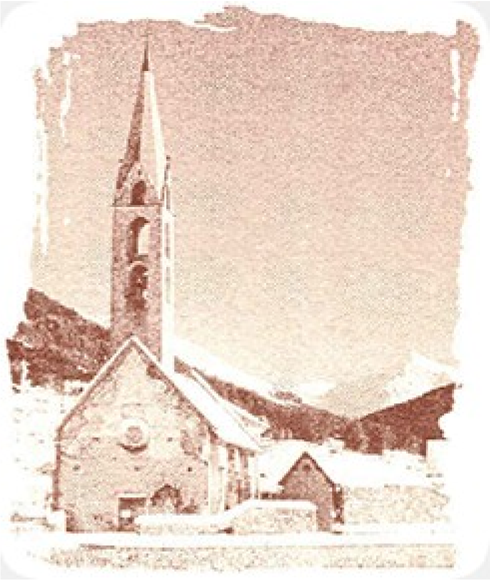


59. International Winter Meeting on Nuclear Physics

23.–27. Jan. 2023
Bormio, Italy

PionMAID for pion photoproduction on nucleons

V. L. Kashevarov (Institut für Kernphysik, Mainz)

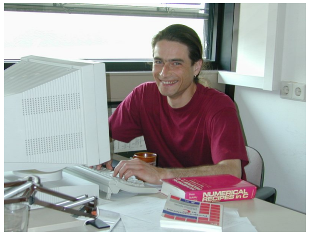
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

- **History of MAID**
- **Details of model**
- **Data base**
- **Selected results**
- **Application**
- **Summary**

History of MAID

The first MAID program appeared in 1998:


1. D. Drechsel, O. Hanstein, S. S. Kamalov, L. Tiator
   Unitary isobar model for pion photoproduction and electroproduction on the proton up to 1-GeV, Nucl. Phys. A 645 (1999) 145.

Soon afterwards the Dubna-Mainz-Taipei (DMT) dynamical model was developed:

2. S. S. Kamalov, Shin Nan Yang,
   Pion cloud and the Q**2 dependence of gamma* N <---> Delta transition form-factors,
   Phys. Rev. Lett. 83 (1999) 4494.

3. S. S. Kamalov, S. N. Yang, D. Drechsel, O. Hanstein, L. Tiator,
   Gamma* N ---> Delta transition form-factors: A New analysis of the JLab data ...
   Phys. Rev. C 64 (2001) 032201.

History of MAID

https://maid.kph.uni-mainz.de

M A I D

Photo- and Electroproduction of Pions, Eta, Etaprime and Kaons on the Nucleon

Institut für Kernphysik, Universität Mainz

Mainz, Germany

MAID2007	unitary isobar model for (e,e'π)
DMT2001	dynamical model for (e,e'π)
KAON-MAID	isobar model for (e,e'K)
ETA-MAID	EtaMAID2000 isobar model for (e,e'η) EtaMAID2018 isobar model for (γ,η) and (γ,η') NEW
Chiral MAID	chiral perturbation theory approach for (e,e'π)
2-PION-MAID	isobar model for (γ,ππ)
archive	MAID2000 DMT2001original EtaMAID2003 ETAprime2003

History of MAID

Now MAID is part of research program of

MTZ Collaboration

Mainz:	M. Gorchteyn, V. L. Kashevarov, M. Ostrick, L. Tiator
Tuzla:	M. Hadžimehmedović, R. Omerović, H. Osmanović, J. Stahov
Zagreb:	Alfred Švarc

History of MAID

EtaMAID 2018

Institut für Kernphysik, Universität Mainz, Germany

update info NEW

An Isobar Model for Eta and Etaprime Photoproduction on the Nucleon

Victor Kashevarov and Lothar Tiator

Reference:

L. Tiator, M. Gorchtein, V.L. Kashevarov, K. Nikonov, M. Ostrick (Mainz),
M. Hadzimehmedovic, R. Omerovic, H. Osmanovic, J. Stahov (Tuzla),
and A. Svarc (Zagreb),arXiv:1807.04525,
Eur. Phys. J. A (2018) 54: 210

- **Electromagnetic Multipoles ($E_{l\pm}$, $M_{l\pm}$)**
- **CGLN and Helicity Amplitudes ($F_1,\ldots,F_4$, $H_1,\ldots,H_4$)**
- **Observables (with beam, target and recoil polarization)**
- **Total Cross Sections**

Details of model

All the most well-known models (MAID, SAID, BnGa, JüBo etc.) are applicable for data analysis and predictions only in the resonance energy region (W <2.5 GeV).

There are several models for higher energies (JPAC and various Regge models). However, all of them are applicable for energies (W >2.5 GeV), for scattering angles only to forward or backward, and for specific reaction channels.

New version of PionMAID2021 allows data analysis for energies up to W = 6 GeV, at any scattering angles, and for 4 reaction channels:

1. $\gamma\, p \rightarrow \pi^0\, p$
2. $\gamma\, n \rightarrow \pi^0\, n$
3. $\gamma\, p \rightarrow \pi^+\, n$
4. $\gamma\, n \rightarrow \pi^-\, p$

Photon polarization		Target polarization		
		X	Y	Z
unpolarized	σ		T	
linear	$-\Sigma$	H	-P	-G
circular		-F		-E

Details of model

1. PionMAID 2021 is an unitarity isobar model.

2. 22 nucleon + 14 delta resonances, parameterized with Breit-Wigner shapes.
   MAID2007: 7 nucleon + 6 delta resonances.

3. The model includes Born terms in s and u channels, and meson exchange in t channel with damping factors (DF).

3. Total number of data points involved in the fit: 28819

4. Fit parameters:
   - Background: only DF parameters.
   - Resonances: mass M, total width G, branching ratio β(piN), photon helicity amplitudes $pA_{1/2}$, $nA_{1/2}$ and $A_{3/2}$, damping parameters for piN and gN vertices: X and Xg

Details of model: list of resonances

Resonances included in the fit

1. $P_{11}(1440)$ **** MAID 2007
2. $D_{13}(1520)$ **** MAID 2007
3. $S_{11}(1535)$ **** MAID 2007
4. $S_{11}(1650)$ **** MAID 2007
5. $D_{15}(1675)$ **** MAID 2007
6. $F_{15}(1680)$ **** MAID 2007
7. $D_{13}(1700)$ ***
8. $P_{11}(1710)$ ****
9. $P_{13}(1720)$ **** MAID 2007
10. $F_{15}(1860)$ **
11. $D_{13}(1875)$ ***
12. $P_{11}(1880)$ **
13. $S_{11}(1895)$ ****
14. $P_{13}(1900)$ ***
15. $F_{17}(1990)$ **
16. $F_{15}(2000)$ **
17. $D_{15}(2060)$ **
18. $P_{11}(2100)$ **
19. $D_{13}(2120)$ **
20. $G_{17}(2190)$ ****
21. $G_{19}(2250)$ ****
22. $H_{19}(2220)$ ****

1. $P_{33}(1232)$ **** $(99.4 \pm\ )\%$ MAID 2007
2. $P_{33}(1600)$ **** $(16 \pm 8)\%$
3. $S_{31}(1620)$ **** $(30 \pm 5)\%$ MAID 2007
4. $D_{33}(1700)$ **** $(15 \pm 5)\%$ MAID 2007
5. $S_{31}(1900)$ *** $(8 \pm 4)\%$
6. $F_{35}(1905)$ **** $(9 \pm 3)\%$ MAID 2007
7. $P_{31}(1910)$ **** $(23 \pm 8)\%$ MAID 2007
8. $P_{33}(1920)$ *** $(65 \pm 5)\%$
9. $D_{35}(1930)$ *** $(10 \pm 5)\%$
10. $F_{37}(1950)$ **** $(40 \pm 5)\%$ MAID 2007
11. $F_{35}(2000)$ ** $(7 \pm 4)\%$
12. $G_{37}(2200)$ *** $(5 \pm 3)\%$
13. $H_{39}(2300)$ ** $(4 \pm 4)\%$
14. $G_{39}(2400)$ ** $(6 \pm 3)\%$

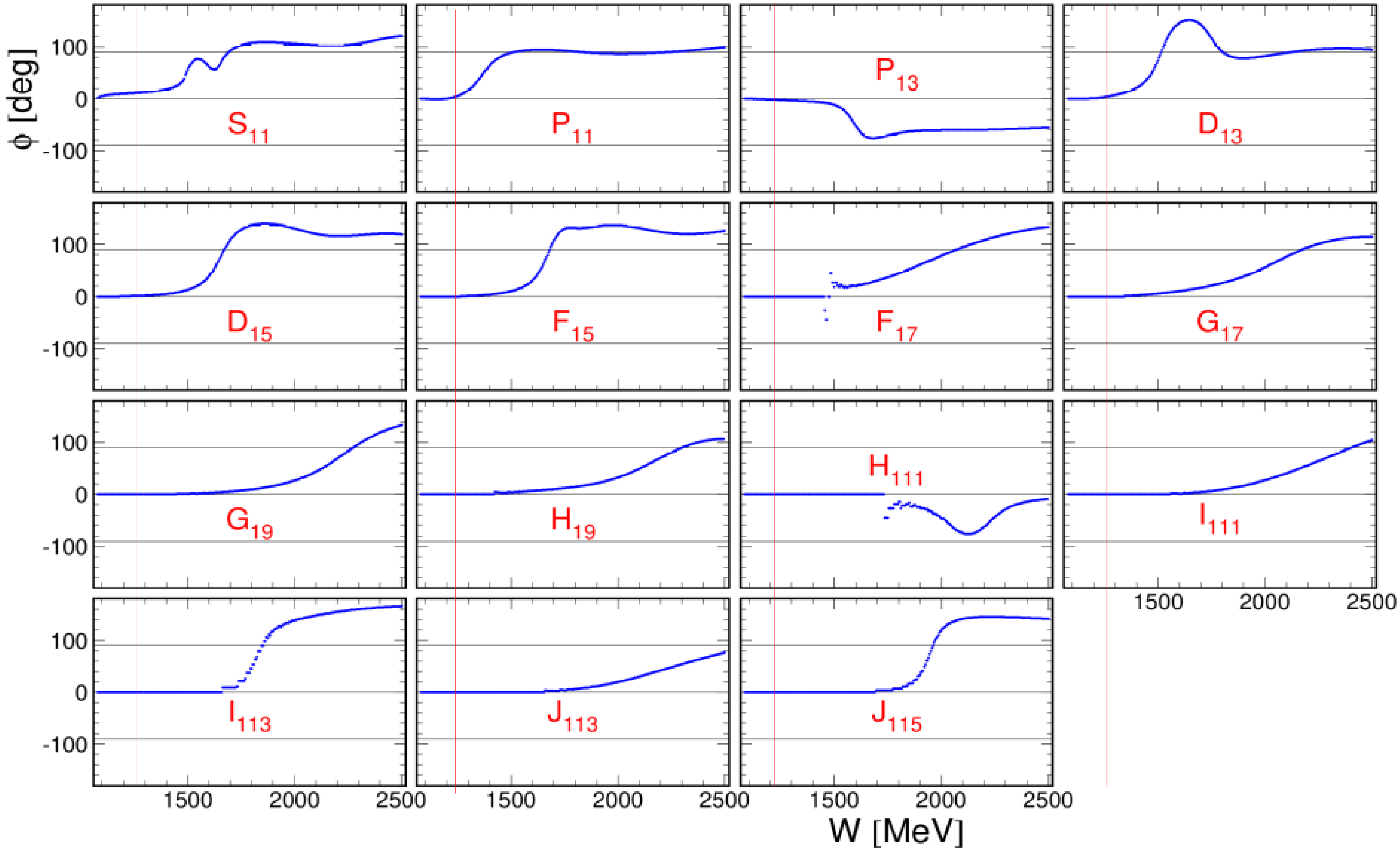
Resonances not included in the fit (PDG overall rating: two stars and more)

1. $D_{15}(2570)$ **
2. $H_{111}(2600)$ ***
3. $I_{113}(2700)$ **

1. $D_{33}(1940)$ ** $(4 \pm 3)\%$

Details of model: unitarization

phases of πN scattering (SAID WI08)



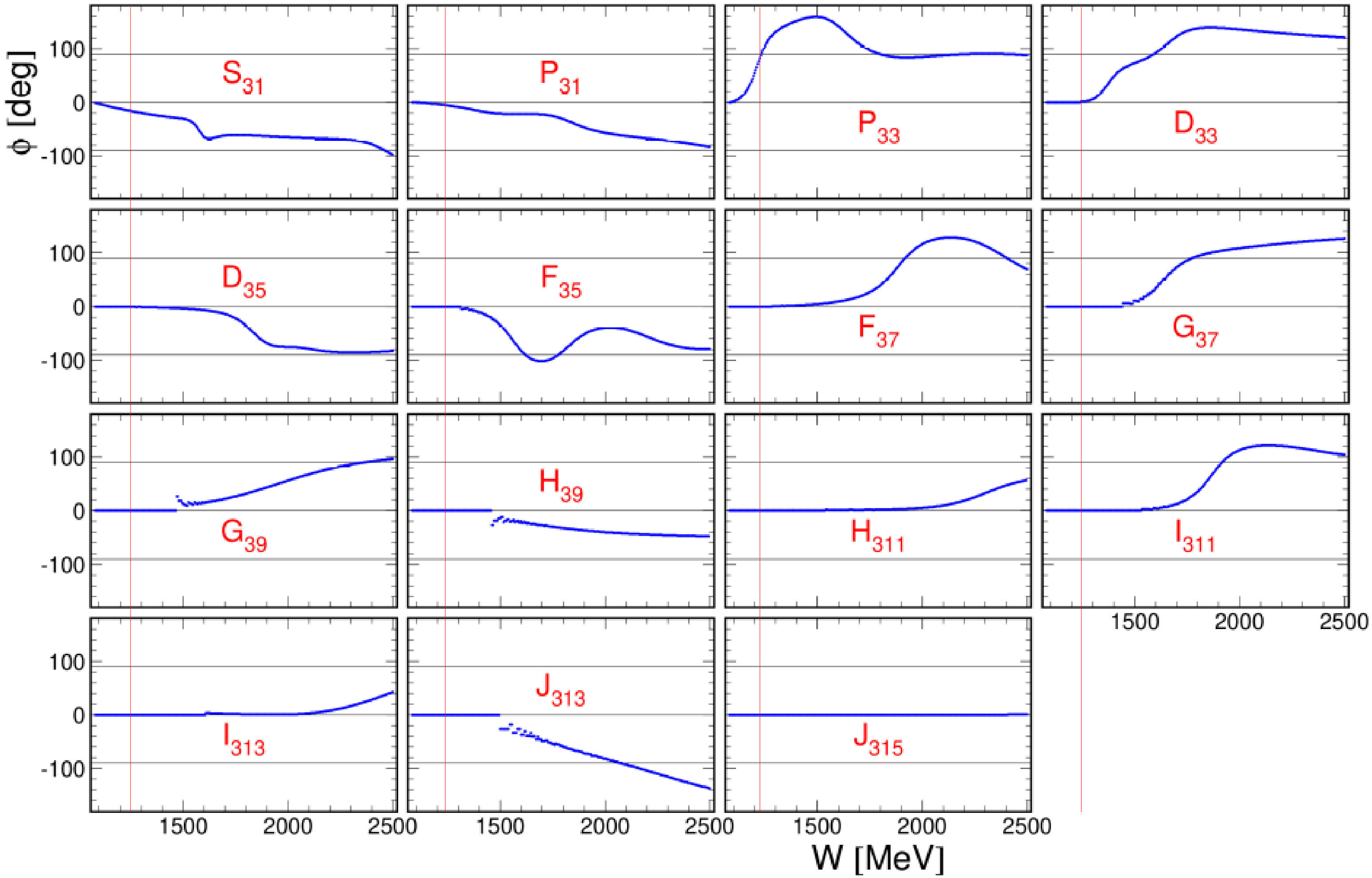
Unitarization was done for S_{11}, P_{11}, P_{13} at W < 1400 MeV, similar as MAID2007

Red lines: W~ 1230 MeV(limit for unitarization according Fermi-Watson theorem)

Horizontal lines: -90°, 0°, +90°

Details of model: unitarization

phases of πN scattering (SAID WI08)



Unitarization was done for S_{31}, P_{31}, P_{33} at W < 1400 MeV (MAID2007: 1600 MeV for P_{33})

Red lines: W~ 1230 MeV (limit for unitarization according Fermi-Watson theorem)
Horizontal lines: -90°, 0°, +90°

Details of model: background

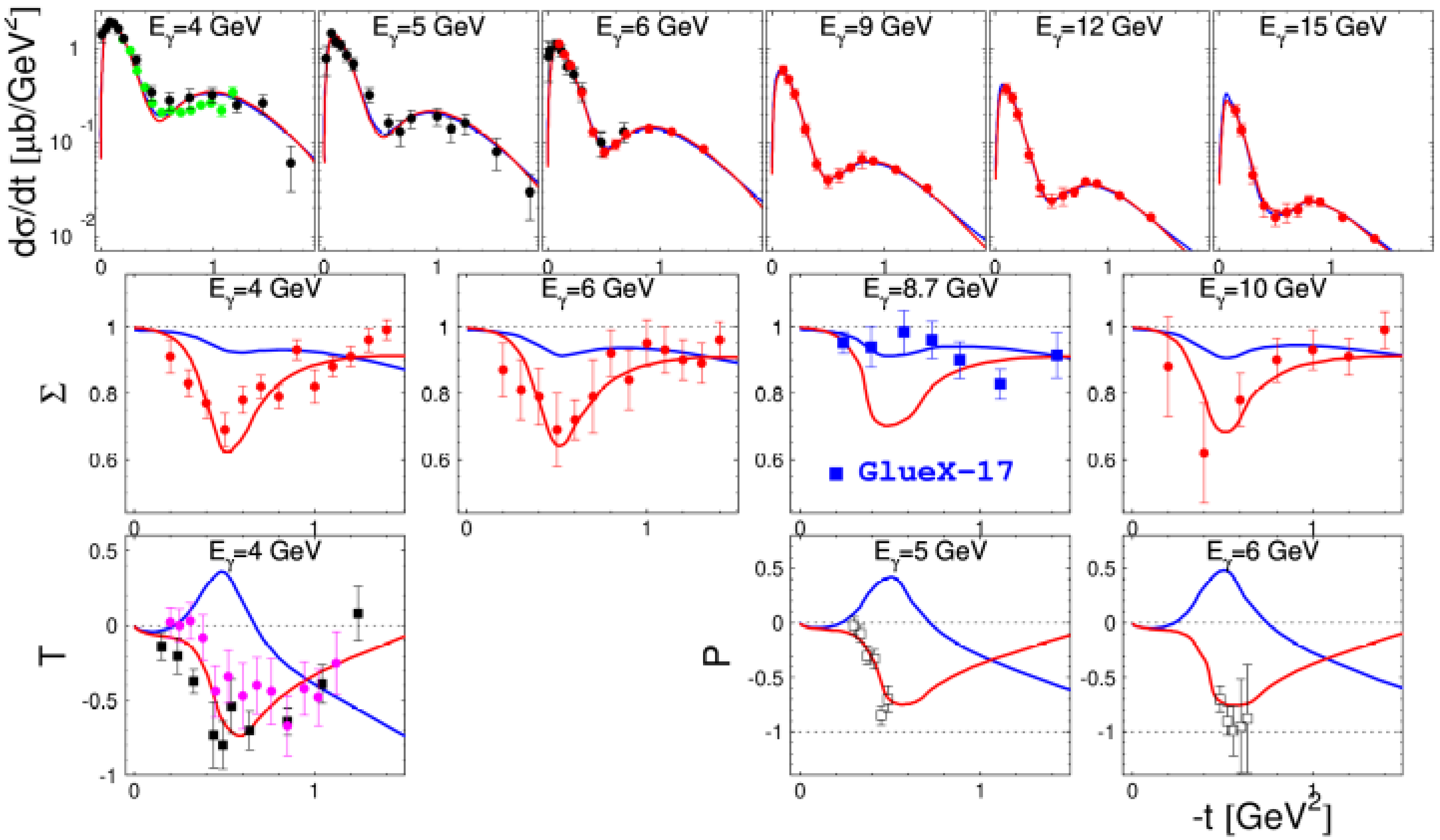
1. For η, η´, and π^0 photoproduction in t channel was used Regge cut model:
   V. Kashevarov, M. Ostrick, L. Tiator, RRC 96 (2017) 035207.

2. For π^+ and π^- photoproduction our model based on publications:
   M.Guidal, J.-M. Laget, M. Vanderhagen, NP A627 (1997) 645,
   And J.-M. Laget, Prog. Part. Nucl. Phys., Vol. 111 (2020) 103737.

3. New:
   - saturation of all Regge trajectories was added
   - nucleon trajectory in u channel was added

Details of model: background

V. L. Kashevarov, M. Ostrick, L. Tiator , Phys. Rev. C**96** (2017) 045207



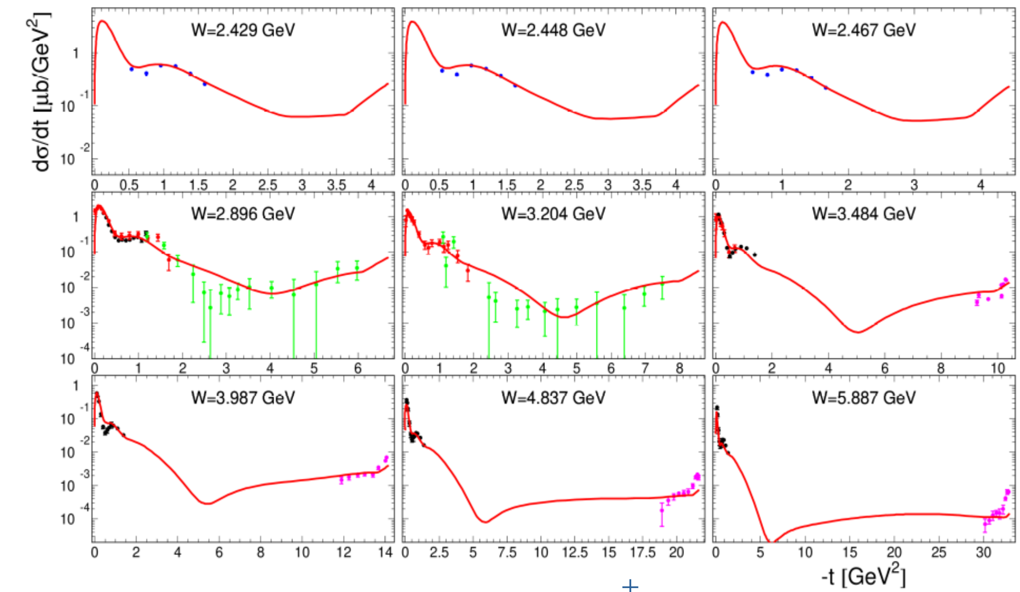
Red lines: our fit result to all data

Blue lines: fit to ds/dt and GlueX-17 data.

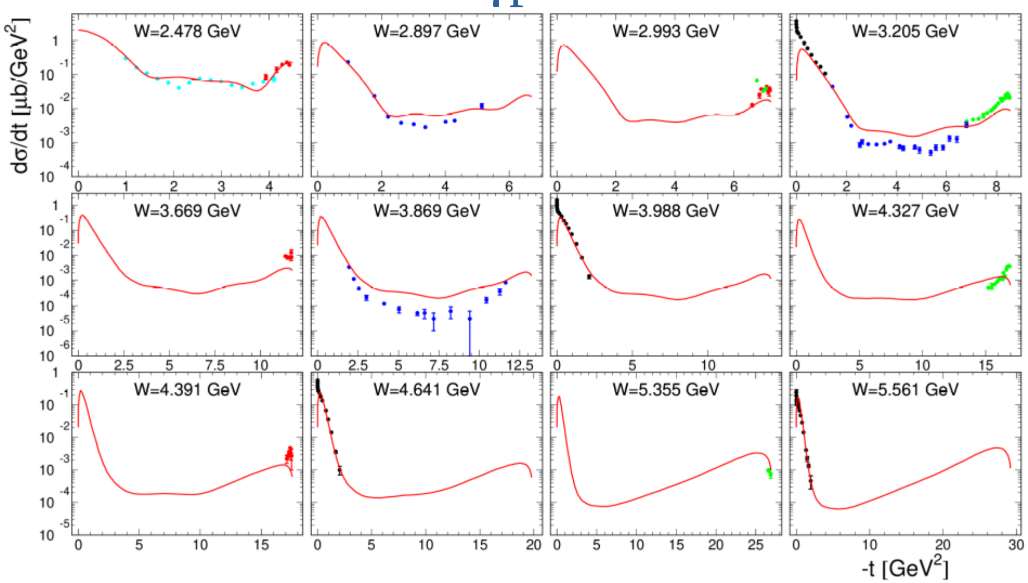
DATA: ds/dt and Σ: SLAC-71 (red), DESY-68 (black discs), DESY-73 (Nucl.Phys. B51) (green)
T: Daresbury-72 (magenta)}, DESY-73 (PL B46) (black full squares),
P: CEA-73 (black open squares).

Details of model: background

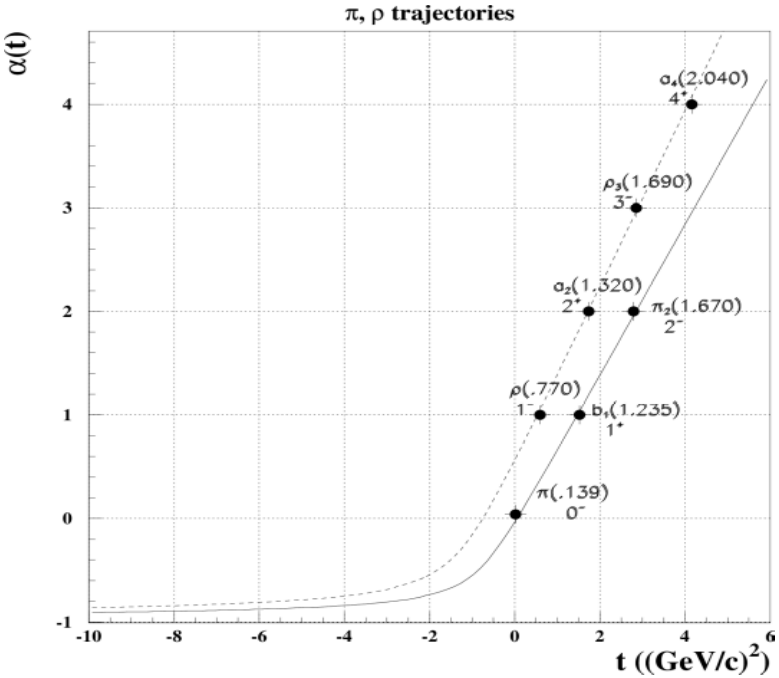
$\gamma p \to \pi^0 p$



$\gamma p \to \pi^+ n$



Saturation of Regge trajectories.
Example for ρ and π reggeons.

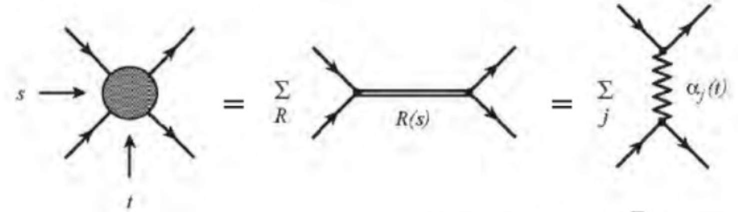


Duality

from quark-hadron duality it is known:

sum over all s-channel resonances is equivalent to sum over all t-channel resonances

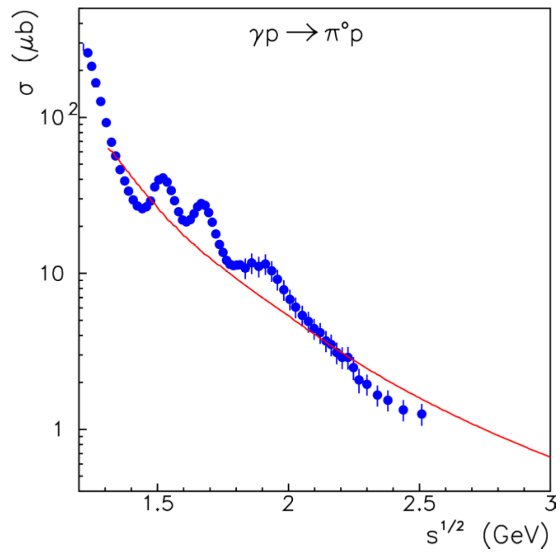
therefore: keeping both leads to double counting



$$M = \sum_{i=1}^{\infty} M_s^{Res_i} = \sum_{i=1}^{\infty} M_t^{Res_i} = \sum_{i=1}^{N} M_s^{Res_i} + \left[\sum_{i=1}^{\infty} M_t^{Res_i} - \sum_{i=1}^{N} M_s^{Res_i}\right]$$

$$\approx \sum_{i=1}^{N} M_s^{Res_i} + M^{Regge} \cdot F_d(W)$$: our approach

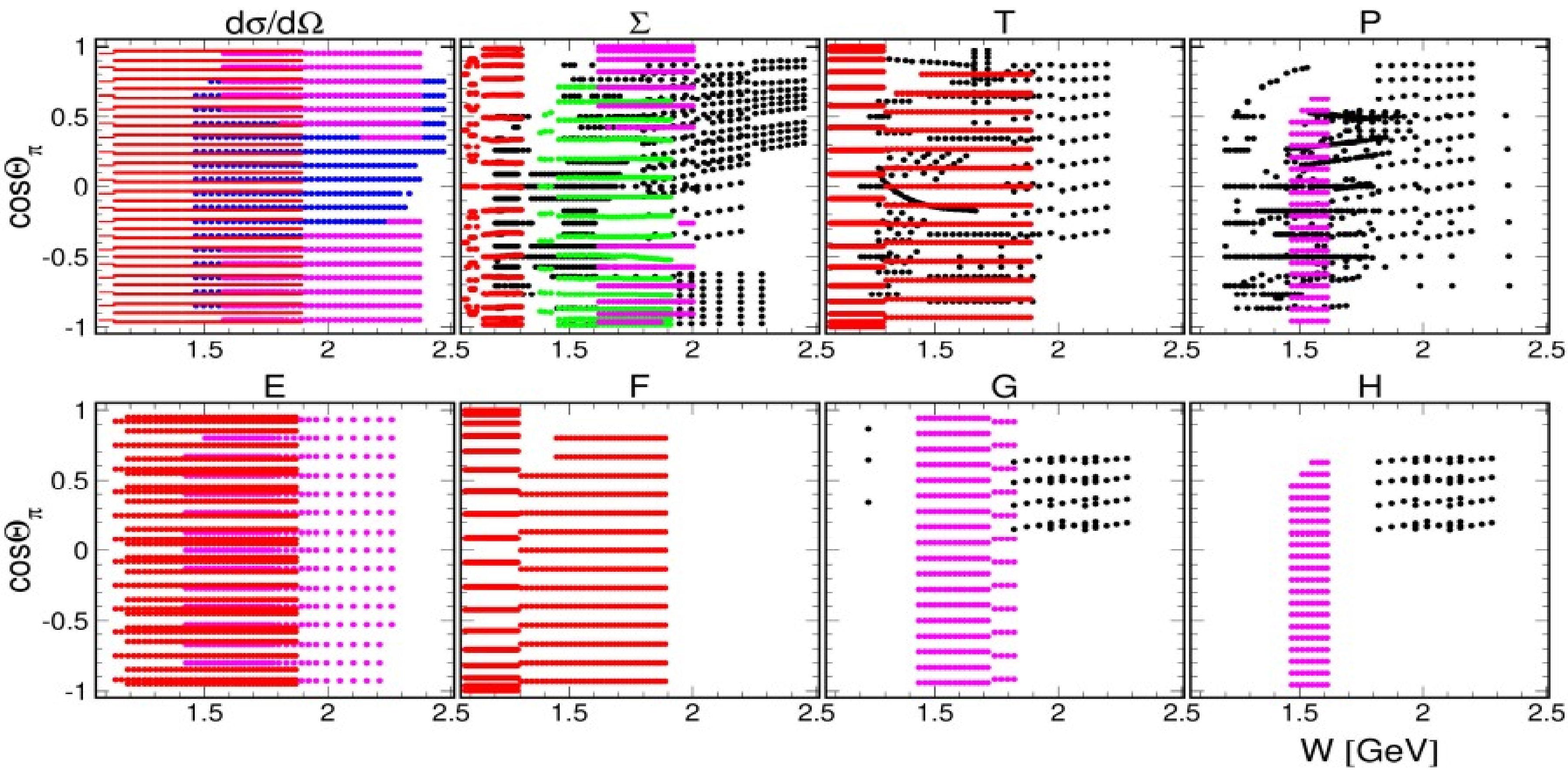
$$f_d(W) = 1 - e^{-\frac{W - W_{thr}}{\Lambda}}$$



A.~Sibirtsev, J.~Haidenbauer, S.~Krewald, U.~G.~Meissner, and A.~W.~Thomas, EPJA {\bf 41}, 71 (2009)

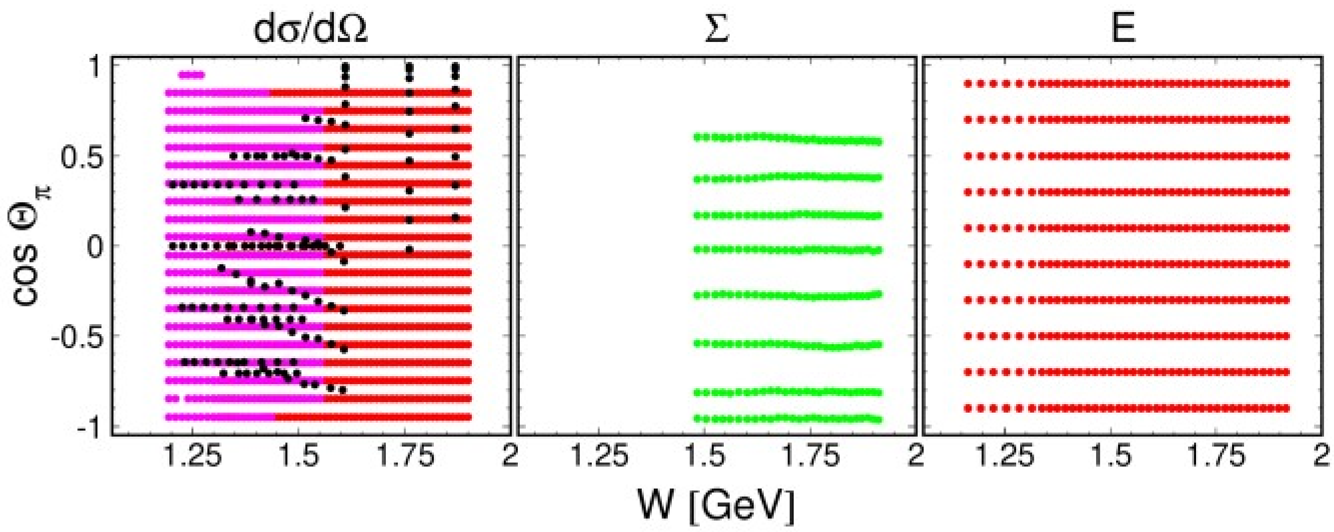
Data base for $\gamma p \rightarrow \pi^0 p$

The biggest data set exist for this reaction:
200 publications with experimental results for 10 observables.
For $d\sigma/d\Omega$ in resonance region used only latest data from A2MAMI and CLAS Collaborations



A2MAMI (red), CB/ELSA (magenta), CLAS (blue), GRAAL (green).
Black points correspond to the old data.

Data base for $\gamma n \rightarrow \pi^0 n$



Black: old data [1], Red: A2MAMI [2, 5], Magenta: A2MAMI [3], Green: GRAAL [4]

References

1. Clinesmith, PhD theris, CIT (1967);
   C. Bacciet et al., Phys. Lett. C 39, 559 (1972);
   Y. Hemmiet et al., Nucl. Phys. B 55, 333 (1973);
   A. Ando et al., Physik Daten, Fachinformationszentrum, Karlsruhe (1977).
2. M. Dieterle et al. (A2 Collaboration at MAMI), Phys. Rev. C 97, 065205 (2018).
3. W. J. Briscoe et al. (A2 Collaboration at MAMI), Phys. Rev. C 100, 065205 (2019).
4. R. Di Salvo et al. (GRAAL Collaboration), Eur. Phys. J. A 42, 151 (2009).

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5. F. Cividini et al., (A2 Collaboration at MAMI), Eur. Phys. J. A 58:113 (2022).

New publication (data are not presented in the plot) :

6. C. Mullen et al. (A2 Collaboration at MAMI), Eur. Phys. J. A 57, 205 (2021).
   (Σ asymmetry, W = 1271 – 1424 MeV, cosΘ = from -0.85 to 0.66, 189 experimental points.)

Data base for $\gamma n \to \pi^0 n$

Problem: no data on free neutron!

SAID: inludes in the fit only data corrected for Fermi motion and FSI.
BnGa: there are two solution, for free and quasi free neutron.

Conclusions from Ref. 5:

- E asymmetry data for free and quasi-free pion photoproduction on proton are rather close to each other and agree within statistical and systematic uncertainties.
- This feature is confirmed by corresponding calculations (A.Fix).

It means, that effects of Fermi motion and FSI are not significant for E asymmetry.

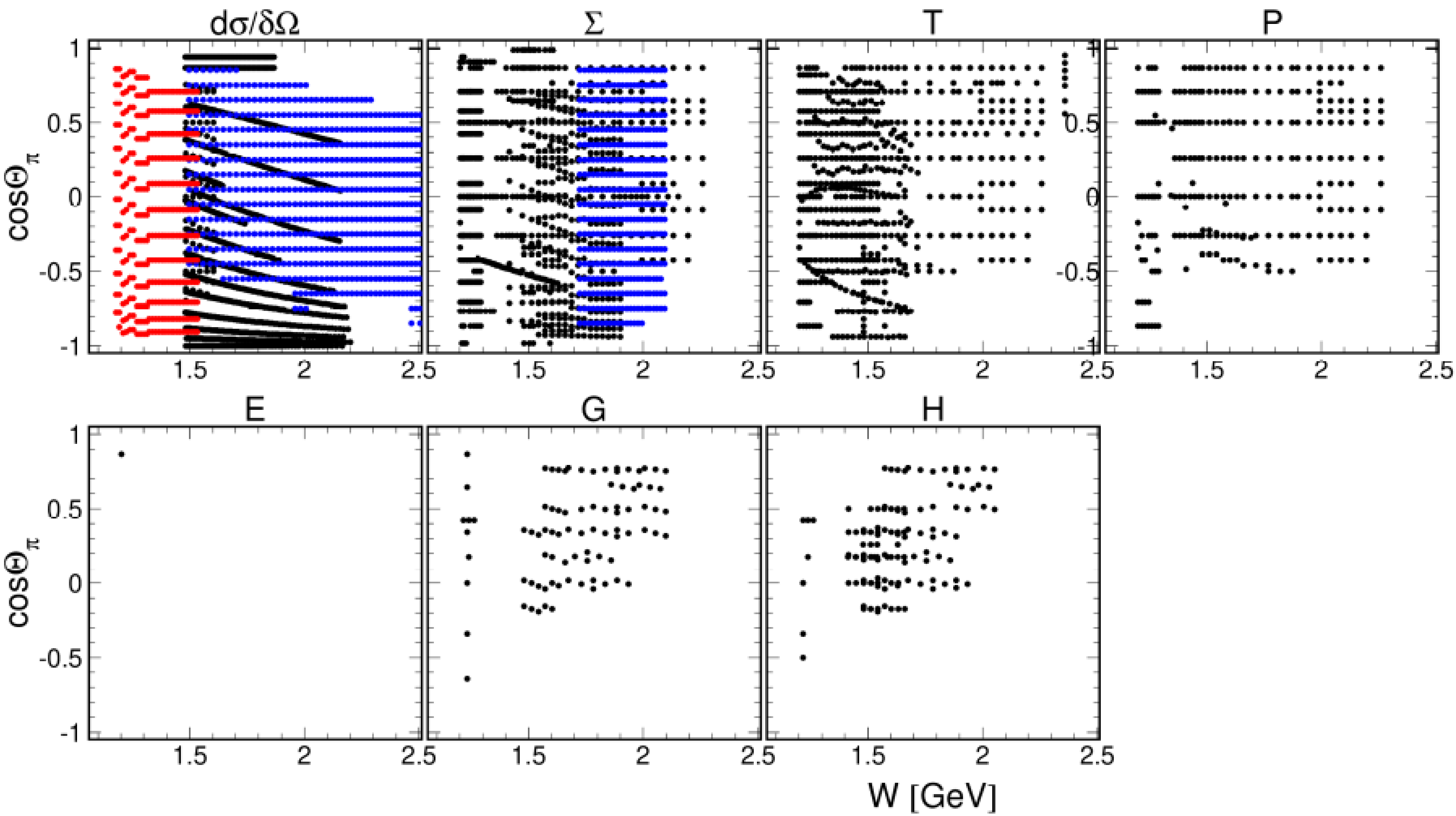
We can hope, that it should be the same for neutron data.

Also we supposed that it hould be work for others asymmetries.

So, we include in the fit both E and Σ data.

We included also $d\sigma/d\Omega$ from Ref. 3, were correction for Fermi motion and FSI was done.

Data base for $\gamma p \to \pi^+ n$



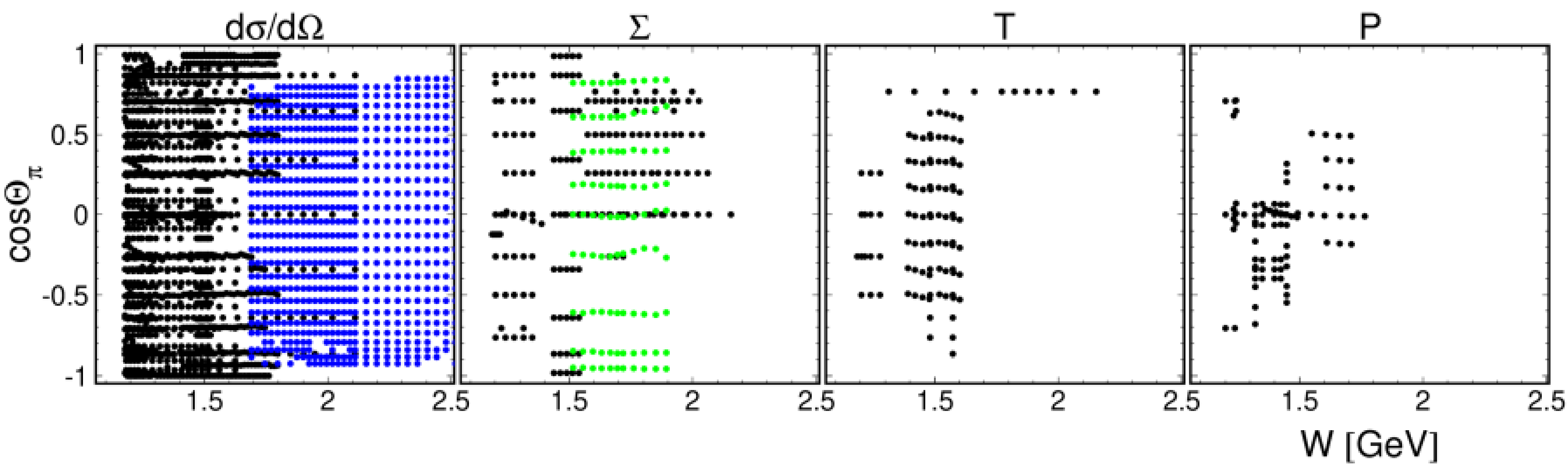
A2MAMI (red), CLAS (blue)
Black points correspond to the old data.

Data base for $\gamma n \rightarrow \pi^- p$

For this reaction beside results on quasi-free neutron target exist data from reaction:

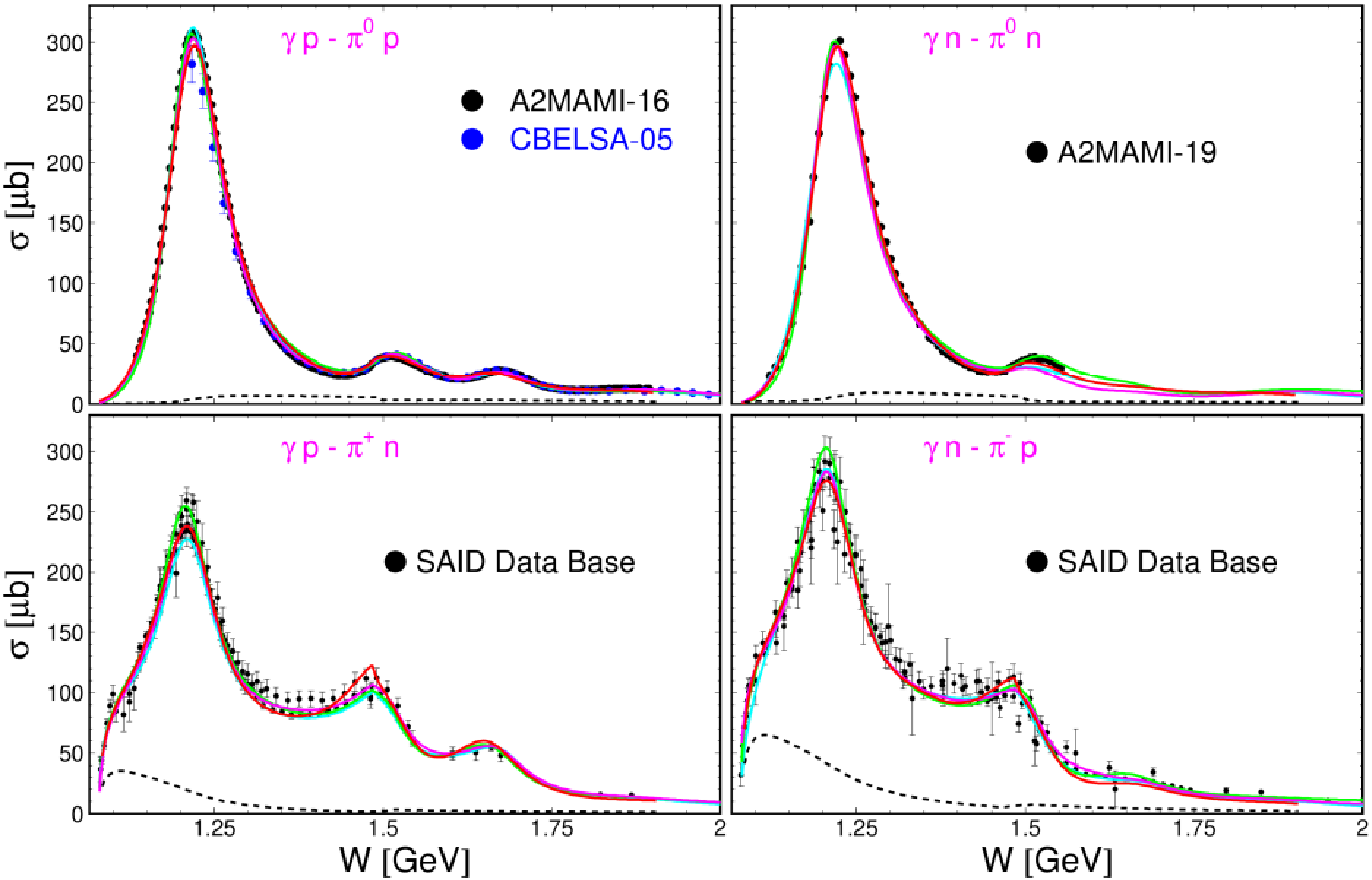
$$\pi^- p \rightarrow \gamma n$$

that are in good agreement with neutron data.
So, we use both of them.



A2MAMI (red), CLAS (blue), GRAAL (green).
Black points correspond to the old data.

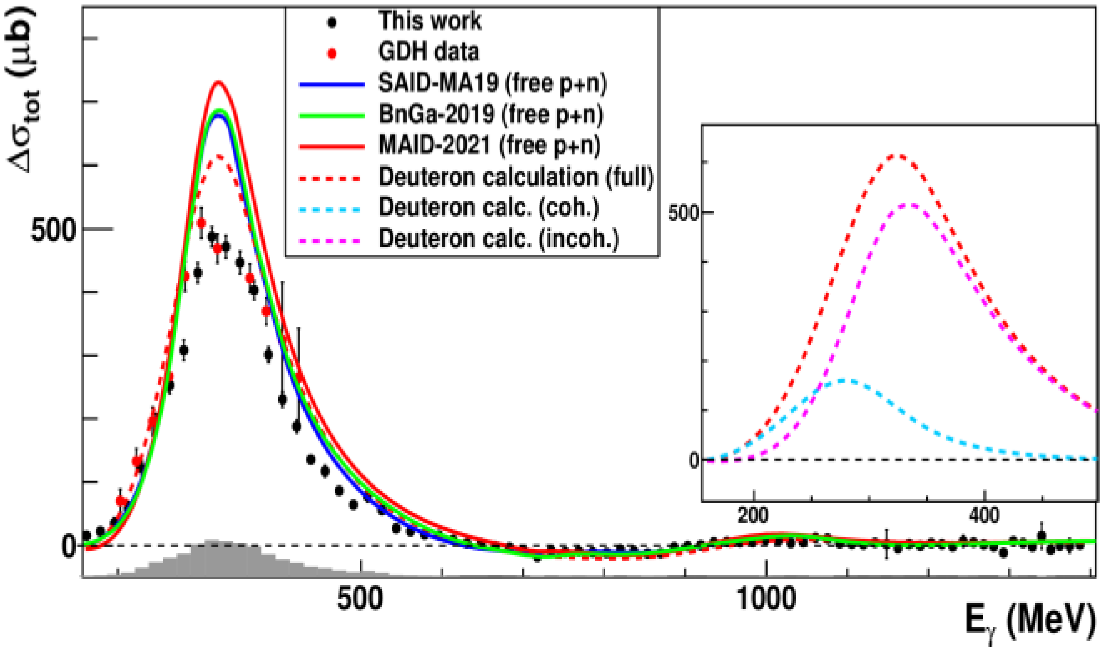
Selected results: total cross sections



Red line: PionMAID2021, green: MAID2007, magenta: SAID SM12, cyan: BnGa-2019

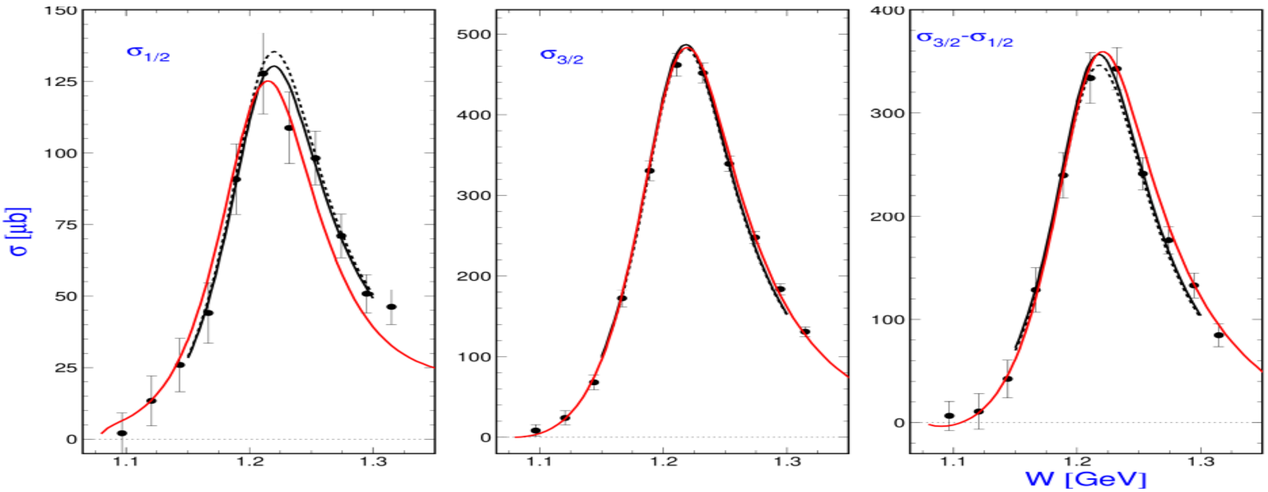
Black dashed: PionMAID2021 background

Selected results: polarized total cross sections



F. Cividini et al., Eur. Phys.J. A (2022) 58:113

$\gamma\ n \rightarrow \pi^0\ n$



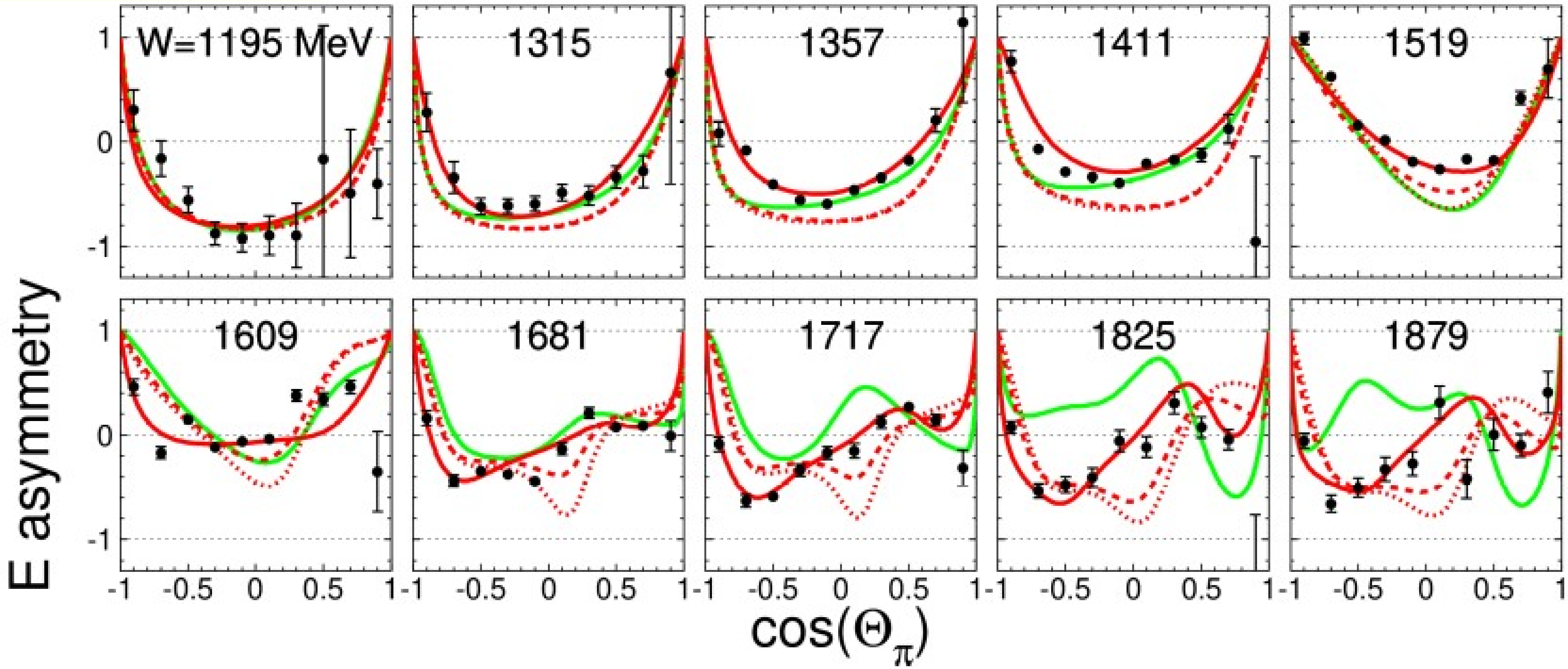
Red: PionMAID2021,

black: BnGa-2019 (solid),
BnGa-2014-02(dashed).

Data: A2MAMI-04

Selected results: helicity asymmetry E

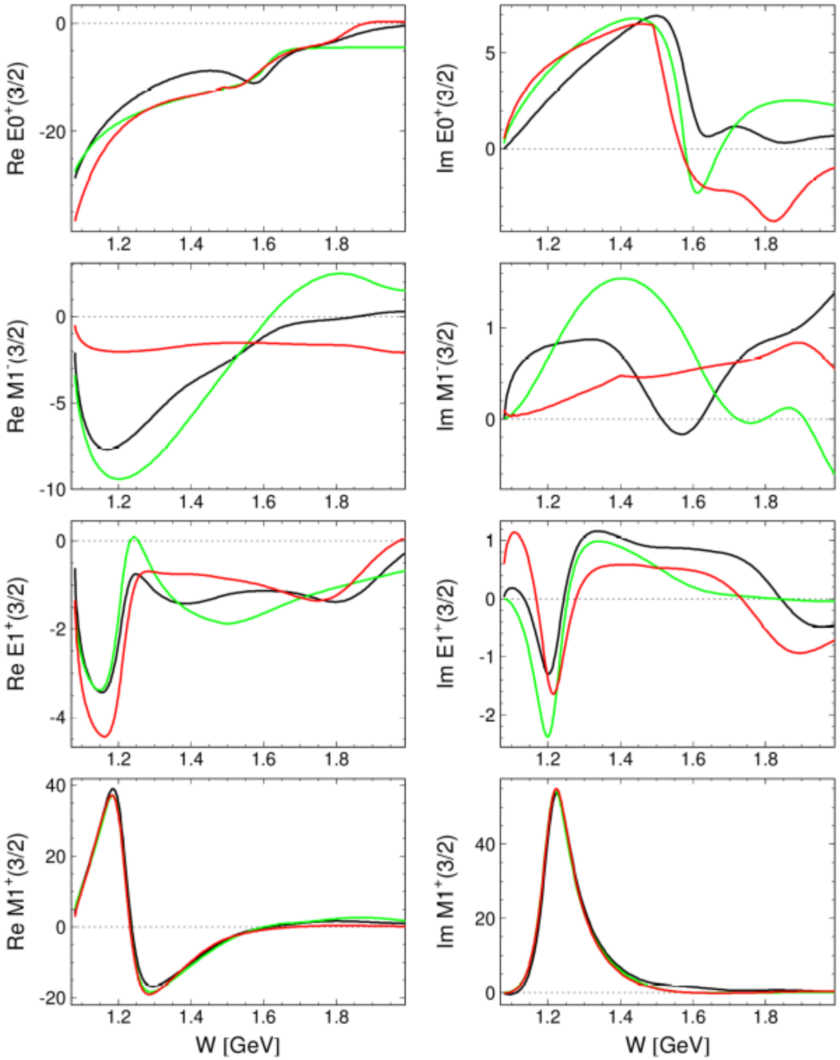
$\gamma\ n \rightarrow \pi^0\ n$



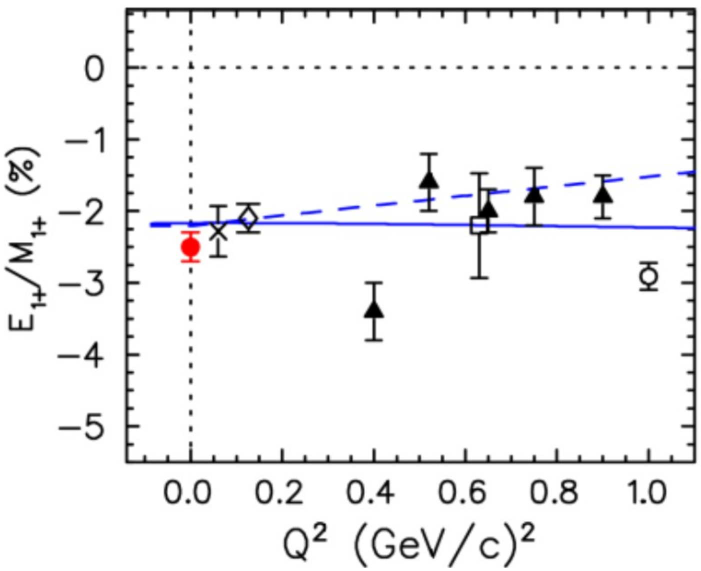
Red lines: PionMAID2021
dotted – all pi0 data not fitted
dashed – new A2MAMI not fitted
solid – full solution

green: MAID2007

Selected results: multipoles



Red line: PionMAID2021,
green: MAID2007,
black:BnGa-2019



solid line: MAID2007
dashed line: MAID2003

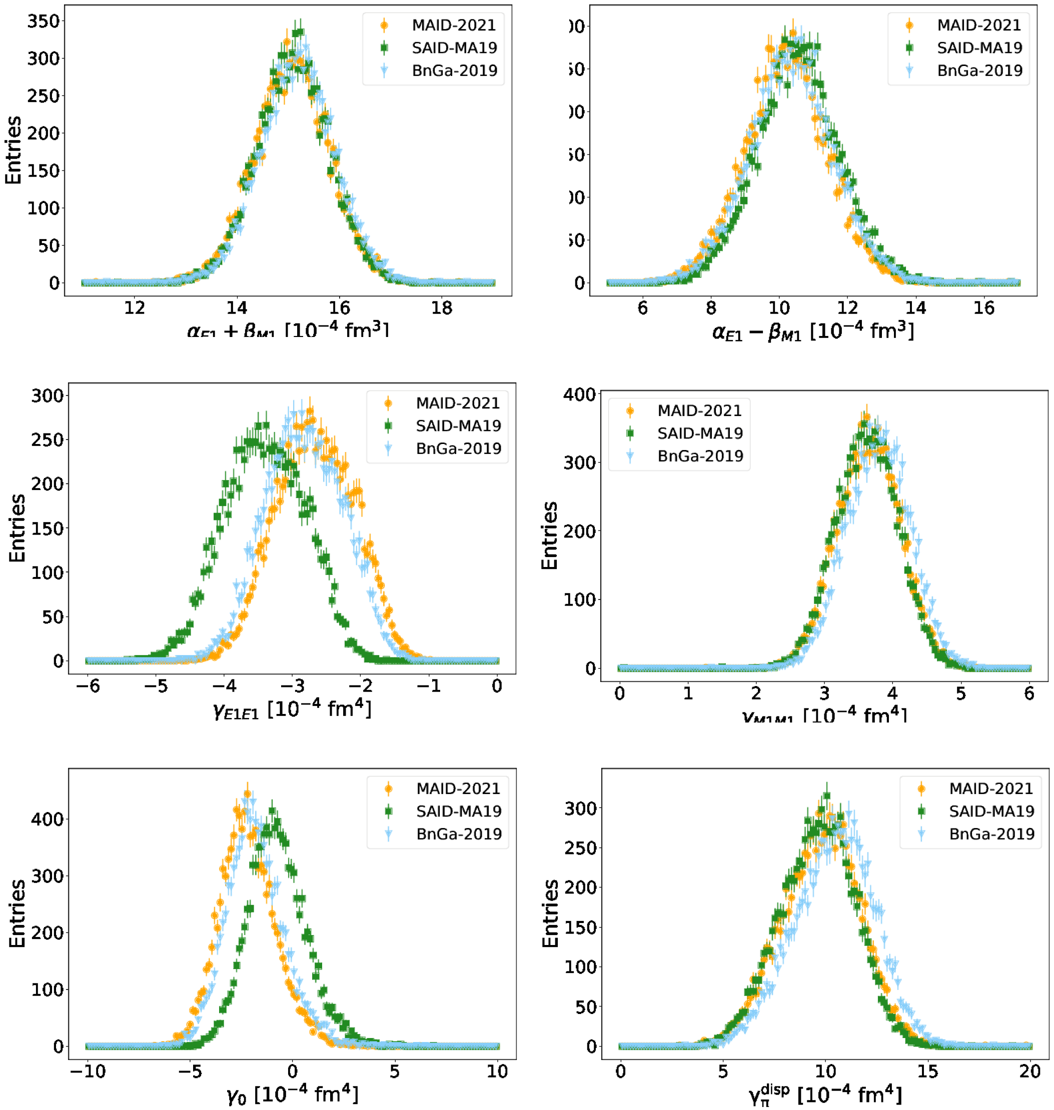
Red point: (- 2.5 ± 0.2 ± 0.2)%
R. Beck et al., PRL 78 (1997) 606

PDG2020 estimate: (-2.5 ± 0.5) %
(min: -3.9%, max: -1.6%)

Our result:
E_{1+}/M_{1+} = - 2.66 %

Application of PionMAID: proton polarizabilities

E. Mornacchi, S. Rodini, B. Pasquini, and P. Pedroni, Phys. Rev. Lett. 129, no.10, 102501 (2022)



Distribution of fit parameters obtained with different PWA

The fit was performed using a bootstrap-based technique combined with a fixed-t subtracted dispersion relation model, using three different PWA solutions as input.

Summary and conclusions

1. New version of unitarity isobar PionMAID model for pion photoproduction on protons and neutrons is finished.

2. The model describes experimental data from threshold up to W = 6 GeV (E_γ =15 GeV). It is main difference from all others PWA models.

3. The well-known duality problem is addressed in a new approach with a damping factor removing most of Regge background in the resonance region.

4. PionMAID was used to analyze new A2MAMI data for pi0 photoproduction on deuteron. Impact of new data for E asymmetry was shown.

5. PionMAID was successfully applied for extraction scalar and spin polarizabilities of proton.

6. Next steps: - combining PionMAID2021 and EtaMAID2017 for multichannel fit (8 reaction channels)
   - adding in ω N, K Λ, and K Σ channels in the multichannel fit .

7. Further steps: electoproduction for all channels.

Resonance parameters

	M[MeV]	G[MeV]	B(pi)	B(eta)	B(eta´)	B(KL)	B(KS)	B(om)	A12p	A32p	A12n	A32n
1 P11(1440)	1421.	450.	75.2	0.0	0.0	0.0	0.0	0.0	-63.	0.	103.	0.
PDG20 ****	1440.	350.	65.0	0.0	0.0	0.0	0.0	0.0	-65.	0.	45.	0.
2 D13(1520)	1520.	116.	70.5	0.1	0.0	0.0	0.0	0.0	-21.	148.	-22.	-148.
PDG20 ****	1515.	110.	60.0	0.1	0.0	0.0	0.0	0.0	-25.	140.	-50.	-115.
3 S11(1530)	1537.	195.	42.6	42.0	0.0	0.0	0.0	0.0	132.	0.	-108.	0.
PDG20 ****	1530.	150.	42.0	42.0	0.0	0.0	0.0	0.0	105.	0.	-75.	0.
4 S11(1650)	1673.	150.	47.6	25.0	0.0	10.0	0.0	0.0	32.	0.	16.	0.
PDG20 ****	1650.	125.	60.0	25.0	0.0	10.0	0.0	0.0	45.	0.	-10.	0.
5 D15(1675)	1680.	173.	39.3	1.0	0.0	0.0	0.0	0.0	23.	61.	-38.	-100.
PDG20 ****	1675.	145.	40.0	1.0	0.0	0.0	0.0	0.0	18.	22.	-60.	-85.
6 F15(1680)	1690.	149.	62.5	1.0	0.0	0.0	0.0	0.0	-15.	152.	27.	-19.
PDG20 ****	1685.	120.	65.0	1.0	0.0	0.0	0.0	0.0	-10.	135.	30.	-35.
7 D13(1700)	1666.	268.	7.9	1.0	0.0	0.0	0.0	22.0	1.	143.	-25.	-13.
PDG20 ***	1720.	200.	12.0	1.0	0.0	0.0	0.0	22.0	41.	-37.	25.	-32.
8 P11(1710)	1754.	210.	16.4	30.0	0.0	15.0	1.0	3.0	170.	0.	-70.	0.
PDG20 ****	1710.	140.	10.0	30.0	0.0	15.0	1.0	3.0	50.	0.	-40.	0.
9 P13(1720)	1690.	400.	9.1	3.0	0.0	5.0	0.0	26.0	101.	-51.	-50.	-48.
PDG20 ****	1720.	250.	11.0	3.0	0.0	5.0	0.0	26.0	100.	135.	80.	-140.
10 F15(1860)	1756.	450.	19.1	4.0	0.0	0.0	0.0	0.0	51.	-12.	25.	-106.
PDG20 **	1860.	270.	20.0	4.0	0.0	0.0	0.0	0.0	-17.	29.	21.	34.
11 D13(1875)	1888.	264.	10.0	10.0	0.0	1.1	0.7	13.0	31.	-112.	0.	-34.
PDG20 ***	1875.	200.	7.0	10.0	0.0	1.1	0.7	13.0	15.	-5.	10.	-20.
12 P11(1880)	1800.	450.	5.1	25.0	0.0	2.0	17.0	20.0	-209.	0.	94.	0.
PDG20 **	1880.	300.	6.0	25.0	0.0	2.0	17.0	20.0	21.	0.	-60.	0.
13 S11(1895)	1920.	188.	25.2	25.0	20.0	10.0	10.0	10.0	-43.	0.	17.	0.
PDG20 ****	1895.	120.	10.0	25.0	20.0	10.0	10.0	10.0	-16.	0.	13.	0.
14 P13(1900)	1933.	184.	4.5	10.0	6.0	14.0	5.0	15.0	82.	5.	-36.	30.
PDG20 ***	1920.	200.	10.0	10.0	6.0	14.0	5.0	15.0	24.	-67.	10.	-60.
15 F17(1990)	2100.	400.	4.8	1.0	0.0	6.0	0.0	0.0	104.	152.	-69.	37.
PDG20 **	2020.	300.	4.0	1.0	0.0	6.0	0.0	0.0	40.	57.	-45.	-52.
16 F15(2000)	2100.	326.	8.0	2.0	0.0	0.0	0.0	18.0	18.	56.	17.	6.
PDG20 **	2060.	390.	8.0	2.0	0.0	0.0	0.0	18.0	31.	-43.	-18.	-35.
17 D15(2060)	2030.	346.	7.2	4.0	0.0	15.0	3.0	4.0	83.	3.	-44.	93.
PDG20 **	2100.	400.	10.0	4.0	0.0	15.0	3.0	4.0	62.	62.	25.	-37.
18 P11(2100)	2105.	231.	13.5	30.0	8.0	1.0	0.0	15.0	-1.	0.	-5.	0.
PDG20 **	2100.	260.	12.0	30.0	8.0	1.0	0.0	15.0	10.	0.	10.	0.
19 D13(2120)	2108.	314.	10.1	3.0	4.0	8.0	0.0	12.0	-55.	96.	20.	-48.
PDG20 **	2120.	300.	10.0	3.0	4.0	8.0	0.0	12.0	130.	160.	110.	40.
20 G17(2190)	2140.	500.	14.9	4.0	0.0	0.6	0.0	14.0	0.	62.	-11.	-22.
PDG20 ****	2180.	400.	15.0	4.0	0.0	0.6	0.0	14.0	-71.	27.	-15.	-34.
21 G19(2250)	2250.	303.	10.0	5.0	0.0	2.0	0.0	0.0	68.	70.	-40.	-4.
PDG20 ****	2280.	500.	10.0	5.0	0.0	2.0	0.0	0.0	10.	10.	10.	10.
22 H19(2220)	2200.	500.	26.3	0.0	0.0	0.0	0.0	0.0	15.	42.	10.	-14.
PDG20 ****	2250.	400.	25.0	0.0	0.0	0.0	0.0	0.0	10.	10.	10.	10.
23 D15(2570)	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.	0.	0.	0.
PDG20 **	2570.	250.	1.0	0.0	0.0	0.0	0.0	0.0	10.	10.	10.	10.
24 H111(2600)	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.	0.	0.	0.
PDG20 ***	2600.	650.	5.0	0.0	0.0	0.0	0.0	0.0	10.	10.	10.	10.
25 I113(2700)	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.	0.	0.	0.
PDG20 **	2700.	350.	4.0	0.0	0.0	0.0	0.0	0.0	10.	10.	10.	10.

Resonance parameters

	M[MeV]	G[MeV]	B(pi)	B(KS)	A12	A32
1 P33(1232)	1231.	115.	99.4	0.0	-129.	-265.
PDG20 ****	1232.	117.	99.4	0.0	-135.	-255.
2 P33(1600)	1503.	420.	14.6	0.0	54.	86.
PDG20 ****	1570.	250.	16.0	0.0	-45.	-35.
3 S31(1620)	1570.	290.	21.3	0.0	141.	0.
PDG20 ****	1610.	130.	30.0	0.0	50.	0.
4 D33(1700)	1704.	363.	18.6	0.0	105.	120.
PDG20 ****	1710.	300.	15.0	0.0	130.	130.
5 S31(1900)	1840.	180.	4.1	1.0	98.	0.
PDG20 ***	1860.	250.	8.0	1.0	65.	0.
6 F35(1905)	1918.	373.	13.6	0.0	44.	42.
PDG20 ****	1880.	330.	12.0	0.0	22.	-45.
7 P31(1910)	1950.	315.	23.0	9.0	14.	0.
PDG20 ****	1900.	300.	20.0	9.0	20.	0.
8 P33(1920)	1870.	360.	3.7	4.0	149.	-102.
PDG20 ***	1920.	300.	12.0	4.0	110.	-105.
9 D35(1930)	1955.	494.	5.1	0.0	-96.	85.
PDG20 ***	1950.	300.	10.0	0.0	-7.	5.
10 D33(1940)	0.	0.	0.0	0.0	0.	0.
PDG20 **	2000.	400.	4.0	0.0	170.	150.
11 F37(1950)	1927.	279.	35.0	0.4	-55.	-91.
PDG20 ****	1930.	285.	40.0	0.4	-70.	-90.
12 F35(2000)	2065.	350.	7.0	0.0	36.	-40.
PDG20 **	2000.	500.	7.0	0.0	-61.	158.
13 G37(2200)	2150.	500.	4.6	4.0	23.	150.
PDG20 ***	2200.	350.	5.0	4.0	107.	-131.
14 H39(2300)	2200.	500.	4.0	0.0	25.	104.
PDG20 **	2300.	400.	4.0	0.0	10.	10.
15 G39(2400)	2459.	314.	5.2	0.0	59.	-51.
PDG20 **	2400.	500.	6.0	0.0	-128.	-115.

Details of model: resonance exctruction

Breit-Wigner ansatz for s-channel resonance excitations:

$$\mathcal{M}_{\ell\pm}(W) = \bar{\mathcal{M}}_{\ell\pm} f_{\gamma N}(W) \frac{M_R \Gamma_{\text{tot}}(W)}{M_R^2 - W^2 - i M_R \Gamma_{\text{tot}}(W)} f_{\pi N}(W) C_{\pi N}$$

$$f_{\pi N}(W) = \zeta_{\pi N} \left[\frac{1}{(2J+1)\pi} \frac{k}{q} \frac{M_N}{W} \frac{\Gamma_{\pi N}(W)}{\Gamma_{\text{tot}}(W)^2} \right]^{1/2}$$

$$f_{\gamma N}(W) = \left(\frac{k}{k_R} \right)^2 \left(\frac{X^2 + k_R^2}{X^2 + k^2} \right)^2$$

$C_{\pi N}$ is an isospin factor:

$$C_{\pi N} = \begin{cases} -1/\sqrt{3} & : \ I = 1/2 \\ \sqrt{3/2} & : \ I = 3/2 \end{cases}$$

for η and η' production:

$$C_{\eta N} = C_{\eta' N} = -1$$

$\zeta_{\pi N}$ is a relative phase of an individual resonance:

$$\zeta_{\pi N} = 1, \ \zeta_{\eta N} = \pm 1, \ \zeta_{\eta' N} = \pm 1$$