Analytic two-loop soft and beam functions for leading-jet p_T

Samuel Abreu
CERN & The University of Edinburgh

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Together with J. Gaunt, P. Monni, L. Rottoli and R. Szafron

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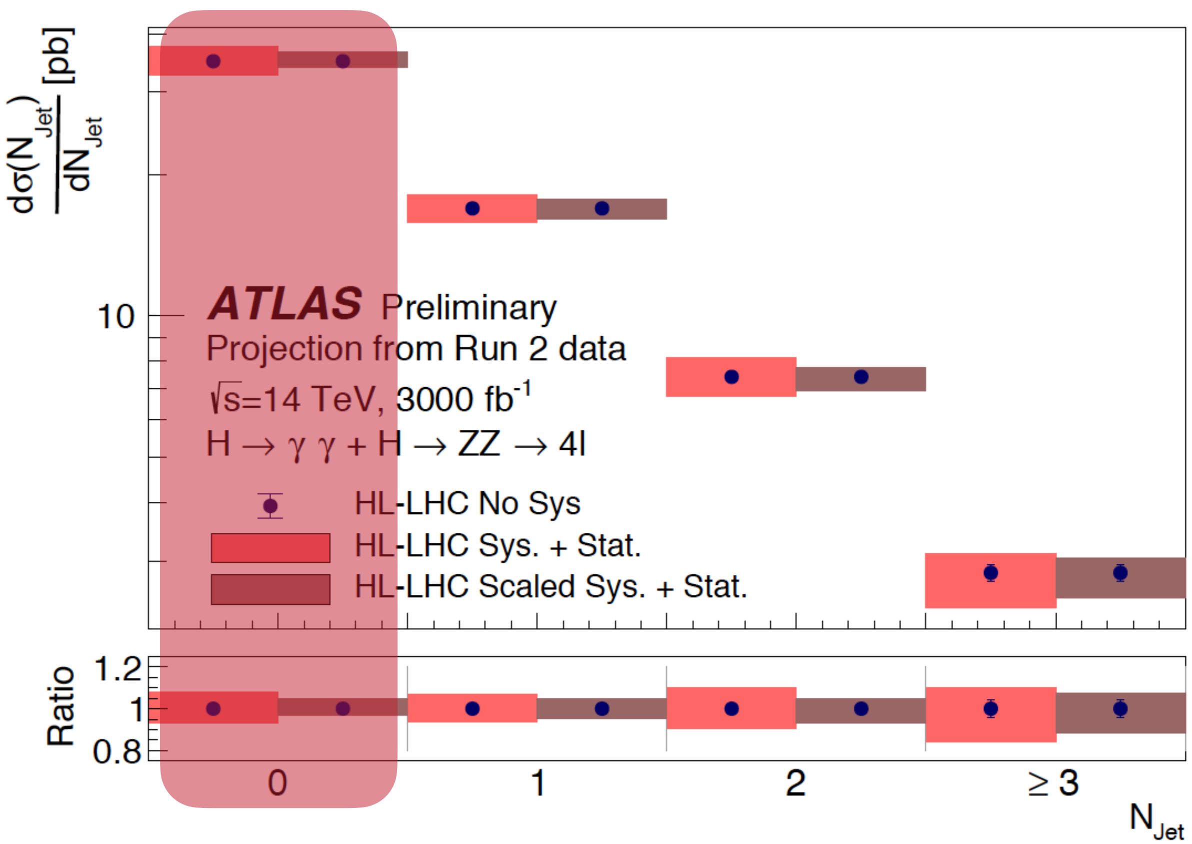
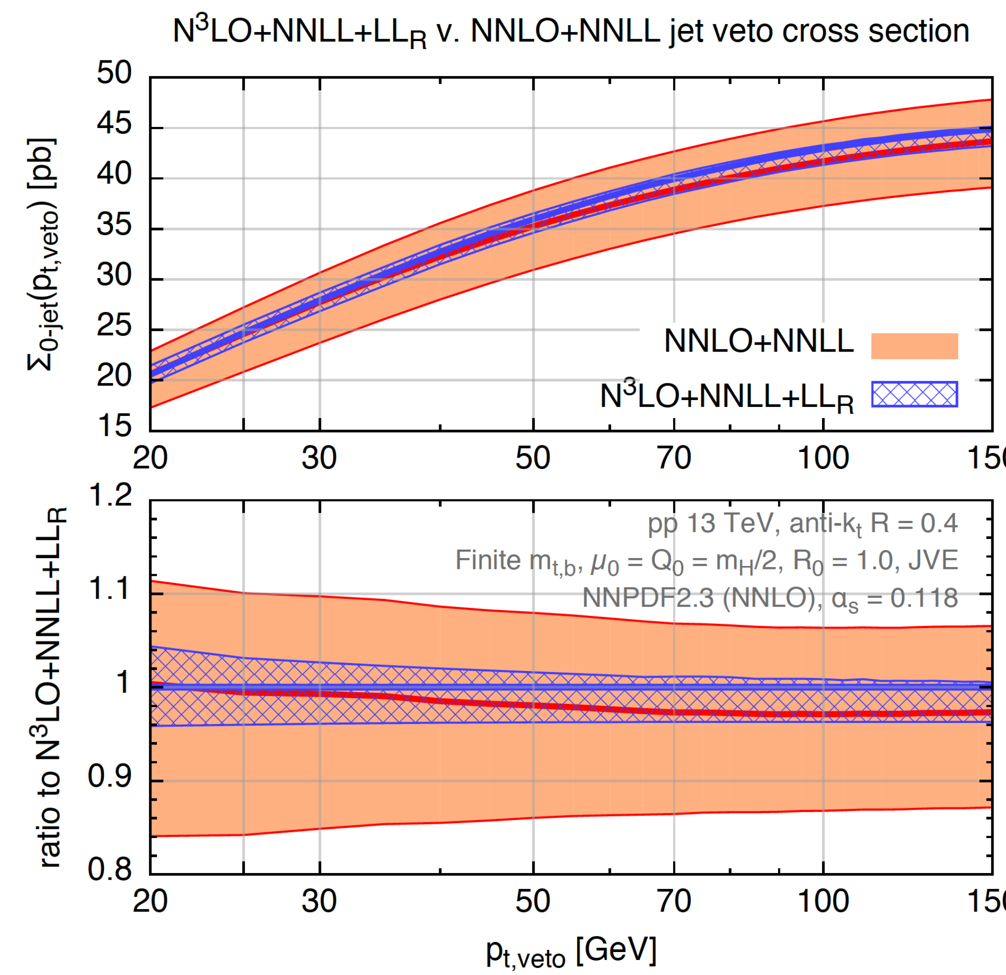
Motivation

- Jet-vetoed cross sections: reduce backgrounds in Higgs analyses
  - ✓ e.g. $H \to WW$ vs $t\bar{t}$



Formalism for resummation
[Banfi, Monni, Salam, Zanderighi 12;
Becher, Neubert 12;
Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13]

Higgs jet-vetoed cross section



State of the art: N^{3}LO+NNLL [Banfi et al 16]

Goal: improve resummation to N^{3}LL

Cross-section factorisation in SCET

- Colour-singlet production with jet veto $p_T^{\text{jet}} < p_T^{\text{veto}}$

$$\frac{d\sigma(p_T^{\text{veto}})}{d\Phi_{\text{Born}}} \longrightarrow \text{logarithms} \quad \log(p_T^{\text{veto}}/Q)$$

- In the limit $p_T^{\text{veto}} \ll Q$

[Becher, Neubert 12;
Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13]

$$\frac{d\sigma(p_T^{\text{veto}})}{d\Phi_{\text{Born}}} = |A_{\text{Born}}^F|^2\, \mathscr{H}(Q;\mu)\, \mathscr{B}_n(x_1, Q, p_T^{\text{veto}}, R^2;\mu,\nu)\, \mathscr{B}_{\bar{n}}(x_2, Q, p_T^{\text{veto}}, R^2;\mu,\nu)\, \mathscr{S}(p_T^{\text{veto}}, R^2;\mu,\nu)$$

- ✓ Hard function: $\mathscr{H}(Q;\mu)$
- ✓ Beam functions: $\mathscr{B}_{n,\bar{n}}(x, Q, p_T^{\text{veto}}, R^2;\mu,\nu)$
- ✓ Soft function: $\mathscr{S}(p_T^{\text{veto}}, R^2;\mu,\nu)$
- ✓ Valid up to power corrections: $\mathcal{O}\left((p_T^{\text{veto}}/Q)^r\right)$

Evolution equations

[Becher, Neubert, Rothen 13; Stewart, Tackmann, Walsh, Zuberi 13]

- Hard function $\mathscr{H}(Q;\mu) = |\mathscr{C}(Q;\mu)|^2$

$$\frac{d}{d\ln\mu}\ln\mathscr{C}(Q;\mu) = \Gamma_{\text{cusp}}(\alpha_s(\mu))\ln\frac{-Q^2}{\mu^2} + \gamma_H(\alpha_s(\mu))$$

- Soft function

$$\frac{d}{d\ln\mu}\ln\mathscr{S}(p_T^{\text{veto}}, R^2;\mu,\nu) = 4\,\Gamma_{\text{cusp}}(\alpha_s(\mu))\ln\frac{\mu}{\nu} + \gamma_S(\alpha_s(\mu))$$

$$\frac{d}{d\ln\nu}\ln\mathscr{S}(p_T^{\text{veto}}, R^2;\mu,\nu) = -4\int_{p_T^{\text{veto}}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu')) + \gamma_\nu(p_T^{\text{veto}}, R^2)$$

- Beam functions

$$\frac{d}{d\ln\mu}\ln\mathscr{B}_n(x, Q, p_T^{\text{veto}}, R^2;\mu,\nu) = 2\,\Gamma_{\text{cusp}}(\alpha_s(\mu))\ln\frac{\nu}{Q} + \gamma_B(\alpha_s(\mu))$$

$$\frac{d}{d\ln\nu}\ln\mathscr{B}_n(x, Q, p_T^{\text{veto}}, R^2;\mu,\nu) = 2\int_{p_T^{\text{veto}}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu')) - \frac{1}{2}\gamma_\nu(p_T^{\text{veto}}, R^2)$$

Missing for N^3LL: two-loop $\mathscr{S}$ and $\mathscr{B}_n$, three-loop γ_ν

SOFT FUNCTION

CALCULATION AND RESULTS

The soft function

- Operator definition:

$$\boxed{\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu) = \frac{1}{d_F} \sum_{X_s} \text{Tr} \left\{ \mathcal{M}(p_T^{\text{veto}}, R^2) \langle 0 | Y_n^\dagger Y_{\bar{n}} | X_s \rangle \langle X_s | Y_{\bar{n}}^\dagger Y_n | 0 \rangle \right\}}$$

  - ✓ Soft Wilson lines: $Y_{n,\bar{n}}$
  - ✓ Measurement function: $\mathcal{M}(p_T^{\text{veto}}, R^2)$

$$\mathcal{M}(p_T^{\text{veto}}, R^2) = \Theta(p_T^{\text{veto}} - \max\{p_T^{\text{jet}_i}\})\Theta_{\text{cluster}}(R^2)$$

- Regularisation of divergences:
  - ✓ UV/IR/Coll. divergences: dimensional regularisation
  - ✓ Rapidity divergences: exponential regulator

- Exponential regulator: modify phase-space integration measure

$$\boxed{\prod_i d^d k_i \delta(k_i^2)\theta(k_i^0) \to \prod_i d^d k_i \delta(k_i^2)\theta(k_i^0) \exp\left[\frac{-e^{-\gamma_E}}{\nu}(n \cdot k_i + \bar{n} \cdot k_i)\right]} \qquad \boxed{\nu \to \infty}$$

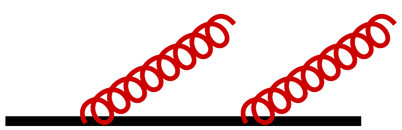
[Li, Neill, Zhu 16]

Soft function decomposition

- **Reference observable**:

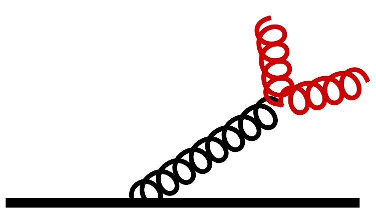
$$\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu) = \mathcal{S}_\perp(p_T^{\text{veto}}, \mu, \nu) + \Delta\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$$

[e. g. : Banfi, Salam, Zanderighi 12
Gangal, Gaunt, Stahlhofen, Tackmann 16
Bauer, Manohar, Monni 20]



- ✓ $\mathcal{S}_\perp(p_T^{\text{veto}}, \mu, \nu)$: soft function for p_T resummation

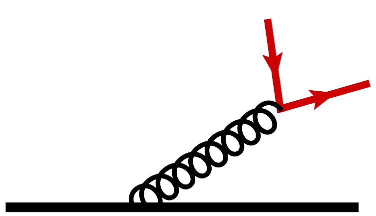
[with the same regulator:
Li, Neill, Zhu 16; Li, Zhu 16]



- ✓ $\Delta\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$: remainder

$$\Delta\mathcal{M}(p_T^{\text{veto}}, R^2) \equiv \Theta(p_T^{\text{veto}} - \max\{p_T^{\text{jet}_i}\})\Theta_{\text{cluster}}(R^2) - \Theta\left(p_T^{\text{veto}} - \left|\sum_{X_s} p_T^{\text{jet}_i}\right|\right)$$

- ✓ Remainder: only rapidity divergences, work in four dimensions



- ✓ $\Delta\mathcal{S}^{(2)}(p_T^{\text{veto}}, R^2; \mu, \nu)$: double-real diagrams with two soft gluons or a soft quark-antiquark pair

[Campbell, Glover 98
Dokshitzer, Lucenti, Marchesini, Salam 97
Catani, Grazzini 99]

- Two-loop correlated and uncorrelated contributions

$$\Delta\mathcal{S}^{(2)}(p_T, R^2; \mu, \nu) = \Delta S^{\text{corr.}}(p_T, R^2; \mu, \nu) + \Delta S^{\text{uncorr.}}(p_T, R^2; \mu, \nu)$$

- ✓ $\Delta S^{\text{uncorr.}}(p_T, R^2; \mu, \nu)$: two emissions are widely separated in rapidity
- ✓ $\Delta S^{\text{corr.}}(p_T, R^2; \mu, \nu)$: $\to 0$ when two emissions are widely separated

Calculation: setup

- Momenta parametrisation:

$$k_i = k_{i\perp}\left(\cosh\eta_i\,,\cos\phi_i\,,\sin\phi_i, \sinh\eta_i\right), \quad i = 1{,}2$$

$$\{k_{2\perp}, \eta_2, \phi_2\} \to \{\zeta \equiv k_{2\perp}/k_{1\perp}, \eta \equiv \eta_1 - \eta_2, \phi \equiv \phi_1 - \phi_2\}$$

- Squared amplitudes:

$$\mathscr{A}^{\text{cor./uncor.}}(k_1, k_2) = \frac{1}{k_{1\perp}^4}\frac{1}{\zeta^2}\mathscr{D}^{\text{cor./uncor.}}(\zeta, \eta, \phi)$$

- Integrals to compute:

$$\boxed{\int \frac{dk_{1\perp}}{k_{1\perp}}\, d\eta_1\, \frac{d\zeta}{\zeta}\, d\eta\, \frac{d\phi}{2\pi}\, e^{-2k_{1\perp}\frac{e^{-\gamma_E}}{\nu}[\cosh(\eta_1)+\zeta\cosh(\eta-\eta_1)]}\, \mathscr{D}(\zeta, \eta, \phi)\, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)}$$

- Measurement function, after some manipulation

$$\boxed{\Delta\mathscr{M}(p_T^{\text{veto}}, R^2) \equiv \left[\Theta(p_T^{\text{veto}} - k_{1\perp}\max\{1,\zeta\}) - \Theta\left(p_T^{\text{veto}} - k_{1\perp}\sqrt{1+\zeta^2+2\zeta\cos\phi}\right)\right]\Theta(\eta^2+\phi^2-R^2)}$$

- Goal: $\Delta\mathcal{S}^{(2)}(p_T, R^2; \mu, \nu)$ as a series in powers of R^2

Calculation: correlated contribution

- Rapidity divergences:

$$\mathscr{D}^{\text{cor.}}(\zeta,\eta,\phi) \to 0 \text{ for } \eta = \eta_1 - \eta_2 \to \infty \quad \Rightarrow$$

Only η_1 integral needs rapidity regulation

- Integrate over η_1 (easy!), keep terms that survive when $\nu \to \infty$

$$I\left(p_T^{\text{veto}}/\nu, R^2\right) = \int \frac{dk_{1\perp}}{k_{1\perp}} \frac{d\zeta}{\zeta} d\eta \frac{d\phi}{2\pi} \Omega\left(\frac{k_{1\perp}}{\nu},\zeta,\eta\right) \mathscr{D}^{\text{cor.}}(\zeta,\eta,\phi)\, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)$$

$$\Omega\left(\frac{k_{1\perp}}{\nu},\zeta,\eta\right) = \eta + 2\ln\frac{\nu}{k_{1\perp}} - \ln\left(1+\zeta e^{\eta}\right) - \ln\left(\zeta + e^{\eta}\right)$$

- Measurement function: $\Theta\left(\eta^2 - R^2 + \phi^2\right) = \underbrace{\Theta\left(\phi^2 - R^2\right)}_{\text{part } A} + \underbrace{\Theta\left(R^2-\phi^2\right)\Theta\left(\eta^2 - R^2 + \phi^2\right)}_{\text{part } B}$

  - ✓ $I_A\left(p_T^{\text{veto}}/\nu, R^2\right)$: full R^2 dependance, expand in powers of R^2
  - ✓ $I_B\left(p_T^{\text{veto}}/\nu, R^2\right)$: regular at R^2=0
    - ➡ Compute $\frac{\partial}{\partial R^2} I_B$ order by order in R^2
    - ➡ Solve differential equation order by order in R^2, $I\left(p_T^{\text{veto}}/\nu, 0\right)$

[**HypExp**, Huber, Maitre 05
PolyLogTools, Duhr, Dulat 19]

Calculation: uncorrelated contribution

- Rapidity divergences: both on η and η_1!

$$\mathscr{D}^{\text{uncor.}}(\zeta,\eta,\phi) = 16C_R^2$$

$$\int \frac{dk_{1\perp}}{k_{1\perp}}\, d\eta_1\, \frac{d\zeta}{\zeta}\, d\eta\, \frac{d\phi}{2\pi}\, e^{-2k_{1\perp}\frac{e^{-\gamma_E}}{\nu}[\cosh(\eta_1)+\zeta\cosh(\eta-\eta_1)]}\, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)$$

- More subtle η and η_1 integration of exponential regulator
  - Set $w = e^{\eta}$, $x = e^{\eta_1}$, take Laplace transform
  - In Laplace space, expand exponential regulator in distributions
  - Take inverse Laplace transform, keep terms that survive when $\nu \to \infty$

$$\int \frac{dx}{x\, w} e^{-k_{1\perp}\frac{e^{-\gamma_E}}{\nu x}[1+w\zeta+\frac{x^2}{w}(w+\zeta)]} \to 4\delta(w)\ln\left(\frac{k_{1\perp}}{\nu}\right)\ln\left(\frac{\zeta\, k_{1\perp}}{\nu}\right) + \left[\frac{1}{w}\right]_+ \ln\left(\frac{\nu^2 w}{k_{1\perp}^2(w+\zeta)(1+\zeta w)}\right) + \mathscr{O}\left(\frac{1}{\nu^2}\right)$$

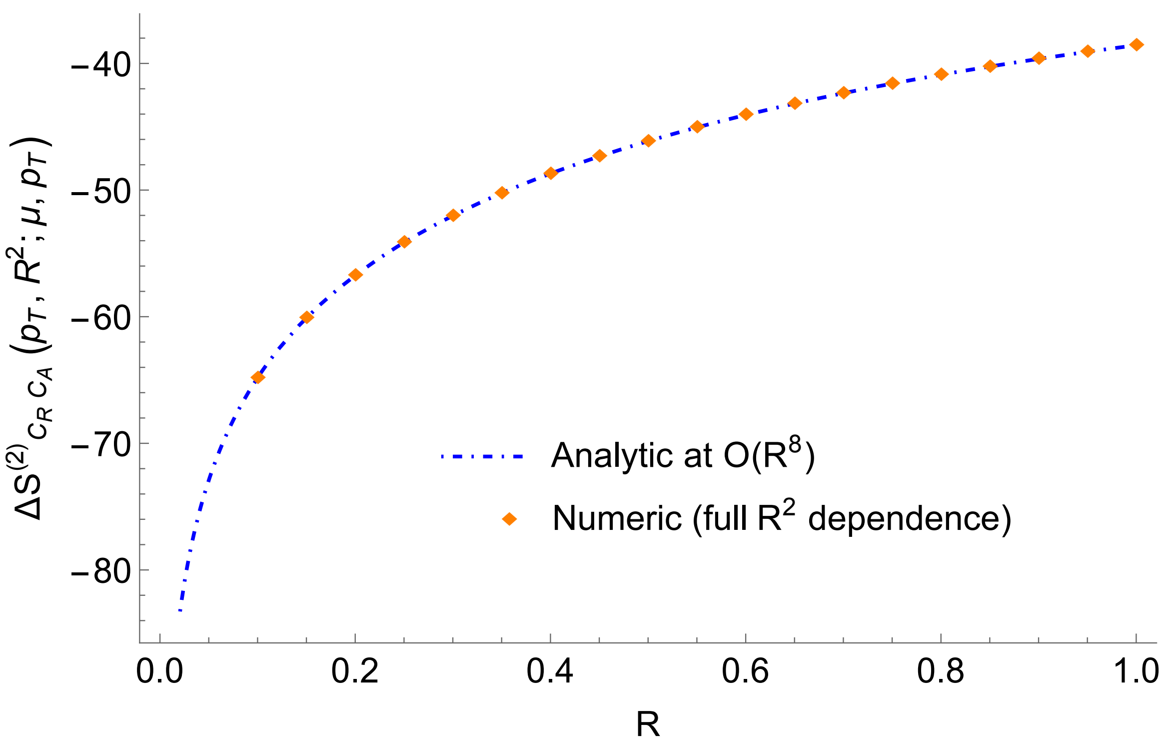
- Continue as for the correlated contributions for remaining integrals

Calculation: numerical calculation, full R dependence

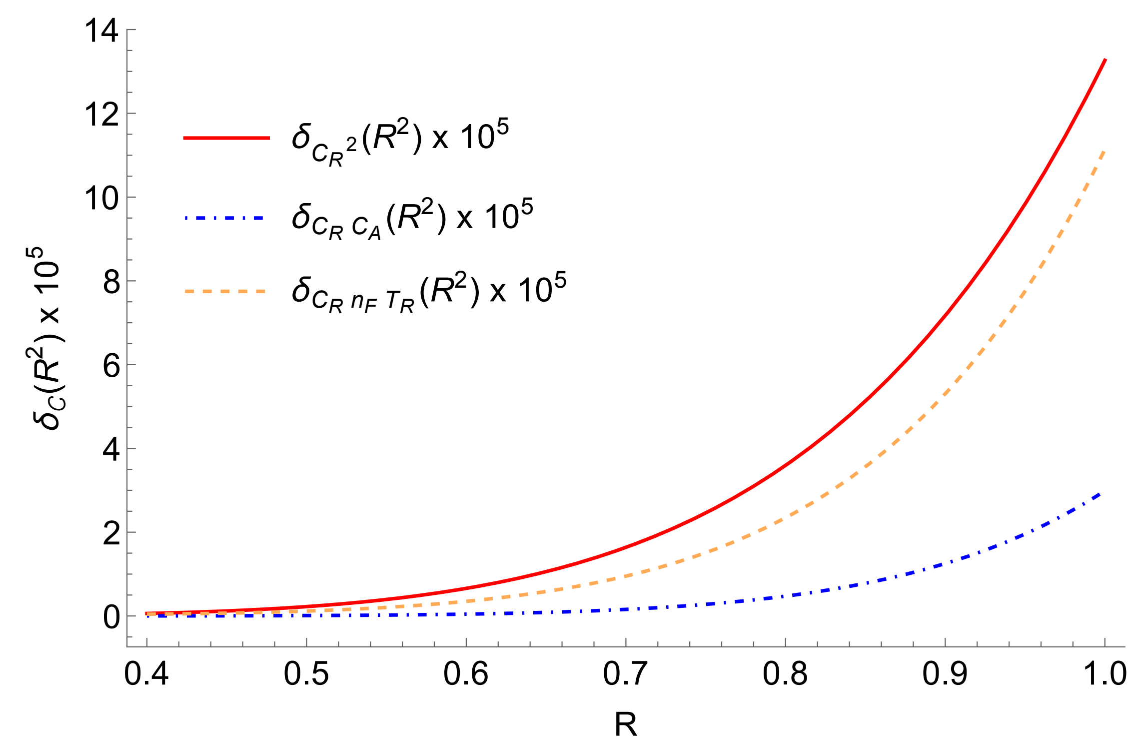
- Correlated contributions
  - Variables: $\phi, \ \eta, \ \eta_t = \frac{1}{2}(\eta_1 + \eta_2), \ z = \frac{k_{1\perp}^2}{k_{1\perp}^2 + k_{2\perp}^2}, \ \mathscr{K}_T^2 = k_{1\perp}^2 + k_{2\perp}^2$
  - Exponential regulator: as in analytic calculation
  - Analytic integrations: $\eta_t, \ \mathscr{K}_T^2$
  - Numerical integrations: $\eta, \ \phi, \ z$
- Uncorrelated corrections
  - Variables: $\phi, \ \eta_1, \ \eta_2, \ z = \frac{k_{1\perp}^2}{k_{1\perp}^2 + k_{2\perp}^2}, \ \mathscr{K}_T^2 = k_{1\perp}^2 + k_{2\perp}^2$
  - Exponential regulator: as in analytic calculation
  - Analytic integrations: $\eta_1, \ \eta_2, \ \mathscr{K}_T^2$
  - Numerical integrations: $\phi, \ z$
- Retain full R dependence, per mille precision in numerical evaluation

Results and checks

- Analytic results for $\Delta\mathcal{S}^{(2)}(p_T^{\rm veto}, R^2; \mu, \nu)$ to $\mathcal{O}(R^8)$ (also $\mathcal{S}_\perp^{(2)}(p_T^{\rm veto}, R^2; \mu, \nu)$)
  - ✓ see also numerical evaluation in different scheme [Bell, Rahn, Talbert 18,20]
- Reproduce known rapidity anomalous dimension [Banfi, Monni, Salam, Zanderighi 12; Becher, Neubert, Rothen 13; Stewart, Tackmann, Walsh, Zuberi 13]
- Verify suitability of R^2 expansion for $0 < R < 1$



Comparison with numerics, with full R^2 dependence



Check $\mathcal{O}(R^8)$ corrections are negligible

$$\delta_{\mathscr{C}}(R) = \left| 1 - \frac{\Delta\mathcal{S}^{(2)}_{\mathscr{C}}(p_T, R^2; \mu, p_T)|_{R^6}}{\Delta\mathcal{S}^{(2)}_{\mathscr{C}}(p_T, R^2; \mu, p_T)|_{R^8}} \right|$$

BEAM FUNCTION

CALCULATION AND RESULTS

The beam functions

[e. g. , Becher, Neubert 12;
Luo, Wang, Xu, Yang, Yang, Zhu 19;
Luo, Yang, Zhu, Zhu, 19]

- Operator definition:

$$\mathscr{B}_q(x,Q,p_T^{\text{veto}},R^2;\mu,\nu) = \frac{1}{2\pi}\sum_{X_C}\int dt e^{-ixt\bar{n}\cdot p}\mathscr{M}(p_T^{\text{veto}},R^2)\left\langle P(p)\left|\bar{\chi}_n(t\bar{n})\frac{\bar{n}\!\!\!/}{2}\right|X_C\right\rangle\left\langle X_C\left|\chi_n(0)\right|P(p)\right\rangle$$

$$\mathscr{B}_g(x,Q,p_T^{\text{veto}},R^2;\mu,\nu) = -\frac{x\bar{n}\cdot p}{2\pi}\sum_{X_C}\int dt e^{-ixt\bar{n}\cdot p}\mathscr{M}(p_T^{\text{veto}},R^2)\left\langle P(p)\left|\mathscr{A}_\perp^{\mu,a}(t\bar{n})\right|X_C\right\rangle\left\langle X_C\left|\mathscr{A}_{\perp,\mu}^{a}(0)\right|P(p)\right\rangle$$

  - ✓ χ_n, $\mathscr{A}_\perp^{\mu,a}$: collinear gauge invariant fields
  - ✓ Measurement function: as for Soft function

- Must be perturbatively matched to PDFs

$$\mathscr{B}_F(x,Q,p_T^{\text{veto}},R^2;\mu,\nu) = \sum_{F'}\int_x^1\frac{dz}{z}\, I_{FF'}(z,Q,p_T^{\text{veto}},R^2;\mu,\nu)\, f_{F'/P}(x/z,\mu) + \mathscr{O}(\Lambda_{\text{QCD}}/p_T^{\text{veto}})$$

- Regularisation of divergences: as for Soft function

  - ✓ Exponential regulator:

$$\prod_i d^dk_i\delta(k_i^2)\theta(k_i^0) \to \prod_i d^dk_i\delta(k_i^2)\theta(k_i^0)\exp\left[\frac{-e^{-\gamma_E}}{\nu}(n\cdot k_i + \bar{n}\cdot k_i)\right]$$

Beam function calculation

- Reference observable:

$$\boxed{\mathscr{B}(x, p_T^{\text{veto}}, R^2; \mu, \nu) = \mathscr{B}_\perp(x, p_T^{\text{veto}}, \mu, \nu) + \Delta\mathscr{B}(x, p_T^{\text{veto}}, R^2; \mu, \nu)}$$

- Same approach as Soft function: decompose into different contributions
  - ✓ Several channels and several colour factors

$$\boxed{\int \frac{dk_{1\perp}}{k_{1\perp}}\, d\eta_1 \frac{d\zeta}{\zeta}\, d\eta \frac{d\phi}{2\pi}\, e^{-2k_{1\perp}\frac{e^{-\gamma_E}}{\nu}[\cosh(\eta_1)+\zeta\cosh(\eta-\eta_1)]}\, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)\, \mathscr{D}_{1\to 3}(\zeta, \eta, \phi)\, \delta(k_1^\pm + k_2^\pm - (1-x)p^\pm)}$$

[Catani, Grazzini 99]

- Structure of our results

$$\Delta\mathscr{B}^{(2)}(x, R^2) = \delta(1-x)\, f_1(R^2) + \left[\frac{1}{1-x}\right]_+ \Big(f_2(x, R^2) + f_3(x) \Big)$$

Series in R^2, up to $\mathcal{O}(R^8)$, analytic

Numerical grid at $R=0$, 3-fold integral, per mille precision

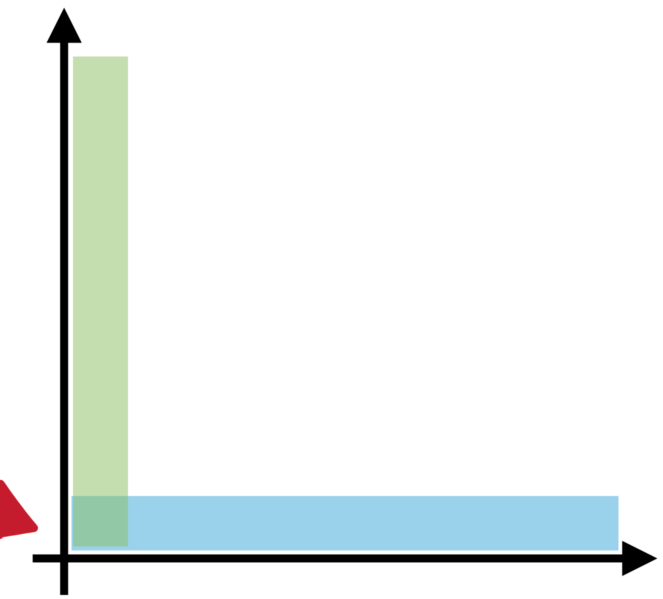
- Also perform numerical calculation with full R dependence
- Calculation in Mellin space/different scheme also available [Bell, Brune, Das, Wald 22]

Zero-Bin Subtraction and Mixing Terms

- Goal: remove soft modes from beam functions

$$\boxed{\mathscr{B}_{cc} = \mathscr{B} - \mathscr{B}_{sc} - \mathscr{B}_{cs} + \mathscr{B}_{ss}}$$

Remove surviving soft contributions



- Can this be consistently done in SCET?
  - If not, SCET factorisation is broken by soft-collinear mixing terms
  - Observable dependent answer: ok at NLO, contested beyond [Becher, Neubert, Rothen 13] [Tackmann, Walsh, Zuberi 12, Stewart, Tackmann, Walsh, Zuberi 13]
  - We show that two-loop mixing terms cancel (with exponential regulator)
    - SCET factorisation holds at NNLO and reproduces QCD

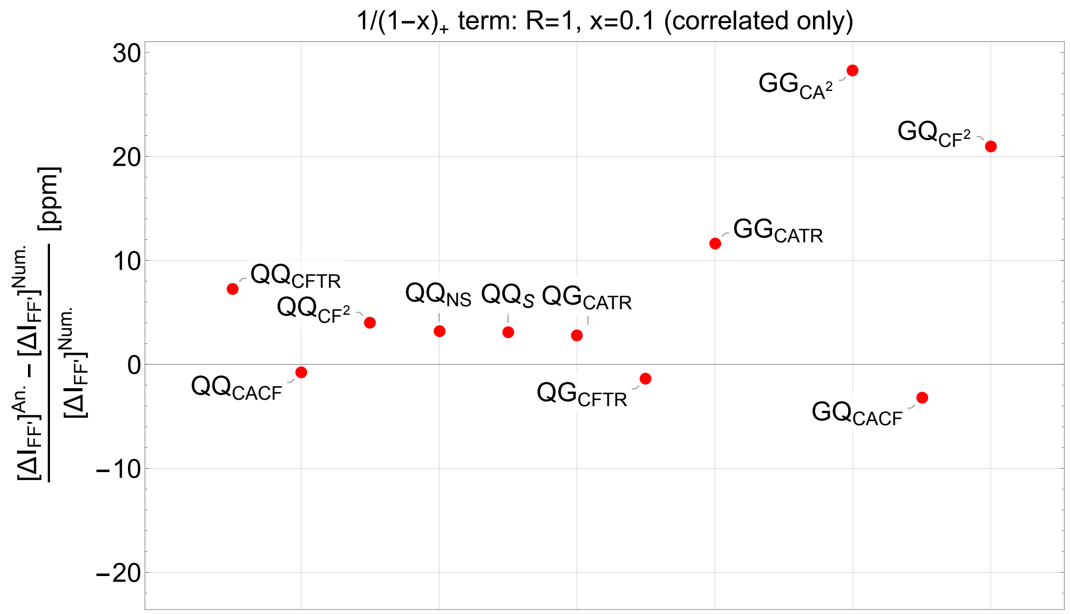
Results and checks

- Reproduce known rapidity anomalous dimension
  (requires 0-bin subtraction and soft-collinear mixing)

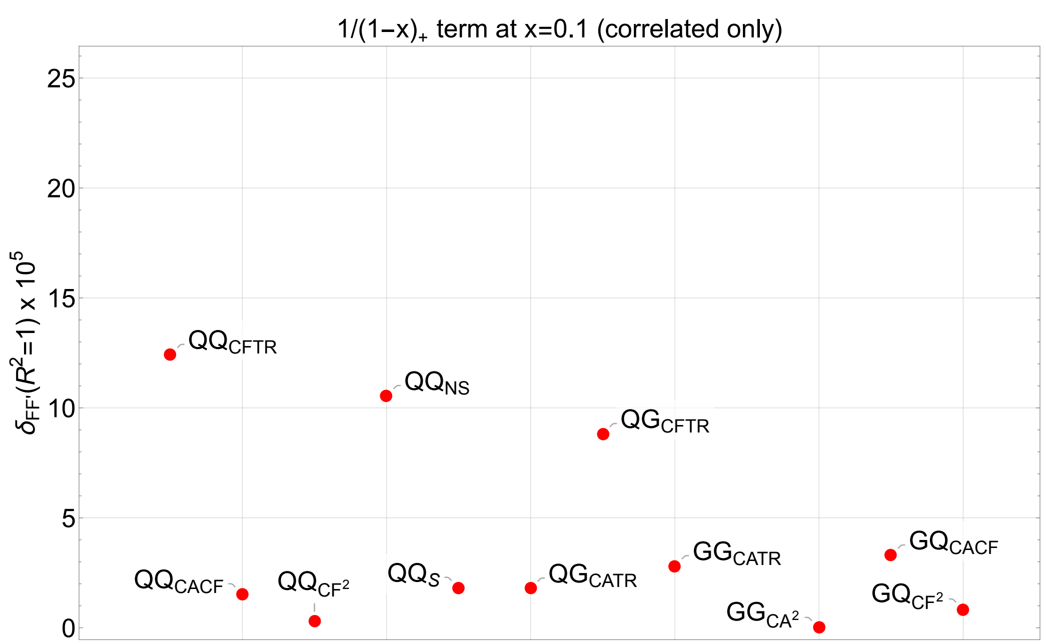
[Banfi, Monni, Salam, Zanderighi 12;
Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13]

- Analytic vs Numerical calculation

$$\Delta\mathscr{B}^{(2)}(x,R^2) = \delta(1-x)\, f_1(R^2) + \left[\frac{1}{1-x}\right]_+ \Big(f_2(x,R^2) + f_3(x)\Big)$$



- Convergence of R^2 expansion



$$\delta_{FF'}(R^2) = \left| 1 - \frac{\Delta I^{(2)}_{FF'}|_{R^6}}{\Delta I^{(2)}_{FF'}|_{R^8}} \right|$$

APPLICATIONS AND MORE CHECKS: LEADING-JET p_T SLICING

Leading-jet p_T slicing

- All ingredients for NNLO slicing with p_T^{jet} for colour singlet production

$$\mathrm{d}\sigma^{F}_{\text{NNLO}} = \mathbb{H}^{\text{NNLO}}_{\text{veto}} \otimes \mathrm{d}\sigma_{\text{Born}} + \lim_{p_{T,\text{cut}}^{\text{jet}} \to 0} \int_{p_{T,\text{cut}}^{\text{jet}}}^{+\infty} \mathrm{d}p_T^{\text{jet}} \left(\frac{\mathrm{d}^2\sigma^{F+\text{jet}}_{\text{NLO}}}{\mathrm{d}p_T^{\text{jet}}} - \left.\frac{\mathrm{d}^2\sigma(p_T^{\text{veto}})}{\mathrm{d}p_T^{\text{veto}}}\right|^{(\alpha_s^2)}_{p_T^{\text{veto}}=p_T^{\text{jet}}} \right)$$

n3loxs

[Baglio, Duhr, Mistleberger, Szafron 19]

Finite terms in two-loop soft and beam functions

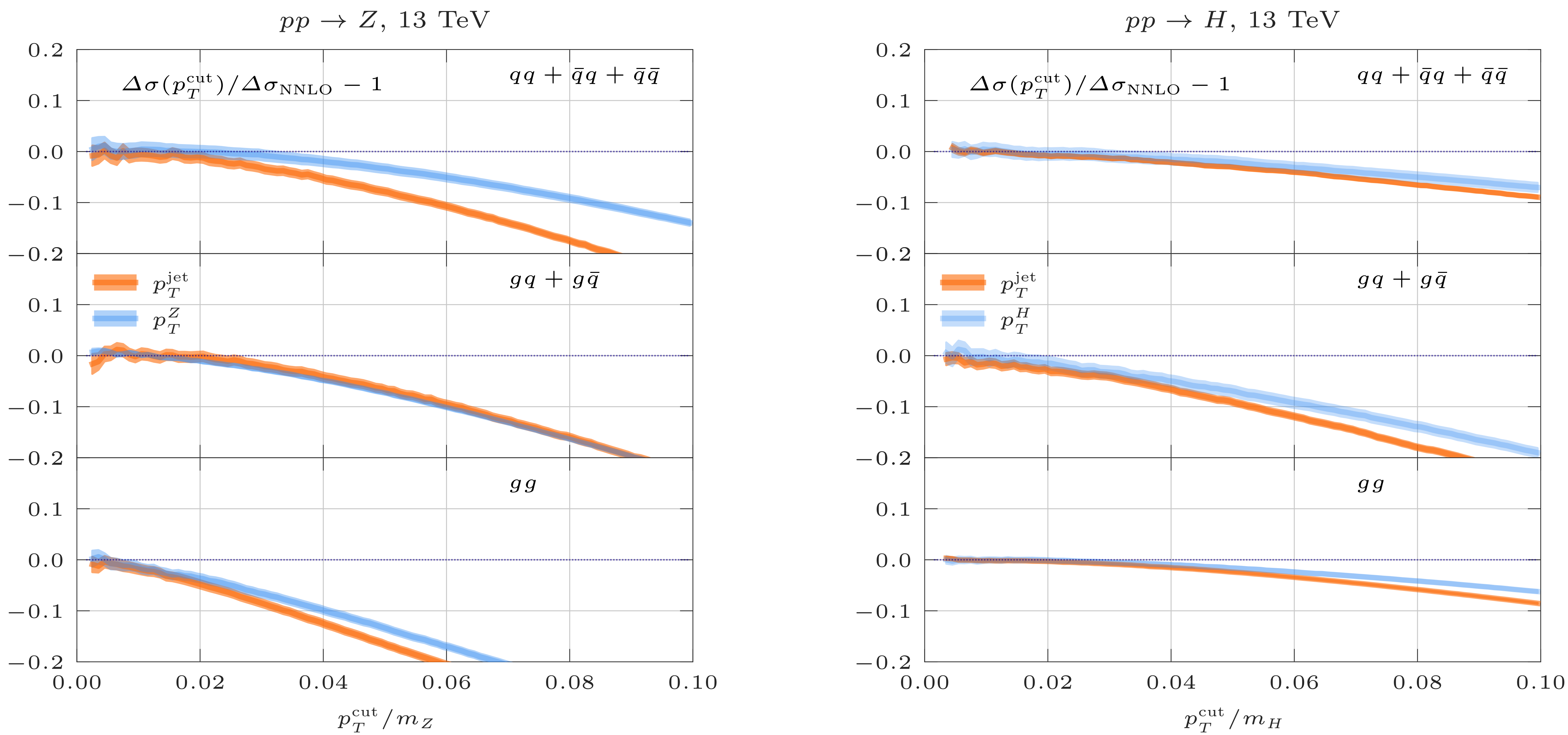
From e.g. MCFM

[Campbell, Neumann 19]

Two-loop anomalous dimensions

- Implemented in RadISH
- Reproduce know NNLO cross-section: very strong check
- Dependence on p_T^{jet}: determine power corrections to SCET

Leading-jet p_T slicing: channel decomposition



- Convergence at low p_T^{cut}: correct two-loop anomalous dimensions
- Convergence to 0: correct finite terms in two-loop soft and beam functions
- Shape of curves: form of power corrections
- Leading-jet p_T slicing competitive with q_T slicing

CONCLUSION AND OUTLOOK

Conclusion and outlook

- Fully analytic two-loop soft function for jet-vetoed cross sections
- Analytic two-loop beam functions (up to boundary condition at $R = 0$)
- Validated finite terms of soft/beam functions by reproducing NNLO DY and Higgs cross sections
- To do: Determine the three-loop γ_ν
- To do: Setup N^{3}LL resummation of jet-vetoed cross section

THANK YOU!