Hadron Spectroscopy: The Next Big Steps
14–25 Mar 2022
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Europe/Berlin timezone

From the line shape of $X(3872)$ and $T_{cc}(3875)$ to their structure

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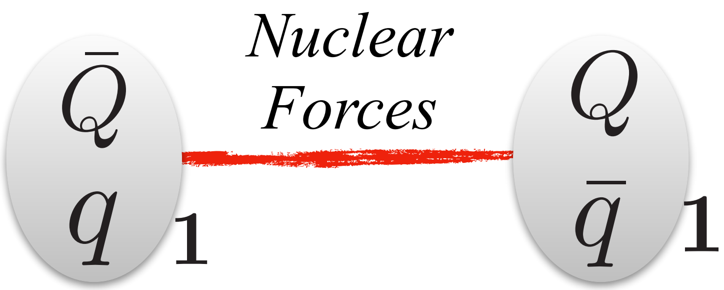
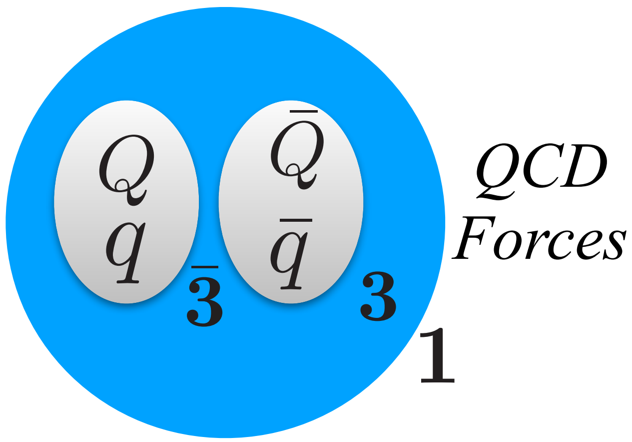
Introduction



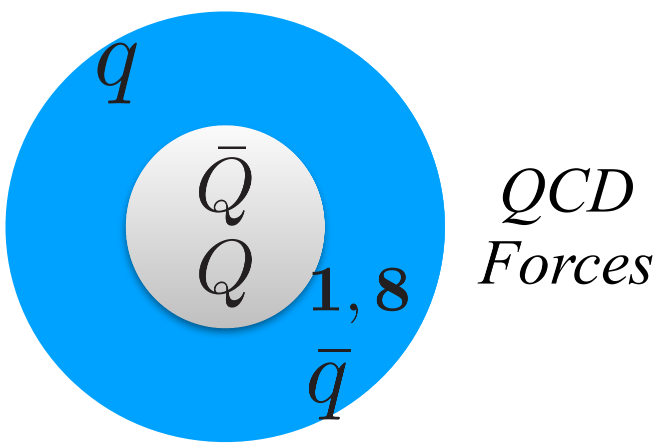
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; **D 89** (2014) 114010.



HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B.Voloshin, Phys. Lett. **B 666**,(2008) 344.

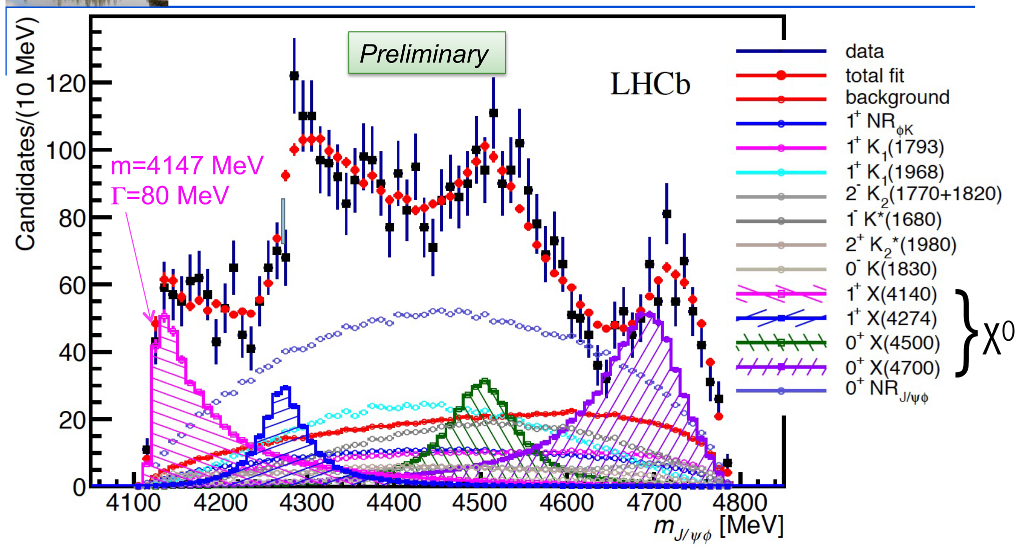
E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. **D 90** (2014) 01404

For a review, see:
A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)

1. Exotics: the new wave

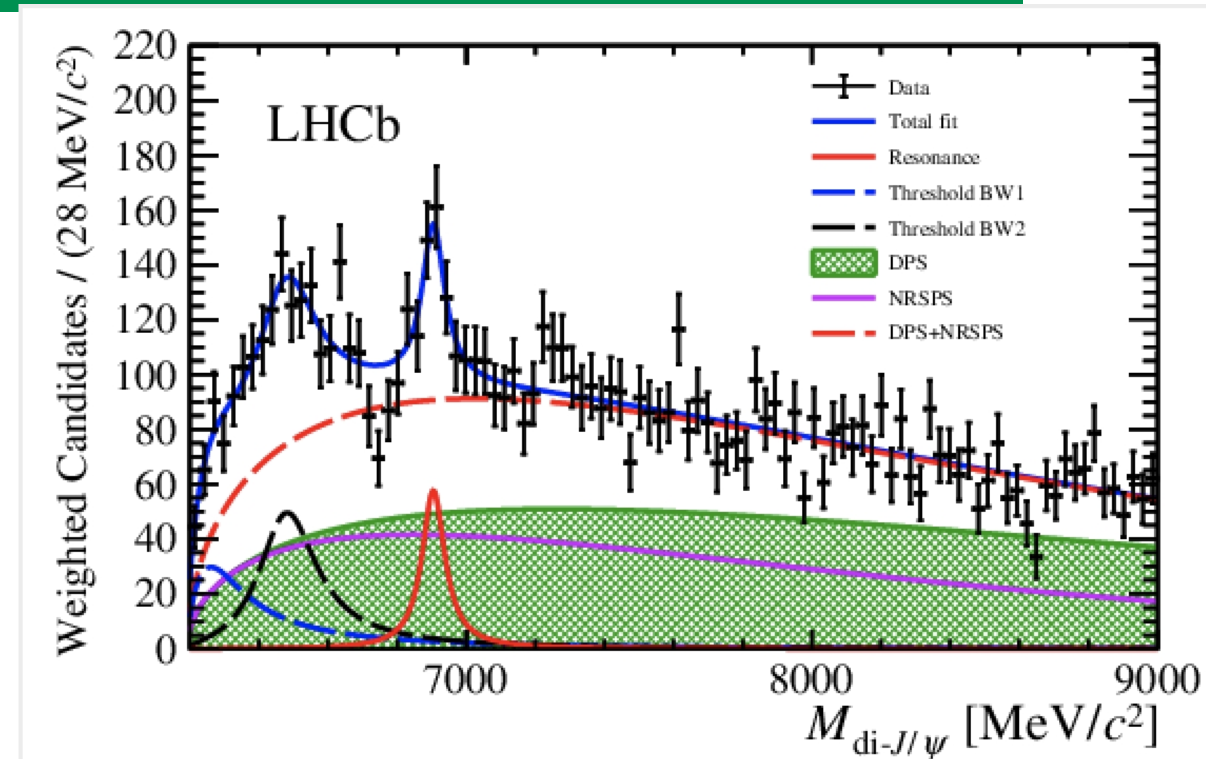
$B^+ \to K^+ + X(4140) \to K^+ + \phi\ \Psi$, etc .

Results of fit: m(J/ψφ)



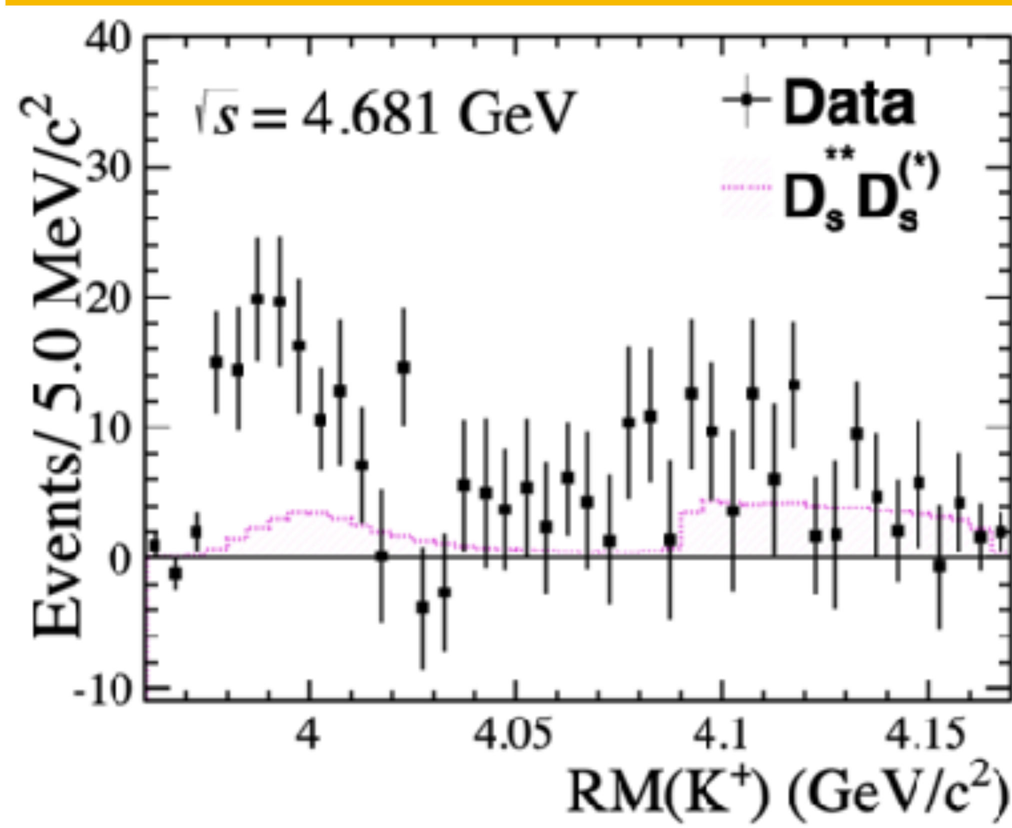
- 4 visible structures fit with BW amplitudes

LHCb (2016): $\Psi\ \phi$ resonances (2016)

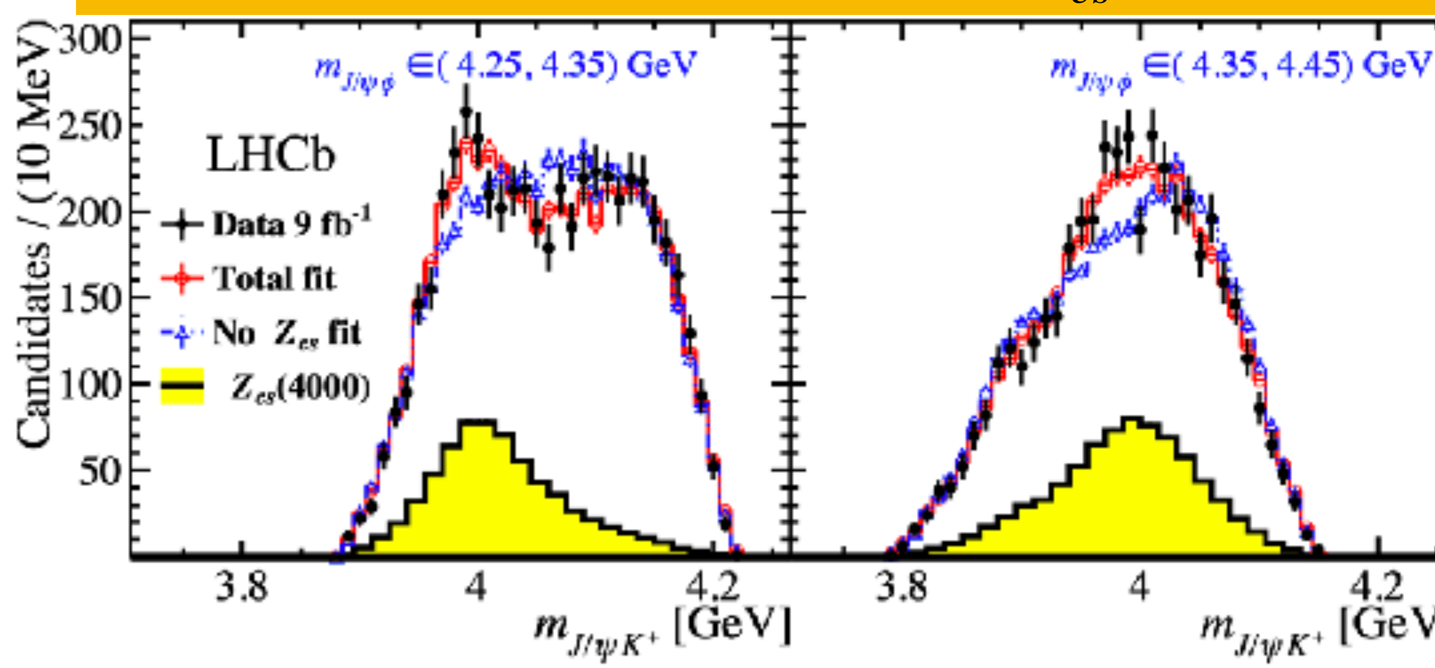


LHCb (2020): di-Ψ resonance(s) spectrum

BES III (2021): $e^+e^- \to K^+ + Z_{cs}^-(3985) \to K^+(D_s^* D^0 + D_s^- D^{*0})$



LHCb (2021): $B \to \Psi + K^+ + \phi \to Z_{cs}(4003) + \phi$



new wave (cont'd)

- Starting from 2016, new kinds of exotic hadrons have been discovered:

  - $J/\Psi\ \phi$ resonances, $di - J/\Psi$ resonances,

  - open strangeness Exotics: $Z_{cs}(3082)$ and $Z_{cs}(4003)$

- No pion exchange forces could bind them as hadron molecules made by color singlet mesons

- molecular models applied to the have to stand on the existence of "phenomenological forces" with undetermined parameters

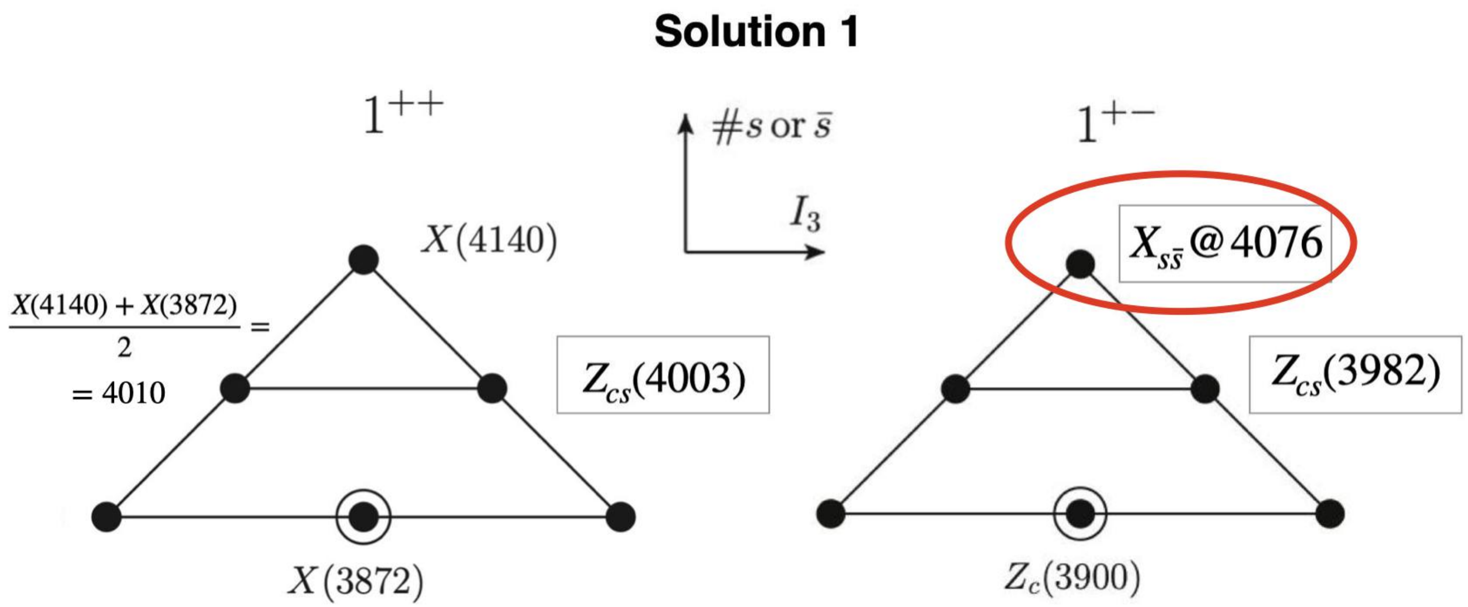
- The New Exotics arise very naturally as $([cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']^{\mathbf{3}})_{\mathbf{1}}$ bound in color singlet

- the compact tetraquark model makes a firm prediction: hidden charm tetraquarks must form ***complete multiplets flavor SU(3)***, with mass differences determined by the quark mass difference $m_s - m_u$.

- ***with $Z_{cs}(3082)$ and $Z_{cs}(4003)$ we can almost fill two tetraquark nonets with the expected scale of mass differences***

Two nonets: Solution 1 (preferred)

L. Maiani, A. D. Polosa and V. Riquer, Sci. Bull. **66** (2021), 1616, arXiv:2103.08331

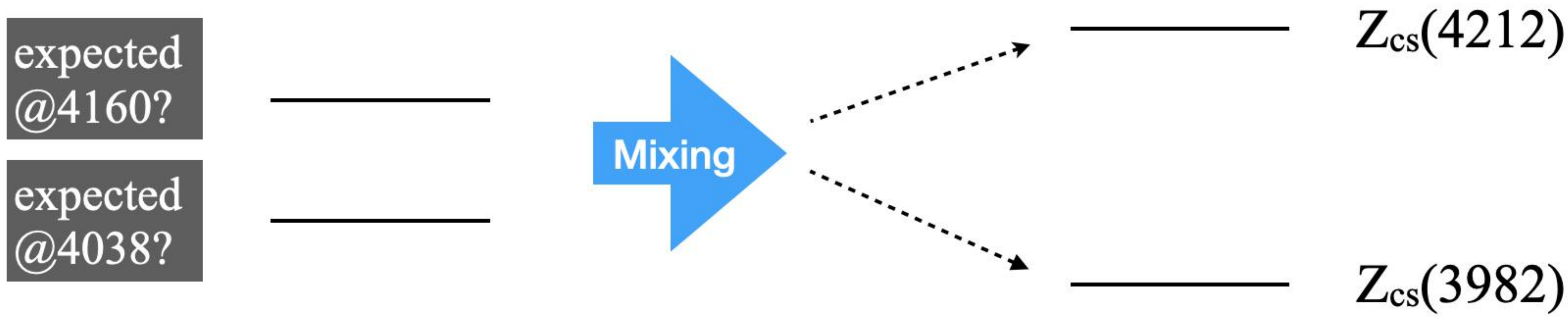


A well defined shopping list towards completion:

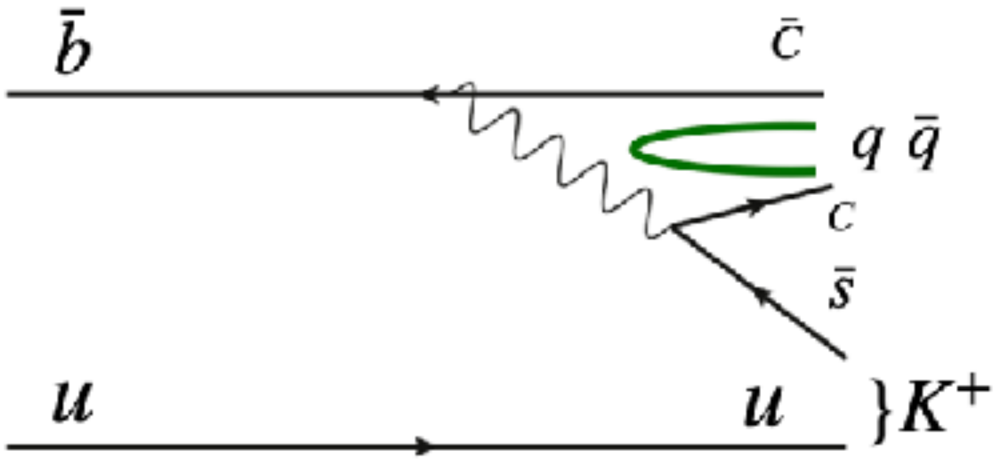
- $X_{s\bar{s}}$, $M = 4076$ (Sol . 2 : 4121), decays: $\eta\psi$, $\eta_c\ \phi$, $D_s^*\bar{D}_s$ (if phase space allows)
- the I=1 partner of X(3872), decays: $X^+ \to J/\psi\ \rho^\pm \to J/\psi\ \pi^+\pi^0$
- the I=0 partners of $Z_c(3900)$ and $Z_c(4020)$, possibly decaying into: $J/\psi + f_0(500)$ (aka $\sigma(500)$)

•There is a ***third nonet*** associated to $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, Mass=4150 - 4170

•***LHCb sees a $Z_{cs}(4220)$, $J^P = 1^+$or 1^-: is it too heavy ?*** A bold proposal:

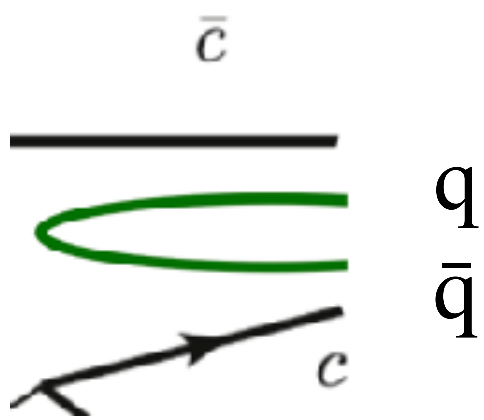


2. Molecule or compact? Back to the fundamentals

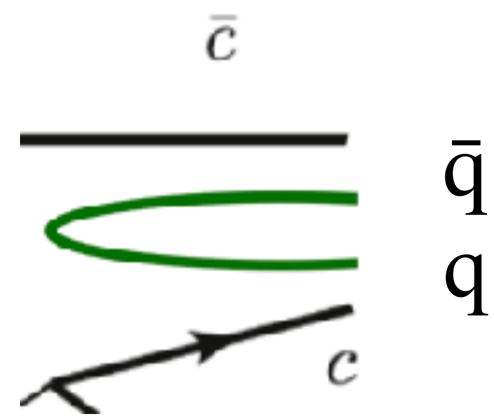


$\to D^*\bar{D} + \bar{D}^*D$

- The interesting part is the upper left corner, which we can specify in two ways



or



or both?

$$[\bar{C}q]^1[C\bar{q}]^1 \to D^*\bar{D} + \bar{D}^*D \qquad [Cq]^{\bar{3}}\ [\bar{C}\bar{q}]_3 \to D^*\bar{D} + \bar{D}^*D$$

- for a neutron-proton pair, the question posed by Weinberg was;
  is the deuteron a bound state or is there an "elementary" dibaryon ?

Molecule or compact? the QCD framework

- We know for sure that QCD produces hidden charm, confined hadron states: charmonia, $D^*\bar{D} + \bar{D}^*D$. ??? Do confined tetraquarks exist??
- Suppose we switch off the interactions between confined hadrons. The space of possible hidden charm states is made by two components
  - discrete energy states: charmonia and possibly tetraquarks: $|C><C| + |T><T|$
  - continuum charmed meson pairs: $\int d\alpha\, |D^*\bar{D}(\alpha)><D^*\bar{D}(\alpha)|$

In this limit: $<X|X> = 1 = Z + \int d\alpha\, |<X|D^*\bar{D}(\alpha)>|^2$, where,

$Z = |<X|C>|^2 + |<X|T>|^2$

There are ***two regimes*** Z=0: corresponds to a pure molecular state: X results from $D^* - \bar{D}$ interactions only (like the deuteron)

- $Z \neq 0$: some compact, discrete state ***must*** exist
- unlike charmonium states, X decays violate isospin: $\Gamma(\Psi\rho) \sim \Gamma(\Psi\omega)$

so that:

- $Z \neq 0$ ➜ ***Tetraquark with X quantum numbers most likely exists***.

How can we know?

- The key is the $D^*\bar{D}$ scattering amplitude, *f*, that near threshold (k=center of mass momentum~0) can be parametrised as

$$f^{-1} = k\cot\delta(k) - ik = -\kappa_0 + \frac{1}{2}r_0k^2 - ik + \dots$$

- With Weinberg, we find

S. Weinberg, Phys.Rev. **137**, (1965) B672

$$\kappa_0^{-1} = 2\frac{1-Z}{2-Z}\kappa^{-1} + O(1/m_\pi);\ \ r_0 = -\frac{Z}{1-Z}\kappa^{-1} + O(1/m_\pi)$$

valid for a shallow resonance

$$\kappa^{-1} = \sqrt{2\mu B},\ B = M(D^*) + M(D) - M(X)\ \text{(the “binding energy”)}$$

- It turns out that ***the parameters*** κ_0, r_0 ***can be determined from the X(3872) (or*** T^+_{cc}***) line-shape***

R. Aaij *et al*. (LHCb), PRD **102**, (2020) 092005

- in the molecular case (Z=0) one has $r_0 = O(1/m_\pi)$…
- … and, for attractive potentials, one can show that the unspecified part of $O(1/m_\pi)$ is positive:

$$r_0 > 0$$

Reported as the solution to a Problem in Landau and Lifshitz, *Quantum Mechnics*
H.Bethe, 1949, gives a concise demonstration
see A. Esposito *et al*. Phys. Rev. **D 105** (2022), L031503

3. X lineshape: from Breit-Wigner to scattering lengths

- Consider $D^{*0}\bar{D}^0$scattering above threshold. If there is a resonance slight below, the amplitude takes the Breit-Wigner form

$$f = -\frac{\frac{1}{2}g^2_{\mathrm{BW}}}{E - m_{\mathrm{BW}} + \frac{i}{2}g^2_{\mathrm{BW}}k} \quad \text{(A)}$$

- for E= $\dfrac{k^2}{2\mu}$, the BW has the same form of the scattering amplitude

$$f = \frac{1}{\cot\delta(k) - ik} = \frac{1}{-\kappa_0 + \frac{1}{2}r_0k^2 - ik + \dots} \quad \text{(B)}$$

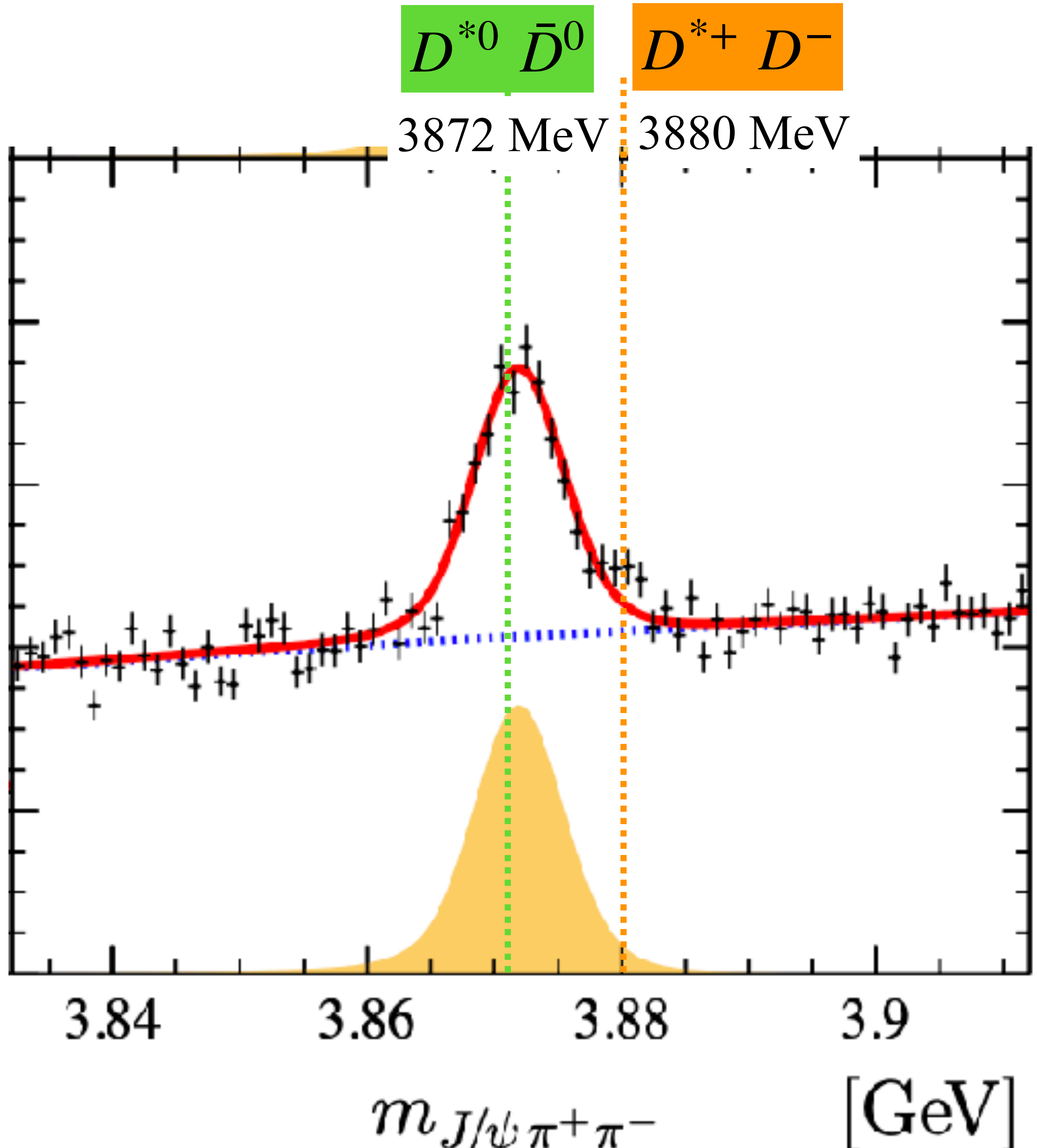
and from the parameters of the line-shape (A) we can determine κ_0 and r_0 (B)

- neglecting experimental errors on the parameters, for the X(3872) and the LHCb data, we find:
- $\kappa_0 \simeq 6.92$ MeV; $r_0 = -5.3$ fm, well into the compact tetraquark region.
- using a more recent error analysis, the effective radius is found to be in the range

$$-1.6\ \text{fm} > r_0 > -5.3\ \text{fm}$$

A. Esposito *et al*. Phys. Rev. **D 105** (2022), L031503 [arXiv:2108.11413v2 [hep-ph]].

V. Baru et al., arXiv:2110.07484



$D^{*0}\,\bar{D}^0$
$D^{*+}\,D^-$
3872 MeV
3880 MeV
3.84
3.86
3.88
3.9
$m_{J/\psi\,\pi^+\pi^-}$
[GeV]

Details

- Flattte' function to fit the X(3972) lineshape to determine the parameters m_X^0, g_{LHCb}

$$f\left(X \to J/\psi\pi^+\pi^-\right) = -\frac{N}{E - m_X^0 + \frac{i}{2}g_{\rm LHCb}\left(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)}\right) + \frac{i}{2}\left(\Gamma_\rho^0(E) + \Gamma_\omega^0(E) + \Gamma_0^0\right)}$$

$$\mu = \frac{D^{*0}\bar{D}^0}{D^{*0} + \bar{D}^0} = 967 \text{ MeV} \qquad \mu^+ = \frac{D^{*+}D^-}{D^{*+} + D^-} = 969 \text{ MeV} \qquad \delta = D^{*+} + D^- - D^{*0} - \bar{D}^0 = 8.3 \text{ MeV} >> E$$

- Parametrization of the denominator

$$Den = E - m_X^0 + \frac{i}{2}g_{\rm LHCb}\left(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)}\right) \sim \frac{2}{g_{\rm LHCb}}(T - m_X^0) - \sqrt{2m_+\delta} + T\sqrt{\frac{m_+}{2\delta}} + ik \qquad T = \frac{k^2}{2\mu}$$

$$\kappa_0 = -\frac{2m_X^0}{g_{\rm LHCb}} - \sqrt{2\mu_+\delta} \simeq 6.92 \text{ MeV}$$

$$r_0 = -\frac{2}{\mu g_{\rm LHCb}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34\ fm$$

best fit:

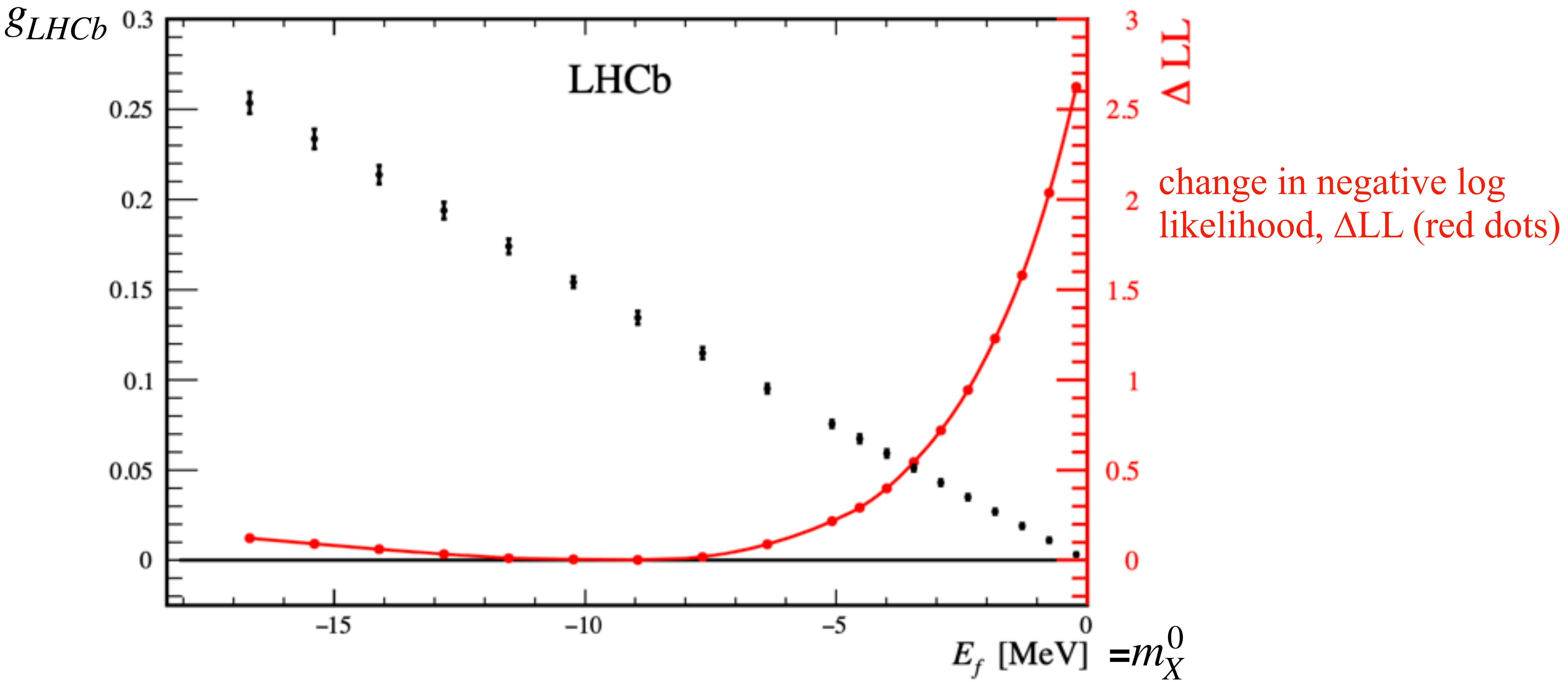
$g_{LHCb} = 0.108$

$m_X^0 = -7.18$ MeV

$$-\frac{2m_X^0}{g_{LHCb}} = 133 \text{ MeV} \qquad \sqrt{2\mu_+\delta} = 127 \text{ MeV}$$

- Taking into account the error on g_{LHCb}:

$$-1.7 \text{ fm} > r_0 > -5.3 \text{ fm}$$



- Log Likelihood is very insensitive to the value of g_{LHCb}
- $10 > g_{LHCb} > 0.108$
- leads to the range of r_0

4. The doubly charmed Tetraquark, T_{cc}^{+}

- The existence of doubly charmed tetraquarks, $[QQ\bar{q}\bar{q}]$, was considered in 2013 by Esposito et al.

Esposito *et al*, PRD **88**(2013) 054029)

- Starting from the mass of the doubly charmed baryon, Karliner and Rosner estimated of the mass of the lowest lying, I=0 state at $M(T_{cc}^{+}) = 3882 \pm 12$ MeV, 7 MeV above the $D^0 D^{*+}$ threshold.

M. Karliner and J. L. Rosner, PRL **119**(2017) 202001.

- A similar value was obtained by Eichten and Quigg

E. J. Eichten and C. Quigg, PRL **119** (2017) 202002

- A value close to the $D^0 D^* \gamma$ threshold is obtained in the Born -Oppenheimer Approximation, using constituent quark masses derived from the meson spectrum (recently re-evaluated !!)

L. Maiani et al., PRD **100** (2019) 074002

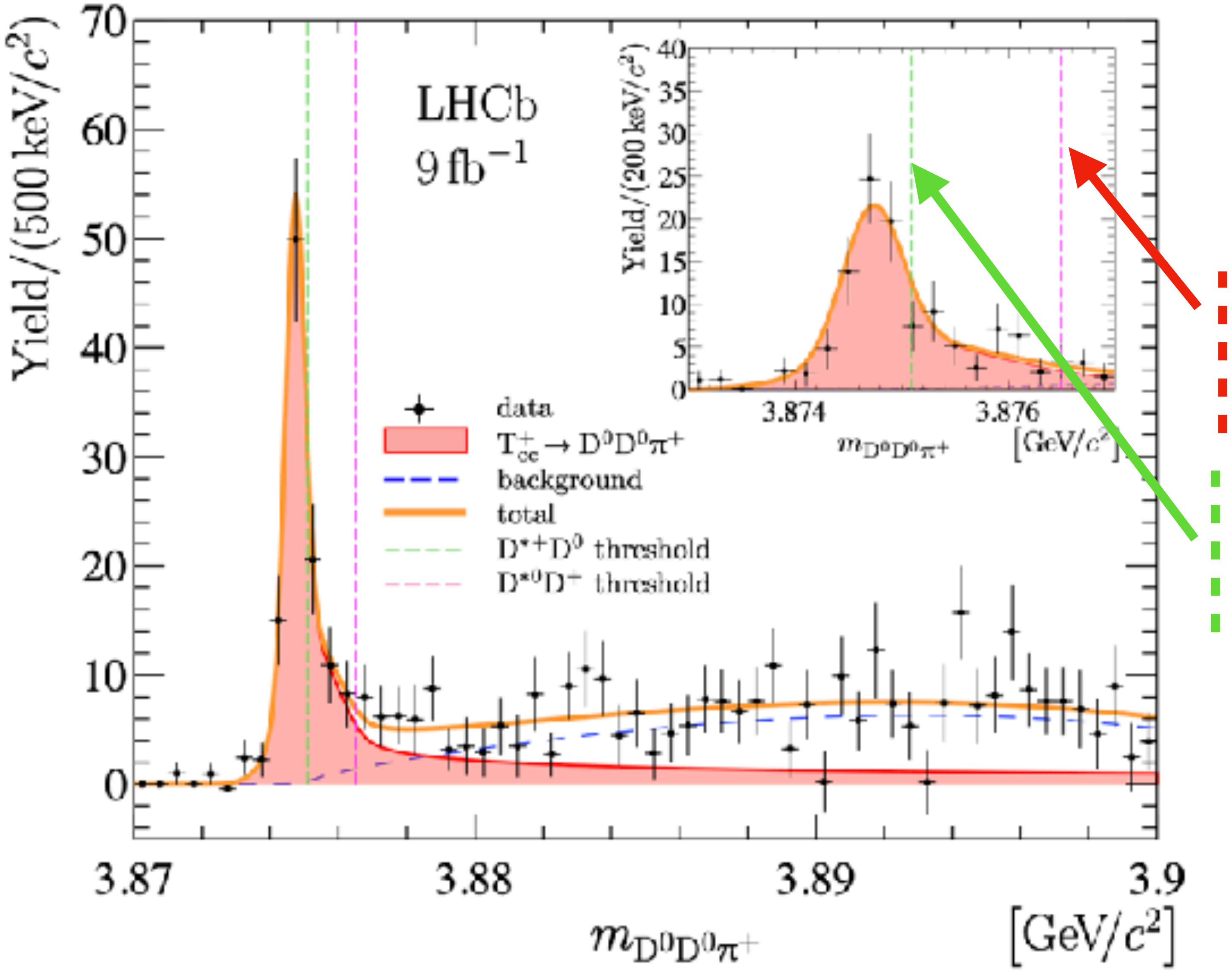
- The value $M(T_{cc}^{+}) - M(D^0 D^{+}) = -23 \pm 11$ MeV is obtained in lattice QCD calculation

P. Junnarkar *et al*, PRD **99**(2019) 034507

- The closeness to the $D^0 D^{*+}$ threshold has nonetheless invited speculations about a molecular, $D^0 D^{*+}$, nature of T_{cc}^{+}.

Latest: double charm tetraquark

LHCb arXiv:2109.01056v2



Mass : $D^{*0}D^+ = 3876.5$ MeV

Mass : $D^{*+}D^0 = 3875.1$ MeV

for $\mathcal{T}_{cc}^{+}$

$$\mu = \frac{D^{*+}D^0}{D^{*+} + \bar{D}^0} = 967.5 \text{ MeV} \quad \mu^+ = \frac{D^{*0}D^+}{D^{*0} + D^+} = 968.0 \text{ MeV} \quad \delta = D^{*0} + D^+ - D^{*+} - \bar{D}^0 = 1.7 \text{ MeV}$$

$$(r_0)_{u.l.} = \sqrt{\frac{\mu_+}{2\mu^2\delta}} = -3.4 \text{ fm}$$

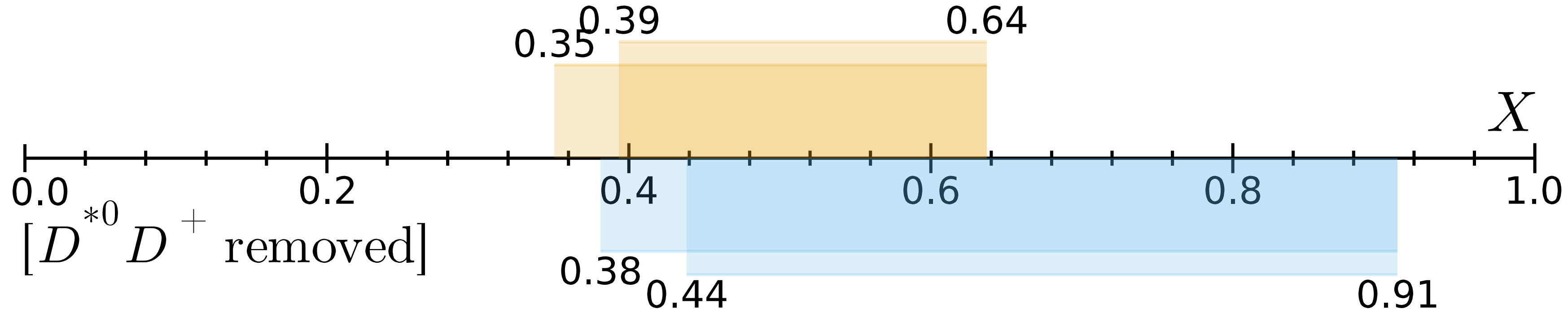
lower limit
$(r_0)_{l.l.} = -11.9 \text{ fm}$

agrees with Mikhasenko's result: $r_0 = -3.7 \text{ fm}$
ArXiv:2203.04622v1

LHCb, arXiv:2109.01056

from Mikhasenko's paper (Arxiv:2203.04622v1)

$$X = 1 - Z = \frac{1}{\sqrt{1 - 2r_0\kappa_0}}$$



$$-1.8 > r_0 > -11 \text{ fm}$$

my estimate from Mikasenko's figures !!!

5. The value of Z

- From previous Weinberg formulae, we derive

$$Z = \frac{-r_0\kappa}{1 - r_0\kappa}$$

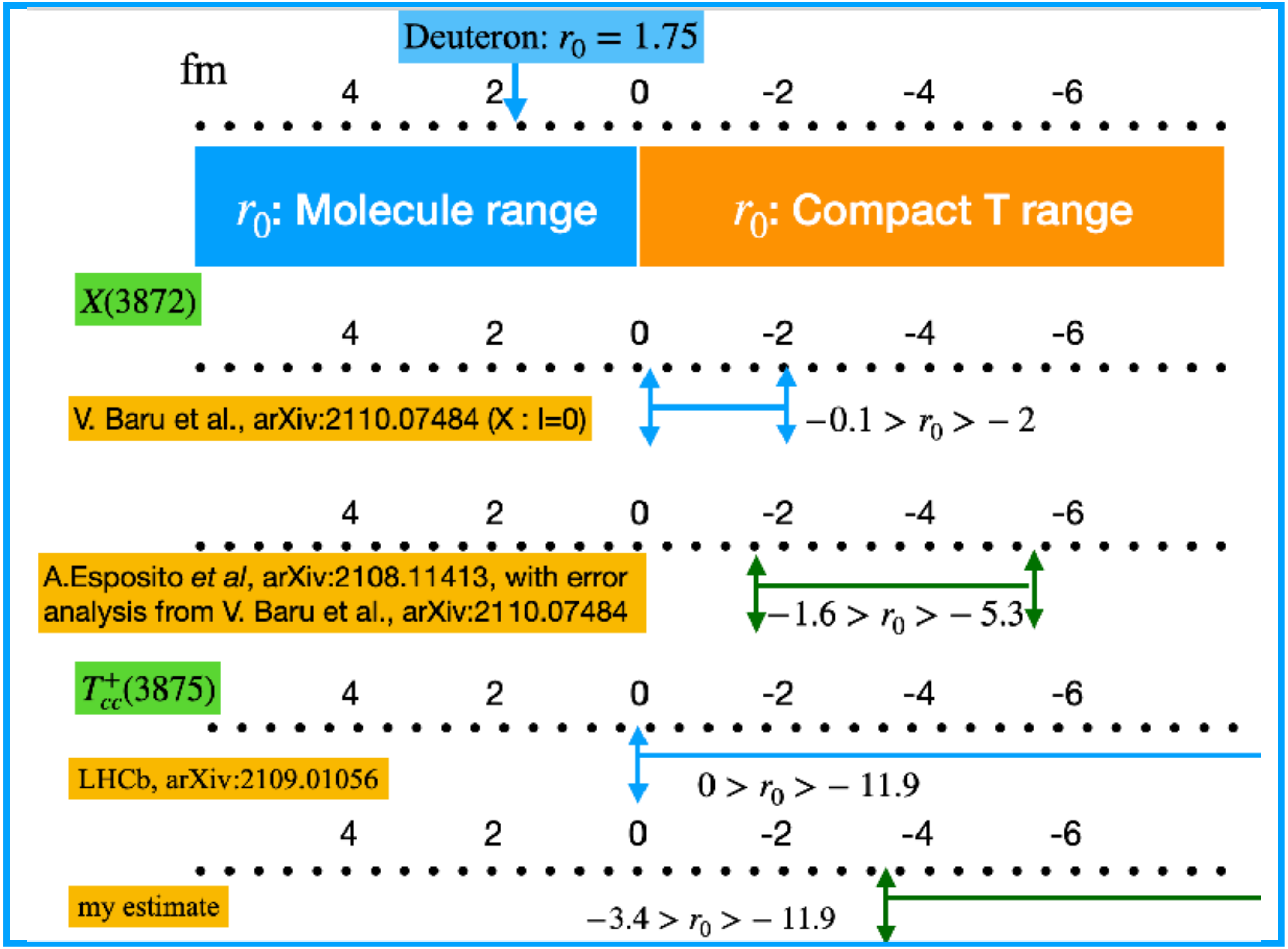
and with $\kappa \simeq \kappa_0$, we find, for X(3872):

$$0.14 > Z > 0.052 > 0$$

- Z is often identified with the admixture of X with the compact (tetraquark) state. In this case one would say that X is "essentially" a molecule
- However, the interpretation of Z as mixing coefficient, ***holds true in the free theory only***. With interaction, the state vector corresponding to the compact state may be renormalized and the strenght of Z losses its meaning.

- We think that what counts is that Z is non vanishing, indicating that ***there are***, in the Hilbert space, discrete states different from the D D* continuum to which X has a non-vanishing projection. This is stated clearly in Weinberg's paper:
  - *the true token that the deuteron is composite is an effective range r_0 small and positive rather than large and negative*
  - *an elementary deuteron would have* 0<Z<1.
- $0 < Z < 1$ does not say anything about the existence of bound states in the inter-hadron $D^*\bar{D}$ potential, i.e. molecules: the interaction could be driven by the compact state only and be consistent with no bound molecule at all.

My Summary about r_0



A new analysis by the Valencia group claims $r_0 \sim +1$ fm for T_{cc}^+.

No consensus yet, but it seems we are on a very promising road.
Stay tuned!!