The HISH (Holographic Inspired Stringy Hadron) model: Spectra, Decay processes and exotic hadrons

with Dorin Weissman Mainz March 2022



Summary The HISH model

- The derivation of the HISH (Holography Inspired Stringy Hadron) model will be briefly reviewed at the end of the talk. The main results related to the spectra and strong decay processes are:
- A **meson** is described as an open string with massive quark and an anti-quark on its ends.
- A baryon is an open string with a quark on one side and a baryonic vertex (bv.) + a di-quark on the other side
- A **glueball** is a **close** string
- A tetra-quark is an open string with a bv. + a diquark on one side and an anti bv. and anti- di-quark on the other side.

Light stringy hadrons in holography and HISH

 Light stringy mesons and baryons in holography and HISH





Summary The HISH model

The spectra of the hadrons include: Ground state-

The tension is balanced by a ``repulsive Casimir force"



Summary The HISH model

Excited states with angular momentum Classically for a rotating string with massive endpoints



Small and large msep approximations

• For small msep, and $\beta_i \rightarrow 1$ the modified Regge trajectory is

$$J = \alpha' E^{2} \times \left(1 - \sum_{i=1}^{2} \left(\frac{4\sqrt{\pi}}{3} \left(\frac{m_{i}}{E}\right)^{3/2} + \frac{2\sqrt{\pi^{3}}}{10\sqrt{2}} \left(\frac{m_{i}}{E}\right)^{5/2} + \cdots\right)\right)$$

• For large msep and $\beta_i \rightarrow 0$,

$$J = \frac{4\pi}{3\sqrt{3}} \alpha' \sqrt{\frac{m_1 m_2}{m_1 + m_2}} (E - m_1 - m_2)^{3/2} + \frac{7\sqrt{2\pi}}{27\sqrt{3}} \alpha' \frac{m_1^2 - m_1 m_2 + m_2^2}{m_1 m_2 \sqrt{(m_1 + m_2)^3}} (E - m_1 - m_2)^{5/2} + \cdots$$

Summary The HISH model

The intercept is related to the Casimir energy of the quantized string. For an ordinary string with no massive endpoints

$$E_{Casimir} \equiv \frac{1}{2} \sum_{n=1}^{n=\infty} w_n = \frac{\pi (D-2)}{2L} \sum_{n=1}^{n=\infty} n = -\frac{(D-2)}{24} \frac{\pi}{L} = a \frac{\pi}{L}$$

 For a string with quarks on its ends the eigen frequencies are given by _____q=m/TL



Summary The HISH model

• "Radial" excited string states-

• The excited states of ordinary string with no angular momentum have a spectrum of

$$N = \alpha' M^2 + a$$

• For string with massive endpoints

$$w_N = \alpha' (M - m_1 - m_2)^2 + a_m$$



The strong decay width

• A string calculation for the break of the string yields linear dependence on the length L

(b)

$$\Gamma = \frac{\pi}{2} ATL(M, m_1, m_2, T) \,.$$

• For short strings with important role of the massive endpoints we add a **phase space factor**

$$\Gamma = \frac{\pi}{2} A \times \Phi(M) \times TL(M, m_1, m_2, T).$$

The phase space factor

$$\Phi(M, M_1, M_2) \equiv 2\frac{|p_f|}{M} = \sqrt{\left(1 - \left(\frac{M_1 + M_2}{M}\right)^2\right)\left(1 - \left(\frac{M_1 - M_2}{M}\right)^2\right)}$$

The branching ratios

Assuming first that the string stretches in flat spacetime we found using both a string beads model and a continues one that

$$\Gamma = \text{Const} \quad \exp\left(-1.0\frac{z_B^2}{\alpha'_{\text{eff}}}\right) = \exp\left(-2\pi\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

 There are further corrections due to the curvature and due to the massive endpoints.

•
$$\Gamma = \exp\left(-2\pi C(T_{\text{eff}}, M, m_i)\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

$$C(T_{\text{eff}}, M, m_i) \approx 1 + c_c \frac{M^2}{T_{\text{eff}}} + \sum_{i=1}^2 c_{m_i} \frac{m_i}{M}$$

The branching ratios

- What is the probability that the string breaks and a pair of quark anti-quark is created.
- In holography this translates to the probability that a hadronic string hit a flavor brane



Outline:

- I. A brief review of the HISH model
- A. Hadrons in holographic backgrounds and in HISH
- B. Mesons open string between massive endpoints
- C. Baryons- open strings between a quark and a di-quark
- D. Glueballs –closed strings
- E. Exotic hadrons
- F. Width of hadronic decays
- II. Fitting with PDG data and predictions
- A. Spectra of Mesons
- B. Spectra of Baryons
- C. Glueballs
- D. Exotic hadrons
- E. Decay Width

Maldacena correspondence



- The curvature in string-size units sets the 't Hooft coupling: $\lambda \sim \left(\frac{R}{l_{\text{tring}}}\right)^4$.
- $R \gg l_{\text{string}}$: strings reduce to gravity: gauge/gravity correspondence.
- We will make this relation precise in a moment.

From Ads/CFT to general string(gravity)/gauge duality

- The basic duality relates the string theory on Ads5xS5 to N=4 SYM. Both sides are invariant under the maximal super-conformal symmetries.
- To get to non-susy YM theory we need to break all the supersymmetries.
- The need to introduce a scale in the gauge theory translates to deforming the bulk into a non-Ads one.
- What are bulk geometries that correspond to confining gauge theories? Confining means here admitting an area law Wilson line

Geometry encodes 4d physics

- The 4d physics is encoded in the higher-dimensional geometry.
- We now know which geometries yield confining or finite-T duals.



Witten's model of confining background



Adding flavor

- We would like to introduce flavor degrees of freedom
- We add N_f flavor branes to a non-supersymmetric confining background.
- A natural candidate is therefore *Witten's model*.
- For N_f<< N_c the flavor brane do not back-react on the background thus they are probe branes
- To assure chiral symmetry one adds D8 branes and anti D8 branes
- Their U shape profile associate with a spontaneous breaking of the UV chiral flavor global symmetry of U(Nf)xU(Nf) to a U(Nf)D in the IR

Stringy holographic Hadrons

(1) The rotating holographic string meson

• The structure of a holographic meson is a string connected to a flavor branes



 The string is the classical solution of the Nambu-Goto action defined in confining holographic background

Adding flavor: The Sakai Sugimoto model

Adding Nf D8 anti-D8 branes into Witten's model
In the cigar geometry the flavor brane have a U shape profile



Example: The B meson



(2) Stringy Baryons

• How do we identify a baryon in holography ?

- Since a quark corresponds to an end of a string, the baryon has to be a structure with N_c strings connected to it.
- The proposed **baryonic vertex** in holographic background is is a wrapped Dp brane over a p cycle
- Because of the RR flux in the background the wrapped brane has to be connected to Nc strings

Dynamical baryon



A possible baryon layout

 A possible dynamical baryon - Nc strings connected in a symmetric way to the flavor brane and to the baryonic vertex which is also on the flavor brane.



Nc-1 quarks around the Baryonic vertex

An asymmetric possible layout is that of one quark connected with a string to the baryonic vertex to which the rest of the Nc-1 quarks are attached.



(3) Glueballs as closed strings

- Mesons are open strings connected to flavor branes.
- Baryons are Nc open strings connected to a baryonic vertex on one side and to a flavor brane on the other one.
- What are glue balls?
- Since they do not incorporate quarks it is natural to assume that they are rotating closed strings
- Angular momentum associates with rotation of folded closed strings



(4) Holographic exotic hadrons

 A configuration of a bryonic vertex connected to a u c di-quark and connected to an anti-baryonic vertex which is connected to anti- u and anti- c



Hadrons of the (H15H)

Holographic Inspired stringy



HISH- Holography Inspired Stringy Hadron

- The construction of the HISH model is based on the following steps.
- (i) Analyzing classical string configurations in confining holographic string models that correspond to hadrons.
- (ii) Performing a transition from the holographic regime (for fields) of large Nc and large λ to the real world that bypasses expansions in $\frac{1}{N_c}$ and $\frac{1}{\lambda}$
- (iii) Proposing a model of stringy hadrons in **flat four dimensions with massive endpoint particles** that is **inspired** by the corresponding holographic model
- (iv)Dressing the endpoint particles with baryonic vertex, charge, spin etc
- (v) Confronting the outcome of the models with **experimental data** .

The HISH map of a stringy hadron

• The basic idea is to approximate the classical holographic spinning string by a string in flat space time with massive endpoints. The masses are m_{sep_1} and m_{sep_2}



String end-point mass

• We define the string end-point quark mass

$$m_{sep} = T \int_{u_0}^{u_f} g(u) du = T \int_{u_0}^{u_f} \sqrt{G_{00}G_{uu}} du$$

The boundary equation of motion is

 $T_{eff}(1 - v^2) = m_q \omega^2 R_0$ $T_{eff} = Tf = TG_{00}$

$$\frac{T_{eff}}{\gamma} = m_{sep} \gamma \omega^2 R_0$$

• This simply means that the tension is balanced by the (relativistic) centrifugal force.

HMRT- HISH Modified classical Regge trajectory

- For strings with massive endpoints one determines the solution of the classical EOM that corresponds to a rotating string
- The classical energy and angular momentum

$$E = \sum_{i=1,2} \left(\gamma_i m_i + T\ell_i \frac{\arcsin \beta_i}{\beta_i} \right)$$

$$J = \sum_{i=1,2} \left[\gamma_i m_i \beta_i \ell_i + \frac{1}{2} T \ell_i^2 \left(\arcsin \beta_i - \beta_i \sqrt{1 - \beta_i^2} \right) \right]$$

Small and large mass approximations

We can get a direct relation in the limits of
Small mass

$$J = \alpha' E^2 - \alpha' \frac{4\pi^{1/2}}{3} (m_1^{3/2} + m_2^{3/2}) \sqrt{E}$$

Large mass

$$J_4 = \frac{2m^{1/2}}{T_3\sqrt{3}} (E - 2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T} (E - 2m)^{5/2} - \frac{1003}{\sqrt{2332803}Tm^{3/2}} (E - 2m)^{7/2}$$

Holographic mesons and glueballs and their map



(ii) The HISH Baryons


(3) Glueballs as closed strings

- Mesons are open strings connected to flavor branes.
- Baryons are Nc open strings connected to a baryonic vertex on one side and to a flavor brane on the other one.
- What are glue balls?
- Since they do not incorporate quarks it is natural to assume that they are rotating closed strings
- Angular momentum associates with rotation of folded closed strings



On the quantization of the HISH

The passage from the classical to quantum bosonic string with no massive endpoints in D=26 is

$$J = \alpha' M^2 \qquad \rightarrow \qquad J = \alpha' M^2 + a$$

• For the excited states with excitation number n

$$n + J = \alpha' M^2 + a$$

• a the intercept is given by the Casimir energy

$$E_{Casimir} \equiv \frac{1}{2} \sum_{n=1}^{n=\infty} w_n = \frac{\pi (D-2)}{2L} \sum_{n=1}^{n=\infty} n = -\frac{(D-2)}{24} \frac{\pi}{L} = a \frac{\pi}{L}$$

Quantum modified Regge trajectory

• With massive endpoints the intercept is modified

$$a \equiv -\frac{D-2}{2\pi} \sum_{n=1}^{\infty} \omega_n$$

• The modified intercept changes the trajectory

$$\delta J - \frac{L}{2}\delta E = a$$

• For the massless case that goes back to the usual liner Regge trajectory

The intercept for a string with massive endpoints

• The eigenfrequencies for a static massive string

$$\tan(\omega_n) = \frac{2q\omega_n}{q^2\omega_n^2 - 1} \quad q = m/TL$$

We computed the intercept using a contour integral

$$a = (D-3)a_t + a_p + a_{PS} \approx 1 - \frac{26 - D}{12\pi} (\frac{2m}{TL})^{1/2} + \frac{199 - 14D}{240\pi} (\frac{2m}{TL})^{3/2}$$



The negative intercept assumption

- In nature the intercept associated with all the hadrons whether mesons or baryons is **negative** when it is defined in relation to the orbital and not the total angular momentum.
- For instance ρ has a 0.5 and S=1 s so for L=J-S we get a= -0.5
- The attempt to explain this universal feature is probably the most important problem of the hadronic spectra.
- To account for it we study strings with different masses, electric charges and spins at their ends.
- We can get negative intercept but not yet in a fully satisfactory manner
- At present we take it as one of the HISH ingredients

Closed strings versus open strings

• The spectrum of states of a **closed** string admits

$$M^2 = \frac{2}{\alpha'} \left(N + \tilde{N} + A + \tilde{A} \right)$$

• The spectrum of an open string

$$M^2_{open} = \frac{1}{\alpha'} \left(N + A \right)$$

• The slope of the closed string is ½ of the open

The closed string ground states has

$$M^2=\frac{2}{\alpha'}(A+\tilde{A})=\frac{2-D}{6\alpha'}$$

• The intercept is 2 that of an open string

Decay width of Stringy

holographic and 4754



The decay of a long string

The decay of a hadron is in fact the breaking of a string into two strings

• A type I open string can undergo such a split



The decay of a long string in critical flat space-time

 The total decay width is related by the optical theorem to the imaginary part of the self-energy diagram

$$2 \operatorname{Im}\left(--\left(\right)\right) = \Sigma_{f} \left|--\left(\right)^{2}\right|^{2}$$

A trick that Polchinski et al used is to compactify one space coordinate and consider incoming and outgoing strings that wrap this coordinate so one can avoid an annulus open string diagram and instead compute a disk diagram with simple vertex operator of a closed string

The string amplitude



The decay of a long string in critical flat space-time

 We would like to determine the dependence of the string amplitude on the string length L



Determination of the suppression factor

Assuming first that the string stretches in flat spacetime we found using both a string beads model and a continues one that

$$\Gamma = \text{Const} \quad \exp\left(-1.0\frac{z_B^2}{\alpha'_{\text{eff}}}\right) = \exp\left(-2\pi\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

 There are further corrections due to the curvature and due to the massive endpoints.

•
$$\Gamma = \exp\left(-2\pi C(T_{\text{eff}}, M, m_i)\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

$$C(T_{\text{eff}}, M, m_i) \approx 1 + c_c \frac{M^2}{T_{\text{eff}}} + \sum_{i=1}^2 c_{m_i} \frac{m_i}{M}$$

The decay of a long string in flat space-time

• After substituting into the integral the amplitude yields

$$\begin{split} i\mathcal{A}_2 &= \frac{iTN\kappa^2}{2\pi g^2} \lim_{t \to 0} \frac{\Gamma(t-1)\Gamma(1-\tilde{J})}{\Gamma(t-\tilde{J})} & \tilde{J} \equiv \frac{L^2T}{2\pi} - 2 \\ & \text{regulator} \\ &= \frac{iTN\kappa^2}{2\pi g^2} \left(\tilde{J}\partial_{\tilde{J}} \ln[\Gamma(-\tilde{J})] + \lim_{t \to 0} \frac{\tilde{J}}{t} \right) \\ & \text{regulator} \end{split}$$

• The imaginary part $\sum_k \pi k \delta(J - k)$ for k = 1,Using Stirling approximation

$$\mathrm{Im}\mathcal{A}_2 = -\frac{iTN\kappa^2}{2g^2}\tilde{J}$$

Check of the linear dependence on L

 The final result for long strings is a linear dependence on the length L

$$\Gamma = \frac{\pi}{2} ATL(M, m_1, m_2, T) \,.$$

• For short strings with important role of the massive endpoints we add a **phase space factor**

$$\Gamma = \frac{\pi}{2}A \times \Phi(M) \times TL(M, m_1, m_2, T)$$

The phase space factor

$$\Phi(M, M_1, M_2) \equiv 2\frac{|p_f|}{M} = \sqrt{\left(1 - \left(\frac{M_1 + M_2}{M}\right)^2\right)\left(1 - \left(\frac{M_1 - M_2}{M}\right)^2\right)}$$

The suppression factor for stringy holographic hadrons

- The horizontal segment of the stringy hadron fluctuates and can reach flavor branes
- When this happens the string may **break up** , and the two new endpoints connect to a flavor brane



Part 2:

Fits of the 4754 model to

the experimental data and

predictions about yet

undiscovered states.

Fits and predictions of the HISH model

- We compare the predictions of the HISH model with the PDG data. We extract the optimal values of the tension (or α'), endpoints masses, and intercepts
- We determine the chi square of the fits of the spectra of
- Mesons
- Baryons
- Glueballs
- Exotic Hadrons
- We fit the total decay width of hadrons including Zweig suppressed decays
- We determine branching ratios

1. Fits of Mesonic Spectra

and Predictions for higher

excited states

The Fits of the meson trajectories

Traj.	J^{PC}	Exp.	Calc.	Traj.	J^{PC}	Exp.	Calc.
π/b	1^{+-}	1229	1257	K^*	1-	892	892
	2^{-+}	1672	1650		2^{+}	1426	1415
	3^{+-}	2032	1965		3^{-}	1776	1783
	4^{-+}	2250	2236		4^{+}	2045	2084
ho/a	1	776	776		5^{-}	2382	2345
	2^{++}	1318	1324	ϕ/f'	1	1020	1019
	3	1689	1701		2^{++}	1525	1514
	4^{++}	1996	2008		$3^{}$	1854	1870
	$5^{}$	2330	2274	D	0^{-}	1865	1862
	6^{++}	2450	2511		1^{+}	2421	2408
η/h	0^{-+}	548	545		2^{-}	2737	2752
	1^{+-}	1170	1206	D_s^*	1-	2112	2112
	2^{-+}	1617	1612		2^{+}	2572	2563
	3^{+-}	2025	1933		3^{-}	2862	2881
	4^{-+}	2328	2208	Ψ	1	3097	3080
ω/f	1	783	768		1^{++}	3494	3535
	2^{++}	1275	1319		1	3778	3824
	3	1667	1698				
	4^{++}	2018	2006				
	$5^{}$	2250	2271				
	6^{++}	2469	2509				







J









4





Toward a universal model

 The fit results for several trajectories simultaneously. The (J, M²) trajectories of ρ, ω, K*, φ D, and Ψ mesons
 We take the string endpoint masses in MeV

$$m_{u/d} = 60, m_s = 220, m_c = 1500$$

• Only the intercept was allowed to change. We got

$$\alpha' = 0.899$$

 $a_{\rho} = 0.51, a_{\omega} = 0.52, a_{K^*} = 0.49$
 $a_{\phi} = 0.44, a_D = 0.80, a_{\Psi} = 0.94$

Optimization in the m α ' plane : s quark



Figure 3. Left: χ^2 as a function of two masses for the K^* trajectory. a and α' are optimized for each point. The red line is the curve $m_1^{3/2} + m_2^{3/2} = 2 \times (160)^{3/2}$ along which the minimum (approximately) resides. The minimum is $\chi_m^2/\chi_l^2 = 0.925$ and the entire colored area has $\chi_m^2/\chi_l^2 < 1$. On the right is χ^2 as a function of α' and m for the (J, M^2) trajectory of the ϕ . The intercept a is optimized. The minimum is at $\alpha' = 1.07, m = 400$ with $\chi_m^2/\chi_l^2 < 10^{-4}$ at the darkest spot. The lightest colored zone still has $\chi_m^2/\chi_l^2 < 1$, and the coloring is based on a logarithmic scale.

Comments of light mesons

- The trajectories of mesons containing u and d quarks are nearly linear.
- For mesons with strange quarks there are significant deviations from linearity.
- The strength of the HISH model it describes well hadrons with light-light and light-heavy quarks.
- The pion is notable as a meson that does not reside exactly on his trajectory probably since it is a goldstone boson but all his partners do.
- Similar story occurs with the pseudo scalar Kaons
- There is a puzzle with the K3 and K4 that should be according the model lighter than those discovered
- The K* furnish nicely a HMRT (HISH modified R T)

Light-Heavy mesons

- For mesons with one heavy (c or b) quark and one light (u/d or s) the HMRT works well.
- We have trajectories of D and D_s^* including their decay width and also for B, B^* , B_s , and B_s^* mesons.
- We have predictions for higher excited states
- We can assign quantum numbers for states that were discovered but not fully identified. For instance D(2740) and $B_J(5970)$ should be $2^$ states,

Charmonioum and Bottomonioum

- The spectrum of quarkonia has been extensively studied
 Here we see some deviations from the light quark trajectories. For *cc̄* still the slope is 0.9 GeV⁻² for (*J*, *M*²) but 0.60 GeV⁻² for (*n*, *M*²) For *bb̄* the slopes are 0.55 GeV⁻² and 0.42 GeV⁻².
- For heavy quarkonia the HISH approximation of the holographic hadrons is not as good as for the light ones.
- The reason for the disparity between the slopes of the trajectories of (J, M^2) and (n, M^2) is that for the latter the spectrum eigenvalues are w_n and not n anymore
- The Bc state with $J^P = 0^-$ at 6275 MeV interpolates between the charmonioum and botomonium slopes.

Predictions on mesons with higher J

Trajectory	Quarks	$J^{P[C]}$	Mass	Width	$J^{P[C]}$	Mass	Width
π/b	I = 1	5^{+-}	2480	240	6^{-+}	2700	270
η/h	I = 0	5^{+-}	2470	260	6^{-+}	2690	290
ρ/a	I = 1	7	2720	260	8++	2920	280
ω/f	I = 0	7	2710	320	8++	2910	350
K	$sar{q}$	3^{+}	2050	220	4^{-}	2330	250
K^*	$sar{q}$	6^{+}	2620	230	7^{-}	2840	250
ϕ	$s\bar{s}$	4^{++}	2260	130	$5^{}$	2520	150
D	c ar q	3^{+}	3030	70	4^{-}	3270	90
D^*	$car{q}$	4^{+}	3070	100	5^{-}	3310	120
D_s	$c\bar{s}$	2^{-}	2890	-	3^{+}	3160	-
D_s^*	$c\overline{s}$	4^{+}	3160	120	5^{-}	3400	140
Ψ	$c\overline{c}$	4^{++}	4020	90	$5^{}$	4230	130
η_c	$c\overline{c}$	2^{-+}	3790	-	3^{+-}	4030	-
B	$bar{q}$	2^{-}	5980	-	3^{+}	6210	-
B^*	$bar{q}$	3^{-}	6000	-	4^{+}	6230	-
B_s	$b\bar{s}$	2^{-}	6080	-	3^{+}	6320	-
B_s^*	$b\bar{s}$	3^{-}	6100	-	4^{+}	6330	-
Ϋ́	$b ar{b}$	4^{++}	10420	Narrow	$5^{}$	10630	-
η_b	$b\overline{b}$	2^{-+}	10180	Narrow	3^{+-}	10410	Narrow

Predictions on excited mesons with higher n

Traj.	Quarks	J^{PC}	n	Mass	Width	n	Mass	Width
π	I = 1	0^{-+}	5	2610	300	6	2830	330
π_2	I = 1	2^{-+}	3	2520	300	4	2740	350
a_1	I = 1	1^{++}	2	1990	350	4	2520	390
h_1	I = 0	1	4	2470	400	5	2700	450
ω	I = 0	1	5	2560	360	6	2780	390
ω_3	I = 0	$3^{}$	3	2510	230	4	2740	250
ϕ	$s\overline{s}$	1	2	2000	100	4	2570	120
η_c	$c\bar{c}$	0^{-+}	2	4020	-	3	4330	-
Ψ	$c\bar{c}$	1	4	4620	110	5	4860	120
χ_{c1}	$c\bar{c}$	1^{++}	1	3920	-	2	4240	-
Υ	$b\overline{b}$	1	6	11310	90	7	11510	100
χ_{b1}	$b\overline{b}$	1^{++}	3	10800	-	4	11040	-



and Predictions
Trajectories of N and Δ



Trajectories of Λ and Σ



Trajectories of $\Xi \quad \Lambda_c \text{ and } \Xi_c$



Trajectories of Ω_c and Λ_b



Predictions on baryons with higher J

Traj.	Quarks	J^P	Mass	Width	J^P	Mass	Width
N	qqq	$15/2^{-}$	2950	690	$17/2^+$	3050	580
Δ	qqq	$17/2^{-}$	3180	450	$19/2^+$	3160	490
Λ	qqs	$11/2^{-}$	2610	120	$13/2^+$	2810	140
Σ	qqs	$9/2^{+}$	2450	160	$11/2^{-}$	2660	180
Σ	qqs	$9/2^{-}$	2310	200	$11/2^+$	2530	230
Ξ	qss	$7/2^{-}$	2340	-	$9/2^+$	2570	-
Ω	<u>888</u>	$5/2^{-}$	2070	-	$7/2^{+}$	2370	-
Λ_c	qqc	$7/2^{-}$	3140	-	$9/2^+$	3350	-
Σ_c	qqc	$3/2^{-}$	2760	-	$5/2^{+}$	3020	-
Σ_c	qqc	$5/2^{-}$	2820	-	$7/2^{+}$	3060	-
Ξ_c	qsc	$5/2^{+}$	3070	-	$7/2^{-}$	3300	-
Ω_c	(ss)c	$5/2^{+}$	3310	-	$7/2^{-}$	3540	-
Ω_c	s(sc)	$5/2^{+}$	3350	-	$7/2^{-}$	3590	-
Ω_c	(ss)c	$7/2^{+}$	3360	-	$9/2^{-}$	3580	-
Ω_c	s(sc)	$7/2^{+}$	3390	-	$9/2^{-}$	3620	-
Ξ_{cc}	(qc)c	$3/2^{-}$	3870	Narrow?	$5/2^{+}$	4090	-
Ξ_{cc}	q(cc)	$3/2^{-}$	4000	-	$5/2^{+}$	4270	-
Λ_b	qqb	$5/2^{+}$	6140	-	$7/2^{-}$	6340	-
Σ_b	qqb	$3/2^{-}$	6060	-	$5/2^{+}$	6260	-
Σ_b^*	qqb	$5/2^{-}$	6070	-	$7/2^{+}$	6280	-
Ξ_b	qsb	$3/2^{-}$	6060		$5/2^{+}$	6280	-
Ω_b	(ss)b	$3/2^{-}$	6340	-	$5/2^{+}$	6580	-
Ω_b	s(sb)	$3/2^{-}$	6310	-	$5/2^{+}$	6520	-

Predictions on excited baryons with higher n

Trai	Quarka	J^P	~	Mass	~	Mass
Traj.	Quarks	J	n	Mass	n	mass
N	qqq	$1/2^{+}$	4	2330	5	2560
N	qqq	$3/2^{-}$	3	2380	4	2610
N	qqq	$5/2^{+}$	2	2260	3	2490
N	qqq	$1/2^{-}$	2	2150	3	2400
N	qqq	$3/2^{+}$	2	2290	3	2520
N	qqq	$5/2^{-}$	2	2270	3	2510
Δ	qqq	$3/2^{+}$	3	2210	4	2450
Λ_b	qqb	$1/2^{+}$	1	6070	2	6420
Λ_b	qqb	$3/2^{-}$	1	6290	2	6600
Σ_b	qqb	$1/2^{+}$	1	6210	2	6530
Σ_b^*	qqb	$3/2^{+}$	1	6230	2	6540
Ξ_b	qsb	$1/2^{+}$	2	6560	3	6840
Ω_b	(ss)b	$1/2^{+}$	1	6470	2	6790
Ω_b	s(sb)	$1/2^{+}$	1	6520	2	6870

On the Ω spectra

The spectrum of the excited Ω baryons sss could have helped us to determine the mass of the ss diquark directly, but there is not much data on the excited states.

- Our model predicts the first orbitaly excited state of the Ω to have a mass of around 2050 MeV, but there is no confirmed state there.
- The states that are known $\Omega(2250)$, $\Omega(2380)$, and $\Omega(2470)$ could be higher excitations. The $\Omega(2380)$ is at the right mass for the $\frac{7}{2}$ +
- One possibility is that the Ω trajectories exhibit the same even-odd effect as the N and Δ . Then, one might claim that there is no missing state, and the $\Omega(2250)$ as the $\frac{5}{2}^{-}$ belongs to a separate trajectory

Charmed Baryons

• The charmed baryons Λ_c and Ξ_c have trajectories with 3 and 2 states with the same slope as of the light baryons.

- On the Σ_c states $\frac{1}{2}^+$ and $\frac{3}{2}^+$ are confirmed. The state found $\Sigma_c(2800)$ fits the first excited state of $\frac{1}{2}^+$
- Recently 5 narrow resonances of Ω_c (css) were discovered. Their masses fit the HISH predictions for the first orbital excited states of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ gs.
- The di-quark of the Ω_c is (cs) or (ss). The fits prefer the (cs) option. We have predictions for both.

Doubly charmed baryons

- For doubly charmed baryons only the gs of Ξ_{cc}^+ and Ξ_{cc}^{++} are listed in the PDG. The status of the former is uncertain so we use only the later,
- The di-quark could be (cu)/(cd) or (cc). In the former case we have two m_c masses on both sides of the string and in the later one light mass and one (cc) di-quark mass which depends on the location of the baryonic vertex.
- We may estimate the decay width of the excited states from that of a single charm baryon states
- The first excited state will decay to the gs plus pions.
- The second excited state will be able to decay to a charmed meson and a charmed baryon $\Lambda_c D$, we estimate the width to be around 20 Mev.

Bottom Baryons

- The spectra of the excited states of bottom baryons is largely unexplored.
- The only confirmed excited state is the $\Lambda_b^0(5930)\frac{3}{2}^-$ of the gs $\Lambda_b^0(5620)\frac{1}{2}^+$
- The corresponding slope is the universal one for baryons $\alpha' = 0.95 \text{ GeV}^{-2}$.
- We predict the excited states of Σ_b , Ξ_b , and Ω_b .
- Recently $\Sigma_b(6097)$ has been discovered. Its mass is slightly higher than our prediction 6060. It can be identified as a 1P state.
- LHCB has found recently a state of Ξ_b^- which can either be 1P or 2S.





Fits of (potential) glueball spectra

• A rotating and exciting folded closed string admits in flat space-time a linear Regge trajectory

$$J + n = \alpha'_{gb}M^2 + a \qquad \qquad \alpha'_{gb} = \frac{1}{2}\alpha'$$

- The basic candidates of glueballs are flavorless hadrons f₀ of 0++ and f₂ of 2++. There are 9 (+3) fo and 12 (+5) f2.
- The question is whether one can fit all of them into meson and separately some glueball trajectories.
- We found various different possibilities of fits.

Possible scalar glueball trajectories

	Т	rajectories			Predicte	ed st	ates	
α'	Type	Assigned states	n	Mass	Width	n	Mass	Width
	Glueball	980	2	2470	180	4	3350	240
0.78	Light	1370, 1710, 2100, 2330	4	2620	200	4	2850	250
	$s\bar{s}$	1500, 2020	2	2300	300	3	2590	300
	Glueball	1370	2	2510	> 700	4	3290	> 900
0.89	Light	1500, *1800, 2100, 2330	4	2580	200	5	2790	250
	$s\bar{s}$	1710, 2200	2	2390	200	3	2630	250
	Glueball	1500	2	2600	180	4	3350	240
0.89	Light	1370, *1800, 2020, 2330	4	2540	350	5	2760	400
	$s\bar{s}$	1710, 2100	2	2360	250	3	2610	250
	Glueball	1710	2	2800	220	4	3570	280
0.82	Light	1370, *1800, 2100, 2330	4	2610	300	5	2840	300
	$s\bar{s}$	1500, 2020, 2200	3	2270	350	4	2550	350

Table 5: The different assignments of the f_0 into radial trajectories and predicted hig states. The slope α' (in units of GeV⁻²) was fitted for each assignment separately, as done in [4], but is common to all three types of trajectories. Widths are provided as estima based on proportionality of the width to the string length.

Gluebal trajectory built on the 980 gs

• Scalar glueball trajectory with the 980 gs

n	Mass	Width
0	990 ± 20	70 ± 30
2	2515 ± 85	180 ± 75
4	3415 ± 115	$240{\pm}105$
6	4120 ± 140	290 ± 125

J	Mass	Width
0	990 ± 20	70 ± 30
2	2385 ± 70	170 ± 75
4	3225 ± 95	$230{\pm}100$
6	3885 ± 115	275 ± 120

The 980 state can potentially be a KK bound state
So probably the more reliable glueball gs are the 1370 and the 1500.

n	Mass	Width
0	1505 ± 6	109 ± 7
2	2755 ± 95	200 ± 15
4	3595 ± 120	260 ± 20
6	4275 ± 145	310 ± 20

J	Mass	Width
0	1505 ± 6	109 ± 7
2	2640 ± 80	190 ± 15
4	3415 ± 100	245 ± 20
6	4050 ± 120	295 ± 20

Gluebal trajectory built on the 1370 gs

• Assignment with $f_0(1380)$ as the glueball ground-state

Light :	1500, *1800, 2200
$s\bar{s}$:	1710,2100
Glue :	1370, *2060

n	Mass	Width
0	1350 ± 150	350 ± 150
2	2675 ± 120	695 ± 310
4	3535 ± 130	915 ± 405
6	4220 ± 150	1095 ± 485

J	Mass	Width
0	1350 ± 150	350 ± 150
2	2555 ± 110	660 ± 295
4	3350 ± 115	870 ± 385
6	3995 ± 130	$1035 {\pm} 460$

Glueball o++ fits of experimental data

• The meson and glueball trajectories based on $f_0(1380)$ as a glueball lowest state.



Tensor glueballs f 2++

• The meson and glueball trajectories of the f2++

Trajectories			Predicted states							
Type	Assigned states	n	Mass	Width	n	Mass	Width	n	Mass	Width
Light	1270,1640,1950	3	2260	400	4	2510	450	5	2730	500
$s\overline{s}$	1525,2010,2300	3	2540	150	4	2780	200	5	2990	200
2nd light	1810, 2150	2	2380	200	3	2610	250	4	2830	250
Glueball?	$f_2(1430)$	2	2610	-	4	3390	-	6	4030	-
Glueball?	$f_J(2220)$	2	3110	-	4	3790	-	6	4370	-

Table 10: The meson trajectories of f_2 as discussed in [4], with their excited states. The fitted slope here is 0.85 GeV⁻², and half this value is used for the glueball trajectories. Widths are provided as estimates, based on proportionality of the width to the string length. However, the glueball candidates have very small widths, 13 MeV for the $f_2(1430)$ and 23 MeV for the $f_J(2220)$. The predicted widths for the excited states based on our model are between 30 to 50 MeV, but it is unlikely that they remain so narrow.

On the identification of glueball trajectory

- Unfortunately there exists no unambiguous way to assign the known flavorless hadrons into trajectories of mesons and glueballs,
- But it is clear that one cannot sort all the known resonances into meson trajectories alone.
- One of the main problems in identifying glueball trajectories is simply the lack of experimental data, particularly in the mass region between 2.4 GeV and the cc threshold, where we expect the first excited states of the glueballs to be found.
- It is because of this that we cannot find a glueball trajectory in the angular momentum plane.

Glueballs made out of baryonic vertices

- In addition to ordinary closed string there is a zoo of stringy configurations without quarks built from BVs and anti-BVs.
- In general glueballs must have

BVs= # anti-Bvs

• These configurations look differently for different Nc

• The simplest configuration is



• The mass of such a glueball is

0

$$M_{gb} = N_c TL \qquad T = \frac{2\pi a}{L^2} \qquad \rightarrow \alpha' M_{gb}^2 = N_c |a|$$

The corresponding slope $T_{gb} = N_c T \rightarrow \alpha'_{gb} = \frac{1}{N_c} \alpha'$

Glueballs made out of baryonic vertices

- In a similar way we can have a closed loop with n BVs and n anti-BVs with k and Nc-k strings from each BV.
- There also 3d configuraions depending on Nc



(4) Fits and predictions of

"Tetra quarks" and other exotics

Exotic configurations.

- We can have alternative ways to connect the baryonic vertex with strings to flavor branes and thus getting exotic hadrons.
- Obviously one has to perform a stability check to such configuration.
- One possibility is to connect the baryonic vertex to an anti-baryonic vertex thus forming a "tetra quark"

Mesons, bayons and tetra-quarks in holography and HISH

• We demonstrate the structures for **charmed hadrons**

Holography



С

S

U/D



HISH

C



b.Baryon Ξ cc



Holographic tetra quark

 A configuration of a bryonic vertex connected to a u c di-quark and connected to an anti-baryonic vertex which is connected to anti- u and anti- c



Types of tetra-quarks

- In the construction of a tetra quark as a string with a di-quark on one end and an anti-diquark on the other end, there are three types of tetra quarks. Altogether there 225 possibilities
- **Symmetric** the anti di-quark is made out of the anti-quarks that make up the di-quark There are obviously 15 of this type like
- Semi-symmetric- one pair of quark and antiquark of the same flavor and one with different flavors. There are 100 such tetra quark for in $(cu)(\bar{c}\bar{s})$
- Asymmetric both pairs are of different flavor. There are 110 of this kind like $(cs)(\bar{u}\bar{d})$

Regge-like trajectories of tetra-quarks.

- Since the structure of the tetra quark is of a single string with a BV+ a di-quark on one side and an anti-BV and an anti-di-quark on the other side, it has to admit a Regge like trajectories like mesons and baryons in J and n.
- We computed the spectra along these trajectories.
 Discovering a trajectory is a clear indication that the exotic object is a genuine tetra quark and not a molecule.
- A particular trajectory includes the Y_c (4630) and its Yb analog

Predictions of trajectory of charmed tetra quarks

• Based on the Y(4630) that was observed to decay predominantly to $\Lambda_c^+ \Lambda_c^-$. If we assume that it is on a Regge-like trajectory and we borrow the slop and the endpoint masses from the J/Ψ trajectory we get

n	Mass	Width	J^{PC}	Mass	Width
0	4634_{-11}^{+9}	92^{+41}_{-32}	1	4634_{-11}^{+9}	92^{+41}_{-32}
1	$4902{\pm}95$	$103{\pm}46$	2^{++}	$4791{\pm}64$	$98{\pm}44$
2	$5148{\pm}99$	$114{\pm}51$	3	$4939{\pm}66$	$105{\pm}47$
3	$5378{\pm}104$	$124{\pm}55$	4++	$5080{\pm}67$	111 ± 49
4	$5594{\pm}109$	$134{\pm}60$	$5^{}$	$5215{\pm}69$	117 ± 52

• The gs is 1-- thus easy to create in e+ e- collisions

Predictions for the trajectory of bottom tetra quarks

• In a similar manner we predict a trajectory of Yb tetra quark that decays predominantly to $\Lambda_b \overline{\Lambda}_b$

n	Mass
"-2"	10870 ± 50
"-1"	11080 ± 50
0	$11280{\pm}40$
1	11460 ± 40
2	11640 ± 40
3	11810 ± 40
4	$11980{\pm}40$

J^{PC}	Mass
1	$11280{\pm}40$
2^{++}	11410 ± 40
3	$11550 {\pm} 40$
4++	11670 ± 40
5	11800 ± 40

A test case : cccc tetra-quarks



Figure 1: Location of peaks in the LHCb data. Adapted from figure 7 in [1]. The 7.2 GeV state appears to be almost exactly on the $\Xi_{cc} \bar{\Xi}_{cc}$ threshold, which is at 7242 MeV.

The *cccc̄* tetra-quarks

• Two states have been identified one at 6.9 GeV which is below the threshold to decay to $\Xi_{cc}\bar{\Xi}_{cc}$

• Another state was discovered at 7.2 Gev above this threshold and hence we predict that a channel of decay to $\Xi_{cc}\bar{\Xi}_{cc}$ should be discovered.

Possible decays of a stringy tetra quark

- If the mass of the tetra quark is above the threshold of the mass of a baryon and anti-baryon, it will decay via the standard **breaking** of a string.
- If the mass is below this threshold but above the threshold of a pair of mesons it will decay via an annihilation process of the BV and anti-BV
- If it is below this threshold it will be strong interaction **stable**.
- Using the stringy structure one can determine the conditions for these 3 possibilities based on properties of the spectra of mesons and baryons.

Possible decays of the tetra-quarks



Decays of the tetra quarks

- The tetra quark can naturally decay into a by baryon anti-baryon tearing apart the string that connects them and creating a quark anti quark pair
- For instance a creation of a d anti-d pair at the endpoints of the torn apart string between a baryonic vertex that connects to a uc di-quark and a similar anti- baryonic vertex we get a pair of Λc and anti Λc



(5) Predictions about the

decay width of strong hadronic

decays

A test case: The K

• We compare our model to the decays of K* trajectory

State	J^P	Mass	Width	Γ/M	Decay modes ⁴
$K^{*}(892)$	1-	$891.66 {\pm} 0.26$	50.8 ± 0.9	$(5.7\pm0.1)\%$	$K\pi$ (100%)
$K_2^*(1430)$	2^{+}	1425.6 ± 1.5	$98.5 {\pm} 2.7$	$(6.9 \pm 0.2)\%$	$K\pi$ (50%), $K^*\pi$ (25%),
					$K^*\pi\pi$ (13%), $K\rho$ (9%),
$K_3^*(1780)$	3-	1776 ± 7	159 ± 21	$(9.0\pm1.1)\%$	$K\rho$ (31%), $K^*\pi$ (20%),
					$K\pi$ (19%), $K\eta$ (~30%),
$K_4^*(2045)$	4^{+}	2045 ± 9	198 ± 30	$(9.7 \pm 1.5)\%$	$K\pi$ (10%), $K^*\pi\pi$ (9%),
					5 more modes (7% or less), \ldots
$K_5^*(2380)$	5^{-}	2382 ± 24	178 ± 50	$(7.5\pm2.1)\%$	$K\pi$ (6%), no other measured
					modes.

test case: The K*



2.6

3

3.5
Fit results: the meson trajectories

• Fits of the decay width of other Mesons

Trajectory (No.	of states)	a (from spectrum)	A (fitted value)	$\sqrt{\chi^2/DOF}$
ρ	$5^{[a]}$	-0.46	0.097	1.76
ω	$5^{[a]}$	-0.40	0.120	2.31
ρ and ω (avg.)	6	-0.46	0.108	1.14
π	$3^{[a]}$	-0.34	0.100	1.66
η	$3^{[a]}$	-0.29	0.108	1.56
π and η (avg.)	4	-0.29	0.109	1.52
K^*	5	-0.25	0.098	0.77
ϕ	3	-0.10	0.074	0.50
D	2	-0.20	0.072	0.87
D_s^*	2	-0.03	0.076	1.44

Fit results: the meson trajectories





The decay width of baryons

• For baryons the linearity with L is somewhat modified.

Trajectory (No. of states)		a (from spectrum)	A (fitted value)	$\sqrt{\chi^2/DOF}$
N (even)	2	-0.77	0.080	3.33
$N \pmod{1}$	3	-1.11	0.082	2.43
Δ (even)	3	-1.37	0.101	1.90
Λ	4	-0.46	0.041	2.33
$\Sigma (S = 1/2)$	2	-0.95	0.052	0.96
$\Sigma (S = 3/2) \qquad \qquad 3$		-1.22	0.100	1.57
a				

Exponential suppression of pair creation

• The ratio of the decay width to a strange pair versus to a light quark pair is

$$\lambda_s = \exp\left(-2\pi C(m_s^2 - m_{u/d}^2)/T_{\text{eff}}\right) \approx 0.3$$

Hadron	J^P	Light channel		$s\bar{s}$ channel		Ratio	λ_s
$\rho_3(1690)$	3^{-}	$\omega\pi$	$16{\pm}6\%$	$K\bar{K}\pi$	$3.8{\pm}1.2\%$	$0.24{\pm}0.12$	$0.30{\pm}0.15$
$K_4^*(2045)$	4+	$K^*\pi\pi\pi$	$7\pm5\%$	ϕK^*	$1.4{\pm}0.7\%$	$0.20{\pm}0.17$	0.32 ± 0.28

• In radiative decays

$$\frac{\Gamma(J/\Psi \to \gamma f_2'(1525))}{\Gamma(J/\Psi \to \gamma f_2(1270))} = 0.31 \pm 0.06 \,.$$

$$\frac{\Gamma(\Upsilon \to \gamma f_2'(1525))}{\Gamma(\Upsilon \to \gamma f_2(1270))} = 0.38 \pm 0.10$$

Zweig suppressed decays and the string length

The probability of a meson to decay via annihilation of the quark and antiquark

$$\Gamma = \Gamma_Z \exp(-T_Z L^2/2)$$

The decays of upsilon

State	Full width [keV]	B(ggg)	$B(\gamma gg)$	Partial width [keV]	Best fit [keV
$\Upsilon(1S)$	54.02 ± 1.25	$81.7 {\pm} 0.7\%$	$2.2{\pm}0.6\%$	45.3 ± 1.3	45.2
$\Upsilon(2S)$	$31.98 {\pm} 2.63$	$58.8 {\pm} 1.2\%$	$1.87{\pm}0.28\%$	$19.4{\pm}1.7$	20.6
$\Upsilon(3S)$	$20.32{\pm}1.85$	$35.7{\pm}2.6\%$	$0.97{\pm}0.18\%$	$7.5 {\pm} 0.9$	7.1

Decays of glueballs

 Recall that the width of the decay of a meson into two mesons is

$$\Gamma \propto L e^{-m_q^2/T}$$

 In a similar way the width for the decay of a glueball into two mesons is

$$\Gamma \propto L \exp(-\frac{m_q^2}{T}) \exp(-\frac{m_{q'}^2}{T})$$

Thus we get the following hierarchy for the decay of glueballs

 $\Gamma(Gb \to 2 \text{ light}) : \Gamma(Gb \to K\bar{K}) : \Gamma(Gb \to \phi\phi) = 1 : e^{-1} : e^{-2}$

 $\Gamma(GB \to \omega\omega) : \Gamma(GB \to K^{*0}K^{*0}) : \Gamma(GB \to \phi\phi) = 1 : 0.30 : 0.07.$

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Summary and Outlook

Ask HISH

 If you encounter physics phenomena of Hadrons that has to be explained or quantitative calculation of certain properties.

Ask HISH

We may be able to provide an answer
HISH can hint where to look for new physics within the standard model: for instance glueballs and exotic hadrons

Summary and outlook

- Hadron spectra fit much better strings in holographic backgrounds rather than the spectra of bulk fields like fluctuations of flavor branes.
- Holographic Regge trajectories can be mapped into trajectories of strings with massive endpoints.
- Heavy quark mesons are described in a much better way by the holographic trajectories (or massive) than the original linear trajectories.
- Even for the u and d quark there is a non vanishing string endpoint mass of ~60 Mev.

Summary and outlook

- Baryons are also straight strings with tension which is the same as the one of mesons.
- The baryonic vertex is still mysterious since data prefers it to be massless.
- Glueballs can be described as rotating folded closed strings
- Open questions:
- Quantizing a string with massive endpoints
- Accounting for the spin and for the intercept
- Scattering amplitudes of mesons and baryons like (proton-proton scattering)
- Nuclear interaction and nuclear matter
- Incorporating leptons....

Stringy inspired questions about hadrons

- The intercept- theoretical determination.
- Identifying glue-balls. Exotic glueballs
- Tetra quarks and other exotic hadrons. The sister states. Decays into baryon anti-baryons. Regge trajectories of tetra quarks
- Magnetic moments of baryons
- Totem experiment and the total cross section
- Multi baryon states like the sexaquark
- Strong decay width of excited hadrons. Branching ratio
- The quark di-quark structure of the baryon
- Hadronization in heavy ion collisions.
- Confinement de-confinement phase transition in the early universe.
- From stringy to partonic and then to pomeron scattering amplitude and cross section.
- Jets. Factorization the Lund model
- The hidden region between 2.5 and 3 Gev

- Identifying glueballs. Exotic glueballs
- Tetra quarks and other exotic hadrons. The sister states.
- From stringy to partonic and then to pomeron scattering amplitude and cross section.
- Jets. Factorization the Lund model
- The role of hadrons in the evolution of the universe
- Hadronization in heavy ion collisions.
- Confinement de-confinement phase transition in the early universe.
- From stringy to partonic and then to pomeron scattering amplitude and cross section.

Decays of glueballs versus mesons



Predictions for glueballs

- Glueballs are closed strings. They form linear trajectories with the slope of $\alpha'_{ab} = \frac{1}{2} \alpha'_{meson}$.
- Unfortunately there are only few confirmed flavorless hadrons with higher J and higher n.
- When we use the glueball slope we can fit at most 2 points. Higher points are already in a mass range where not much states have been confirmed.
- We can predict the locations of the higher glueballs and their width based on

$$\Gamma = M^2 \frac{\Gamma_0}{M_0^2}$$