QED Corrections in Leptonic B-Meson Decays

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Invitation - a simple computation for a simple process

With **LFU tests** being en vogue these days, $B \to \ell\nu$ is one possible corroborating channel. QED corrections **violate LFU** due to their sensitivity to the mass.

Also: with Belle II on track to measure $B \to \mu\nu$ at $\sim 7\%$, it can be an exclusive determination of V_{ub} $\Rightarrow$ a precise prediction is in order.

[Belle II physics book]

The **leading corrections** will be double-logarithms $L_\ell = \log m_\ell^2/m_B^2$. These arise in both virtual and real corrections.

In the exclusive case, one puts a **cut on photon energy**: $E_\gamma < E_s/2$. This gives additional logarithms $L_s = \log E_s^2/m_\ell^2$.

All of the aforementioned corrections arise from **radiation too soft to resolve** the structure of the meson.

So, treat meson as a **charged scalar** with $\mathcal{L}_y = y\phi_B\,(\bar{\ell}P_L\nu)$ and compute NLO QED decay rate:

$$\Gamma_{\rm NLO} = \Gamma_{\rm LO}\left\{1 + \frac{\alpha}{2\pi}\left[\frac{3}{2}L_\mu - L_\ell^2 - L_\ell L_s - \frac{7}{2}L_\ell - 2L_s - \frac{\pi^2}{3} + 2\right]\right\}$$

Exponentiation of L_ℓ^2 straight-forward. But what about $L_\ell \cdot L_s$? What about the single-logs? Are we missing something in this crude treatment? **Can we do better?**

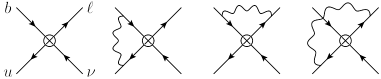
Yes, but things will get **much** more complicated (and fun!).

The process $B \to \ell\nu$ depends on a multitude of scales ($\ell = \mu$ for all of this talk)

m_W

m_B

$\mu_{\rm hc} = \sqrt{m_B \Lambda_{\rm QCD}}$

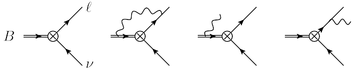


description of meson by $|b\bar{q}\rangle + \ldots$ [Beneke et al (*2019*), JHEP10 232]

$m_\mu \sim \Lambda_{\rm QCD}$

description of meson by point-like scalar

E_s



$\mu_{\rm sc} = \frac{E_s m_\mu}{m_B}$

[Dai et al (*2022*), Phys.Rev.D 105 3]

Can treat all of these thresholds

The plan

The program is as follows:

- Find **all relevant scales** and the appropriate effective description at each scale, complete with their matching coefficients and renormalization group equations

- Derive a **factorization theorem** to break the multiscale process into a **product of single-scale objects**

- Use the **renormalization group** to evaluate each object at its natural scale and evolve them to a common scale to **resum logarithms**

Matching step 1 - $\mu \sim m_b$

The story starts as usual. HQET for the b-quark:

$$b(x) \to e^{im_b(v\cdot x)} h_v(x)$$

Power-counting: $\lambda = m_\ell / m_B \sim \Lambda_{\mathrm{QCD}} / m_b$.

QED corrections: Photon exchange **between the partons** and the final state **lepton**.

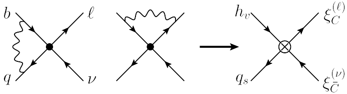
Relevant momentum modes - my notation is $p^\mu = (\bar{n}\cdot p, n\cdot p, p_\perp)$:

$p \sim (1, \lambda^2, \lambda)$	"collinear" - leptons have $p^2 \sim m_\ell^2 \sim \mathcal{O}\left(\lambda^2\right)$
$p \sim (1, \lambda, \sqrt{\lambda})$	"hard-collinear" - soft and collinear x-talk $p^2 \sim m_\ell m_b$
$p \sim (\lambda, \lambda, \lambda)$	"soft" - spectator quarks with $p^2 \sim \Lambda_{\mathrm{QCD}}^2$

Standard program: **SCET I** construction that we then match **to SCET II**.

Regions of the one-loop graphs

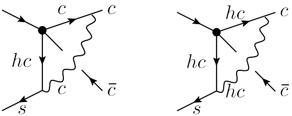
Loops with virtuality $p_h^2 \sim \mathcal{O}(m_b^2)$ match onto four-fermion operators in SCET I:



Contribute at tree-level → call them "**A-type**" (direct) contributions.

Indirect contributions ("**B-type**") from loops in the EFT.

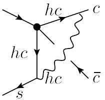
Consider photon exchange **between spectator and lepton**, **collinear** and **hard-collinear**:



Contributions at **separate virtualities** belonging to SCET I/SCET II matrix elements.
→ Tree and one-loop matching at μ_{hc}.

The virtual lepton

In the hard-collinear loop (of full QED) the **virtual lepton** propagator enters as



$$= \int \frac{d^d l}{(2\pi)^d} \cdots \frac{1}{(l+q)^2 - m_\ell^2}$$

The object is **not homogeneous** in power-counting:

$$\frac{1}{(l+q)^2 - m_\ell^2} = \frac{1}{(l+q_+)^2} \left[1 + \frac{m_\ell^2 - (n \cdot q)(\bar{n} \cdot l + \bar{n} \cdot q)}{(l+q_+)^2} + \dots \right] .$$

The effective theory needs to reproduce **both types** of **power-corrections**.

Massive SCET Lagrangians

A hard-collinear lepton has $q^2 > m_\ell^2 \quad \to$ **off-shell**.

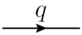
On-shell leptons are collinear, so $q^2 \sim m_\ell^2$.

Their Lagrangians **look identical**

$$\mathcal{L}_{(\ell)} = \bar{\xi}^{(\ell)}(in \cdot D)\frac{\not{\bar{n}}}{2}\xi^{(\ell)} + \bar{\xi}^{(\ell)}(i\not{D}_\perp - m_\ell)\frac{1}{i\bar{n} \cdot D}(i\not{D}_\perp + m_\ell)\frac{\not{\bar{n}}}{2}\xi^{(\ell)} \, .$$

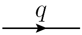
But their power-counting is not:

Collinear lepton:

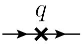


$$= \frac{i\not{n}}{2}\frac{\bar{n} \cdot q}{q^2 - m_\ell^2}$$

Hard-Collinear lepton:



$$= \frac{i\not{n}}{2}\frac{\bar{n} \cdot q}{q^2}$$



$$= -\frac{i\not{\bar{n}}}{2}\frac{m^2}{\bar{n} \cdot q}$$

Mixed-collinear interactions

But there is another type of term in the propagator expansion:

The **small components** of collinear and hard-collinear momenta are **not homogeneous**: $n \cdot q < n \cdot l$.

For the EFT to reproduce these power-corrections, we need to have **both** hard-collinear and collinear modes.

This goes **against the usual** SCET I $\to$ SCET II treatments, where:

- Above the **hard-collinear** scale, the only collinear modes are hard-collinear.
- At $\mu \sim \mu_{hc}$, we **lower the virtuality**, and integrate out hard-collinear modes $\psi_C \to \psi_c + \psi_c \psi_s + \ldots$
- The **collinear modes** of SCET II are then thought to be contained **in the hard-collinear** modes of SCET I.

Mixed-collinear interactions

Keeping **both** modes, we obtain a **mixed Lagrangian** $\mathcal{L}_{Cc}$ which describes the interactions between **collinear** and **hard-collinear** modes.

These interactions are power-suppressed, starting at $\mathcal{O}\left(\sqrt{\lambda}\right)$.

Similar to the soft-collinear Lagrangian, the **collinear field** needs to be **multipole-expanded**:

$$\xi_c(x) \to \left[\bar{\xi}_c + (x_\perp \cdot \partial_\perp)\bar{\xi}_c + \frac{1}{2}x_\perp^\mu x_\perp^\nu \partial_\mu^\perp \partial_\nu^\perp \bar{\xi}_c + (x_+ \cdot \partial_-)\bar{\xi}_c + \dots\right](x_-)$$

Lagrangian is a copy of the **hard-collinear Lagrangian** with one hard-collinear field **replaced** by the multipole-expanded **collinear field**.

With the following power-counting:

$$h_v, q_s \sim \mathcal{O}\left(\lambda^{3/2}\right) \quad \chi_C \sim \mathcal{O}\left(\lambda^{1/2}\right) \qquad \chi_c \sim \mathcal{O}\left(\lambda\right) \quad \mathcal{A}_c^{\perp} \sim \mathcal{O}\left(\lambda\right)$$

we can define our operator basis. Two types:

direct (A-type) $\quad \mathcal{O}_A \sim (\bar{q}_s \ldots h_v)\left(\bar{\chi}_c^{(l)} \ldots \bar{\chi}_{\bar{c}}^{(\nu)}\right)$

indirect (B-type) $\quad \mathcal{O}_B \sim (\bar{\chi}_C \ldots h_v)\left(\bar{\chi}_C^{(l)} \ldots \bar{\chi}_{\bar{c}}^{(\nu)}\right)$

The process $B \to \ell\nu$ is **chirality-suppressed**. Process starts at **subleading power** in λ-counting*!

$\rightarrow$ B-type contributions from **lower-order operators** with **subleading Lagrangian** insertions.

SCET I operators: A-type

The A-type (direct) operators generate the process starting at **tree-level in SCET I**, encodes all hard corrections:

$$\mathcal{O}_A^{(5)} = \left(\bar{q}_s \gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_c^{(\ell)} \gamma_\mu^\perp P_L \chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{O}_{A,1}^{(6)} = \frac{m_\ell}{m_b}\left(\bar{q}_s \frac{\bar{\not{n}}}{2} P_L h_v\right)\left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{O}_{A,2}^{(6)} = \frac{m_\ell}{m_b}\left(\bar{q}_s \frac{\not{n}}{2} P_L h_v\right)\left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{O}_{A,3}^{(6)} = \frac{1}{m_b}\left(\bar{q}_s i\overleftarrow{D}_{s\perp}^\mu \frac{\bar{\not{n}}}{2} P_L h_v\right)\left(\bar{\chi}_c^{(\ell)} \gamma_\mu^\perp P_L \chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{O}_{A,4}^{(6)} = \frac{1}{m_b}\left(\bar{q}_s i\overleftarrow{D}_{s\perp}^\mu \frac{\not{n}}{2} P_L h_v\right)\left(\bar{\chi}_c^{(\ell)} \gamma_\mu^\perp P_L \chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{O}_{A,5}^{(6)} = \frac{1}{m_b}\left(\bar{q}_s (v \cdot i\overleftarrow{D}_s) \gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_c^{(\ell)} \gamma_\mu^\perp P_L \chi_{\bar{c}}^{(\nu)}\right)$$

Not all have overlap with the pseudoscalar meson.

SCET I operators: B-type

Operators with more than one hard-collinear fermion:

$$\mathcal{O}_B^{(7/2)} = \left(\bar{\chi}_C^{(q)}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_C^{(\ell)}\gamma_\mu^\perp P_L \chi_c^{(\nu)}\right)$$

$$\mathcal{O}_{B,1}^{(4)} = \left(\bar{\chi}_C^{(q)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}i\overleftarrow{\not\partial}_\perp\frac{\not\bar{n}}{2}P_L h_v\right)\left(\bar{\chi}_C^{(\ell)}P_L\chi_c^{(\nu)}\right)$$

$$\mathcal{O}_{B,2}^{(4)} = \left(\bar{\chi}_C^{(q)}\frac{\not\bar{n}}{2}P_L h_v\right)\left(\bar{\chi}_C^{(l)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}i\overleftarrow{\not\partial}_\perp P_L\chi_{\bar{c}}^{(\nu)}\right)$$

$$\mathcal{O}_B^{(9/2)} = m_\ell\left(\bar{\chi}_C^{(q)}\frac{\not\bar{n}}{2}P_L h_v\right)\left(\bar{\chi}_C^{(l)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}P_L\chi_c^{(\nu)}\right)$$

Since we are only interested in the $\mathcal{O}(\alpha)$ result, we content ourselves with **tree-level matching** for these operators.

From RPI we know:

$$\mathcal{C}_B^{(7/2)} = -\mathcal{C}_{B,1}^{(4)} = -\frac{1}{2}\mathcal{C}_{B,2}^{(4)} = -\frac{1}{2}\mathcal{C}_B^{(9/2)}$$

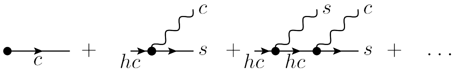
We now remove all hard-collinear modes.

$$p_c \sim (1, \lambda^2, \lambda)\,, \qquad p_s \sim (\lambda, \lambda, \lambda)\,, \qquad p_c^2 \sim p_s^2 \sim \mathcal{O}\left(\lambda^2\right)\,.$$

⇒ Now a pure SCET II type construction.

Integrating the hard-collinear modes:

$$\psi_C \quad \rightarrow \quad \psi_c \;+\; \psi_c \cdot \psi_s \;+\; \psi_c \cdot \psi_s^2 \;+\; \ldots$$



The intermediate propagators introduce **non-localities**, even in soft operator products:

$$\frac{1}{n\cdot\partial}q_s, \quad \left(\frac{1}{n\cdot\partial}\mathcal{A}^{\mu}_{\perp s}\right)\left(\frac{1}{n\cdot\partial}q_s\right), \quad \ldots$$ ⇒ more fields, same order

SCET II – non-localities

Inverse-derivative operators can probe meson structure:

$$\left\langle 0 \middle| \left(\frac{1}{n\cdot\partial} q_s\right) \ldots h_v \middle| B \right\rangle \sim \frac{1}{\lambda_B} \sim \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^{-1}\right)$$

Can overcome the power-suppression: $\dfrac{m_\ell}{\lambda_B} \sim \mathcal{O}(1)$

This is precisely what is happening in $B_s \to \mu\mu$. What about $B \to \ell\nu$?

The $\sim 1/\omega$ terms come with Dirac structures, that are **fully evanescent** for left-handed currents:

$$\left(\bar{v}\frac{\not{n}}{2}\gamma_\perp^\mu\gamma_\perp^\nu P_L u\right)_h \left(\bar{u}\gamma_\mu^\perp\gamma_\nu^\perp \left[\frac{v-a\gamma_5}{2}\right] v\right)_\ell = 2(v-a)\left(\bar{v}\frac{\not{n}}{2}P_L u\right)_h \left(\bar{u}P_R v\right)_\ell + \mathcal{O}(\epsilon)$$

For us $v = a$ so this is evanescent.

Of course, the $\mathcal{O}(\epsilon)$ structures **can still contribute** to physical processes, once these operators are inserted into loop graphs:

$$\mathcal{O}\left(\epsilon\right) \cdot \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^4} \quad = \quad \mathcal{O}\left(1\right)$$

Such contribution is usually absorbed into **finite counterterm** of the physical operators.

Here: Enhanced structure-dependent contributions ...

- ... are evanescent $\rightarrow$ means they might not be log-enhanced but finite corrections could still exist!
- ... and fully cancel between matrix elements and matching coefficients $\rightarrow$ means they vanish identically.

SCET II operators

Operator basis now:

$$O_A^{(5)} = \left(\bar{q}_s(sn)\gamma_\mu^\perp P_L h_v(0)\right)\left(\bar{\chi}_c^{(\ell)}(t\bar{n})\gamma_\perp^\mu P_L \chi_{\bar{c}}^{(\nu)}(un)\right)$$

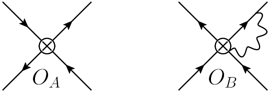
$$O_{A,1}^{(6)} = \frac{m_\ell}{m_B}\left(\bar{q}_s(sn)\frac{\not{n}}{2}P_L h_v(0)\right)\left(\bar{\chi}_c^{(\ell)}(t\bar{n})P_L \chi_{\bar{c}}^{(\nu)}(un)\right)$$

$$O_{A,2}^{(6)} = \frac{m_\ell}{m_B}\left(\bar{q}_s(sn)\frac{\not{\bar{n}}}{2}P_L h_v(0)\right)\left(\bar{\chi}_c^{(\ell)}(t\bar{n})P_L \chi_{\bar{c}}^{(\nu)}(un)\right)$$

$$O_{B,1}^{(6)} = \left(\bar{q}_s(sn)\frac{\not{n}}{2}P_L h_v(0)\right)\left(\bar{\chi}_c^{(\ell)}(t_1\bar{n})\not{\mathcal{A}}_c^\perp(t_2\bar{n})P_L \chi_{\bar{c}}^{(\nu)}(un)\right)$$

$$O_{B,2}^{(6)} = \left(\bar{q}_s(sn)\frac{\not{\bar{n}}}{2}P_L h_v(0)\right)\left(\bar{\chi}_c^{(\ell)}(t_1\bar{n})\not{\mathcal{A}}_c^\perp(t_2\bar{n})P_L \chi_{\bar{c}}^{(\nu)}(un)\right)$$

Matrix elements $\langle O_A \rangle$ and $\langle O_B \rangle$ start at tree-level and one-loop, respectively:



Matching step 3 - the muon mass

Below the muon mass, we treat the **muon as infinitely heavy**, and thus integrate it out.

In this matching step, **all loops with collinear photons** (which have virtualities $q^2 \sim m_\mu^2$, but $\bar{n} \cdot q \sim m_B$) are integrated out.

As the muon gets integrated out, it is **replaced by an HQET-like field**. Consequently, the only remaining scales in the theory are the photon cut E_s and the lowest scale $\mu_{\rm sc} = E_s m_\mu / m_B$.

This means, there are **no virtual corrections** in this EFT!

Matching step 4 - the hadronic scale

Below $\mu \sim \Lambda_{\mathrm{QCD}}$ we are passing to an **effective description** of a Yukawa theory:

$$\mathcal{L}_{\mathrm{y}} = y\, e^{-im_B(v\cdot x)} \varphi_B \left(\bar{\chi}_{v_\ell} P_L \chi_{\bar{c}}^{(\nu)} \right) + \mathrm{h.c.}$$

The Yukawa coupling is then fixed by **matching hadronic matrix elements** between our previous description and this:

$$\langle \ell\, \nu \,|\, \mathcal{L}_{\mathrm{SCET\,II}\otimes\mathrm{HQET}} \,|\, B \rangle \quad = \quad \langle \ell\, \nu \,|\, \mathcal{L}_{\mathrm{SCET\,II}\otimes\mathrm{HSET}} \,|\, B \rangle$$

Understanding the low-energy theory

In order to understand the **degrees of freedom** of the low-energy theory, a region analysis is in order.

Remember the cut on extra photons was: $E_\gamma < E_s/2 \sim \mathcal{O}\left(\lambda^2\right)$.

Which scalings can the photon now have?

- **At least one** component needs to **probe the cut** and thus be $\sim E_s$.
- Either **all components** are of this order **or other components** have to be **smaller**.
- The photon sees **only one collinear direction**, so the largest component (if exists) points in this direction.

General scaling for photon with virtuality $q^2 \sim \lambda^t$:

$$q \sim (\lambda^2,\ \lambda^{t-2},\ \lambda^{t/2}) \qquad \text{find} \qquad \begin{aligned} t = 4 &: \quad q \sim (\lambda^2, \lambda^2, \lambda^2) \\ t = 6 &: \quad q \sim (\lambda^2, \lambda^4, \lambda^3) \end{aligned}$$

Understanding the low-energy theory

To understand the soft-collinear scaling $q \sim (\lambda^2, \lambda^4, \lambda^3)$, boost it to the rest frame of the muon:

$$q_{\rm sc} \sim m_B(\lambda^2, \lambda^4, \lambda^3) \quad \rightarrow \quad q'_{\rm sc} \sim m_B(\lambda^3, \lambda^3, \lambda^3) \sim m_\ell(\lambda^2, \lambda^2, \lambda^2)$$

This shows that the **new mode** is just an **ultrasoft photon** to the heavy lepton, just as the ultrasoft scaling $q_{\rm s} \sim m_B(\lambda^2, \lambda^2, \lambda^2)$ is to the meson.

We now understand the lepton needs to be described by a **boosted HQET** construction ⇒ “bHLET”.

This new region gives rise to logarithms $L_\mu^{\rm sc} = \log \mu^2/\mu_{\rm sc}^2$ of the **soft-collinear scale** $\mu_{\rm sc} = \frac{E_s m_\ell}{m_B}$:

$$\Gamma_{(\rm sc)} = \Gamma_0 \frac{\alpha}{2\pi} \left[-\frac{1}{\epsilon^2} + \frac{1 - L_\mu^{\rm sc}}{\epsilon} - \frac{1}{2} \left(L_\mu^{\rm sc} \right)^2 + L_\mu^{\rm sc} - \frac{\pi^2}{12} \right]$$

The low-energy theory is now given by (HSET $\otimes$ bHLET), with the fields:

$$\ell(x) = e^{-im_\ell\,(v_\ell \cdot x)} \chi_{v_\ell}(x)\,, \qquad \Phi_B(x) = e^{-im_B\,(v \cdot x)} \varphi_B$$

Interactions with ultrasoft and soft-collinear photons can be moved into Wilson lines by the HQET decoupling transformations, with:

$$Y_v^{(\mathrm{s})}(x) = \mathcal{P}\exp\left\{ie\int_{-\infty}^{0} ds\, v \cdot A_s(x+sv)\right\}$$
$$Y_v^{(\mathrm{sc})}(x) = \mathcal{P}\exp\left\{ie\int_{-\infty}^{0} ds\, v \cdot A_{\mathrm{sc}}(x+sv)\right\}$$

leading to the operator:

$$O_\varphi = Y_n^{(\mathrm{s})}(x_-)Y_v^{(\mathrm{s})\dagger}(x)Y_{v_\ell}^{(\mathrm{sc})}(x)Y_{\bar{n}}^{(\mathrm{sc})\dagger}(x_+) \cdot \varphi_B(x)\left(\bar{\chi}_{v_\ell} P_L \xi_{\bar{c}}^{(\nu)}\right)$$

Real radiation and soft function

With radiation decoupled, **real corrections** are fully described by matrix elements of the Wilson lines.

Ultrasoft and soft-collinear functions:

$$W_s(\omega_s,\mu) = \left[\sum_{n_s=0}^{\infty}\prod_{i=1}^{n_s}\int d\Pi_i(q_i)\right]\left|\langle n_s\gamma_s(q_i)|\, Y_v^{(\mathrm{s})}Y_n^{(\mathrm{s})\dagger}\,|0\rangle\right|^2\delta\left(\omega_s - q_0^{(\mathrm{s})}\right)\,,$$

$$W_{sc}(\omega_{sc},\mu) = \left[\sum_{n_{sc}=0}^{\infty}\prod_{j=1}^{n_s}\int d\Pi_j(q_j)\right]\left|\langle n_{sc}\gamma_{sc}(q_j)|\, Y_{\bar{n}}^{(\mathrm{sc})\dagger}Y_{v_l}^{(\mathrm{sc})}\,|0\rangle\right|^2\delta\left(\omega_{sc} - q_0^{(\mathrm{sc})}\right)\,,$$

When integrated over a **measurement function**, they combine to the **soft function** of the process:

$$S(E_s,\mu) = \int_0^\infty d\omega_s\int_0^\infty d\omega_{sc}\;\theta\left(\frac{E_s}{2}-\omega_s-\omega_{sc}\right)W_s(\omega_s,\mu)W_{sc}(\omega_{sc},\mu)$$

Soft function at one-loop

This can be integrated with the measurement function over $\omega_{s,sc}$ in Laplace space:

$$\tilde{S}_0(s,\mu) = \int_0^\infty dE_s e^{-sE_s} S(E_s,\mu) = \frac{1}{s}\tilde{W}_{\rm s}(2s,\mu)\tilde{W}_{\rm sc}(2s,\mu)\,,$$

with:

$$\tilde{W}_s^0(2s,\mu) = 1 + \frac{\alpha}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{1+\tilde{L}_s}{\epsilon} + \frac{\tilde{L}_s^2}{2} + \tilde{L}_s + \frac{\pi^2}{12} + 1\right)\,,$$

$$\tilde{W}_{sc}^0(2s,\mu) = 1 + \frac{\alpha}{2\pi}\left(-\frac{1}{\epsilon^2} + \frac{1-\tilde{L}_{sc}}{\epsilon} - \frac{\tilde{L}_{sc}^2}{2} + \tilde{L}_{sc} - \frac{5\pi^2}{12}\right)\,,$$

$$\tilde{L}_s = \log \mu^2 s^2 e^{2\gamma_E}\,, \qquad \tilde{L}_{sc} = \log\frac{\mu^2 s^2 e^{2\gamma_E}}{r_l^2}.$$

Each of these can now be renormalized to perform the resummation of the soft and soft-collinear logs.

We can then write a **factorization formula**, which is really **two nested** factorization formulae.

The low-energy factorization formula describes all real radiation and is a **factorization** at the level of the **rate**:

$$\Gamma = \Gamma_0 \, |y(\mu)|^2 \, |\mathcal{F}(\mu)|^2 \; W_s(\mu) \otimes W_{sc}(\mu)$$

Collinear loops

Real radiation
(soft + soft-collinear)

$$y(\mu) = C_{\mathrm{L}}(\mu) \times \mathcal{H}(\mu) \times \mathcal{J}(\mu)$$

Fermi theory   Hard loops   Hard-collinear loops

Conclusions

Conclusions

- Our factorization formula separates **all scales** in the process and allows for the **resummation of all logarithms** that appear.
- This is achieved by a series of **effective theories**, allowing to be **systematically extended** to higher orders in α_i and λ.
- The EFT constructions relied on a combination of **HQET**, **SCET I**, **SCET II** and **boosted HQET**.
- As a charged-current decay, this process **does not feature** log-enhanced **structure-dependent** corrections that were seen in $B_s \to \mu\mu$.
- Channels with **other lepton flavors** (e, τ) are **not related** by replacing the lepton mass. They have different scale hierarchies, matching thresholds, EFT constructions...

Bonus slides

There are no bonus slides.