# Model building with sterile neutrinos 

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Workshop "Crossroads of Neutrino Physics", 03.08.2015, Mainz, Germany

## Outline

- Status of experimental indications for sterile neutrinos $\nu_{s}$
- Model building with eV-scale $\nu_{s}$
- Model building with keV-scale $\nu_{s}$
- Four generations of (all) leptons?
- Model building with $\nu_{s}$ in GUTs
- Summary


## Experimental indications for eV -scale $\nu_{s}$

Several experiments observe anomalies

- LSND, MiniBooNE
- reactor neutrino fluxes
- Gallium anomaly
- however, $\nu_{\mu}$ disappearance experiments do not seem to be compatible with anomalies
- however, standard cosmology seems to be fine with only three light states


## Experimental indications for eV -scale $\nu_{s}$

Results of two different global fits (Giunti etal. ('13), Kopp et al. ('13))



## Experimental indications for keV -scale $\boldsymbol{\nu}_{s}$

## 3.5 keV line?

(Boyarsky et al. ('14))



## Experimental indications for keV -scale $\boldsymbol{\nu}_{s}$

## 3.5 keV line?

(Bulbul et al. ('14))



## Model building with $\boldsymbol{\nu}_{s}$

- in principle you can just add a new state to an existing model
- however, you should be able to give reason for mass as well as mixing of the new state
- according to the experimental indications the latter are
- eV-scale $\nu_{s}: m_{s} \sim 1 \mathrm{eV}$ and $\theta \sim 10^{-1}$
- keV-scale $\nu_{s}: m_{s} \sim 7 \mathrm{keV}$ and $\theta \sim 10^{-5}$
- probably you should also address the question why there is just one such (neutral) state and not three
- possible reasons: symmetry protects one state, additional state is really different (under flavor group)


## eV -scale $\boldsymbol{\nu}_{s}$ in $\boldsymbol{A}_{4}$ model

Idea and main features

- starting point is $A_{4}$ model leading to tri-bimaximal mixing
(Altarelli/Feruglio ('05))
- neutrino masses come from Weinberg operator
- charged lepton mass hierarchy is explained with Froggatt-Nielsen symmetry $U(1)_{F N}$
- lepton mixing is predicted, independent from neutrino mass spectrum
- add one gauge singlet $\nu_{s}^{c}$ that is a singlet of $A_{4}$ and only carries FN charge

$$
\nu_{s}^{c} \sim(\mathbf{1}, 1,6) \text { under }\left(A_{4}, Z_{3}, U(1)_{F N}\right)
$$

## eV -scale $\boldsymbol{\nu}_{s}$ in $\boldsymbol{A}_{\mathbf{4}}$ model

Details about the flavor group $A_{4}$

- this group is isomorphic to the group of even permutations of four different objects
- it has 12 elements
- it possesses four irreducible representations:

1 is real; $\mathbf{1}^{\prime}, \mathbf{1}^{\prime \prime}$ are complex conjugated;
3 is real

- it is a subgroup of $S O(3)$
- it belongs to a series of groups $A_{n}, n=1,2, \ldots$ of which, however, only $A_{4}$ and $A_{5}$ are "useful" for flavor model building


## eV -scale $\boldsymbol{\nu}_{s}$ in $\boldsymbol{A}_{\mathbf{4}}$ model

Particle content of the model

$$
\left(\omega=e^{2 \pi i / 3}\right)
$$

| Field | $L$ | $e^{c}$ | $\mu^{c}$ | $\tau^{c}$ | $\nu_{s}^{c}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A_{4}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}^{\prime \prime}$ | $\mathbf{1}^{\prime}$ | $\mathbf{1}$ |
| $Z_{3}$ | $\omega$ | $\omega^{2}$ | $\omega^{2}$ | $\omega^{2}$ | 1 |
| $U(1)_{F N}$ | 0 | 3 | 1 | 0 | 6 |
| Field | $h_{u, d}$ | $\varphi$ | $\varphi^{\prime}$ | $\xi$ | $\theta$ |
| $A_{4}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $Z_{3}$ | 1 | 1 | $\omega$ | $\omega$ | 1 |
| $U(1)_{F N}$ | 0 | 0 | 0 | 0 | -1 |

## eV -scale $\boldsymbol{\nu}_{\boldsymbol{s}}$ in $\boldsymbol{A}_{4}$ model

Lagrangian involving $\nu_{s}^{c}$

$$
\begin{aligned}
\mathcal{L}_{\nu_{s}^{c}}= & \frac{x_{e}}{\Lambda^{2}}\left(\frac{\theta}{\Lambda}\right)^{6} \xi\left(\varphi^{\prime} L h_{u}\right) \nu_{s}^{c}\left(+\frac{x_{f}}{\Lambda^{2}}\left(\frac{\theta}{\Lambda}\right)^{6}\left(\varphi^{\prime} \varphi^{\prime} L h_{u}\right) \nu_{s}^{c}\right) \\
& +\theta\left(\frac{\theta}{\Lambda}\right)^{11} \nu_{s}^{c} \nu_{s}^{c}+\text { h.c. }
\end{aligned}
$$

- take as expansion parameter VEV/ $\Lambda \approx 0.03$
- mass of $\nu_{s}^{c}$ appropriately suppressed by large FN charge: $m_{s} \sim\langle\theta\rangle \lambda^{11} \approx\langle\theta\rangle 10^{-17} \sim 1 \mathrm{eV}$
- coupling of $\nu_{s}^{c}$ also suppressed: $e \sim v_{u} \lambda^{8} \approx 0.1 \mathrm{eV}$
- lepton mixing also gets perturbed


## eV -scale $\boldsymbol{\nu}_{s}$ in $\boldsymbol{A}_{4}$ model

Mass matrix of neutrinos and $\nu_{s}$

$$
\mathcal{M}_{\nu}^{4 \times 4}=\left(\begin{array}{cccc}
a+2 d / 3 & -d / 3 & -d / 3 & e \\
\cdot & 2 d / 3 & a-d / 3 & e \\
\cdot & \cdot & 2 d / 3 & e \\
\cdot & \cdot & \cdot & m_{s}
\end{array}\right)
$$

We can extract as mixing matrix

$$
U=\left(\begin{array}{ccccc} 
& & & 0 \\
& \text { TB } & & 0 \\
& & & 0 \\
0 & 0 & 0 & 1
\end{array}\right)+\left(\begin{array}{ccc} 
& & \\
0 & & e / m_{s} \\
& & \\
0 & -\sqrt{3} e / m_{s} & 0
\end{array}\right)+\ldots
$$

## eV -scale $\boldsymbol{\nu}_{\boldsymbol{s}}$ in $\boldsymbol{A}_{\mathbf{4}}$ model

$$
\cdots+\left(\begin{array}{cccc}
0 & -\sqrt{3} e^{2} /\left(2 m_{s}^{2}\right) & 0 & 0 \\
0 & -\sqrt{3} e^{2} /\left(2 m_{s}^{2}\right) & 0 & 0 \\
0 & -\sqrt{3} e^{2} /\left(2 m_{s}^{2}\right) & 0 & 0 \\
0 & 0 & 0 & -3 e^{2} /\left(2 m_{s}^{2}\right)
\end{array}\right)
$$

We find

$$
\theta_{13}=0, \sin ^{2} \theta_{12,23} \text { are corrected at } \mathcal{O}\left(\left(e / m_{s}\right)^{2}\right)
$$

and
$U_{e 4}=U_{\mu 4}=U_{\tau 4} \quad$ and $\quad \sin ^{2} \theta_{i 4}=\mathcal{O}\left(\left(e / m_{s}\right)^{2}\right) \sim 10^{-2}$ with $i=1,2,3$

## eV -scale $\boldsymbol{\nu}_{s}$ and induced $\boldsymbol{\theta}_{13}$

The preceding model could not give rise to $\theta_{13}$ sufficiently large. This problem can be solved, if one uses a more general structure of vacuum of the field coupling $\nu_{s}^{c}$ to $L$ (Merle et al. ('14)).
In particular, one can find the relation (Merle et al. ('14))
$\sin \theta_{14}\left(\sin \theta_{24} e^{-i \beta}+\sin \theta_{34} e^{-i \gamma}\right) \approx \frac{\sqrt{2 \Delta m_{31}^{2}}}{m_{s}} e^{-i\left(\alpha-\delta_{2}\right)} \sin \theta_{13} \cos \theta_{13}$
See also very recent paper (Rivera-Agudelo/Perez-Lorenzana ('15)).

## keV-scale $\boldsymbol{\nu}_{s}$ in $\boldsymbol{A}_{\mathbf{4}}$ model

A side remark:
instead of an eV -scale $\nu_{s}$ one can easily implement a keV -scale $\nu_{s}$ that can play the role of Warm Dark Matter (Barry et al. ('11)).
For this to work FN charge of $\nu_{s}^{c}$ as well as scales in the model (VEVs, $\Lambda$ ) need to be adjusted.

## $\nu \mathrm{MSM}:$ characteristics

(Asaka et al. ('05), Asaka/Shaposhnikov ('05))

- add three right-handed (RH) neutrinos $N_{I}, I=1,2,3$, to Standard Model (SM)

$$
\delta \mathcal{L}=\bar{N}_{I} i \partial_{\mu} \gamma^{\mu} N_{I}-F_{\alpha I} \bar{L}_{\alpha} N_{I} \Phi-\frac{M_{I}}{2} \bar{N}_{I}^{c} N_{I}+\text { h.c. }
$$

with $L_{\alpha}, \alpha=e, \mu, \tau$, lepton doublets and $\Phi$ Higgs field

- 15 free parameters in $F_{\alpha I}$


## $\nu \mathrm{MSM}:$ characteristics

(Asaka et al. ('05), Asaka/Shaposhnikov ('05))
Phenomenology

- Dark Matter candidate $N_{1}$ with $M_{1} \sim$ few keV
- two heavier states, $1 \mathrm{GeV} \lesssim M_{2,3} \lesssim 10 \mathrm{GeV}$ that are highly degenerate in mass $\Delta M / M \sim 10^{-6}$ can explain baryon asymmetry of the Universe
- Yukawa couplings $F_{\alpha I}$ must be very suppressed and exhibit hierarchy
- light neutrino masses are strongly hierarchical, both orderings are admitted


## $\nu \mathrm{MSM}:$ characteristics

(Asaka et al. ('05), Asaka/Shaposhnikov ('05))

## Experimental tests

- experimental indications for $N_{1}, \mathrm{keV}$-scale $\nu_{s}$, have been found in 2014 (Boyarsky et al. ('14), Bulbul et al. ('14))
- states $N_{2,3}$ with masses of a few GeV can be tested at SHiP (Alekhin et al. ('15))


## $\nu \mathrm{MSM}$ with global lepton number

Idea and main features

- assume global lepton number with the following assignment

$$
N_{1}: q, N_{2}:-1, N_{3}: 1, L_{i}:-1 \text { and } E_{k}:-1
$$

with $q \neq 0, \pm 1$

- state $N_{1}$ is massless
- states $N_{2,3}$ have same mass: $M \bar{N}_{2}^{c} N_{3}$
- no mixing of state $N_{1}$ with active neutrinos, only $h_{k 2} \bar{L}_{k} N_{2} \Phi$
- masses of active neutrinos all vanish, lepton mixing is undetermined


## $\nu \mathrm{MSM}$ with global lepton number

Explicit breaking of global lepton number

$$
L_{\text {break }}=-\frac{1}{2} \bar{N}^{c} \Delta M N-\bar{L} \Delta F N \Phi
$$

$$
\Delta M=\left(\begin{array}{ccc}
m_{11} e^{i \alpha} & m_{12} & m_{13} \\
m_{12} & m_{22} e^{i \beta} & 0 \\
m_{13} & 0 & m_{33} e^{i \gamma}
\end{array}\right) \text { and } \Delta F=\left(\begin{array}{ccc}
h_{11} & 0 & h_{13} \\
h_{21} & 0 & h_{23} \\
h_{31} & 0 & h_{33}
\end{array}\right)
$$

with $m_{i j} \ll M$ and $h_{i 1}, h_{j 3} \ll h_{k 2}$.
The order of the lepton number breaking terms should be suppressed by $\epsilon \sim\left(10^{-4} \div 10^{-3}\right)$ relative to the invariant terms.
If $m_{i j} \sim \epsilon^{n} M$, then $\left|M_{2}-M_{3}\right| \sim M_{1}$.
Lepton mixing is adjusted correctly by choice of parameters $h_{i j}$.

## Version of $\boldsymbol{\nu} \mathrm{MSM}$ with $\boldsymbol{L}_{e}-\boldsymbol{L}_{\mu}-\boldsymbol{L}_{\tau}$

Idea and main features

- neutrino masses receive in this model also a contribution from type-II seesaw mechanism (Bezrukov et al. ('09))
- assume $L_{e}-L_{\mu}-L_{\tau}$ as exact symmetry at leading order

$$
\begin{gathered}
L_{e L}: 1, L_{\mu L}:-1, L_{\tau L}:-1, e_{R}: 1, \mu_{R}:-1, \tau_{R}:-1, \\
N_{1 R}: 1, N_{2 R}:-1, N_{3 R}:-1
\end{gathered}
$$

- scalar fields $\phi$ and $\Delta$ do not transform under $L_{e}-L_{\mu}-L_{\tau}$
- model leads to realistic results, if $L_{e}-L_{\mu}-L_{\tau}$ is explicitly (softly) broken


## Version of $\boldsymbol{\nu} \mathrm{MSM}$ with $\boldsymbol{L}_{e}-\boldsymbol{L}_{\mu}-\boldsymbol{L}_{\tau}$

## Phenomenology

- if $L_{e}-L_{\mu}-L_{\tau}$ is unbroken, one neutrino is massless and two have degenerate masses (active as well as sterile ones)
- breaking $L_{e}-L_{\mu}-L_{\tau}$ leads to keV -scale $\nu_{s}$ and active neutrino masses with inverted ordering
- if $L_{e}-L_{\mu}-L_{\tau}$ is unbroken, lepton mixing angles are fixed:
$\theta_{13}=0, \theta_{23}=\pi / 4$ and $\theta_{12}=\pi / 4$
- breaking $L_{e}-L_{\mu}-L_{\tau}$ introduces corrections to mixing, in particular coming from the charged lepton sector (most relevant for the solar mixing angle)

Version of $\boldsymbol{\nu} \mathrm{MSM}$ with $\boldsymbol{L}_{e}-\boldsymbol{L}_{\boldsymbol{\mu}}-\boldsymbol{L}_{\tau}$


$$
L_{e}-L_{\mu}-L_{\tau}
$$

$$
L_{\rho}-L_{\tau}<I_{\tau}
$$


(Merle/Niro ('11))

## $\boldsymbol{\nu}$ MSM with flavor symmetry $\boldsymbol{Q}_{6}$

Idea and main features

- use finite, discrete, non-abelian symmetry $Q_{6} \simeq D_{3}^{\prime}$, because it is smallest group with complex singlet and real doublet
- assign sterile neutrinos to particular representations of $Q_{6}$ :
- keV-scale $\nu_{s}$ separated from others: singlet; choose complex singlet in order to suppress mass term
- GeV-scale $\nu_{s}$ have almost same mass: doublet; choose real doublet in order to have mass unsuppressed
- hierarchy of mixing between active and different sterile states $N_{1}$ and $N_{2,3}$ is explained


## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

Details about the flavor group $Q_{6}$

- this group has 12 elements
- it possesses six irreducible representations:
$1, \mathbf{1}^{\prime}$ are real; $\mathbf{1}^{\prime \prime}, \mathbf{1}^{\prime \prime \prime}$ are complex conjugated;
2 is pseudo-real and $2^{\prime}$ is real
- it is a subgroup of $S U(2)$
- it belongs to a series of groups $Q_{2 n} \simeq D_{n}^{\prime}, n=1,2, \ldots$, that all have similar properties


## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

## Particle content of the model

$$
\left(\omega=e^{2 \pi i / 3}\right)
$$

| Field | $L_{1}$ | $L_{D}$ | $E_{1}$ | $E_{D}$ | $N_{1}$ | $N_{D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{6}$ | $\mathbf{1}$ | $\mathbf{2}^{\prime}$ | $\mathbf{1}^{\prime}$ | $\mathbf{2}$ | $\mathbf{1}^{\prime \prime}$ | $\mathbf{2}^{\prime}$ |
| $Z_{3}$ | 1 | 1 | $\omega^{2}$ | $\omega$ | 1 | 1 |
| $Z_{2}$ | + | + | + | + | + | - |


| Field | $H$ | $S_{x}$ | $S_{y}$ | $S_{z}$ | $D$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{6}$ | $\mathbf{1}$ | $\mathbf{1}^{\prime \prime}$ | $\mathbf{1}^{\prime \prime \prime}$ | $\mathbf{1}$ | $\mathbf{2}^{\prime}$ |
| $Z_{3}$ | 1 | $\omega^{2}$ | $\omega^{2}$ | 1 | 1 |
| $Z_{2}$ | + | + | + | - | - |

## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

## Lagrangian of sterile neutrinos

$\mathcal{L}_{M}=m_{a}\left(N_{D} N_{D}\right)_{\mathbf{1}}+\frac{m_{b}}{\Lambda^{2}}\left(N_{D} N_{D}\right)_{\mathbf{2}^{\prime}}(D D)_{\mathbf{2}^{\prime}}+\frac{m_{c}}{\Lambda^{2}} N_{1} N_{1} S_{x} S_{y}^{\star}+$ h.c.
leads to mass matrix

$$
M_{R}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & m_{a} \\
0 & m_{a} & 0
\end{array}\right)+\frac{1}{\Lambda^{2}}\left(\begin{array}{ccc}
m_{c}\left\langle S_{x} S_{y}\right\rangle & 0 & 0 \\
0 & m_{b}\left\langle D_{2}\right\rangle^{2} & 0 \\
0 & 0 & m_{b}\left\langle D_{1}\right\rangle^{2}
\end{array}\right)
$$

## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

Lagrangian of sterile neutrinos
$\mathcal{L}_{M}=m_{a}\left(N_{D} N_{D}\right)_{\mathbf{1}}+\frac{m_{b}}{\Lambda^{2}}\left(N_{D} N_{D}\right)_{\mathbf{2}^{\prime}}(D D)_{\mathbf{2}^{\prime}}+\frac{m_{c}}{\Lambda^{2}} N_{1} N_{1} S_{x} S_{y}^{\star}+$ h.c.

- assume $m_{a, b, c} \sim \mathcal{O}(\mathrm{GeV})$
- masses $M_{2,3}$ almost degenerate and of order GeV
- $M_{1} \ll M_{2,3}$ suppressed by $\left\langle S_{x} S_{y}\right\rangle / \Lambda^{2} \sim 10^{-6}: M_{1} \sim \mathcal{O}(\mathrm{keV})$
- splitting $M_{2,3}$ is of order $\left\langle D_{1,2}\right\rangle^{2} / \Lambda^{2} \sim 10^{-6}$
- if $\left\langle D_{1}\right\rangle=\left\langle D_{2}\right\rangle$, maximal 2-3 mixing


## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

Charged leptons

- all arise at the non-renormalizable level

$$
m_{e} \sim \frac{\left\langle S_{x}\right\rangle^{2}}{\Lambda^{2}} v, m_{\mu, \tau} \sim \frac{\left\langle S_{x}\right\rangle}{\Lambda} v, \frac{\left\langle S_{y}\right\rangle}{\Lambda} v
$$

- explanation of $m_{e} \ll m_{\mu}$ via hierarchy in operators
- instead $m_{\mu} \ll m_{\tau}$ is fine-tuned
- $m_{\tau} \ll v$ is explained
- estimate

$$
\frac{\left\langle S_{x}\right\rangle}{\Lambda}, \frac{\left\langle S_{y}\right\rangle}{\Lambda} \sim\left(10^{-3} \div 10^{-2}\right)
$$

## $\boldsymbol{\nu} \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

Terms generating Dirac mass matrix

$$
\begin{aligned}
\mathcal{L}_{\nu}= & \frac{\alpha}{\Lambda} \bar{L}_{1} \tilde{H}\left(N_{D} D\right)_{\mathbf{1}}+\frac{\beta}{\Lambda}\left(\bar{L}_{D} \tilde{H} N_{D}\right)_{\mathbf{2}^{\prime}} D+\frac{\gamma}{\Lambda}\left(\bar{L}_{D} \tilde{H} N_{D}\right)_{\mathbf{1}} S_{z} \\
& +\frac{\delta}{\Lambda^{3}} \bar{L}_{1} \tilde{H} N_{1} S_{x}^{3}\left(+\frac{\epsilon}{\Lambda^{3}}\left(\bar{L}_{D} \tilde{H} N_{D}\right)_{\mathbf{1}^{\prime}} S_{x} S_{y}^{\star} S_{z}\right)+\text { h.c. }
\end{aligned}
$$

leads to mass matrix

$$
M_{D}=\frac{v}{\Lambda}\left(\begin{array}{ccc}
0 & \alpha\left\langle D_{2}\right\rangle & \alpha\left\langle D_{1}\right\rangle \\
0 & \beta\left\langle D_{1}\right\rangle & \gamma\left\langle S_{z}\right\rangle \\
0 & \gamma\left\langle S_{z}\right\rangle & \beta\left\langle D_{2}\right\rangle
\end{array}\right)+\frac{v}{\Lambda^{3}}\left(\begin{array}{ccc}
\delta\left\langle S_{x}\right\rangle^{3} & 0 & 0 \\
0 & 0 & \ldots \\
0 & \ldots & 0
\end{array}\right)
$$

## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

- if $\left\langle D_{1}\right\rangle=\left\langle D_{2}\right\rangle$, lepton mixing angles $\theta_{23}=\pi / 4$ and $\theta_{13}=0$
- relative suppression of coupling of $N_{1}$ to light neutrinos compared to couplings of $N_{2,3}$
- however, suppression of all couplings is not sufficient: $\left\langle S_{i}, D_{i}\right\rangle / \Lambda \sim 10^{-3}$; thus, we need small couplings $\alpha, \ldots, \delta$ of order $10^{-4.5}$
- mixing of keV -scale $\nu_{s}$ with active neutrinos depends on $\delta$; can be different from $\alpha, \ldots, \gamma$


## $\nu \mathrm{MSM}$ with flavor symmetry $\boldsymbol{Q}_{6}$

Phenomenology of light neutrinos

- strongly hierarchical neutrino masses with normal ordering and $m_{1} \approx 0$
- atmospheric and reactor mixing angles

$$
\tan \theta_{23} \approx 1+\mathcal{O}\left(\frac{\left\langle D_{1}\right\rangle-\left\langle D_{2}\right\rangle}{\left\langle D_{1}\right\rangle+\left\langle D_{2}\right\rangle}\right) \text { and } \sin \theta_{13} \approx \mathcal{O}\left(\frac{\left\langle D_{1}\right\rangle-\left\langle D_{2}\right\rangle}{\left\langle D_{1}\right\rangle+\left\langle D_{2}\right\rangle}\right)
$$

- no prediction for solar mixing angle; is function of $\alpha, \beta$ and $\gamma$


## keV -scale $\boldsymbol{\nu}_{s}$ more general

- using a "general" global $U(1)$ symmetry (Froggatt-Nielsen symmetry) one can achieve a spectrum with one keV-scale $\nu_{s}$, two states with much larger masses as well as describe light neutrino masses and lepton mixing correctly
(Merle/Niro ('11))
- alternative idea: "split seesaw mechanism" (Kusenko et al. ('10)) use extra dimension and different localization of RH neutrinos in order to achieve split spectrum; exponential suppression of masses is possible ( $M_{i} \sim e^{-2 m_{i} l}$ ), also of couplings ( $\lambda_{i} \sim e^{-m_{i} l}$ ) [numerical example: $m_{2} \simeq 2.3 l^{-1}$ leads to $M_{2} \sim 10^{12} \mathrm{GeV}$, whereas $m_{1} \simeq 24 l^{-1}$ leads to $M_{1} \sim \mathrm{keV}$ ]


## Split seesaw mechanism



## Split seesaw mechanism with $\boldsymbol{A}_{4}$

(Adulpravitchai/Takahashi ('11))

- challenge 1: RH neutrino masses should be split, but in traditional $A_{4}$ models (Altarelliferuglio ('05)) these are assigned to 3 of $A_{4}$ in order to achieve (close to) tri-bimaximal mixing
- splitting of RH neutrino masses achieved via new particle assignment

$$
N_{R 1} \sim \mathbf{1}, \quad N_{R 2} \sim \mathbf{1}^{\prime}, \quad N_{R 3} \sim \mathbf{1}^{\prime \prime}
$$

- lepton mixing angles are partly predicted

$$
\theta_{13}=0, \theta_{23}=\pi / 4 \text { and } \theta_{12} \text { arbitrary }
$$

## Split seesaw mechanism with $\boldsymbol{A}_{4}$

- challenge 2: lightest RH neutrino with mass of order keV can only be Dark Matter candidate, if additional contribution to light neutrino masses exists
- so, neutrino masses arise from type I+II seesaw mechanism
- predictions for lepton mixing angles are maintained

In case you do not like extra dimensions:
a similar model has also been considered in four dimensions only with the flavor symmetry $A_{4}$ (Barry et al. ('11)).
The main features are the same: RH neutrinos in singlets of $A_{4}$ and mass hierarchy among these is achieved via Froggatt-Nielsen symmetry.

## Four generations of (all) leptons?

- $A_{5}$ model with four chiral lepton generations, three vector-like charged leptons and six RH neutrinos
(Chen et al. ('10))
- models with four chiral families and four RH neutrinos
(Schmidt/Smirnov ('11))
- main purpose of this study: analysis of different contributions to light neutrino masses
- sketches of models with flavor group $\operatorname{SG}(20,3)$ :
$L \sim 4, e_{R} \sim \mathbf{1}_{\mathbf{2}}+\mathbf{1}_{\mathbf{3}}+\mathbf{1}_{\mathbf{4}}+\mathbf{1}_{\mathbf{1}}$
and RH neutrinos transform either as

$$
N_{R} \sim \mathbf{1}_{\mathbf{2}}+\mathbf{1}_{\mathbf{3}}+\mathbf{1}_{\mathbf{4}}+\mathbf{1}_{\mathbf{1}} \quad \text { or } \quad N_{R} \sim \mathbf{4}
$$

## Model building with $\nu_{s}$ in GUTs

In $S U(5)$ RH neutrinos appear in different GUT multiplets:

$$
\Psi_{10}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccccc}
0 & u_{3}^{c} & -u_{2}^{c} & u^{1} & d^{1} \\
-u_{3}^{c} & 0 & u_{1}^{c} & u^{2} & d^{2} \\
u_{2}^{c} & -u_{1}^{c} & 0 & u^{3} & d^{3} \\
-u^{1} & -u^{2} & -u^{3} & 0 & e^{c} \\
-d^{1} & -d^{2} & -d^{3} & -e^{c} & 0
\end{array}\right) \quad \Psi_{\overline{5}}=\left(\begin{array}{c}
d_{1}^{c} \\
d_{2}^{c} \\
d_{3}^{c} \\
e \\
-\nu
\end{array}\right) \quad \Psi_{1}=\nu^{c}
$$

$\mathcal{L}=y_{u} \Psi_{10} \Psi_{10} H_{5}+y_{d, 1} \Psi_{\overline{5}} \Psi_{10} H_{\overline{5}}+y_{d, 2} \Psi_{\overline{5}} \Psi_{10} H_{\overline{45}}+y_{\nu} \Psi_{\overline{5}} \Psi_{1} H_{5}+M \Psi_{1} \Psi_{1}$
In $S O(10)$ they are naturally included in the representation $\mathbf{1 6}$ :

$$
\Psi_{16}=\mathbf{1 0}_{S U(5)}+\overline{\mathbf{5}}_{S U(5)}+\mathbf{1}_{S U(5)}
$$

## Model building with $\nu_{s}$ in GUTs

However, there can be additional singlets $S_{i}$ in $S O(10)$ : double seesaw mechanism (Mohapatra ('866), Mohapatra/Valle ('86)).
This mechanism is interesting, since it can explain the difference between charged fermions and neutrinos;
in particular, the huge difference between the up quark and light neutrino mass matrix. (Het al. ('08))

$$
\begin{aligned}
& \Psi_{16}=\mathbf{1 0}_{S U(5)}+\overline{\mathbf{5}}_{S U(5)}+\mathbf{1}_{S U(5)} \quad \text { and gauge singlets } S_{i} \\
& \mathcal{L}=y_{u} \Psi_{16} \Psi_{16} H_{10}+y_{16 S} \Psi_{16} S H_{\overline{16}}+M_{S S} S S \\
& m_{D}=y_{u}\left\langle H_{10}\right\rangle \propto M_{u}, \quad M_{N S}=y_{16 S}\left\langle H_{\overline{16}}\right\rangle \\
& \left\langle H_{\overline{16}}\right\rangle \propto M_{G U T} \quad \text { and } \quad M_{S S} \propto M_{\text {Planck }}
\end{aligned}
$$

## Model building with $\nu_{s}$ in GUTs

However, there can be additional singlets $S_{i}$ in $S O(10)$ : double seesaw mechanism (Mohapatra ('866), Mohapatra/Valle ('86)).
This mechanism is interesting, since it can explain the difference between charged fermions and neutrinos; in particular, the huge difference between the up quark and light neutrino mass matrix.

$$
\begin{gathered}
\left(\nu_{L}, N, S\right)\left(\begin{array}{ccc}
0 & m_{D} & 0 \\
m_{D}^{T} & 0 & M_{N S} \\
0 & M_{N S}^{T} & M_{S S}
\end{array}\right)\left(\begin{array}{c}
\nu_{L} \\
N \\
S
\end{array}\right) \\
m_{\nu}^{D S}=m_{D}\left(M_{N S}^{-1 T} M_{S S} M_{N S}^{-1}\right) m_{D}^{T}
\end{gathered}
$$

## Model building with $\nu_{s}$ in GUTs

However, there can be additional singlets $S_{i}$ in $S O(10)$ : double seesaw mechanism (Mohapatra ('86), Mohapatra/Valle ('86)).
This mechanism is interesting, since it can explain the difference between charged fermions and neutrinos;
in particular, the huge difference between the up quark and light neutrino mass matrix.

$$
m_{\nu}^{D S}=m_{D}\left(M_{N S}^{-1 T} M_{S S} M_{N S}^{-1}\right) m_{D}^{T}
$$

If $M_{N S} \propto m_{D}^{T}$, then
$m_{\nu}^{D S} \propto M_{S S}$ and thus hierarchy is (partly) cancelled (Hetal. ('08))
Examples for suitable flavor symmetries: $T_{7}$ and $\Sigma(81)$.

## Summary

- some ideas exist in the literature for explaining properties of eV-scale or keV-scale $\nu_{s}$
- however, model building is very challenging and by now no satisfactory model has been proposed
- $\nu_{s}$ can also have interesting effects in GUTs
- surely, more work on symmetries and models is needed

Thanks for your attention.

