MITP workshop : PEOL

The LBK theorem in QED at one loop

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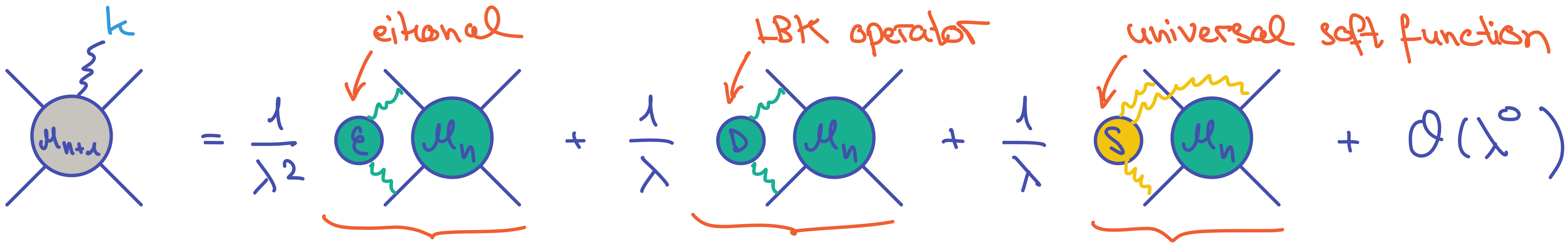
Overview

scale hierarchy $\quad k \sim \lambda \ll Q^2, m^2$

∇ QED

- no collinear scale
- ⇝ hard & soft d.o.f.
- ⇝ HQET



@ all orders [YFS 61] | @ tree level [Low 58; Burnett, Kroll 68] | @ one loop [TE, Signer, Ulrich 24]

[Beneke, Smirnov 98]

- MoR @ one loop : LBK $\hat{=}$ hard region
- theorem @ level of squared amplitude

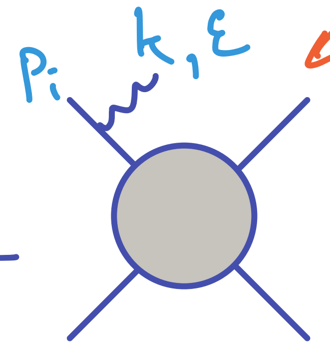
not true @ ampl. level

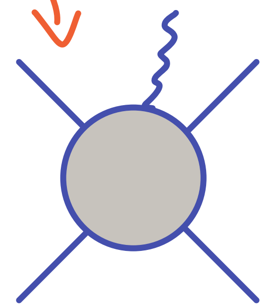
- massless case : $\nexists$ soft region, $\exists$ coll. region $\hat{=}$ "radiative jet"

↳ talk L. Vernazza

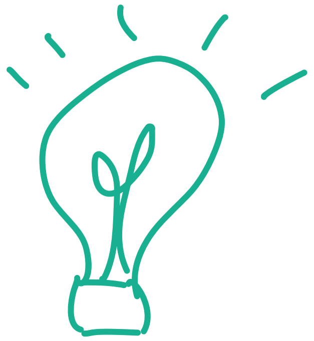
LBK @ tree level

not gauge inv. gauge inv.

$\mathcal{A}^{(0)}_{n+1} = \sum_i$ 

$+$  $= \underbrace{(\mathcal{A}^{ext} + K)}_{\mathcal{A}^{I}\cdot\varepsilon} + \underbrace{(\mathcal{A}^{int} - K)}_{\mathcal{A}^{II}\cdot\varepsilon} + \mathcal{O}(\lambda)$

just expand → $\mathcal{A}^{ext}$ $\mathcal{A}^{int}$?



$$k\cdot\mathcal{A}^{II} \sim \mathcal{O}(\lambda^2) \implies \mathcal{A}^{II}_\mu \sim \mathcal{O}(\lambda)$$

[Adler 66]

no $1/k$, i.e. $\mathcal{O}(\lambda^0)$ k-indep.

$$\boxed{\mathcal{M}^{(0)}_{n+1}(\{p\},k) = \sum_{ij} Q_i Q_j \left(-\frac{1}{\lambda^2}\frac{p_i\cdot p_j}{(k\cdot p_i)(k\cdot p_j)} + \frac{1}{\lambda}\frac{p_j\cdot D_i}{k\cdot p_j}\right)\mathcal{M}^{(0)}_n(\{s\}) + \mathcal{O}(\lambda^0)}$$

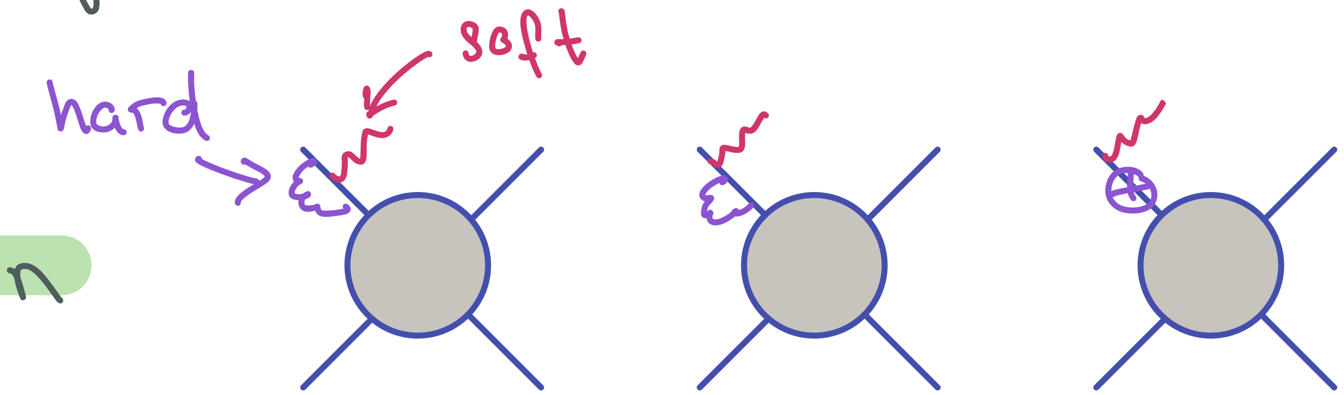
with $D_i^\mu = \sum_L \left(\frac{p_i^\mu}{k\cdot p_i}\, k\cdot\frac{\partial s_L}{\partial p_i} - \frac{\partial s_L}{\partial p_{i,\mu}}\right)\frac{\partial}{\partial s_L}$

⚠ ambiguous since $\sum_i p_i = k \neq 0$

→ be consistent!

LBK @ one loop

method of regions hard & soft contribution



hard region ↯ ! cancel @ level of squared ampl.

$$\mathcal{M}^{(1),\text{hard}}_{n+1}(\{p\},k) = \sum_{ij} Q_i Q_j \left(-\frac{1}{\lambda^2}\frac{p_i\cdot p_j}{(k\cdot p_i)(k\cdot p_j)} + \frac{1}{\lambda}\frac{p_j\cdot D_i}{k\cdot p_j}\right)\mathcal{M}^{(1)}_n(\{s\}) + \mathcal{O}(\lambda^0)$$

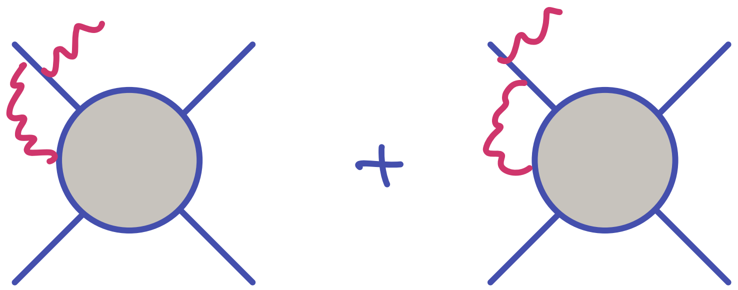
with $$D_i^\mu = \sum_L \left(\frac{p_i^\mu}{k\cdot p_i}\, k\cdot\frac{\partial s_L}{\partial p_i} - \frac{\partial s_L}{\partial p_{i,\mu}}\right)\frac{\partial}{\partial s_L}$$

soft region $k \sim l \sim \lambda \ll Q^2, m^2$ ↯ new scale hierarchy

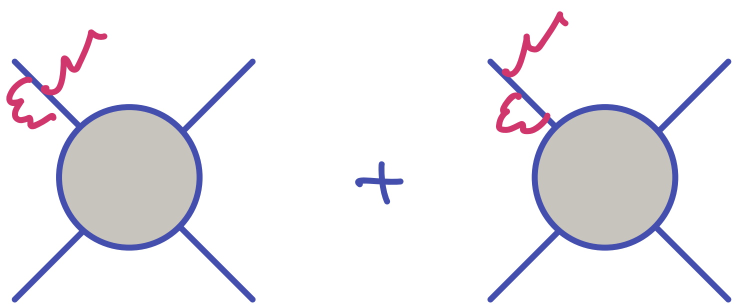
⇝ new approach

Soft contribution

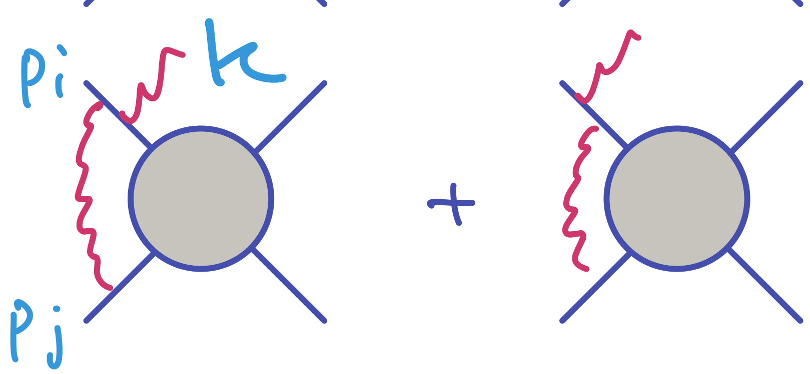
$$\nexists \;\; (\ell+p_i-k)^2-m_i^2 = \lambda\left(2\ell\cdot p_i - 2k\cdot p_i\right)+\mathcal{O}(\lambda^2) \Rightarrow \text{scaleless}$$

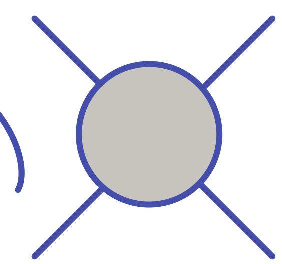
$\Rightarrow$ 

$$= \lambda^0\times 0+\mathcal{O}(\lambda)$$



$$= \frac{1}{\lambda}\times 0+\lambda^0\times 0+\mathcal{O}(\lambda)$$



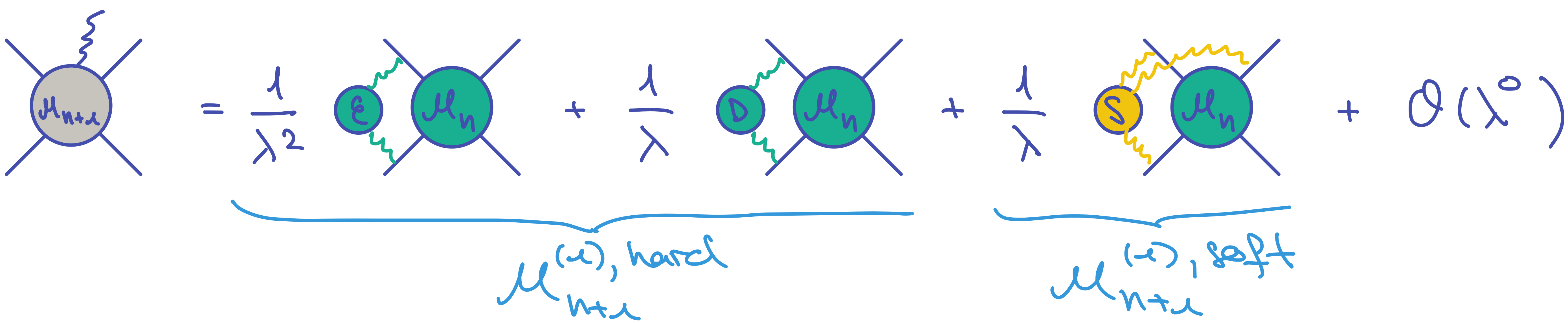
$$= \frac{\ell}{\lambda}\times 0+\lambda^0\times S(p_i,p_j,k)$$ 

$$\Rightarrow A^{(1),\,\mathrm{soft}}_{n+1} = \sum_{i\neq j} S(p_i,p_j,k)\, A^{(0)}_n$$

$$\sim p_i\cdot p_j\, I_1 + m_j^2\, k\cdot p_i\, I_2$$

$$\begin{cases} I_1 = \displaystyle\int d\ell \, \frac{1}{[\ell^2][\ell\cdot p_i - k\cdot p_i]} \\[2ex] I_2 = \displaystyle\int d\ell \, \frac{}{[\ell^2][-\ell\cdot p_j][\ell\cdot p_i - k\cdot p_i]} \end{cases}$$

The theorem



$$\mathcal{M}^{(1),\,\mathrm{hard}}_{n+1} = \sum_{ij} Q_i Q_j \left(-\frac{1}{\lambda^2} \frac{p_i \cdot p_j}{(k\cdot p_i)(k\cdot p_j)} + \frac{1}{\lambda} \frac{p_j \cdot D_i}{k \cdot p_j} \right) \mathcal{M}_n^{(1)} + \mathcal{O}(\lambda^0)$$

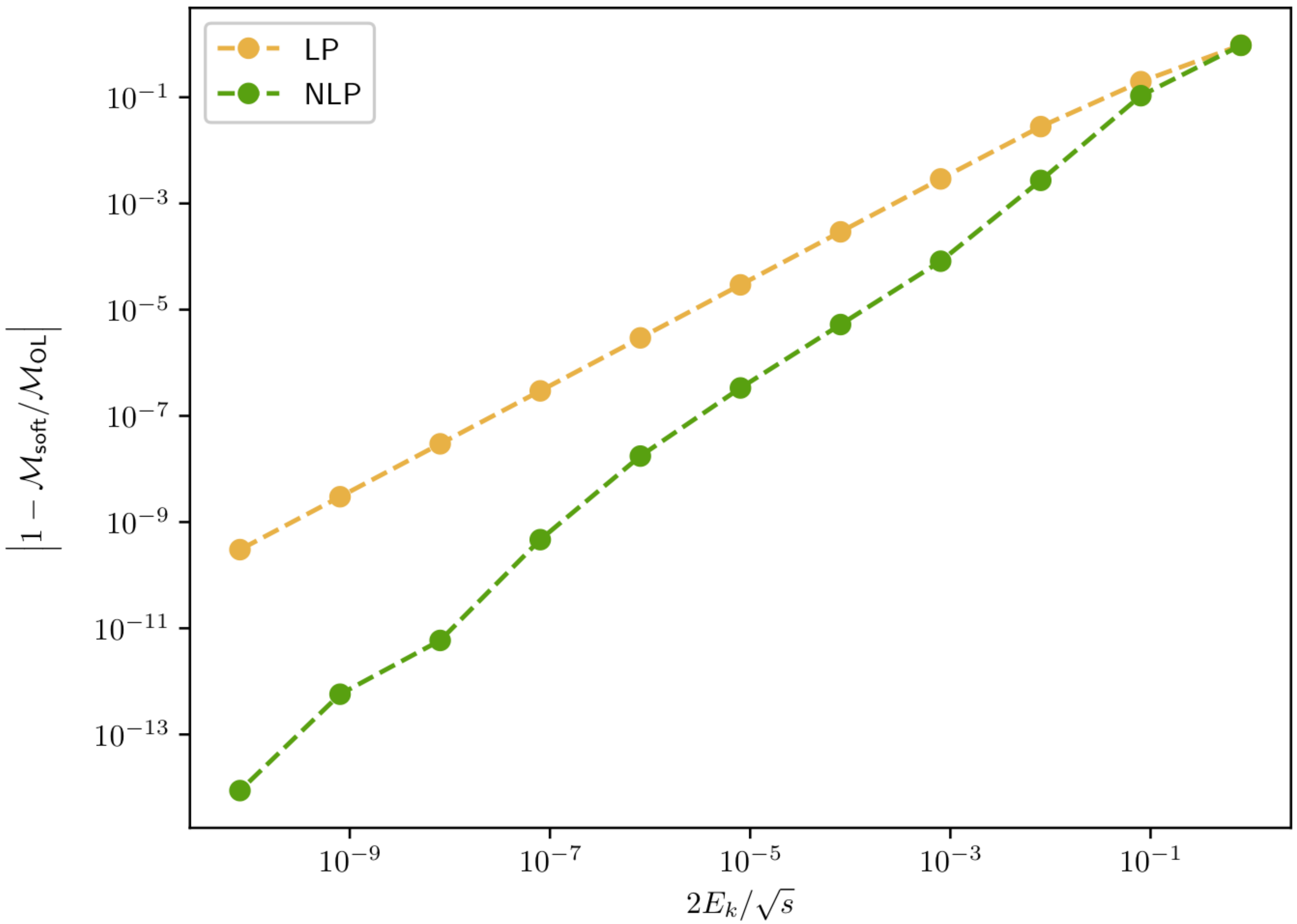
$$\mathcal{M}^{(1),\,\mathrm{soft}}_{n+1} = \frac{1}{\lambda} \sum_{\ell} \sum_{i \neq j} Q_i^2 Q_j Q_\ell \left(\frac{p_i \cdot p_\ell}{(k\cdot p_i)(k\cdot p_\ell)} - \frac{p_j \cdot p_\ell}{(k\cdot p_j)(k\cdot p_\ell)} \right) 2\, S(p_i, p_j, k)\, \mathcal{M}_n^{(0)} + \mathcal{O}(\lambda^0)$$

Validation

process $e^+e^- \to e^+e^-\gamma\gamma$ @ one loop

reference OpenLoops [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang 19]

in quad. mode



Conclusion & outlook

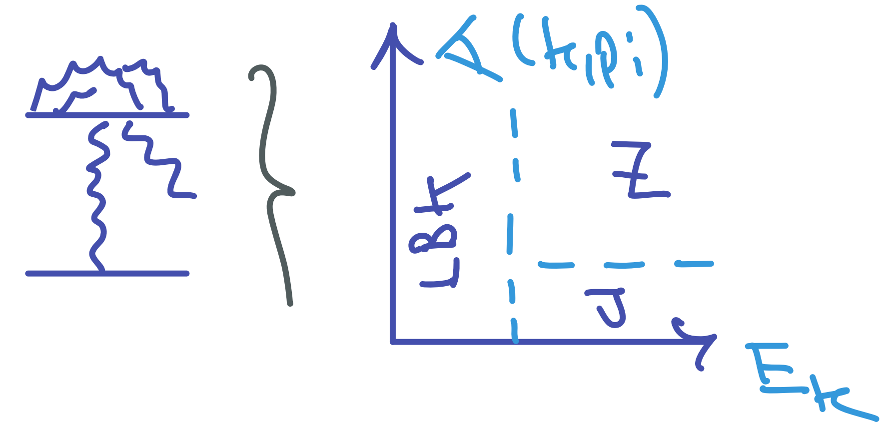
$k \ll Q^2, m^2$

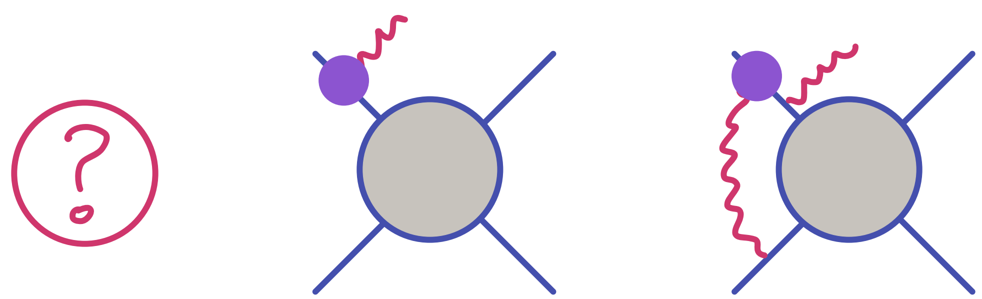
- LBK theorem extended to one loop

  $\rightsquigarrow$ Tree-level theorem $\hat{=}$ hard contribution

  new contribution from soft region $\rightsquigarrow$ universal !

- generalisation beyond one loop ? $\rightarrow$





- Proper formulation in HQET ?