Model building with sterile neutrinos

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Outline

- Status of experimental indications for sterile neutrinos ν_s
- Model building with eV-scale ν_s
- Model building with keV-scale ν_s
- Four generations of (all) leptons?
- Model building with ν_s in GUTs
- Summary



Experimental indications for eV-scale ν_s

Several experiments observe anomalies

- LSND, MiniBooNE
- reactor neutrino fluxes
- Gallium anomaly
- however, ν_{μ} disappearance experiments do not seem to be compatible with anomalies
- however, standard cosmology seems to be fine with only three light states



Experimental indications for eV-scale ν_s

Results of two different global fits (Giunti et al. ('13), Kopp et al. ('13))



Experimental indications for keV-scale ν_s



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Experimental indications for keV-scale ν_s



Model building with u_s

- in principle you can just add a new state to an existing model
- however, you should be able to give reason for mass as well as mixing of the new state
- according to the experimental indications the latter are
 - eV-scale ν_s : $m_s \sim 1 \, {\rm eV}$ and $\theta \sim 10^{-1}$
 - keV-scale ν_s : $m_s \sim 7 \, {\rm keV}$ and $\theta \sim 10^{-5}$
- probably you should also address the question why there is just one such (neutral) state and not three
- possible reasons: symmetry protects one state, additional state is really different (under flavor group)



Idea and main features

(Barry et al. ('11))

- starting point is A₄ model leading to tri-bimaximal mixing (Altarelli/Feruglio ('05))
 - neutrino masses come from Weinberg operator
 - charged lepton mass hierarchy is explained with Froggatt-Nielsen symmetry $U(1)_{FN}$
 - lepton mixing is predicted, independent from neutrino mass spectrum
- add one gauge singlet ν_s^c that is a singlet of A_4 and only carries FN charge

 $\nu_s^c \sim (\mathbf{1}, 1, 6)$ under $(A_4, Z_3, U(1)_{FN})$



Details about the flavor group A_4

- this group is isomorphic to the group of even permutations of four different objects
- it has 12 elements
- it possesses four irreducible representations:
 1 is real; 1', 1" are complex conjugated;
 3 is real
- it is a subgroup of SO(3)
- it belongs to a series of groups A_n , n = 1, 2, ... of which, however, only A_4 and A_5 are "useful" for flavor model building



Particle content of the model

$$(\omega = e^{2\pi i/3})$$

Field		(e^{c}	μ^c	$ au^c$	ν_s^c
A_4	3		1	1 ''	1'	1
Z_3	ω	ω^2		ω^2	ω^2	1
$U(1)_{FN}$	0	3		1	0	6
Field	$h_{u,d}$		φ	φ'	ξ	θ
A_4	1		3	3	1	1
Z_3	1		1	ω	ω	1
$U(1)_{FN}$	0		0	0	0	$\parallel -1$

Lagrangian involving ν_s^c

$$\mathcal{L}_{\nu_s^c} = \frac{x_e}{\Lambda^2} \left(\frac{\theta}{\Lambda}\right)^6 \xi(\varphi' L h_u) \nu_s^c \left(+ \frac{x_f}{\Lambda^2} \left(\frac{\theta}{\Lambda}\right)^6 (\varphi' \varphi' L h_u) \nu_s^c \right) \\ + \theta \left(\frac{\theta}{\Lambda}\right)^{11} \nu_s^c \nu_s^c + \text{h.c.}$$

- take as expansion parameter ${\rm VEV}/\Lambda \approx 0.03$
- mass of ν_s^c appropriately suppressed by large FN charge: $m_s \sim \langle \theta \rangle \, \lambda^{11} \approx \langle \theta \rangle \, 10^{-17} \sim 1 \, \text{eV}$
- coupling of ν_s^c also suppressed: $e \sim v_u \, \lambda^8 \approx 0.1 \, \mathrm{eV}$
- lepton mixing also gets perturbed

Mass matrix of neutrinos and ν_s

$$\mathcal{M}_{\nu}^{4 \times 4} = \begin{pmatrix} a + 2d/3 & -d/3 & -d/3 & e \\ . & 2d/3 & a - d/3 & e \\ . & . & 2d/3 & e \\ . & . & . & m_s \end{pmatrix}$$

We can extract as mixing matrix

$$U = \begin{pmatrix} & & 0 \\ & \mathsf{TB} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} & & & e/m_s \\ & & & e/m_s \\ & & & e/m_s \\ 0 & -\sqrt{3}e/m_s & 0 & 0 \end{pmatrix} + \dots$$



$$\dots + \begin{pmatrix} 0 & -\sqrt{3}e^2/(2m_s^2) & 0 & 0 \\ 0 & -\sqrt{3}e^2/(2m_s^2) & 0 & 0 \\ 0 & -\sqrt{3}e^2/(2m_s^2) & 0 & 0 \\ 0 & 0 & 0 & -3e^2/(2m_s^2) \end{pmatrix}$$

We find

$$heta_{13} = 0$$
 , $\sin^2 heta_{12,23}$ are corrected at $\mathcal{O}((e/m_s)^2)$

and

$$U_{e4} = U_{\mu4} = U_{\tau4}$$
 and $\sin^2 \theta_{i4} = \mathcal{O}((e/m_s)^2) \sim 10^{-2}$ with $i = 1, 2, 3$



eV-scale ν_s and induced θ_{13}

The preceding model could not give rise to θ_{13} sufficiently large. This problem can be solved, if one uses a more general structure of vacuum of the field coupling ν_s^c to *L* (Merle et al. ('14)). In particular, one can find the relation (Merle et al. ('14))

$$\sin\theta_{14} \left(\sin\theta_{24} e^{-i\beta} + \sin\theta_{34} e^{-i\gamma} \right) \approx \frac{\sqrt{2\Delta m_{31}^2}}{m_s} e^{-i(\alpha - \delta_2)} \sin\theta_{13} \cos\theta_{13}$$

See also very recent paper (Rivera-Agudelo/Perez-Lorenzana ('15)).



A side remark:

instead of an eV-scale ν_s one can easily implement a keV-scale ν_s that can play the role of Warm Dark Matter (*Barry et al. ('11)*). For this to work FN charge of ν_s^c as well as scales in the model (VEVs, Λ) need to be adjusted.



ν MSM: characteristics

(Asaka et al. ('05), Asaka/Shaposhnikov ('05))

• add three right-handed (RH) neutrinos N_I , I = 1, 2, 3, to Standard Model (SM)

$$\delta \mathcal{L} = \overline{N}_I i \,\partial_\mu \,\gamma^\mu \,N_I - F_{\alpha I} \,\overline{L}_\alpha N_I \,\Phi - \frac{M_I}{2} \,\overline{N}_I^c N_I + \text{h.c.}$$

with L_{α} , $\alpha = e, \mu, \tau$, lepton doublets and Φ Higgs field

• 15 free parameters in $F_{\alpha I}$



ν MSM: characteristics

(Asaka et al. ('05), Asaka/Shaposhnikov ('05))

Phenomenology

- Dark Matter candidate N_1 with $M_1 \sim \text{few keV}$
- two heavier states, $1 \text{ GeV} \lesssim M_{2,3} \lesssim 10 \text{ GeV}$ that are highly degenerate in mass $\Delta M/M \sim 10^{-6}$ can explain baryon asymmetry of the Universe
- Yukawa couplings $F_{\alpha I}$ must be very suppressed and exhibit hierarchy
- light neutrino masses are strongly hierarchical, both orderings are admitted



ν MSM: characteristics

(Asaka et al. ('05), Asaka/Shaposhnikov ('05))

Experimental tests

- experimental indications for N_1 , keV-scale ν_s , have been found in 2014 (Boyarsky et al. ('14), Bulbul et al. ('14))
- states $N_{2,3}$ with masses of a few GeV can be tested at SHiP (Alekhin et al. ('15))



ν MSM with global lepton number

Idea and main features

(Shaposhnikov ('06))

assume global lepton number with the following assignment

 N_1 : q , N_2 : -1 , N_3 : 1 , L_i : -1 and E_k : -1

with $q \neq 0, \pm 1$

- state N₁ is massless
- states $N_{2,3}$ have same mass: $M \overline{N}_2^c N_3$
- no mixing of state N_1 with active neutrinos, only $h_{k2} \overline{L}_k N_2 \Phi$
- masses of active neutrinos all vanish, lepton mixing is undetermined



ν MSM with global lepton number

Explicit breaking of global lepton number

$$L_{\text{break}} = -\frac{1}{2} \overline{N}^{c} \Delta M N - \overline{L} \Delta F N \Phi$$
$$\Delta M = \begin{pmatrix} m_{11} e^{i \alpha} & m_{12} & m_{13} \\ m_{12} & m_{22} e^{i \beta} & 0 \\ m_{13} & 0 & m_{33} e^{i \gamma} \end{pmatrix} \text{ and } \Delta F = \begin{pmatrix} h_{11} & 0 & h_{13} \\ h_{21} & 0 & h_{23} \\ h_{31} & 0 & h_{33} \end{pmatrix}$$

with $m_{ij} \ll M$ and h_{i1} , $h_{j3} \ll h_{k2}$. The order of the lepton number breaking terms should be suppressed by $\epsilon \sim (10^{-4} \div 10^{-3})$ relative to the invariant terms. If $m_{ij} \sim \epsilon^n M$, then $|M_2 - M_3| \sim M_1$. Lepton mixing is adjusted correctly by choice of parameters h_{ij} .

Version of ν MSM with $L_e - L_\mu - L_\tau$

Idea and main features

(Lindner et al. ('10))

- neutrino masses receive in this model also a contribution from type-II seesaw mechanism (Bezrukov et al. ('09))
- assume $L_e L_\mu L_\tau$ as exact symmetry at leading order

$$L_{eL}$$
 :1 , $L_{\mu L}$:-1 , $L_{\tau L}$:-1 , e_R :1 , μ_R :-1 , τ_R :-1 ,

$$N_{1R}$$
 : 1 , N_{2R} : -1 , N_{3R} : -1

- scalar fields ϕ and Δ do not transform under $L_e L_\mu L_\tau$
- model leads to realistic results, if $L_e L_\mu L_\tau$ is explicitly (softly) broken



Version of ν MSM with $L_e - L_\mu - L_\tau$

Phenomenology

- if $L_e L_\mu L_\tau$ is unbroken, one neutrino is massless and two have degenerate masses (active as well as sterile ones)
- breaking $L_e L_\mu L_\tau$ leads to keV-scale ν_s and active neutrino masses with inverted ordering
- if $L_e L_\mu L_\tau$ is unbroken, lepton mixing angles are fixed: $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \pi/4$
- breaking $L_e L_\mu L_\tau$ introduces corrections to mixing, in particular coming from the charged lepton sector (most relevant for the solar mixing angle)



Version of ν MSM with $L_e - L_\mu - L_\tau$

 $M_3 \approx M_2$ $M_2 = M_3 \approx \text{GeV}$ $M_2 \approx \text{GeV}$

 $L_e - L_\mu - L_\tau$ $L_e - L_\tau$

 $-M_1 \sim \text{keV}$ $-M_1 \equiv 0$

(Merle/Niro ('11))



Idea and main features

(Araki/Li ('11))

- use finite, discrete, non-abelian symmetry $Q_6 \simeq D'_3$, because it is smallest group with complex singlet and real doublet
- assign sterile neutrinos to particular representations of Q_6 :
 - keV-scale ν_s separated from others: singlet; choose complex singlet in order to suppress mass term
 - GeV-scale ν_s have almost same mass: doublet; choose real doublet in order to have mass unsuppressed
 - hierarchy of mixing between active and different sterile states N_1 and $N_{2,3}$ is explained



Details about the flavor group Q_6

- this group has 12 elements
- it possesses six irreducible representations:
 1, 1' are real; 1", 1" are complex conjugated;
 2 is pseudo-real and 2' is real
- it is a subgroup of SU(2)
- it belongs to a series of groups $Q_{2n} \simeq D'_n$, n = 1, 2, ..., that all have similar properties



Particle content of the model

$$(\omega = e^{2\pi i/3})$$

Field	L_1	L_D	E_1	E_D	N_1	N_D
Q_6	1	2 '	1 '	2	$1^{\prime\prime}$	2'
Z_3	1	1	ω^2	ω	1	1
Z_2	+	+	+	+	+	—

Field	H	S_x	S_y	S_z	D
Q_6	1	1″	$1^{\prime\prime\prime}$	1	2 '
Z_3	1	ω^2	ω^2	1	1
Z_2	+	+	+	_	_



Lagrangian of sterile neutrinos

$$\mathcal{L}_M = m_a \left(N_D N_D \right)_{\mathbf{1}} + \frac{m_b}{\Lambda^2} \left(N_D N_D \right)_{\mathbf{2}'} \left(DD \right)_{\mathbf{2}'} + \frac{m_c}{\Lambda^2} N_1 N_1 S_x S_y^{\star} + \text{h.c.}$$

leads to mass matrix

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_a \\ 0 & m_a & 0 \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} m_c \langle S_x S_y \rangle & 0 & 0 \\ 0 & m_b \langle D_2 \rangle^2 & 0 \\ 0 & 0 & m_b \langle D_1 \rangle^2 \end{pmatrix}$$



Lagrangian of sterile neutrinos

$$\mathcal{L}_{M} = m_{a} (N_{D} N_{D})_{\mathbf{1}} + \frac{m_{b}}{\Lambda^{2}} (N_{D} N_{D})_{\mathbf{2}'} (DD)_{\mathbf{2}'} + \frac{m_{c}}{\Lambda^{2}} N_{1} N_{1} S_{x} S_{y}^{\star} + \text{h.c.}$$

• assume
$$m_{a,b,c} \sim \mathcal{O}(\text{GeV})$$

- masses M_{2,3} almost degenerate and of order GeV
- $M_1 \ll M_{2,3}$ suppressed by $\langle S_x S_y \rangle / \Lambda^2 \sim 10^{-6}$: $M_1 \sim \mathcal{O}(\text{keV})$
- splitting $M_{2,3}$ is of order $\langle D_{1,2} \rangle^2 / \Lambda^2 \sim 10^{-6}$
- if $\langle D_1 \rangle = \langle D_2 \rangle$, maximal 2-3 mixing



Charged leptons

all arise at the non-renormalizable level

$$m_e \sim \frac{\langle S_x \rangle^2}{\Lambda^2} v , \ \ m_{\mu,\tau} \sim \frac{\langle S_x \rangle}{\Lambda} v , \ \frac{\langle S_y \rangle}{\Lambda} v$$

- explanation of $m_e \ll m_\mu$ via hierarchy in operators
- instead $m_\mu \ll m_ au$ is fine-tuned
- $m_{ au} \ll v$ is explained
- estimate

$$\frac{\langle S_x \rangle}{\Lambda}$$
, $\frac{\langle S_y \rangle}{\Lambda} \sim (10^{-3} \div 10^{-2})$



Terms generating Dirac mass matrix

$$\mathcal{L}_{\nu} = \frac{\alpha}{\Lambda} \overline{L}_{1} \tilde{H} (N_{D}D)_{1} + \frac{\beta}{\Lambda} (\overline{L}_{D} \tilde{H} N_{D})_{2'} D + \frac{\gamma}{\Lambda} (\overline{L}_{D} \tilde{H} N_{D})_{1} S_{z} + \frac{\delta}{\Lambda^{3}} \overline{L}_{1} \tilde{H} N_{1} S_{x}^{3} \left(+ \frac{\epsilon}{\Lambda^{3}} (\overline{L}_{D} \tilde{H} N_{D})_{1'} S_{x} S_{y}^{\star} S_{z} \right) + \text{h.c.}$$

leads to mass matrix

$$M_D = \frac{v}{\Lambda} \begin{pmatrix} 0 & \alpha \langle D_2 \rangle & \alpha \langle D_1 \rangle \\ 0 & \beta \langle D_1 \rangle & \gamma \langle S_z \rangle \\ 0 & \gamma \langle S_z \rangle & \beta \langle D_2 \rangle \end{pmatrix} + \frac{v}{\Lambda^3} \begin{pmatrix} \delta \langle S_x \rangle^3 & 0 & 0 \\ 0 & 0 & \dots \\ 0 & \dots & 0 \end{pmatrix}$$



- if $\langle D_1 \rangle = \langle D_2 \rangle$, lepton mixing angles $\theta_{23} = \pi/4$ and $\theta_{13} = 0$
- relative suppression of coupling of N_1 to light neutrinos compared to couplings of $N_{2,3}$
- however, suppression of all couplings is not sufficient: $\langle S_i, D_i \rangle / \Lambda \sim 10^{-3}$; thus, we need small couplings α , ..., δ of order $10^{-4.5}$
- mixing of keV-scale ν_s with active neutrinos depends on δ ; can be different from α , ..., γ



Phenomenology of light neutrinos

- strongly hierarchical neutrino masses with normal ordering and $m_1 \approx 0$
- atmospheric and reactor mixing angles

$$\tan \theta_{23} \approx 1 + \mathcal{O}\left(\frac{\langle D_1 \rangle - \langle D_2 \rangle}{\langle D_1 \rangle + \langle D_2 \rangle}\right) \quad \text{and} \quad \sin \theta_{13} \approx \mathcal{O}\left(\frac{\langle D_1 \rangle - \langle D_2 \rangle}{\langle D_1 \rangle + \langle D_2 \rangle}\right)$$

• no prediction for solar mixing angle; is function of α , β and γ



keV-scale ν_s more general

- using a "general" global U(1) symmetry (Froggatt-Nielsen symmetry) one can achieve a spectrum with one keV-scale ν_s , two states with much larger masses as well as describe light neutrino masses and lepton mixing correctly (Merle/Niro ('11))
- alternative idea: "split seesaw mechanism" (Kusenko et al. ('10)) use extra dimension and different localization of RH neutrinos in order to achieve split spectrum; exponential suppression of masses is possible ($M_i \sim e^{-2m_i l}$), also of couplings ($\lambda_i \sim e^{-m_i l}$) [numerical example: $m_2 \simeq 2.3 l^{-1}$ leads to $M_2 \sim 10^{12}$ GeV, whereas $m_1 \simeq 24 l^{-1}$ leads to $M_1 \sim \text{keV}$]



Split seesaw mechanism



(Merle/Niro ('11))



Split seesaw mechanism with A_4

(Adulpravitchai/Takahashi ('11))

- challenge 1: RH neutrino masses should be split, but in traditional A₄ models (*Altarelli/Feruglio ('05)*) these are assigned to 3 of A₄ in order to achieve (close to) tri-bimaximal mixing
- splitting of RH neutrino masses achieved via new particle assignment

$$N_{R1} \sim \mathbf{1} \ , \ N_{R2} \sim \mathbf{1'} \ , \ N_{R3} \sim \mathbf{1''}$$

lepton mixing angles are partly predicted

$$\theta_{13}=0$$
, $\theta_{23}=\pi/4$ and θ_{12} arbitrary



Split seesaw mechanism with A_4

- challenge 2: lightest RH neutrino with mass of order keV can only be Dark Matter candidate, if additional contribution to light neutrino masses exists
- so, neutrino masses arise from type I+II seesaw mechanism
- predictions for lepton mixing angles are maintained

In case you do not like extra dimensions:

a similar model has also been considered in four dimensions only with the flavor symmetry A_4 (Barry et al. ('11)).

The main features are the same: RH neutrinos in singlets of A_4 and mass hierarchy among these is achieved via Froggatt-Nielsen symmetry.

Four generations of (all) leptons?

- A₅ model with four chiral lepton generations, three vector-like charged leptons and six RH neutrinos (*Chen et al. ('10)*)
- models with four chiral families and four RH neutrinos (Schmidt/Smirnov ('11))
 - main purpose of this study: analysis of different contributions to light neutrino masses
 - sketches of models with flavor group SG(20, 3): $L \sim 4, e_R \sim \mathbf{1_2} + \mathbf{1_3} + \mathbf{1_4} + \mathbf{1_1}$ and RH neutrinos transform either as $N_R \sim \mathbf{1_2} + \mathbf{1_3} + \mathbf{1_4} + \mathbf{1_1}$ or $N_R \sim 4$



In SU(5) RH neutrinos appear in different GUT multiplets:

$$\Psi_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ -u_3^c & 0 & u_1^c & u^2 & d^2 \\ u_2^c & -u_1^c & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & e^c \\ -d^1 & -d^2 & -d^3 & -e^c & 0 \end{pmatrix} \quad \Psi_{\bar{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix} \quad \Psi_1 = \nu^c$$

 $\mathcal{L} = y_u \Psi_{10} \Psi_{10} H_5 + y_{d,1} \Psi_{\bar{5}} \Psi_{10} H_{\bar{5}} + y_{d,2} \Psi_{\bar{5}} \Psi_{10} H_{\overline{45}} + y_\nu \Psi_{\bar{5}} \Psi_1 H_5 + M \Psi_1 \Psi_1$

In SO(10) they are naturally included in the representation 16: $\Psi_{16} = \mathbf{10}_{SU(5)} + \mathbf{\overline{5}}_{SU(5)} + \mathbf{1}_{SU(5)}$



However, there can be additional singlets S_i in SO(10): double seesaw mechanism (Mohapatra ('86), Mohapatra/Valle ('86)). This mechanism is interesting, since it can explain the difference between charged fermions and neutrinos; in particular, the huge difference between the up quark and light

neutrino mass matrix. (H et al. ('08))

$$\begin{split} \Psi_{16} &= \mathbf{10}_{SU(5)} + \overline{\mathbf{5}}_{SU(5)} + \mathbf{1}_{SU(5)} & \text{ and gauge singlets } S_i \\ \mathcal{L} &= y_u \Psi_{16} \Psi_{16} H_{10} + y_{16S} \Psi_{16} S H_{\overline{16}} + M_{SS} S S \\ m_D &= y_u \langle H_{10} \rangle \propto M_u , \quad M_{NS} = y_{16S} \langle H_{\overline{16}} \rangle \\ \langle H_{\overline{16}} \rangle \propto M_{GUT} & \text{and} \quad M_{SS} \propto M_{Planck} \end{split}$$

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$$(\nu_L, N, S) \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_{NS} \\ 0 & M_{NS}^T & M_{SS} \end{pmatrix} \begin{pmatrix} \nu_L \\ N \\ S \end{pmatrix}$$

 $m_{\nu}^{DS} = m_D \left(M_{NS}^{-1\,T} \, M_{SS} \, M_{NS}^{-1} \right) m_D^T$



However, there can be additional singlets S_i in SO(10):

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$$m_{\nu}^{DS} = m_D \left(M_{NS}^{-1\,T} \, M_{SS} \, M_{NS}^{-1} \right) m_D^T$$

If $M_{NS} \propto m_D^T$, then

 $m_{
u}^{DS} \propto M_{SS}$ and thus hierarchy is (partly) cancelled (H et al. ('08))

Examples for suitable flavor symmetries: T_7 and $\Sigma(81)$.



Summary

- some ideas exist in the literature for explaining properties of eV-scale or keV-scale ν_s
- however, model building is very challenging and by now no satisfactory model has been proposed
- ν_s can also have interesting effects in GUTs
- surely, more work on symmetries and models is needed

Thanks for your attention.

