

MITP workshop : PEOL

The LBK theorem in QED
at one loop

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Overview

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scale hierarchy

$k \ll \lambda \ll Q^2, m^2$

QED

} no collinear scale
no hard & soft d.o.f.
no HQET

$$\begin{aligned} \text{Diagram: } & \text{A grey circle } M_{n+1} \text{ with a wavy line } k \text{ entering it, connected to a green circle } M_n \text{ by a solid line.} \\ & = \frac{1}{\lambda^2} \underbrace{\text{Diagram: } \text{eitalon}}_{\substack{@ \text{ all orders} \\ [\text{YFS 61}]}} + \frac{1}{\lambda} \underbrace{\text{Diagram: } \text{LBK operator}}_{\substack{@ \text{ tree level} \\ [\text{Feynman 58; Burnett, Kroll 68}]}} + \frac{1}{\lambda} \underbrace{\text{Diagram: } \text{universal soft function}}_{\substack{@ \text{ one loop} \\ [\text{ITE, Signer, Ulrich 21}]}} + \mathcal{O}(\lambda^0) \end{aligned}$$

- Mot @ one loop : LBK $\hat{=}$ hard region not true @ ampl. level
- theorem @ level of squared amplitude
- massless case : \nexists soft region, \exists coll. region $\hat{=}$ "radiative jet"
↳ talk L. Vernazza

LBK @ tree level

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$$\Delta_{n+1}^{(G)} = \sum_i \underbrace{\text{Diagram with } p_i, k, \epsilon}_{\Delta^{\text{ext}}} + \text{Diagram with } k \text{ only} = (\Delta^{\text{ext}} + \kappa) + (\Delta^{\text{int}} - \kappa) + Q(\lambda)$$

not gauge inv.

gauge inv.

just expand $\rightarrow \Delta^{\text{ext}}$

Δ^{int} ?



$$k \cdot \Delta^{\text{II}} \sim \mathcal{O}(\lambda^2) \Rightarrow \Delta_k^{\text{II}} \sim \mathcal{O}(\lambda) \quad [\text{Adder 66}]$$

no $1/k$, i.e. $\mathcal{O}(\lambda^\circ)$ k -indep.

$$\mathcal{M}_{n+1}^{(G)}(\{p_i\}, k) = \sum_{i,j} Q_i Q_j \left(-\frac{1}{\lambda^2} \frac{p_i \cdot p_j}{(k \cdot p_i)(k \cdot p_j)} + \frac{1}{\lambda} \frac{p_j \cdot D_i}{k \cdot p_j} \right) \mathcal{M}_n^{(G)}(\{s_i^2\}) + \mathcal{O}(\lambda^\circ)$$

! ambiguous since
 $\sum_i p_i = k \neq 0$
 we be consistent!

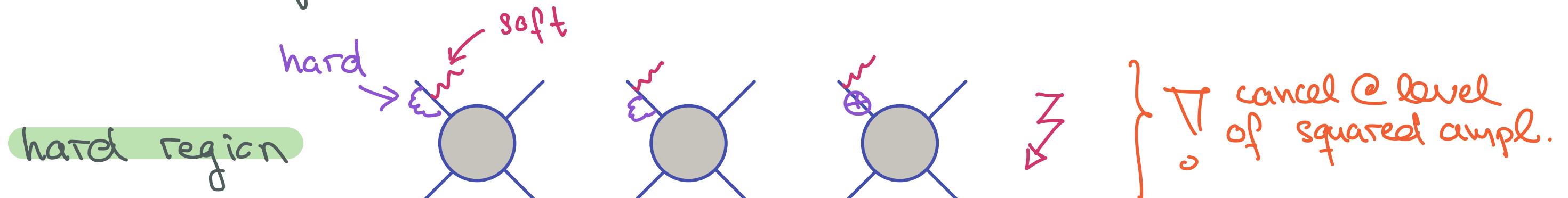
with $D_i^{\text{II}} = \sum_L \left(\frac{p_i \cdot \epsilon}{k \cdot p_i} k \cdot \frac{\partial s_L}{\partial p_i} - \frac{\partial s_L}{\partial p_{i,L}} \right) \frac{\partial}{\partial s_L}$

LBK @ one loop

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method of regions

hard & soft contribution



$$\mathcal{M}_{n+1}^{(u), \text{hard}}(\{p_i\}, k) = \sum_{i,j} Q_i Q_j \left(-\frac{1}{\lambda^2} \frac{\vec{p}_i \cdot \vec{p}_j}{(k \cdot p_i)(k \cdot p_j)} + \frac{1}{\lambda} \frac{\vec{p}_j \cdot D_i}{k \cdot p_j} \right) \mathcal{M}_n^{(u)}(\{s_i\}) + O(\lambda^0)$$

with $D_i^L = \sum \left(\frac{p_i \cdot k}{k \cdot p_i} k \cdot \frac{\partial S_L}{\partial q_i} - \frac{\partial S_L}{\partial q_{i,L}} \right) \frac{\partial}{\partial S_L}$

soft region $k \sim l \sim \lambda \ll Q^2, m^2$ \rightsquigarrow new scale hierarchy
 \rightsquigarrow new approach

Soft contribution

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$$\not \equiv (l + p_i - k)^2 - m_i^2 = \lambda (2l \cdot p_i - 2k \cdot p_i) + \mathcal{O}(\lambda^2) \Rightarrow \text{scaleless}$$

$$\text{SD} \quad \begin{array}{c} \text{Diagram: } \\ \text{Two grey circles with red wavy lines attached to them.} \end{array} + \begin{array}{c} \text{Diagram: } \\ \text{Two grey circles with red wavy lines attached to them.} \end{array} = \lambda^0 \times 0 + \mathcal{O}(\lambda)$$

$$= \frac{\lambda}{\lambda} \times 0 + \lambda^0 \times 0 + \mathcal{O}(\lambda)$$

$$\begin{array}{c} \text{Diagram: } \\ \text{Two grey circles with red wavy lines attached to them. The wavy lines are labeled } p_i \text{ and } k \text{ on the left circle, and } p_j \text{ on the right circle.} \end{array} + \begin{array}{c} \text{Diagram: } \\ \text{Two grey circles with red wavy lines attached to them.} \end{array} = \frac{\lambda}{\lambda} \times 0 + \lambda^0 \times S(p_i, p_j, k) \begin{array}{c} \text{Diagram: } \\ \text{One grey circle with a red wavy line attached to it.} \end{array}$$

$$\text{SD} \quad A_{n+\epsilon}^{(1), \text{soft}} = \sum_{i \neq j} S(p_i, p_j, k) A_n^{(0)} \quad \left\{ \begin{array}{l} I_1 = \int dl \frac{\epsilon}{[l^2] [l \cdot p_i - k \cdot p_i]} \\ I_2 = \int dl \frac{\epsilon}{[l^2] [l \cdot p_j] [l \cdot p_i - k \cdot p_i]} \end{array} \right.$$

$\sim p_i \cdot p_j I_1 + m_i^2 k \cdot p_i I_2$

The theorem

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$$\mathcal{M}_{n+1} = \frac{1}{\lambda^2} \mathcal{M}_n^{(u), \text{hard}} + \frac{1}{\lambda} \mathcal{M}_n^{(u), \text{soft}} + \frac{1}{\lambda} \mathcal{M}_n^{(s)} + O(\lambda^0)$$

$\mathcal{M}_{n+1}^{(u), \text{hard}}$

$\mathcal{M}_{n+1}^{(u), \text{soft}}$

$$\mathcal{M}_{n+1}^{(u), \text{hard}} = \sum_{i,j} Q_i Q_j \left(-\frac{1}{\lambda^2} \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{(\mathbf{k} \cdot \mathbf{p}_i)(\mathbf{k} \cdot \mathbf{p}_j)} + \frac{1}{\lambda} \frac{\mathbf{p}_j \cdot \mathbf{D}_i}{\mathbf{k} \cdot \mathbf{p}_j} \right) \mathcal{M}_n^{(u)} + O(\lambda^0)$$

$$\mathcal{M}_{n+1}^{(u), \text{soft}} = \frac{1}{\lambda} \sum_{\ell} \sum_{i \neq j} Q_i^2 Q_j Q_\ell \left(\frac{\mathbf{p}_i \cdot \mathbf{p}_\ell}{(\mathbf{k} \cdot \mathbf{p}_i)(\mathbf{k} \cdot \mathbf{p}_\ell)} - \frac{\mathbf{p}_j \cdot \mathbf{p}_\ell}{(\mathbf{k} \cdot \mathbf{p}_j)(\mathbf{k} \cdot \mathbf{p}_\ell)} \right) 2 S(p_i, p_j, \mathbf{k}) \mathcal{M}_n^{(0)} + O(\lambda^0)$$

Validation

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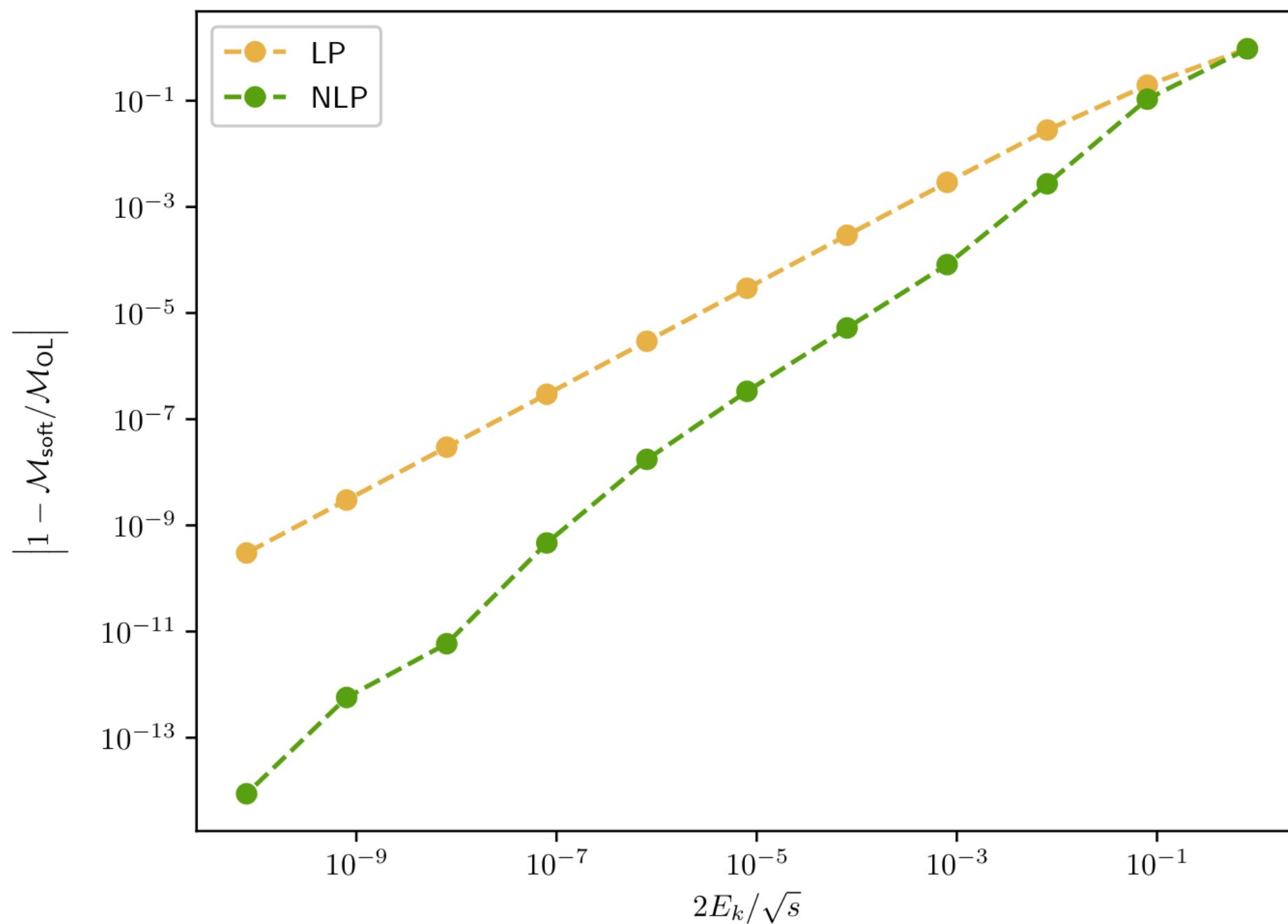
process

$e^+ e^- \rightarrow e^+ e^- \gamma\gamma$ @ one loop

reference

OpenLoops [Buccini, Lang, Lindert, Mäderhöfer, Pazzorini, Zhang 19]

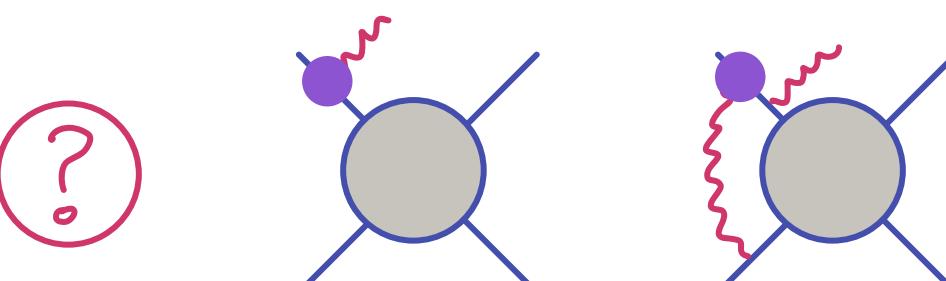
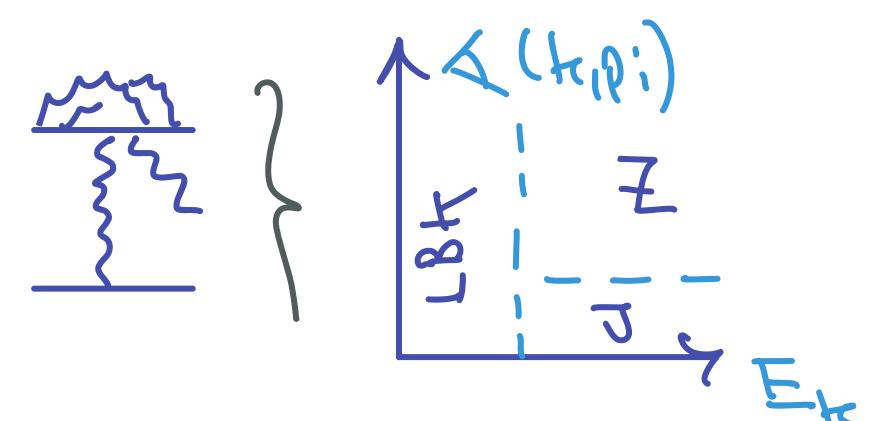
in quck.
mode



Conclusion & outlook

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$$k \ll Q^2, m^2$$

- LBK theorem extended to one loop
 - ↳ Tree-level theorem $\hat{=}$ hard contribution
new contribution from soft region ∇ universal !
- generalisation beyond one loop ? \rightarrow


- Proper formulation in HQET ?