AUTOMATING CALCULATIONS IN SOFT-COLLINEAR EFFECTIVE THEORY

[GUIDO BELL]

UNIVERSITÄT SIEGEN

Introduction

SCET provides a systematic framework for precision studies of collider observables

- N^3LL / N^4LL resummations for benchmark observables

  threshold resummation, p_T resummation, event shapes, . . .

- Automated NNLL resummations for global observables

  [ARES], SCETlib, SoftSERVE, . . .

- Factorisation at NLP

  subleading interactions, operator bases, endpoint singularities, . . .

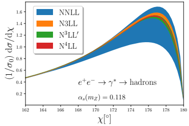
- Jet physics

  non-global logarithms, super-leading logarithms, . . .

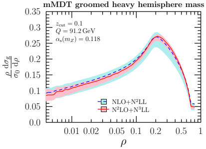
- Factorisation violation

  Glauber exchanges, BFKL, . . .

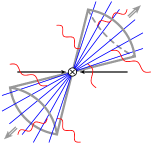
[Duhr, Mistlberger, Vita 22]



[Kardos, Larkoski, Trócsányi 20]



[Becher, Neubert, Rothen, Shao 16]



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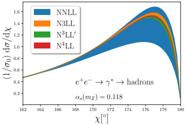

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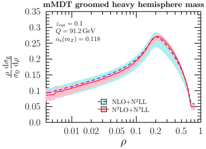

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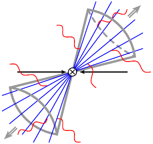


Current status

Structure of LP factorisation theorems

$$\mathrm{d}\sigma = H(\mu) \cdot \prod_i B_{i/h}(\mu) \otimes \prod_j J_j(\mu) \otimes S(\mu) + \mathcal{O}(\lambda)$$

- hard functions

  QCD-SCET matching $\rightarrow$ virtual amplitudes with full colour information

- soft functions [GB, Rahn, Talbert 18,20; GB, Dehnadi, Mohrmann, Rahn wip]

  public code for computation of dijet soft functions

  formalism has been extended for N-jet soft functions

  soft functions for massive particle production under development

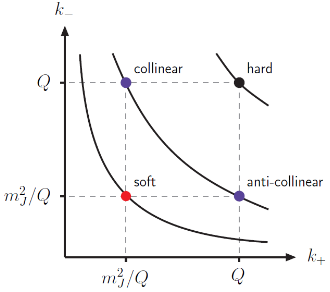
- jet functions [GB, Brune, Das, Wald 21+wip]

  formalism exists for computation of quark and gluon jet functions

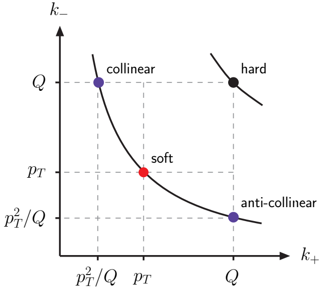
- beam functions [GB, Brune, Das, Wald 22+wip]

  formalism for computation of beam-function matching kernels in moment space

Momentum modes



$\mu_S \ll \mu_J$



$\mu_S = \mu_J$

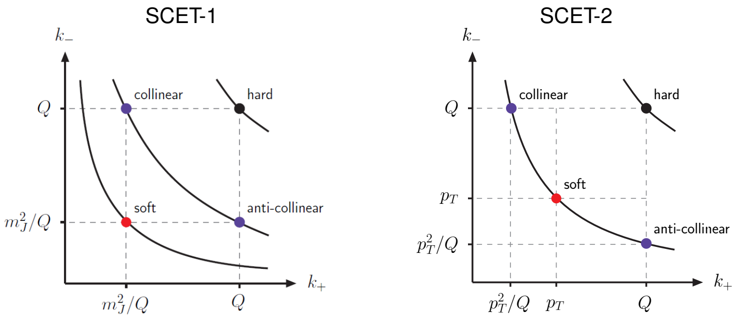
In SCET-2 one cannot distinguish soft from collinear modes when radiated into jet direction

$\Rightarrow$ need additional regulator that distinguishes modes by their rapidities

$$\int d^4k \;\delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^dk \left(\frac{\nu}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

[Becher, GB 11]

Momentum modes



$\mu_S \neq \mu_J$ $\qquad$ $\mu_S = \mu_J$

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[Becher, GB 11]

SCET-1

Three-scale problem: $\mu_S \ll \mu_J \ll \mu_H$

$\mu_H \sim Q$

$\mu_J \sim m_J$

$\mu_S \sim \frac{m_J^2}{Q}$

$$\boxed{d\sigma \;\simeq\; H(Q,\mu)\;\; J(m_J,\mu) \,\otimes\, S(m_J^2/Q,\mu)}$$

$$\ln^2 \frac{Q^2}{m_J^2} = \frac{1}{2}\ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{m_J^2}{\mu^2} + \frac{1}{2}\ln^2 \frac{m_J^4/Q^2}{\mu^2}$$

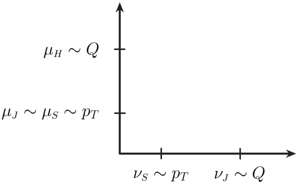
Sudakov resummation with standard RG techniques

$$\frac{dH(Q,\mu)}{d\ln\mu} = \left[2\,\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + 4\gamma_H(\alpha_s)\right] H(Q,\mu)$$

- anomalous dimensions: $\Gamma_{\rm cusp}$, γ_H, γ_J, γ_S
- matching corrections: c_H, c_J, c_S

SCET-2

Two-dimensional problem: $\mu_S \sim \mu_J \ll \mu_H, \quad \nu_S \ll \nu_J$



$$\boxed{d\sigma \simeq H(Q,\mu)\; J(p_T,\mu,Q,\nu) \otimes S(p_T,\mu,p_T,\nu)}$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - 2\ln\frac{p_T^2}{\mu^2}\ln\frac{Q^2}{\nu^2} - \ln^2\frac{p_T^2}{\mu^2} - 2\ln\frac{p_T^2}{\mu^2}\ln\frac{\nu^2}{p_T^2}$$

Exponentiation of rapidity logarithms

- Collinear anomaly [Becher, Neubert 10]

$$\mathcal{J}(x_T,\mu,Q,\nu)\,\mathcal{S}(x_T,\mu,x_T,\nu) = \left(Q^2 x_T^2\right)^{-F(x_T,\mu)}\; W(x_T,\mu)$$

- Rapidity renormalisation group [Chiu, Jain, Neill, Rothstein 11]

$$\frac{d\ln\mathcal{J}(x_T,\mu,Q,\nu)}{d\ln\nu} = \gamma_\nu^J(x_T,\mu) \qquad\qquad \frac{d\ln\mathcal{J}(x_T,\mu,Q,\nu)}{d\ln\mu} = \gamma_\mu^J(x_T,\mu,Q,\nu)$$

Counting logs

Perturbative expansion

$$\frac{d\sigma}{\sigma_0} = 1 + \frac{\alpha_s}{4\pi}\left\{\# L^2 + \# L + \#\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{\# L^4 + \# L^3 + \# L^2 + \# L + \#\right\} + \ldots$$

$$= \exp\left\{\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots\right\}$$

LL NLL NNLL

Accuracy	Γ_{cusp}	$\gamma_H, \begin{cases}\gamma_J, \gamma_S \\ F\end{cases}$	$c_H, \begin{cases} c_J, c_S \\ W\end{cases}$
NLL	2-loop	1-loop	tree
NLL′	2-loop	1-loop	1-loop
NNLL	3-loop	2-loop	1-loop
NNLL′	3-loop	2-loop	2-loop
N^3LL	4-loop	3-loop	2-loop

SCET-1

SCET-2

$\Leftarrow$ State of the art pre-SCET

OUTLINE

SOFT FUNCTIONS

- DIJET SOFT FUNCTIONS
- N-JET EXTENSION
- MASSIVE PARTICLE PRODUCTION

JET FUNCTIONS

BEAM FUNCTIONS

OUTLINE

SOFT FUNCTIONS

DIJET SOFT FUNCTIONS

N-JET EXTENSION

MASSIVE PARTICLE PRODUCTION

JET FUNCTIONS

BEAM FUNCTIONS

Dijet soft functions

Definition

$$S(\tau,\mu) = \frac{1}{N_c} \sum_{i\in X} \mathcal{M}(\tau;\{k_i\}) \,\mathrm{Tr}\, \langle 0|S_{\bar n}^\dagger S_n|X\rangle\langle X|S_n^\dagger S_{\bar n}|0\rangle$$

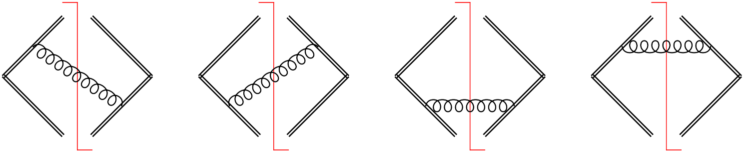
- soft Wilson lines $S_n, S_{\bar n}$ with $n^2 = \bar n^2 = 0$ and $n\cdot\bar n = 2$
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- SCET-1 and SCET-2 observables
- relevant for $e^+e^- \to 2$ jets, $e^-p \to 1$ jet, $pp \to 0$ jets

Structure of divergences is independent of the observable

$\Rightarrow$ isolate singularities with universal phase-space parametrisation

$\Rightarrow$ compute observable-dependent integrations numerically

NLO calculation



NLO calculation

One gluon emission

$$S^{(1)}(\tau,\mu) \sim \int d^dk \, \left(\frac{\nu}{k_+ + k_-}\right)^\alpha \delta(k^2)\,\theta(k^0)\; \mathcal{M}_1(\tau;k)\; \big|\mathcal{A}(k)\big|^2$$

- $n \leftrightarrow \bar{n}$ symmetrised version of phase-space regulator
- matrix element $\big|\mathcal{A}(k)\big|^2 \sim \dfrac{1}{k_+ k_-}$

Phase-space parametrisation

$$k_T = \sqrt{k_+ k_-} \qquad y_k = \frac{k_+}{k_-} \qquad t_k = \frac{1-\cos\theta_k}{2}$$

- k_T is only dimensionful variable
- measurement vector $v^\mu \;\rightarrow$ one angle in transverse plane: $\theta_k \sphericalangle (\vec{k}_\perp, \vec{v}_\perp)$

Measurement function

Generic form

$$\mathcal{M}_1(\tau; k) = \exp\Big(-\tau\, k_T\, y_k^{n/2}\, f(y_k, t_k)\Big)$$

- assumes Laplace transform with $[\tau] = 1/\text{mass}$
- parameter n is fixed by requirement that $f(y_k, t_k)$ is finite and non-zero as $y_k \to 0$

Measurement function

Generic form

$$\mathcal{M}_1(\tau; k) = \exp\Big(-\tau\, k_T\, y_k^{n/2}\, f(y_k, t_k) \Big)$$

Observable	n	$f(y_k, t_k)$
Thrust	1	1
Angularities	$1-A$	1
Recoil-free broadening	0	$1/2$
Threshold Drell-Yan	-1	$1+y_k$
W@large p_T	-1	$1+y_k-2\sqrt{y_k}\,\cos\theta_k$
e^+e^- transverse thrust	1	$\frac{1}{s\sqrt{y_k}}\Bigg(\sqrt{\Big(c\cos\theta_k+\Big(\frac{1}{\sqrt{y_k}}-\sqrt{y_k}\Big)\frac{s}{2}\Big)^2+1-\cos^2\theta_k}\; -\Big\lvert c\cos\theta_k+\Big(\frac{1}{\sqrt{y_k}}-\sqrt{y_k}\Big)\frac{s}{2}\Big\rvert\Bigg)$

$\cos\theta_k = 1-2t_k$

NLO master formula

After performing the observable-independent integrations one finds

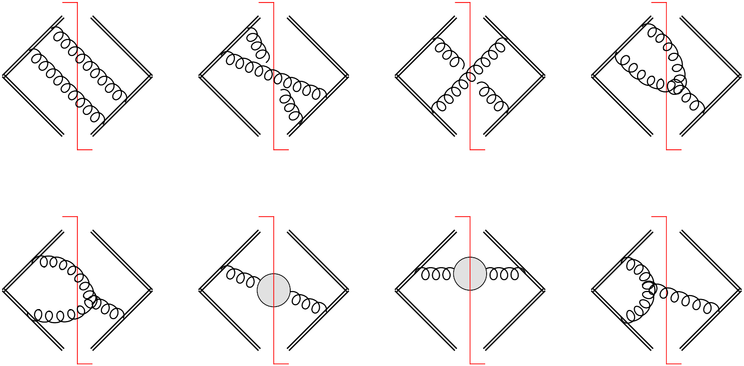
$$S^{(1)}(\tau,\mu) \sim \Gamma(-2\varepsilon-\alpha) \int_0^1 dy_k \; \frac{y_k^{-1+n\varepsilon+\alpha/2}}{(1+y_k)^\alpha} \int_0^1 dt_k \; (4t_k\bar{t}_k)^{-1/2-\varepsilon} \; f(y_k,t_k)^{2\varepsilon+\alpha}$$

- soft ($k_T \to 0$) and collinear ($y_k \to 0$) singularities are factorised
- additional regulator is needed only for $n=0$ ($\to$ SCET-2)

Isolate singularities with standard subtraction techniques

$$\int_0^1 dx \; x^{-1+n\varepsilon} \; f(x) = \int_0^1 dx \; x^{-1+n\varepsilon} \left[\underbrace{f(x)-f(0)}_{\text{finite}} + \underbrace{f(0)}_{1/\varepsilon} \right]$$

NNLO calculation



- real-virtual contribution follows allong the same lines as the NLO calculation

NNLO calculation

Double real emission

$$S_{RR}^{(2)}(\tau,\mu) \sim \int d^d k \left(\frac{\nu}{k_+ + k_-}\right)^{\alpha} \delta(k^2)\,\theta(k^0) \int d^d l \left(\frac{\nu}{l_+ + l_-}\right)^{\alpha} \delta(l^2)\,\theta(l^0)\;\; \mathcal{M}_2(\tau;k,l)\;\; \left|\mathcal{A}(k,l)\right|^2$$

- higher dimensional phase-space integrations
- three colour structures: $\underbrace{C_F C_A,\; C_F T_F n_f,}_{\text{correlated}}\;\; \underbrace{C_F^2}_{\text{uncorrelated}}$

Non-trivial matrix element

$$\left|\mathcal{A}(k,l)\right|^2_{C_F T_F n_f} \;\sim\; \frac{2k\cdot l\,(k_- + l_-)\,(k_+ + l_+) - (k_- l_+ - k_+ l_-)^2}{(k_- + l_-)^2\,(k_+ + l_+)^2\,(2k\cdot l)^2}$$

$\Rightarrow$ complex singularity structure with overlapping divergences

Correlated emissions

[GB, Rahn, Talbert 18]

Phase-space parametrisation

$$p_T = \sqrt{(k_+ + l_+)(k_- + l_-)} \qquad y = \frac{k_+ + l_+}{k_- + l_-} \qquad a = \sqrt{\frac{k_- l_+}{k_+ l_-}} \qquad b = \sqrt{\frac{k_- k_+}{l_- l_+}}$$

- p_T is only dimensionful variable
- three angles in transverse plane: $\theta_k \sphericalangle (\vec{k}_\perp, \vec{v}_\perp)$, $\theta_l \sphericalangle (\vec{l}_\perp, \vec{v}_\perp)$, $\theta_{kl} \sphericalangle (\vec{k}_\perp, \vec{l}_\perp)$

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Measurement function

$$\boxed{\mathcal{M}_2^{corr}(\tau; k, l) = \exp\Big(-\tau\, p_T\, y^{n/2}\, F(a, b, y, t_k, t_l, t_{kl})\Big)}$$

- p_T dependence fixed on dimensional grounds
- $F(a, b, y, t_k, t_l, t_{kl})$ is finite and non-zero for $y \to 0$
- constraints from infrared-collinear safety

soft limit: $F(a, 0, y, t_k, t_l, t_{kl}) = f(y, t_l)$ collinear limit: $F(1, b, y, t_l, t_l, 0) = f(y, t_l)$

Uncorrelated emissions

[GB, Rahn, Talbert 20]

Trivial as long as measurement function respects non-abelian exponentiation

$$\left.\begin{aligned} \left|\mathcal{A}_{RR}^{(CF)}(k,l)\right|^2 &\sim \frac{1}{k_+ k_- l_+ l_-} \\ \mathcal{M}_2(\tau;k,l) &= \mathcal{M}_1(\tau;k)\,\mathcal{M}_1(\tau;l) \end{aligned}\right\} \quad \Rightarrow \quad S_{RR}^{(C_F)}(\varepsilon,\alpha) = \frac{1}{2}\left[S_R(\varepsilon,\alpha)\right]^2$$

$\Rightarrow$ we are interested in a more general class of observables

Uncorrelated emissions

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Phase-space parametrisation

$$y_k = \frac{k_+}{k_-} \qquad q_T = \sqrt{k_+k_-}\left(\frac{\sqrt{l_+l_-}}{l_-+l_+}\right)^{-n} + \sqrt{l_+l_-}\left(\frac{\sqrt{k_+k_-}}{k_-+k_+}\right)^{-n}$$

$$y_l = \frac{l_+}{l_-} \qquad b = \sqrt{\frac{k_+k_-}{l_+l_-}}\left(\frac{\sqrt{k_+k_-}}{k_-+k_+}\right)^{n}\left(\frac{\sqrt{l_+l_-}}{l_-+l_+}\right)^{-n}$$

- q_T is only dimensionful variable
- again three angles in transverse plane: θ_k, θ_l, θ_{kl}

Uncorrelated emissions

[GB, Rahn, Talbert 20]

Trivial as long as measurement function respects non-abelian exponentiation

$$\left.\begin{array}{l} |\mathcal{A}_{RR}^{(CF)}(k,l)|^2 \sim \dfrac{1}{k_+k_-l_+l_-} \\[2ex] \mathcal{M}_2(\tau;k,l) = \mathcal{M}_1(\tau;k)\,\mathcal{M}_1(\tau;l) \end{array}\right\} \quad \Rightarrow \quad S_{RR}^{(C_F)}(\varepsilon,\alpha) = \tfrac{1}{2}\left[S_R(\varepsilon,\alpha)\right]^2$$

$\Rightarrow$ we are interested in a more general class of observables

Measurement function

$$\boxed{\mathcal{M}_2^{unc}(\tau;k,l) = \exp\Big(-\tau\, q_T\, y_k^{n/2}\, y_l^{n/2}\, G(y_k,y_l,b,t_k,t_l,t_{kl})\Big)}$$

- q_T dependence fixed on dimensional grounds
- $G(y_k,y_l,b,t_k,t_l,t_{kl})$ is finite and non-zero for $y_k \to 0$ and $y_l \to 0$
- constraints from infrared-collinear safety

soft: $G(y_k,y_l,0,t_k,t_l,t_{kl}) = \dfrac{f(y_l,t_l)}{(1+y_k)^n}$ collinear: $G(y_l,y_l,b,t_l,t_l,0) = \dfrac{f(y_l,t_l)}{(1+y_l)^n}$

Recap

Considered class of soft functions is characterised by

- parameter n $\rightarrow$ power counting of modes
- $f(y_k, t_k)$ $\rightarrow$ one emission
- $F(a, b, y, t_k, t_l, t_{kl})$ $\rightarrow$ correlated emissions
- $G(y_k, y_l, b, t_k, t_l, t_{kl})$ $\rightarrow$ uncorrelated emissions (only for NAE-breaking observables)

Constraints from infrared-collinear safety

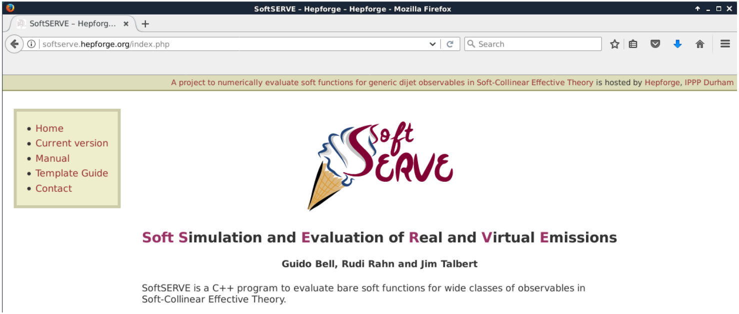
- soft: $F(a, 0, y, t_k, t_l, t_{kl}) = f(y, t_l)$ $\qquad G(y_k, y_l, 0, t_k, t_l, t_{kl}) = \dfrac{f(y_l, t_l)}{(1 + y_k)^n}$
- collinear: $F(1, b, y, t_l, t_l, 0) = f(y, t_l)$ $\qquad G(y_l, y_l, b, t_l, t_l, 0) = \dfrac{f(y_l, t_l)}{(1 + y_l)^n}$

SoftSERVE

[GB, Rahn, Talbert 18,20]

Customised C++ program for numerical evaluation of soft functions

- uses Divonne integrator from Cuba library
- phase-space remappings to improve numerical convergence
- supports multi-precision variables (boost, GMP/MPFR)
- bash scripts for renormalisation in Laplace and cumulant space



Selected results

e^+e^- event-shape variables

- Thrust
  [Kelley et al 11; Monni et al 11]
- C-parameter
  [Hoang et al 14]
- Recoil-free broadening
  [Becher, GB 12]
- Angularities
  [—]
- Hemisphere masses
  [Kelley et al 11; Hornig al 11]

known analytically
known numerically
new result

hadron collider observables

- Threshold Drell-Yan
  [Belitsky 98]
- W at large p_T
  [Becher et al 12]
- p_T resummation
  [Becher, Neubert 10]
- p_T jet veto
  [Banfi et al 12; Becher et al 13; Stewart et al 13]
- Rapidity dependent jet vetoes
  [Gangal et al 16]
- Soft-drop jet groomer
  [—]
- Transverse thrust
  [Becher at al 15]

Performance

C-parameter	$c_2^{C_A}$	$c_2^{n_f}$	runtime*
standard setting	-57.893 ± 0.039	43.817 ± 0.004	25 sec
precision setting	-57.973 ± 0.004	43.818 ± 0.001	20 min
EVENT2	-58.16 ± 0.26	43.74 ± 0.06	[Hoang et al 14]

* on a single 8-core machine

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analytic	-57.976	43.818	[GB et al 18]

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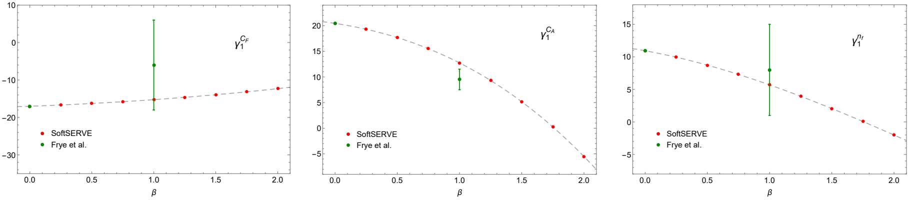
W at large p_T	$c_2^{C_A}$	$c_2^{n_f}$	runtime*
standard setting	-2.660 ± 0.075	-25.313 ± 0.009	30 sec
precision setting	-2.651 ± 0.005	-25.307 ± 0.001	9 h
analytic	-2.650	-25.307	[Becher et al 12]

* on a single 8-core machine

Soft-drop jet mass

Jet grooming removes soft radiation from jets [Frye, Larkoski, Schwartz, Yan 16]

- parameter β controls aggressiveness of groomer
- observable violates non-abelian exponentiation
- confirm and extend existing NNLO results



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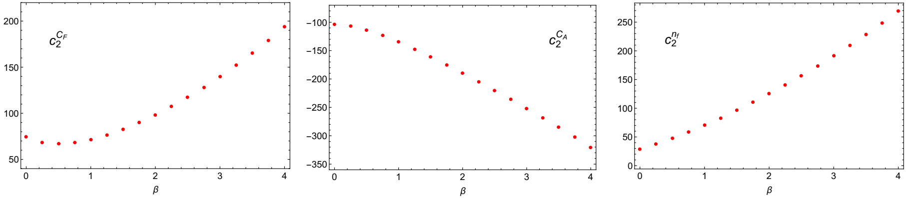


$\Rightarrow$ ingredients allowed for N^{3}LL resummation [Kardos, Larkoski, Trócsányi 20]

N-jet soft functions

[GB, Dehnadi, Mohrmann,Rahn wip]

Definition

$$S(\tau,\mu) = \sum_{i\in X} \mathcal{M}(\tau;\{k_i\})\, \langle 0|(S_{n_1} S_{n_2} S_{n_3} \dots)^\dagger |X\rangle\langle X| S_{n_1} S_{n_2} S_{n_3} \dots |0\rangle$$

- soft Wilson lines S_{n_i} with $n_i^2 = 0$, non-back-to-back
- soft function is a matrix in colour space
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- current working assumptions: SCET-1 and NAE

Main complications

- angular parametrisations more complicated for non-back-to-back dipoles
- 3-particle real-virtual contribution (requires at least four hard partons)
- renormalisation in colour space

N-jettiness

Definition [Stewart, Tackmann, Waalewijn 10]

$$\mathcal{T}_N = \sum_m \min_i \{n_i \cdot k_m\} \qquad i \in \{\underbrace{1, 2}_{\text{beams}}, \underbrace{3, \dots, N+2}_{\text{jets}}\}$$

- slicing variable for fixed-order perturbative computations [Boughezal et al 15; Gaunt et al 15]
- resolution variable in `Geneva` Monte-Carlo framework [Alioli et al 15]
- jet substructure studies [Thaler, van Tilburg 10]

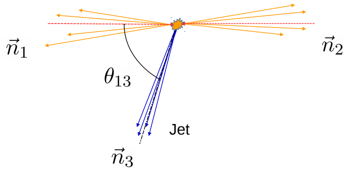
Status

- 1-jettiness NNLO soft function known [Boughezal et al 15; Campbell et al 17]
- 2-jettiness NNLO soft function known for certain kinematic configurations [GB et al 18; Jin, Liu 19]
- general case $N \geq 3$ only known to NLO [Jouttenus, Stewart, Tackmann, Waalewijn 11]

$\Rightarrow$ devise a method for the NNLO N-jettiness soft function for arbitrary N

1-jettiness

Kinematics

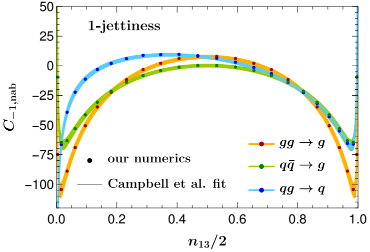


$$n_{12} \equiv n_1 \cdot n_2 = 2$$

$$n_{13} \equiv n_1 \cdot n_3 = 1 - \cos\theta_{13}$$

$$n_{23} \equiv n_2 \cdot n_3 = 1 + \cos\theta_{13}$$

Two-loop matching correction in distribution space

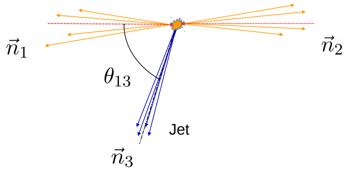


- overall very good agreement
- Campbell et al do not provide correlations

  $\Rightarrow$ inflates theory uncertainties
- logarithmic growth at endpoints?

1-jettiness

Kinematics



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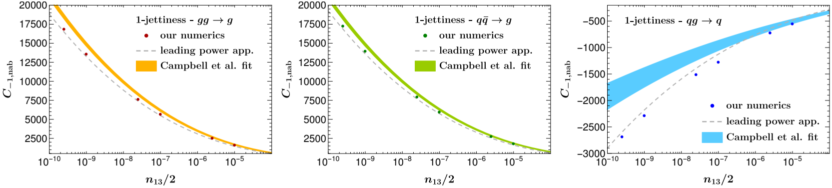
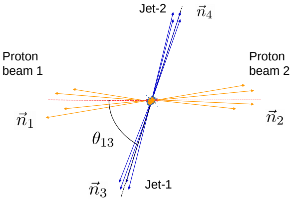
Two-loop matching correction in distribution space



2-jettiness

We sampled the phase space of the hard emitters in terms of $\sim 35,000$ points

$\Rightarrow$ simplified kinematics for illustration

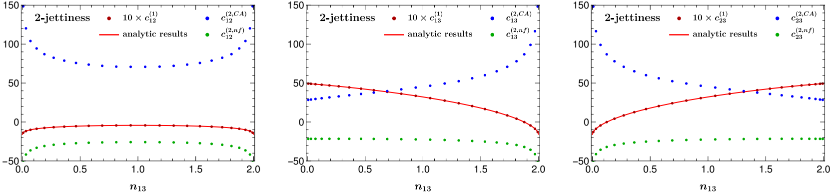


$$n_{12} \equiv n_1 \cdot n_2 = n_3 \cdot n_4 = 2$$

$$n_{13} \equiv n_1 \cdot n_3 = n_2 \cdot n_4 = 1 - \cos\theta_{13}$$

$$n_{14} \equiv n_1 \cdot n_4 = n_2 \cdot n_3 = 1 + \cos\theta_{13}$$

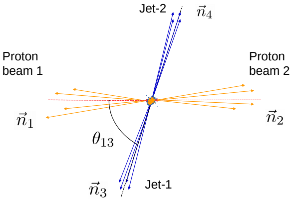
Matching corrections in Laplace space



2-jettiness

We sampled the phase space of the hard emitters in terms of $\sim 35,000$ points

$\Rightarrow$ simplified kinematics for illustration



$$n_{12} \equiv n_1 \cdot n_2 = n_3 \cdot n_4 = 2$$

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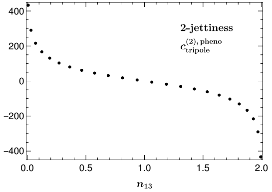
$$n_{14} \equiv n_1 \cdot n_4 = n_2 \cdot n_3 = 1 + \cos\theta_{13}$$

Tripole contribution



$$S(\tau,\mu) = 2\pi \sum_{i\neq j\neq k} f_{ABC}\, T_i^A T_j^B T_k^C \left(\frac{\lambda_{ij}\Gamma_0}{n} \left[\frac{\Gamma_0}{3} L^3 + (\gamma_{jk}^{(0)} + \Gamma_0 \tilde{L}_{jk}) L^2 + \left(\Gamma_0 \tilde{L}_{jk}^2 + 2\gamma_{jk}^{(0)} \tilde{L}_{jk} + n\, c_{jk}^{(1)} \right) L \right] + c_{ijk}^{(2,\mathrm{pheno})} \right) + \ldots$$

$$\sum_{i\neq j\neq k} f_{ABC}\, T_i^A T_j^B T_k^C\, c_{ijk}^{(2,\mathrm{pheno})} = f_{ABC}\, T_1^A T_2^B T_3^C \cdot c_{\mathrm{tripole}}^{(2),\mathrm{pheno}}$$

$$L = \ln \mu\bar{\tau},\; \tilde{L}_{jk} = \frac{n}{2} \ln \frac{n_{jk}}{2}$$

Massive particle production

[GB, Broggio, Dehnadi, Lim, Rahn wip]

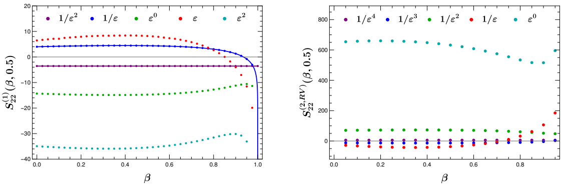
Definition

$$S(\tau,\mu) \;=\; \sum_{i\in X} \mathcal{M}(\tau;\{k_i\})\;\langle 0|(S_{n_1}S_{n_2}S_{v_1}S_{v_2})^\dagger|X\rangle\langle X|S_{n_1}S_{n_2}S_{v_1}S_{v_2}|0\rangle$$

- light-like Wilson lines S_{n_i} with $n_i^2=0$ and time-like Wilson lines S_{v_i} with $v_i^2=1$
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- soft matrix elements are more complicated but less singular

  $\Rightarrow$ numerical approach is ideally suited for this purpose

0-jettiness soft function for $t\bar{t}$ production



OUTLINE

SOFT FUNCTIONS

DIJET SOFT FUNCTIONS

N-JET EXTENSION

MASSIVE PARTICLE PRODUCTION

JET FUNCTIONS

BEAM FUNCTIONS

Jet functions

[GB, Brune, Das, Wald 21+wip]

Definitions

$$\left[\frac{\not{n}}{2}\right]_{\beta\alpha} J_q(\tau,\mu) = \frac{1}{\pi}\sum_{i\in X}(2\pi)^d\,\delta\Big(Q-\sum_i k_i^-\Big)\,\delta^{(d-2)}\Big(\sum_i k_i^\perp\Big)\,\langle 0|\chi_\beta|X\rangle\,\langle X|\bar\chi_\alpha|0\rangle\,\mathcal{M}(\tau;\{k_i\})$$

$$-g_\perp^{\mu\nu} g_s^2\, J_g(\tau,\mu) = \frac{Q}{\pi}\sum_{i\in X}(2\pi)^d\,\delta\Big(Q-\sum_i k_i^-\Big)\,\delta^{(d-2)}\Big(\sum_i k_i^\perp\Big)\,\langle 0|\mathcal{A}_\perp^\mu|X\rangle\,\langle X|\mathcal{A}_\perp^\nu|0\rangle\,\mathcal{M}(\tau;\{k_i\})$$

- collinear field operators $\chi = W^\dagger\,\frac{\not{n}\not{\bar{n}}}{4}\psi\,,\ \mathcal{A}_\perp^\mu = W^\dagger\left(iD_\perp^\mu W\right)$
- phase-space constraints fix jet energy and jet axis
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$

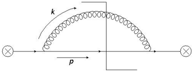
Main challenges

- three-particle phase space
- highly non-trivial jet-axis constraint

Technical aspects

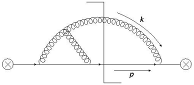
Jet functions in a nutshell

- matrix elements are given by splitting functions



NLO: $P^{(0)}_{q^*\to gq}$ NNLO-RV: $P^{(1)}_{q^*\to gq}$ NNLO-RR: $P^{(0)}_{q^*\to ggq}$

- ansatz for measurement function

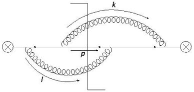


$$\mathcal{M}_1(\tau;k) = \exp\left\{ -\tau\, k_T \left(\frac{k_T}{z_k Q}\right)^n f(z_k,t_k)\right\} \qquad z_k = \frac{k_-}{Q}$$

- complicated singularity structure

$\Rightarrow$ sector decomposition, selector functions, non-linear transformations

$\Rightarrow$ calculation splits into $\mathcal{O}(100)$ contributions



- overlapping zeroes in measurement function
- two independent `pySecDec` implementations [Borowka et al 17]

Results

SCET-1

- Thrust
  [Becher, Neubert 06; Becher, GB 10]
- Angularities
  [GB, Hornig, Lee, Talbert 18]
- Transverse thrust
  [—]

SCET-2

- winner-take-all-axis broadening
  [—]

known analytically
known numerically
new result

Results

SCET-1

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  [GB, Hornig, Lee, Talbert 18]
- Transverse thrust
  [—]

SCET-2

- winner-take-all-axis broadening
  [—]

known analytically
known numerically
new result

Thrust: quark jet function

	$c_2^{n_f}$	$c_2^{C_F}$	$c_2^{C_A}$
this work	-10.785 ± 0.009	4.663 ± 0.120	-2.092 ± 0.135
analytic	-10.787	4.655	-2.165

Results

SCET-1

- Thrust
  [Becher, Neubert 06; Becher, GB 10]
- Angularities
  [GB, Hornig, Lee, Talbert 18]
- Transverse thrust
  [—]

SCET-2

- winner-take-all-axis broadening
  [—]

known analytically

known numerically

new result

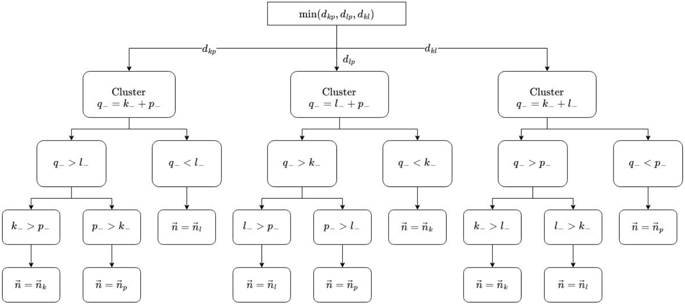
Thrust: gluon jet function

	$c_2^{n_f^2}$	$c_2^{C_F n_f}$	$c_2^{C_A n_f}$	$c_2^{C_A^2}$
this work	2.014 ± 0.001	0.904 ± 0.050	-13.727 ± 0.069	3.195 ± 0.168
analytic	2.014	0.900	-13.725	3.197

Winner-take-all-axis broadening

Measure broadening with respect to a recoil-free jet axis [Bertolini, Chan, Thaler 13]

- SCET-2 observable
- jet axis is determined in a pairwise recombination algorithm
- k_T-type distance measure $d_{ij} = 2\min(E_i^2, E_j^2)\,(1-\cos\theta_{ij})$



Winner-take-all-axis broadening

Measure broadening with respect to a recoil-free jet axis [Bertolini, Chan, Thaler 13]

- SCET-2 observable
- jet axis is determined in a pairwise recombination algorithm
- k_T-type distance measure $d_{ij} = 2\min(E_i^2, E_j^2)\,(1-\cos\theta_{ij})$

quark jet	$c_2^{n_f}$	$c_2^{C_F}$	$c_2^{C_A}$
this work	62.958 ± 0.003	511.09 ± 0.30	-240.44 ± 0.27

gluon jet	$c_2^{n_f^2}$	$c_2^{C_F n_f}$	$c_2^{C_A n_f}$	$c_2^{C_A^2}$
this work	17.435 ± 0.001	-85.222 ± 0.058	135.97 ± 0.02	293.96 ± 0.07

OUTLINE

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MASSIVE PARTICLE PRODUCTION

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BEAM FUNCTIONS

Beam functions

[GB, Brune, Das, Wald 22+wip]

Definitions

$$\left[\frac{\not{n}}{4}\right]_{\beta\alpha} \mathcal{B}_{q/h}(x,\tau,\mu) = \sum_{i\in X} \delta\Big((1-x)P^- - \sum_i k_i^-\Big) \,\langle\, h(P)\,|\bar\chi_\alpha|X\rangle\,\langle X|\chi_\beta|\,h(P)\,\rangle\,\mathcal{M}(\tau;\{k_i\})$$

$$-g_s^2\,\mathcal{B}_{g/h}(x,\tau,\mu) = \frac{x\,P^-}{2\pi}\sum_{i\in X} \delta\Big((1-x)P^- - \sum_i k_i^-\Big)\,\langle\, h(P)\,|\mathcal{A}_\perp^\mu|X\rangle\,\langle X|\mathcal{A}_{\perp,\mu}|\,h(P)\,\rangle\,\mathcal{M}(\tau;\{k_i\})$$

- hadronic matrix elements of collinear field operators
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$

As long as $\tau \ll 1/\Lambda_{\rm QCD}$ beam functions can be matched onto pdfs

$$\mathcal{B}_{i/h}(x,\tau,\mu) = \sum_k \int_x^1 \frac{dz}{z}\,\mathcal{I}_{i\leftarrow k}\Big(\frac{x}{z},\tau,\mu\Big)\,f_{k/h}(z,\mu)$$

⇒ resolve convolutions by additional Mellin transform

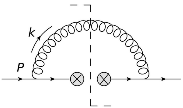
⇒ determine matching kernels $\hat{\mathcal{I}}_{i\leftarrow k}(N,\tau,\mu)$ in moment space

Technical aspects

Beam functions in a nutshell

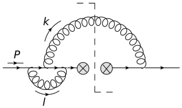
- matrix elements are given by crossed splitting functions

  NLO: $P^{(0)}_{q\to gq^*}$ NNLO-RV: $P^{(1)}_{q\to gq^*}$ NNLO-RR: $P^{(0)}_{q\to ggq^*}$



- ansatz for measurement function

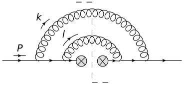
$$\mathcal{M}_1(\tau;k) = \exp\left\{ -\,\tau\, k_T \left(\frac{k_T}{(1-x)P^-} \right)^n f(t_k) \right\}$$



- unconstrained two-particle phase space

  $\Rightarrow$ similar but simpler singularity structure

  $\Rightarrow$ calculation splits into $\mathcal{O}(50)$ contributions



- overlapping zeroes in measurement function
- two independent `pySecDec` implementations

[Borowka et al 17]

Results

SCET-1

- N-jettiness
  [Gaunt, Stahlhofen, Tackmann 14]

known analytically

known numerically

new result

SCET-2

- p_T resummation
  [Gehrmann, Lübbert, Yang 14]
- p_T veto
  [—]
- Transverse thrust
  [—]

Two-loop matching for quark beam function

$$\hat{I}^{(2)}_{q\leftarrow q}(N) = C_F^2\,\hat{I}^{(2,C_F)}_{q\leftarrow q}(N) + C_F C_A\,\hat{I}^{(2,C_A)}_{q\leftarrow q}(N) + C_F T_F n_f\,\hat{I}^{(2,n_f)}_{q\leftarrow q}(N) + C_F T_F\,\hat{I}^{(2,T_F)}_{q\leftarrow q}(N)$$

$$\hat{I}^{(2)}_{q\leftarrow g}(N) = C_F T_F\,\hat{I}^{(2,C_F)}_{q\leftarrow g}(N) + C_A T_F\,\hat{I}^{(2,C_A)}_{q\leftarrow g}(N)$$

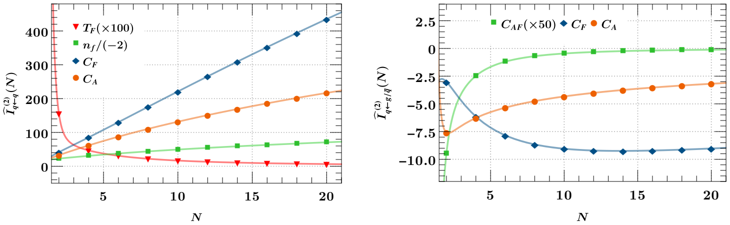
$$\hat{I}^{(2)}_{q\leftarrow \bar{q}}(N) = C_F(C_A - 2C_F)\,\hat{I}^{(2,C_{AF})}_{q\leftarrow \bar{q}}(N) + C_F T_F\,\hat{I}^{(2,T_F)}_{q\leftarrow q}(N)$$

$$\hat{I}^{(2)}_{q\leftarrow q'}(N) = \hat{I}^{(2)}_{q\leftarrow \bar{q}'}(N) = C_F T_F\,\hat{I}^{(2,T_F)}_{q\leftarrow q}(N)$$

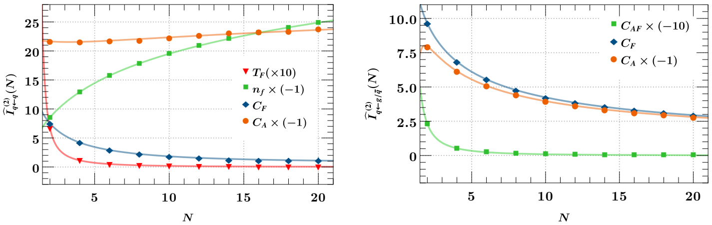
$\Rightarrow$ seven independent matching kernels

Validity checks

N-jettiness

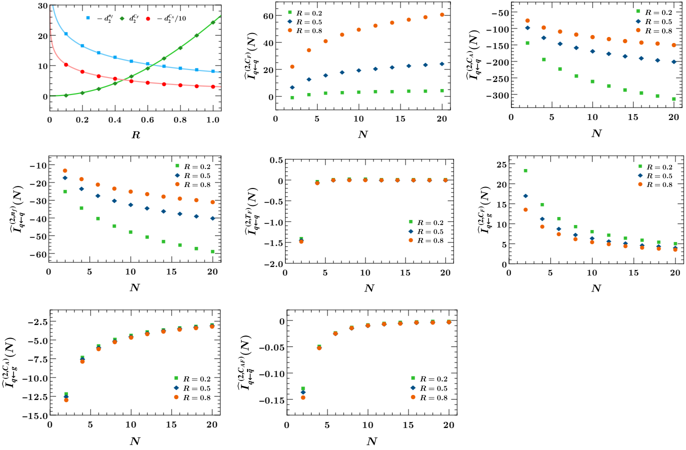


p_T resummation



Jet veto

Matching kernels depend on jet radius R



- independent calculation in momentum space

[Abreu, Gaunt, Monni, Rottoli, Szafron 22]

Conclusions

Numerical framework for automated computation of SCET ingredients

- public code for dijet soft functions
- extension to higher particle multiplicities and massive partons
- first results for quark and gluon jet functions
- beam-function calculations currently limited to moment space

Outlook

- development of `JetSERVE` and `BeamSERVE`
- massive jet functions? fragmenting jet functions? NLP ingredients?
- automated resummations

Backup slides

SCET-1 soft functions

Multiplicative renormalisation in Laplace space

$$\frac{d\,S(\tau,\mu)}{d\ln\mu} = -\frac{1}{n}\left[4\,\Gamma_{\text{cusp}}(\alpha_s)\,\ln(\mu\bar{\tau}) - 2\gamma^S(\alpha_s)\right] S(\tau,\mu)$$

Two-loop solution with $L = \ln(\mu\bar{\tau})$

$$S(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left\{-\frac{2\Gamma_0}{n}L^2 + \frac{2\gamma_0^S}{n}L + c_1^S\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{2\Gamma_0^2}{n^2}L^4 - 4\Gamma_0\left(\frac{\gamma_0^S}{n^2} + \frac{\beta_0}{3n}\right)L^3\right.$$
$$\left. -2\left(\frac{\Gamma_1}{n} - \frac{(\gamma_0^S)^2}{n^2} - \frac{\beta_0\gamma_0^S}{n} + \frac{\Gamma_0 c_1^S}{n}\right)L^2 + 2\left(\frac{\gamma_1^S}{n} + \frac{\gamma_0^S c_1^S}{n} + \beta_0 c_1^S\right)L + c_2^S\right\}$$

Results will be presented in the form

$$\gamma_1^S = \gamma_1^{C_A}\,C_F C_A + \gamma_1^{n_f}\,C_F T_F n_f + \gamma_1^{C_F}\,C_F^2$$

$$c_2^S = c_2^{C_A}\,C_F C_A + c_2^{n_f}\,C_F T_F n_f + c_2^{C_F}\,C_F^2$$

C-parameter

1) Derive measurement functions

$$n = 1 \qquad F_A(a,b,y) = \frac{ab}{a(a+b)+(1+ab)y} + \frac{a}{a+b+a(1+ab)y} \qquad F_B(a,b,y) = F_A(1/a,b,y)$$

2) To apply numerical improvement techniques, `SoftSERVE` needs another parameter

$$F_A(a,b,y) \xrightarrow{y\to 0} c_0 + c_1\, y^m + \ldots \qquad \Rightarrow \quad m = 1$$

3) Set up input files

- `Input_Common.cpp` $\Rightarrow$ n, m
- `Input_Measurement_Correlated.cpp` $\Rightarrow$ $F_A(a,b,y),\ F_B(a,b,y)$
- `Input_Integrator.cpp` $\Rightarrow$ Divonne settings
- `Input_Parameters.h` $\Rightarrow$ user-specified parameters

C-parameter

4) Compile and run `./execsftsrvNAE -o Cparameter`

```
Terminal
Datei   Bearbeiten   Ansicht   Suchen   Terminal   Hilfe
NLO result
eps^-2 coefficient: -4  +/-  -2.13956*10^-11, probability of incorrect error estimate: 0
eps^-1 coefficient: 5.33555*10^-09  +/-  3.26761*10^-06, probability of incorrect error estimate: 0
eps^0 coefficient: -3.28987  +/-  2.45348*10^-05, probability of incorrect error estimate: 0
eps^1 coefficient: -20.8357  +/-  0.000194464, probability of incorrect error estimate: 0
eps^2 coefficient: -2.43523  +/-  0.000751913, probability of incorrect error estimate: 0

Cf Ca structure
eps^-4 coefficient: 0 +/- .00000001319166403453, probability of incorrect error estimate: 0
eps^-3 coefficient: -3.66665000266778000000 +/- .00016311518256774873, probability of incorrect error estimate: .00000288631000000000
eps^-2 coefficient: -4.15465 +/- .00085497368857166592, probability of incorrect error estimate: .00000297784000000000
eps^-1 coefficient: -4.1653 +/- .00576508967850076032, probability of incorrect error estimate: .00000297784000000000
eps^0 coefficient: -134.29081 +/- .03918343739337961656, probability of incorrect error estimate: .00000297784000000000
Cf Tf nf structure
eps^-3 coefficient: 1.33333  +/-  -5.77909*10^-06, probability of incorrect error estimate: 0
eps^-2 coefficient: 2.22223  +/-  0.000101468, probability of incorrect error estimate: 1.11022*10^-16
eps^-1 coefficient: 6.34143  +/-  0.000691237, probability of incorrect error estimate: 9.39956*10^-06
eps^0 coefficient: 71.5982  +/-  0.00396634, probability of incorrect error estimate: 9.40345*10^-06
~
                                                                              7,0-1        Alles
```

C-parameter

5) Renormalise with `./laprenormNAE -o Cparameter -n 1`

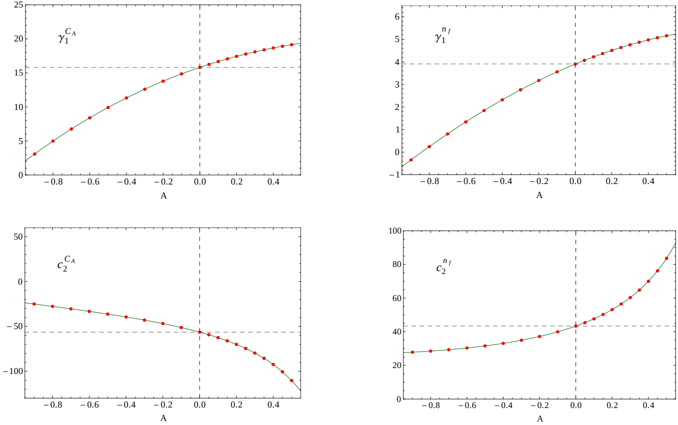
Terminal

Datei Bearbeiten Ansicht Suchen Terminal Hilfe

```
Result for Laplace space renormalised soft function for Cparameter (non-abelian exponentiation assumed)

Anomalous dimension, Ca part: 15.795
Error estimate for Ca part: 0.012
Anomalous dimension, Nf part: 3.910
Error estimate for Nf part: 0.001

Finite part, Ca structure: -57.893
Error estimate for Ca part: 0.039
Finite part, Nf structure: 43.817
Error estimate for Nf part: 0.004


Pole cancellation check values, these must be compatible with zero:
One loop: 0.000 +- -0.000000
CF CA, leading: 0.000 +- 0.000000
CF CA, first subleading: 0.000 +- 0.000163
CF CA, second subleading: -0.000 +- 0.000855
CF TF nf, leading: -0.000 +- -0.000006
CF TF nf, first subleading: 0.000 +- 0.000101
~
                                                                        13,0-1        Alles
```

Angularities

e^+e^- event shape that interpolates between thrust ($A = 0$) and broadening ($A = 1$)



$\Rightarrow$ last missing ingredient for NNLL resummation

[GB, Hornig, Lee, Talbert 18]

NNLO subtractions

Double-soft integral for nested soft-collinear subtractions [Caola, Delto, Frellesvig, Melnikov 18]