Power Expansions on the Lightcone: From Theory to Phenomenology

QED calculations at NNLO (and beyond)

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non-exhaustive list of experiments as implemented in McMule

process	experiment	physics motivation
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge
$e^-e^- \to e^-e^-$	Prad 2	normalisation
	MOLLER, ...	$\sin\theta_W^2$ at low Q^2
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio
	Belle	τ properties
$ee \to \gamma\gamma$	Daphne	dark searches
	any e^+e^- collider	luminosity measurement
$\mu \to \nu\bar{\nu}e$	MEG	ALP searches
	DUNE	beam-line profiling

QCD @ LHC	$\Leftrightarrow$	QED @ low & medium energy	
non-abelian	$\gtrsim$	abelian	matrix elements somewhat easier
non-abelian	$\gg$	abelian	IR structure much easier ①
massless fermions	$\ll$	massive fermions	loop amplitudes much harder ②
jets	$<$	exclusive w.r.t.	numerics harder $\supset \log(m^2/Q^2) \equiv L$
		collinear radiation	much harder for small masses ③

stealing from QCD

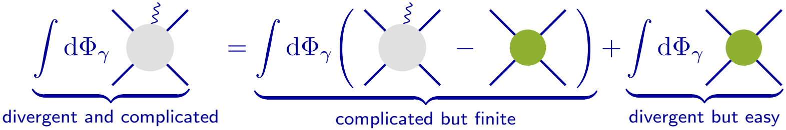
- master integrals (reduction and computation), automated tools, EFT methods
- use dimensional regularisation for IR singularities, not photon mass
- use subtraction method for phase-space integration, not slicing method
- for the future: match fixed-order result to parton shower

soft singularities exponentiate [Yennie, Frautschi, Suura 61]

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} = \text{finite}$

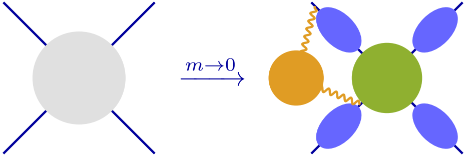
use this to construct an all-order subtraction scheme FKS$^\ell$

- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$

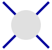


② SCET-ish IR factorisation

universality of collinear singularities → calculate up to $\mathcal{O}(m^2/Q^2)$



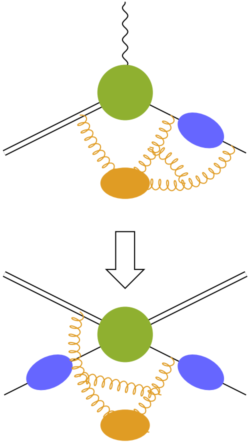
$= S \times Z \times Z \times Z \times Z \times H$

- H: hard function ∼  $\big|_{m=0}$
- Z: process independent jet function
- S: simple soft function

simple process ($\mu \to e\nu\nu$ or $t \to b\ell\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

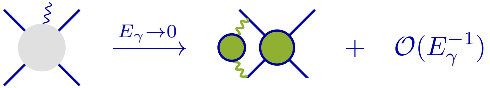
[Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]



different process ($\mu e \to \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

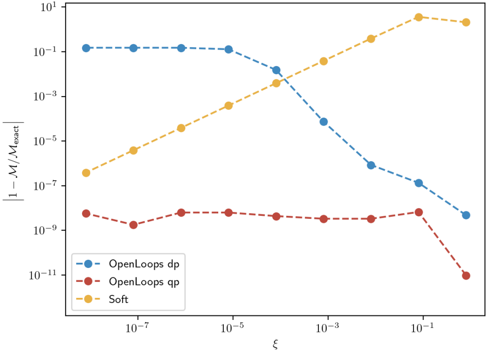
real-virtual corrections trivial in principle, extremely delicate numerically



example $ee \to ee\gamma$

[Banerjee, Engel, Schalch, Signer, YU 21]

- soft limit (of collinear emission)
  $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs dp, qp, eikonal, NTS
- stability problem



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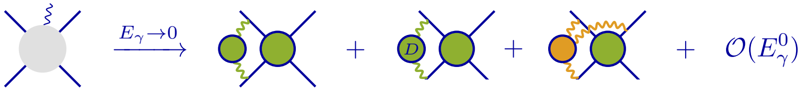
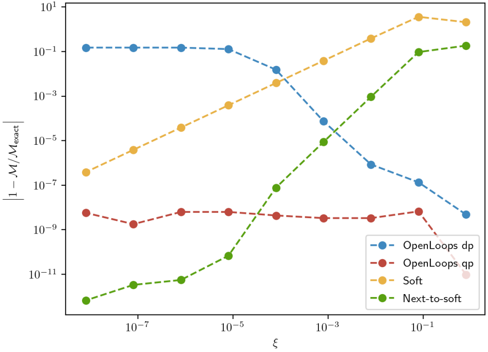


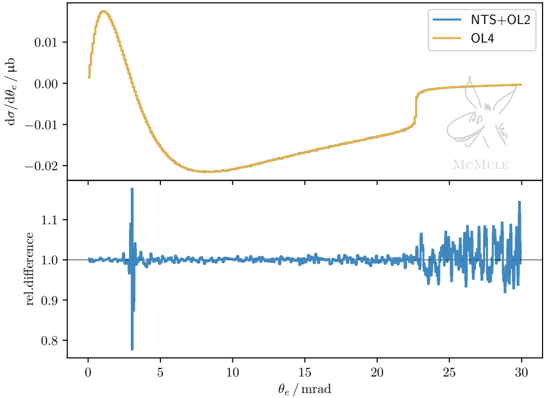
example $ee \to ee\gamma$

[Banerjee, Engel, Schalch, Signer, YU 21]

- soft limit (of collinear emission) $E_\gamma = \xi\sqrt{s}/2$
- arbitrary prec. calculation vs dp, qp, eikonal, NTS
- stability problem solved & speed-up



test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \to \mu e$ real-virtual



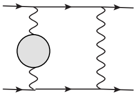
- same statistics, same result
- 70 days vs 4 days
- integrated results for different cuts

⇒ this is not an approximation but a numerical tool

NTS	OL4
$-0.29268(4)$	$-0.29267(4)$
$-0.44789(6)$	$-0.44778(6)$
$-0.64662(9)$	$-0.64649(9)$

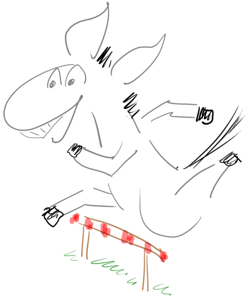
a few more hurdles

- VP diagrams for $e/\mu/\tau/\text{had}/\ldots$ numerically with full mass dependence



- collinear pseudo-singularities $\lim_{\to 0} \sphericalangle(p_\gamma, p_i) \;\Rightarrow\; L$
- phase-space tuning s.t. $\cos\sphericalangle \sim x_i$

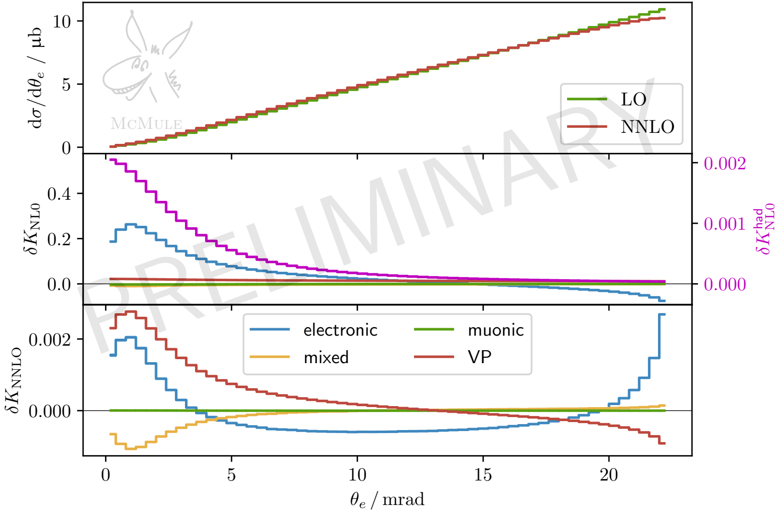
$\Rightarrow$ at most one small angle $\to$ FKS partitioning



[Signer 22]

pheno example: full NNLO for $e\mu \to e\mu$

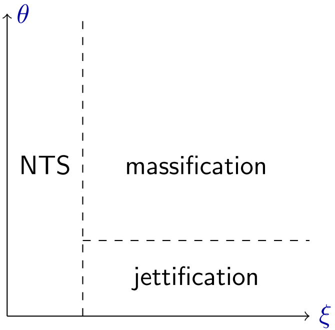
$E^{\mu}_{\text{beam}} = 150\,\text{GeV},\ E_e > 1\,\text{GeV},\ \theta_\mu > 0.3\,\text{mrad}$ [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]



$ee \to \gamma^*$ can be taken to N^3 LO

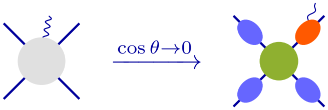
- VVV: known
  [Fael, Lange, Schönwald, Steinhauser 22]
- RRR: "trivial"
- RRV: OpenLoops + NTS stabilisation
- RVV: massless known (three-jet @ NNLO), massive (DiffExp?)

$\Rightarrow$ LBK + jettification at two-loop



jettification

- expand for small emission angles



$= J \times Z \times Z \times Z \times H$

N^3 LO workstop in Durham

f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (PSI), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (→ Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)

McMule

mule-tools.gitlab.io