Relativistic fluid dynamics of multiple conserved charges in heavy ion collisons

59th International Winter Meeting on Nuclear Physics

Bormio, Italy

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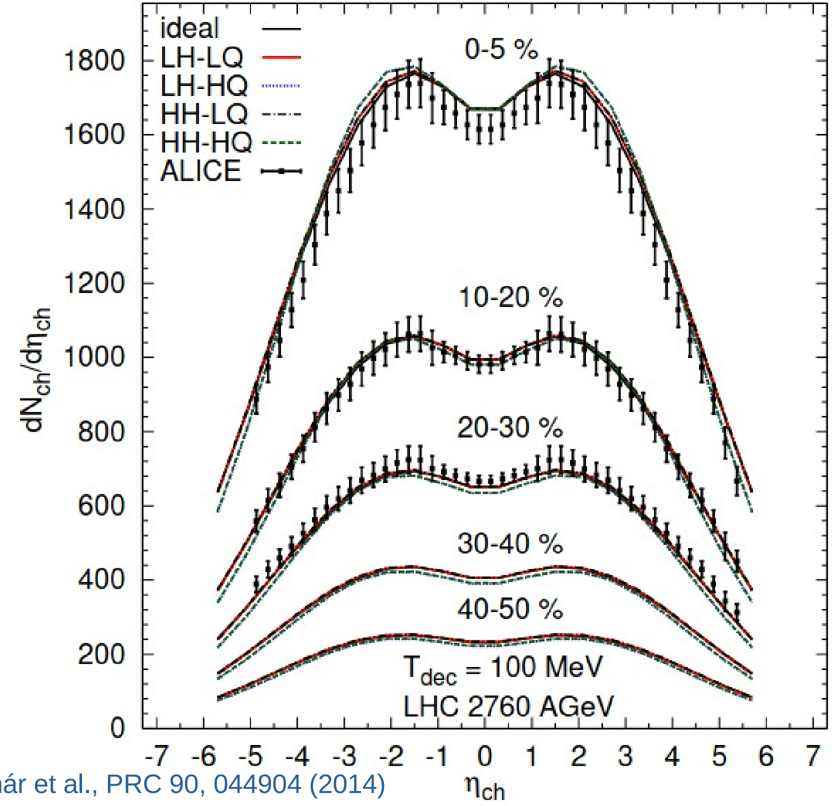
Hydrodynamics applied to heavy ion collisions

Hydro has been very successful in the **effective** description of the evolution of heavy-ion collisions ...



Molnár et al., PRC 90, 044904 (2014)

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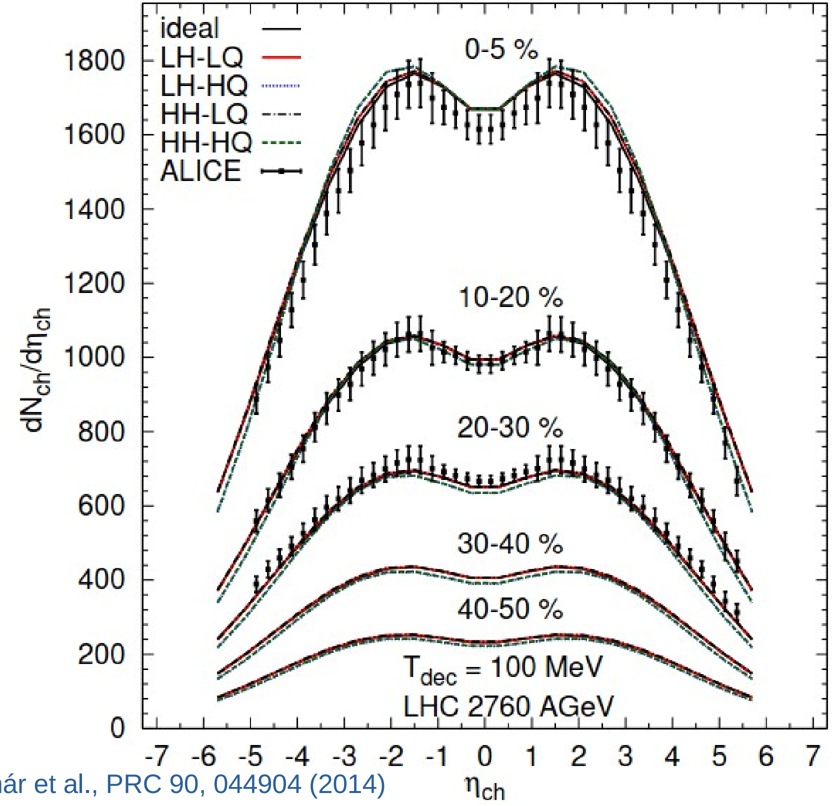
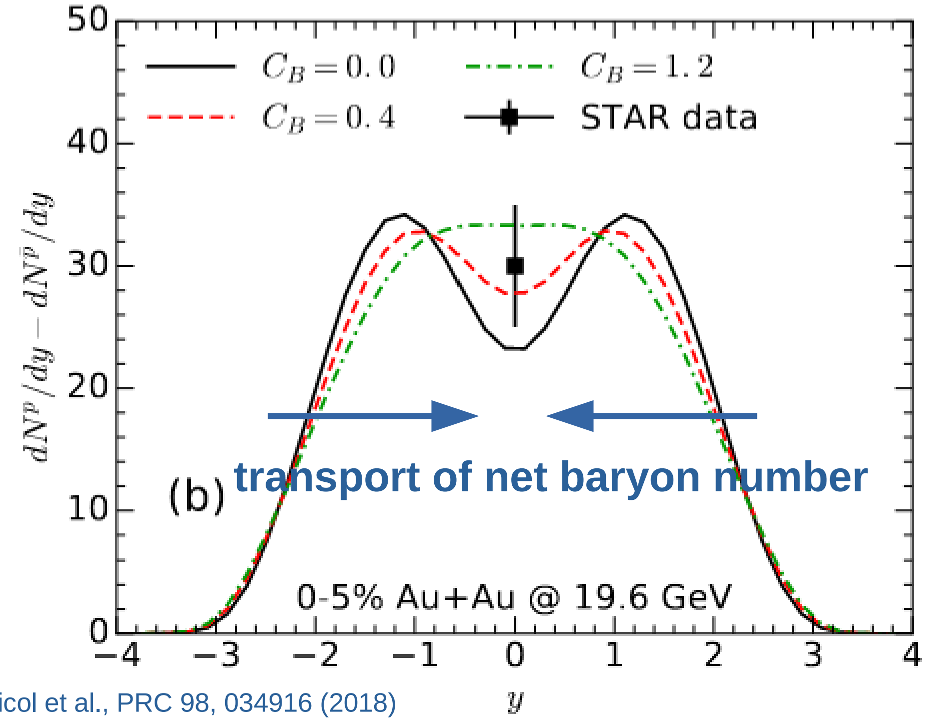


Molnár et al., PRC 90, 044904 (2014)

… but the theory has to be extended in order to explicitly account for effects and is dependent on the knowledge of the underlying microscopic properties.



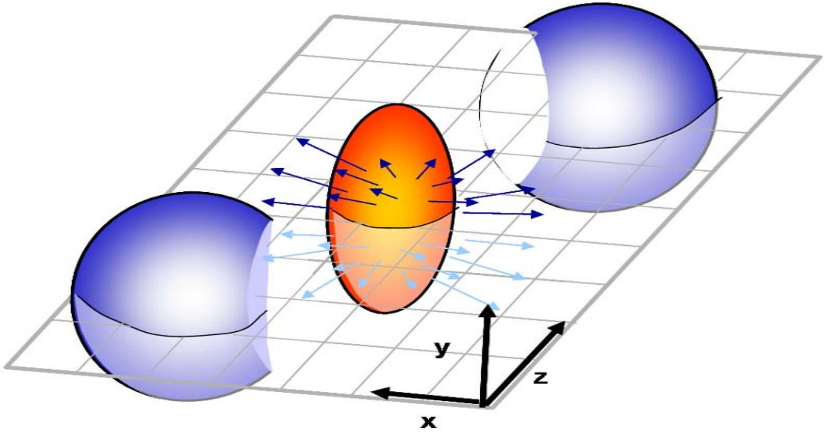
Denicol et al., PRC 98, 034916 (2018)

The multi-component nature of nuclear matter

Traditionally:

Viewed as ‘blob’ of one type of matter (single component) with one velocity field

- usually ‘blob’ of energy
  with conserved particle number



https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

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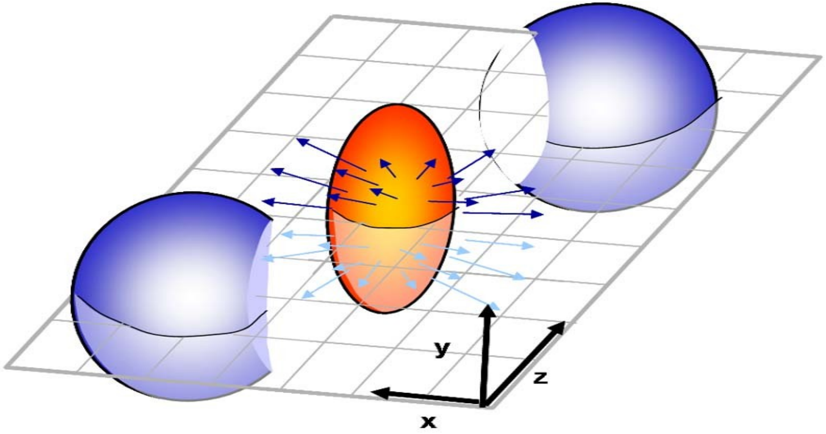

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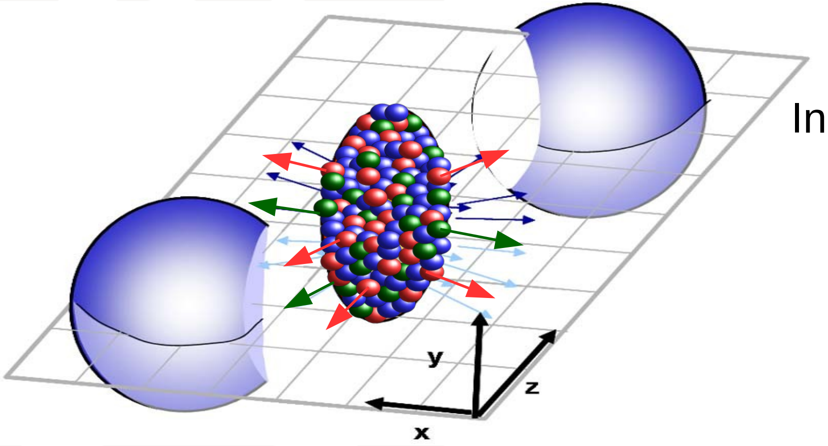

In general:

Consists of multiple components with <u>various properties</u> with multiple velocity fields

- with **multiple conserved quantities**
(e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry → **<u>coupled charge currents!</u>**

Increasing interest in the recent years …

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Simulation with baryon number

- Denicol et al., PRC 98, 034916 (2018)
- Li et al., PRC 98, 064908 (2018)
- Du et al., Comp. Phys. Comm. 251 (2020) 107090

Diffusion coefficients with BQS

- **Greif et al., PRL 120, 242301 (2018)**
- **Rose et al., PRD 101, 114028 (2020)**
- **Fotakis et al., PRD 104, 034014 (2021)**
- Das et al., arXiv:2109.01543

Simulation with multiple charges

- **Fotakis et al., PRD 101, 076007 (2020)**
- Chen et al., arXiv:2203.04685

Theory with multiple conserved charges

- Monnai et al., Nucl. Phys. A847:283-314 (2010)
- Kikuchi et al., PRC 92, 064909 (2015)
- **Fotakis et al., PRD 106, 036009 (2022)**

BQS equation of state

- Noronha-Hostler et al., PRC 100, 064910 (2019)
- Monnai et al., arXiv:2101.11591

On last years QM conference ...

- Plaschke et al., Poster Session 1 T02/T03
- Mishra et al., Poster Session 2 T03
- Almaalol et al., Poster Session 2 T14_2
- Pihan et al., Poster Session 2 T07_2
- Weickgenannt., Plenary Session VII

… and many more!

Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field u^μ

Conservation of Energy and Momentum: $\partial_\mu T^{\mu\nu} = 0$

Conservation of charge: $\partial_\mu N^\mu_q = 0$

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q-th conserved charge (eg. B,Q,S)

Conservation of Energy and Momentum: $\partial_\mu T^{\mu\nu} = 0$

$$T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Conservation of charge: $\partial_\mu N_q^\mu = 0$

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$10 + 4N_{\rm ch}$ degrees of freedom, $4 + N_{\rm ch}$ equations $\rightarrow$ $6 + 3N_{\rm ch}$ unknowns

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What needs to be known:

- Equation of state $P_0 = P_0(\epsilon, n_q)$, $T = T(\epsilon, n_q)$, $\alpha_q = \mu_q/T = \alpha_q(\epsilon, n_q)$
- Equations of motion for dissipative fields & <u>transport coefficients</u> $\Pi, V_q^\mu, \pi^{\mu\nu}$
- Initial state
- Final state: freeze-out and δf-correction

Deriving fluid dynamics from kinetic theory

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Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009

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relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

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2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

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equilibrium — off-equilibrium

$$f_{i,\mathbf{k}} = f_{i,\mathbf{k}}^{(0)} + \delta f_{i,\mathbf{k}}$$

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equilibrium off-equilibrium

$$f_{i,\mathbf{k}} = f_{i,\mathbf{k}}^{(0)} + \delta f_{i,\mathbf{k}}$$

Irreducible off-equilibrium moments obey Boltzmann eq.:

$$\rho_{i,n}^{\mu\nu} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^n k_i^{\langle\mu} k_i^{\nu\rangle} \delta f_{i,\mathbf{k}}$$

<u>Problem:</u> infinitely many coupled PDEs.

<u>Aim:</u> Truncate in a well-defined manner (“perturbation theory”)

Deriving fluid dynamics from kinetic theory

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“Order-of-magnitude approximation”:
relate off-equilibrium moments to the dissipative fields

$$\rho_{i,n}^{\mu\nu} = \frac{\eta_{i,n}}{\eta}\pi^{\mu\nu} + \mathcal{O}(2)$$

Counting scheme:

Gradients in velocity, temperature etc. $\sigma^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Kn})$
Dissipative fields $\pi^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Rn}^{-1})$

Equations of motion with multiple conserved charges

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

$$\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} + (\text{higher-order terms})$$

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Navier-Stokes theory / Fick's law

$$\text{"}\ \mathbf{V} = -D\nabla n\ \text{"}$$

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- Relaxation equations (Israel-Stewart type) ⟶ in principle causal
- Mixed chemistry couples diffusion currents (coupled charge-transport)
- 2nd-order terms: couples all currents to each other; depend on all gradients
- Explicit expressions for transport coefficients!

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Diffusion coefficients: extensively studied!

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

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Computation of transport coefficients (Example: diffusion coefficients)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\rm species}} \left(\mathcal{C}^{-1}\right)^{(1)}_{ji,0n} q_j \left(q'_i \mathcal{J}^{(i)}_{n+1,1} - \frac{n_{q'}}{\epsilon + P_0} \mathcal{J}^{(i)}_{n+2,1} \right)$$

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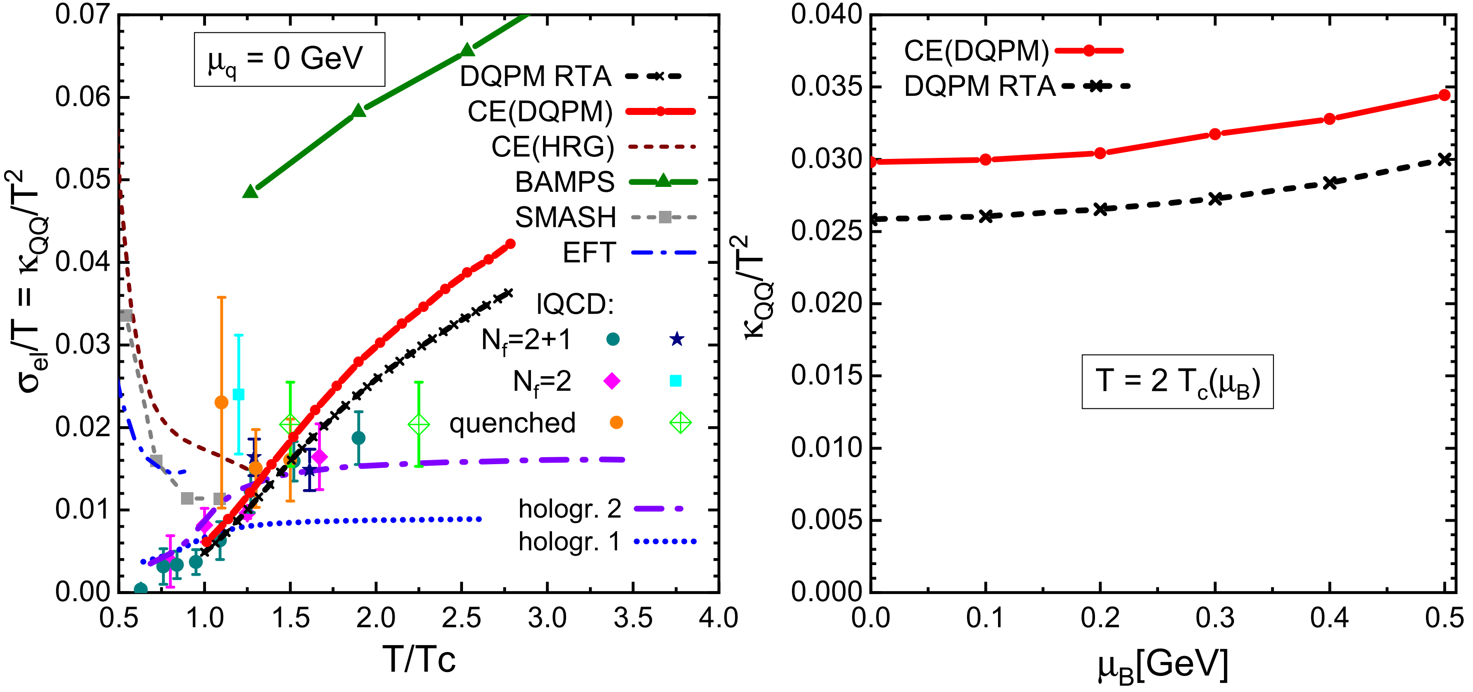
Example: introduction of features from LQCD via the usage of DQPM



Coupled charge-transport

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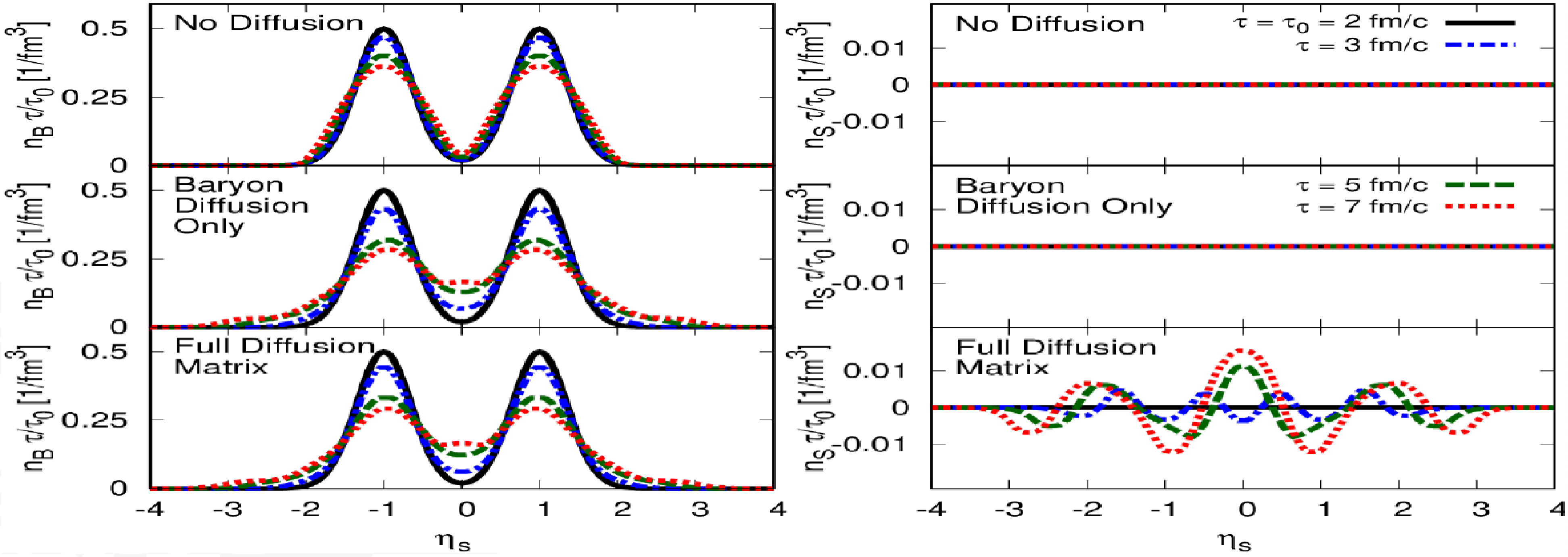

Simplistic case study: no viscosity, diffusion only, no 2nd-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation

$$\Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

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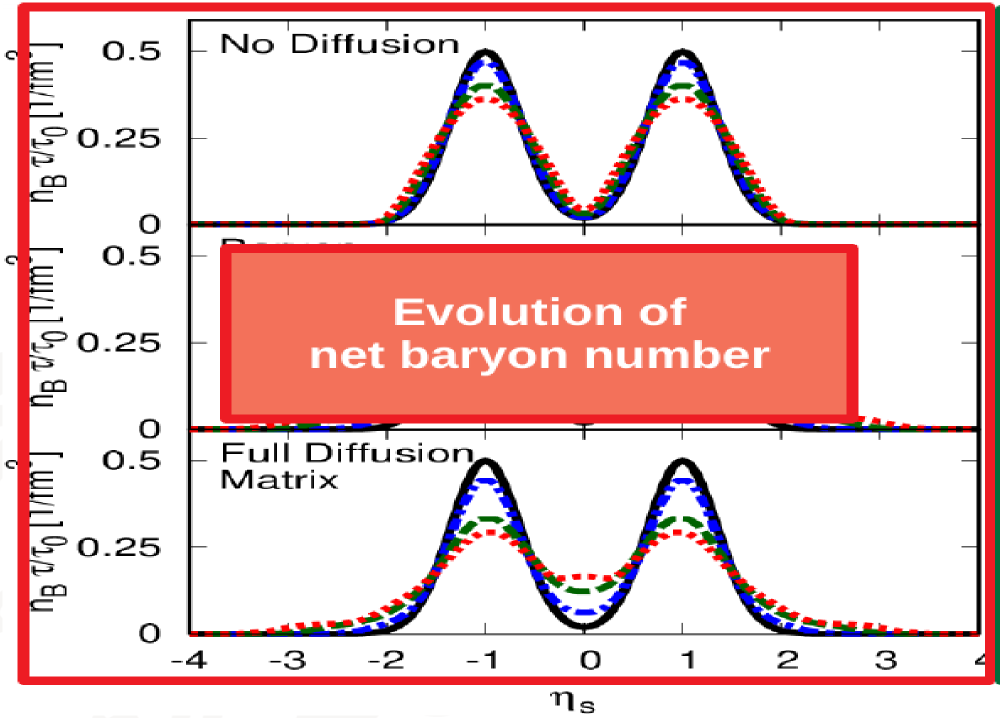

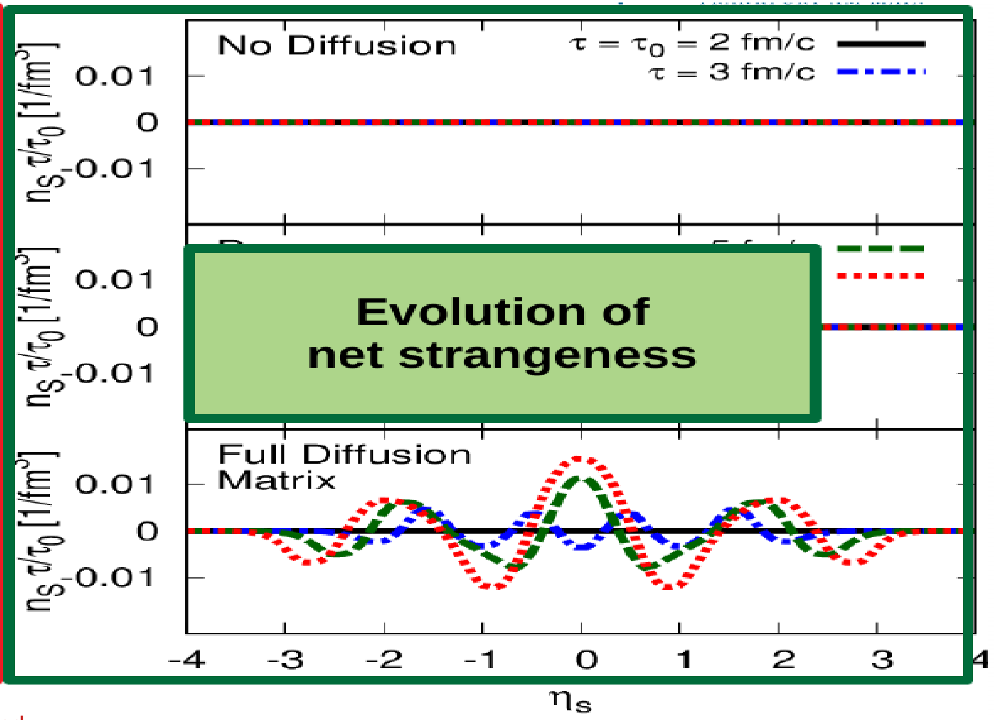

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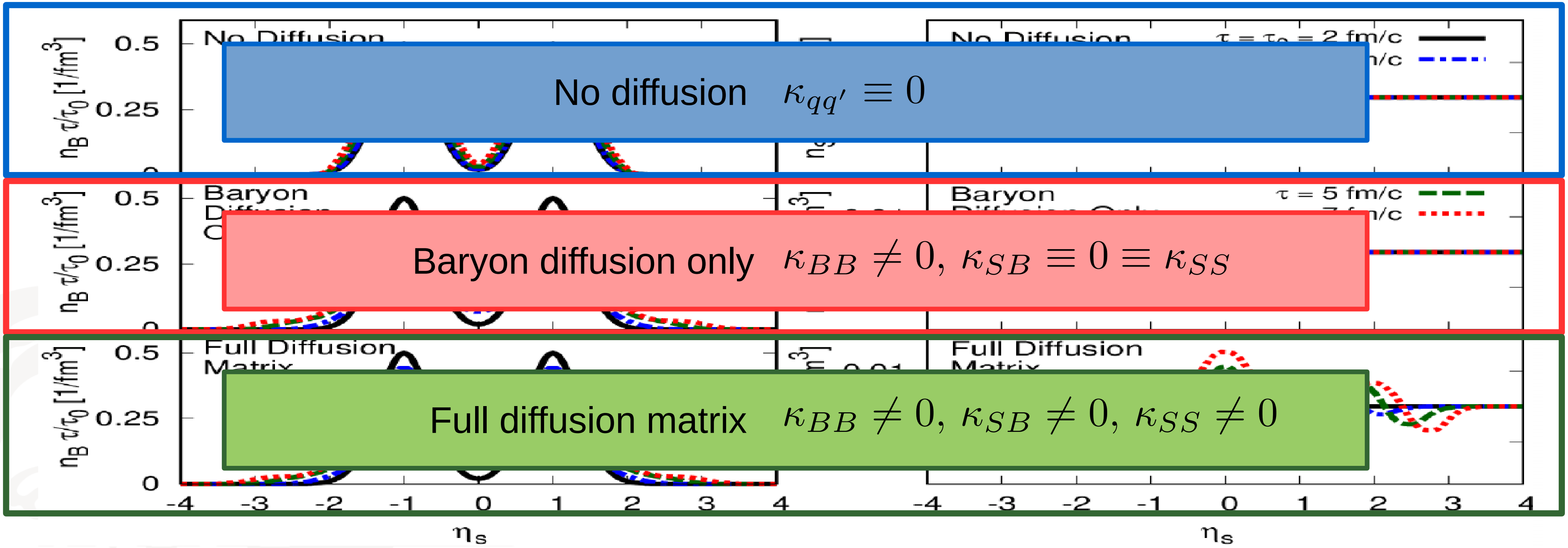

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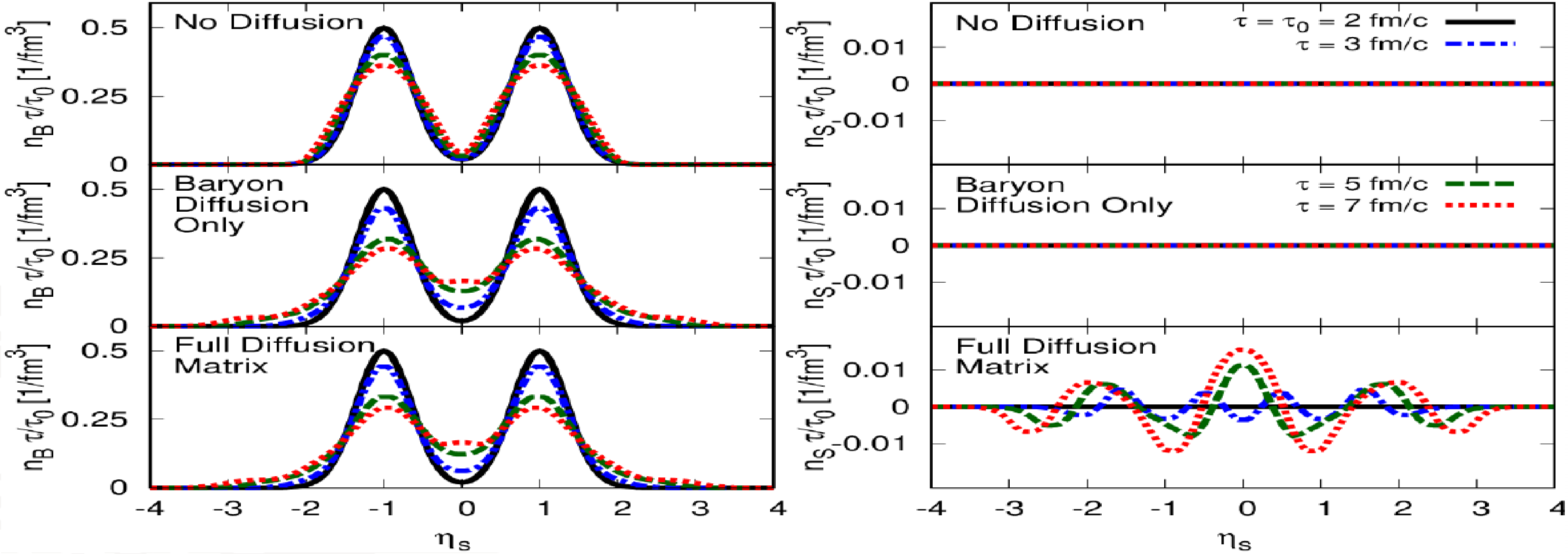


Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS

e.g.: $\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$

$\nabla^\mu \alpha_S \sim \nabla^\mu n_B$

→ Generation of domains of non-vanishing local net charge (here net strangeness)!

Conclusion

- Derived 2^{nd}-order relativistic fluid dynamic theory
  for multicomponent systems from the Boltzmann equation
- Transport coefficients given explicitly containing all information about particle interactions
- Mixed chemistry couples diffusion currents to each other $\longrightarrow$ coupled charge-transport
- Consistency of equation of state, 1^{st}- and 2^{nd}-order transport coefficients is important!
- Thermal features from LQCD can be adapted in transport coefficients with quasi-particle models
- Implemented derived fluid dynamic theory in (3+1)D-hydro code

Outlook

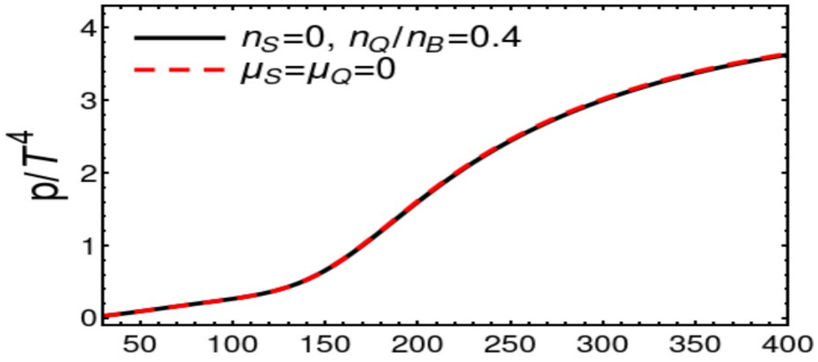
- Evaluate 2^{nd} order transport coefficients for more realistic systems
- Use more realistic initial state and equation of state
- Apply freeze-out routines, take δf-correction
- Find observables sensitive to charge-coupling $\longrightarrow$ investigate impact

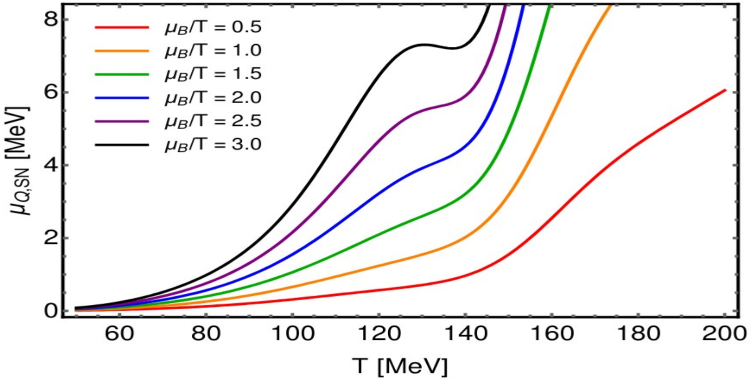
Backup

Equation of state with multiple conserved charges

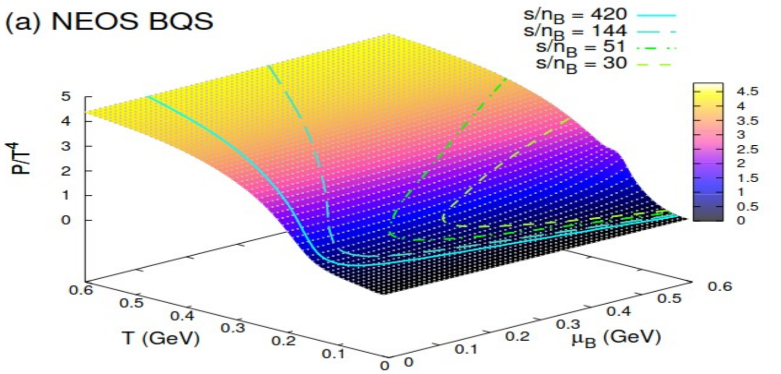
$$P_0(T) \quad \rightarrow \quad P_0(T, \mu_\mathrm{B}, \mu_\mathrm{Q}, \mu_\mathrm{S})$$

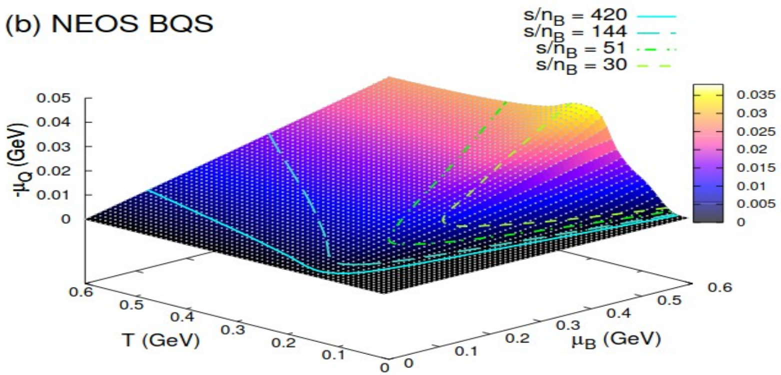
Noronha-Hostler et al., PRC 100, 064910 (2019)



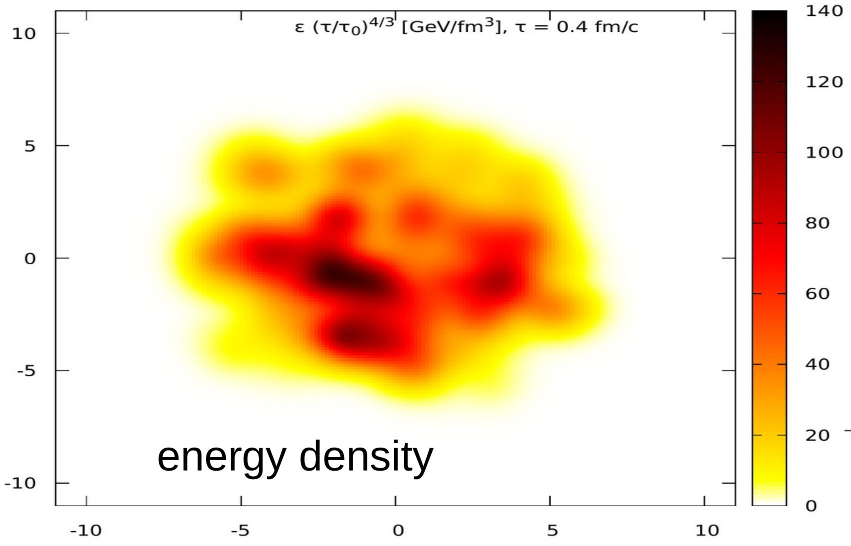


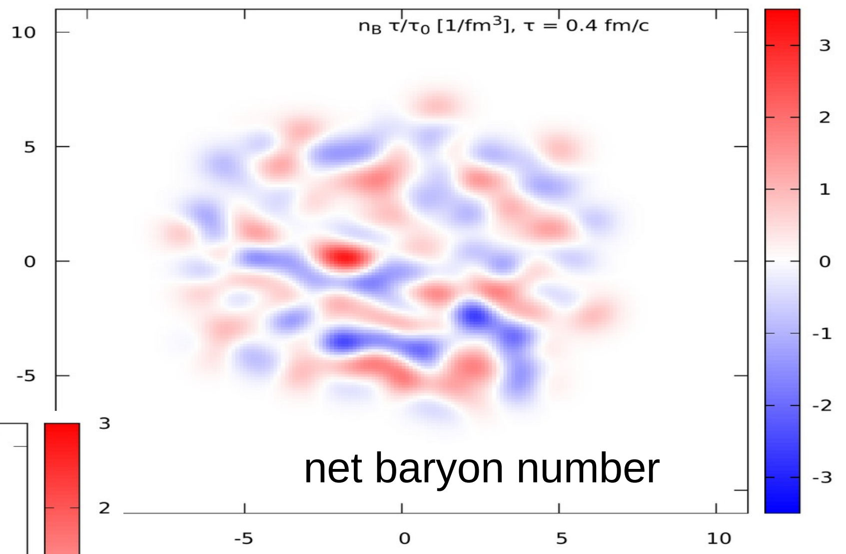
Monnai et al., PRC 100, 024907 (2019)

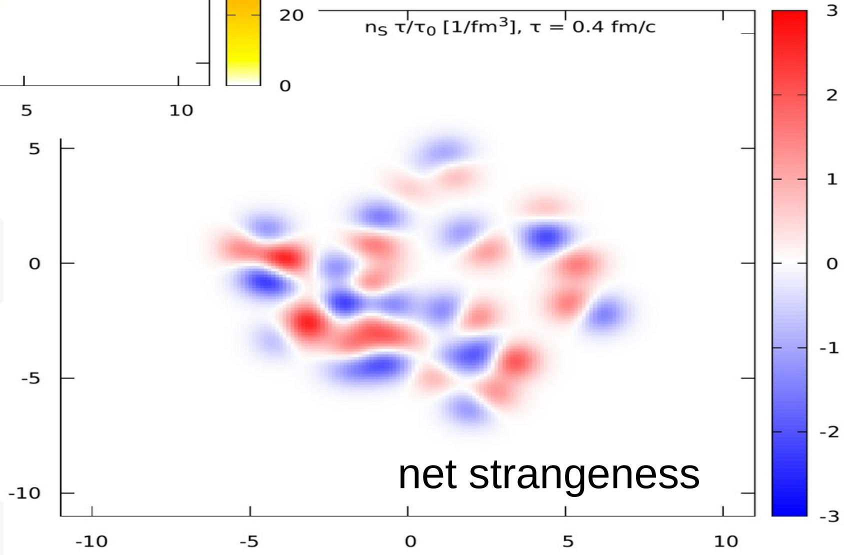




Initial state with multiple conserved charges







Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

Martinez et al., arXiv:1911.10272, arXiv:1911.12454

Plots not published!

Computation of transport coefficients (Example: diffusion coefficients)

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On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

$$\mathcal{C}_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} \mathcal{C}_i[f_i]$$

$$= -\sum_{m=0}^{\infty} \sum_j \mathcal{C}_{ij,nm}^{(1)} \rho_{j,m}^\mu + \text{non-linear terms}$$

Entries of „collision matrix“ (for diffusive moments)

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Entries of „collision matrix" (for diffusive moments)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(\mathcal{C}^{(1)}\right)^{-1}_{ij,0n} q_i \left(q_j' J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1} \right)$$

Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Equation of State - details

- Hadronic system including lightest 19 species

$$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^\pm, \bar{\Sigma}^\pm$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density and net charge densities

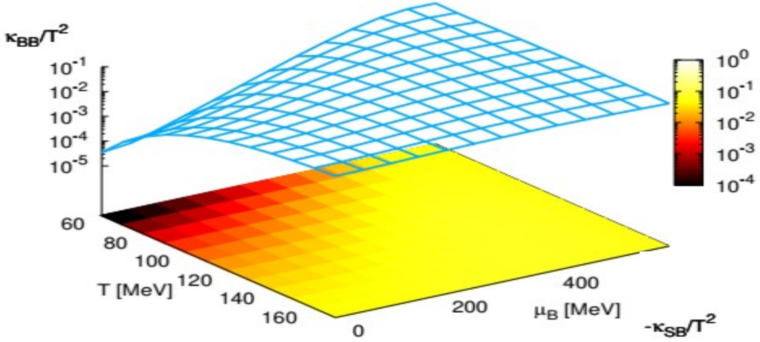
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix - details

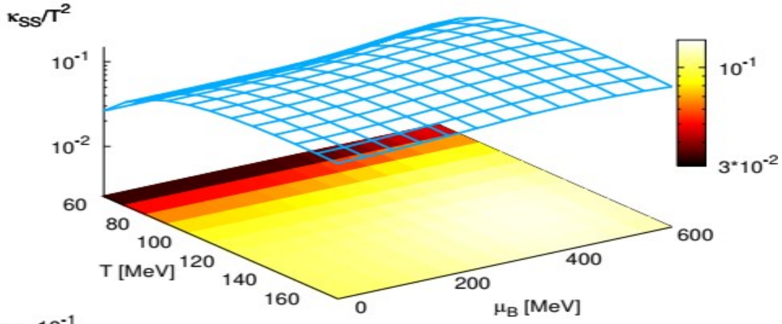
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. **37**, 405 (1931) & Phys. Rev. **38**, 2265 (1931)

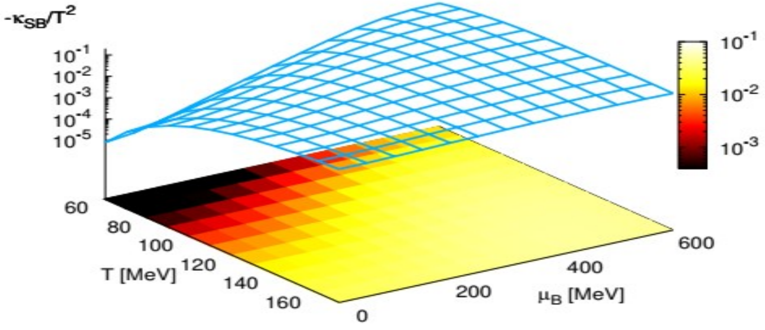


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is negative and has similar magnitude as κ_{BB}

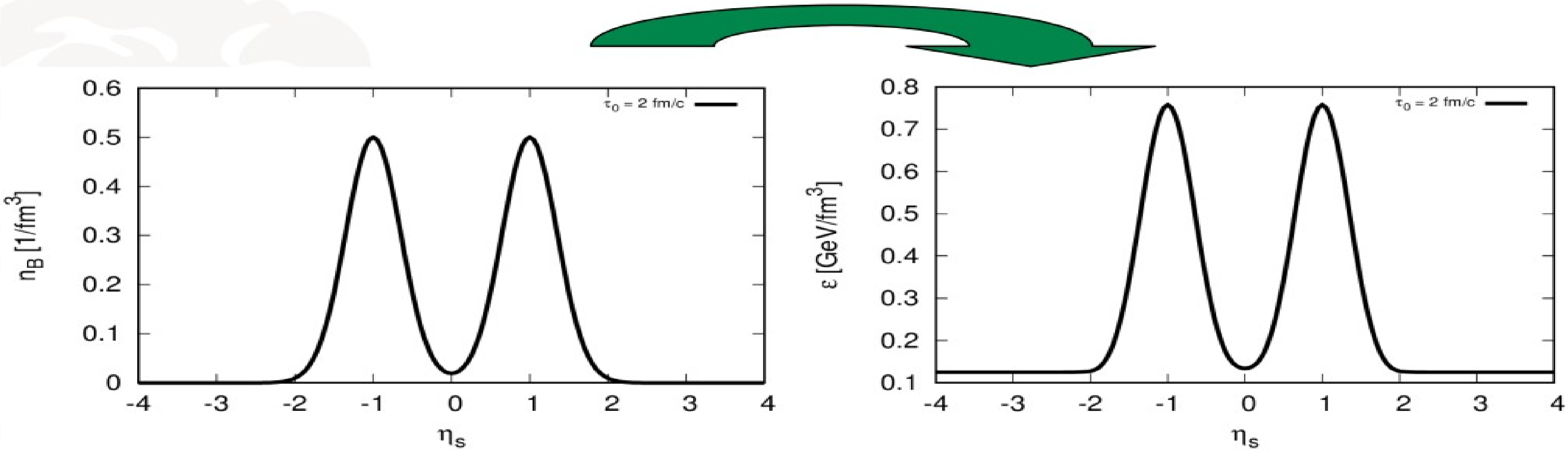
$\Longrightarrow$ significant coupling?



- Tabulate coefficient matrix over $T,\ \mu_B,\ \mu_S$
- $\mu_Q = 0$

Initial conditions - details

- $\tau_0 = 2\ \mathrm{fm/c}$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density



Equations of motion with multiple conserved charges

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

upcoming publication!

$$\tau_\Pi \dot{\Pi} + \Pi = S_\Pi$$
$$\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^\mu = S_q^\mu$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = S_\pi^{\mu\nu}$$

Relaxation equations (Israel-Stewart-type causal theory)

$$S_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - \sum_{q'} \tau_{qq'} V_{q',\nu}\omega^{\nu\mu} - \sum_{q'} \delta_{VV}^{(q,q')} V_{q'}^\mu \theta - \sum_{q'} \lambda_{VV}^{(q,q')} V_{q',\nu}\sigma^{\mu\nu}$$
$$- \ell_{V\Pi}^{(q)} \nabla^\mu \Pi + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q)} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q)} \pi^{\mu\nu} \dot{u}_\nu$$
$$+ \sum_{q'} \lambda_{V\Pi}^{(q,q')} \Pi \nabla^\mu \alpha_{q'} - \sum_{q'} \lambda_{V\pi}^{(q,q')} \pi^{\mu\nu} \nabla_\nu \alpha_{q'}$$

Diffusion coefficients: extensively studied!

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1st order term

2nd order terms: couples all currents to each other; depend on all gradients!

Explicit expressions for transport coefficients!

Single-component vs. Multi-component system

Potentially problematic terms in single-component systems $S_q^{\mu} = (\dots) + \ell_{V\pi}^{(q)} \Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda + (\dots)$

Ultrarelativistic, classical system with hard-sphere interactions:

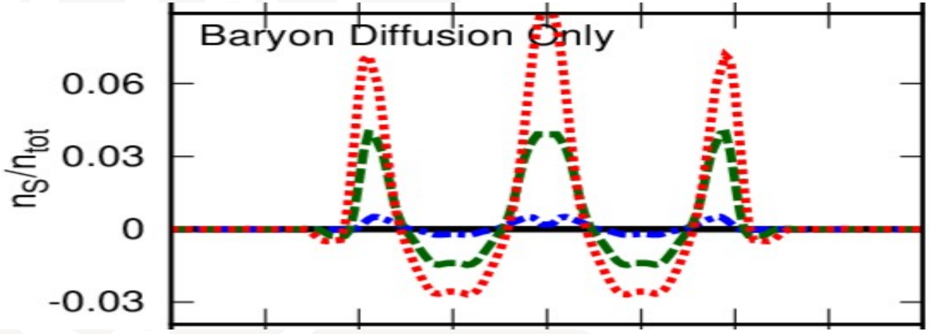
Denicol et al., PRD 85, 114047 (2012)

Used in simulations of heavy-ion collisions!

TABLE I. The coefficients for the particle diffusion for a classical gas with constant cross section in the ultrarelativistic limit, in the 14-moment approximation. The transport coefficient $\tau_{n\pi}$ was incorrectly listed as being zero in Ref. [1]

κ	$\tau_n[\lambda_{\mathrm{mfp}}]$	$\delta_{nn}[\tau_n]$	$\lambda_{nn}[\tau_n]$	$\lambda_{n\pi}[\tau_n]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	1	$3/5$	$\beta_0/20$	$\beta_0/20$	$\beta_0/80$

$$\tau_n \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q,\nu}\omega^{\nu\mu} - \tau_n V_{q'}^\mu \theta - \frac{3\tau_n}{5} V_{q,\nu}\sigma^{\mu\nu} + \boxed{\frac{\tau_n}{20T}\Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda - \frac{\tau_n}{20T}\pi^{\mu\nu}\dot{u}_\nu} - \frac{\tau_n}{20T}\pi^{\mu\nu}\nabla_\nu \alpha_q$$



Second-order transport coefficients not consistent with assumed system

→ generation of unphysical charge currents

Consistency is important in charge transport!
Use multi-component expressions.

Single-component vs. Multi-component system

Potentially problematic terms in single-component systems $$\mathrm{S}_q^\mu = (\ldots) + \ell_{V\pi}^{(q)}\,\Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda + (\ldots)$$

Ultrarelativistic, classical system with ha… …14047 (2012)

$$\ell_{V\pi}^{(q')} = \frac{9}{80\sigma_{\mathrm{tot}} P}\mathbf{c}_{q'} \overset{\text{single}}{\longrightarrow} \frac{\beta}{20}\tau_q$$

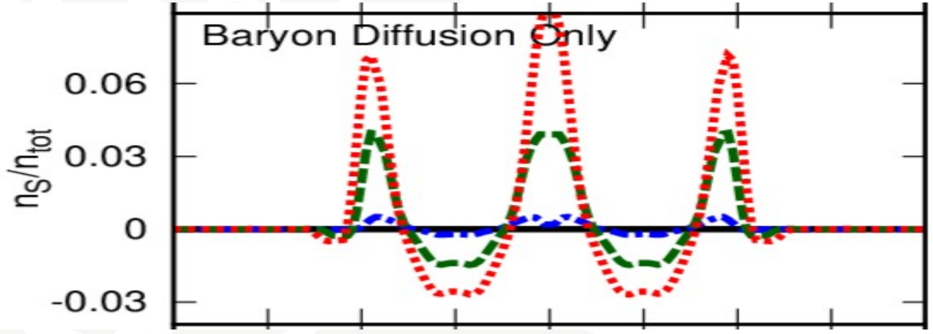
TABLE I. The coefficients f… …n the ultrarelativistic limit, in the 14-moment approximation. T… …f. [1]

$$\tau_{V\pi}^{(q')} = \ell_{V\pi}^{(q')} \overset{\text{single}}{\longrightarrow} \frac{\beta}{20}\tau_q\,,$$

κ	$\tau_n[\lambda_{\mathrm{mfp}}]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	$\beta_0/20$	$\beta_0/80$

Used in simulations of heavy-ion collisions!

$$\tau_n \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'}\,\nabla^\mu \alpha_{q'} - V_{q,\nu}\omega^{\nu\mu} - \tau_n V_{q'}^\mu \theta - \frac{3\tau_n}{5} V_{q,\nu}\sigma^{\mu\nu} + \boxed{\frac{\tau_n}{20T}\,\Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda - \frac{\tau_n}{20T}\,\pi^{\mu\nu}\dot{u}_\nu} - \frac{\tau_n}{20T}\,\pi^{\mu\nu}\nabla_\nu \alpha_q$$



Second-order transport coefficients not consistent with assumed system

→ generation of unphysical charge currents

Consistency is important in charge transport!
Use multi-component expressions.

Yet another hydro code - „Hydra“

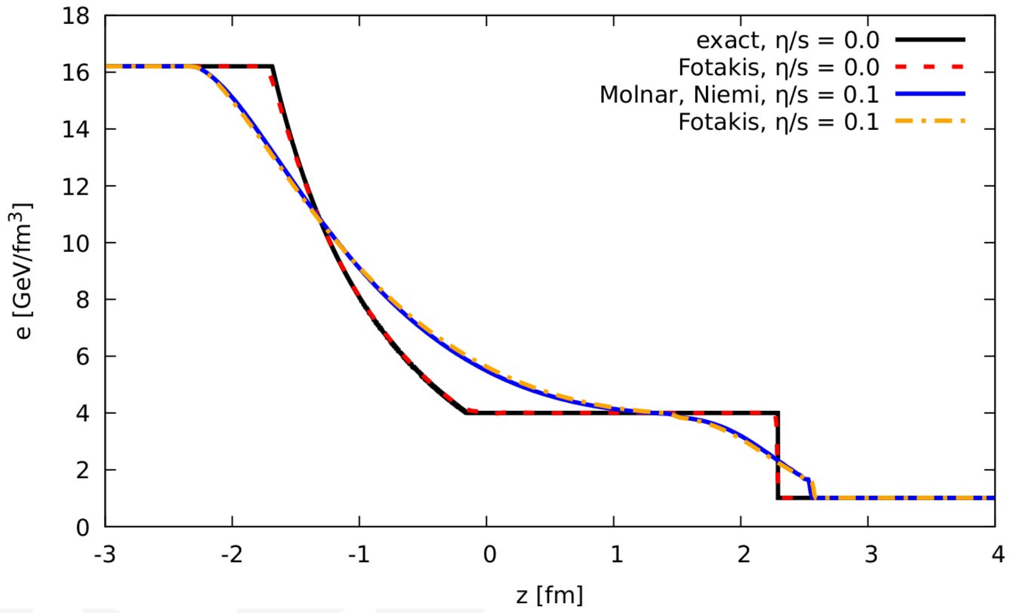
Core features:

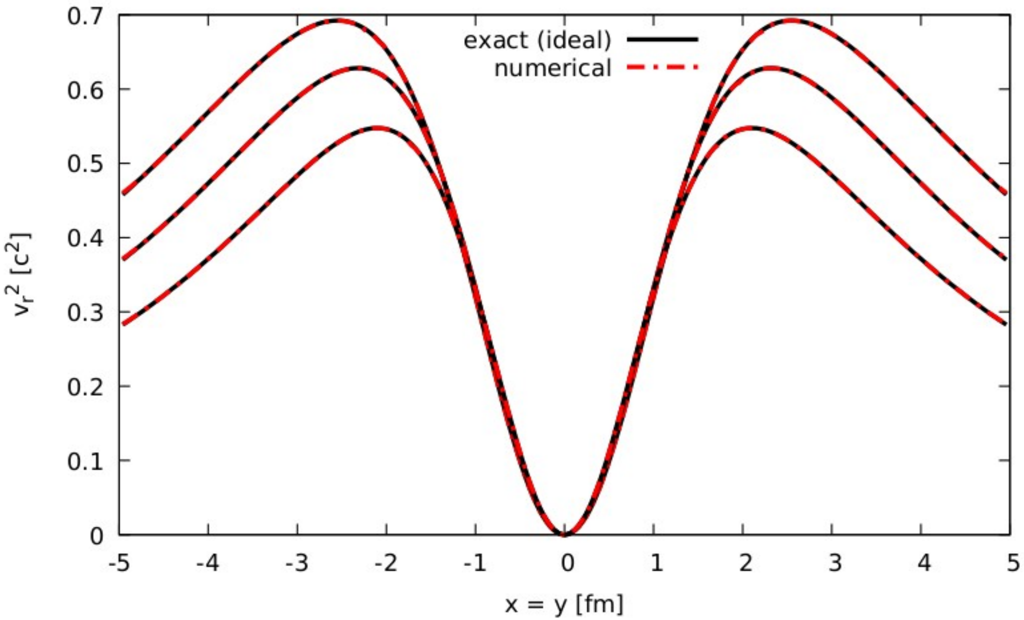
- (3+1)D-hydro – optimized reduction to 2D and 1D
- (v)SHASTA solver
- Shear-stress and multiple conserved charges (2 charges)
- Ultrarelativistic, tabled and/or any user-defined equations of state
- DNMR theory, this theory, and/or any user-defined theory
- any (tabled, user-defined) transport coefficients
- Curve-linear geometry (so far Cartesian and Hyperbolic coordinates)
- state of the art unit and physical tests
- available in the CRC-TR211 collaboration soon (hopefully)

Yet another hydro code - „Hydra“

Core features:

- (3+1)D-hydro – optimized reduction to 2D
- (v)SHASTA solver





ı soon (hopefully)