QCD Beyond Leading Power

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MITP Workshop
Power Expansions on the Lightcone:
From theory to phenomenology
Mainz, 19 September 2022

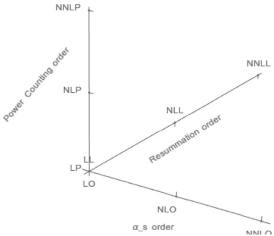
For reference:

"QCD Beyond Leading Power"

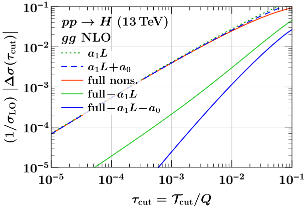
arXiv: 2008.10606

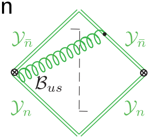
Outline

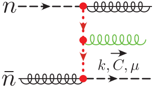
- Introduction:
  - Motivation for going beyond Leading Power
  - Systematic Expansion of QCD using Soft and Collinear Effective Theory

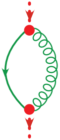


- SCET at Subleading Power:
  - Computing Power Corrections at Fixed Order
  - Subleading Power Regularization and Renormalization
  - Leading Log Resummation at subleading power
  - NLP Phenomena: Glauber Quarks



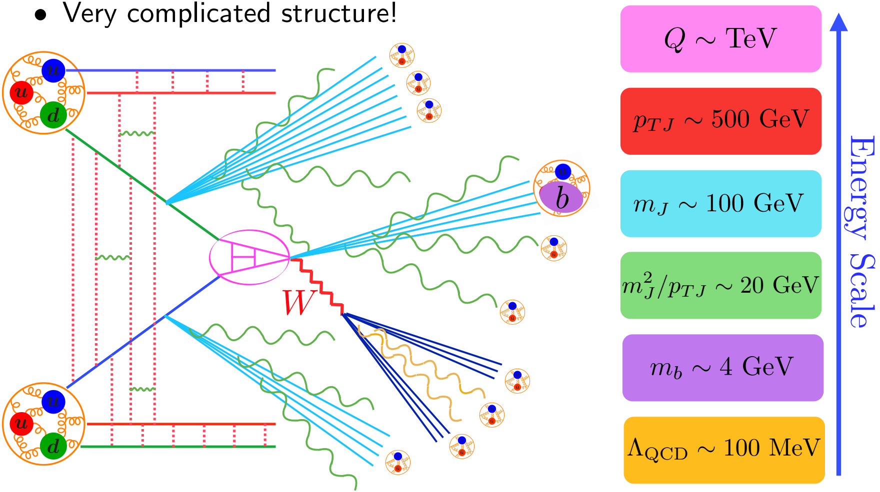






An LHC Collision

- Very complicated structure!

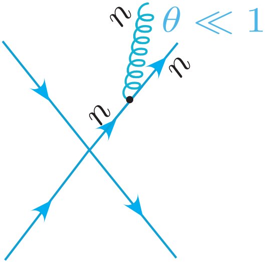


- Involves interactions at many hierarchical energy scales.
- It is very complicated to obtain precise theoretical predictions

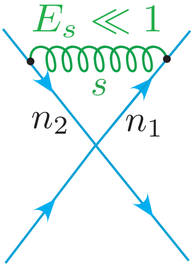
Limits of QCD

- Significant progress in understanding QCD made by considering limits where we have a power expansion in some small kinematic quantity.

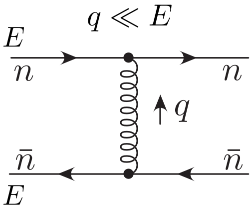
Collinear



Soft



Regge



Power expansion for generic $\mathcal{O}$ observable

- A large class of observables $\mathcal{O}$ (q_T, event shapes, angularities, etc.) exhibit singularities in perturbation theory as $\mathcal{O} \to 0$.
- Standard factorization theorems describe only leading power term.
- To be concrete let's take $\mathcal{O} = p_T^2$.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \frac{\log^m \frac{p_T^2}{Q^2}}{p_T^2}$$

Leading Power (LP)

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Leading Power (LP)

$$+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(1)} \log^m \frac{p_T^2}{Q^2}$$

Next to Leading Power (NLP)

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Next to Leading Power (NLP)

$$+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \, p_T^2 \log^m \frac{p_T^2}{Q^2} + \cdots$$

$$= \frac{d\sigma^{(0)}}{dp_T^2} + \frac{d\sigma^{(1)}}{dp_T^2} + \frac{d\sigma^{(2)}}{dp_T^2} + \cdots$$

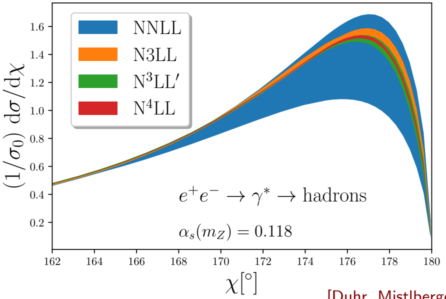
Leading Power

Leading power well understood for a wide variety of observables.

- We can prove factorization theorems

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\mathcal{O}} = H^{(0)} J^{(0)} \otimes J^{(0)} \otimes S^{(0)} + \ldots$$

- We can resum logs to very high logarithmic accuracy



e.g. we now have reached **N^{4}LL accuracy** at LP for event shapes!
Also plenty of results at N^{3}LL and N^{3}LL′ accuracy...

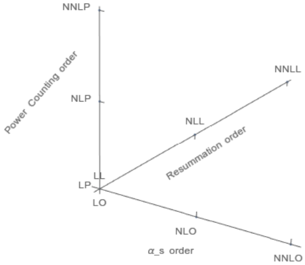
I'll talk about this N^{4}LL result and recent developements on LP resummation next Tuesday!

[Duhr, Mistlberger, **GV** '22]

So, why bother going beyond leading power?

NLP field theoretical motivations

- **Power counting** is a different **direction** in which amplitudes and cross sections can be **expanded**



- Various interesting field theoretical questions to answer at subleading power:
  - What is the **structure of factorization theorems** at each power?

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\mathcal{O}} = \sum_j H_j^{(n_{Hj})} \otimes J_j^{(n_{Jj})} \otimes S_j^{(n_{Sj})}$$

  - What is the degree of **universality**?
  - Appearance of universal structures, e.g. $\Gamma_{\text{cusp}}(\alpha_s)$?
  - Appearance of **new RGE structures**, functions, objects, etc
  - Appearance of **new processes** forbidden at leading power

Application: Fixed Order Computations via Slicing

- IR divergences in fixed order calculations can be regulated using slicing parameter (e.g. q_T [Catani,Grazzini], N-jettiness [Gaunt et. al], [Boughezal et al.]).

$$\sigma(X) = \int_0 dq_T \frac{d\sigma(X)}{dq_T} = \int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \int_{q_T^{\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T}$$

- q_T subtraction has been applied to many processes in pp at **NNLO**: $pp \to Z$, $pp \to W$, $pp \to H$, $pp \to \gamma\gamma$, $pp \to Z\gamma$, $pp \to W\gamma$, $pp \to ZZ$, $pp \to WW$, $pp \to WZ$ [Matrix collaboration]

- N-jettiness subtraction also applied to $W/Z/H + 1$ jet @NNLO

- Error, $\Delta\sigma(q_T^{\text{cut}})$, (or computing time) can be exponentially improved by analytically computing power corrections.

$$\Delta\sigma(q_T^{\text{cut}}) = \int_0^{q_T^{\text{cut}}} dq_T \left(\frac{d\sigma(X)}{dq_T} - \frac{d\sigma(X)^{\text{LP}}}{dq_T} \right) \equiv \sigma^{\text{non sing.}}(q_T^{\text{cut}})$$

- Understanding of power corrections crucial for applications to more complicated processes (fully differential N^{3}LO calculations, H + jets, Z/W + jets)

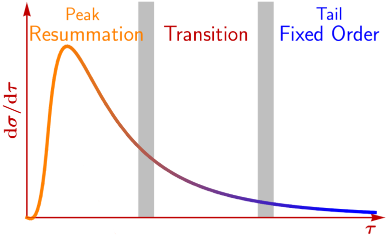
Applications

Matching resummation with FO

If observable τ needs resummation:

- Use Leading Power EFT for resummed XS at small τ

$$\frac{d\sigma}{d\tau} \underset{\tau\to 0}{\sim} \alpha_s^n \frac{\log^m \tau}{\tau} \xrightarrow[\text{resummation}]{\text{EFT}} \frac{e^{-\alpha_s^k \log^{2k}\tau}}{\tau}$$



- For large τ use Fixed Order calculation to get full $\mathcal{O}(\alpha_s^n)$ contribution
- Need matching procedure in transition region between the two.
- Computing **Power Corrections** analytically **improves** convergence of the EFT at larger values of τ

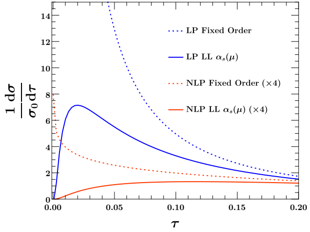
  $\Longrightarrow$ **smaller transition regions**

  $\Longrightarrow$ smaller uncertainties from matching procedure

Taming log divergence of NLP

- Fixed order power correction (NLP) exhibits an integrable divergence for $\tau \to 0$
- If Leading Power (singular) is resummed and NLP is not, the NLP (integrable) divergence dominates.

$$\alpha_s^n \frac{\log^m \tau}{\tau} \longrightarrow \frac{e^{-\alpha_s^k \log^{2k}\tau}}{\tau} \quad \text{vs} \quad \alpha_s^n \log^m \tau$$



[Moult, Stewart, Vita, Zhu]

Bootstrap for observables

- Bootstrap approaches aim to completely reconstruct amplitudes or cross sections from limits.

- Intensively applied for amplitudes in $\mathcal{N} = 4$.

- Recenty, some success in QCD for soft matrix elements [Zhu et al.]

Remaining Parameters in Symbol of 6-Point MHV Remainder Function

Constraint	$L=2$	$L=3$	$L=4$
1. Integrability	75	643	5897
2. Total S_3 symmetry	20	151	1224
3. Parity invariance	18	120	874
4. Collinear vanishing (T^0)	4	59	622
5. OPE leading discontinuity	0	26	482
6. Final entry	0	2	113
7. Multi-Regge limit	0	2	80
8. Near-collinear OPE (T^1)	0	0	4
9. Near-collinear OPE (T^2)	0	0	0

LL All Powers → (row 5)

NLP, NNLP → (rows 8, 9)

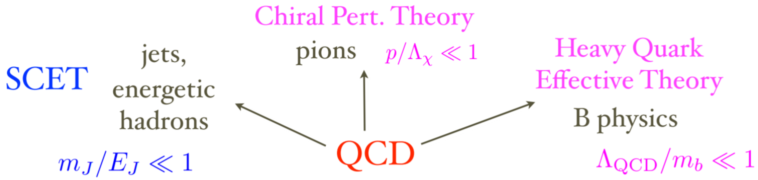
[Dixon et al.], [Basso, Sever, Vieira]

- Can the bootstrap be extended from amplitudes to cross section?
  For example, can we bootstrap an event shape observable using the information from limits at leading and subleading power?

SCET Beyond Leading Power

Soft and Collinear Effective Theory [Bauer, Fleming, Pirjol, Stewart]

Soft and Collinear Effective Theory (SCET) is limit of QCD



- Results derived with SCET must be equivalent to results derived directly from QCD.
- SCET systematizes the power expansion from the start
  $\rightarrow$ explicit power counting at any step
- Simplifies field theoretic derivation of factorization formulae
  $\rightarrow$ Scales separated in building the EFT once and for all, recycled among different processes
- Resummation of large logs from deriving anomalous dimensions of hard, collinear or soft operators $\rightarrow$ logs coming from IR poles in pQCD get related to UV divergences in SCET, hence we can define $\overline{\rm MS}$-like counterterms, anomalous dimensions, RGEs, etc..

Mode setup in SCET

- Light cone coordinates: $k^\mu = \frac{\bar{n}^\mu}{2} k^+ + \frac{n^\mu}{2} k^- + k_\perp^\mu \equiv (k^+,\, k^-,\, k_\perp)$

EFT expansion: $\lambda \ll 1$

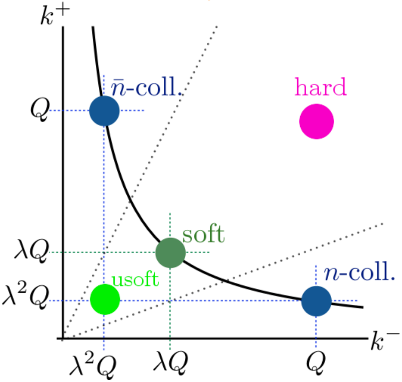
$(k^+, k^-, k_\perp)$

n-collinear: $k_n^\mu \sim Q(\lambda^2,\, 1,\, \lambda)$

$\bar{n}$-collinear: $k_{\bar{n}}^\mu \sim Q(1,\, \lambda^2,\, \lambda)$

SCET$_{\text{II}}$ $\rightarrow$ soft: $k_s^\mu \sim Q(\lambda,\, \lambda,\, \lambda)$

SCET$_{\text{I}}$ $\rightarrow$ usoft: $k_{us}^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$



hard scale: $k_{hard}^\mu \sim Q(1, 1, 1)$ (integrated out)

- Allows for a factorized description: Hard, Jet, Beam, Soft radiation

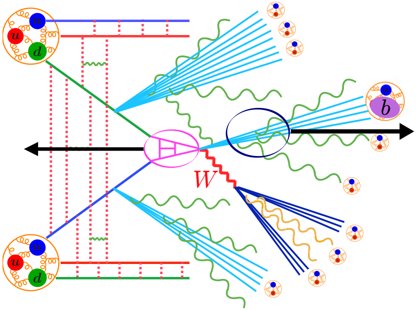
From Standard Model to SCET

$$\mathcal{L}_{SM} \to \mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_{\mathrm{hard}} + \mathcal{L}_{\mathrm{dyn}} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\mathrm{hard}} + \sum_{i \geq 0} \mathcal{L}^{(i)}_{\mathrm{dyn}} + \mathcal{L}^{(0)}_{\mathcal{G}}$$

$\mathcal{L}_{\mathrm{hard}}$ describes the hard scattering/the partonic interaction.

e.g. how to go from gg to $H + 2$ partons.

Note: it can come from non-QCD interactions



$\mathcal{L}_{\mathrm{dyn}}$ describes the evolution of the strongly interacting final/initial states

e.g. how to go from 2 partons to 2 jets/ how the jets evolve

EFT of pure QCD

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim \sigma_0 H(Q,\mu) \otimes J(Q,\tau,s,\mu) \otimes S(s,\mu) + \dots$$

Hard scattering vs dynamical evolution

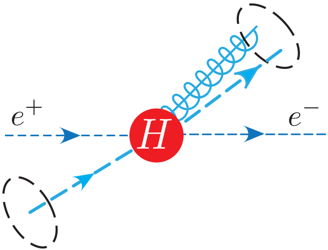
- Fields have a definite power counting in the power counting parameter $\lambda \ll 1$

Operator	$\mathcal{B}^{\mu}_{n_i\perp}$	χ_{n_i}	$\mathcal{P}^{\mu}_{\perp}$	ψ_{us}	$\mathcal{B}^{\mu}_{us}$	∂^{μ}_{us}	$Y_n/\bar{\mathcal{P}}/W_n$
Power Counting	λ	λ	λ	λ^3	λ^2	λ^2	λ^0

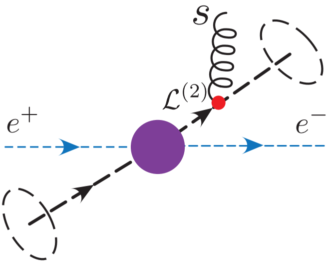
- Expansion for Lagrangians naturally derived from power counting of fields and derivatives

$$\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_{\mathrm{hard}} + \mathcal{L}_{\mathrm{dyn}} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\mathrm{hard}} + \sum_{i\geq 0} \mathcal{L}^{(i)} + \mathcal{L}^{(0)}_{\mathcal{G}}$$

Subleading Hard Scattering Operators



Subleading Lagrangians



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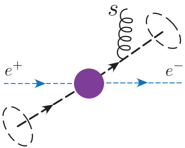
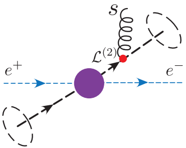
Example: ultrasoft gluon emission in $e^+e^- \to 2$ jets

$\mathcal{L}^{(0)} \supset \bar{\chi}_{\bar{n}} Y^{\dagger}_{\bar{n}} \Gamma \mathrm{Y}_{\mathrm{n}} \chi_n$



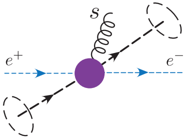
eikonal soft emission from Wilson line

$\mathcal{L}^{(2)}_{\rm dyn} \supset \bar{\chi}_n \mathcal{B}^{\perp}_{us} \chi_n$



non-eikonal soft emission from collinear quark

$\mathcal{L}^{(2)}_{\rm hard} \supset \bar{\chi}_{\bar{n}} Y^{\dagger}_{\bar{n}} \Gamma \bar{\mathrm{n}} \cdot \mathrm{B}_{\mathrm{us}} Y_n \chi_n$



Subleading hard scattering operator with B_{us}

Soft-Collinear Factorization at Subleading Power

- Factorization is defined order by order in the EFT parameter λ (at all orders in α_s)
- BPS field redefinition decouples leading power soft and collinear interactions. (but not in $\mathcal{L}_G^{(0)}$)
- Subleading power Lagrangians enter as T-products:

$$\langle 0|T\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\,\mathcal{L}_{\rm dyn}]\}|X\rangle$$
$$=\langle 0|T\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\,(\mathcal{L}^{(0)}+\mathcal{L}^{(1)}+\mathcal{L}^{(2)}+\cdots)]\}|X\rangle$$
$$=\langle 0|T\Big\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\mathcal{L}^{(0)}]\Big(1+i\int d^4y\mathcal{L}^{(1)}+\frac{1}{2}(i\int d^4y\mathcal{L}^{(1)})(i\int d^4z\mathcal{L}^{(1)})+i\int d^4z\mathcal{L}^{(2)}+\cdots\Big)\Big\}|X\rangle$$
$$=\langle 0|T\Big\{\tilde{O}_j^{(k)}(0)\Big(1+i\int d^4y\mathcal{L}^{(1)}+\frac{1}{2}(i\int d^4y\mathcal{L}^{(1)})(i\int d^4z\mathcal{L}^{(1)})+i\int d^4z\mathcal{L}^{(2)}\Big)\Big\}|X\rangle_{\mathcal{L}^{(0)}}+\cdots\,.$$

- Decoupling of leading power dynamics $\implies$ states still factorize (assuming Glaubers from $\mathcal{L}_G^{(0)}$ cancel...)

$$|X\rangle=|X_n\rangle|X_s\rangle$$

- Only a *finite* number of subleading power insertions contribute at a given order in λ.
- Even if subleading Lagrangians couple soft and collinear fields, each insertion contributes to a *different* order in λ
- The **only** interaction that can break factorization is soft and collinear coupling at **leading power**, i.e. $\mathcal{L}_G^{(0)}$.

Subleading Power SCET for collider observables

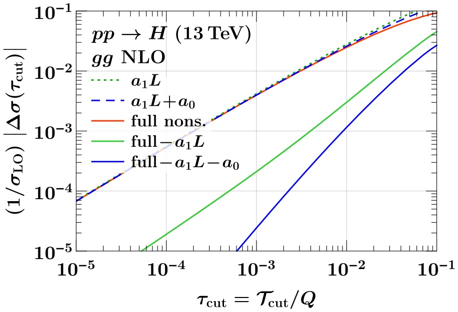
	SCET_I ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	SCET_II soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	Quark Regge, ✓
Fixed Order (fully differential)	LL, NLL	NLL
Hard Scattering Operators	ggH, $Vq\bar{q}$, $Hq\bar{q}$, N-jet	✓
Resummation	$H \to gg$, Soft Quark Sudakov, $H \to gq$	EEC

Lagrangians	Fixed Order	Hard Scattering	Resummation
[Stewart et al.] [Beneke et al.] (2002-2004)	[Moult et al.] LL at $\mathcal{O}(\alpha_s^2)$ 1612.00450 ($q\bar{q}V$), 1710.03227 (ggH)	[Moult, Stewart, GV] 1703.03408 [Moult et al.] $q\bar{q}V$ 1703.03411	[Moult, Stewart, GV, Zhu] $H \to gg$, Soft Quark Sudakov
[Moult, Solon, Stewart, GV] Fermionic Glauber Operators	[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764, $\mathcal{T}_0$ (beam thrust) at NLL NLP	[Chang, Stewart, GV] $q\bar{q}H$ 1712.04343	[Moult, GV, Yan] EEC in $\mathcal{N}=4$
[Chang, Stewart, GV] Subleading Lagrangians in SCET_II	[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1812.08189, q_t at NLL NLP	[Beneke et al.] (N-jet operators) 1712.04416, 1808.04742	[Beneke et al.] gq in Higgs Thrust
		[Chang, Stewart, GV]	

Other works on at subleading powers in different contexts:

- *B*-**physics**: [Lee, Stewart],[Neubert, Becher, Paz, Hill] [Beneke, Feldmann] [Tackmann, Mannel] (and many others)
- **Threshold** (only soft radiation): [Bonocore, Laenen, Magnea, Vernazza, White] (next-to-eikonal), [Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza] (resummation), [Anastasiou, Duhr, Dulat Furlan, Gehrmann, Herzog, Mistlberger]
- **Inclusive fixed order**: [Boughezal, Liu, Petriello], [Boughezal, Isgrò, Petriello], [Cieri, Oleari, Rocco]
- **Subleading power in light quark mass expansion**: [Liu, Penin], [Liu, Neubert], [Liu, Mecaj, Neubert, Wang, Fleming]
- ...

Power corrections at Fixed Order



(Ebert, Moult, Stewart, Tackmann, GV, Zhu) [1807.10764]

Power corrections at FO: General Setup

- Take as example the **fully differential cross section** $\frac{d\sigma}{dQ^2 dY d\mathcal{T}}$ for color singlet production (0-jettiness) including $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\mathcal{T}/Q)$ corrections .

- Power corrections in $\mathcal{O}(\mathcal{T}/Q)$:
  - **Perturbative**
  - **NOT** higher twist PDFs/non-perturbative power corrections.

Beam Thrust (0-jettiness)

$$\mathcal{T}_0 = \sum_{k \in \text{event}} \min(p_k^+, p_k^-)$$

- Used as a slicing parameter for FO calculations
- Represents the "crossed" version of thrust

- $\mathcal{O}(\mathcal{T}/Q)$ corrections contained in:
  - **Phase space:** $\Phi = \Phi^{(0)} + \frac{\mathcal{T}}{Q}\Phi^{(2)} + \mathcal{O}(\frac{\mathcal{T}^2}{Q^2})$
  - **Matrix element squared:** $|\mathcal{M}|^2 = A^{(0)} + \frac{\mathcal{T}}{Q}A^{(2)} + \mathcal{O}(\frac{\mathcal{T}^2}{Q^2})$

Schematically: $$\frac{d\sigma}{dQ^2 dY d\mathcal{T}} \sim \int \frac{dz}{z}\left[A^{(0)}\Phi^{(0)} + \frac{\mathcal{T}}{Q}A^{(0)}\Phi^{(2)} + \frac{\mathcal{T}}{Q}A^{(2)}\Phi^{(0)}\right] + \mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}, \alpha_s^2\right)$$

Power corrections at FO: Cross section results

- Combining **soft** and **collinear** kernels, $\frac{1}{\epsilon}$ poles cancel (consistency check) and the differential cross section takes the form:

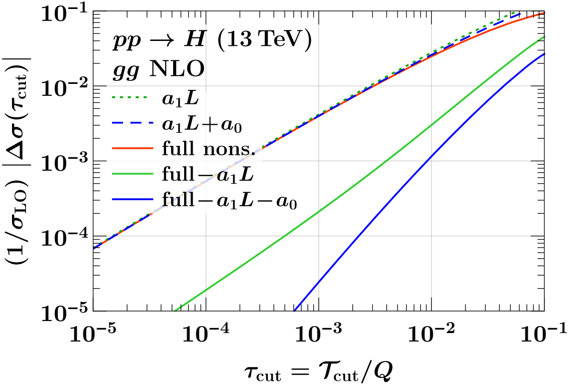
$$\frac{\mathrm{d}\sigma^{(2,n)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} = \hat{\sigma}^{\mathrm{LO}}\left(\frac{\alpha_s}{4\pi}\right)^n \int_{x_a}^1\int_{x_b}^1 \frac{\mathrm{d}z_a}{z_a}\frac{\mathrm{d}z_b}{z_b}\left[f_i f_j C^{(2,n)}_{f_i f_j}(z_a,z_b,\mathcal{T}) + \frac{x_a}{z_a} f_i' f_j C^{(2,n)}_{f_i' f_j}(z_a,z_b,\mathcal{T}) + \frac{x_b}{z_b} f_i f_j' C^{(2,n)}_{f_i f_j'}(z_a,z_b,\mathcal{T})\right]$$

- At **Leading Log** the kernel must have trivial z_a, z_b dependence (by consistency with soft kinematic).
- However, at **NLL**: non-trivial z_a, z_b dependence.
- Example for gg channel in H production at NLL:

$$C^{(2,1)}_{f_g' f_g}(z_a,z_b,\mathcal{T}) = 4C_A\,\frac{\rho}{Qe^Y}\,\delta(1-z_a)\left[\left(-\ln\frac{\mathcal{T}e^Y}{Q\rho}-1\right)\delta(1-z_b) + \frac{(1+z_b)(1-z_b+z_b^2)^2}{2z_b^2}\,\mathcal{L}_0(1-z_b)\right]$$
$$+\,4C_A\,\frac{e^Y}{Q\rho}\,\frac{(1-z_a+z_a^2)^2}{2z_a}\,\delta(1-z_b)$$

Power corrections at FO: full NLO results for $pp \to H$

[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764

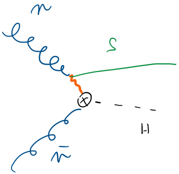


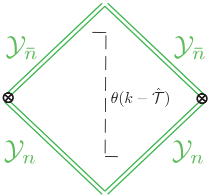
$F_{\rm NLO}(\tau) = \frac{\mathrm{d}}{\mathrm{d}\ln\tau}\Big\{\tau\big[a_1 \ln\tau + a_0 + \mathcal{O}(\tau)\big]\Big\}$

Numerical fit at percent level matches analytic calculation within 1 σ

NLO $\mathcal{T}_0^{\rm lep}$ $gg \to Hg$	a_1	a_0
earlier fit	$+0.6090 \pm 0.0060$	$+0.1824 \pm 0.0043$
analytic	$+0.6040$	$+0.1863$

New features of Regularization and Renormalization at Subleading Power





Regularization of subleading power Rapidity divergences

(Ebert, Moult, Stewart, Tackmann, GV, Zhu)

[1812.08189]

Renormalization with θ functions

(Moult, Stewart, GV, Zhu)

[1804.04665]

Rapidity Divergences

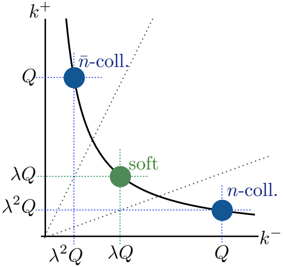
- Large class of observables e.g. $\vec{q}_T$, broadening, EEC, p_T^{veto}, ... belong to the class of SCET$_{\text{II}}$ observables
- SCET$_{\text{II}}$ calculations are affected by **Rapidity Divergences**
- Measurement fixes $\perp$ component of momentum, i.e. $k^+k^- \sim k_\perp^2$ hyperbola

Light cone coordinates: $k^\mu = (k^+,\, k^-,\, \vec{k_\perp})$

n-collinear: $p_n \sim Q(\lambda^2,\, 1,\, \lambda)$

$\bar{n}$-collinear: $p_{\bar{n}} \sim Q(1,\, \lambda^2,\, \lambda)$

soft: $p_s \sim Q(\lambda,\, \lambda,\, \lambda)$



- Example of massless soft real emission with SCET$_{\text{II}}$ measurement:

$$\int \mathrm{d}^d k\, \delta_+(k^2)\delta^{(d-2)}(\vec{q}_\perp - \vec{k}_\perp) f(k^+, k^-, \vec{k}_\perp) = q_T^{-2\epsilon} \int_0^\infty \frac{\mathrm{d}k^-}{k^-} f(k^-, \vec{q}_\perp)$$

- Divergence when modes overlap

$$k^\pm \to 0\,, \qquad y = 1/2 \log(k^+/k^-) \to \pm\infty\,,$$

not regulated by dimensional regularization $\Longrightarrow$ need a rapidity regulator

Rapidity Divergences beyond leading power

- **Leading Power** (in $q_T^2 \ll Q^2$) representative rapidity divergent integral:

$$\frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}q_T^2} \sim \frac{1}{q_T^{2+2\epsilon}} \int_0^Q \frac{\mathrm{d}k^-}{k^-}$$

  - ◇ **Log divergent**, from eikonal propagators from Wilson Lines. (typically...)
  - ◇ It can be regulated in many ways: [Collins] , [Beneke, Feldmann, Chiu, Manohar, ...], [Becher, Bell] [Bell, Rahn, Talbert], [Chiu, Jain, Neill, Rothstein] [Rothstein,Stewart], [Chiu, Fuhrer, Hoang, Kelley, Manohar], [Echevarria, Idilbi, Scimemi], [Li, Neill, Zhu], ...

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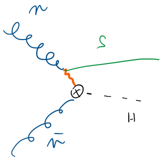
- **Subleading Power**: much broader class of rapidity divergent integrals appearing
  - ◇ Prototypical integrals take the form: $\frac{\mathrm{d}\sigma^{\mathrm{NLP}}}{\mathrm{d}q_T^2} \sim \frac{1}{q_T^{2\epsilon}} \int_0^Q \frac{\mathrm{d}k^-}{(k^-)^{\alpha}}$
  - ◇ α can be negative, hence not only log divergences

$$\int \frac{\mathrm{d}k^-}{(k^-)^2}, \quad \int \frac{\mathrm{d}k^-}{(k^-)^3} \quad \Longrightarrow \quad \textbf{Power Law Rapidity Divergences}$$

  - ◇ **Regulating only Wilson lines is not sufficient**.
    Note that this is also true at LP for Glaubers, see [Rothstein, Stewart]
  - ◇ Divergences also from soft-quark emissions, hard-collinear propagators, phase space expansion.



Rapidity Regularization at Subleading Power

Hence, at Subleading Power:

- Regulating only Wilson lines **is not sufficient**.
- Regularization should conveniently treat power law rapidity divergent integrals
- Common simplifications always used at Leading Power **no longer true**

Example: non-homogeneous regulators (as k^0 or η regulator with $|k_z|$) generate power corrections!

n collinear regulator: $$\left(\frac{k_n^- + k_n^+}{\nu}\right)^{-\eta} = \nu^\eta \left(k_n^- + \frac{k_T^2}{k_n^-}\right)^{-\eta} = \left(\frac{k_n^-}{\nu}\right)^{-\eta}\left[1 - \eta\,\frac{k_T^2}{(k_n^-)^2} + \mathcal{O}(\lambda^4)\right]$$

$$\mathcal{I}_n^{(0)} = \underbrace{\nu^\eta \int_0^Q \mathrm{d}k^- \frac{g_n(k^-/Q)}{(k^-)^{1+\eta}}}_{\text{LP collinear integral}} - \underbrace{k_T^2 \nu^\eta \int_0^Q \mathrm{d}k^- \,\eta\, \frac{g_n(k^-/Q)}{(k^-)^{3+\eta}}}_{\text{NLP integral induced by non homogeneous reg.}} + \mathcal{O}(\lambda^4)$$

The NLP integral induced by the regulator is $\frac{1}{\eta}$ divergent $\Longrightarrow$ the η prefactor cancels out and the term does NOT vanish for $\eta \to 0$

Introduce the *pure rapidity regulator*

$$\int \mathrm{d}^d k \to \int \mathrm{d}^d k\, \omega^2 \upsilon^\eta \left|\frac{\bar{n}\cdot k}{n\cdot k}\right|^{-\eta/2} = \int \mathrm{d}^d k\, \omega^2 \upsilon^\eta e^{-y_k \eta}$$

- It doesn't introduce power corrections
- It breaks boost symmetry in the most minimal way.
- Includes **dimensionless** (pure) rapidity scale υ (**upsilon**)

Leading-Logarthmic power corrections

- Compute power corrections in q_T^2/Q^2 in the n-collinear, $\bar{n}$-collinear and soft limits (soft is scaless for homogeneous regulators)
- Sum together results
- Rapidity divergences cancel between sectors, finite terms add up.
  (In rapidity regularization this is trivial since $g_n(\eta) = g_{\bar{n}}(-\eta)$)

At Leading Log the result is quite simple. Here a couple of examples:

- Drell Yan production ($q\bar{q} \to Vg$)

$$\frac{\mathrm{d}\sigma^{(2),\mathrm{LL}}_{q\bar{q}\to Vg}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \hat{\sigma}^{\mathrm{LO}}_{q\bar{q}\to V}(Q) \times \frac{\alpha_s C_F}{4\pi}\frac{2}{Q^2}\ln\frac{Q^2}{q_T^2}\left[f^{q\bar{q}}_{\mathrm{uni}}(x_a, x_b)\right],$$

- Gluon fusion Higgs production ($gg \to Hg$)

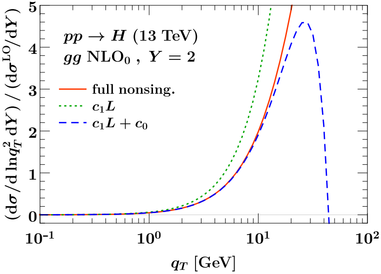
$$\frac{\mathrm{d}\sigma^{(2),\mathrm{LL}}_{gg\to Hg}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \hat{\sigma}^{\mathrm{LO}}_{gg\to H}(Q) \times \frac{\alpha_s C_A}{4\pi}\frac{2}{Q^2}\ln\frac{Q^2}{q_T^2}\left[8 f_g(x_a) f_g(x_b) + f^{gg}_{\mathrm{uni}}(x_a, x_b)\right],$$

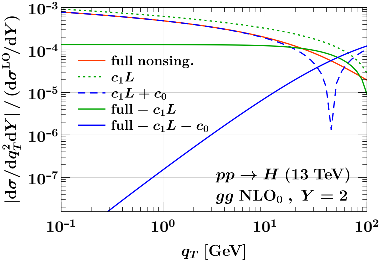
- Common factor

$$f^{ij}_{\mathrm{uni}}(x_a, x_b) = -x_a f_i'(x_a) f_j(x_b) - f_i(x_a)\, x_b f_j'(x_b) + 2 x_a f_i'(x_a)\, x_b f_j'(x_b)$$

Next Leading-Logarithmic power corrections

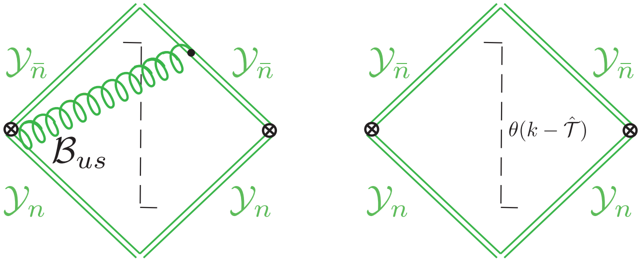
- We computed also the NLL kernels at $\mathcal{O}(\alpha_s)$ for all channels both in DY and ggH.
- z_a, z_b kernels pretty complicated. They involve $\mathcal{L}_0^{++}(1 - z_a)$, etc.
- Remainder is q_T^2/Q^2 suppressed
- Describes q_T distribution up to 10 GeV





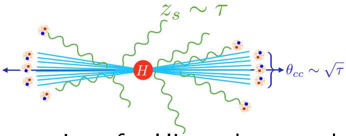
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2} - \frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}Y\mathrm{d}q_T^2} = c_1(Y)\ln\frac{Q^2}{q_T^2} + c_0(Y) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Renormalization at subleading powers

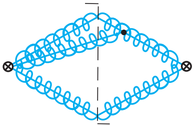


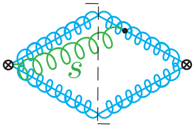
(Moult, Stewart, GV, Zhu) [1804.04665]

Fixed Order Calculation of Thrust



- Compute power corrections for Higgs thrust at lowest order





$$\frac{1}{\sigma_0}\frac{d\sigma^{(2)}}{d\tau} = 8C_A\left(\frac{\alpha_s}{4\pi}\right)\left[\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{Q^2\tau}\right) - \left(\frac{1}{\epsilon} + \log\frac{\mu^2}{Q^2\tau^2}\right)\right]\theta(\tau) + \mathcal{O}(\alpha_s^2)$$

$$= 8C_A\left(\frac{\alpha_s}{4\pi}\right)\log\tau\ \theta(\tau) + \mathcal{O}(\alpha_s^2)$$

- No virtual corrections at lowest order ($\delta(\tau) \sim 1/\tau$).
- Divergences cancel between soft and collinear.
- Log appears at first non-vanishing order:
  - At LP, $\log(\tau)/\tau$ arises from RG evolution of $\delta(\tau)$
  - At NLP $\log(\tau)$ arises from RG evolution of "nothing"?

Elements of Subleading Power Factorization [Moult, Stewart, GV, Zhu]

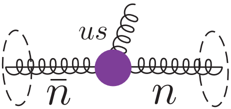
- Analogously to what we have seen at FO power corrections at the operator level arise from two distinct sources:
    - Power corrections to scattering amplitudes.
    - Power corrections to kinematics.

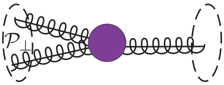
Note: Fields/Lagrangians have a definite power counting in λ!

Operator	$\mathcal{B}^{\mu}_{n\perp}$	χ_n	$\mathcal{P}^{\mu}_{\perp}$	$\mathcal{B}^{\mu}_{us}$	ψ_{us}	∂^{μ}_{us}
Power C.	λ	λ	λ	λ^2	λ^3	λ^2

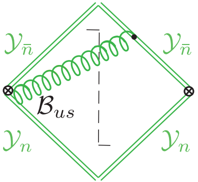
$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{hard}} + \mathcal{L}_{\text{dyn}} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\text{hard}} + \sum_{i\geq 0} \mathcal{L}^{(i)} + \mathcal{L}^{(0)}_{\mathcal{G}}$$

- Power corrections to scattering amplitudes can be computed from subleading SCET operators [Moult, Stewart, GV]

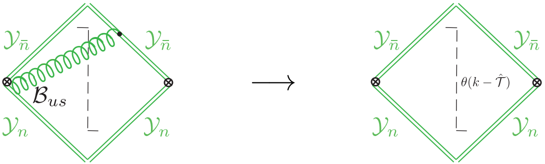




- They give rise to new jet and soft functions, whose renormalization was not previously known



Renormalization of Subleading Soft Functions



- The subleading soft function satisfies a 2×2 mixing RG

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{11}(y,\mu) & \gamma_{12} \\ 0 & \gamma_{22}(y,\mu) \end{pmatrix} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix}$$

- It mixes with "θ-soft" functions

$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\theta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

  - It is power suppressed due to $\theta(\tau) \sim 1$ instead of $\delta(\tau) \sim 1/\tau$.
  - In collinear sector, analogous subleading Jet functions and θ-jet functions appear
  - We find this type of mixing is a generic behavior at subleading power.

Resummed Soft Function

- Solve RGE mixing equation to renormalize the operators, and resum subleading power logarithms.
- We find the final result for the renormalized subleading power soft function:

$$S^{(2)}_{g,\mathcal{B}_{us}}(Q\tau,\mu) = \theta(\tau)\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right)e^{\frac{1}{2}\gamma_{11}\log^2\left(\frac{\mu}{Q\tau}\right)}$$

- Expanded perturbatively, we see a simple series:

$$S^{(2)}_{g,\mathcal{B}_{us}}(Q\tau,\mu) = \theta(\tau)\left[\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right) + \frac{1}{2}\gamma_{12}\gamma_{11}\log^3\left(\frac{\mu}{Q\tau}\right) + \cdots\right]$$

- In particular, we find:
  - First log generated by mixing with the θ function operators.
  - The single log is then dressed by Sudakov double logs from the diagonal anomalous dimensions.
- Example also useful for understanding power suppressed RG consistency.

LL Resummation for Thrust at NLP [Moult, Stewart, Vita, Zhu]

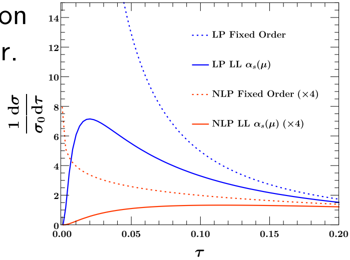
- Complete result given by sum of two contributions.

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{kin,LL}}}{\mathrm{d}\tau} + \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{hard,LL}}}{\mathrm{d}\tau}$$

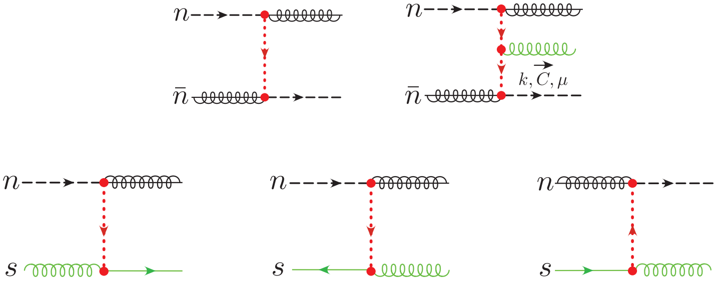
- Both have same Sudakov $\Longrightarrow$ can be directly added.
- Obtain the LL resummed result for pure glue $H \to gg$ thrust

$$\boxed{\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8 C_A \log(\tau) e^{-\frac{\alpha_s}{4\pi}\Gamma^g_{\mathrm{cusp}}\log^2(\tau)}}$$

- Provides the first all orders resummation for an event shape at subleading power.
- Very simple result. Subleading power LL driven by cusp anomalous dimension!



Regge Limit Beyond Leading Power: Glauber Quarks



"Fermionic Glauber Operators and Quark Reggeization"

Ian Moult, Mikhail P. Solon, Iain W. Stewart, **GV**

[1709.09174]

EFT for forward scattering

SCET can be used to treat forward limit:

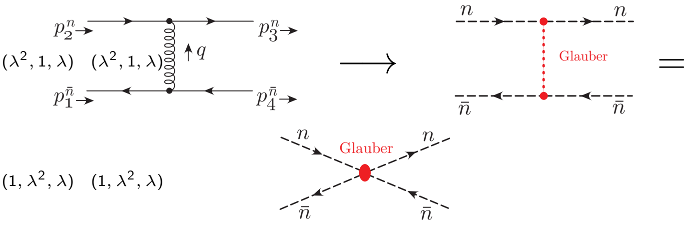
- Take $2 \to 2$ scattering with forward condition:

$$\bar{n} \cdot p_2^n = \bar{n} \cdot p_3^n \qquad \text{and} \qquad n \cdot p_1^{\bar{n}} = n \cdot p_4^{\bar{n}}$$

- Exchanged **gluon** in the t channel has **Glauber scaling**:

$$q^\mu \sim Q(\lambda^2, \lambda^2, \lambda) \implies t = q^2 = q_\perp^2 + \underbrace{\ldots}_{\text{higher orders in } \lambda}$$

- Integrate out Glauber modes, get Glauber potentials/operators



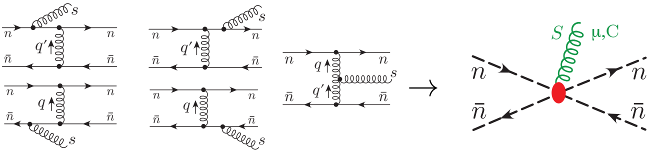
"An Effective Field Theory for Forward Scattering and Factorization Violation"
I.Rothstein and I.W.Stewart [hep-ph/1601.04695]

Leading power Glauber Lagrangians

The complete set of Glauber operators at LP gives **Glauber Lagrangian**

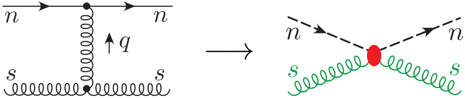
$$\mathcal{L}_G^{\text{II}(0)} = \sum_{n,\bar{n}} \mathcal{O}_{n\bar{n}s} + \sum_{n} \mathcal{O}_{ns}$$

- **3-rapidity** *sector operators*: $\mathcal{O}_{n\bar{n}s} = \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC}$

e.g. 

(Lipatov vertex)

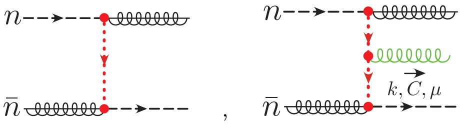
- **2-rapidity** *sector operators*: $\mathcal{O}_{ns} = \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$

e.g. 

Glauber quark Lagrangians

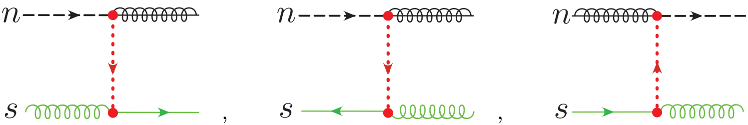
- If we consider process with **fermion number flip** in a collinear sector, the exchanged particle carries fermion number.
- In this situation one can study the *Reggeization* of a quark
- We derived the complete set of Glauber operators responsible for LL **quark Reggeization**:
- **3-rapidity** *sector operators*: $\mathcal{L}^{\text{II}(1)} \supset \sum_{n,\bar{n}} \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^{n\bar{n}} \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_{\bar{n}} \equiv \mathcal{O}_{n\bar{n}}$

e.g.



(call it Fadin-Sherman vertex)

- **2-rapidity** *sector operators*: $\mathcal{L}^{\text{II}(1/2)} \supset \sum_{n} \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^{n} + \text{h.c.} \equiv \mathcal{O}_{ns}$



Glauber quark Lagrangian operators

$$\sum_{n,\bar{n}} \underbrace{\bar{\mathcal{O}}_n}_{\lambda^2} \frac{1}{\mathcal{P}\!\!\!/_\perp} \underbrace{\mathcal{O}_s^{n\bar{n}}}_{\lambda} \frac{1}{\mathcal{P}\!\!\!/_\perp} \underbrace{\mathcal{O}_{\bar{n}}}_{\lambda^2} + \sum_{n} \underbrace{\bar{\mathcal{O}}_n}_{\lambda^2} \frac{1}{\mathcal{P}\!\!\!/_\perp} \underbrace{\mathcal{O}_s^{n}}_{\lambda^{3/2}} + \text{h.c.}$$

Forward scattering **collinear** operators with fermion number:

$$\mathcal{O}_n = \mathcal{B}\!\!\!/_{\perp n}\chi_n \qquad \mathcal{O}_{\bar{n}} = \mathcal{B}\!\!\!/_{\perp \bar{n}}\chi_{\bar{n}}$$
$$\bar{\mathcal{O}}_n = \bar{\chi}_n\mathcal{B}\!\!\!/_{\perp n} \qquad \bar{\mathcal{O}}_{\bar{n}} = \bar{\chi}_{\bar{n}}\mathcal{B}\!\!\!/_{\perp \bar{n}}$$

Single index **soft** operators (2-rapidity sector Lagrangian):

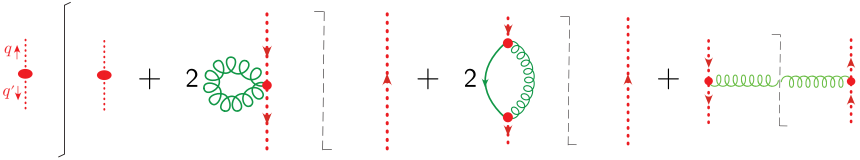
$$\mathcal{O}_s^n = -4\pi\alpha_s \mathcal{B}\!\!\!/^n_{\perp S}\psi^n_S \qquad \bar{\mathcal{O}}_s^n = -4\pi\alpha_s \bar{\psi}^n_S \mathcal{B}\!\!\!/^n_{\perp S}\,.$$

Two index **soft** operator (3-rapidity sector Lagrangian):

$$\mathcal{O}_s^{n\bar{n}} = -2\pi\alpha_s \left[S_n^\dagger S_{\bar{n}} \mathcal{P}\!\!\!/_\perp + \mathcal{P}\!\!\!/_\perp S_n^\dagger S_{\bar{n}} - S_n^\dagger S_{\bar{n}} g \mathcal{B}\!\!\!/^{\bar{n}}_{S\perp} - g\mathcal{B}\!\!\!/^{n}_{S\perp} S_n^\dagger S_{\bar{n}} \right]$$
$$= -4\pi\alpha_s \left[S_n^\dagger S_{\bar{n}} \mathcal{P}\!\!\!/_\perp - g\mathcal{B}\!\!\!/^{n}_{S\perp} S_n^\dagger S_{\bar{n}} \right] \qquad \text{via } \mathcal{B}\!\!\!/^{n}_{S\perp} \text{ definition}$$

- RGE determined by 1-loop calculation of quark regge Soft Function S^q ("squared" soft glauber operator)

$S^q_{1\text{-l}} =$



- Subtract rapidity divergences via counterterm $Z_{S^q}(q_\perp, k_\perp)$.
- Bare Soft function is independent of $\nu \implies$ derive RGE

$$0 = \nu \frac{d}{d\nu} S^q_{\rm bare}(q_\perp, q'_\perp) = \nu \frac{d}{d\nu} \int d^2k_\perp \, Z^{-1}_{S^q}(q_\perp, k_\perp) \, S^q(k_\perp, q'_\perp, \nu)$$

- RGE for S^q is BFKL equation with C_F instead of C_A

$$\nu \frac{d}{d\nu} S^q(q_\perp, q'_\perp, \nu) = \frac{2C_F\,\alpha_s(\mu)}{\pi^2} \int d^2k_\perp \left[\frac{S^q(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} S^q(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

Future Directions

- Fixed order calculation of power corrections for jet observables and diboson production
- Resummation for q_T distributions beyond leading power
- Resummation beyond NLP LL: NLP NLL? NNLP LL?
- Systematic application of fixed order techniques (IBPs, DE, etc.) to calculate EFT objects at high loop order beyond leading power
- Regge/Small-x/Forward/high-energy limit beyond leading power
- Factorization beyond leading power and Factorization breaking effects
- Subleading power observables

Conclusions

- Described developements for collider observables at subleading power

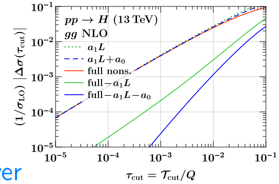
	SCET$_{\text{I}}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	SCET$_{\text{II}}$ soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	Quark Regge, ✓
Fixed Order (fully differential)	LL, NLL	NLL
Hard Scattering Operators	ggH, $Vq\bar{q}$, $Hq\bar{q}$, N-jet	✓
Resummation	$H \to gg$, Soft Quark Sudakov, $H \to gq$	EEC

Lagrangians	Fixed Order	Hard Scattering	Resummation
[Stewart et al.] [Beneke et al.] (2002-2004)	[Moult et al.] LL at $\mathcal{O}(\alpha_s^2)$ 1612.00450 ($q\bar{q}V$), 1710.03227 (ggH)	[Moult, Stewart, GV] 1703.03408 [Moult et al.] $q\bar{q}V$ 1703.03411	[Moult, Stewart, GV, Zhu] $H \to gg$, Soft Quark Sudakov
[Moult, Solon, Stewart, GV] Fermionic Glauber Operators	[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764, $\mathcal{T}_0$ (beam thrust) at NLL NLP	[Chang, Stewart, GV] $q\bar{q}H$ 1712.04343	[Moult, GV, Yan] EEC in $\mathcal{N}=4$
[Chang, Stewart, GV] Subleading Lagrangians in SCET$_{\text{II}}$	[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1812.08189, q_T at NLL NLP	[Beneke et al.] (N-jet operators) 1712.04416, 1808.04742	[Beneke et al.] gq in Higgs Thrust

- Studied how to implement rapidity regularization at subleading powers and proposed new regulator *purely* based on rapidity

$$\int \mathrm{d}^d k \to \int \mathrm{d}^d k \, \omega^2 \upsilon^\eta \left| \frac{\bar{n} \cdot k}{n \cdot k} \right|^{-\eta/2}$$

- Computed full $\mathcal{O}(\alpha_s)$ power correction of differential distribution for color singlet production



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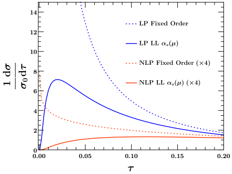
- Showed EFT treatment for quark reggeization and derived fermionic BFKL



Conclusions

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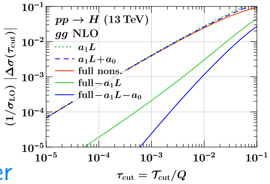

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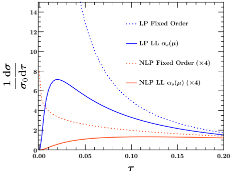


THANK YOU!

Backup slides

Factorization with subleading Hard operators

$$\sigma^{(n)} \sim \sum H \otimes B \otimes B \otimes S$$

universal!

	Operators	Factorization	Beam n	Beam $\bar{n}$	Soft
$\mathcal{O}(\lambda^0)$	$O_{\mathcal{B}}^{(0)} O_{\mathcal{B}}^{(0)}$	$H_g^{(0)} B_g^{(0)} B_g^{(0)} S_g^{(0)}$	$\mathcal{B}_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}}$	$\mathcal{Y}_n^T \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n$
$\mathcal{O}(\lambda^2)$	$O_{\mathcal{B}\bar{n}}^{(1)} O_{\mathcal{B}\bar{n}}^{(1)}$	$H_{g1}^{(0)} B_q^{(0)} B_{qgg}^{(2)} S_q^{(0)}$	$\bar{\chi}_n \hat{\delta} \chi_n$	$\bar{\chi}_{\bar{n}} \mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}} \chi_{\bar{n}}$	$Y_{\bar{n}}^\dagger Y_n \widehat{\mathcal{M}}^{(0)} Y_n^\dagger Y_{\bar{n}}$
	$O^{(0)} O_{\mathcal{B}1}^{(2)}$	$H_{g2}^{(0)} B_{gqq}^{(2)} B_g^{(0)} S_g^{(0)}$	$\bar{\chi}_n \mathcal{B}_n \chi_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}}$	$\mathcal{Y}_n^T \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n$
	$O^{(0)} O_{\mathcal{P}\chi}^{(2)}$	$H_{g3}^{(0)} B_g^{(0)} B_{gqP}^{(2)} S_g^{(0)}$	$\mathcal{B}_n \hat{\delta} \mathcal{B}_n$	$\bar{\chi}_{\bar{n}} [\mathcal{P}_\perp \chi_{\bar{n}}] \hat{\delta} \mathcal{B}_{\bar{n}}$	$\mathcal{Y}_n^T \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n$
	$O^{(0)} O_{\mathcal{P}\mathcal{B}}^{(2)}$	$H_{g4}^{(0)} B_g^{(0)} B_{ggP}^{(2)} S_g^{(0)}$	$\bar{\mathcal{B}}_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} [\mathcal{P}_\perp \mathcal{B}_{\bar{n}}] \hat{\delta} \mathcal{B}_{\bar{n}}$	$\mathcal{Y}_n^T \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n$
	$O^{(0)} O_{4g2}^{(2)}$	$H_{g5}^{(0)} B_g^{(0)} B_{gg}^{(2)} S_g^{(0)}$	$\mathcal{B}_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} \mathcal{B}_{\bar{n}} \mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}}$	$\mathcal{Y}_n^T \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n$
	$O^{(0)} O_{\mathcal{B}(us)0}^{(2)}$	$H_{g6}^{(0)} B_g^{(0)} B_g^{(0)} S_{g\mathcal{B}}^{(2)}$	$\mathcal{B}_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}}$	$\mathcal{B}_{us(n)0} \mathcal{Y}_n \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}} \mathcal{Y}_n$
	$O^{(0)} O_{\partial(us)0}^{(2)}$	$H_{g7}^{(0)} B_g^{(0)} B_g^{(0)} S_{g\partial 0}^{(2)}$	$\mathcal{B}_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}}$	$\partial_{us(n)0} \mathcal{Y}_n \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}} \mathcal{Y}_n$
	$O^{(0)} O_{\partial(us)\bar{0}}^{(2)}$	$H_{g8}^{(0)} B_g^{(0)} B_g^{(0)} S_{g\partial\bar{0}}^{(2)}$	$\mathcal{B}_n \hat{\delta} \mathcal{B}_n$	$\mathcal{B}_{\bar{n}} \hat{\delta} \mathcal{B}_{\bar{n}}$	$\partial_{us(n)\bar{0}} \mathcal{Y}_n \mathcal{Y}_{\bar{n}} \widehat{\mathcal{M}}^{(0)} \mathcal{Y}_{\bar{n}} \mathcal{Y}_n$

Operator basis at subleading powers for $gg \to H$ (Moult, Stewart, **GV**) [1703.03408]

Order	Category	Operators	# helicity configs	# of color	$\sigma_{2j}^{\mathcal{O}(\lambda^2)} \neq 0$
$\mathcal{O}(\lambda^0)$	Hgg	$O^{(0)ab}_{\mathcal{B}\lambda_1\lambda_1} = \mathcal{B}^a_{n\lambda_1}\mathcal{B}^a_{\bar n\lambda_1} H$	2	1	✓
$\mathcal{O}(\lambda)$	$Hq\bar q g$	$O^{(1)a\,\bar\alpha\beta}_{\mathcal{B}n,\bar n\lambda_1(\lambda_i)} = \mathcal{B}^a_{n,\bar n\lambda_1} J^{\bar\alpha\beta}_{n\bar n\,\lambda_j} H$	4	1	✓
$\mathcal{O}(\lambda^2)$	$Hq\bar q Q\bar Q$	$O^{(2)\bar\alpha\beta\bar\gamma\delta}_{qQ1(\lambda_1;\lambda_2)} = J^{\bar\alpha\beta}_{(q)n\lambda_1} J^{\bar\gamma\delta}_{(Q)\bar n\lambda_2} H$	4	2	
		$O^{(2)\bar\alpha\beta\bar\gamma\delta}_{qQ2(\lambda_1;\lambda_1)} = J^{\bar\alpha\beta}_{(q\bar Q)n\lambda_1} J^{\bar\gamma\delta}_{(Q\bar q)\bar n\,\lambda_1} H$	2	2	
		$O^{(2)\bar\alpha\beta\bar\gamma\delta}_{qQ3(\lambda_1;-\lambda_1)} = J^{\bar\alpha\beta}_{(q)n\bar n\lambda_1} J^{\bar\gamma\delta}_{(Q)n\bar n-\lambda_1} H$	2	2	
	$Hq\bar q q\bar q$	$O^{(2)\bar\alpha\beta\bar\gamma\delta}_{qq1(\lambda_1;\lambda_2)} = J^{\bar\alpha\beta}_{(q)n\lambda_1} J^{\bar\gamma\delta}_{(q)\bar n\lambda_2} H$	3	2	
		$O^{(2)\bar\alpha\beta\bar\gamma\delta}_{qq3(\lambda_1;-\lambda_1)} = J^{\bar\alpha\beta}_{(q)n\bar n\lambda_1} J^{\bar\gamma\delta}_{(q)n\bar n-\lambda_1} H$	1	2	
	$Hq\bar q gg$	$O^{(2)ab\,\bar\alpha\beta}_{\mathcal{B}1\lambda_1\lambda_2(\lambda_3)} = \mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar n\lambda_2} J^{\bar\alpha\beta}_{n\,\lambda_3} H$	4	3	✓
		$O^{(2)ab\,\bar\alpha\beta}_{\mathcal{B}2\lambda_1\lambda_2(\lambda_3)} = \mathcal{B}^a_{\bar n\lambda_1}\mathcal{B}^b_{\bar n\lambda_2} J^{\bar\alpha\beta}_{n\,\lambda_3} H$	2	3	
	$Hgggg$	$O^{(2)abcd}_{4g1\lambda_1\lambda_2\lambda_3\lambda_4} = S\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{n\lambda_2}\mathcal{B}^c_{\bar n\lambda_3}\mathcal{B}^d_{\bar n\lambda_4} H$	3	9	
		$O^{(2)abcd}_{4g2\lambda_1\lambda_2\lambda_3\lambda_4} = S\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar n\lambda_2}\mathcal{B}^c_{\bar n\lambda_3}\mathcal{B}^d_{\bar n\lambda_4} H$	2	9	✓
	$\mathcal{P}_\perp$	$O^{(2)a\,\bar\alpha\beta}_{\mathcal{P}\chi\lambda_1(\lambda_2)[\lambda_\mathcal{P}]} = \mathcal{B}^a_{n\lambda_1} \{J^{\bar\alpha\beta}_{\bar n\,\lambda_2} (\mathcal{P}_\perp^{\lambda_\mathcal{P}})^\dagger\} H$	4	1	✓
		$O^{(2)abc}_{\mathcal{P}\mathcal{B}\lambda_1\lambda_2\lambda_3[\lambda_\mathcal{P}]} = S\,\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar n\lambda_2} \left[\mathcal{P}_\perp^{\lambda_\mathcal{P}} \mathcal{B}^c_{\bar n\lambda_3}\right] H$	4	2	✓
	Ultrasoft	$O^{(2)a\,\bar\alpha\beta}_{\chi(us(n))0:(\lambda_1)} = \mathcal{B}^a_{us(n)0} J^{\bar\alpha\beta}_{n\bar n\,\lambda_1} H$	2	1	
		$O^{(2)a\,\bar\alpha\beta}_{\chi(us(\bar n))0:(\lambda_1)} = \mathcal{B}^a_{us(\bar n)0} J^{\bar\alpha\beta}_{n\bar n\,\lambda_1} H$	2	1	
		$O^{(2)\,\bar\alpha\beta}_{\partial\chi(us(i))\lambda_1:(\lambda_2)} = \{\partial_{us(i)\lambda_1} J^{\bar\alpha\beta}_{n\bar n\,\lambda_2}\} H$	4	1	
		$O^{(2)abc}_{\mathcal{B}(us(n))\lambda_1:\lambda_2\lambda_3} = \mathcal{B}^a_{us(n)\lambda_1}\mathcal{B}^b_{n\,\lambda_2}\mathcal{B}^c_{\bar n\,\lambda_3} H$	2	2	✓
		$O^{(2)abc}_{\mathcal{B}(us(\bar n))\lambda_1:\lambda_2\lambda_3} = \mathcal{B}^a_{us(\bar n)\lambda_1}\mathcal{B}^b_{n\,\lambda_2}\mathcal{B}^c_{\bar n\,\lambda_3} H$	2	2	✓
		$O^{(2)ab}_{\partial\mathcal{B}(us(i))\lambda_1:\lambda_2\lambda_3} = \left[\partial_{us(i)\lambda_1}\mathcal{B}_{n\,\lambda_2}\right]\mathcal{B}_{\bar n\,\lambda_3} H$	4	1	✓

Operator basis at subleading powers for $e^+e^- \to$ dijet

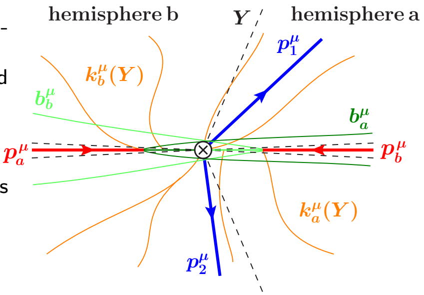
[1703.03411 Stewart, Moult et al.]

Order	Category	Operators	# helicity configs	$\sigma_{2j}^{\mathcal{O}(\lambda^2)} \neq 0$
$\mathcal{O}(\lambda^0)$	$e\bar{e}q\bar{q}$	$O^{(0)\bar{\alpha}\beta}_{(\lambda_1;\pm)} = J^{\bar{\alpha}\beta}_{n\bar{n}\lambda_1} J_{e\pm}$	4	✓
$\mathcal{O}(\lambda)$	$e\bar{e}q\bar{q}g$	$O^{(1)a\,\bar{\alpha}\beta}_{n\bar{n}1,2\lambda_1(\lambda_2;\pm)} = \mathcal{B}^a_{n,\bar{n}\lambda_1} J^{\bar{\alpha}\beta}_{n\bar{n}-\lambda_1} J_{e\pm}$	8	✓
		$O^{(1)a\,\bar{\alpha}\beta}_{\bar{n}\lambda_1(\lambda_2:\pm)} = \mathcal{B}^a_{n\lambda_1} J^{\bar{\alpha}\beta}_{\bar{n}\lambda_2} J_{e\pm}$	8	✓
	$e\bar{e}ggg$	$O^{(1)abc}_{\mathcal{B}\lambda_1\lambda_2\lambda_3(\pm)} = S\, \mathcal{B}^a_{n\lambda_1} \mathcal{B}^b_{\bar{n}\lambda_2} \mathcal{B}^c_{\bar{n}\lambda_3} J_{e\pm}$	8	✓
$\mathcal{O}(\lambda^2)$	$e\bar{e}q\bar{q}Q\bar{Q}$	$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ1(\lambda_1;\lambda_2:\pm)} = J^{\bar{\alpha}\beta}_{(q)n\lambda_1} J^{\bar{\gamma}\delta}_{(Q)\bar{n}\lambda_2} J_{e\pm}$	8	
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ2(\lambda_1;\lambda_1:\pm)} = J^{\bar{\alpha}\beta}_{(q\bar{Q})n\lambda_1} J^{\bar{\gamma}\delta}_{(Q\bar{q})\bar{n}\,\lambda_1} J_{e\pm}$	4	
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ3(\lambda_1;-\lambda_1;\pm)} = J^{\bar{\alpha}\beta}_{(q)n\bar{n}\lambda_1} J^{\bar{\gamma}\delta}_{(Q)n\bar{n}-\lambda_1} J_{e\pm}$	4	
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ4(\lambda_1:\lambda_2;\pm)} = J^{\bar{\alpha}\beta}_{(q)\bar{n}\lambda_1} J^{\bar{\gamma}\delta}_{(Q)n\bar{n}\,\lambda_2} J_{e\pm}$	8	✓
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ5(\lambda_1:\lambda_2;\pm)} = J^{\bar{\alpha}\beta}_{(q)\bar{n}\lambda_1} J^{\bar{\gamma}\delta}_{(Q)\bar{n}n\lambda_2} J_{e\pm}$	8	✓
	$e\bar{e}q\bar{q}q\bar{q}$	$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qq1(\lambda_1;\lambda_2:\pm)} = J^{\bar{\alpha}\beta}_{(q)n\lambda_1} J^{\bar{\gamma}\delta}_{(q)\bar{n}\lambda_2} J_{e\pm}$	8	
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qq3(\lambda_1;-\lambda_1;\pm)} = J^{\bar{\alpha}\beta}_{(q)n\bar{n}\lambda_1} J^{\bar{\gamma}\delta}_{(q)n\bar{n}-\lambda_1} J_{e\pm}$	2	
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qq4(\lambda_1:\lambda_2;\pm)} = J^{\bar{\alpha}\beta}_{(q)\bar{n}\lambda_1} J^{\bar{\gamma}\delta}_{(q)n\bar{n}\,\lambda_2} J_{e\pm}$	8	✓
		$O^{(2)\bar{\alpha}\beta\bar{\gamma}\delta}_{qq5(\lambda_1:\lambda_2;\pm)} = J^{\bar{\alpha}\beta}_{(q)\bar{n}\lambda_1} J^{\bar{\gamma}\delta}_{(q)\bar{n}n\lambda_2} J_{e\pm}$	8	✓
	$e\bar{e}q\bar{q}gg$	$O^{(2)ab\,\bar{\alpha}\beta}_{\mathcal{B}1\lambda_1\lambda_2(\lambda_3;\pm)} = S\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{n\lambda_2} J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_3} J_{e\pm}$	8	✓
		$O^{(2)ab\,\bar{\alpha}\beta}_{\mathcal{B}2\lambda_1\lambda_2(\lambda_3;\pm)} = S\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{n\lambda_2} J^{\bar{\alpha}\beta}_{\bar{n}n\,\lambda_3} J_{e\pm}$	8	✓
		$O^{(2)ab\,\bar{\alpha}\beta}_{\mathcal{B}3\lambda_1\lambda_2(\lambda_3;\pm)} = \mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar{n}\lambda_2} J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_3} J_{e\pm}$	12	✓
		$O^{(2)ab\,\bar{\alpha}\beta}_{\mathcal{B}4\lambda_1\lambda_2(\lambda_3:\pm)} = \mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar{n}\lambda_2} J^{\bar{\alpha}\beta}_{n\,\lambda_3} J_{e\pm}$	8	
		$O^{(2)ab\,\bar{\alpha}\beta}_{\mathcal{B}5\lambda_1\lambda_2(\lambda_3:\pm)} = \mathcal{B}^a_{\bar{n}\lambda_1}\mathcal{B}^b_{\bar{n}\lambda_2} J^{\bar{\alpha}\beta}_{n\,\lambda_3} J_{e\pm}$	4	

Category	Operators	# helicity configs	$\sigma_{2j}^{\mathcal{O}(\lambda^2)} \neq 0$
$e\bar{e}gggg$	$O^{(2)abcd}_{4g1\lambda_1\lambda_2\lambda_3\lambda_4(\pm)} = S\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{n\lambda_2}\mathcal{B}^c_{\bar{n}\lambda_3}\mathcal{B}^d_{\bar{n}\lambda_4} J_{e\pm}$	6	
	$O^{(2)abcd}_{4g2\lambda_1\lambda_2\lambda_3\lambda_4(\pm)} = S\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar{n}\lambda_2}\mathcal{B}^c_{\bar{n}\lambda_3}\mathcal{B}^d_{\bar{n}\lambda_4} J_{e\pm}$	4	
$\mathcal{P}_\perp$	$O^{(2)a\,\bar{\alpha}\beta}_{\mathcal{P}2\lambda_1(\lambda_2:\pm)[\lambda_\mathcal{P}]} = \mathcal{B}^a_{n\lambda_1} \{J^{\bar{\alpha}\beta}_{\bar{n}\,\lambda_2}(\mathcal{P}^{\lambda_\mathcal{P}}_\perp)^\dagger\} J_{e\pm}$	8	
	$O^{(2)a\,\bar{\alpha}\beta}_{\mathcal{P}1n,\bar{n}\lambda_1(\lambda_2;\pm)[\lambda_\mathcal{P}]} = \left[\mathcal{P}^{\lambda_\mathcal{P}}_\perp \mathcal{B}^a_{n,\bar{n}\lambda_1}\right] J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_2} J_{e\pm}$	24	✓
	$O^{(2)abc}_{\mathcal{P}\mathcal{B}\lambda_1\lambda_2\lambda_3(\pm)[\lambda_\mathcal{P}]} = S\,\mathcal{B}^a_{n\lambda_1}\mathcal{B}^b_{\bar{n}\lambda_2} \left[\mathcal{P}^{\lambda_\mathcal{P}}_\perp \mathcal{B}^c_{\bar{n}\lambda_3}\right] J_{e\pm}$	8	
Ultrasoft	$O^{(2)a\,\bar{\alpha}\beta}_{\mathcal{B}(us(i))\lambda_1:(\lambda_2;\pm)} = \mathcal{B}^a_{us(i)\lambda_1} J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_2} J_{e\pm}$	8	
	$O^{(2)a\,\bar{\alpha}\beta}_{\mathcal{B}(us(i))0:(\lambda_1;\pm)} = \mathcal{B}^a_{us(i)0} J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1} J_{e\pm}$	8	✓
	$O^{(2)\,\bar{\alpha}\beta}_{\partial(us(i))\lambda_1:(\lambda_2;\pm)} = \{\partial_{us(i)\lambda_1} J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_2}\} J_{e\pm}$	8	
	$O^{(2)\,\bar{\alpha}\beta}_{\partial(us(i))0,\bar{0}:(\lambda_1;\pm)} = \{\partial_{us(i)0,\bar{0}} J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1}\} J_{e\pm}$	8	✓
	$O^{(2)abc}_{(us(i))\lambda_1:\lambda_2\lambda_3(\pm)} = \mathcal{B}^a_{us(i)\lambda_1} \mathcal{B}^b_{n\,\lambda_2} \mathcal{B}^c_{\bar{n}\,\lambda_3} J_{e\pm}$	24	
	$O^{(2)ab}_{\partial\mathcal{B}(us(i))\lambda_1:\lambda_2\lambda_3(\pm)} = \left[\partial_{us(i)\lambda_1}\mathcal{B}_{n\,\lambda_2}\right] \mathcal{B}_{\bar{n}\,\lambda_3} J_{e\pm}$	24	

Power corrections at FO: PDF expansion

- Need to keep track of $\mathcal{O}(\mathcal{T})$ component of momenta: both for phase space expansion and mandelstams entering $|\mathcal{M}|^2$.
- Solving Q and Y measurements uniquely fixes how factors of $\mathcal{T}$ enter the PDFs.



Example n-collinear emission, $k^+ \sim \mathcal{T}$, $k^- \sim Q$:

$$p_a^\mu = Qe^Y\left[\left(1+\frac{k^- e^{-Y}}{Q}\right)+\frac{\mathcal{T}}{Q}\frac{k^-}{2Q}+\mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}\right)\right]\frac{n^\mu}{2}$$

$$p_b^\mu = Qe^{-Y}\left[1+\frac{\mathcal{T}}{Q}\left(e^Y+\frac{k^-}{2Q}\right)+\mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}\right)\right]\frac{\bar{n}^\mu}{2}$$

$$n^\mu = (1,0,0,1)$$
$$\bar{n}^\mu = (1,0,0,-1)$$

- At subleading power **both** PDF momenta contain power corrections **regardless** of the direction of the emission $\Longrightarrow$ derivative of both PDFs

$\mathcal{T}$ power corrections from residual momenta in PDFs for an n-collinear emission:

$$f_a\left(\frac{p_a}{E_{cm}}\right) \sim f_a\left(\frac{x_a}{z_a}+\frac{\mathcal{T}}{Q}\Delta_a\right) = f_a\left(\frac{x_a}{z_a}\right)+\frac{\mathcal{T}}{Q}\Delta_a f_a'\left(\frac{x_a}{z_a}\right)$$

$$f_b\left(\frac{p_b}{E_{cm}}\right) \sim f_b\left(x_b+\frac{\mathcal{T}}{Q}\Delta_b\right) = f_b(x_b)+\frac{\mathcal{T}}{Q}\Delta_b f_b'(x_b)$$

Power corrections at FO: Master formulae

- Expansion of **phase space** and **matrix element squared** in **soft** and **collinear** limits has a general (universal) structure

n-Collinear Master Formula for 0-Jettiness power corrections

$$\frac{\mathrm{d}\sigma_n^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \int_{x_a}^{1} \frac{\mathrm{d}z_a}{z_a} \frac{z_a^{\epsilon}}{(1-z_a)^{\epsilon}} \left(\frac{Q\mathcal{T}e^Y}{\rho}\right)^{-\epsilon} \Bigg\{ f_a f_b\, A^{(2)}(Q, Y, z_a) + \frac{e^Y}{\rho} A^{(0)} \frac{\mathcal{T}}{Q} \left[f_a f_b \frac{(1-z_a)^2 - 2}{2z_a} + x_a \frac{1-z_a}{2z_a} f_a' f_b + x_b \frac{1+z_a}{2z_a} f_a f_b' \right] \Bigg\}$$

Soft Master Formula for 0-Jettiness power corrections

$$\frac{\mathrm{d}\sigma_s^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \frac{1}{\epsilon} \frac{\mathcal{T}^{-2\epsilon}}{Q} \Bigg\{ \bar{A}^{(0)}(Q, Y) \left[f_a f_b \left(-\frac{\rho}{e^Y} - \frac{e^Y}{\rho} \right) + x_a \frac{\rho}{e^Y} f_a' f_b + x_b \frac{e^Y}{\rho} f_a f_b' \right] + f_a f_b \left[\rho Q\, \bar{A}_+^{(2)}(Q, Y) + \frac{Q}{\rho}\, \bar{A}_-^{(2)}(Q, Y) \right] \Bigg\}$$

How to treat power law divergences

- Consider rapidity divergent integral $$\int_x^1 \mathrm{d}z\, \frac{g(z)}{(1-z)^{a+\eta}}\,.$$
- When $g(z)$ is not known analytically (eg. when it involves PDFs), need to extract **pole** as $\eta \to 0$ without computing the integral.
- For $a = 1$, use standard distributional identity

$$\frac{1}{(1-z)^{1+\eta}} = -\frac{\delta(1-z)}{\eta} + \mathcal{L}_0(1-z) + \mathcal{O}(\eta)\,, \qquad \mathcal{L}_0(y) = [\theta(y)/y]_+\,,$$

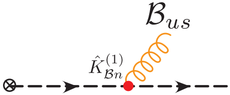
- For $a > 1$, these distributions need to be generalized to higher-order plus distributions subtracting higher derivatives as well. For example, for $a = 2$ one obtains

$$\boxed{\frac{1}{(1-z)^{2+\eta}} = \frac{\delta'(1-z)}{\eta} - \delta(1-z) + \mathcal{L}_0^{++}(1-z) + \mathcal{O}(\eta)}\,,$$

  where the second-order plus function $\mathcal{L}_0^{++}(1-z)$ acts on a test function $g(z)$ as a double subtraction.
- Power law divergences generate new PDF derivatives

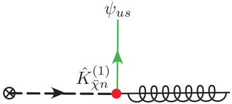
$$\int_{x_a}^1 \mathrm{d}z_a\, \frac{f(x_a/z_a)f(x_b/z_b)}{(1-z_a)^{2+\eta}} = \frac{f'(x_a)f(x_b/z_b)}{\eta} + \mathcal{O}(\eta^0)$$

- Subleading Power Lagrangian insertions give rise to Radiative Function



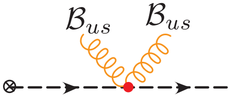
$$\mathcal{L}^{(1)}_{\chi_n} = g\bar{\chi}_n \mathcal{B}^{\perp}_{us(n)} \cdot \mathcal{P}_\perp \frac{\bar{n}\!\!\!/}{\bar{\mathcal{P}}}\chi_n = \hat{K}^{(1)\mu}_{\mathcal{B}_n} \mathcal{B}^{\perp}_{us\mu}$$

non-eikonal single soft emission from a collinear quark



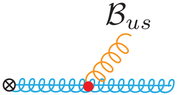
$$\mathcal{L}^{(1)\mathrm{BPS}}_{\chi_n\psi_{us}} = \bar{\chi}_n g \mathcal{B}\!\!\!/_{n\perp} \psi^{(n)}_{us} = \hat{K}^{(1)\bar{\alpha}}_{\chi_n} \psi^{\alpha}_{us(n)}$$

ultrasoft quark emission from a collinear field



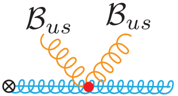
$$\mathcal{L}^{(2)}_{\chi\mathcal{B}\mathcal{B}} = \bar{\chi}_n \left[T^a \gamma^\mu_\perp \frac{1}{\bar{\mathcal{P}}} T^b \gamma^\nu_\perp \right] \frac{\bar{n}\!\!\!/}{2} \chi_n g\mathcal{B}^{a\mu}_{us(n)} g\mathcal{B}^{b\nu}_{us(n)}$$

λ^2 non-eikonal double soft emission from a collinear quark



$$\mathcal{L}^{(2)}_g \supset ig \left[\partial^{[\mu}_\perp \mathcal{B}^{\nu]}_{us} \right] \left[\mathcal{B}^\perp_{n\mu}, \mathcal{B}^\perp_{n\nu} \right]$$

λ^2 non-eikonal single soft emission from a collinear gluon



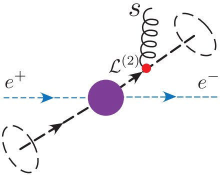
$$\mathcal{L}^{(2)}_g \supset g^2 \mathrm{Tr} \left([\mathcal{B}^{\mu\perp}_{us}, \mathcal{B}^{\nu\perp}_{us}][\mathcal{B}^\perp_{n\mu}, \mathcal{B}^\perp_{n\nu}] \right)$$

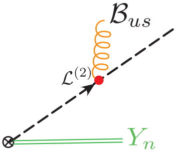
λ^2 non-eikonal double soft emission from a collinear gluon

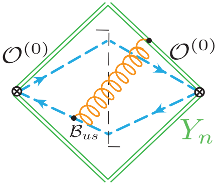
Radiative Jet Function contribution to Power Corrections in Thrust

Leading Power $\quad\quad\quad\quad$ $\mathcal{O}(\lambda)$ typically vanishes

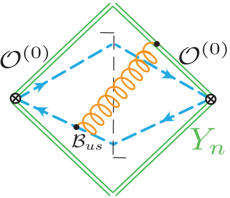
$$d\sigma \sim \langle O^{(0)}O^{(0)}\rangle + \langle O^{(1)}O^{(0)}\rangle + \langle O^{(0)}\mathcal{L}^{(1)}O^{(0)}\rangle + \dots$$
$$+ \langle O^{(0)}O^{(2)}\rangle + \langle O^{(1)}O^{(1)}\rangle + \langle O^{(0)}\mathcal{L}^{(2)}O^{(0)}\rangle + \langle O^{(0)}\mathcal{L}^{(1)}\mathcal{L}^{(1)}O^{(0)}\rangle + \dots$$

Example: 

- LP hard scattering operator: $\mathcal{O}^{(0)} = \bar{\chi}_{\bar{n}}\gamma_\perp^\mu \chi_n$
- Subleading Lagrangian insertion on χ_n dynamics: 

- Cross section: 

RJF contribution to Power Corrections in Thrust

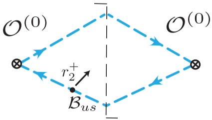


$$\sim \int \mathrm{d}^4 y \langle 0| \overbrace{\left[\bar\chi_n(x) Y_n \gamma_\perp^\mu Y_{\bar n}\chi_{\bar n}(x)\right]}^{\mathcal{O}^{(0)}_{\mathrm{BPS}}(x)} \overbrace{\left[\bar\chi_n T^a g \not{\mathcal{B}}_{us\perp}\frac{1}{\bar{\mathcal{P}}} i\not{\partial}_{us\perp}\frac{\bar{\not{n}}}{2}\chi_n + \dots\right]}^{\mathcal{L}^{(2)}(y)\text{ insertion}} \underbrace{\left[\bar\chi_{\bar n}(0) Y_{\bar n}^\dagger \gamma_\perp^\mu Y_n \chi_n(0)\right]}_{\mathcal{O}^{(0)\dagger}_{\mathrm{BPS}}(0)} |0\rangle$$

After fierzing, color algebra, reducing the allowed form of the convolutions, using simmetry to reduce the number of allowed object that appear we get a factorized expression in terms of matrix elements of soft and collinear fields.

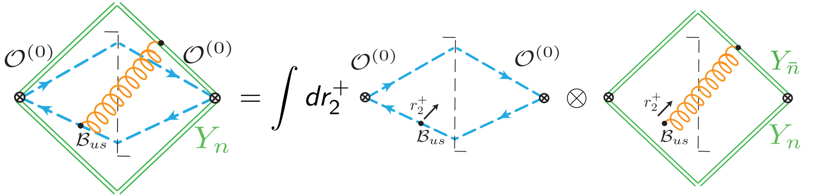
Define **Radiative Jet Function**: $J_{\mathcal{B}}^{(2)}$. In picture, combine it with the LP jet function on $\bar n$ to give

$$J(\tau_{\bar n}) J_{\mathcal{B}}^{(2)}(\tau_n, r_2^+) =$$

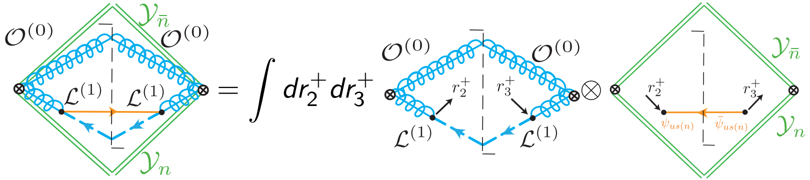


Factorization in Pictures

- Allows all orders factorization for Lagrangian insertions.
- Integral over soft and collinear matrix elements:



Other example: double insertion of soft quark emission



- Can separately compute radiative corrections to each matrix element
- Valid to all orders in α_s, but you need to address convergence and closure issues.