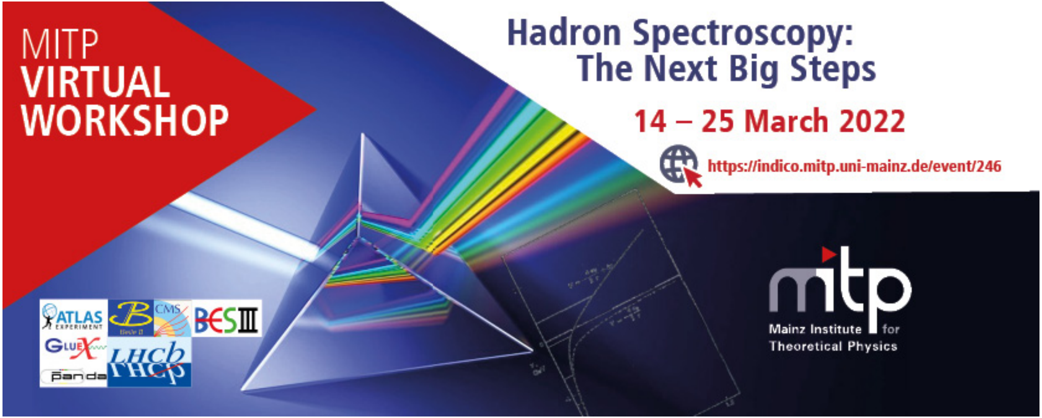
Symmetries and Dynamics: Theoretical Keys to Hadron Spectroscopy

Chris Quigg

Fermilab



A voice from the past

> *If you know the elementary particles and their interactions, and you call yourself a physicist, you ought to be able to calculate the consequences—or at least you should feel guilty if you can't!*
>
> *CQ, Les Houches Lectures (1981)*

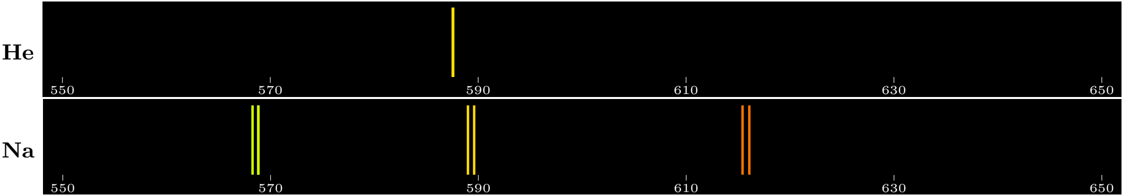
Aspirations:

- Compute the properties of hadrons, explain the absence of unseen species, and predict the existence of new varieties of hadrons;
- Explain why quarks and the quanta of the color force, gluons, are not observed;
- Derive the interactions among hadrons as a collective effect of the interactions among constituents.

Heidelberg Hauptstraße . . .

In this house in 1859 Kirchhoff turned to the Sun and stars the spectral analysis he founded with Bunsen and thus opened up the chemistry of the Universe.

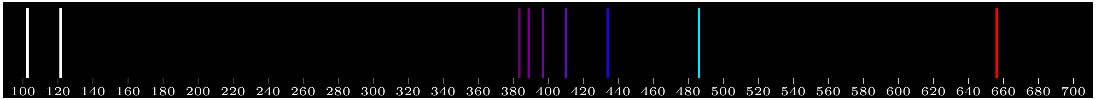
1868: Janssen & Lockyer independently notice a new line in solar prominences



Detection on Earth: 1882, Luigi Palmieri in Vesuvian lava;

1894, William Ramsay in Clevite (U ore); named helium

H



THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

JULY 1913.

I. *On the Constitution of Atoms and Molecules.* *By* N. BOHR, *Dr. phil. Copenhagen**.

Introduction.

IN order to explain the results of experiments on scattering of α rays by matter Prof. Rutherford† has given a theory of the structure of atoms. According to this theory, the atoms consist of a positively charged nucleus surrounded by a system of electrons kept together by attractive forces from the nucleus; the total negative charge of the electrons is equal to the positive charge of the nucleus. Further, the nucleus is assumed to be the seat of the essential part of the mass of the atom, and to have linear dimensions exceedingly small compared with the linear dimensions of the whole atom. The number of electrons in an atom is deduced to be approximately equal to half the atomic weight. Great interest is to be attributed to this atom-model; for, as Rutherford has shown, the assumption of the existence of nuclei, as those in question, seems to be necessary in order to account for the results of the experiments on large angle scattering of the α rays‡.

In an attempt to explain some of the properties of matter on the basis of this atom-model we meet, however, with difficulties of a serious nature arising from the apparent

* Communicated by Prof. E. Rutherford, F.R.S.
† E. Rutherford, Phil. Mag. xxi. p. 669 (1911).
‡ See also Geiger and Marsden, Phil. Mag. April 1913.

Phil. Mag. S. 6. Vol. 26. No. 151. *July* 1913. B

Letters to the Editor.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, nor to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

Spinning Electrons and the Structure of Spectra.

So far as we know, the idea of a quantised spinning of the electron was put forward for the first time by A. K. Compton (*Journ. Frankl. Inst.*, Aug. 1921, p. 145), who pointed out the possible bearing of this idea on the origin of the natural unit of magnetism. Without being aware of Compton's suggestion, we have directed attention in a recent note (*Naturwissenschaften*, Nov. 20, 1925) to the possibility of applying the spinning electron to interpret a number of features of the quantum theory of the Zeeman effect, which were brought to light by the work especially of van Lohuizen, Sommerfeld, Landé and Pauli, and also of the analysis of complex spectra in general. In this letter we shall try to show how our hypothesis enables us to overcome certain fundamental difficulties which have hitherto hindered the interpretation of the results arrived at by those authors.

To start with, we shall consider the effect of the spin on the manifold of stationary states which corresponds to motion of an electron round a nucleus. On account of its magnetic moment, the electron will be acted on by a couple just as if it were placed at rest in a magnetic field of magnitude equal to the vector product of the nuclear electric field and the velocity of the electron relative to the nucleus divided by the velocity of light. This couple will cause a slow precession of the spin axis, the conservation of the angular momentum of the atom being ensured by a compensating precession of the orbital plane of the electron. This complexity of the motion requires that, corresponding to each stationary state of an imaginary atom, in which the electron has no spin, there shall in general exist a set of states which differ in the orientation of the spin axis relative to the orbital plane, the other characteristics of the motion remaining unchanged. If the spin corresponds to a one-quantum rotation there will be in general two such states. Further, the energy difference of these states will, as a simple calculation shows, be proportional to the fourth power of the nuclear charge. It will also depend on the quantum numbers which define the state of motion of the non-spinning electron in a way very similar to the energy differences connected with the rotation of the orbit in its own plane arising from the relativity variation of the electronic mass. We are indebted to Dr. Heisenberg for a letter containing some calculations on the quantitative side of the problem.

This result suggests an essential modification of the explanation hitherto given of the fine structure of the hydrogen-like spectra. As an illustration we may consider the energy levels corresponding to electronic orbits for which the principal quantum number is equal to three. The scheme on the left side of the accompanying figure (Fig. 1) corresponds to the results to be expected from Sommerfeld's theory. The so-called azimuthal quantum number k is defined by the quantity of moment of momentum of the electron about the nucleus, $kh/2\pi$, where $k=1, 2, 3$. According to the new theory, depicted in the scheme[1] on the right, this moment of momentum is given by $Kh/2\pi$, where $K=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. The total angular momentum of the atom is $Jh/2\pi$, where $J=1, 2, 3$. The symbols K and J correspond to those used by Landé in his classification of the Zeeman effects of the optical multiplets. The letters S, P, D also relate to the analogy with the structure of optical spectra which we consider below. The dotted lines represent the position of the energy levels to be expected in the absence of the spin of the electron. As the arrows indicate, this spin now splits each level into two, with the exception of the level $K=\frac{1}{2}$, which is only displaced.

In order to account for the experimental facts, the resulting levels must fall in just the same places as the levels given by the older theory. Nevertheless, the two schemes differ fundamentally. In particular, the new theory explains at once the occurrence of certain components in the fine structure of the hydrogen spectrum and of the helium spark spectrum

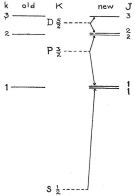


FIG. 1.

which according to the old scheme would correspond to transitions where K remains unchanged. Unless these transitions could be ascribed to the action of electric forces in the discharge which would perturb the electronic motion, their occurrence would be in disagreement with the correspondence principle, which only allows transitions in which the azimuthal quantum number changes by one unit. In the new scheme we see that, in the transitions in question, K will actually change by one unit and only J will remain unchanged. Their occurrence is, therefore, quite in conformity with the correspondence principle.

The modification proposed is specially important for explaining the structure of X-ray spectra. These spectra differ from the hydrogen-like spectra by the appearance of the so-called "screening" doublets, which are ascribed to the interaction of the electrons within the atom, effective mainly through reducing the effect of the nuclear attraction. In our view, these screening doublets correspond to pairs of levels which have the same angular momentum J but different azimuthal quantum numbers K. Consequently, the orbits will penetrate to different distances

[1] Quite independently of the ideas discussed here, a scheme of levels corresponding to this figure has been previously proposed by the writers (*Physica*, 5, 266, 1925), on the ground of the formal analogy between spectral structures. From similar formal considerations, this scheme has recently also been arrived at by J. C. Slater (*Proc. Washington Acad.*, December 1925).

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Goudsmit & Uhlenbeck (1925–6)

HARVARD OBSERVATORY MONOGRAPHS

HARLOW SHAPLEY, EDITOR

No. 1

STELLAR ATMOSPHERES

A CONTRIBUTION TO THE OBSERVATIONAL STUDY OF HIGH TEMPERATURE IN THE REVERSING LAYERS OF STARS

BY

CECILIA H. PAYNE

PUBLISHED BY THE OBSERVATORY

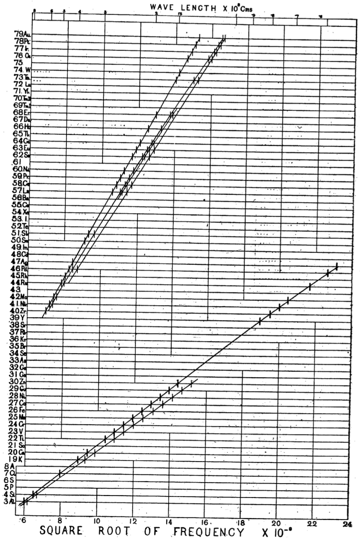
CAMBRIDGE, MASSACHUSETTS

1925

Moseley (1913): x-ray spectra determined by Atomic Number (Rutherford)

High-Frequency Spectra of the Elements. 709

Fig. 3.



Summary.

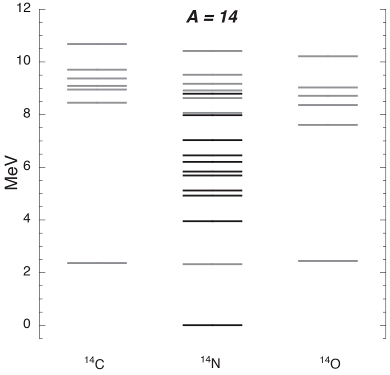
1. Every element from aluminium to gold is characterized by an integer N which determines its X-ray spectrum. Every detail in the spectrum of an element can therefore be predicted from the spectra of its neighbours.
2. This integer N, the atomic number of the element, is identified with the number of positive units of electricity contained in the atomic nucleus.
3. The atomic numbers for all elements from Al to Au have been tabulated on the assumption that N for Al is 13.
4. The order of the atomic numbers is the same as that of the atomic weights, except where the latter disagrees with the order of the chemical properties.
5. Known elements correspond with all the numbers between 13 and 79 except three. There are here three possible elements still undiscovered.
6. The frequency of any line in the X-ray spectrum is approximately proportional to $A(N-b)^2$, where A and b are constants.

$SU(2)_{\text{isospin}}$, the first internal symmetry

$m_p \approx m_n$

Charge-independence of nuclear force: binding of ^{3}H(pnn) and ^{3}He(ppn)

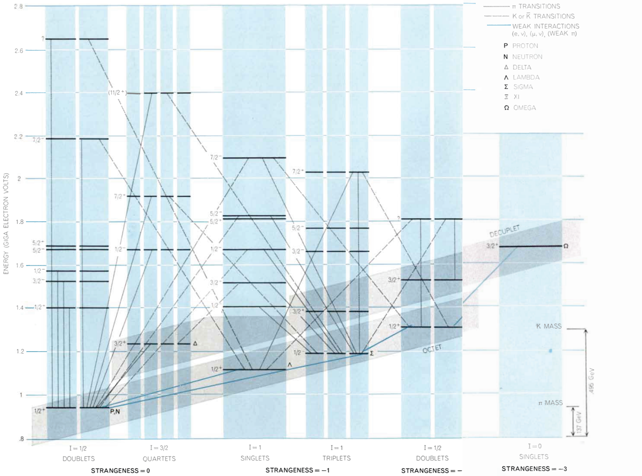
Level structures in mirror nuclei; isobaric analogue states



(n - p mass difference, Coulomb energy removed)

Hadrons, the third spectroscopy

Weisskopf (1968)



BARYON SPECTRUM is composed of the nucleon (P, N) and its various excited states. The states are arranged in columns according to their multiplicity and strangeness. The letter I denotes isotopic spin; the multiplicity is given by $2I + 1$. Strangeness is an intrinsic quantum property. In the subnuclear spectrum of the baryon the ground state is taken to be the mass energy of the proton, .938 GeV. The number to the left of each state indicates spin angular momentum and parity (+ or −). The symbol to the right is the name of the state. The quanta emitted in certain transitions are shown in the key. Photon emissions are omitted; they generally link the same states linked by pions if there is no change in charge. Colored lines indicate transitions that are mediated by weak interactions: lepton pairs or weak pion emissions. Transitions go from every member of a multiplet to every member of another, but for simplicity only one such transition is shown for each pair of states. The masses of pions and kaons appear at the right. The states in the octet and decuplet exhibit certain internal symmetries. Each baryon state shown here also exists in an antimatter state, so that there is a similar spectrum of antibaryons.

24

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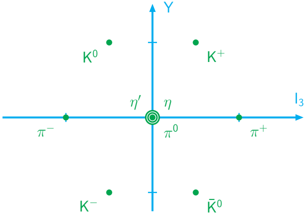
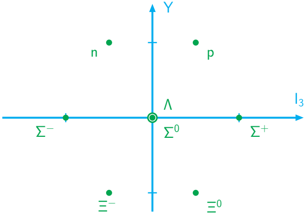
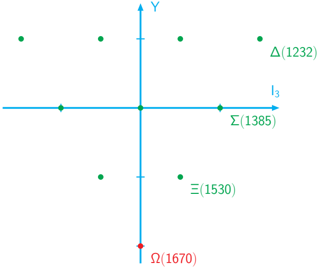
Ancient History: Hadron Spectroscopy & SU(3)$_{\text{flavor}}$

Gell-Mann, "The Eightfold Way" & Ne'eman, Nucl. Phys. **26**, 229 (1961): SU(3)$_\mathrm{f}$ classification symmetry







REVIEWS OF MODERN PHYSICS VOLUME 35, NUMBER 4 OCTOBER, 1963

The Octet Model and its Clebsch-Gordan Coefficients

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CERN, Geneva

1. INTRODUCTION

In trying to understand the structure of the strong interactions, several higher symmetry schemes have been proposed.[1,2] These higher symmetries should conserve the isospin I and the hypercharge Y. Especially interesting in this respect is the octet model (unitary symmetry) proposed independently by Gell-Mann[3] and Ne'eman.[4] In this model one assumes the strongest interactions to be invariant under transformations belonging to $SU(3)$, i.e., under unimodular unitary transformations in some three-dimensional complex linear vector space ("unitary spin space"). The symmetry of these strong interactions is broken by some unknown weaker mechanism, but in such a way that the isospin and the hypercharge are still conserved. A still weaker interaction, the electromagnetic interaction, breaks this lower symmetry in such a way that only the hypercharge and the third component of isospin are conserved. In this unitary symmetry model one assigns groups of strongly interacting particles with the same quantum numbers (not the same are I, Y, I_3, and directly related ones as strangeness, charge, G parity, etc.), to irreducible representations (IR's) of the group $SU(3)$. The lowest nontrivial IR in the octet model, which is physically possible (i.e., has integer quantum numbers for the hypercharge), is the IR $\{8\}$. The eight well-known baryons N,Λ,Σ, and Ξ, as well as the eight pseudoscalar mesons, K,η,π, and $\overline{K}$, are assigned to IR's $\{8\}$. One assumes, moreover, the existence of eight vector mesons which belong to such a representation. Perhaps the mesons ρ,ω,K^*, and $\overline{K}^*$ constitute this octet. A difficulty here is which K^* to take. There seem to be two $(K\pi)$ resonances, one[5] at 730 MeV and the other[6] at 888 MeV. One favors the 888-MeV resonance because it seems to have all the correct quantum numbers. The next higher IR can contain 10 particles. It is suggested[7] that the familiar (3,3) pion-nucleon resonance, the Y_1^* (1385 MeV), the recently discovered[8,9] $I=\frac{1}{2}$, $\Xi\pi$ resonance at 1532 MeV and a still unknown baryon $\Omega^-(Y=-2,\ I=0,\pm 1685$ MeV) belong to this IR $\{10\}$. A discovery of this Ω^- would be a great triumph for this octet model. Okubo[10] has derived a mass formula for the different members belonging to the same IR. For the octets (IR $\{8\}$), this formula reduces to a mass relation between the different members. This mass relation is very well satisfied for the baryons and for the pseudoscalar mesons. However, for the vector mesons, neither the 888-MeV nor the 730-MeV $(K\pi)$ resonance fulfills this relation. For the IR $\{10\}$ this mass formula is again very well satisfied. Coleman and Glashow[11] have given a relation connecting the electromagnetic mass differences within the baryon octet. This relation is also very well satisfied.

The main purpose of this paper is to derive the

* On leave from the University of Nijmegen, Nijmegen, The Netherlands.

[1] A very nice survey of the different higher symmetry schemes in strong interactions is given by R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. **34**, 1 (1962). The reader is referred there to the large existing literature about this subject.

[2] D. R. Speiser and J. Tarski, Math. Phys. **4**, 588 (1963).

[3] M. Gell-Mann, California Institute of Technology, Report CTSL–20, March, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962).

[4] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[5] G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962).

[6] For extensive references, see, *Proceedings of the 1962 Annual International Conference on High-Energy Physics, at CERN* (CERN, Geneva, 1962), p. 781.

[7] M. Gell-Mann, *Proceedings of the 1962 Annual International Conference on High-Energy Physics, at CERN* (CERN, Geneva, 1962), p. 805.

[8] G. M. Pjerrou, D. J. Prowse, P. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, *Proceedings of the 1962 Annual International Conference on High-Energy Physics, at CERN* (CERN, Geneva, 1962), p. 289.

[9] L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, L. Gray, J. Leitner, S. Lichtman, and J. Westgard, *Proceedings of the 1962 Annual International Conference on High-Energy Physics, at CERN* (CERN, Geneva, 1962), p. 279.

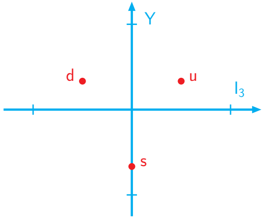
[10] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[11] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

Ancient History: Hadron Spectroscopy & Quarks

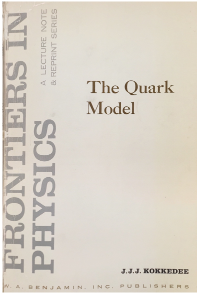
1964: Gell-Mann ("quarks") and Zweig ("aces") noticed that observed multiplets could be constructed from elementary spin-$\frac{1}{2}$ flavor triplets—isospin doublet (u, d) and strange isospin singlet s, according to the rules meson $= q\bar{q}$ and baryon $= qqq$.

Both open to $qq\bar{q}\bar{q}$, $4q\bar{q}$ ...



SU(3)$_f$ algebra:

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8};$$
$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$



"Of course, the whole quark idea is ill-founded. So far, quarks have escaped detection. This fact could simply be taken to mean that they are extremely massive and therefore difficult to produce, but it could also be an indication that quarks cannot exist as individual particles but, like phonons in a crystal, can have meaning only inside the hadrons. In either case, nevertheless, the dynamical system of such quarks binding together to give the observed hadrons that has the properties demanded by the applications, is very difficult to understand in terms of conventional concepts. The quark model should, therefore, at least for the moment, not be taken for more than what it is, namely the tentative and simplistic expression of an as yet obscure dynamics underlying the hadronic world."

—Jaap Kokkedee, 1969

Quark model as formulated conflicts with Exclusion Principle

$\Delta^{++}(uuu)$: symmetric in space (s-wave), spin ($J = 3/2$), isospin ($I = 3/2$)

$\rightsquigarrow$ Add a label ("color") to distinguish the identical quarks

Validated in weak-interaction branching fractions, $\sigma(e^+e^- \to \text{hadrons})$, ...

Generalize from label to continuous symmetry: $\mathrm{SU}(3)_{\text{color}}$

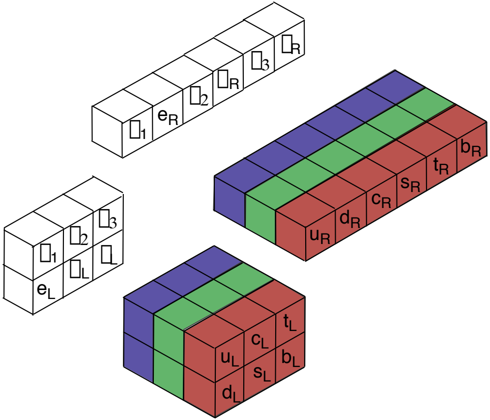
$\mathrm{SU}(3)_{\text{c}}$ gauge symmetry $\rightsquigarrow$ Quantum Chromodynamics

$\rightsquigarrow$ asymptotic freedom, justifying parton model, offering confinement mechanism

Color singlets: maximally attractive channels in one-gluon exchange

Our picture of matter

Pointlike ($r \lesssim 10^{18}$ m) quarks and leptons



Interactions: $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge symmetries

QCD: the basis of hadronic physics

Fundamental fields: quarks and gluons, manifest in

- Proton structure [high resolution, hard scattering, ...]
- Matter at high density
- Lattice calculations

Effective degrees of freedom, manifest in

- Constituent quarks, Goldstone bosons, ...
- Effective field theories
- Isobar (resonance) models
- Nuclei and nuclear structure

Flavor-spin $SU(6)_{fs} = SU(3)_{flavor} \otimes SU(2)_{spin}$

Generalizes Wigner $SU(4) = SU(2)_{isospin} \otimes SU(2)_{spin}$

Construct SU(6) wave functions for polarized nucleons,

$$
\begin{aligned}
|p\uparrow\rangle &= (1/\sqrt{18})\,[2u_\uparrow d_\downarrow u_\uparrow - u_\downarrow d_\uparrow u_\uparrow - u_\uparrow d_\uparrow u_\downarrow - d_\uparrow u_\downarrow u_\uparrow \\
&\quad + 2d_\downarrow u_\uparrow u_\uparrow - d_\uparrow u_\uparrow u_\downarrow - u_\uparrow u_\downarrow d_\uparrow - u_\downarrow u_\uparrow d_\uparrow + 2u_\uparrow u_\uparrow d_\downarrow]\,, \\
|n\uparrow\rangle &= -(1/\sqrt{18})\,[2d_\uparrow u_\downarrow d_\uparrow - d_\downarrow u_\uparrow d_\uparrow - d_\uparrow u_\uparrow d_\downarrow - u_\uparrow d_\downarrow d_\uparrow \\
&\quad + 2u_\downarrow d_\uparrow d_\uparrow - u_\uparrow d_\uparrow d_\downarrow - d_\uparrow d_\downarrow u_\uparrow - d_\downarrow d_\uparrow u_\uparrow + 2d_\uparrow d_\uparrow u_\downarrow]\,,
\end{aligned}
$$

$+$ remaining light-baryon wave functions . . .

Reproduces the pattern of baryon magnetic moments

Why did extravagantly naïve arguments lead to successful predictions?

Colorspin $SU(6)_{cs} = SU(3)_{color} \otimes SU(2)_{spin}$ Jaffe (1977)

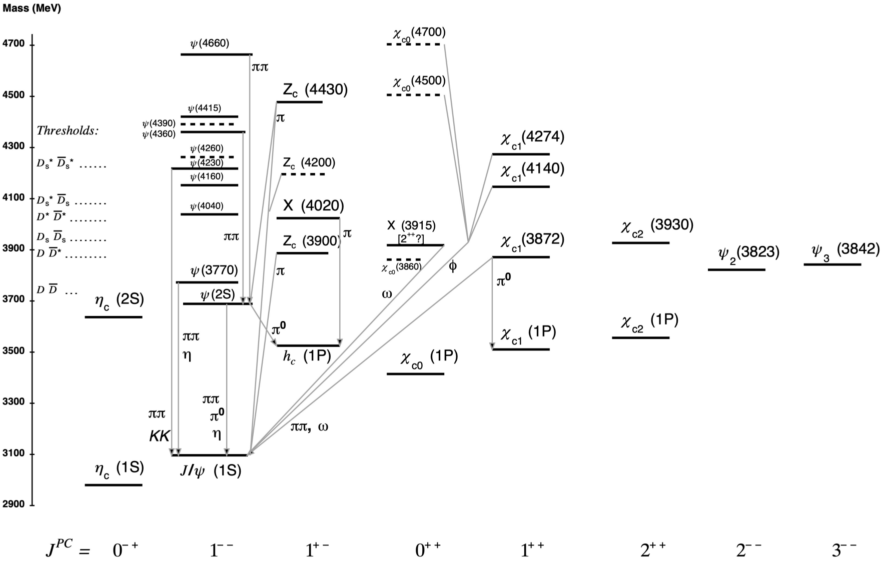
No residual color interaction between color singlets, e.g., $pn, \Lambda\Lambda, \ldots$

A $(uds)^2$ state in the **490** representation 
has an additional attraction—a color hyperfine interaction

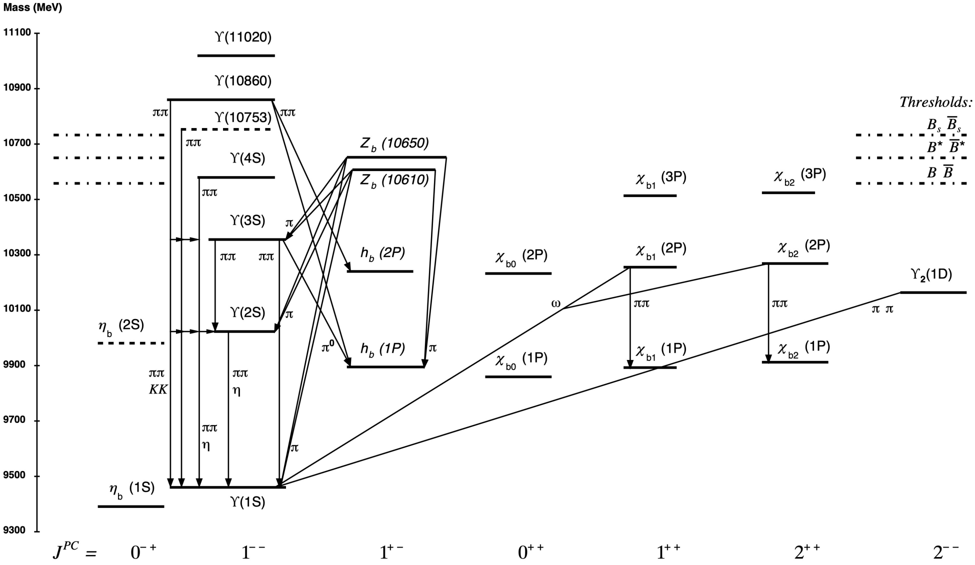
The conjectured H-dibaryon, perhaps to be seen near $\Lambda\Lambda$ threshold

Fate still unknown

Charmonium spectrum



Bottomonium spectrum PDG



Why attack a particular question?

PHYSICAL REVIEW VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Infrared Photons and Gravitons*

STEVEN WEINBERG†
Department of Physics, University of California, Berkeley, California
(Received 1 June 1965)

It is shown that the infrared divergences arising in the quantum theory of gravitation can be removed by the familiar methods used in quantum electrodynamics. An additional divergence appears when infrared photons or gravitons are emitted from noninfrared external lines of zero mass, but it is proved that for infrared gravitons this divergence cancels in the sum of all such diagrams. (The cancellation does not occur in massless electrodynamics.) The formula derived for graviton bremsstrahlung is then used to estimate the gravitational radiation emitted during thermal collisions in the sun, and we find this to be a stronger source of gravitational radiation (though still very weak) than classical sources such as planetary motion. We also verify the conjecture of Dalitz that divergences in the Coulomb-scattering Born series may be summed to an innocuous phase factor, and we show how this result may be extended to processes involving arbitrary numbers of relativistic or nonrelativistic particles with arbitrary spin.

I. INTRODUCTION

THE chief purpose of this article is to show that the infrared divergences in the quantum theory of gravitation can be treated in the same manner as in quantum electrodynamics. However, this treatment apparently does not work in other non-Abelian gauge theories, like that of Yang and Mills. The divergent phases encountered in Coulomb scattering will incidentally be explained and generalized.

It would be difficult to pretend that the gravitational infrared divergence problem is very urgent. My reasons for now attacking this question are:

(1) Because I can. There still does not exist any satisfactory quantum theory of gravitation, and in lieu of such a theory it would seem well to gain what experience we can by solving any problems that can be solved with the limited formal apparatus already at our disposal. The infrared divergences are an ideal case of this sort, because we already know all about the coupling of a very soft graviton to any other particle,[1] and about the external graviton line wave functions[1] and internal graviton line propagators.[2]

(2) Because something might go wrong, and that would be interesting. Unfortunately, nothing does go wrong. In Sec. II we see that the dependence on the infrared cutoffs of real and virtual gravitons cancels just as in electrodynamics.

However, there is a more subtle difficulty that might have been expected. Ordinary quantum electrodynamics would contain unremovable logarithmic divergences if the electron mass were zero, due to diagrams in which a soft photon is emitted from an external electron line with momentum parallel to the electron's.[3] There are no charged massless particles in the real world, but hard neutrinos, photons, and gravitons do carry a gravitational "charge," in that they can emit soft gravitons. In Sec. III we show that diagrams in which a soft graviton is emitted from some other hard massless particle line do contain divergences like the $\ln m_e$ terms in massless electrodynamics, but that these divergences cancel when we sum all such diagrams.[4] However, this cancellation is definitely due to the details of gravitational coupling, and does not save theories (like Yang and Mills's) in which massless particles can emit soft massless particles of spin one.

(3) Because in solving the infrared divergence problem we obtain a formula for the emission rate and spectrum of soft gravitons in arbitrary collision processes, which may (if our experience in electrodynamics is a guide) be numerically the most important gravi-

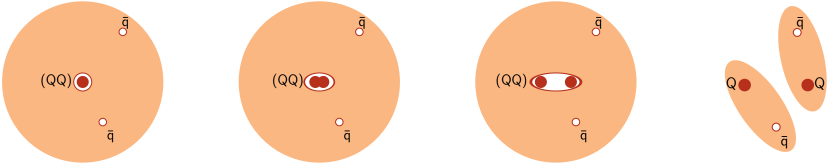
Toward Controlled Approximations to Supplement Models

- NRQCD for heavy-heavy systems $(Q_1\bar{Q}_2)$
  $m_{Q_i} \gg \Lambda_{\mathrm{QCD}}$
  expansion parameter v/c
- HQET for heavy-light systems $(Q\bar{q})$
  $m_Q \gg \Lambda_{\mathrm{QCD}}$; $\vec{\jmath}_q = \vec{L} + \vec{s}_q$
  expansion parameter $\Lambda_{\mathrm{QCD}}/M_Q$
- Chiral symmetry for light quarks $(q_1\bar{q}_2)$
  $m_{q_i} \ll \Lambda_{\mathrm{QCD}}$
  expansion parameter $\Lambda_{\mathrm{QCD}}/4\pi f_\pi$
- Lattice QCD

Seeking the Relevant Degrees of Freedom

Correlations among quarks long known . . .

- $\triangleright$ $x \to 1$ behavior of proton parton distributions: $F_2^n/F_2^p < \frac{2}{3}$; Spin differs from $\mathrm{SU}(6)_{\mathrm{fs}}$ wave functions
- $\triangleright$ $\mathbf{3} \otimes \mathbf{3}$ attractive in $\mathbf{3}^*$ (half as strong as in $\mathbf{3} \otimes \mathbf{3}^* \to \mathbf{1}$?) In extreme heavy-quark limit, surely pointlike $\mathbf{3}^*$ is apt



Growing separation alters $\mathbf{3}^*$, $\mathbf{6}$ mix $\rightsquigarrow$ dissociation

- $\triangleright$ Scalar nonet $f_0(600) = \sigma, \kappa(900), f_0(980), a_0(980)$ as $qq\bar{q}\bar{q}$ organized into diquark–antidiquark $\mathbf{3} \otimes \mathbf{3}^*$

Test, extend idea of diquarks

- $\rightsquigarrow$ QQq baryons (and comparison with $Q\bar{q}$)
- systematics of $(qq)(\bar{q}\bar{q})$ states; extension to $(Qq)(\bar{Q}\bar{q})$, $(QQ)(\bar{q}\bar{q})$
- shape of baryons (at least high-spin?) in lattice QCD
- comparison with $1/N_c$ systematics?
- additional configurations beyond qqq and $\bar{q}q$?
- role of diquarks in color–flavor locking, color superconductivity, etc.
- colorspin as an organizing principle? mass effects . . .

Superposition, Interference, Confusion, . . .

Consider the neighborhood of $X(3872)$

1. Possible charmonium state. Spectrum is influenced by open-charm states (coupled-channel studies by Cornell group). We estimated open-charm content of $(c\bar{c})$ levels, but did not consider interplay with
2. Molecular states ($+-$ and 00 charge configurations), or
3. Tetraquark states, or
4. Examine exhaustively the influence of threshold cusps and such.

One idea might successfully give the big picture, but
a full understanding will require incorporating all relevant possibilities.
The mix will be different for every state. Higher mass, more intricate.

Doubly Heavy Baryons

Spectroscopy

- Analogy: $[QQ^{(\prime)}]_{3^*}q$ and $\bar{Q}q$ as heavy-light systems
- One-gluon-exchange: $V_{[QQ^{(\prime)}]_{3^*}}(r) = \frac{1}{2}V_{(\bar{Q}q)_1}(r)$; deviations beyond?
- Learn about $[QQ^{(\prime)}]_{3^*}$ dynamics through excitation spectrum?
- As in $b\bar{c}$, unequal masses in bcq may expose limitations of NRQM

Weak decays

- Rich set of heavy $\to$ heavy, heavy $\to$ light transitions
- Isolate different pieces of $\mathcal{H}_{\text{weak}}^{\text{eff}}$

Can we find new connections and learn from others?

Stan Brodsky: a QCD supersymmetry relating mesons to baryons.

1. Any lessons from or for supersymmetry in nuclei (Francesco Iachello)?
2. How do relations among mesons, baryons, tetraquarks, pentaquarks relate to heavy-quark symmetry relations such as
$$m(Q_iQ_j\bar{q}_k\bar{q}_l) - m(Q_iQ_jq_m) = m(Q_xq_kq_l) - m(Q_x\bar{q}_m)?$$
3. . . .

QCD could be complete to very high energies

How Might QCD Crack?

(Strong CP Problem)

(Breakdown of factorization)

Free quarks / unconfined color

New kinds of colored matter

Quark compositeness

Larger color symmetry containing QCD

Hadron Spectroscopy is rich in opportunities

- Models are wonderful exploratory tools
- Engage lattice, symmetries at every opportunity
- Build coherent networks of understanding
- Tune between systems: models beyond comfort zones
- Relate mesons to baryons to ...
- Look beyond qqq and $q\bar{q}$: heavy flavors, exotics, matter under unusual conditions

Focus on what we can learn of lasting value