Power corrections in off-forward hard processes

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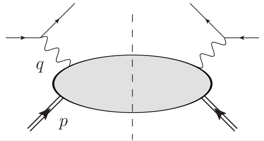
PEOL MITP Program, 21.09.2022

Planar vs. non-planar kinematics

“Natural” separation of longitudinal and transverse d.o.f. in DIS

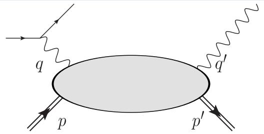


$$\begin{aligned} p &= (p_0, \vec{0}_\perp, p_z) \\ q &= (q_0, \vec{0}_\perp, q_z) \end{aligned}$$

$\Rightarrow$ parton fraction = Bjorken x_B

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"DIS frame"

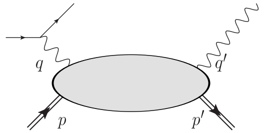
$$\begin{aligned} p &= (p_0, \vec{0}_\perp, p_z) \\ q &= (q_0, \vec{0}_\perp, q_z) \end{aligned}$$

$\Rightarrow$ asymmetry parameter $\xi \simeq x_B/(2-x_B)$

$\Rightarrow$ momentum transfer $\Delta = p' - p$ (almost) transverse

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"Photon frame"

$$\begin{aligned} q' &= (q'_0, \vec{0}_\perp, q'_z) \\ q &= (q_0, \vec{0}_\perp, q_z) \end{aligned}$$

$\Rightarrow$ skewedness parameter $\xi = \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$

$\Rightarrow$ momentum transfer $\Delta = p' - p$ longitudinal

The message:

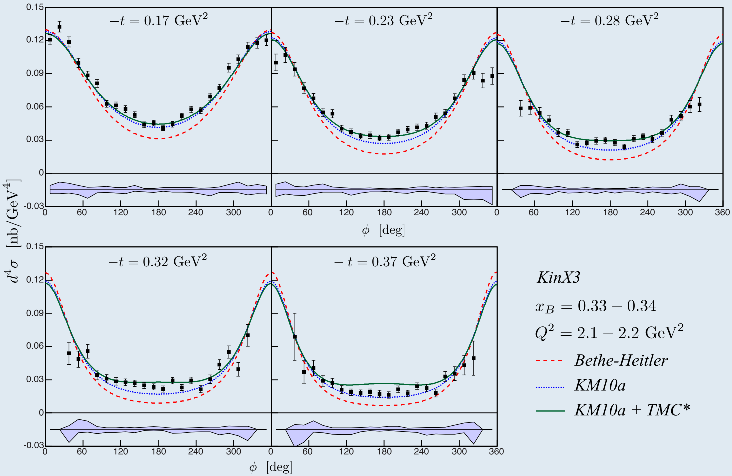
- **noncomplanarity makes separation of collinear directions ambiguous**
  - — hence "leading twist approximation" ambiguous
  - — related to violation of translation invariance and EM Ward identities
- **have to be repaired by adding power corrections of special type, "kinematic" PC**

$$\left(\frac{\sqrt{-t}}{Q}\right)^k \qquad\qquad \left(\frac{M}{Q}\right)^k$$

- **Potentially $\sqrt{-t} \gg \Lambda_{\rm QCD}$, corrections can be large**

Large effects for the DVCS cross section in certain kinematics

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Operator Product Expansion

schematically

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\dots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\dots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\dots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\dots\mu_N}}_{\text{descendants of twist 2}}$$
$$+ C_N^{\mu_1\dots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\dots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\dots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\dots\mu_N}}_{\text{descendants}} + \dots \Big\}$$
$$+ \text{ quark-gluon operators}$$

"kinematic" corrections that repair the frame dependence and Ward identities come from

(1) corrections m/Q and $\sqrt{-t}/Q$ to the ME of twist-two operators (Nachtmann)

(2) higher-twist operators that are obtained from twist-two by adding total derivatives

Operator Product Expansion (2)

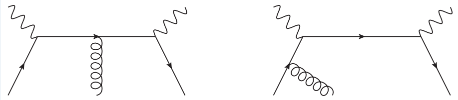
Problem: matrix elements of some descendant operators over free quarks vanish

Ferrara, Grillo, Parisi, Gatto, '71-'73

Example

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}\, g F_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

— Usual procedure to calculate the coefficient functions does not work, use qFq matrix elements



— Is it possible to separate "kinematic" and "genuine" (quark-gluon) contributions?

Guidung principle:

VB, A.Manashov, PRL 107 (2011) 202001

— "kinematic" approximation amounts to the assumption that genuine twist-four matrix elements are zero
— for consistency, they must remain zero at all scales
— they must not reappear at higher scales due to mixing with "kinematic" operators

- **"Kinematic" and "genuine" HT contributions must have autonomous scale-dependence**

[∼ The "kinematic" approximation corresponds to taking into account *all* operators with the same anomalous dimensions as the leading twist operators]

Let $G_{N,k}$ be your favourite set of twist-four operators

$$T\{j(x)j(0)\}^{\mathrm{tw}-4} = \sum_{N,k} c_{N,k}(x) G_{N,k}$$

Let $\mathcal{G}_{N,k}$ be the set of *multiplicatively renormalizable* twist-four operators

$$\mathcal{G}_{N,k} = \sum_{k'} \psi^{(N)}_{k,k'}\, G_{N,k'} \qquad\qquad \mathcal{G}_{N,k=0} \stackrel{!}{=} (\partial\mathcal{O})_N$$

If this relation can be inverted

$$G_{N,k} = \phi^{(N)}_{k,0} (\partial\mathcal{O})_N + \sum_{k'\neq 0} \phi^{(N)}_{k,k'}\, \mathcal{G}_{N,k'}$$

then

$$T\{j(x)j(0)\}^{\mathrm{tw}-4} = \sum_{N,k} c_{N,k}(x) \phi^{(N)}_{k,0}\, (\partial\mathcal{O})_N + \ldots$$

the ellipses stand for the contributions of “genuine” twist-four operators

The problem is that finding $\phi^{(N)}_{k,0}$ requires the knowledge of the full matrix $\psi^{(N)}_{k,k'}$, alias explicit solution of the twist-four RG equations.

Solution:

V.B., A. Manashov , JHEP 01 (2012) 085

Bukhvostov, Frolov, Lipatov, Kuraev, NPB 258 (1985) 601

- Four-particle twist-4 operators have autonomous scale-dependence $\longrightarrow$ irrelevant

V.B., A. Manashov, J. Rohrwild NPB 807 (2009) 89; NPB 826 (2010) 235

- RG equations for three-particle (non-quasipartonic) operators are hermitian w.r.t. a certain scalar product

Hence different solutions are mutually orthogonal w.r.t. a certain weight function:

$$\sum_k \mu_k^{(N)} \psi_{l,k}^{(N)} \psi_{m,k}^{(N)} \sim \delta_{l,m}$$

so that

$$\phi_{k,0}^{(N)} = \psi_{0,k}^{(N)} ||\psi_{0,k}^{(N)}||^{-2}, \qquad ||\psi_{0,k}^{(N)}||^2 = \sum_k \mu_k^{(N)} (\psi_{0,k}^{(N)})^2$$

and finally

V.B., A. Manashov , JHEP 01 (2012) 085

$$T\{j(x)j(0)\}^{\text{tw}-4} = \sum_N \left(\sum_k \frac{c_{N,k}(x)\psi_{0,k}^{(N)}}{||\psi_{0,k}^{(N)}||^2} \right) (\partial\mathcal{O})_N + \text{dynamical higher twist}$$

DVCS at twist-four: t/Q^2 and m^2/Q^2

$$\mathcal{A}_{\mu\nu} = -\, g^{\perp}_{\mu\nu}\, \mathcal{A}^{(0)} + \frac{1}{\sqrt{-q^2}} \left(q_\mu - q'_\mu \frac{q^2}{(qq')} \right) g^{\perp}_{\nu\rho} P^\rho\, \mathcal{A}^{(1)} + \frac{1}{2} \left(g^{\perp}_{\mu\rho} g^{\perp}_{\nu\sigma} - \epsilon^{\perp}_{\mu\rho}\epsilon^{\perp}_{\nu\sigma} \right) P^\rho P^\sigma\, \mathcal{A}^{(2)} + q'_\nu \mathcal{A}^{(3)}_\mu$$

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q'_\nu + q'_\mu q_\nu}{(qq')} + q'_\mu q'_\nu \frac{q^2}{(qq')^2} \qquad \epsilon^{\perp}_{\mu\nu} = \frac{1}{(qq')} \epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta$$

known to

$$\mathcal{A}^{(0)} \sim 1 + \frac{1}{Q^2}$$

$$\mathcal{A}^{(1)} \sim \frac{1}{Q}$$

$$\mathcal{A}^{(2)} \sim \frac{1}{Q^2}$$

- Physical observables including all helicity amplitudes:

A.V.Belitsky, D.Müller and Y.Ji, NPB 878, 214 (2014)

DVCS at twist-four: t/Q^2 and m^2/Q^2 (2)

- **Results:**
  - — translation and gauge invariance restored
  - — factorization valid at twist 4 (IR divergences cancel)
  - — correct threshold behavior $t \to t_{\min}$, $\xi \to 1$
  - — target mass corrections absorbed in the dependence on t_{min}

$$\frac{t + t_{min}}{Q}, \qquad t_{min} = -\frac{\xi^2 m^2}{1-\xi^2}$$

Compare DIS, Nachtmann variable

$$\xi_N = \frac{2x_B}{1+\sqrt{1+\frac{4x_B^2 m^2}{Q^2}}} = x_B\left(1 - \frac{x_B^2 m^2}{Q^2} + \ldots\right)$$

- On a nucleus $m \mapsto Am$, $x_B \mapsto x_B/A$, $\xi \mapsto \xi/A$ target mass corrections are the same

$\to$ factorization not in danger

New project

All orders in $(\sqrt{-t}/Q)^k$, $(m/Q)^k$?

apart from theoretical completeness

- Factor-two effects in some kinematic regions, need resummation to all twists
- Problems with some newer data ?
- Mass corrections in coherent DVCS on ^{4}He ?

The first step:

VB, Yao Ji, A. Manashov, JHEP 51 (2021) 51

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\ldots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\ldots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\ldots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\ldots\mu_N}}_{\text{descendants}} + \ldots \Big\} + \ldots$$

$$\equiv \sum_N C_N^{\mu_1\ldots\mu_N}(x,\partial)\, \mathcal{O}^N_{\mu_1\ldots\mu_N} + \text{ quark-gluon operators}$$

S. Ferrara, A. F. Grillo and R. Gatto, 1971-1973: "Conformally covariant OPE"

In conformal field theories, the CFs of descendants are related to the CFs of twist-2 operators by symmetry and do not need to be calculated directly

$$A_N^{\mu_1\ldots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\ldots\mu_N}(x,\partial)$$

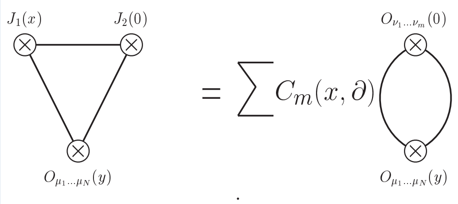
Conformal triangles

A.M. Polyakov, 1970:

$$\langle O_1(x_1)\, O_2(x_2)\rangle = \frac{\text{const}}{|x_1-x_2|^{2\Delta_1}}\,\delta_{\Delta_1\Delta_2}$$

$$\langle O_1(x_1)\, O_2(x_2)\, O_3(x_3)\rangle = \frac{\text{const}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_3+\Delta_2-\Delta_1}}$$

- $\leftarrow$ Δ_k is a scaling dimension (canonical + anomalous)

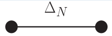


- $\leftarrow$ exact to all orders of perturbation theory and beyond

Tensor operators

$$\vec{\mu}_N = (\mu_1, \ldots, \mu_N)$$

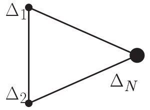
- two-point function



$$= \left\langle \mathcal{O}^{\vec{\mu}_N}_{\Delta_N}(x_1)\mathcal{O}^{\vec{\nu}_N}_{\Delta_N}(x_2)\right\rangle = c_N \mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\Delta_N}(x_{12}), \qquad x_{12} = x_1 - x_2$$

$$\mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\Delta_N}(x) = \frac{1}{|x|^{2\Delta_N}}\Big(\eta^{\mu_1\nu_1}(x)\ldots\eta^{\mu_N\nu_N}(x) + \text{permutations} - \text{traces}\Big)$$

- three-point function with two scalar operators



$$= \left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}^{\vec{\mu}_N}_{\Delta_N}(x_3)\right\rangle = c'_N T^{\vec{\mu}_N}_{\Delta_N}(x_1,x_2,x_3),$$

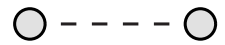
$$T^{\vec{\mu}_N}_{\Delta_N}(x_1,x_2,x_3) = \frac{\Lambda^{\mu_1}(x_1,x_2,x_3)\ldots\Lambda^{\mu_N}(x_1,x_2,x_3) \;-\; \text{traces}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_N+N}|x_{13}|^{\Delta_1+\Delta_N-N-\Delta_2}|x_{23}|^{\Delta_2+\Delta_N-N-\Delta_1}},$$

with

$$\eta^{\mu\nu}(x) = g^{\mu\nu} - \frac{2x^\mu x^\nu}{x^2}, \qquad \Lambda^\mu(x_1,x_2,x_3) = \frac{x^\mu_{13}}{x^2_{13}} - \frac{x^\mu_{23}}{x^2_{23}}$$

Shadow operator formalism   Ferrara et al., 1972

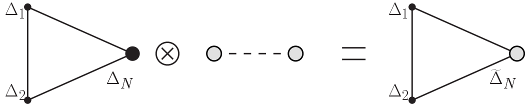
$$\widetilde{\Delta}_N = d - \Delta_N$$



$$= \left\langle \mathcal{O}^{\vec{\mu}_N}_{\tilde{\Delta}_N}(x_1)\mathcal{O}^{\vec{\nu}_N}_{\tilde{\Delta}_N}(x_2)\right\rangle = c_N \mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\tilde{\Delta}_N}(x_{12})$$

$$= \int d^d y\, \mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\Delta_N}(x_1 - y)\mathcal{D}^{\vec{\nu}_N\,\vec{\rho}_N}_{\tilde{\Delta}_N}(y - x_2)$$

$$= c_N\, \delta^{(d)}(x_{12})\left(g^{\mu_1\rho_1}\dots g^{\mu_N\rho_N} + \dots\right)$$



Shadow operator formalism (2)

- The coefficient function of the operator $\mathcal{O}_N^{\vec{\mu}_N}$ (including the descendants) is given by the Fourier transform of the "shadow triangle"

$$C^{\vec{\mu}_N}_{\Delta_N}(x_{12}, ip) = \mathcal{N} \int d^d y\, e^{ipy}\, T^{\vec{\mu}_N}_{\tilde{\Delta}_N}(x_1, x_2, y)$$

$$T^{\vec{\mu}_N}_{\tilde{\Delta}_N}(x_1,x_2,x_3) = \frac{\Lambda^{\mu_1}(x_1,x_2,x_3)\ldots\Lambda^{\mu_N}(x_1,x_2,x_3) \;-\; \text{traces}}{|x_{12}|^{\Delta_1+\Delta_2-\tilde{\Delta}_N+N}|x_{13}|^{\Delta_1+\tilde{\Delta}_N-N-\Delta_2}|x_{23}|^{\Delta_2+\tilde{\Delta}_N-N-\Delta_1}}$$

- Subtlety: contributions of $y \lesssim x_{12}$ must be excluded

How to take the integral?

$$T^{n\dots n}_{\tilde{\Delta}_N}(x_1,x_2,p) = n_{\mu_1}\dots n_{\mu_N}\int d^dy\, e^{ip\cdot y}\, T^{\mu_1\dots\mu_N}_{\Delta_N}(x_1,x_2,y) = ? \qquad n^2=0$$

- Useful representation

$$T^{n\dots n}_{\tilde{\Delta}_N}(x_1,x_2,y) = \frac{2^{-N}}{|x_{12}|^{\Delta_1+\Delta_2-\tilde{\Delta}_N+N}}\int_{\mathfrak{D}} D_{j_1}z_1\int_{\mathfrak{D}} D_{j_2}z_2\,\frac{\bar{z}_{12}^N}{|x_1-y-z_1n|^{4j_1}|x_2-y-z_2n|^{4j_2}},$$

$$4j_1 = \Delta_1+\tilde{\Delta}_N-N-\Delta_2\,, \qquad 4j_2 = \Delta_2+\tilde{\Delta}_N-N-\Delta_1$$

- Integration over z_1, z_2 goes over the unit disks $|z_k|\le 1$ in the complex plane

$$\mathfrak{D} = \{z\in\mathbb{C}, |z|<1\} \qquad\qquad D_jz = \frac{2j-1}{\pi}(1-|z|^2)^{2j-2}d^2z$$

- Identity: “Reproducing operator”

$$\forall f(w) \qquad f(w) = \int_{\mathfrak{D}} D_jz\,(1-w\bar{z})^{-2j}f(z)$$

Result: All-order OPE of two scalar currents

$$\bar{u} = 1 - u\,, \quad x_{12} = x_1 - x_2, \quad x_{21}^u = \bar{u}x_2 + ux_1$$

$$J_\Delta(x_1)J_\Delta(x_2) = \sum_N \frac{c_N}{|x_{12}|^{2\Delta - t_N}} \sum_{k=0}^{N} \frac{N!}{(N-k)!}\Gamma(\varkappa_N - k)\left(\frac{x_{12}^2}{8}\right)^k \times \int_0^1 du\,(u\bar{u})^{j_N-1} C_k^{\varkappa_N - k}(2u-1) \times \sum_{m=0}^{\infty} \frac{(-u\bar{u}x_{12}^2\partial^2/4)^m}{m!\Gamma(\Delta_N + k - d/2 + m + 1)} \mathcal{O}_N^{(k)}(x_{21}^u)\,.$$

$$\Delta_N = N + d - 2 + \gamma_N$$

$$j_N = \tfrac{1}{2}(\Delta_N + N), \quad t_N = \Delta_N - N, \quad \varkappa_N = \tfrac{1}{2}(d - t_N - 1)$$

where

$$\mathcal{O}_N^{(k)}(y) = \partial_y^{\mu_1} \ldots \partial_y^{\mu_k} \mathcal{O}_{\mu_1 \ldots \mu_k \mu_{k+1} \ldots \mu_N}(y) x_{12}^{\mu_{k+1}} \ldots x_{12}^{\mu_N}\,.$$

- The coefficients c_N and scaling dimensions Δ_N are not fixed by symmetry
- k counts applications of the divergence to the leading-twist operator, m counts applications of the Laplace operator, ∂^2
- Result originally derived in [Ferrara:1971vh] in a different form.

Vector currents

$$J^\mu(x_1)\, J^\nu(x_2) = ?$$

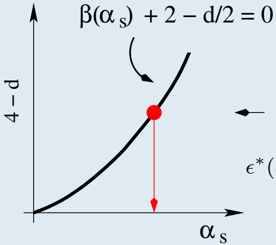
- Four Lorentz structures consistent with conformal symmetry
- Two relations from current conservation $\partial_\mu J^\mu = 0$

$\Rightarrow$

- Two independent structures, coefficients fixed by the two CFs C_2 and C_L in DIS (in $d = 4 - \epsilon$)

QCD?

QCD is not a conformal theory, but



$$\epsilon^*(a_s) = -\beta_0 a_s + \ldots\,, \qquad a_s = \frac{\alpha_s}{4\pi}$$

$$\boxed{\mathcal{A}_{\rm QCD} = \mathcal{A}^{\rm conf}_{\rm QCD} + \mathcal{O}\left(\frac{\beta(\alpha_s)}{\alpha_s}\right)}$$

"Conformal QCD": QCD in $d - 2\epsilon$ at Wilson-Fischer critical point $\beta(\alpha_S) = 0$

V.B., A. Manashov, Eur.Phys.J.C 73 (2013) 2544

Tasks

✓ Resummation of descendant operators in conformal QCD, all powers, all orders

V.B., Yao Ji, A. Manashov, JHEP 03 (2021) 051

✓ Short distance expansion $\to$ nonlocal (light-ray) OPE

(in preparation)

✓ DVCS amplitudes for a scalar target; Cancellation of IR divergences

(in preparation)

not yet done

- Nucleon target
- Observables

Leading order in α_s

- No extra terms $\sim \epsilon^*(a_s) = -\beta_0 a_s + \ldots$

- Sums are truncated

$$J^\mu(x) J^\nu(0) \sim A\frac{1}{x^4} + B\frac{1}{x^2} + \cancel{C + Dx^2 + \ldots}$$

- Polynomials produce delta-functions in momentum space, can be dropped

Local OPE: leading twist and descendants

$\bar{u}=1-u, \quad x_{12}=x_1-x_2, \quad x_{21}^u=\bar{u}x_2+ux_1$

V.B., Yao Ji, A. Manashov, JHEP 03 (2021) 051

$$
\begin{aligned}
\mathrm{T}\{j^\mu(x_1)j^\nu(x_2)\} = \sum_{N>0,\text{even}} r_N \int_0^1 du\,(u\bar{u})^N \Bigg\{ \frac{1}{x_{12}^4}\Bigg[(N+1)g_{\mu\nu}\left(1-\frac{1}{4}\frac{u\bar{u}}{N+1}x_{12}^2\partial^2\right) \\
+\frac{1}{2N}x_{12}^2\left(\partial_1^\mu\partial_2^\nu-\partial_1^\nu\partial_2^\mu\right)+\left(1-\frac{1}{4}\frac{u\bar{u}}{N}x_{12}^2\partial^2\right)\left(\frac{\bar{u}}{u}x_{21}^\mu\partial_1^\nu+\frac{u}{\bar{u}}x_{12}^\nu\partial_2^\mu\right) \\
-\frac{1}{4}\frac{u\bar{u}}{N(N+1)}x_{12}^2\partial^2\left(x_{21}^\nu\partial_1^\mu+x_{12}^\mu\partial_2^\nu\right)-\frac{x_{12}^\mu x_{12}^\nu}{N+1}u\bar{u}\partial^2\left(1-\frac{1}{4}\frac{u\bar{u}}{N+2}x_{12}^2\partial^2\right)\Bigg]\mathcal{O}_N^{(0)}(x_{21}^u) \\
+\frac{1}{x_{12}^2}\Bigg[-\frac{1}{4}N(\bar{u}-u)\,g_{\mu\nu}-\frac{\bar{u}-u}{4(N+1)}\left(x_{21}^\nu\partial_1^\mu+x_{12}^\mu\partial_2^\nu\right) \\
+\frac{1}{2}\left(\bar{u}\,x_{21}^\mu\partial_1^\nu-u\,x_{12}^\nu\partial_2^\mu\right)+\frac{N}{2(N+2)(N-1)}\left(x_{21}^\nu\partial_1^\mu-x_{12}^\mu\partial_2^\nu\right) \\
+\frac{1}{4}\frac{N(N^2+N+2)}{(N+1)(N+2)(N-1)}\left(\frac{u}{\bar{u}}x_{12}^\nu\partial_2^\mu-\frac{\bar{u}}{u}x_{21}^\mu\partial_1^\nu\right) \\
-\frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2}(\bar{u}-u)\frac{N}{N+1}\left(1-\frac{1}{2}\frac{u\bar{u}}{N+2}x_{12}^2\partial^2\right)\Bigg]\mathcal{O}_N^{(1)}(x_{21}^u) \\
+\frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2}\left[\frac{N^2+N+2}{4(N+1)(N+2)}-\frac{u\bar{u}N(N-1)}{(N+1)(N+2)}\right]\mathcal{O}_N^{(2)}(x_{21}^u)\Bigg\}+\ldots
\end{aligned}
$$

Local OPE: leading twist and descendants (2)

where

$$n_{\mu_1}\ldots n_{\mu_N}\mathcal{O}_N^{\mu_1\cdots\mu_N}(y) = \frac{\Gamma(3/2)\Gamma(N)}{\Gamma(N+1/2)}\left(\frac{i\partial_+}{4}\right)^{N-1}\bar{q}(y)\gamma_+ C_{N-1}^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)q(y)\,,$$

and

$$\mathcal{O}_N^{(k)}(y) = \partial_y^{\mu_1}\ldots\partial_y^{\mu_k}\mathcal{O}_{\mu_1\ldots\mu_k\mu_{k+1}\cdots\mu_N}(y)x_{12}^{\mu_{k+1}}\ldots x_{12}^{\mu_N}\,,$$

Local OPE → Light-ray OPE

In what follows set $x_1 = x, x_2 = 0$

- First step:
  For $x^2 \neq 0$ leading-twist $[O_N(ux)]_{lt} \neq O_N(ux)$ even if O_N is symmetrized and traceless in all indices

  Use I.Balitsky, V.B. '89

$$[f(x)]_{lt} = f(x) - \frac{1}{4}x^2 \int_0^1 \frac{dt}{t}\partial_x^2 f(tx) + \mathcal{O}(x^4)\,, \qquad \partial_x^2[f(x)]_{lt} = 0\,.$$

then, e.g.,

$$\int_0^1 du\,(u\bar{u})^N \mathscr{O}_N^{(0)}(ux) =$$
$$= \int_0^1 du\,(u\bar{u})^N \left\{[\mathscr{O}_N^{(0)}(ux)]_{lt} + \frac{1}{4}x^2\partial^2 \frac{u\bar{u}}{N+1}[\mathscr{O}_N^{(0)}(ux)]_{lt} + \frac{1}{2}x^2\frac{N\bar{u}}{N+1}[\mathscr{O}_N^{(1)}(ux)]_{lt} + \mathcal{O}(x^4)\right\}$$

Local OPE → Light-ray OPE (2)

- Second step:
  Find a nonlocal representation

  known:

$$\sum_N \varkappa_N \int_0^1 du\,(u\bar u)^N [O_N^{(0)}(ux)]_{lt} = [\bar q(x)\gamma_+ q(0)]_{lt}\,, \qquad \varkappa_N = \frac{2(2N+1)}{N!}$$

  needed:

$$\sum_N \varkappa_N {\color{red}f(N)} \int_0^1 du\,(u\bar u)^N {\color{red}g(u)} [O_N^{(k)}(ux)]_{lt} = ???$$

  example:

$$\sum_N \varkappa_N \frac{1}{N+1} \int_0^1 du\,(u\bar u)^N \frac{u}{\bar u} [O_N^{(0)}(ux)]_{lt} = \int_0^1 dv\,[\bar q(x)\not{x} q(vx)]_{lt}$$

- Systematic approach developed for a certain class of functions ← $SL(2)$ representations theory

Light-ray OPE; final result

$$
\begin{aligned}
\langle p'|\mathrm{T}\{j^\mu(x)j^\nu(0)|p\rangle = \frac{1}{i\pi^2}\langle p'|\Bigg\{ \frac{1}{x^4}\Bigg[\Big[g^{\mu\nu}(x\partial) - x^\mu\partial^\nu\Big]\int_0^1 du\, \mathscr{O}(\bar u,0) - x^\nu(\partial^\mu - i\Delta^\mu)\int_0^1 dv\, \mathscr{O}(1,v)\Bigg] \\
+ \frac{1}{x^2}\Bigg[\frac{i}{2}\Big(\Delta^\nu\partial^\mu - \Delta^\mu\partial^\nu\Big)\int_0^1 du\int_0^{\bar u} dv\, \mathscr{O}(\bar u,v) - \frac{\Delta^2}{4}x^\mu\partial^\nu\int_0^1 du\, u\int_0^{\bar u} dv\, \mathscr{O}(\bar u,v)\Bigg] \\
+ \frac{\Delta^2}{2}\frac{x^\mu x^\nu}{x^4}\int_0^1 du\,\bar u\int_0^{\bar u} dv\, \mathscr{O}(\bar u,v) + \frac{1}{4x^2}g^{\mu\nu}\Bigg[-\int_0^1 du\int_0^{\bar u} dv\, \mathscr{O}^{(1)}(\bar u,v) + \int_0^1 dv\, \mathscr{O}^{(-)}(1,v)\Bigg] \\
- \frac{1}{4x^2}(x^\nu\partial^\mu + x^\mu\partial^\nu - ix^\mu\Delta^\nu)\int_0^1 du\int_0^{\bar u} dv\,\Bigg(\ln\bar\tau\,\mathscr{O}^{(1)}(\bar u,v) + \frac{v}{\bar v}\,\mathscr{O}^{(-)}(\bar u,v)\Bigg) \\
- \frac{1}{2x^2}(x^\nu\partial^\mu - x^\mu\partial^\nu + ix^\mu\Delta^\nu)\int_0^1 du\int_0^{\bar u} dv\,\frac{\tau}{\bar\tau}\Bigg(-\mathscr{O}^{(1)}(\bar u,v) + \frac{\bar u}{u}\,\mathscr{O}^{(-)}(\bar u,v)\Bigg) \\
- \frac{1}{4x^2}x^\nu(\partial^\mu - i\Delta^\mu)\Bigg[\int_0^1 du\int_0^{\bar u} dv\,\frac{v}{\bar v}\Bigg[-2\Bigg(1+\frac{2\tau}{\bar\tau}\Bigg)\,\mathscr{O}^{(1)}(\bar u,v) + \frac{v}{\bar v}\mathscr{O}^{(-)}(\bar u,v)\Bigg] + \int_0^1 dv\,\frac{v}{\bar v}\mathscr{O}^{(-)}(1,v)\Bigg] \\
- \frac{1}{2x^2}x^\mu\partial^\nu\int_0^1 du\int_0^{\bar u} dv\,\Bigg[(\ln\bar u + u)\,\mathscr{O}^{(1)}(\bar u,v) + \bar u\,\mathscr{O}^{(-)}(\bar u,v) - \frac{1}{2}\Bigg(1+\frac{4\tau}{\bar\tau}\Bigg)\,\mathscr{O}^{(-)}(\bar u,v)\Bigg] \\
- \frac{x^\mu x^\nu}{x^4}\int_0^1 du\int_0^{\bar u} dv\,\Bigg[(\ln\bar\tau + \ln\bar u + u)\,\mathscr{O}^{(1)}(\bar u,v) + \Bigg(\frac{v}{\bar v} + \bar u\Bigg)\,\mathscr{O}^{(-)}(\bar u,v)\Bigg] \\
- \frac{x^\mu x^\nu}{4x^2}\Big[i(\Delta\partial) + \frac{1}{2}\Delta^2\Big]\int_0^1 du\int_0^{\bar u} dv\,\frac{v}{\bar v}\Bigg(\frac{2}{\bar\tau} - 1\Bigg)\,\mathscr{O}^{(1)}(\bar u,v) \\
+ \frac{x^\mu x^\nu}{2x^2}\Big[i(\Delta\partial) + \frac{1}{4}\Delta^2\Big]\int_0^1 du\int_0^{\bar u} dv\,\Bigg(\ln\bar\tau + \frac{2\tau}{\bar\tau}\Bigg)\,\mathscr{O}^{(1)}(\bar u,v)\Bigg\}|p\rangle
\end{aligned}
$$

$\tau = \dfrac{uv}{\bar u\bar v}$ $\partial_\mu = \dfrac{\partial}{\partial x^\mu}$

Helicity amplitudes for a scalar target

- Kinematics:

$$\mathcal{A}_{\mu\nu} = -\, g^{\perp}_{\mu\nu}\,\mathcal{A}^{(0)} + \frac{1}{\sqrt{-q^2}}\left(q_\mu - q'_\mu \frac{q^2}{(qq')}\right) g^{\perp}_{\nu\rho} P^\rho\,\mathcal{A}^{(1)} + \frac{1}{2}\left(g^{\perp}_{\mu\rho} g^{\perp}_{\nu\sigma} - \epsilon^{\perp}_{\mu\rho}\epsilon^{\perp}_{\nu\sigma}\right) P^\rho P^\sigma\,\mathcal{A}^{(2)} + q'_\nu \mathcal{A}^{(3)}_\mu$$

transverse directions are defined vs. q and q':

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q'_\nu + q'_\mu q_\nu}{(qq')} + q'_\mu q'_\nu \frac{q^2}{(qq')^2}\,, \qquad \epsilon^{\perp}_{\mu\nu} = \frac{1}{(qq')}\epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta$$

- Goal:

$$\mathcal{A}^{(0)} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \dots \qquad \checkmark$$

$$\mathcal{A}^{(1)} \sim \frac{1}{Q} + \frac{1}{Q^3} + \dots \qquad \checkmark$$

$$\mathcal{A}^{(2)} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \dots \qquad \checkmark$$

- further terms can be calculated if necessary

Helicity amplitudes for a scalar target (2)

- Two expansion parameters

$$P^\mu = \frac{1}{2}(p+p')^\mu\,, \qquad P_\perp^\mu = g_\perp^{\mu\nu} P^\nu$$

$$\Delta^2 = (p'-p)^2 = t \qquad \xi^2 P_\perp^2 = \xi^2 m^2 \frac{t - t_{\rm min}}{t_{\rm min}} \qquad t_{\rm min} = -\frac{4\xi^2 m^2}{1-\xi^2}$$

- Convolution integral with GPD $H(x,\xi)$

$$H \otimes f = \int_{-1}^{1} \frac{dx}{\xi} H(x,\xi) f\left(\frac{x+\xi}{2\xi}\right)\,, \qquad \xi \to \xi - i0$$

- Useful derivative

$$D_\xi = \xi^2 \frac{\partial}{\partial \xi}$$

Helicity flip amplitudes $\mathcal{A}^{(1)}$, $\mathcal{A}^{(2)}$ for a scalar target

$$\mathcal{A}^{(1)} = \frac{2Q}{(qq')} D_\xi \left(H \otimes \frac{\ln \bar{z}}{z} \right) + \frac{\Delta^2 Q}{(qq')^2} \left[D_\xi \left(H \otimes \frac{\bar{z} - z}{z} \ln \bar{z} \right) + \frac{1}{\xi} D_\xi^2 \left(H \otimes \frac{1}{\bar{z}} (\mathrm{Li}_2(z) - \zeta_2) - \ln \bar{z} \right) \right]$$

$$\mathcal{A}^{(2)} = \frac{8}{(qq')} \left(1 + \frac{\Delta^2}{8(qq')} \right) D_\xi^2 \left(H \otimes \frac{z - \bar{z}}{z} \ln \bar{z} \right) + \frac{1}{(qq')^2} \left(4P_\perp^2 D_\xi^2 - \frac{3}{\xi} \Delta^2 D_\xi + 6\Delta^2 \right) D_\xi^2 \left(H \otimes \frac{1}{\bar{z}} \left(\mathrm{Li}_2(z) - \zeta_2 \right) - \ln \bar{z} \right)$$

- Leading term in $\mathcal{A}^{(1)}$ is known as the twist-three WW contribution

Helicity conserving amplitude $\mathcal{A}^{(0)}$ for a scalar target

$$\mathcal{A}^{(0)} = \underbrace{\mathcal{A}^{(0)}_{00}}_{\mathcal{O}(1)} + \underbrace{\frac{P_\perp^2}{(qq')}\mathcal{A}^{(0)}_{10} + \frac{\Delta^2}{(qq')}\mathcal{A}^{(0)}_{01}}_{\mathcal{O}(1/Q^2)} + \underbrace{\frac{(P_\perp^2)^2}{(qq')^2}\mathcal{A}^{(0)}_{20} + \frac{P_\perp^2\Delta^2}{(qq')^2}\mathcal{A}^{(0)}_{11} + \frac{(\Delta^2)^2}{(qq')^2}\mathcal{A}^{(0)}_{02}}_{\mathcal{O}(1/Q^4)} + \ldots$$

- Leading power

$$\mathcal{A}^{(0)}_{00} = -\,\mathcal{H}\otimes\frac{1}{z}$$

- next-to-leading power

$$\mathcal{A}^{(0)}_{10} = -2D_\xi^2\left(\mathcal{H}\otimes\frac{1}{\bar z}\left(\frac{1}{2}\ln z + \mathrm{Li}_2(z) - \zeta_2\right)\right),$$

$$\mathcal{A}^{(0)}_{01} = \mathcal{H}\otimes\frac{1}{2\bar z}\left(\frac{1}{2} - \ln z\right) + \frac{1}{\xi}D_\xi\left(\mathcal{H}\otimes\frac{1}{\bar z}\left(\frac{1}{2}\ln z + \mathrm{Li}_2(z) - \zeta_2\right)\right)$$

VB, A. Manashov, B. Pirnay, PRD 86 (2012) 014003

Helicity conserving amplitude $\mathcal{A}^{(0)}$ for a scalar target

- next-to-next-to-leading power

PRELIMINARY

$$\mathcal{A}^{(0)}_{20} = -2D_\xi^4 \Big(\mathcal{H} \otimes \Big(3\mathcal{G}_1(z) - 2\mathcal{G}_2(z)\Big)\Big)$$

$$\mathcal{A}^{(0)}_{11} = 2D_\xi^2 \Big(\mathcal{H} \otimes (-5\mathcal{G}_1(z) + 2\mathcal{G}_2(z))\Big) + \frac{4}{\xi} D_\xi^3 \Big(\mathcal{H} \otimes \Big(3\mathcal{G}_1(z) - 2\mathcal{G}_2(z)\Big)\Big)$$

$$\mathcal{A}^{(0)}_{02} = \Big(\mathcal{H} \otimes (\bar{z} - z)\frac{\ln \bar{z}}{4z}\Big) + \frac{1}{\xi} D_\xi \Big(\mathcal{H} \otimes \mathcal{G}_1(z)\Big) - \frac{1}{\xi^2} D_\xi^2 \Big(\mathcal{H} \otimes \Big(3\mathcal{G}_1(z) - 2\mathcal{G}_2(z)\Big)\Big)$$

$$\mathcal{G}_1(z) = \frac{1}{\bar{z}}\Big(\mathrm{Li}_2(z) - \zeta_2\Big) - \ln \bar{z}, \qquad \mathcal{G}_2(z) = \mathrm{Li}_2(z) - \frac{\bar{z}}{4z} \ln \bar{z}.$$

- All IR divergences at $q'^2 \to 0$ cancel

Outlook

- DVCS: Nucleon target; axial-vector contributions, numerical studies in JLAB/EIC kinematics
- Other two-photon processes, e.g. $\gamma^*\gamma \to \pi\pi$
- Conformal triangles with light-ray operators
- Power corrections to NLO in α_s
- Generic $2 \to 2$ scattering amplitudes, heavy quarks,. . .