{2}g^2(\frac{1}{m_{\omega}^2} - \frac{3}{m_{\rho}^2})(p_1+p_3).(p_2+p_4)$	-15.9
		TABLE XII.	Amplitudes for $C = 1$ , $S = -1$ and $I = 1$ .	
J	Amplitude	Contact	V exchange	~Tot
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^{2}$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_{\star}^*}^2} + \frac{g^2}{2} \left(\frac{1}{m_{\omega}^2} + \frac{1}{m_{\rho}^2}\right) (p_1 + p_3).(p_2 + p_4)$	9.7 <i>g</i>
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_{\star}^*}^2} + \frac{g^2}{2} \left(\frac{1}{m_{\omega}^2} + \frac{1}{m_{\rho}^2}\right) (p_1 + p_3).(p_2 + p_4)$	9.9 <i>g</i>
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{g^2}{2}(\frac{1}{m_{\omega}^2} + \frac{1}{m_{\rho}^2})(p_1+p_3).(p_2+p_4)$	15.78
	$T = (\hat{1} - VG)^{-1}V$		$G_i = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M_1^2 + i\epsilon} \frac{1}{(P-q)^2 - M_2^2 + i\epsilon}$	ie

TABLE XI. Amplitudes for C = 1, S = -1 and I = 0.

G is regularized either with a cutoff in the three momentum or dimensional regularization, with qmax, or a subtraction constant  $\alpha$ .

Λ

Decay terms, added to V and iterated in the Bethe Salpeter equation. Through its imaginary part they provide the decay to DKbar



$I[J^P]$	$\sqrt{s_{\text{pole}}}$ (MeV)	Model	Γ (MeV)
0[0+]	2848	A, $\Lambda = 1400 \text{ MeV}$	23
		A, $\Lambda = 1500 \text{ MeV}$	30
		B, $\Lambda = 1000 \text{ MeV}$	25
		B, $\Lambda = 1200 \text{ MeV}$	59
0[1+]	2839	Convolution	3
0[2+]	2733	A, $\Lambda = 1400 \text{ MeV}$	11
		A, $\Lambda = 1500 \text{ MeV}$	14
		B, $\Lambda = 1000 \text{ MeV}$	22
		B, $\Lambda = 1200 \text{ MeV}$	36

TABLE VI. C = 1; S = -1; I = 0. Mass and width for the states with J = 0 and 2.

R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 125, 242001 (2020)

R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 102, 112003 (2020)

 $X_{0}(2866): M = 2866 \pm 7 \text{ and } \Gamma = 57.2 \pm 12.9 \text{ MeV},$   $X_{1}(2900): M = 2904 \pm 5 \text{ and } \Gamma = 110.3 \pm 11.5 \text{ MeV}$  6Decaying to DKbar The state predicted corresponds to the X\_{0}(2866) Revision to the light of experimental results R. Molina, E. O. Phys.Lett.B 811 (2020) 135870



$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state	
0(2+)	2775	38	$D^*\bar{K}^*$	?	
0(1+)	2861	20	$D^*\bar{K}^*$	?	
0(0 <sup>+</sup> )	2866	57	$D^*\bar{K}^*$	$X_0(2866)$	Ν
(A)					I N

No D Kbar decay No D\* Kbar decay The  $\bar{B}^0 \to D^{*+} \bar{D}^{*0} K^-$  reaction to detect the  $I = 0, J^P = 1^+$  partner of the  $X_0(2866)$ L.R. Dai, R. Molina, E. O., Arxiv 2202.00508

# Looking for the exotic $X_0(2866)$ and its $J^P = 1^+$ partner in the

 $\bar{B}^0 \to D^{(*)+} K^- K^{(*)0}$  reactions Arxiv 2202.11973



In R. Molina, T. Branz, E. Oset, PHYSICAL REVIEW D 82, 014010 (2010)

Predictions were done for a  $1^+$  D\* D\* state

TABLE IVpositions, a	<i>C</i> = 2; <i>S</i> = 0; <i>I</i> = 0. Quantum and couplings $g_i$ in units of MeV. Her	numbers, pole e $\alpha = -1.4$ .
$I[J^P]$	$\sqrt{s_{\text{pole}}}$ (MeV)	$g_{D^*D^*}$
0[1+]	3969	16 825

The interaction for D\*D is the same since the contact term is zero for 1+

Thus, we predict a D\*D bound state with mass with 141 MeV less, 3828 MeV. This overcounts the binding because D\*D\* are identical particles, Bose enhancement, and D\* D are not, but we should expect a D\*D bound state in 1+

This state was found as the Tcc of the LHCb collaboration

#### The Tcc discovery by the LHCb collaboration



Spectra without correction by experimental resolution  $m_{exp} = 3875.09 \text{ MeV} + \delta m_{exp},$ 

$$\delta m_{\rm exp} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}. \ \Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}.$$



Spectra corrected by resolution and analyzed with a unitary amplitude

10  $\delta m_{\rm exp} = -360 \pm 40^{+4}_{-0} \text{ keV}, \qquad \Gamma = 48 \pm 2^{+0}_{-14} \text{ keV}.$ 

A. Feijoo, W.H. Liang, Eulogio Oset, Phys.Rev.D 104 (2021) 11, 114015



$$\begin{aligned} \mathcal{L}_{VPP} &= -ig \, \langle [P, \partial_{\mu} P] V^{\mu} \rangle, \\ \mathcal{L}_{VVV} &= ig \, \langle (V^{\nu} \partial_{\mu} V_{\nu} - \partial_{\mu} V^{\nu} V_{\nu}) V^{\mu} \rangle, \\ g &= \frac{M_V}{2 \, f}, \ (M_V = 800 \text{ MeV}, \ f = 93 \text{ MeV}). \end{aligned}$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^{0}}{\sqrt{2}} & \pi^{+} & K^{+} & \bar{D}^{0} \\ \pi^{-} & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^{0}}{\sqrt{2}} & K^{0} & D^{-} \\ K^{-} & \bar{K}^{0} & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D^{-}_{s} \\ D^{0} & D^{+} & D^{+}_{s} & \eta_{c} \end{pmatrix} \qquad V_{\mu} = \begin{pmatrix} \frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^{+} & K^{*+} & \bar{D}^{*0} \\ \rho^{-} & -\frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D^{*-}_{s} \\ D^{*0} & D^{*+} & D^{*+}_{s} & J/\psi \end{pmatrix}_{\mu}$$

# $D^{\ast +}D^0, D^{\ast 0}D^+$ the 1, 2 channels, the interaction that we obtain is

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to different masses there is a bit of isospin breaking

Convolution of the G function: Origin of the width. Spectral function Mass distribution

bution 
$$\operatorname{Im}[D(s_V)] = \operatorname{Im}\left(\frac{1}{s_V - M_V^2 + iM_V\Gamma_V}\right)$$

$$G(\sqrt{s}, M_k, m_k) = \frac{\int\limits_{(M_V - 2\Gamma_V)^2}^{(M_V + 2\Gamma_V)^2} ds_V G(\sqrt{s}, \sqrt{s_V}, m_k) \times \operatorname{Im}[D(s_V)]}{\int\limits_{(M_V - 2\Gamma_V)^2}^{(M_V + 2\Gamma_V)^2} ds_V \operatorname{Im}[D(s_V)]}$$

$$G_{l} = i \int \frac{d^{4}q}{(2\pi)^{4}} \frac{M_{l}}{E_{l}(\mathbf{q})} \frac{1}{k^{0} + p^{0} - q^{0} - E_{l}(\mathbf{q}) + i\epsilon}$$

$$\Gamma_{D^{*+}}(M_{\rm inv}) = \Gamma(D^{*+}) \left(\frac{m_{D^{*+}}}{M_{\rm inv}}\right)^2 \cdot \left[\frac{2}{3} \left(\frac{p_{\pi}}{p_{\pi,\rm on}}\right)^3 + \frac{1}{3} \left(\frac{p'_{\pi}}{p'_{\pi,\rm on}}\right)^3\right]$$

where  $p_{\pi}$  is the  $\pi^+$  momentum in  $D^{*+} \to D^0 \pi^+$  decay  $p'_{\pi}, p'_{\pi,\text{on}}$  are the same magnitudes for  $D^{*+} \to D^+ \pi^0$ .

$$\Gamma_{D^{*0}}(M_{\rm inv}) = \Gamma(D^{*0}) \left(\frac{m_{D^{*0}}}{M_{\rm inv}}\right)^2 \cdot \left[0.647 \left(\frac{p_{\pi}}{p_{\pi,\rm on}}\right)^3 + 0.353\right]$$
$$\dot{D}^{*0} \rightarrow D^0 \pi^0 \qquad D^{*0} \rightarrow D^0 \gamma$$



With mass from unitary reanalysis of LHCb data, Mikhasenko



## Works along the molecular structure of Tcc

L. Meng, G. J. Wang, B. Wang and S. L. Zhu, Phys. Rev. D 104, 051502 (2021)

Xi-Zhe Ling, Ming-Zhu Liu, Li-Sheng Geng, En Wang, Ju-Jun Xie, Phys.Lett.B 826 (2022) 136897

M. Albaladejo, arXiv:2110.02944 [hep-ph]

Meng-Lin Du, Vadim Baru, Xiang-Kun Dong, Arseniy Filin, Feng-Kun Guo, Christoph Hanhart, Alexey Nefediev, Juan Nieves, Qian Wang Phys.Rev.D 105 (2022) 1, 014024

Hong-Wei Ke, Xiao-Hai Liu, Xue-Qian Li, Eur.Phys.J.C 82 (2022) 2, 144

Xiang-Kun Dong, Feng-Kun Guo, Bing-Song Zou, Commun. Theor. Phys. 73 (2021) 12, 125201

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### Prediction of new $T_{cc}$ states of $D^*D^*$ and $D^*_sD^*$ molecular nature



TABLE XVII. Amplitudes for C = 2, S = 0, and I = 1.

J	Amplitude	Contact	V exchange	~Total
0	$D^*D^* \rightarrow D^*D^*$	$-4g^{2}$	$\frac{1}{4}g^{2}(\frac{2}{m_{J/\psi}^{2}} + \frac{1}{m_{\omega}^{2}} + \frac{1}{m_{\rho}^{2}})\{(p_{1} + p_{4}).(p_{2} + p_{3}) + (p_{1} + p_{3}).(p_{2} + p_{4})\}$	$24.3g^2$
1	$D^*D^* \rightarrow D^*D^*$	0	0	0
2	$D^*D^* \to D^*D^*$	$2g^2$	$\frac{1}{4}g^{2}(\frac{2}{m_{J/\psi}^{2}} + \frac{1}{m_{\omega}^{2}} + \frac{1}{m_{\rho}^{2}})\{(p_{1} + p_{4}).(p_{2} + p_{3}) + (p_{1} + p_{3}).(p_{2} + p_{4})\}$	$30.3g^2$

J	Amplitude	Contact	V exchange	~Total
0	$D_s^*D^* \rightarrow D_s^*D^*$	$-4g^{2}$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{\nu^*}^2} + \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{J/\psi^2}}$	$19.0g^{2}$
1	$D_s^*D^* \rightarrow D_s^*D^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{\nu^*}^2} + \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{J/\psi^2}}$	$-19.5g^{2}$
2	$D_s^*D^* \to D_s^*D^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m_{K^*}^2} + \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{J/\psi^2}}$	$25.0g^2$

TABLE XVIII. Amplitudes for C = 2, S = 1, and I = 1/2.

TABLE XIX. Amplitudes for C = 2, S = 2, and I = 0.

J	Amplitude	Contact	V exchange	~Total
0	$D_s^*D_s^* \to D_s^*D_s^*$	$-4g^{2}$	$\frac{g^2}{2}\left(\frac{1}{m_{J/\psi}^2} + \frac{1}{m_{\phi}^2}\right)\left\{(p_1 + p_4).(p_2 + p_3) + (p_1 + p_3).(p_2 + p_4)\right\}$	$15.0g^{2}$
1	$D_s^* D_s^* \to D_s^* D_s^*$	0	0	0
2	$D_s^*D_s^* \to D_s^*D_s^*$	$2g^{2}$	$\frac{g^2}{2}(\frac{1}{m_{J/\psi}^2} + \frac{1}{m_{\phi}^2})\{(p_1 + p_4).(p_2 + p_3) + (p_1 + p_3).(p_2 + p_4)\}$	$21.0g^2$



$$\begin{split} -i\,t &= 4\frac{9}{2}\frac{1}{3}\int \frac{d^4q}{(2\pi)^4}\frac{1}{2E_{D^*}(\boldsymbol{q})}\frac{i}{p_1^0 - q^0 - E_{D^*}(\boldsymbol{q}) + i\epsilon}\frac{1}{2E_D(\boldsymbol{q})} \\ &\times \frac{i}{p_2^0 + q^0 - E_D(\boldsymbol{q}) + i\epsilon}\frac{i}{q^2 - m_\pi^2 + i\epsilon}\frac{i}{(p_2 - p_4 + q)^2 - m_\pi^2 + i\epsilon}\,\boldsymbol{q}^4 \end{split} \qquad \qquad Im\frac{1}{x + i\epsilon} = -i\pi\delta(x)$$

$$ImV_{\rm box} = -6\frac{1}{8\pi} \frac{1}{\sqrt{s}} q^5 E_{D^*}^2 (\sqrt{2}g)^2 \left(\frac{G'}{2}\right)^2 \left(\frac{1}{(p_2^0 - E_D(\boldsymbol{q}))^2 - \boldsymbol{q}^2 - m_\pi^2}\right)^2 F^4(q) F_{HQ}$$

$$q = \frac{\lambda^{1/2}(s, m_{D^*}^2, m_D^2)}{2\sqrt{s}}; \qquad E_{D^*} = \frac{\sqrt{s}}{2} \qquad F(q) = e^{((q^0)^2 - q^2)/\Lambda^2} \qquad q^0 = p_1^0 - E_{D^*}(q) \qquad F_{HQ} = \left(\frac{m_{D^*}}{m_{K^*}}\right)^2$$

 $\Lambda = 1200~{\rm MeV}$ 



Masses and widths of the exotic molecular \$B<sub>(s)</sub><sup>(\*)</sup> B<sub>(s)</sub><sup>(\*)</sup>\$ states L.~R.~Dai, E.~Oset, A.~Feijoo, R.~Molina, L.~Roca, A.~M.~Torres and K.~P.~Khemchandani





TABLE V. States of  $J^P = 1^+$  obtained from different configurations. The binding B is referred to the closest threshold.

States	$M ({ m MeV})$	$B~({ m MeV})$	Γ
$B^*B \ (I=0)$	10583	21	14  eV
$B_s^*B - B^*B_s \ (I = \frac{1}{2})$	10681	11	$45 \mathrm{eV}$
$B^*B^*\ (I=0)$	10630	19	$8 { m MeV}$
$B_s^*B^*~(I=\frac{1}{2})$	10728	12	$0.5 { m MeV}$

## Conclusions

In the recently observed states in the LHCb, Belle, Babar, BesIII, there are many states which qualify as dynamically generated from the interaction of hadron components: molecular states

Many of these states were predicted before. The experiment has served to fine tune some parameters which allow to make more refined predictions for other states not yet found.

The chiral unitary approach in the SU(3) sector has proved to be quite accurate to study the interaction of hadrons and eventually find poles in the t-matrix that correspond to states

The local hidden gauge aproach, with the exchange of vector mesons, is equivalent to the chiral unitary approach in SU(3). An extension of the LHGA has been done to the charm and bottom sectors, which respects heavy quark symmetry and turns out rather accurate interpreting results and making predictions.

More predictions have been made. We hope that they can be tested in the near future.

Attention must also be payed to hybrids of q qbar or qqq and molecular components. J. Nieves, F. K. Guo, David Rodriguez Entem .....



TABLE II. The pole position of the  $T_{cc}^+$  relative to the  $D^{*+}D^0$  threshold and the Riemann sheet (RS) where the pole is located in each scheme (see the text for details). The errors are statistical propagated from fitting to the LHCb data while the uncertainties from the cutoff variation are well within the errors quoted here.

Scheme	Ι	II	III
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$ (RS-I)	$-333^{+41}_{-36} - i(18 \pm 1)$ (RS-II)	$-356^{+39}_{-38} - i(28 \pm 1)$ (RS-II)

## Unveiling the $K_1(1270)$ double-pole structure in the $\overline{B} \rightarrow J/\psi\rho\overline{K}$ and $\overline{B} \rightarrow J/\psi\overline{K}^*\pi$ decays PHYSICAL REVIEW D 103, 116019 (2021)

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# How much is the compositeness of a bound state constrained by a and $r_0$ ? The role of the interaction range

Arxiv 2201.04414

Jing Song,<sup>1,2,\*</sup> L.R.Dai,<sup>3,2,†</sup> and E.Oset<sup>2,‡</sup>

$$V_{\text{eff}} = V_0 + \beta(s - s_0) \qquad T(s) = \frac{1}{[V_0 + \beta(s - s_0)]^{-1} - G(s)} \qquad T(s) = \frac{1}{[\frac{1}{G(s_0)} + \beta(s - s_0)]^{-1} - G(s)}$$

$$G_l = \int_{|q| < q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{w_1(q) + w_2(q)}{2w_1(q)w_2(q)} \frac{1}{s - (w_1(q) + w_2(q))^2 + i\epsilon}$$

$$P_2 = 1 - P_1 = Z = -g^2 G(s_0)^2 \beta \qquad \qquad g^2 = \frac{1}{-G(s_0)^2 \beta - \frac{\partial G}{\partial s}|_{s_0}}$$

$$8\pi\sqrt{s}\left\{\left[\frac{1}{G(s_0)} + \beta(s-s_0)\right]^{-1} - \operatorname{Re}G(s)\right\} \approx \frac{1}{a} - \frac{1}{2}r_0k^2$$



