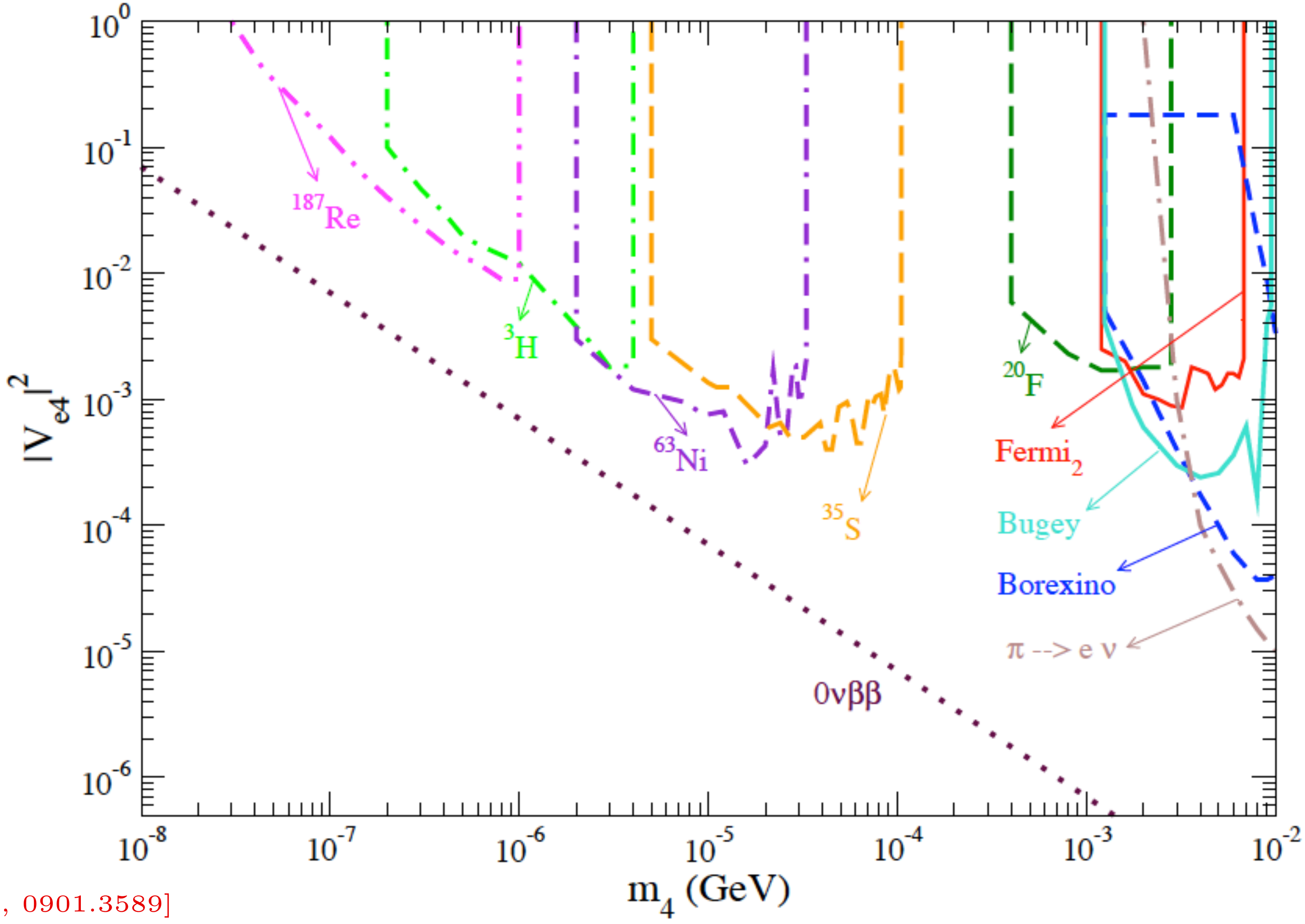
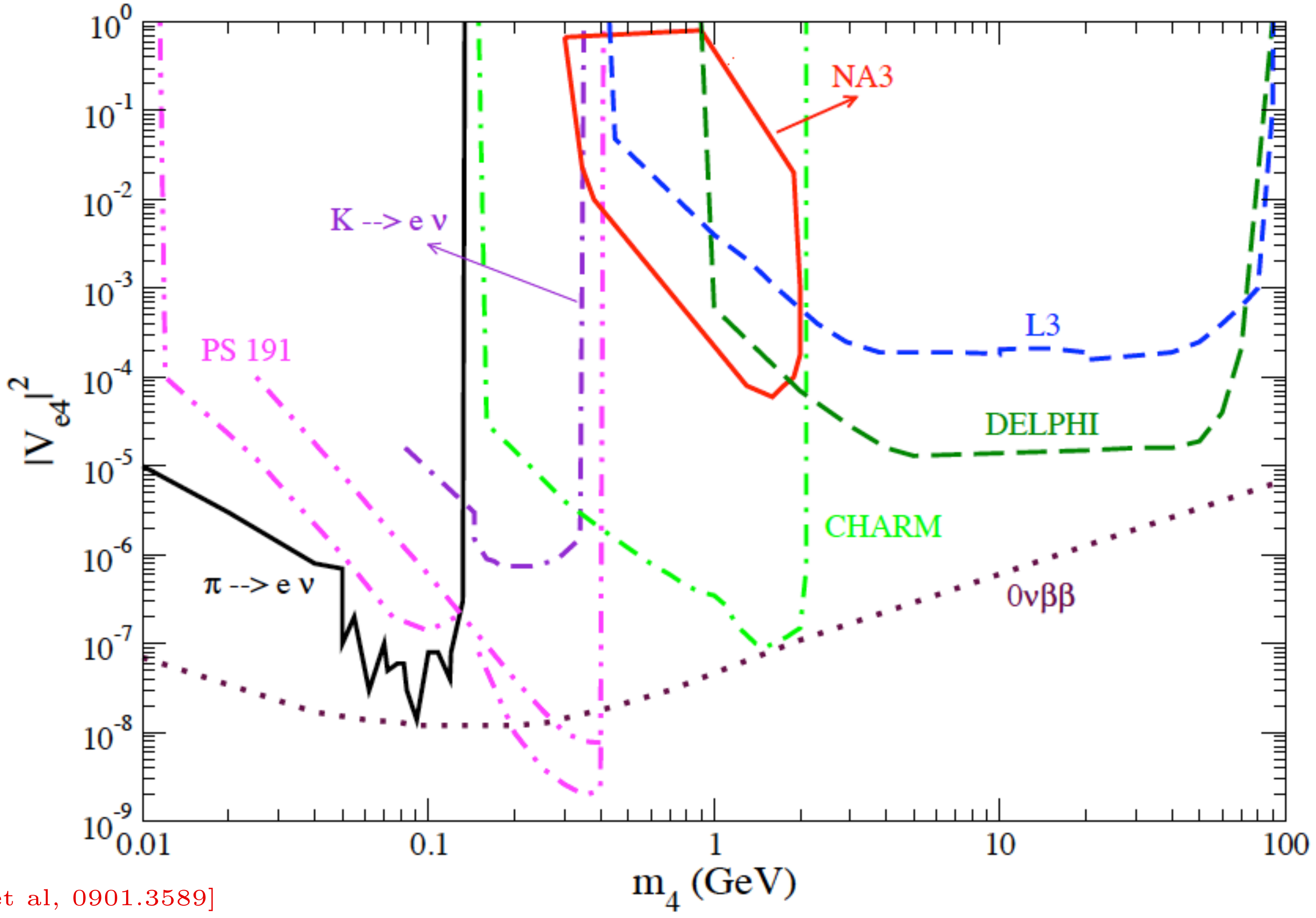


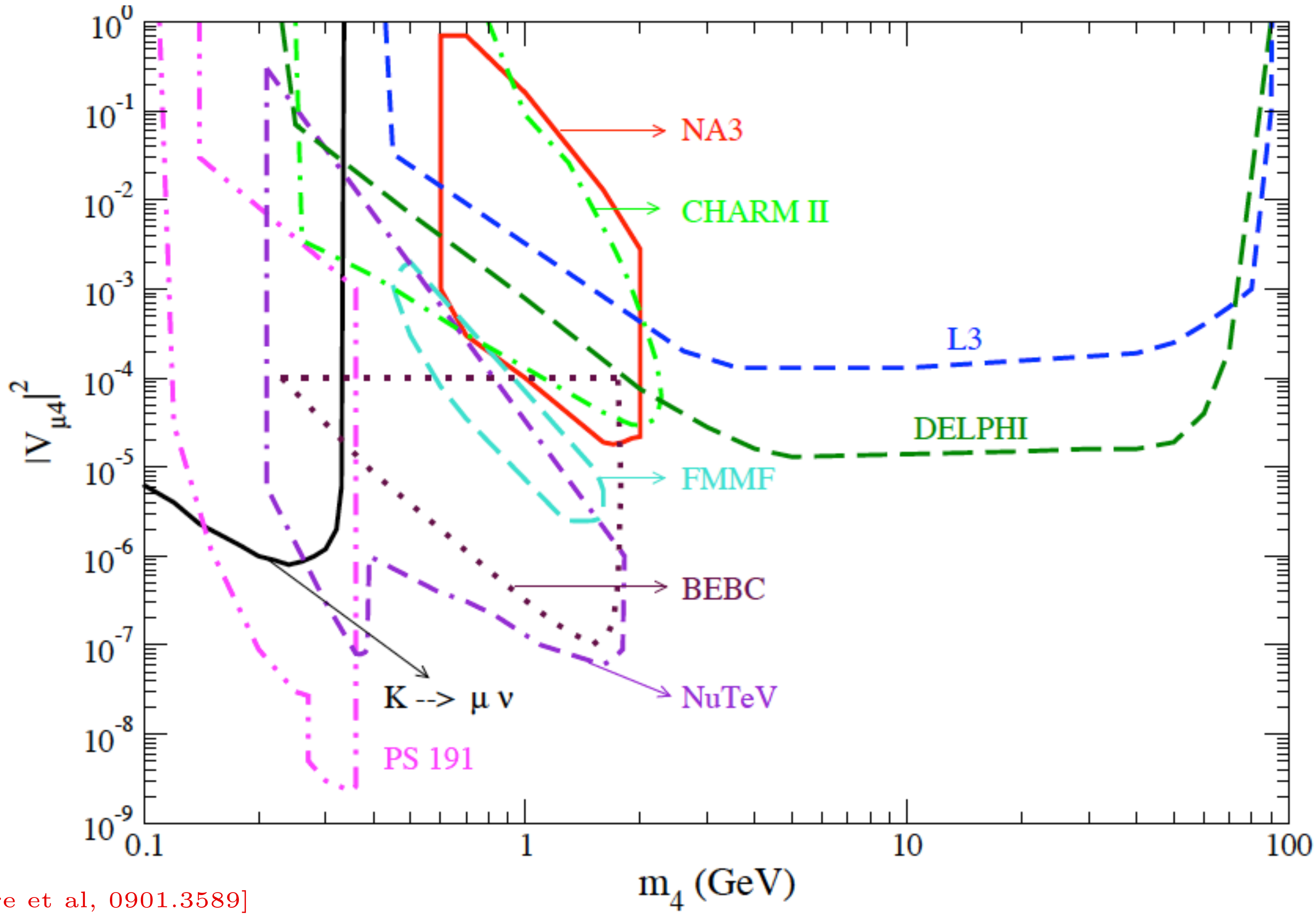
[A. Atre et al, 0901.3589]

Figure 2: Bounds on $|V_{e4}|^2$ versus m_4 in the mass range 10 eV–10 MeV. The excluded regions with contours labeled ^{187}Re [76], ^{3}H [77], ^{63}Ni [78], ^{35}S [79], ^{20}F and Fermi$_2$ [80] refer to the bounds from kink searches. All the limits are given at 95% C.L. except for the ones from Ref. [80] which are at 90% C.L.. The areas delimited by short dashed (blue) contour labeled Borexino and solid (cyan) contour labeled Bugey are excluded at 90% C.L. by searches of N_4 decays from the Borexino Counting Test facility [81] and Ref. [82] respectively. The region with long-dash-dotted (grey) contour, labelled $\pi \to e\nu$, is excluded by peak searches [83]. The dotted (maroon) line labeled $0\nu\beta\beta$ indicates the bound from searches of neutrinoless double beta-decay [84].



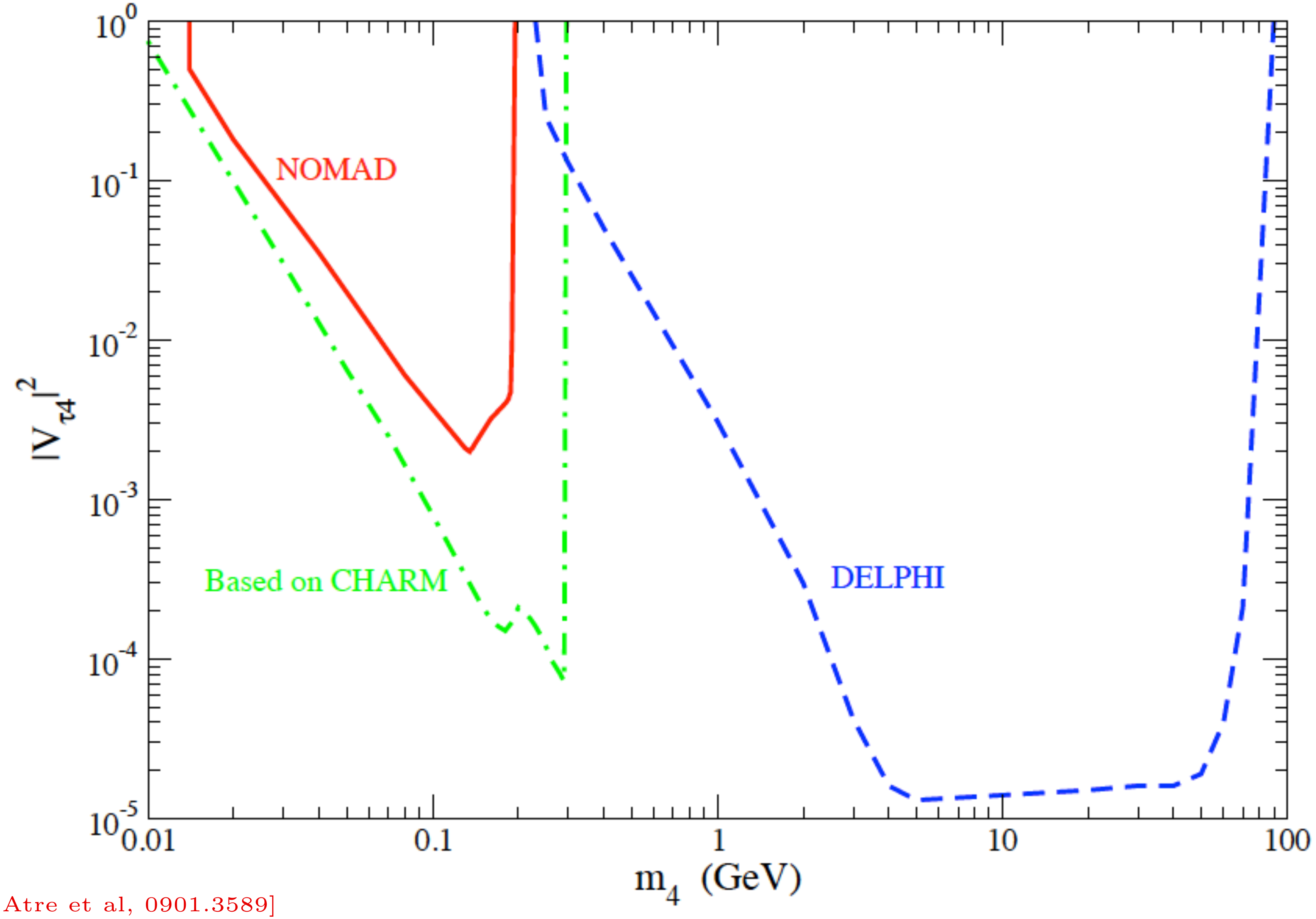
[A. Atre et al, 0901.3589]

Figure 3: Bounds on $|V_{e4}|^2$ versus m_4 in the mass range 10 MeV–100 GeV. The areas with solid (black) contour labeled $\pi \to e\nu$ and double dash dotted (purple) contour labeled $K \to e\nu$ are excluded by peak searches [83, 85]. Limits at 90% C.L. from beam-dump experiments are taken from Ref. [86] (PS191), Ref. [87] (NA3) and Ref. [88] (CHARM). The limits from contours labeled DELPHI and L3 are at 95% C.L. and are taken from Refs. [89] and [90] respectively. The excluded region with dotted (maroon) contour is derived from a reanalysis of neutrinoless double beta decay experimental data [84].



[A. Atre et al, 0901.3589]

Figure 4: Limits on $|V_{\mu 4}|^2$ versus m_4 in the mass range 100 MeV–100 GeV come from peak searches and from N_4 decays. The area with solid (black) contour labeled $K \to \mu\nu$ [92] is excluded by peak searches. The bounds indicated by contours labeled by PS191 [86], NA3 [87], BEBC [93], FMMF [94], NuTeV [95] and CHARMII [96] are at 90% C.L., while DELPHI [89] and L3 [90] are at 95% C.L. and are deduced from searches of visible products in N_4 decays. For the beam dump experiments, NA3, PS191, BEBC, FMMF and NuTeV we give an estimate of the upper limit for the excluded values of the mixing angle.



[A. Atre et al, 0901.3589]

Figure 5: Bounds on $|V_{\tau 4}|^2$ versus m_4 from searches of decays of heavy neutrinos, given in Ref. [97] (CHARM) and in Ref. [98] (NOMAD) at 90% C.L., and in Ref. [89] (DELPHI) at 95% C.L.

Candidate νSM: The One I'll Concentrate On

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\mathrm{SM}} \supset -y_{ij}\frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB: } \mathcal{L}_{\nu\mathrm{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = y_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above *at most* Λ/y.
- Define $y_{\max} \equiv 1 \ \Rightarrow$ data require $\Lambda \sim 10^{14}$ GeV.

What else is this "good for"? Depends on the ultraviolet completion!

The Seesaw Lagrangian

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions.

$\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

[a] Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M.

The data can be summarized as follows: there is evidence for three neutrinos, mostly "active" (linear combinations of ν_e, ν_μ, and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be "sterile."

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large M: $M \gg v$. Popular examples include $M \sim M_{\rm GUT}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV.

we can impose very, very few experimental constraints on M

What We Know About M:

- $M = 0$: the six neutrinos "fuse" into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ $[m \propto 1/\Lambda \Rightarrow \Lambda = M/\mu^2]$.

  This the **seesaw mechanism.** Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though L-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

High-Energy Seesaw: Brief Comments

- This is everyone's favorite scenario.
- Upper bound for M (e.g. Maltoni, Niczyporuk, Willenbrock, hep-ph/0006358):

$$M < 7.6 \times 10^{15}\ \text{GeV} \times \left(\frac{0.1\ \text{eV}}{m_\nu}\right).$$

- Naturalness 'hint' (e.g., Casas, Espinosa, Hidalgo, hep-ph/0410298):

$$M < 10^{7}\ \text{GeV}.$$

- Physics "too" heavy! No observable consequence other than leptogenesis. From thermal leptogenesis $M > 10^9$ GeV. Will we ever convince ourselves that this is correct? (e.g., Buckley, Murayama, hep-ph/0606088)

Low-Energy Seesaw [AdG PRD72,033005)]

The other end of the M spectrum ($M < 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light? [For a possible alternative see Canetti, Shaposhnikov, arXiv: 1006.0133 and reference therein.]
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of M are natural (in the 'tHooft sense). In fact, theoretically, no value of M should be discriminated against!

More Details, assuming three right-handed neutrinos N:

$$m_\nu = \begin{pmatrix} 0 & \lambda v \\ (\lambda v)^t & M \end{pmatrix},$$

M is diagonal, and all its eigenvalues are real and positive. The charged lepton mass matrix also diagonal, real, and positive.

To leading order in $(\lambda v)M^{-1}$, the three lightest neutrino mass eigenvalues are given by the eigenvalues of

$$m_a = \lambda v M^{-1} (\lambda v)^t,$$

where m_a is the mostly active neutrino mass matrix, while the heavy sterile neutrino masses coincide with the eigenvalues of M.

6×6 mixing matrix U $[U^t m_\nu U = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)]$ is

$$U = \begin{pmatrix} V & \Theta \\ -\Theta^\dagger V & 1_{n\times n} \end{pmatrix},$$

where V is the active neutrino mixing matrix (MNS matrix)

$$V^t m_a V = \text{diag}(m_1, m_2, m_3),$$

and the matrix that governs active–sterile mixing is

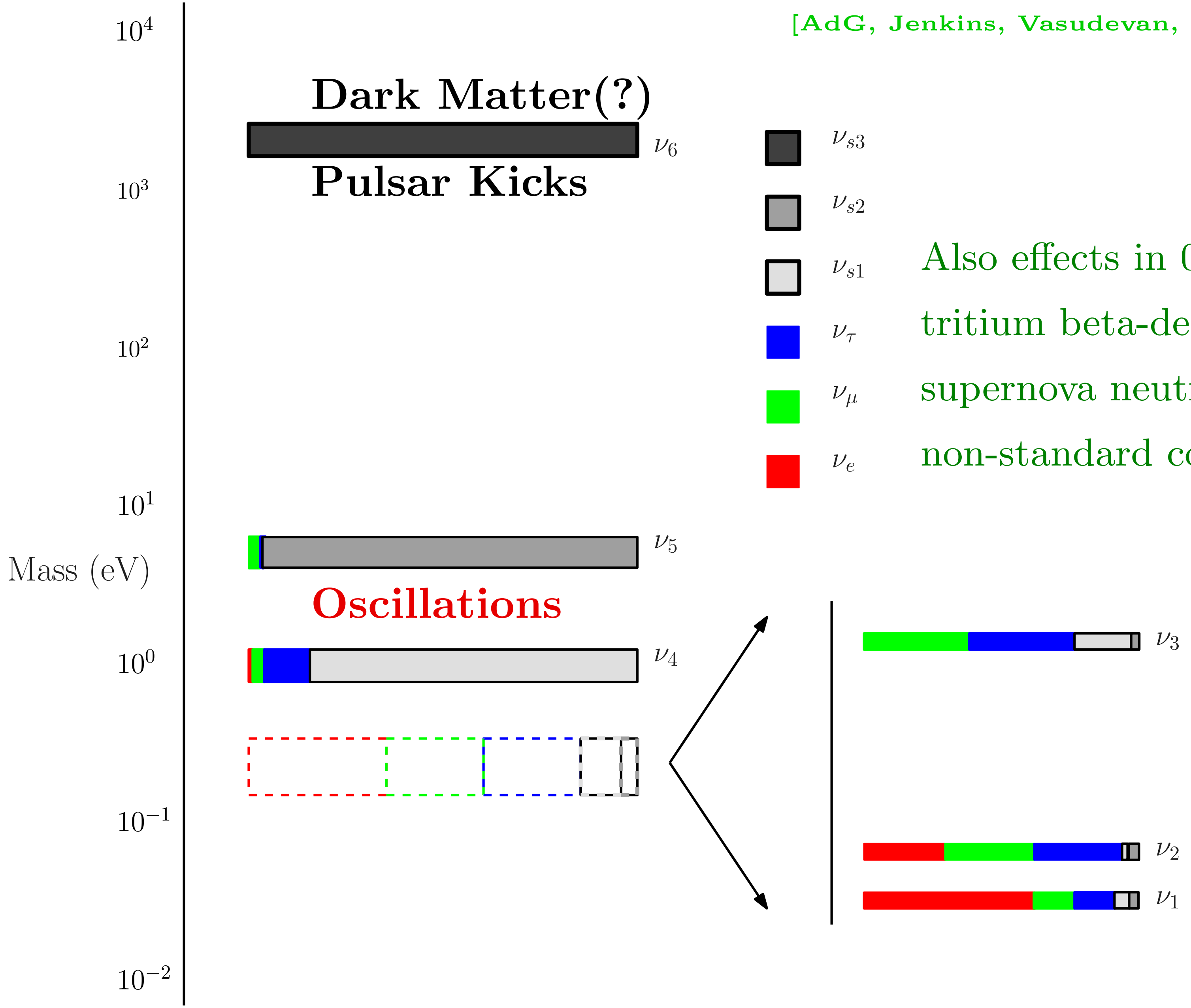
$$\Theta = (\lambda v)^* M^{-1}.$$

One can solve for the Yukawa couplings and re-express

$$\Theta = V\sqrt{\text{diag}(m_1, m_2, m_3)}R^\dagger M^{-1/2},$$

where R is a complex orthogonal matrix $RR^t = 1$.

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



Also effects in $0\nu\beta\beta$, tritium beta-decay, supernova neutrino oscillations, non-standard cosmology.

Predictions: **Neutrinoless Double-Beta Decay**

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \to (Z+2)e^-e^-$.

For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

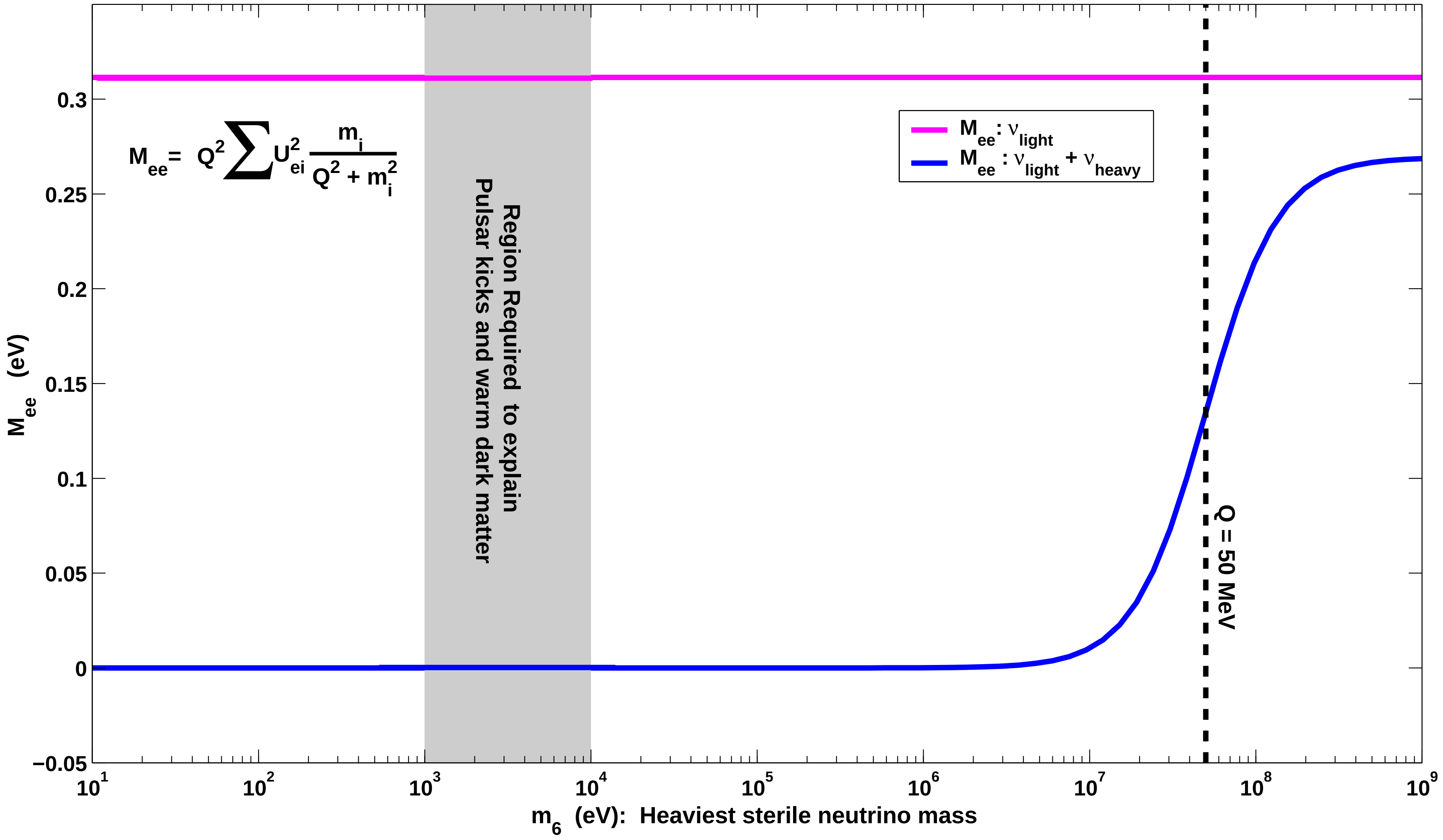
$$m_{ee} = \left|\sum_{i=1}^{6} U_{ei}^2 m_i\right| \sim \left|\sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3} \vartheta_{ei}^2 M_i\right| .$$

However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. **The contribution of light and heavy neutrinos exactly cancels!** This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\left[\ \mathcal{M} = \begin{pmatrix} 0 & \mu^{\mathrm{T}} \\ \mu & M \end{pmatrix} \quad \to \quad m_{ee} \text{ is identically zero!} \ \right]$$

(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



Predictions: **Tritium beta-decay**

Heavy neutrinos participate in tritium β-decay. Their contribution can be parameterized by

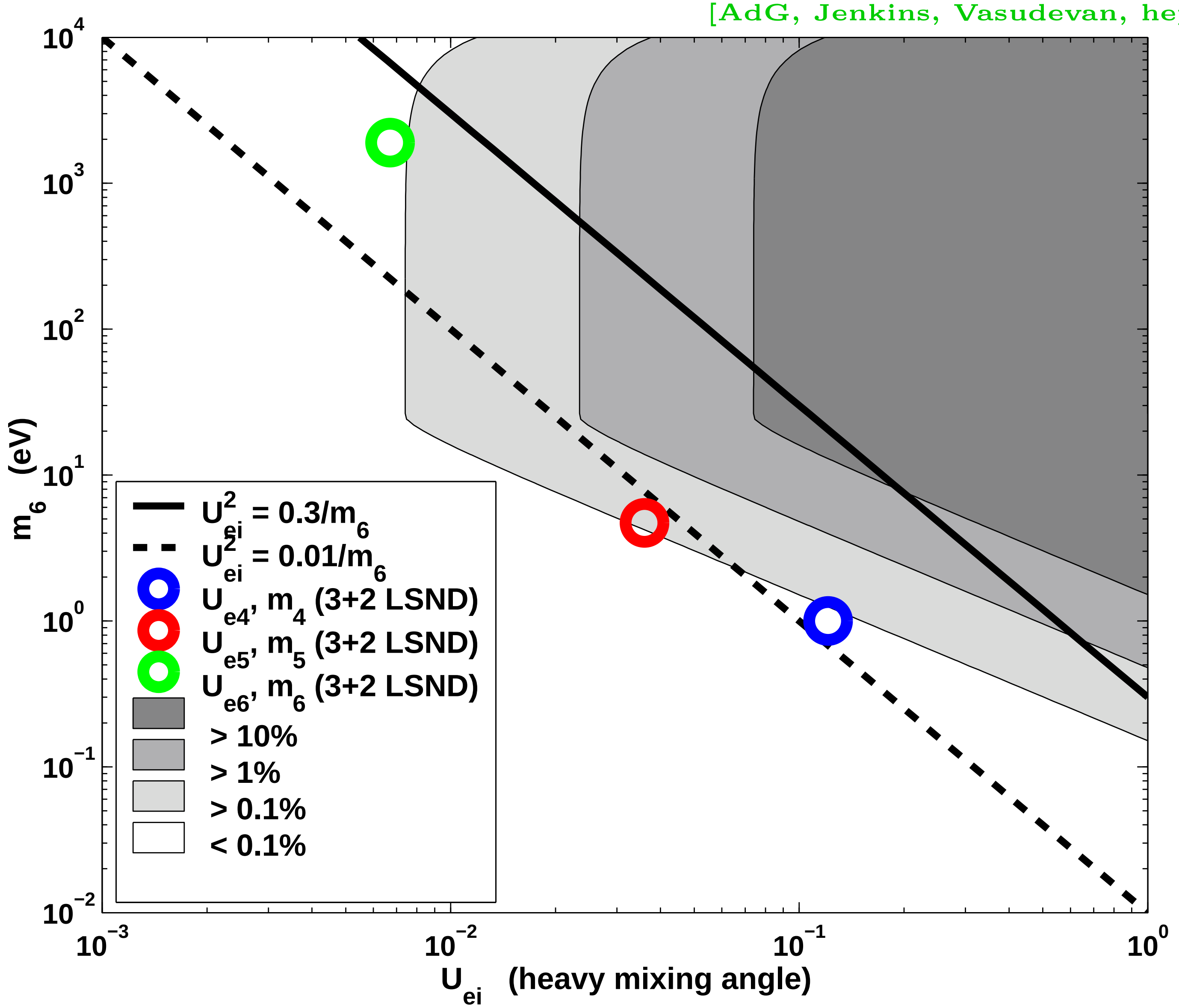
$$m_\beta^2 = \sum_{i=1}^{6} |U_{ei}|^2 m_i^2 \simeq \sum_{i=1}^{3} |U_{ei}|^2 m_i^2 + \sum_{i=1}^{3} |U_{ei}|^2 m_i M_i,$$

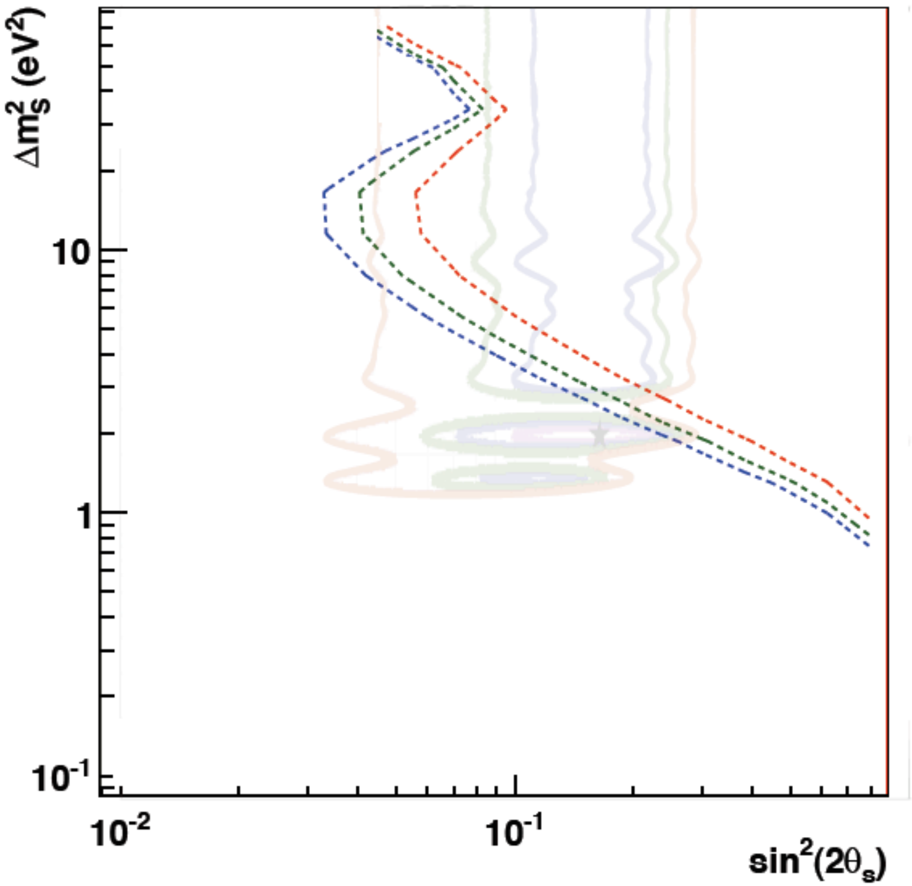
as long as M_i is not too heavy (above tens of eV). For example, in the case of a 3+2 solution to the LSND anomaly, the heaviest sterile state (with mass M_1) contributes the most: $m_\beta^2 \simeq 0.7\ \text{eV}^2 \left(\frac{|U_{e1}|^2}{0.7}\right)\left(\frac{m_1}{0.1\ \text{eV}}\right)\left(\frac{M_1}{10\ \text{eV}}\right)$.

NOTE: next generation experiment (KATRIN) will be sensitive to $O(10^{-1})$ eV^2.

sensitivity of tritium beta decay to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]





[Barrett, Formaggio, 1105.1326]

FIG. 2: Sensitivity of the KATRIN neutrino mass measurement for a sterile neutrino with relatively large mass splitting (dashed contours). Figures shows exclusion curves of mixing angle $\sin^2(2\theta_S)$ versus mass splitting $|\Delta m_S^2|^2$ for the 90% (blue), 95% (green), and 99% (red) C.L. after three years of data taking. Figure 7 from Ref. [2] show in solid curves in the background.

On Early Universe Cosmology / Astrophysics

A combination of the SM of particle physics plus the "concordance cosmological model" severely constrain light, sterile neutrinos with significant active-sterile mixing. Taken at face value, not only is the eV-seesaw ruled out, but so are all oscillation solutions to the LSND anomaly.

Hence, **eV-seesaw → nonstandard particle physics and cosmology**.

On the other hand. . .

- Right-handed neutrinos may make good warm dark matter particles.

  Asaka, Blanchet, Shaposhnikov, hep-ph/0503065.

- Sterile neutrinos are known to help out with r-process nucleosynthesis in supernovae, . . .

- . . . and may help explain the peculiar peculiar velocities of pulsars.

Big Bang Neutrinos are Warm Dark Matter

Planck Collaboration: Cosmological parameters

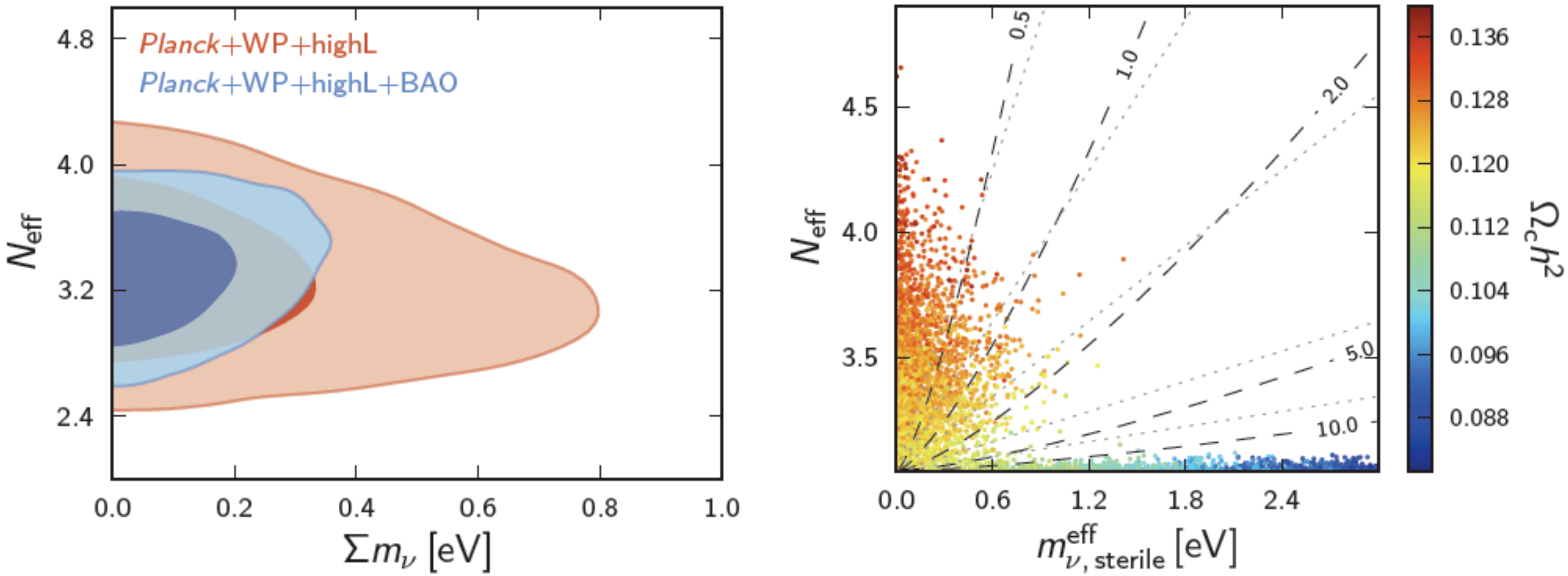


Fig. 28. *Left*: 2D joint posterior distribution between $N_{\rm eff}$ and $\sum m_\nu$ (the summed mass of the three active neutrinos) in models with extra massless neutrino-like species. *Right*: Samples in the $N_{\rm eff}$–$m^{\rm eff}_{\nu,\,\rm sterile}$ plane, colour-coded by $\Omega_c h^2$, in models with one massive sterile neutrino family, with effective mass $m^{\rm eff}_{\nu,\,\rm sterile}$, and the three active neutrinos as in the base ΛCDM model. The physical mass of the sterile neutrino in the thermal scenario, $m^{\rm thermal}_{\rm sterile}$, is constant along the grey dashed lines, with the indicated mass in eV. The physical mass in the Dodelson-Widrow scenario, $m^{\rm DW}_{\rm sterile}$, is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

What if 1 GeV$< M <1$ TeV?

Naively, one expects

$$\Theta \sim \sqrt{\frac{m_a}{M}} < 10^{-5}\sqrt{\frac{1\text{ GeV}}{M}},$$

such that, for $M = 1$ GeV and above, sterile neutrino effects are mostly negligible.

However,

$$\Theta = V\sqrt{\text{diag}(m_1, m_2, m_3)}R^{\dagger}M^{-1/2},$$

and the magnitude of the entries of R can be arbitrarily large [$\cos(ix) = \cosh x \gg 1$ if $x > 1$].

This is true as long as

- $\lambda v \ll M$ (seesaw approximation holds)
- $\lambda < 4\pi$ (theory is "well-defined")

This implies that, in principle, Θ is a quasi-free parameter – independent from light neutrino masses and mixing – as long as $\Theta \ll 1$ and $M < 1$ TeV.

What Does $R \gg 1$ Mean?

It is illustrative to consider the case of one active neutrino of mass m_3 and two sterile ones, and further assume that $M_1 = M_2 = M$. In this case,

$$\Theta = \sqrt{\frac{m_3}{M}} \begin{pmatrix} \cos\zeta & \sin\zeta \end{pmatrix},$$

$$\lambda v = \sqrt{m_3 M} \begin{pmatrix} \cos\zeta^* & \sin\zeta^* \end{pmatrix} \equiv \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix}.$$

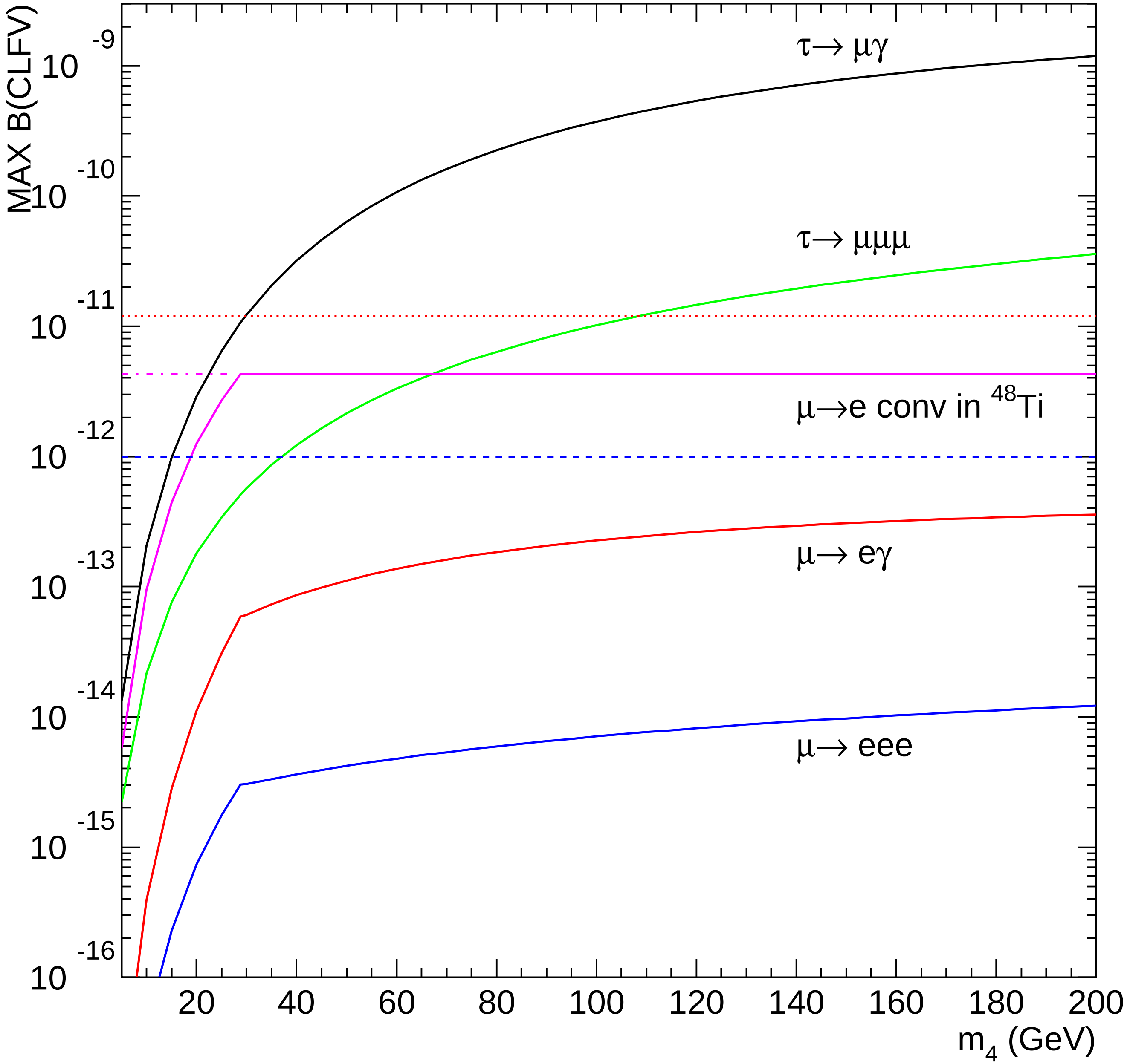
If ζ has a large imaginary part $\Rightarrow$ Θ is (exponentially) larger than $(m_3/M)^{1/2}$, λ_i neutrino Yukawa couplings are much larger than $\sqrt{m_3 M}/v$

The reason for this is a strong cancellation between the contribution of the two different Yukawa couplings to the active neutrino mass
$\Rightarrow$ $m_3 = \lambda_1^2 v^2/M + \lambda_2^2 v^2/M$.

For example: $m_3 = 0.1$ eV, $M = 100$ GeV, $\zeta = 14i \Rightarrow \lambda_1 \sim 0.244, \lambda_2 \sim -0.244i$, while $|y_1| - |y_2| \sim 3.38 \times 10^{-13}$.

NOTE: cancellation may be consequence of a symmetry (say, lepton number). See, for example, the "inverse seesaw" Mohapatra and Valle, PRD34, 1642 (1986).

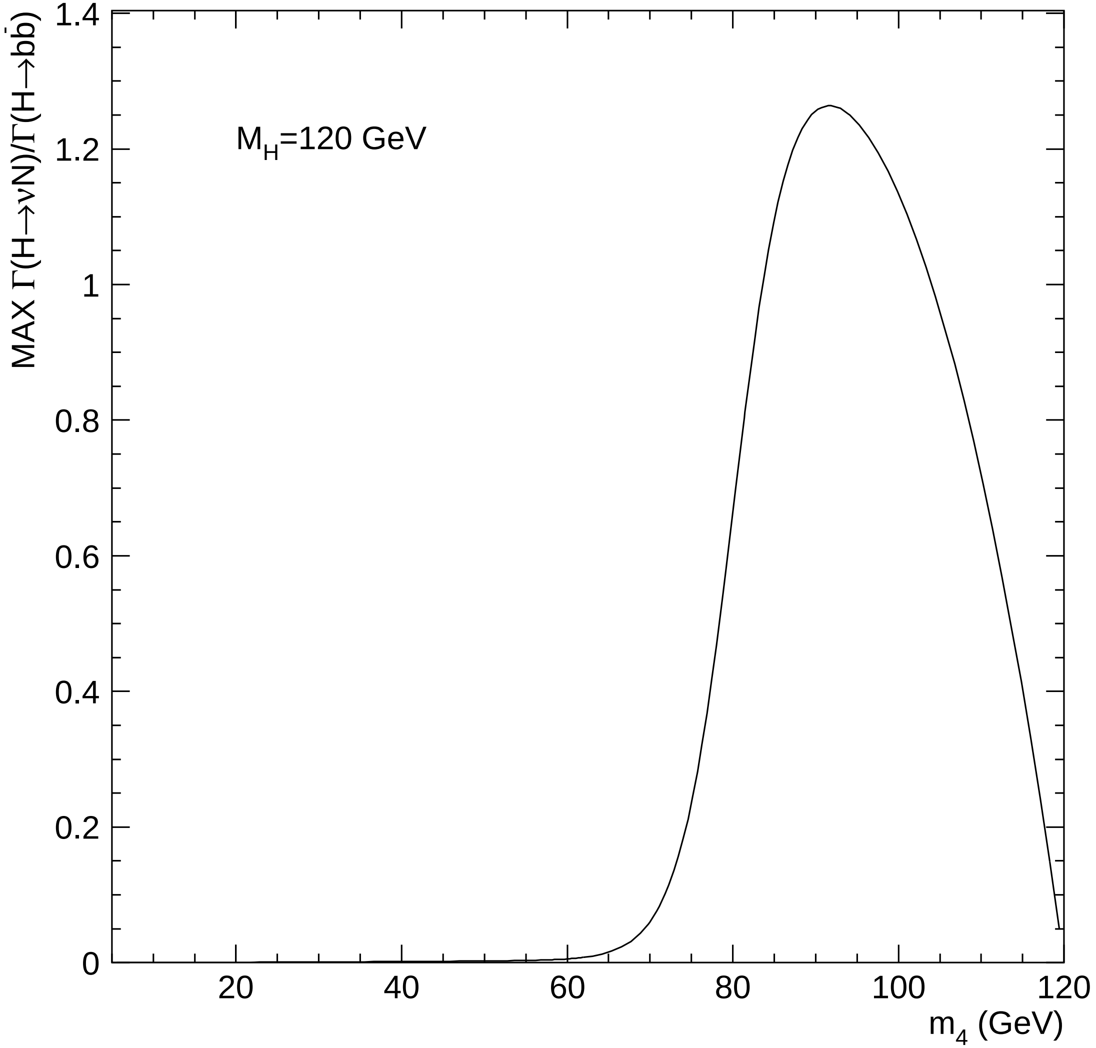
Constraints From Charged Lepton Flavor Violation



arXiv:0706.1732 [hep-ph]

Weak Scale Seesaw, and Accidentally Light Neutrino Masses

[AdG arXiv:0706.1732 [hep-ph]]



What does the seesaw Lagrangian predict for the LHC?

Nothing much, unless. . .

- $M_N \sim 1 - 100$ GeV,
- Yukawa couplings larger than naive expectations.

$\Leftarrow$ $H \to \nu N$ as likely as $H \to b\bar{b}$!

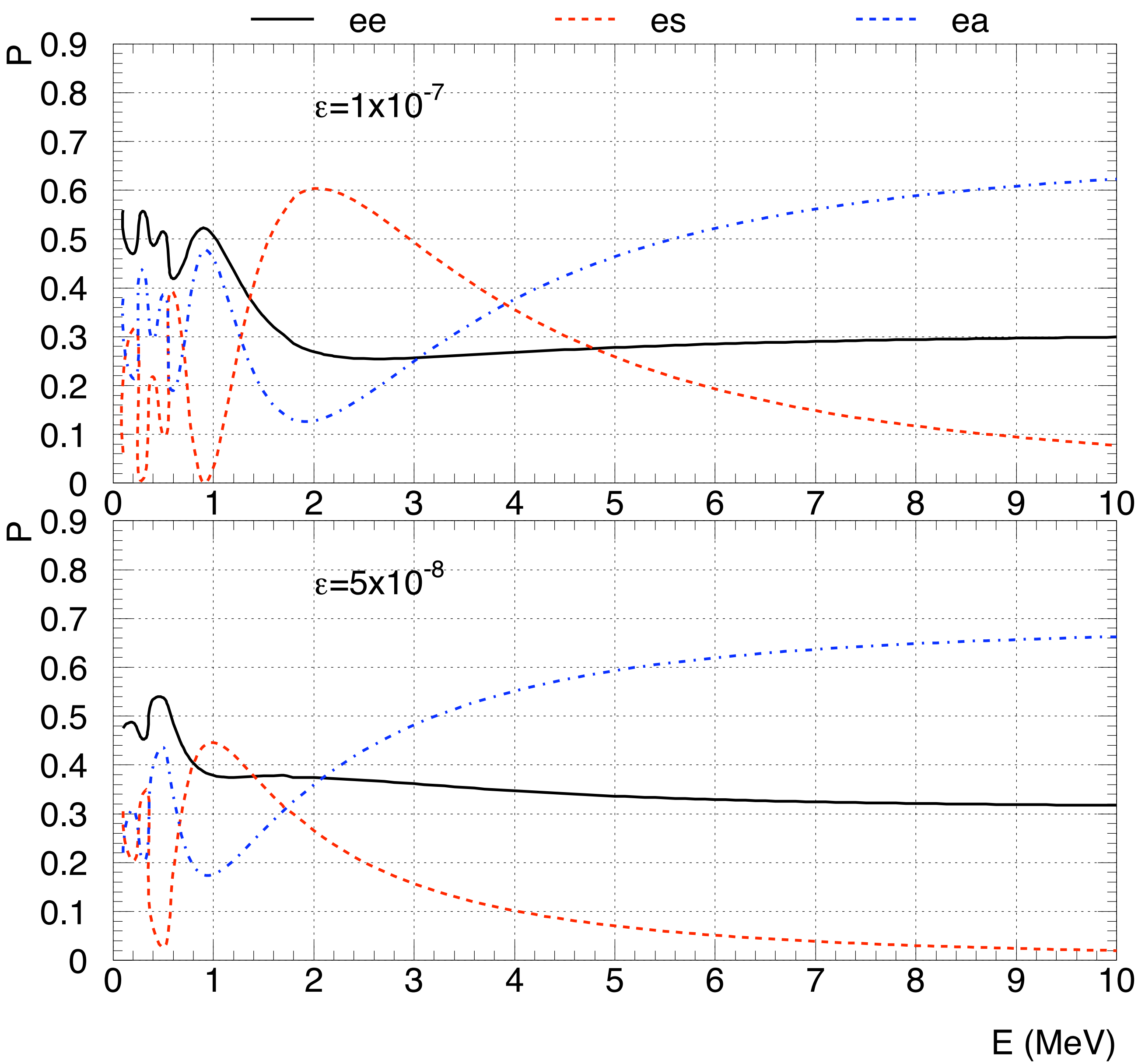
$\Big($NOTE: $N \to \ell q'\bar{q}$ or $\ell\ell'\nu$ (prompt)

"Weird" Higgs decay signature! $\Big)$

Going All the Way: What Happens When $M \ll \mu$?

In this case, the six Weyl fermions pair up into three quasi-degenerate states ("quasi-Dirac fermions").

These states are fifty–fifty active–sterile mixtures. In the limit $M \to 0$, we end up with Dirac neutrinos, which are clearly allowed by all the data.

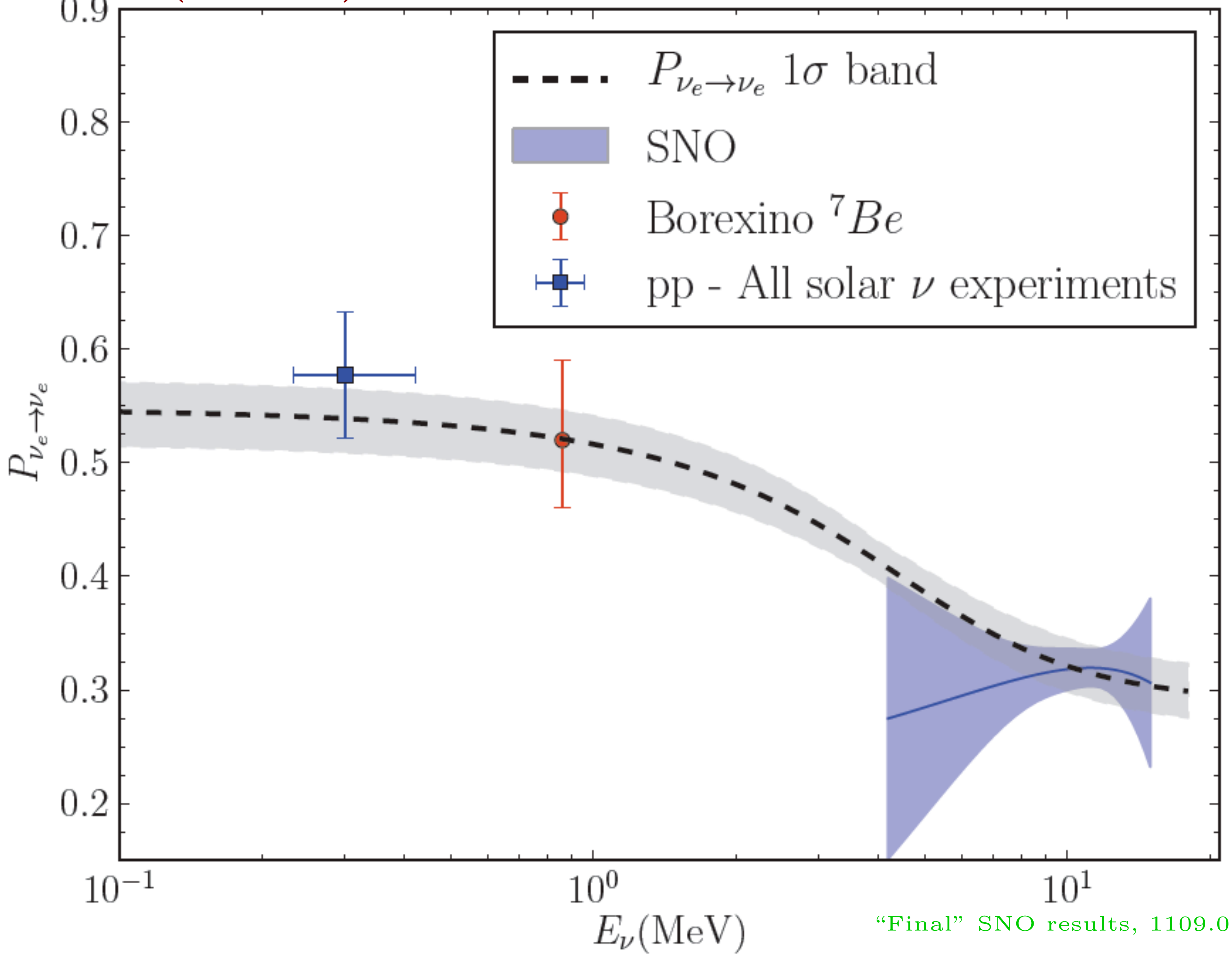


Quasi-Sterile Neutrinos

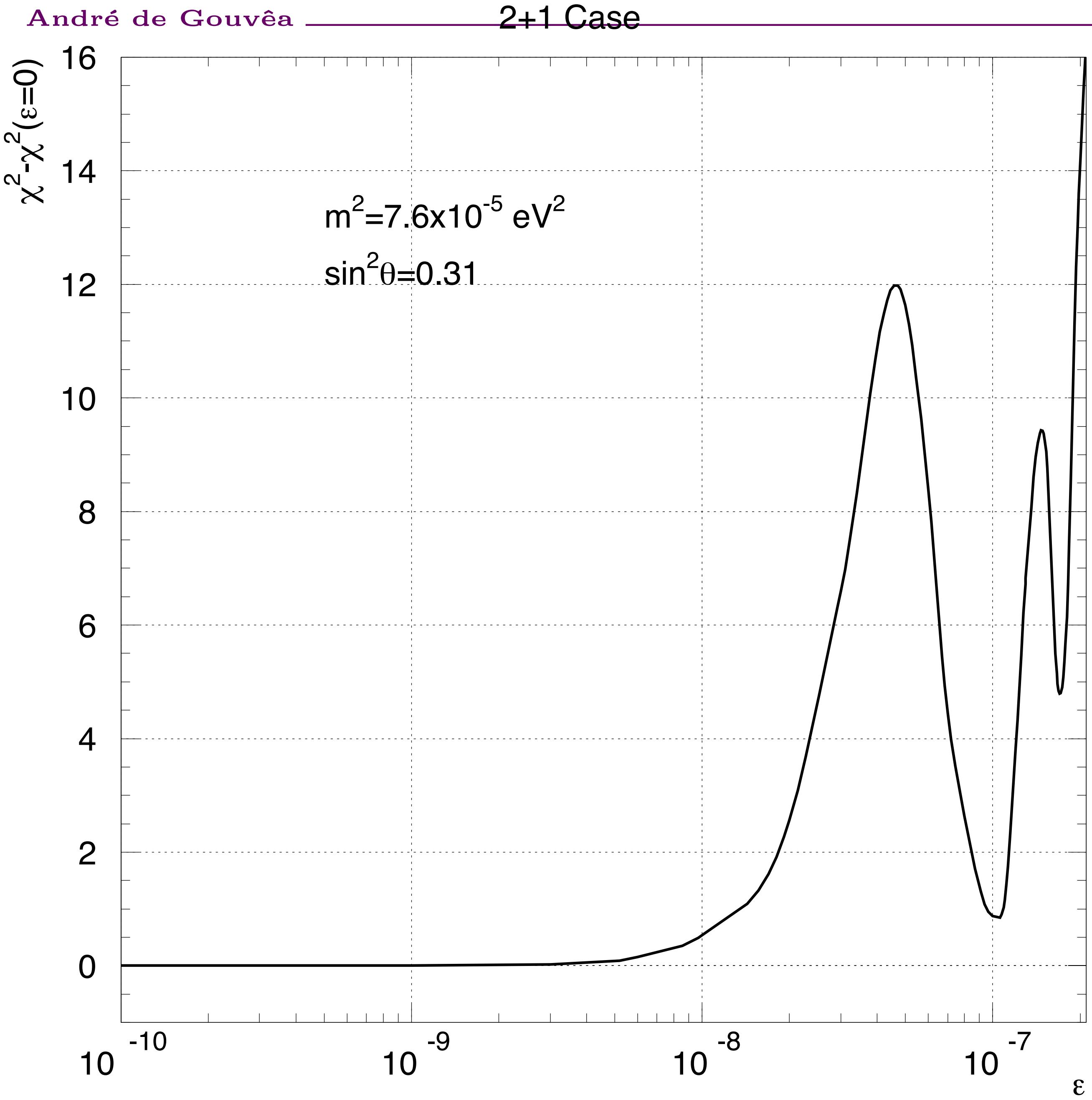
- tiny new $\Delta m^2 = \epsilon \Delta m^2_{12}$,
- maximal mixing!
- Effects in Solar νs

[AdG, Huang, Jenkins, arXiv:0906.1611]

(Almost) All We Know About Solar Neutrinos



"Final" SNO results, 1109.0763



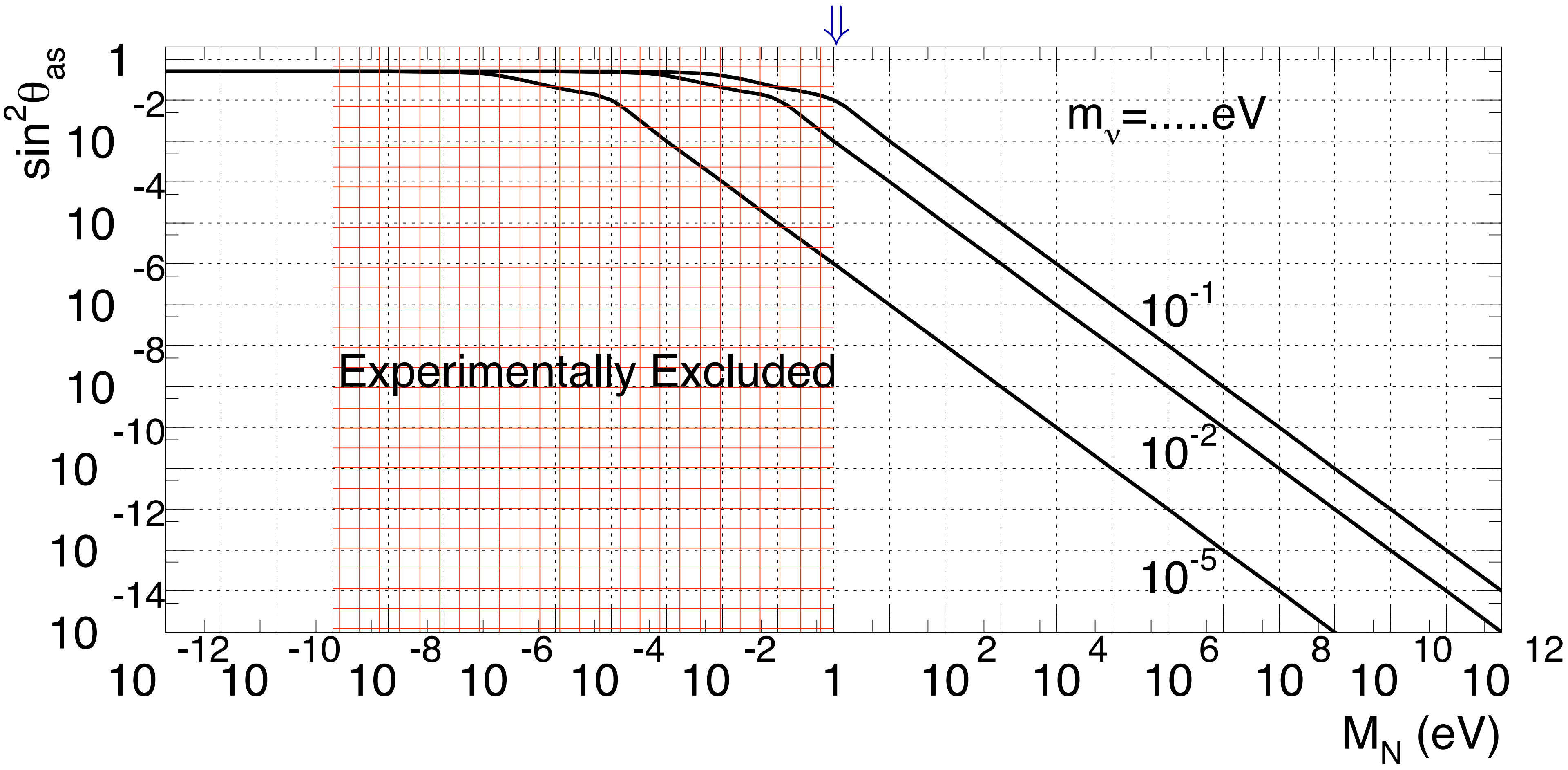
Quasi-Sterile Neutrinos

- tiny new $\Delta m^2 = \epsilon \Delta m^2_{12}$,
- maximal mixing!
- Effects in Solar νs

[AdG, Huang, Jenkins, arXiv:0906.1611]

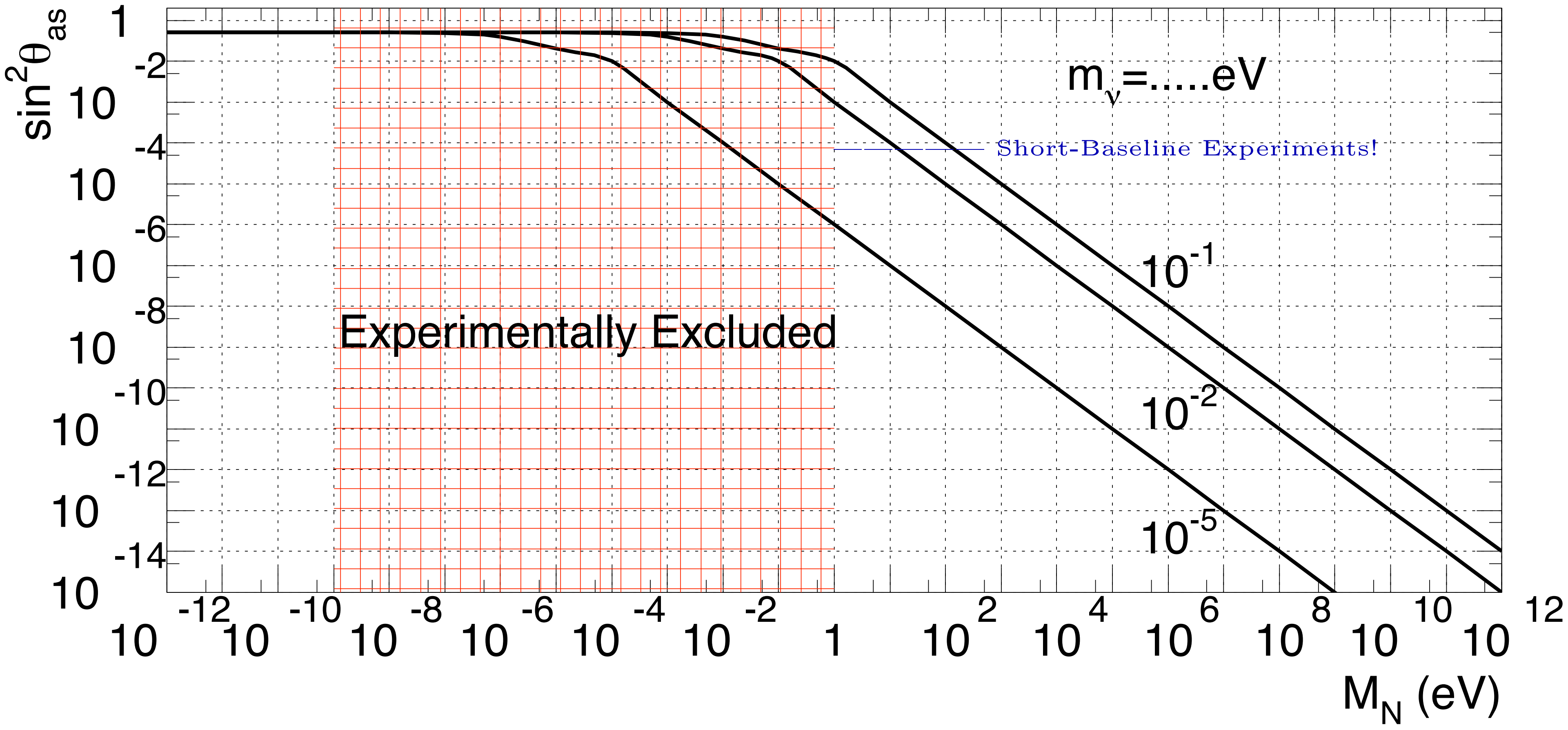
Constraining the Seesaw Lagrangian

[rough upper bound, see Donini et al, arXiv:1106.0064]



[AdG, Huang, Jenkins, arXiv:0906.1611]

Can we improve our sensitivity?



[AdG, Huang, Jenkins, arXiv:0906.1611]

Model independent constraints

Constraints depend, unfortunately, on m_i and M_i and R. E.g.,

$$U_{e4} = U_{e1}A\sqrt{\frac{m_1}{m_4}} + U_{e2}B\sqrt{\frac{m_2}{m_4}} + U_{e3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\mu4} = U_{\mu1}A\sqrt{\frac{m_1}{m_4}} + U_{\mu2}B\sqrt{\frac{m_2}{m_4}} + U_{\mu3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\tau4} = U_{\tau1}A\sqrt{\frac{m_1}{m_4}} + U_{\tau2}B\sqrt{\frac{m_2}{m_4}} + U_{\tau3}C\sqrt{\frac{m_3}{m_4}},$$

where

$$A^2 + B^2 + C^2 = 1.$$

One can pick A, B, C such that two of these vanish. But the other one is maximized, along with $U_{\alpha5}$ and $U_{\alpha6}$.

Can we (a) constrain the seesaw scale with combined bounds on $U_{\alpha4}$ or (b) testing the low energy seesaw if nonzero $U_{\alpha4}$ are discovered?

AdG, Huang arXiv:1110.6122

Concrete Example: 2 right-handed neutrinos

$$X_{\rm normal} = \begin{pmatrix} 0.23e^{i\phi} & 0.1e^{i\delta} \\ (0.25 - 0.02e^{-i\delta})e^{i\phi} & 0.70 \\ -(0.25 + 0.02e^{-i\delta})e^{i\phi} & 0.70 \end{pmatrix} \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix}$$

$$X_{\rm inverted} = \begin{pmatrix} 0.83e^{i\psi} & 0.55 \\ -(0.39 + 0.06e^{-i\delta})e^{i\psi} & 0.59 - 0.04e^{-i\delta} \\ (0.39 - 0.06e^{-i\delta})e^{i\psi} & -0.59 - 0.04e^{-i\delta} \end{pmatrix} \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix}$$

$$\zeta \in C$$

where

$$X_{\rm normal\ (inverted)} = \Theta\sqrt{\frac{m_{\rm heavy}}{m_3\ \ (m_2)}}$$

Some Relevant Examples: [AdG, W-C Huang, arXiv:1110.6122]

$\zeta = 3/4\pi + i$, $\delta = 6/5\pi$, $\phi = \pi/2$ and a normal mass hierarchy,

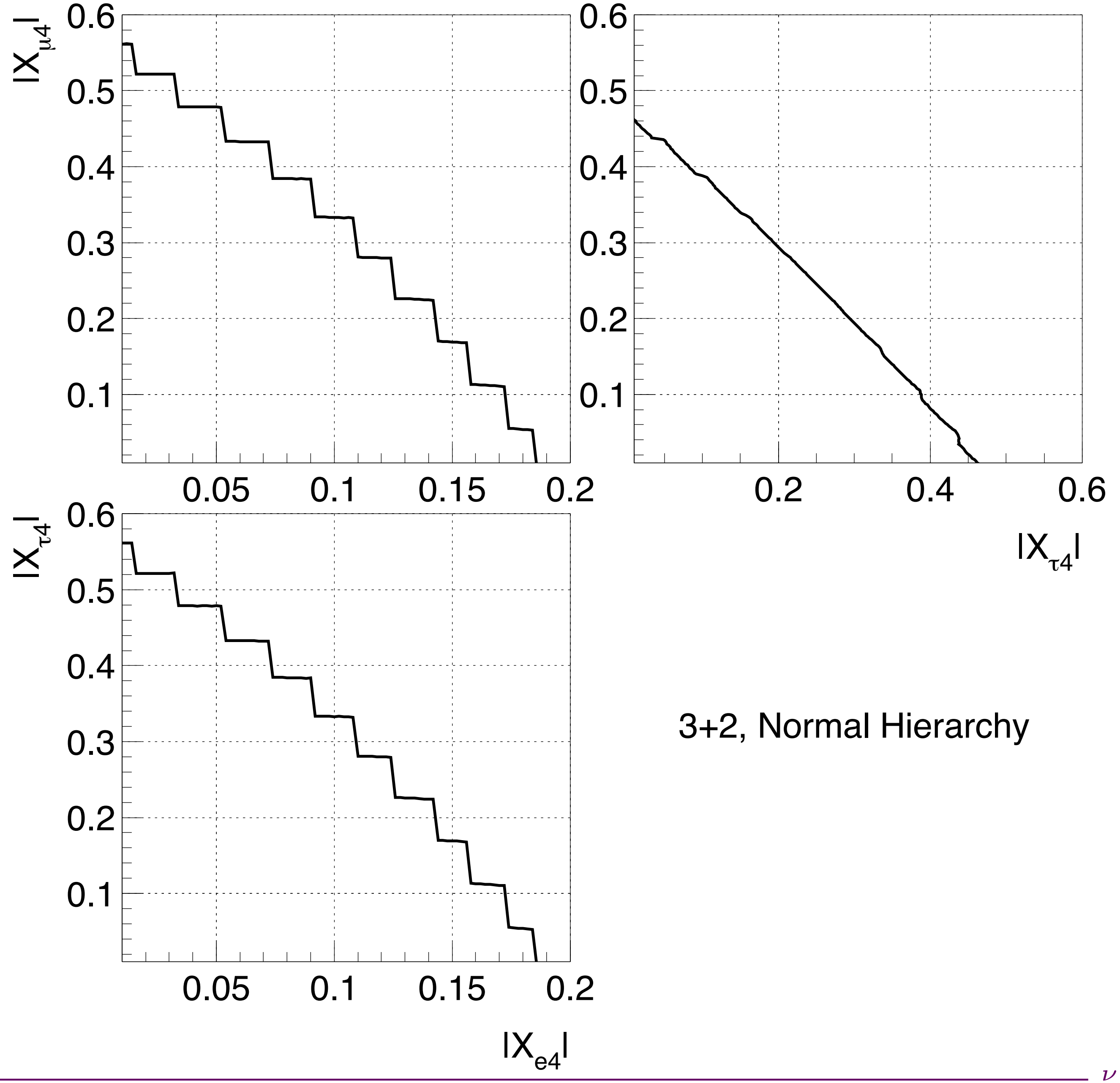
$$X_{\rm normal} = \left(\begin{array}{cc} 0.41e^{-0.66i} & 0.45e^{1.03i} \\ 0.62e^{2.67i} & 0.61e^{-2.62i} \\ 1.27e^{2.44i} & 1.26e^{-2.41i} \end{array} \right).$$

$\zeta = 2/3\pi + 0.3i$, $\delta = 0$, $\psi = \pi/2$, and an inverted mass hierarchy,

$$X_{\rm inverted} = \left(\begin{array}{cc} 0.44e^{-2.24i} & 0.62e^{1.83i} \\ 0.69e^{2.66i} & 0.66e^{-2.14i} \\ 0.71e^{-0.39i} & 0.60e^{0.89i} \end{array} \right).$$

both accommodate 3+2 fit for $m_4^2 = 0.5$ eV^2 and $m_5^2 = 0.9$ eV^2.Furthermore, $|U_{\tau 4}|$ and $|U_{\tau 5}|$ are completely fixed. No more free parameters. They are also both larger than (or at least as large as $|U_{\mu 4}|$ and $|U_{\mu 5}|$).

$\nu_\mu \to \nu_\tau$ MUST be observed if this is the origin of the two mostly sterile neutrinos.



|Xμ4|
0.6
0.5
0.4
0.3
0.2
0.1
0.05
0.1
0.15
0.2
0.6
0.5
0.4
0.3
0.2
0.1
0.2
0.4
0.6
|Xτ4|
|Xτ4|
0.6
0.5
0.4
0.3
0.2
0.1
0.05
0.1
0.15
0.2
|Xe4|
3+2, Normal Hierarchy

Making Predictions, for an inverted mass hierarchy, $m_4 = 1$ eV($\ll m_5$)

- ν_e disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{ee} > 0.02$. An interesting new proposal to closely expose the Daya Bay detectors to a strong β-emitting source would be sensitive to $\sin^2 2\vartheta_{ee} > 0.04$;

- ν_μ disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{\mu\mu} > 0.07$, very close to the most recent MINOS lower bound;

- $\nu_\mu \leftrightarrow \nu_e$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{e\mu} > 0.0004$;

- $\nu_\mu \leftrightarrow \nu_\tau$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{\mu\tau} > 0.001$. A $\nu_\mu \to \nu_\tau$ appearance search sensitive to probabilities larger than 0.1% for a mass-squared difference of 1 eV^2 would definitively rule out $m_4 = 1$ eV if the neutrino mass hierarchy is inverted.