Recent developments and open issues on supernova neutrinos

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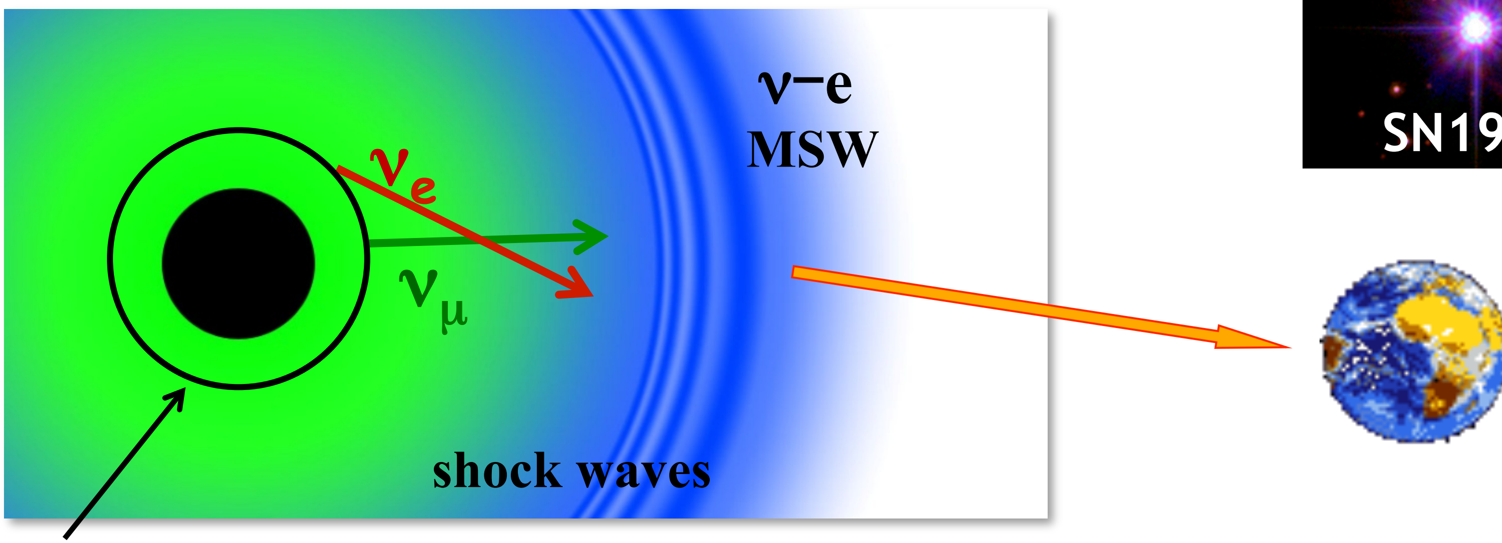
Outline

- ν *flavour conversion in supernovae*
- *Extended neutrino evolution equations*
- *Conclusions*

Flavour conversion in supernovae



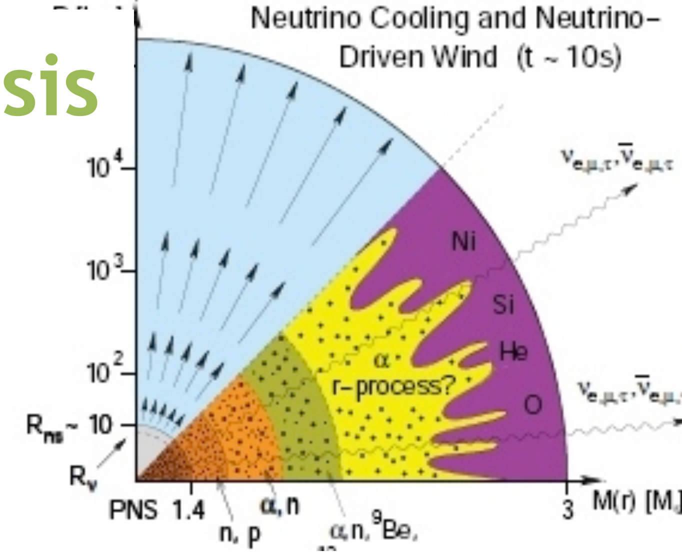
neutrinosphere

Numerous aspects investigated, different from the Sun.
In particular :

- ❑ **the** ν **interaction with** ν and **with matter** (MSW effect).
- ❑ dynamical aspects - **shock waves and turbulence**.

Precisely understanding how, important for observations and …

SN explosions & nucleosynthesis



Supernova neutrinos linked to key issues :

- **How do massive stars explode ?**
  Current simulations :
  multidimensional, realistic ν transport,
  convection and turbulence, hydrodynamical instabilities (SASI).

- **What are the sites where heavy elements are made ?**
  Candidate sites for heavy elements nucleosynthesis :
  supernovae, AD-BH, neutron star mergers.

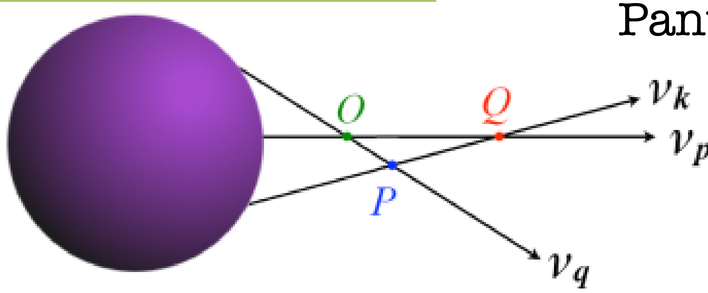
  Neutrinos determine if the site is neutron-rich
  (r-process elements) or proton-rich (νp-process).

see e.g. Focus issue «Nucleosynthesis and the role of neutrinos»,
J.Phys. G 41 (2014)

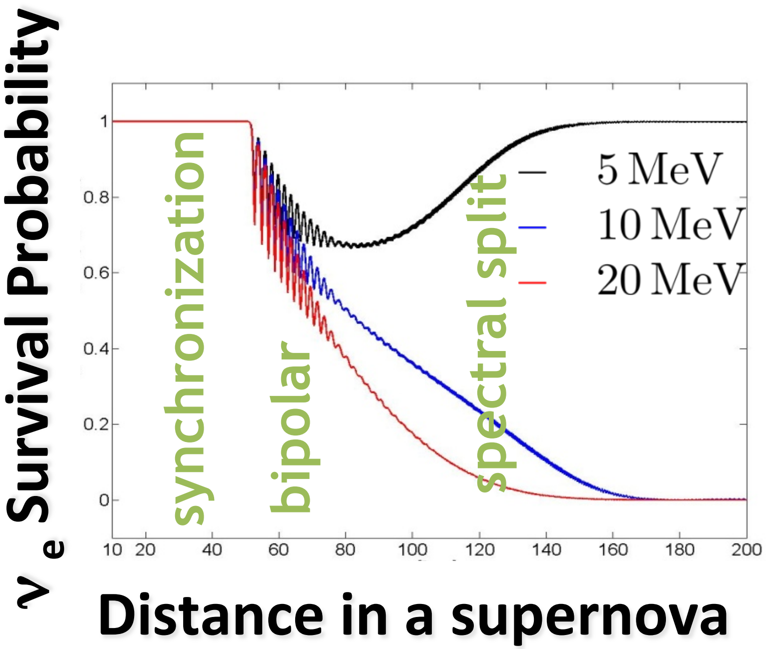
The νν interaction effects

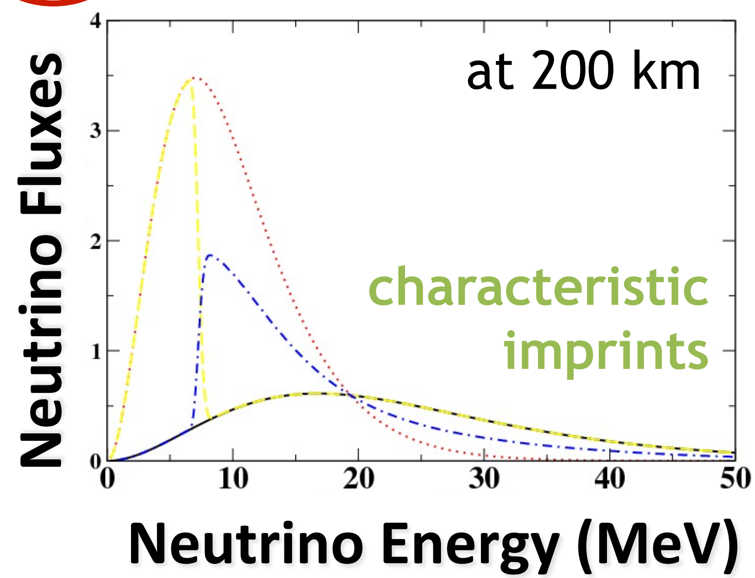
Pantaleone, PLB 287 (1992)

the «bulb» model



$$H = H_{vacuum} + H_{matter} + H_{\nu\nu}$$





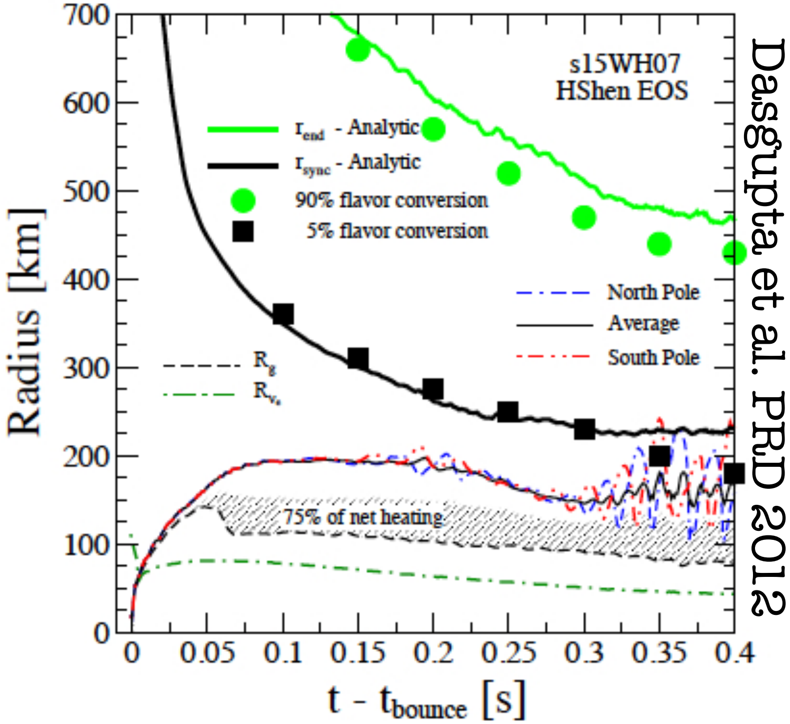
collective stable and unstable ν modes in flavor space

see e.g. Duan, Fuller, Qian, Ann. Rev. 60 (2010)

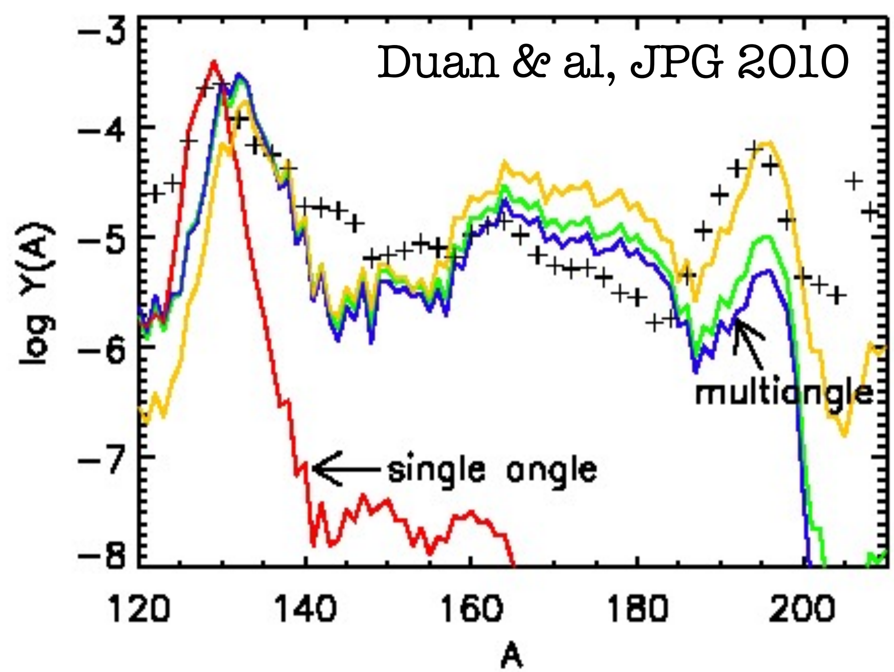
key open questions

Further work needed to finally assess the impact of flavour conversion

- ❑ on the shock wave



- ❑ on nucleosynthesis (r-process, νp process)



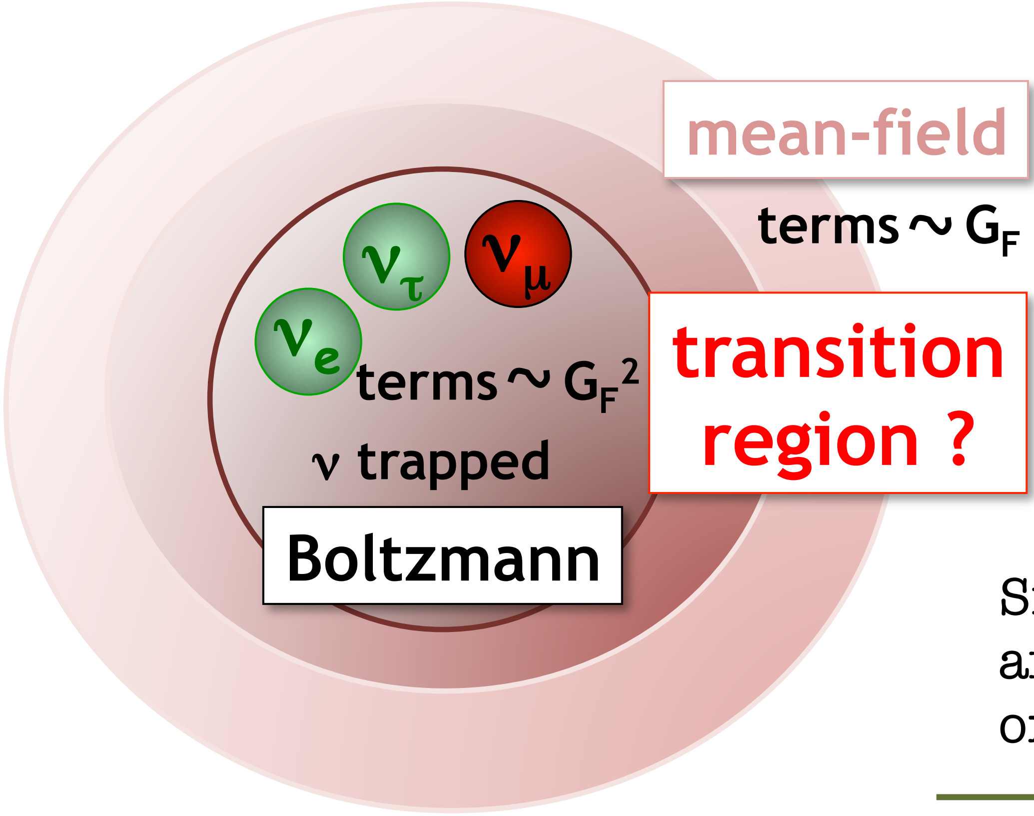
- ❑ role of decoherence, or of symmetry breaking

see e.g. Chakraborty, et al. PRL 107 (2011)

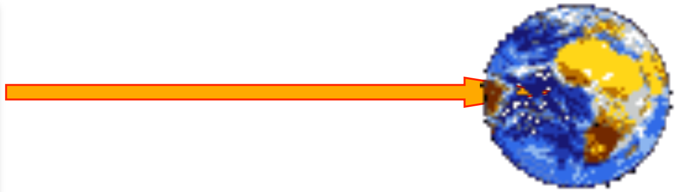
Theoretical description of ν evolution

Evolution equations are based on :

- ❑ mean-field approximation
- ❑ extended mean-field approximation
- ❑ Boltzmann approximation («molecular» chaos assumption)



Corrections discussed :
from collisions,
pairing correlations
and the neutrino mass



Small contributions can be amplified by the non-linearity of the equations.

Theoretical description of ν evolution

Volpe, arXiv : 1506.06222.

Different approaches used to derive evolution equations :

- density matrix formalism, perturbative expansion of interaction terms

  Dolgov, Sov. J. (1981), Stodolsky, Sigl, Raffelt, PRL 51993),
  Sigl, Raffelt, Nucl. Phys. B406 (1993), McKellar, Thomson PRD49 (1994),...

- Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy

  Volpe, Väänänen, Espinoza, PRD87 (2013)
  – ν-antiν pairing correlations, mass contributions.

- neutrino (iso)spins and precession equations

  see e.g. Balantekin, Pehlivan, Kajino, PRD84(2011), PRD90(2014).

- path integral approach

  Balantekin, Pehlivan, JPG 34 (2007)

- Green's functions, Closed-Time-Path and 2PI effective action.

  Yamada, PRD 62 (2000),
  Vlasenko, Fuller, Cirigliano, PRD89 (2014) – spin coherence
  see also Herranen et al, JHEP 1012 (2010), Fidler et al JHEP 1202 (2012)

Extended ν equations for astrophysical media

Serreau and Volpe, PRD 90 (2014), arXiv:1409.3591

General mean-field equations to describe neutrino propagation in an inhomogeneous medium, for massive Dirac (or Majorana) neutrinos.

The most general mean-field Hamiltonian takes the bilinear form :

$$H_{\text{eff}}(t) = \int d^3x\, \bar{\psi}_i(t,\vec{x})\Gamma_{ij}(t,\vec{x})\psi_j(t,\vec{x}),$$

All possible two-point correlators are :

NORMAL DENSITIES

$$\rho_{ij}(t,\vec{q},h,\vec{q}',h') = \langle a_j^\dagger(t,\vec{q}',h')a_i(t,\vec{q},h)\rangle,$$

$$\bar{\rho}_{ij}(t,\vec{q},h,\vec{q}',h') = \langle b_i^\dagger(t,\vec{q},h)b_j(t,\vec{q}',h')\rangle,$$

They usually present off-diagonal terms in flavour due to the mixings. They can also have helicity structure in presence of magnetic fields or of neutrino mass corrections.

ABNORMAL DENSITIES (PAIRING)

$$\kappa_{ij}(t,\vec{q},h,\vec{q}',h') = \langle b_j(t,\vec{q}',h')a_i(t,\vec{q},h)\rangle,$$

$$\kappa_{ij}^\dagger(t,\vec{q},h,\vec{q}',h') = \langle a_j^\dagger(t,\vec{q}',h')b_i^\dagger(t,\vec{q},h)\rangle,$$

The Ehrenfest theorem is used :

$$i\dot{\rho}_{ij}(t,\vec{q},h,\vec{q}',h') = \langle [a_j^\dagger(t,\vec{q}',h')a_i(t,\vec{q},h), H_{\text{eff}}(t)]\rangle.$$

INHOMOGENEOUS BACKGROUND, MASSIVE NEUTRINO CASE

Implementing the neutrino Dirac fields (similarly for Majorana fields)

$$\phi(\vec{x}) = \sum_h \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} [a(\vec{p},h) u_{\vec{p},h} e^{i\vec{p}\cdot\vec{x}} + b^\dagger(\vec{p},h) v_{\vec{p},h} e^{-i\vec{p}\cdot\vec{x}}]$$

the mean-field Hamiltonian has the quadratic form :

$$\boxed{\begin{aligned} H_{\rm eff}(t) &= a^\dagger(t)\cdot\Gamma^{\nu\nu}(t)\cdot a(t) + b(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t)\cdot b^\dagger(t) \\ &\quad + a^\dagger(t)\cdot\Gamma^{\nu\bar{\nu}}(t)\cdot b^\dagger(t) + b(t)\cdot\Gamma^{\bar{\nu}\nu}(t)\cdot a(t), \end{aligned}}$$

note that : $[A\cdot B]_{ij}(\vec{q},h,\vec{q}\,',h') \equiv \int_{\vec{p},s} A_{ik}(\vec{q},h,\vec{p},s) B_{kj}(\vec{p},s,\vec{q}\,',h').$

$$\Gamma^{\nu\nu}_{ij}(t,\vec{q},h,\vec{q}\,',h') = \bar{u}_i(\vec{q},h)\Gamma_{ij}(t,\vec{q}-\vec{q}\,')u_j(\vec{q}\,',h'),$$

The extended equation for the density matrix and ν-antiν correlators :

$$i\dot{\rho}(t) = \Gamma^{\nu\nu}(t)\cdot\rho(\quad i\dot{\rho} = [h(\rho),\rho] \quad + \Gamma^{\nu\bar{\nu}}(t)\cdot\kappa^\dagger(t) - \kappa(t)\cdot\Gamma^{\bar{\nu}\nu}(t).$$

$$i\dot{\kappa}(t) = \Gamma^{\nu\nu}(t)\cdot\kappa(t) - \kappa(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t) - \Gamma^{\nu\bar{\nu}}(t)\cdot\bar{\rho}(t) - \rho(t)\cdot\Gamma^{\nu\bar{\nu}}(t) + \Gamma^{\nu\bar{\nu}}(t)$$

In compact matrix form :

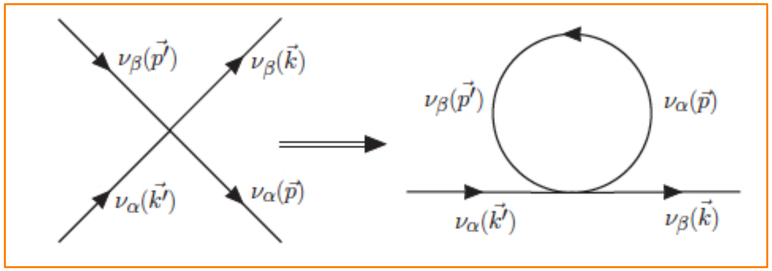
$$\mathcal{R}(t) = \begin{pmatrix} \rho(t) & \kappa(t) \\ \kappa^\dagger(t) & \mathbb{1} - \bar{\rho}(t) \end{pmatrix} \quad i\dot{\mathcal{R}}(t) = [\mathcal{H}(t), \mathcal{R}(t)]. \quad \mathcal{H}(t) = \begin{pmatrix} \Gamma^{\nu\nu}(t) & \Gamma^{\nu\bar{\nu}}(t) \\ \Gamma^{\bar{\nu}\nu}(t) & \Gamma^{\bar{\nu}\bar{\nu}}(t) \end{pmatrix}$$

HOMOGENEOUS BACKGROUND, RELATIVISTIC CASE

Taking e.g. the neutral-current low energy SM Hamiltonian

$$H_{\rm int}^{\rm self} = \frac{G_F}{4\sqrt{2}} \sum_{\alpha,\beta} \int d^3x\, j^\mu_\alpha(t,\vec{x}) j_{\beta,\mu}(t,\vec{x}), \quad j^\mu_\alpha(t,\vec{x}) = \bar{\psi}_\alpha(t,\vec{x})\gamma^\mu(1-\gamma_5)\psi_\alpha(t,\vec{x})$$

the corresponding mean-field kernel is

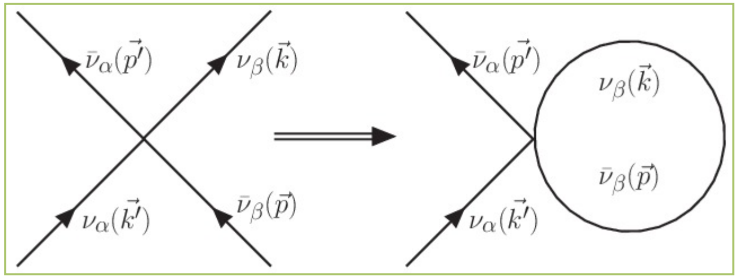


$$\Gamma^{\rm self}_{ij}(t,\vec{x}) = \frac{G_F}{\sqrt{2}}\gamma_\mu(1-\gamma_5)\,\frac{1}{2}\langle\bar{\psi}_j(t,\vec{x})\gamma^\mu(1-\gamma_5)\psi_i(t,\vec{x})\rangle.$$

and the homogeneity condition : $\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}'h',\vec{p}h} = (2\pi)^3 2E_p \delta_{hh'}\delta^3(\vec{p}-\vec{p}')\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}}$.

For the particle-antiparticle correlations,
the homogeneity condition is

$$\kappa^{\nu_\alpha\bar{\nu}_\beta *}_{\vec{k},\vec{p}} = (2\pi)^3 2E_k \delta^3(\vec{p}+\vec{k})\kappa^{\nu_\alpha\bar{\nu}_\beta *}_{\vec{p}}$$



Extended HAMILTONIAN with PAIRING CORRELATIONS

Extended Hamiltonian with mix., int. and particle-antiparticle correlations :

$$\mathcal{H}(t,\vec{q}) = \begin{pmatrix} S(t,q) - \hat{q}\cdot\vec{V}(t) & -\hat{\epsilon}_q^*\cdot\vec{V}(t) \\ -\hat{\epsilon}_q\cdot\vec{V}(t) & \bar{S}(t,q) + \hat{q}\cdot\vec{V}(t) \end{pmatrix}.$$

The off-diagonal term introduces neutrino-antineutrino mixing, if medium anisotropic and pairing correlations present.

The quantities in H are *(trace terms taken out to simplify expressions)* :

$$S(t,q) = h^0(q) + h^{\rm mat}(t) + \sqrt{2}G_F \int_{\vec{p}} \ell(t,\vec{p}) \qquad \ell(t,\vec{q}) = \rho(t,\vec{q}) - \bar{\rho}(t,-\vec{q}).$$

$$\vec{V}(t) = \text{-}\sqrt{2}G_F \int_{\vec{p}} \left\{\hat{p}\,\ell(t,\vec{p}) + \hat{\epsilon}_p\kappa(t,\vec{p}) + \hat{\epsilon}_p^*\kappa^\dagger(t,\vec{p})\right\}.$$

$$\text{-}\ h^{\rm mat}(t) = \sqrt{2}G_F\,\rho_e$$

If medium is anisotropic and ν-ν pairing correlations are negligible, one recovers the « usual » mean-field equations :

$$i\dot{\rho} = [h(\rho),\rho] \qquad h = h^0(q) + h^{\rm mat}(t) + h^{\nu\nu}(t,\hat{q}),$$

$$\text{-}\ h^{\nu\nu}(t,\hat{q}), = \sqrt{2}G_F\int_{\vec{p}}(1-\hat{q}\cdot\hat{p})\ell(t,\vec{p}).$$

Extended Hamiltonian with NEUTRINO MASS CONTRIBUTIONS

Serreau and Volpe, PRD 90 (2014), arXiv:1409.3591

With the same procedure extended equations with neutrino mass terms can be derived. For Majorana neutrinos one has :

$$\rho_M(t,\vec{q}) \to \begin{pmatrix} \rho_M(t,\vec{q}) & \zeta_M(t,\vec{q}) \\ \zeta_M^\dagger(t,\vec{q}) & \bar{\rho}_M^T(t,-\vec{q}) \end{pmatrix} \qquad \zeta_{ij}(t,\vec{q}) = \langle a_j^\dagger(t,\vec{q},+)a_i(t,\vec{q},-)\rangle$$

$$\Gamma_M^{\nu\nu}(t,\vec{q}) \to \begin{pmatrix} H_M(t,\vec{q}) & \Phi_M(t,\vec{q}) \\ \Phi_M^\dagger(t,\vec{q}) & -\bar{H}_M^T(t,-\vec{q}) \end{pmatrix} \quad \begin{aligned} H_M(t,\vec{q}) &= S(t,q) - \hat{q}\cdot\vec{V}(t) - \hat{q}\cdot\vec{V}_m(t), \\ \bar{H}_M(t,\vec{q}) &= \bar{S}(t,q) + \hat{q}\cdot\vec{V}(t) + \hat{q}\cdot\vec{V}_m(t), \end{aligned}$$

$$\Phi_M(t,\vec{q}) = e^{i\phi_q}\hat{\epsilon}_q^* \cdot \left[\vec{V}(t)\frac{m}{2q} + \frac{m}{2q}\vec{V}^T(t)\right].$$

The off-diagonal term introduces neutrino-antineutrino mixing, if medium anisotropic. This is referred to as helicity coherence.

Vlasenko, Fuller, Cirigliano, PRD89 (2014) – spin coherence

First calculation shows it might have an impact under appropriate conditions. Vlasenko, Fuller, Cirigliano, arXiv:1406.6724

Our method used for neutrino electromagnetic interactions in Kartavtsev, Raffelt, H. Vogel, arXiv:1504.03230

Quasi-particle description

Väänänen and Volpe, PRD88 (2013)

Particles $\longrightarrow$ Quasi-particles

Transformation

- By introducing a (generalized) Bogoliubov-Valatin transformation,

$$\begin{cases} \alpha_{\bar{k}} &= v_k^* a_k^\dagger + u_k^* b_{\bar{k}} \\ \alpha_k^\dagger &= z_k a_k^\dagger + w_k b_{\bar{k}} \end{cases}$$

the extended Hamiltonian with pairing correlations can be put in a diagonal form and describe a system of independent quasi-particles.

Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy

The s-reduced density matrix : $\rho_{1\ldots s} = \langle\psi(t)|a_s^\dagger \ldots a_1^\dagger a_1 \ldots a_s|\psi(t)\rangle$

one-body density

$$\rho_1 = \langle\psi(t)|a_1^\dagger a_1|\psi(t)\rangle$$

two-body density

$$\rho_{12} = \langle\psi(t)|a_2^\dagger a_1^\dagger a_1 a_2|\psi(t)\rangle$$

Solving exactly the many-body problem is equivalent to

$$\mathrm{i}\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)}\{[v_{12}, \rho_{12}]\}$$

$$\mathrm{i}\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)}\{[v_{13} + v_{23}, \rho_{123}]\}$$

$$\vdots$$

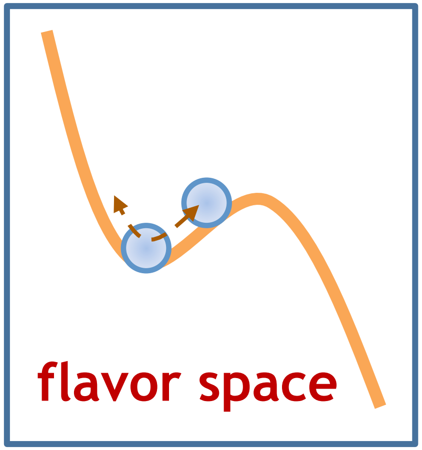
$$\mathrm{i}\dot{\rho}_{1\cdots n} = \left[\sum_{i=1}^{n} t_i + \sum_{j>i=1}^{n} v_{ij}, \rho_{1\cdots n}\right] + \sum_{i=1}^{n} \mathrm{Tr}_{(n+1)}\left\{\left[v_{i(n+1)}, \rho_{1\cdots(n+1)}\right]\right\}$$

hierarchy of equations
for s-reduced
density matrices

A first principle derivation based on BBGKY of mean-field equations and extended mean-field equations given with particle-antiparticle correlations. Contributions from the neutrino mass mentioned.

Volpe, Väänänen, Espinoza, PRD87 (2013)

Flavor collective modes and instabilities



Small amplitude motion

Collective modes and instabilities can be studied with the linearization.

Banerjee et al, PRD84 (2011)

?

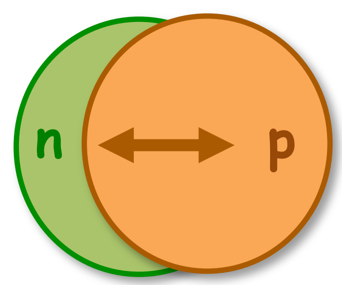
Stability matrix

$$\boxed{\begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix}} \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix} = \omega \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix}$$

S eigenvalues :
-> real : stable collective
-> imaginary : instabilities

Väänänen and Volpe, PRD88 (2013)

connection to collective modes in other many-body systems (nuclei, clusters, ...)



nuclear resonances

Conclusions and perspectives

- Steady progress in our understanding of ν flavour conversion in dense media. Important questions to be addressed.

- Extended equations provided to investigate the transition region. Role of collisions, of correlations and of neutrino mass needs further investigations. Instabilities searched in the heating region.

- **Established connections between a gas of SN ν and other many body systems – nuclei, condensed matter.**


Thank you