Exploring the strong interaction in three-body systems at the LHC

59th International Winter Meeting on Nuclear Physics
January 23 to 27, 2023 Bormio, Italy

Bhawani Singh

Technische Universität München

on behalf of the **ALICE Collaboration**

bhawani.singh@tum.de

January 27, 2022

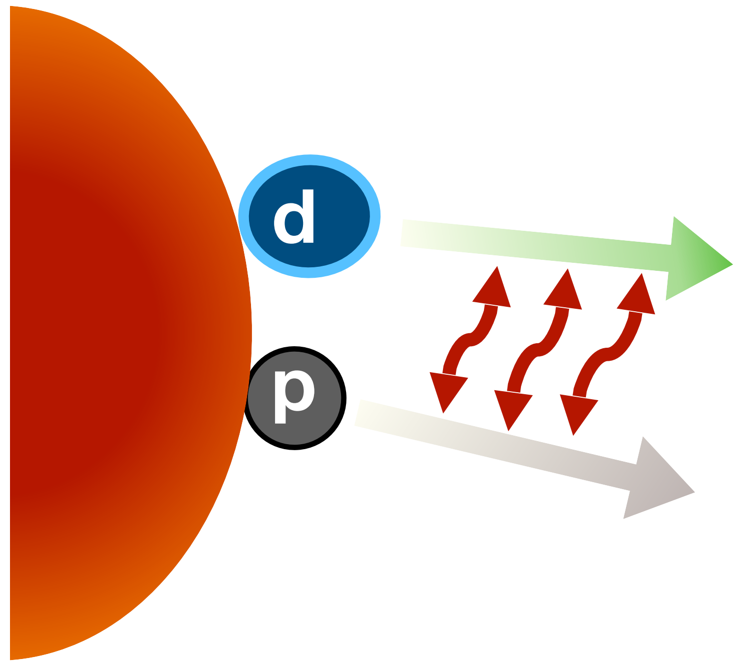
Motivation

- **Proton-deuteron (p—d) interaction**
  - p—d interaction is well constrained from the scattering experiments

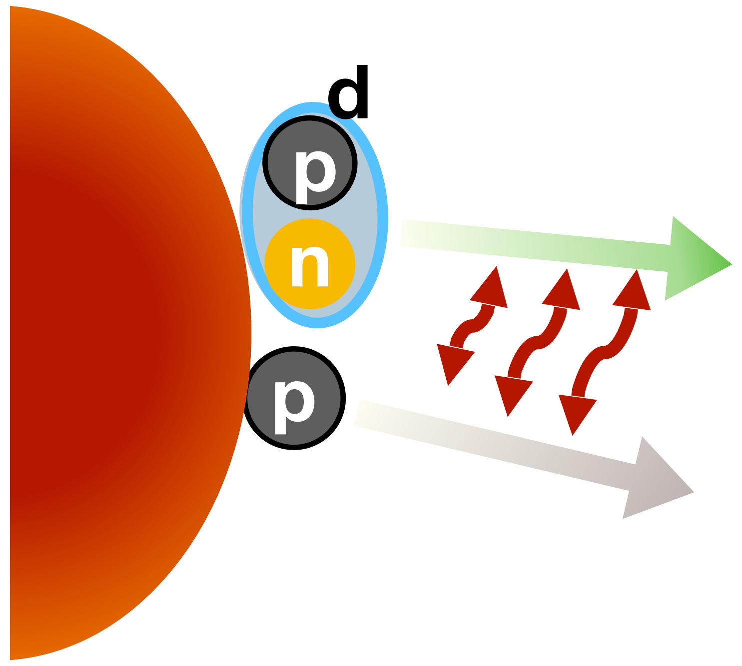
➡ p—d correlations in pp collisions at the LHC provide a new way to explore the interaction of a three-body system at short distances

- **Production mechanism of light nuclei not understood:**
  - What can final-state interaction studies say about the formation of deuterons (antideuterons)?

Pointlike and distinguishable particles

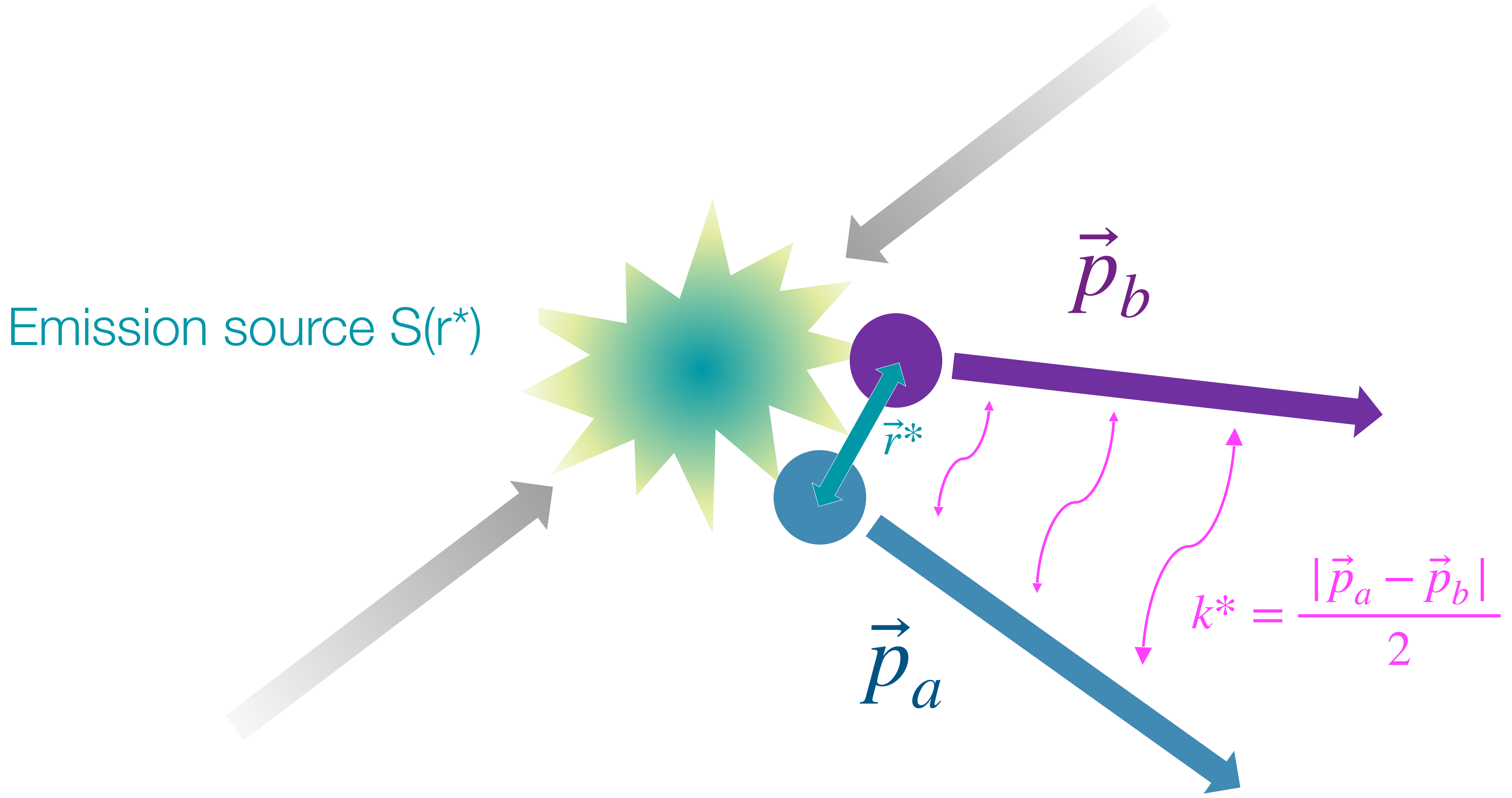


Deuteron as a composite object interacting with the proton



Introduction: Femtoscopy

- **Main observable:** correlation in the relative momentum *k** of a particle pair
  - Emitting source: hypersurface of kinematic freeze-out for final-state particles, in pp collision the source size ~ 1 fm (Gaussian profile)
  - Two-particle relative wave function: expresses the interaction between particles



- Study the emission source if the interaction among the particle pair is known
- Or study the interaction among the particles if emission source is known

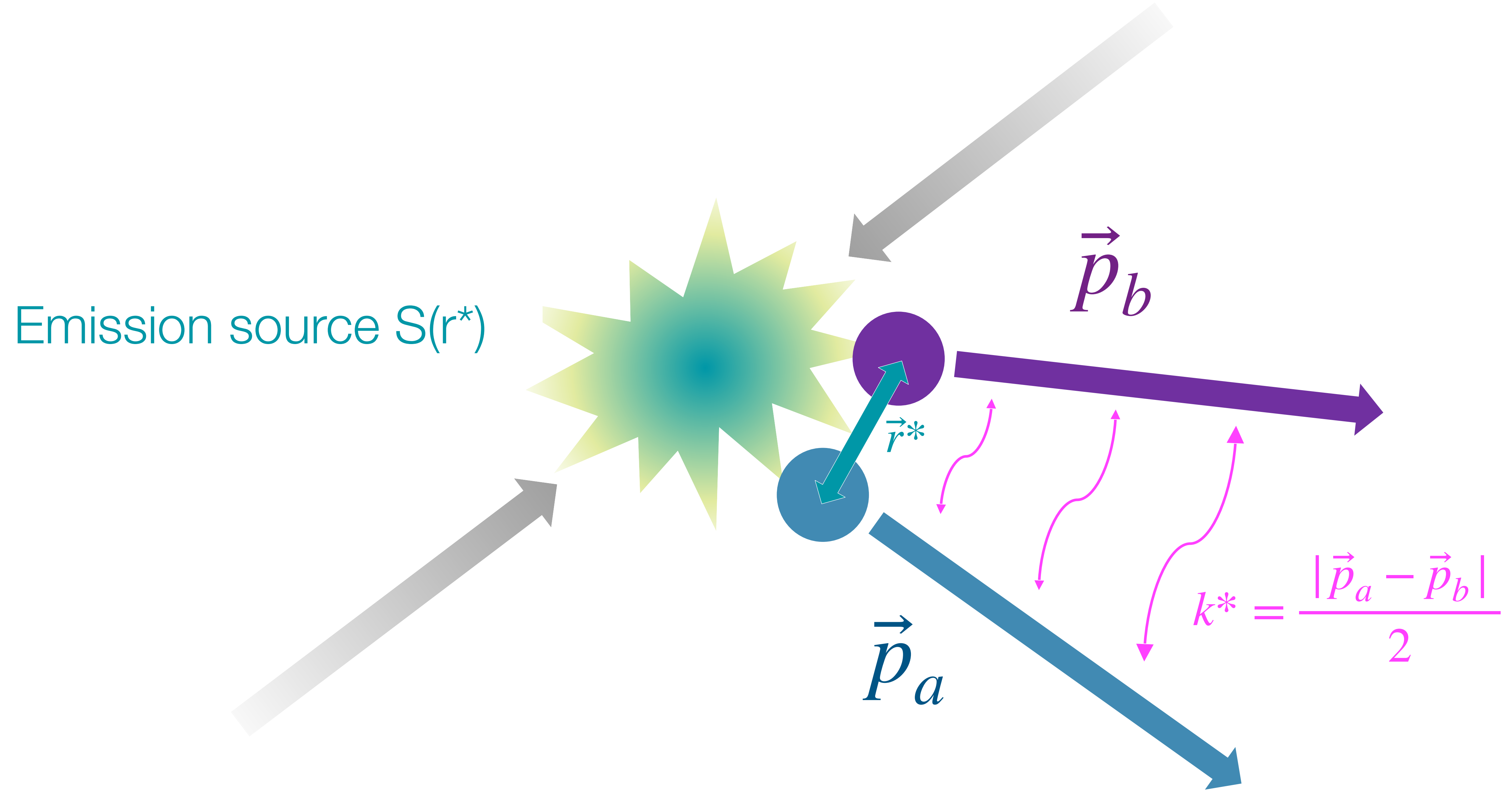
Koonin-Pratt Equation

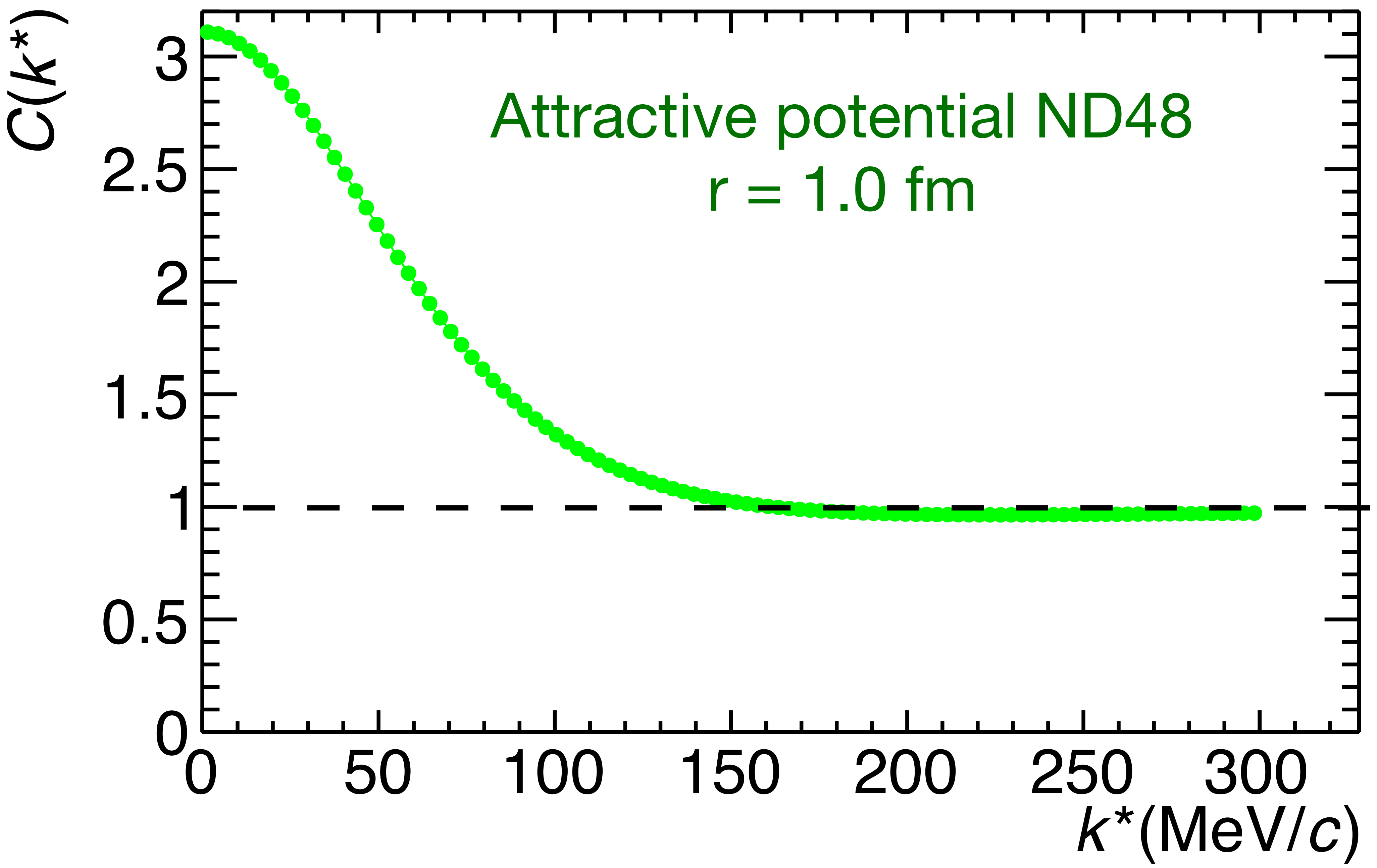
$$C(k^*) = \underbrace{\mathcal{N}\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}}_{\text{experimental definition}} = \underbrace{\int S(\vec{r}^*)\left|\psi(\vec{k}^*,\vec{r}^*)\right|^2 d^3\vec{r}^*}_{\text{theoretical definition}} \xrightarrow{k^*\to\infty} 1$$

CATS Framework: D. Mihaylov et al., EPJ. C78 (2018) 394
S.E. Koonin PLB 70 43 (1977)

Introduction: Femtoscopy

- Correlation rises above 1 for attractive potentials





Koonin-Pratt Equation

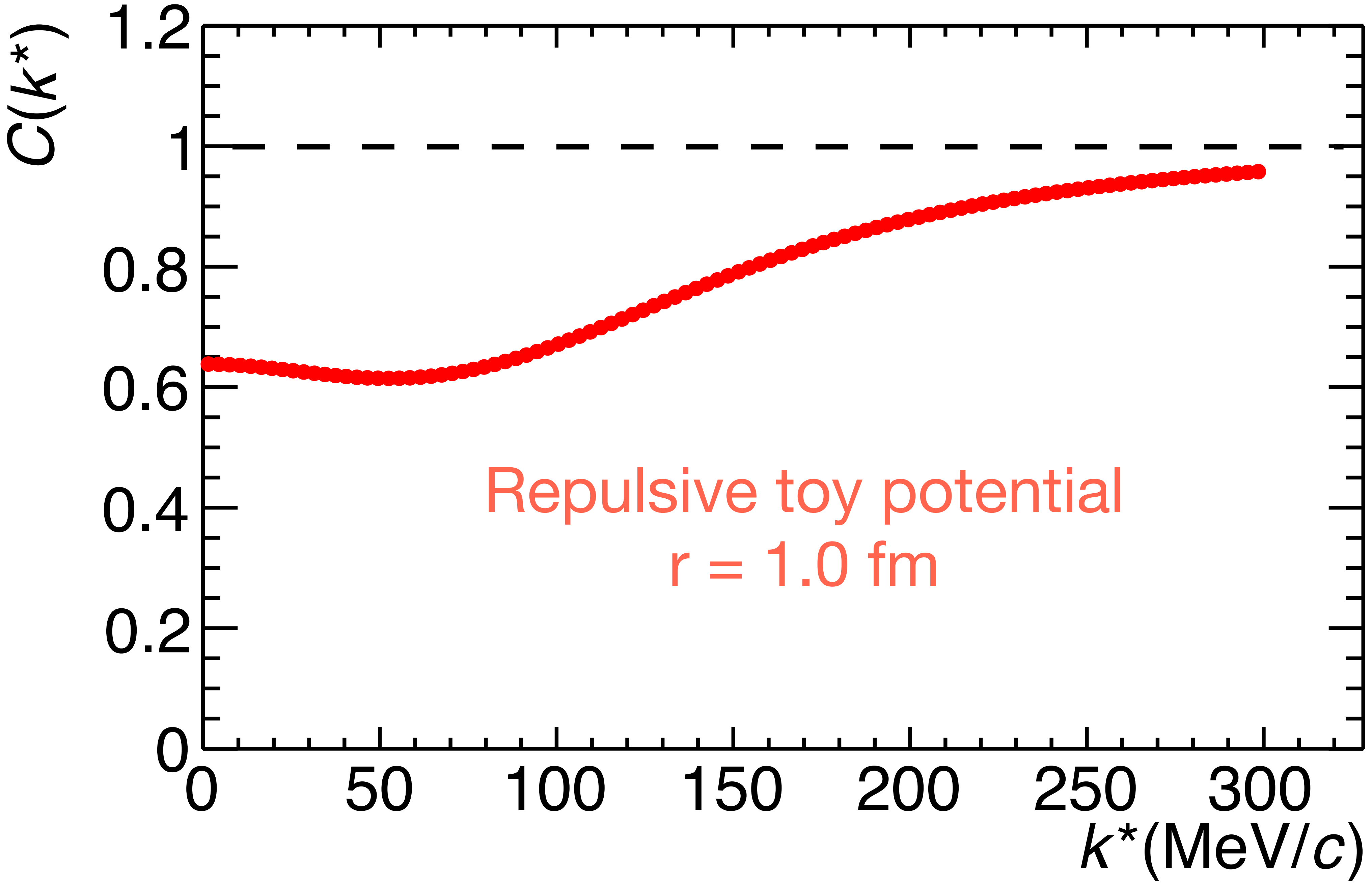
$$C(k^*) = \underbrace{\mathcal{N}\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}}_{\text{experimental definition}} = \underbrace{\int S(\vec{r}^*)\left|\psi(\vec{k}^*,\vec{r}^*)\right|^2 d^3\vec{r}^*}_{\text{theoretical definition}} \quad > 1$$

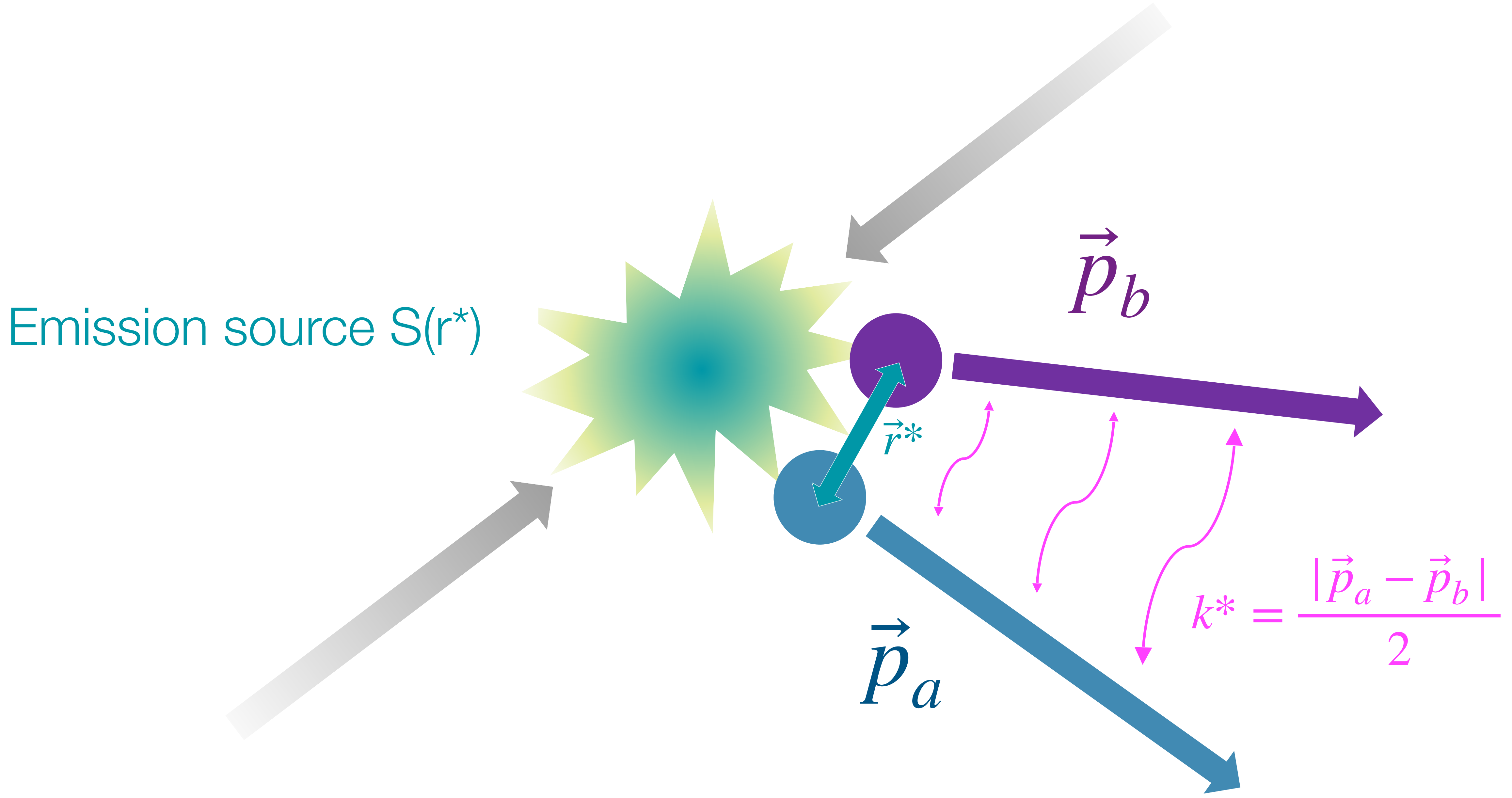
CATS Framework: D. Mihaylov et al., EPJ. C78 (2018) 394
S.E. Koonin PLB 70 43 (1977)

bhawani.singh@tum.de

Introduction: Femtoscopy

- Repulsive interaction brings correlation below 1





Koonin-Pratt Equation

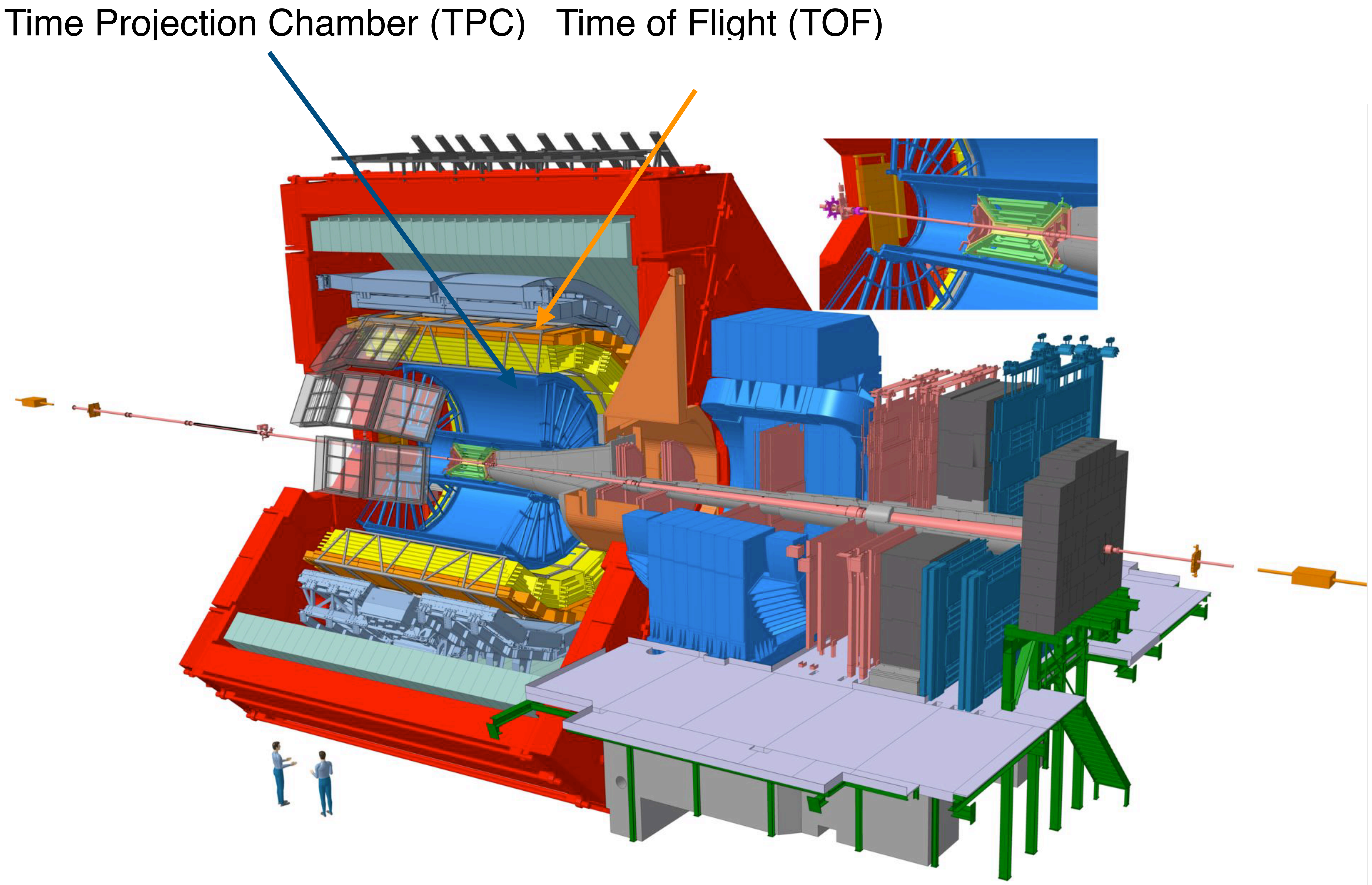
$$\underbrace{C(k^*) = \mathcal{N}\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}}_{\text{experimental definition}} = \underbrace{\int S(\vec{r}^*)\left|\psi(\vec{k}^*,\vec{r}^*)\right|^2 d^3\vec{r}^*}_{\text{theoretical definition}} < 1$$

CATS Framework: D. Mihaylov et al., EPJ. C78 (2018) 394
S.E. Koonin PLB 70 43 (1977)

bhawani.singh@tum.de

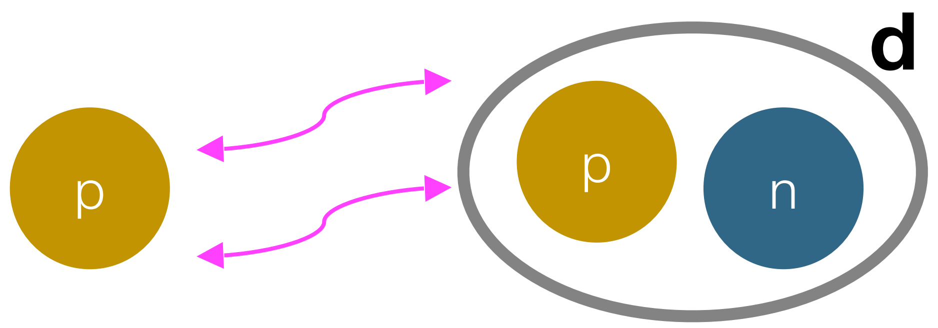
ALICE

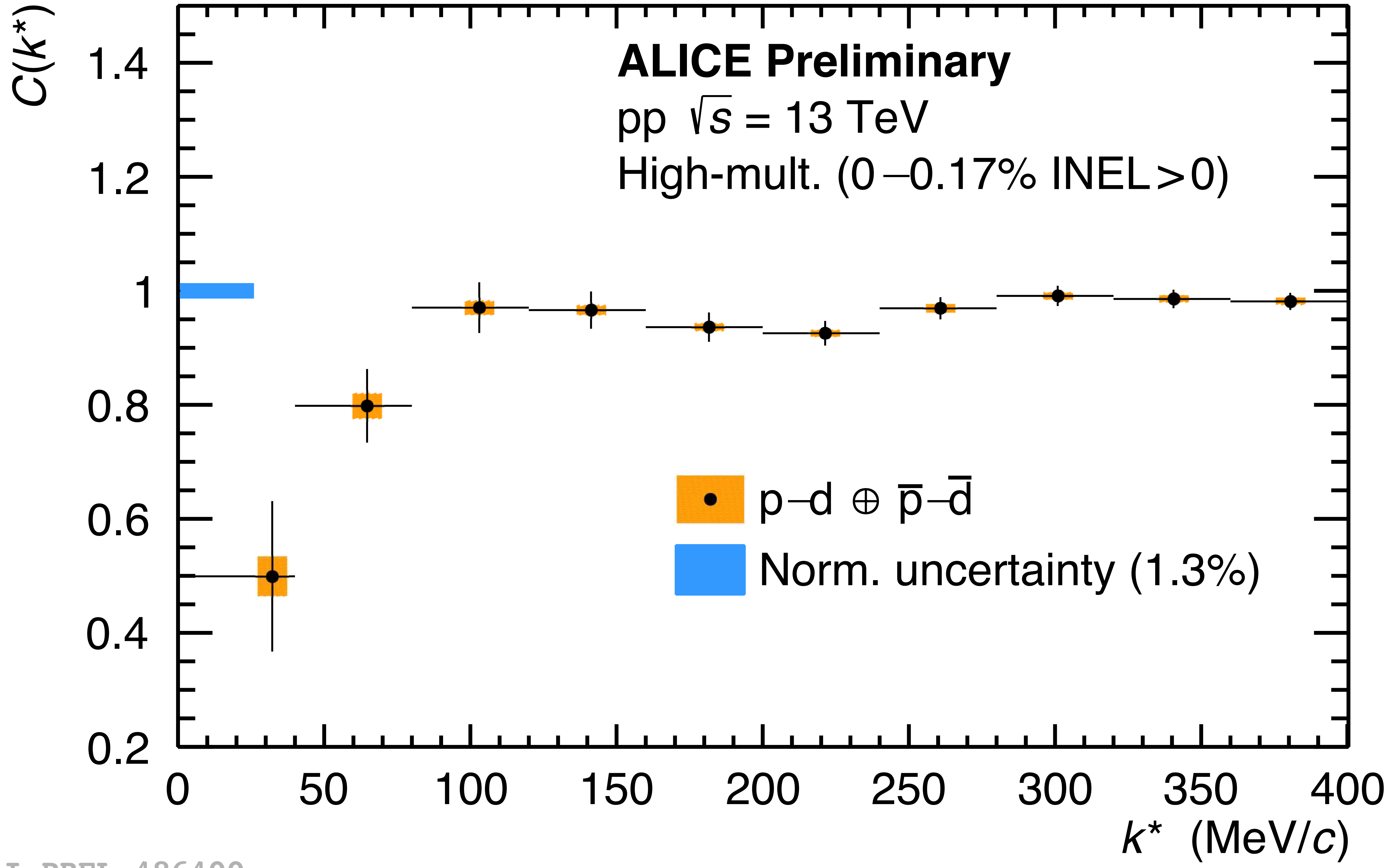
- General purpose heavy-ion experiment
  - Excellent particle identification (PID)
  - Most suited LHC experiment for studying femtoscopic correlations
- Run 2 pp high-multiplicity (HM 0-17% centrality) data
- Number of events: ~1 x10^9
- Particle selection with TPC +TOF
  - p(anti-p) : **98.30% (98.76%)**
  - d(anti-d) : ~**100%**



The first measurement p–d correlations from ALICE

- $\mathrm{p}-\mathrm{d}\oplus\bar{\mathrm{p}}-\bar{\mathrm{d}}$ correlation
  - Measured p-d correlation not flat, shows depletion at low *k**
  - Repulsive type of interaction
  - Accessing spin-isospin dependence of NNN
- Pairs below $k^{*} < 200$ MeV/*c*
  - $\mathrm{p}-\mathrm{d}$: 1747
  - $\bar{\mathrm{p}}-\bar{\mathrm{d}}$: 1250





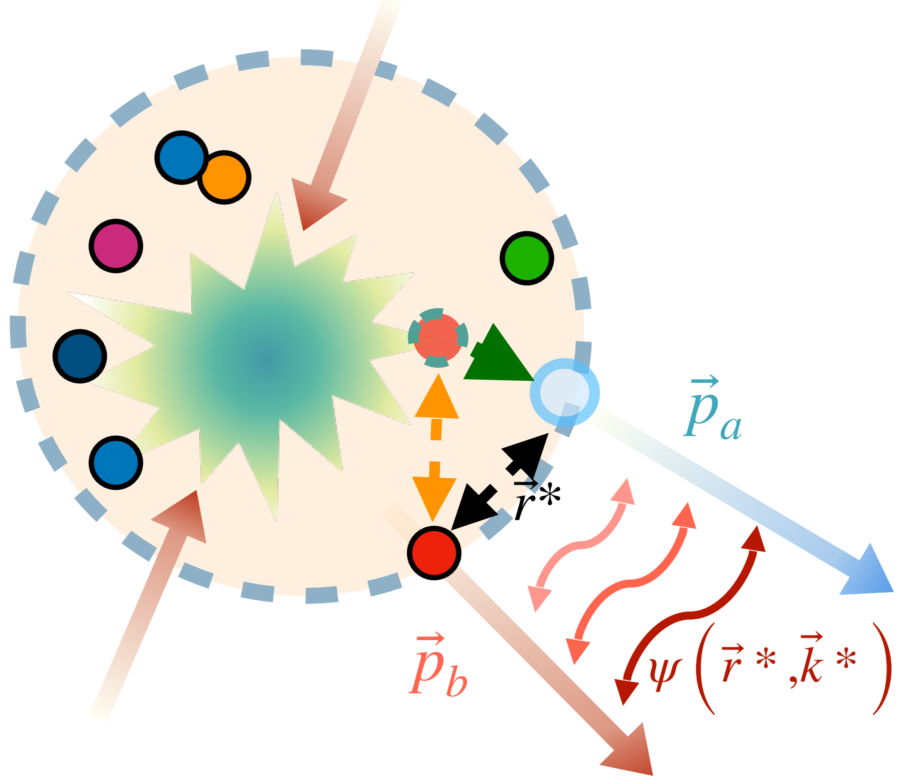
ALI-PREL-486400

bhawani.singh@tum.de

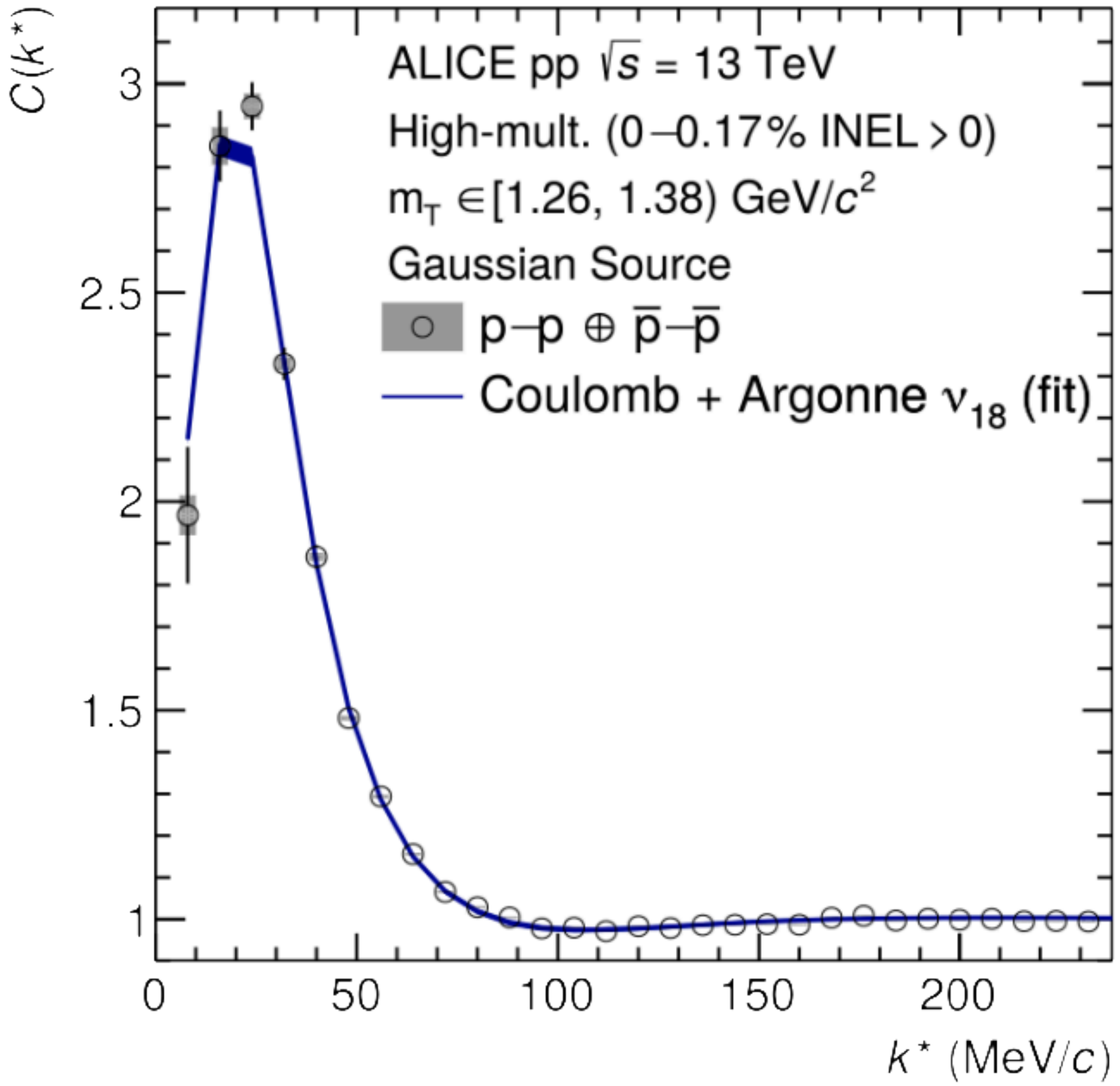
The femtoscopic source

- Short distances in pp collisions
- Particle emission from **Gaussian core** source
- Well constrained theoretical p-p correlation with **AV18 interaction** with Fermi-Dirac statistics, Coulomb and strong interaction
- Extract: the source size as fit parameter in transverse mass (m_T) ranges of pp pairs

Include short-lived strongly decaying resonances (cτ ≈ 1 fm) e.g. Δ-resonances in case of protons



PLB 811 (2020) 1358249



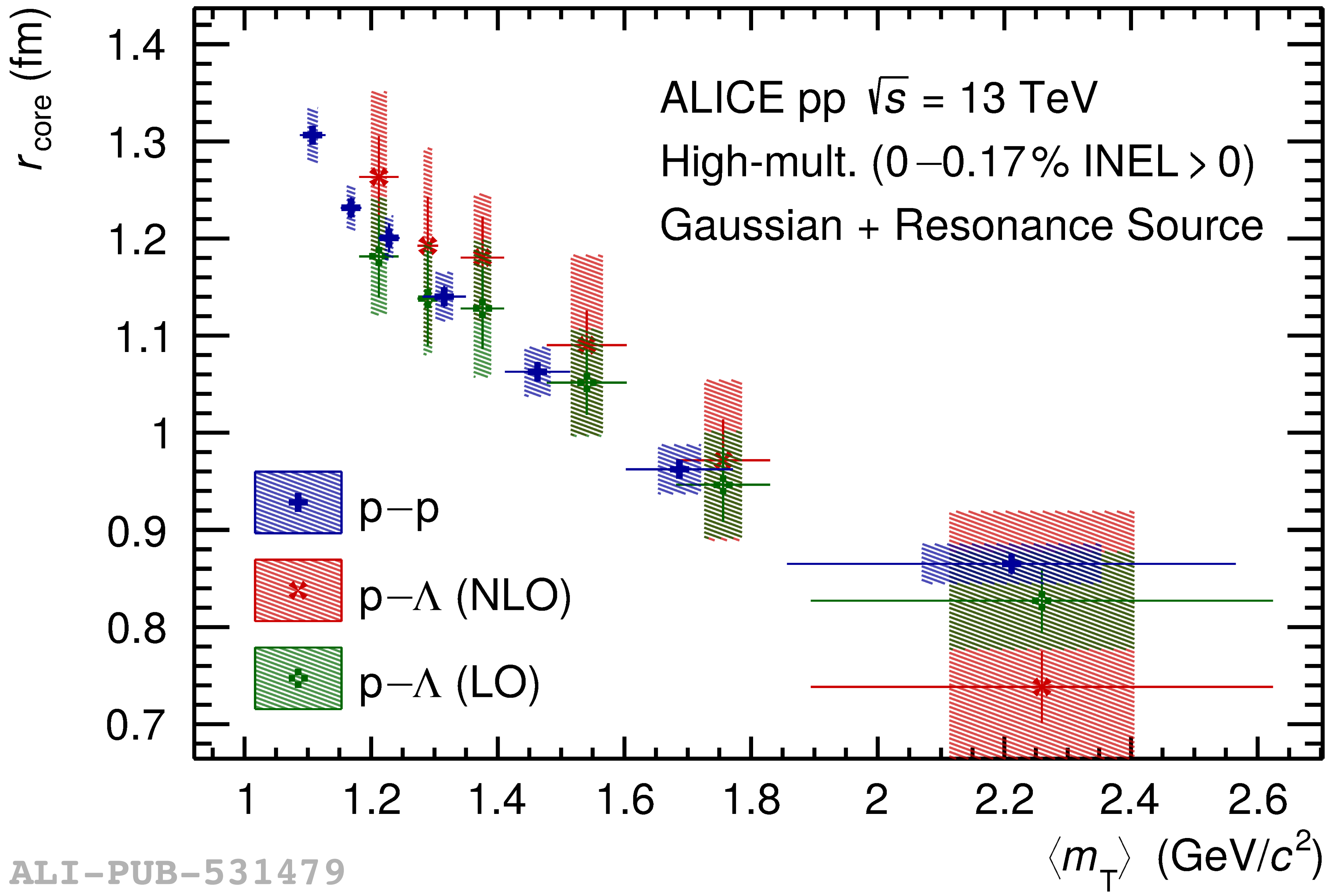
Koonin-Pratt Equation

$$C(k^*) = \int S(\vec{r}^*) \left|\psi(\vec{k}^*, \vec{r}^*)\right|^2 d^3\vec{r}^* = \mathcal{N} \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

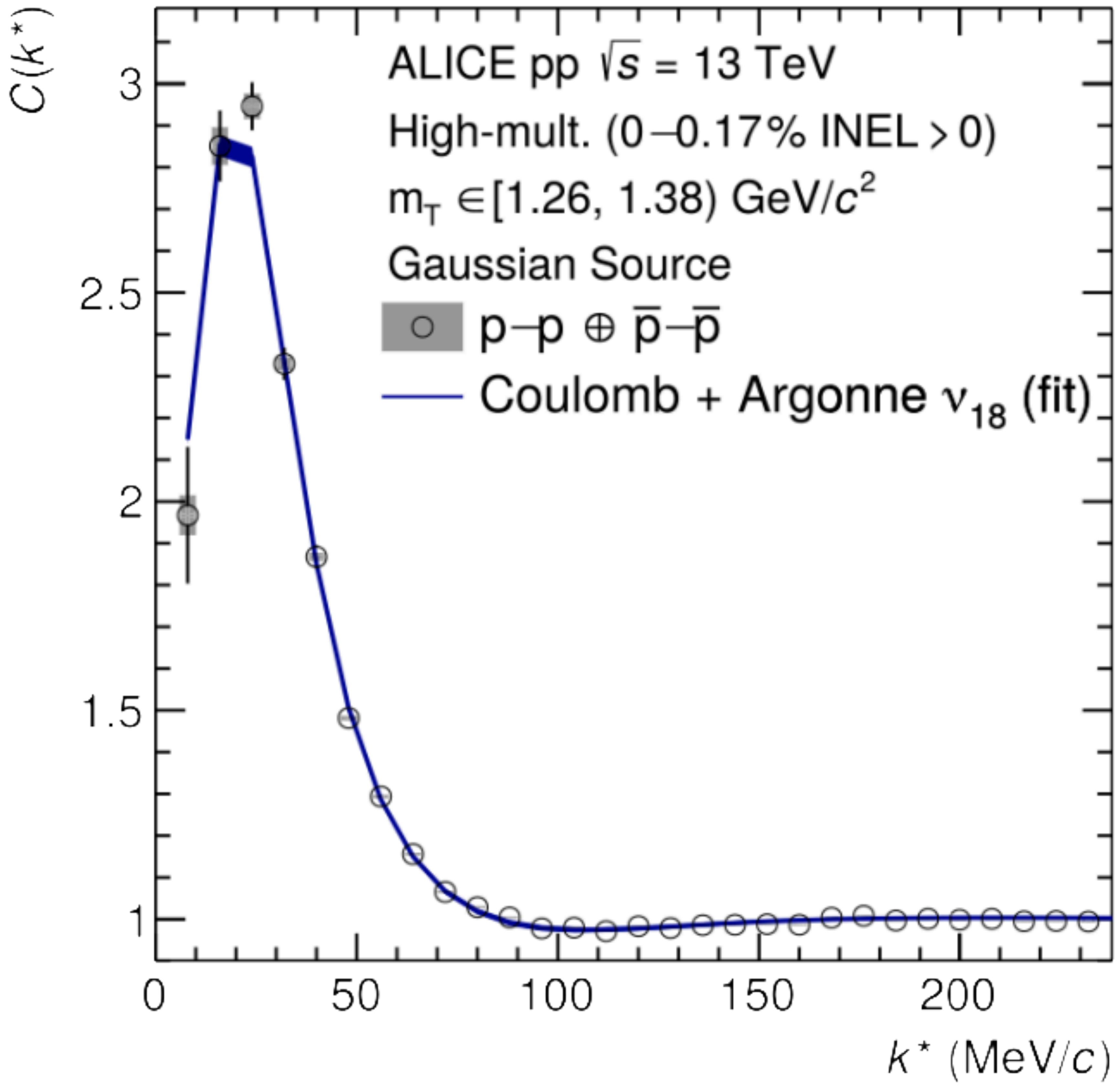
ALICE Coll. PLB 811 135849 (2020)

The femtoscopic source

- Short distances in pp collisions
- Particle emission from **Gaussian core** source
- Well constrained theoretical p-p correlation with **AV18 interaction** with Fermi-Dirac statistics, Coulomb and strong interaction
- Extract: the source size as fit parameter in transerves mass ranges of pp pairs



PLB 811 (2020) 1358249



Koonin-Pratt Equation

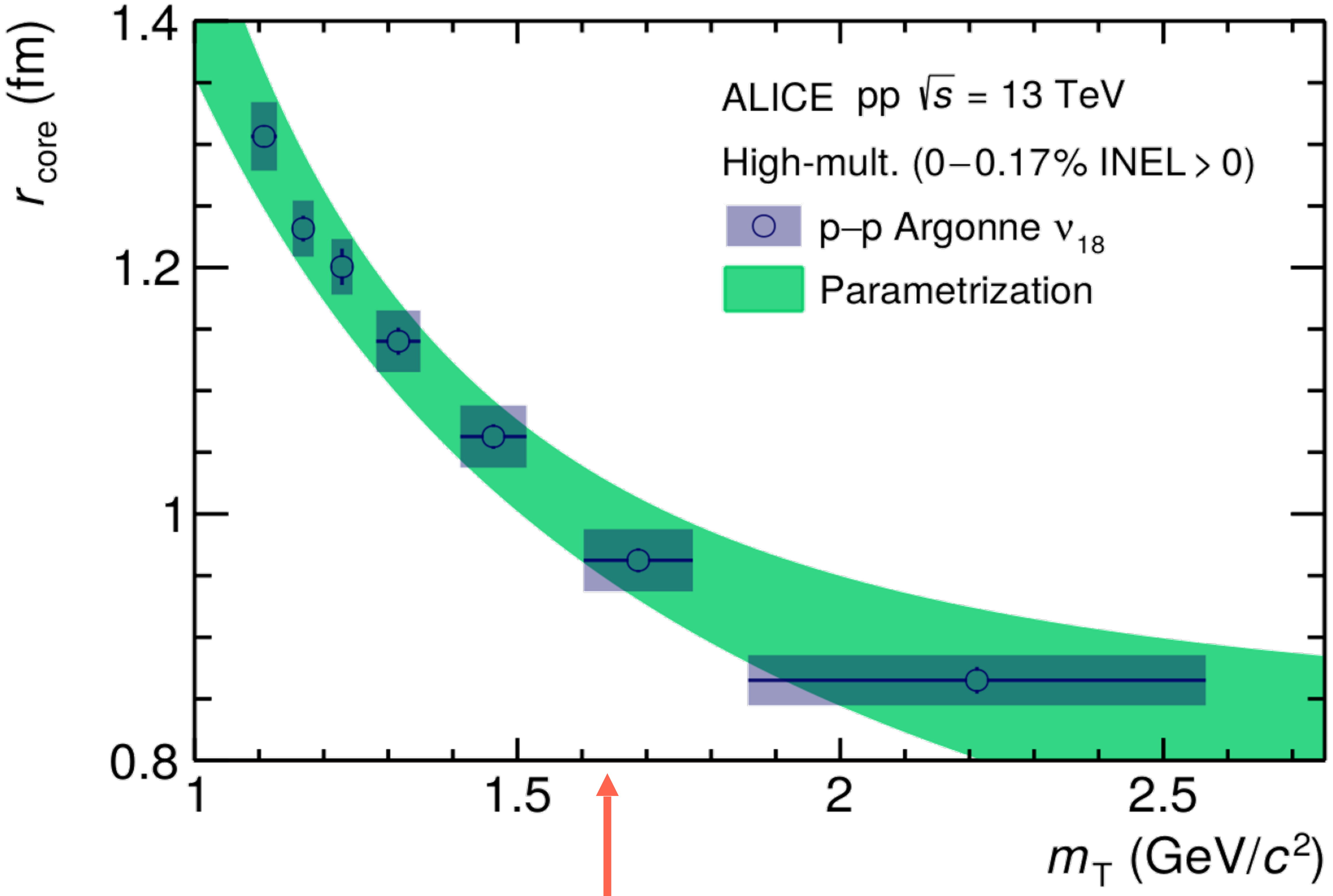
$$C(k^*) = \int S(\vec{r}^*) \left| \psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3\vec{r}^* = \mathcal{N} \frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}$$

ALICE Coll. PLB 811 135849 (2020)

The femtoscopic source

- p–d **Gaussian core** source(r_{core}): using the m_T -scalling

Source size	mean value:pd
r_{core}	0.99±0.05 fm

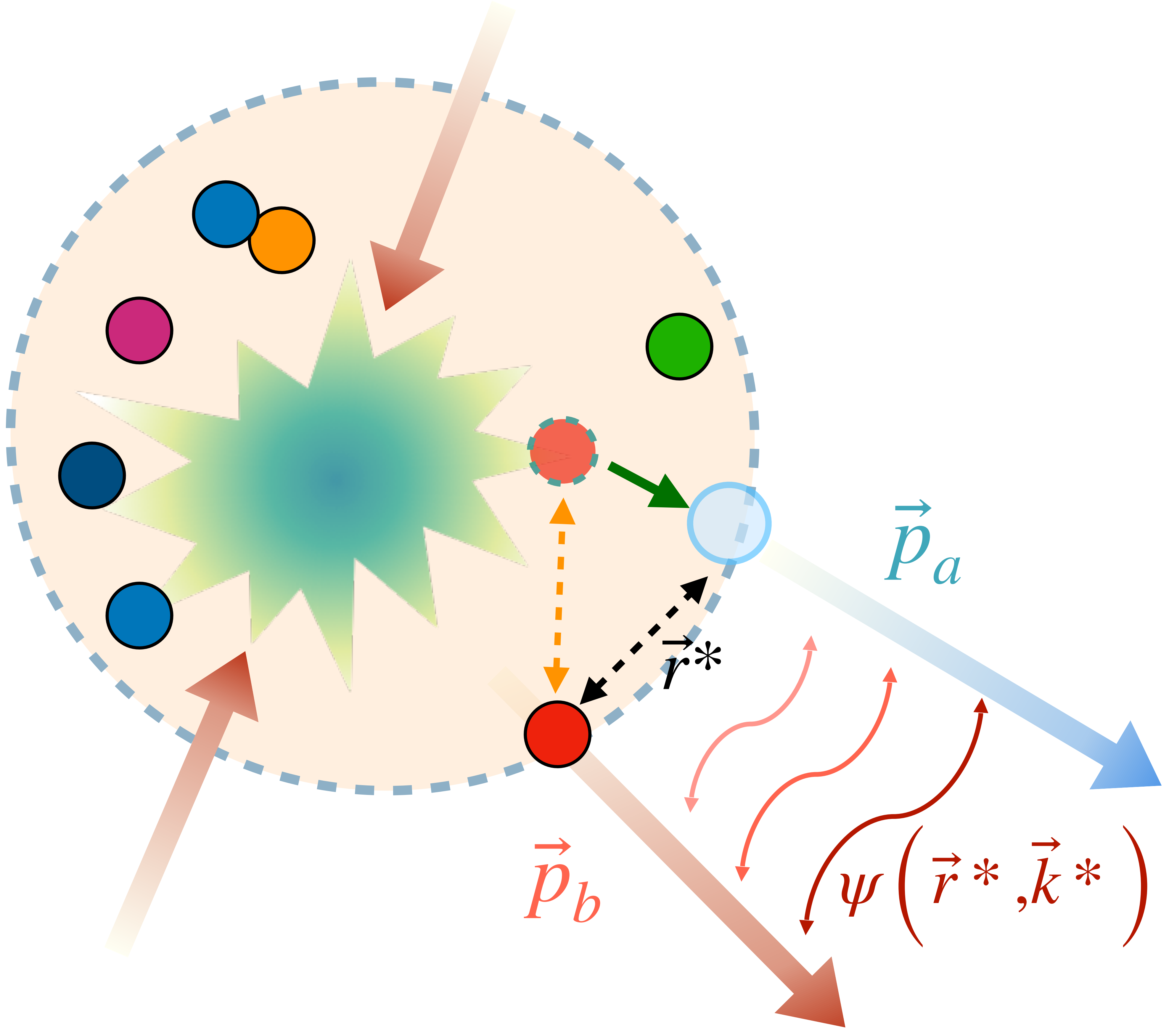


<$m_{T,\ p\text{-}d}$> = 1.64 GeV/c^2

ALICE Coll. PLB 811 135849 (2020)

The femtoscopic source

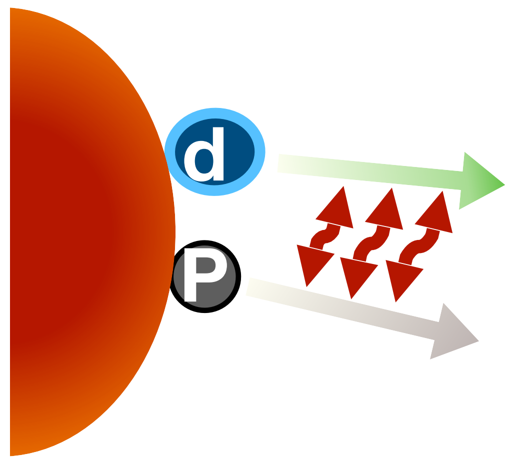
- p–d **Gaussian core** source(r_{core}): using the m_T -scalling
- The source radius is effectively increased by **short-lived strongly decaying resonances** ($c\tau \approx r_{core}$) e.g. Δ-resonances in case of protons



Source size	mean value:pd
r_{core}	0.99±0.05 fm
r_{eff}	1.08±0.06 fm

ALICE Coll. PLB 811 135849 (2020)

bhawani.singh@tum.de



Theoretical model comparison

Lednický model: pointlike deuterons

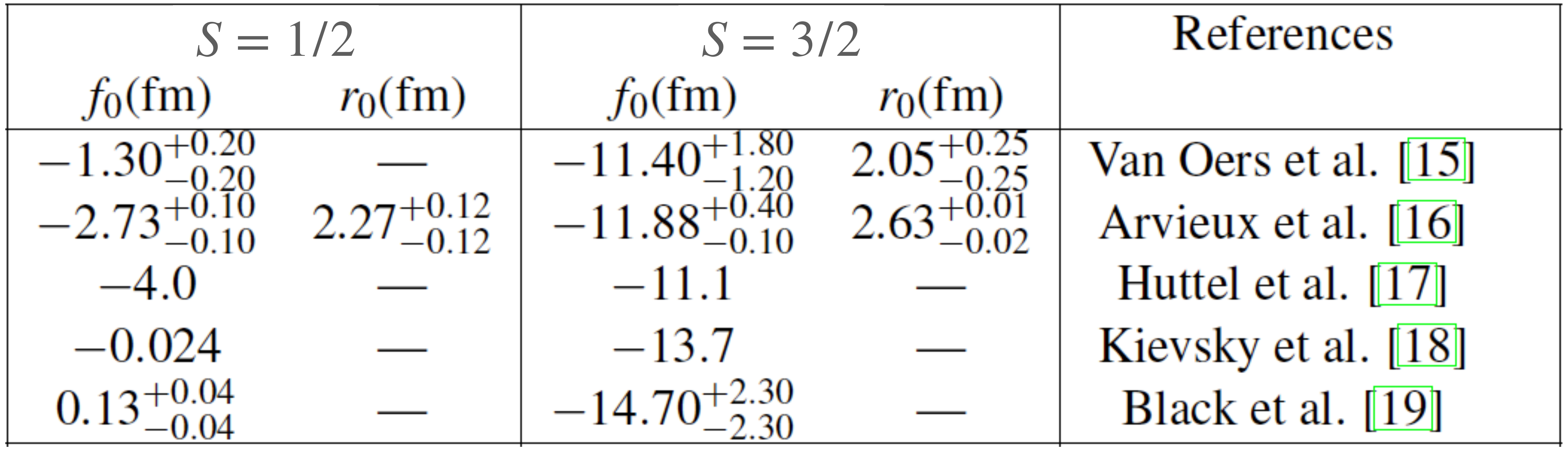
R. Lednický, Phys. Part. Nuclei 40, 307–352 (2009)

bhawani.singh@tum.de

Lednický model: pointlike deuterons

- **For distinguishable pointlike particles**
  - Starting from the scattering parameters ⇒ define the s-wave two-particle relative wave function
  - Considers Coulomb effects
  - Assumption: p and d are pointlike particles!

⇒ **p–d scattering parameters** from fits to p–d scattering data



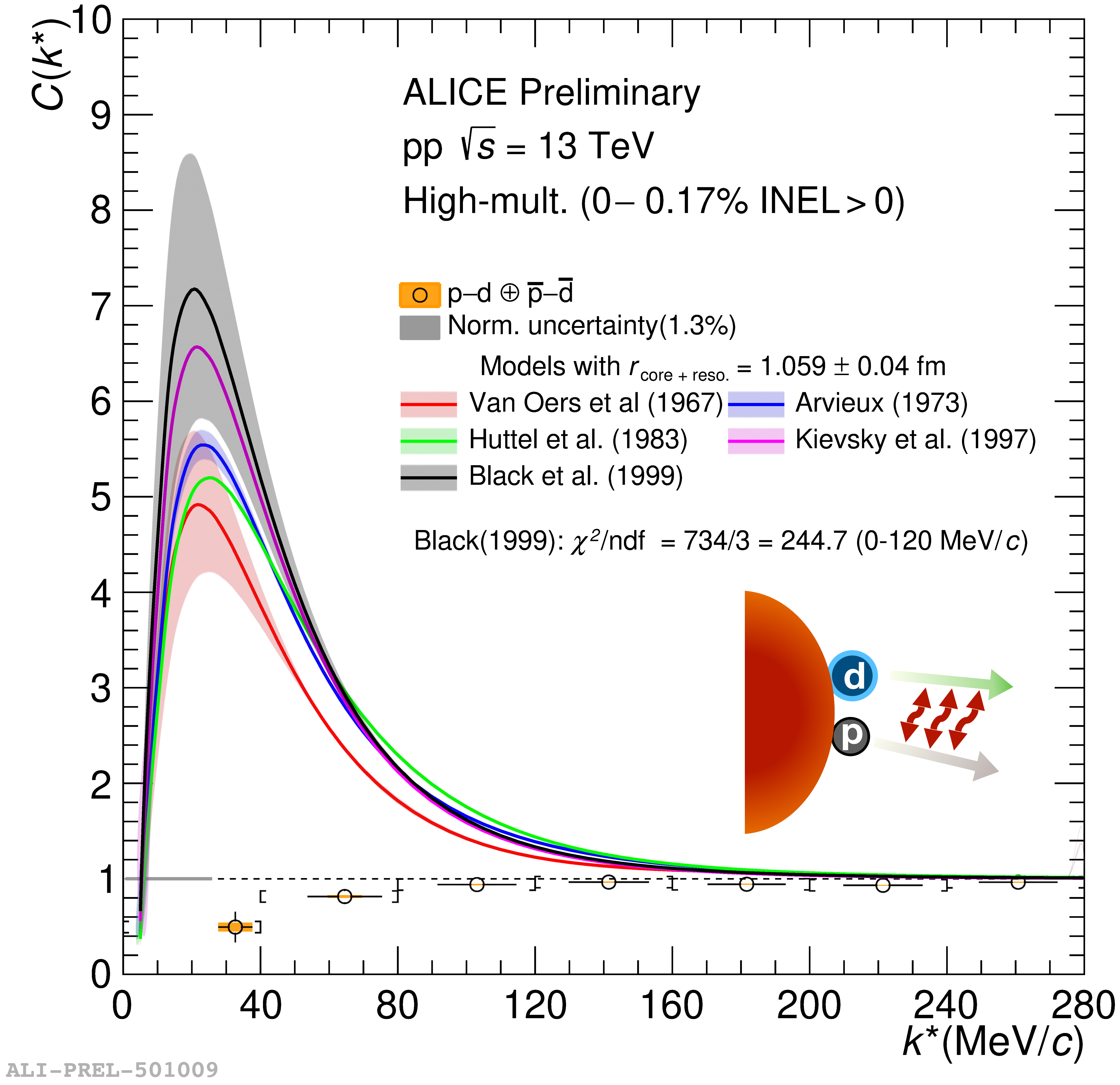
$S = 1/2$		$S = 3/2$		References
f_0(fm)	r_0(fm)	f_0(fm)	r_0(fm)	
$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.80}_{-1.20}$	$2.05^{+0.25}_{-0.25}$	Van Oers et al. [15]
$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.40}_{-0.10}$	$2.63^{+0.01}_{-0.02}$	Arvieux et al. [16]
-4.0	—	-11.1	—	Huttel et al. [17]
-0.024	—	-13.7	—	Kievsky et al. [18]
$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—	Black et al. [19]

Convention sign: In this presentation positive (negative) f_0 means attractive (repulsive) interaction

R. Lednicky, Phys. Part. Nuclei 40, 307–352 (2009)

bhawani.singh@tum.de

Lednický model vs ALICE data



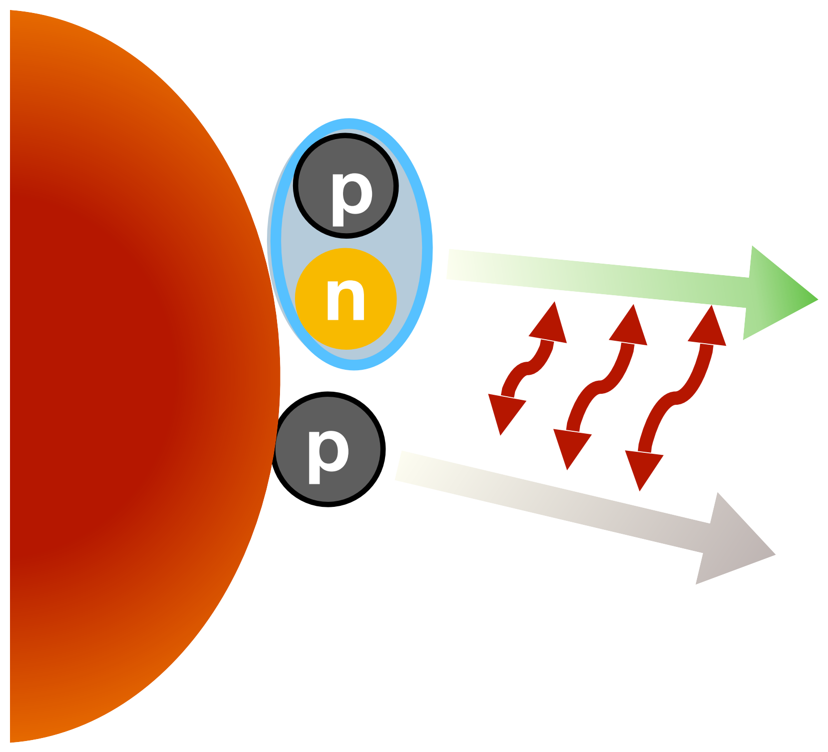
$$r_{\mathrm{eff}}^{\mathrm{pd}} = 1.08^{+0.06}_{-0.06}\ \mathrm{fm}$$

pd data not described

⇒ p–d can't be treated as effective two-body system

Considering protons and deuterons as distinguishable pointlike particles leads to huge discrepancy!

Van Oers,Brockmann et al. Nucl. Phys. A 561-583 (1967)
J.Arvieux et al. Nucl. Phys. A 221 253-268 (1973)
E.Huttel et al. Nucl. Phys. A 406 443-455 (1983)
A.Kievsky et al. PLB 406 292-296 (1997)
T.C.Black et al. PLB 471 103-107 (1999)



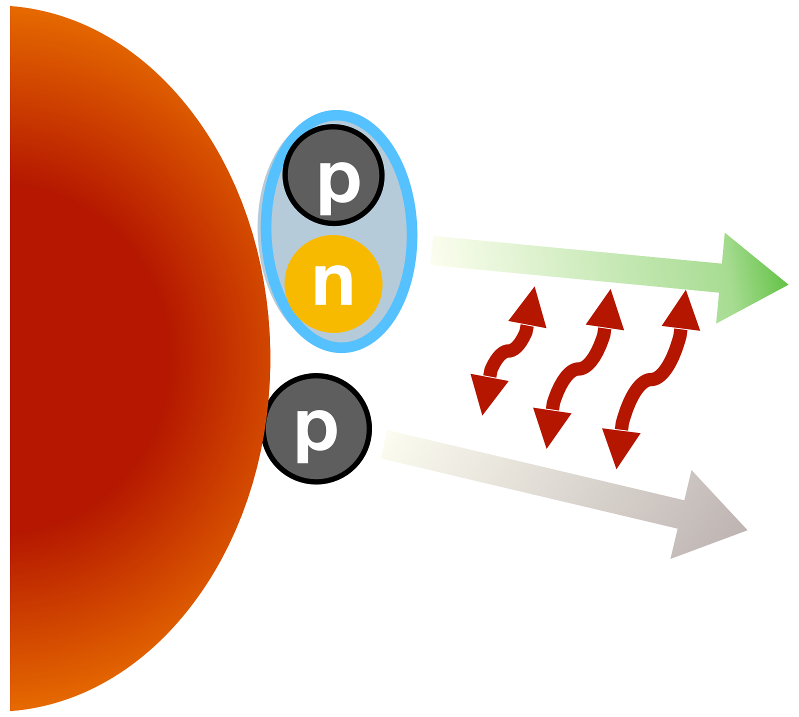
Pisa model: **p–d as three-body system**

Calculation done by PISA theory group: Michele Viviani, Alejandro Kievsky and Laura Marcucci

Kievsky et al, Phys. Rev. C 64 (2001) 024002
Kievsky et al, Phys. Rev. C 69 (2004) 014002
Deltuva et al, Phys. Rev. C71 (2005) 064003

p–d correlation with d as composite object

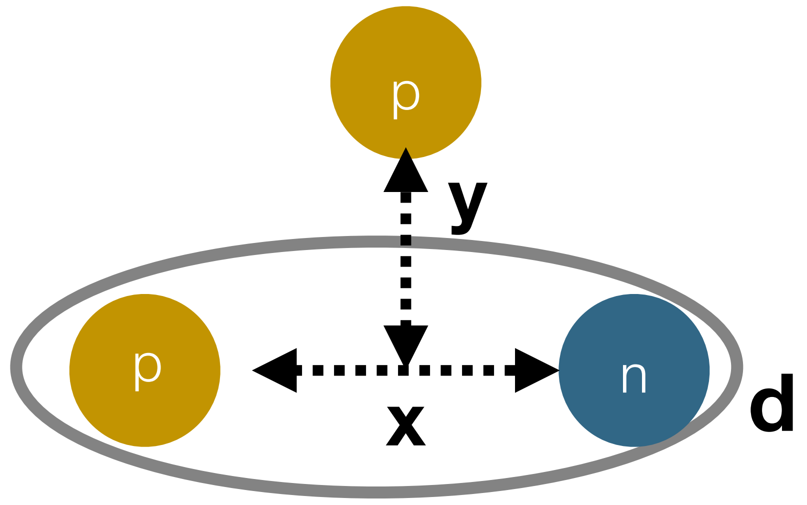
The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p–d state.

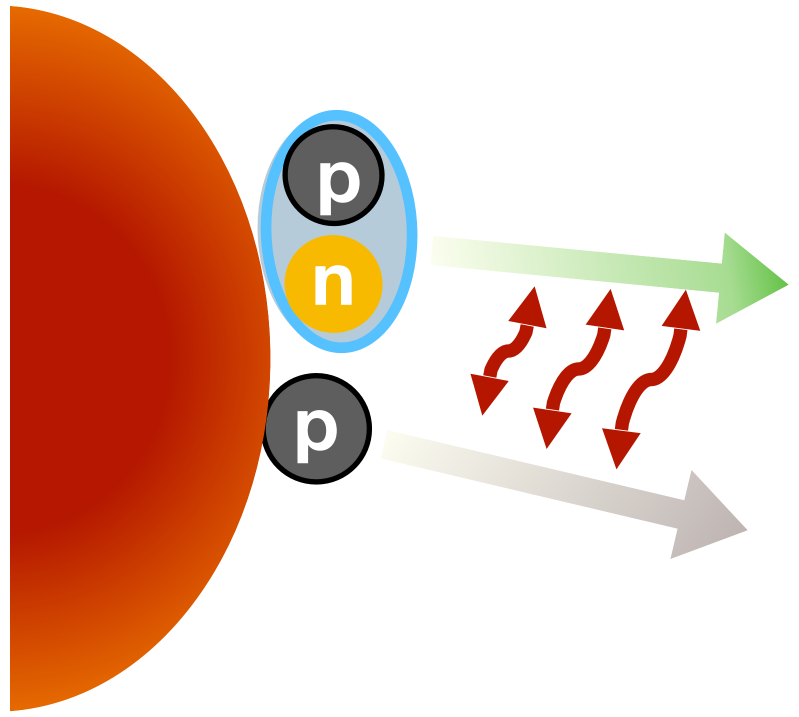


p–d correlation with d as composite object

The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p–d state.

- **Three–body wavefunction for p–d:** $\Psi_{m_2, m_1}(x, y)$ describing three-body dynamics, anchored to p–d scattering observables.
  - x = distance of p-n system within the deuteron
  - y = p-d distance
  - m_2 and m_1 deuteron and proton spin

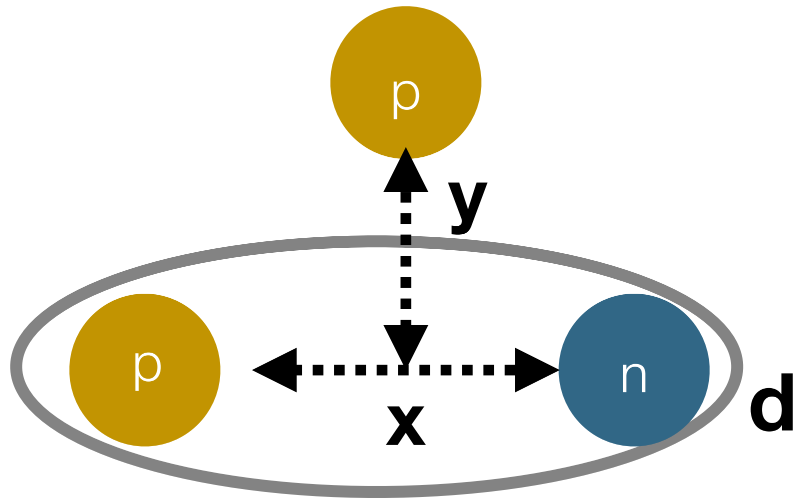




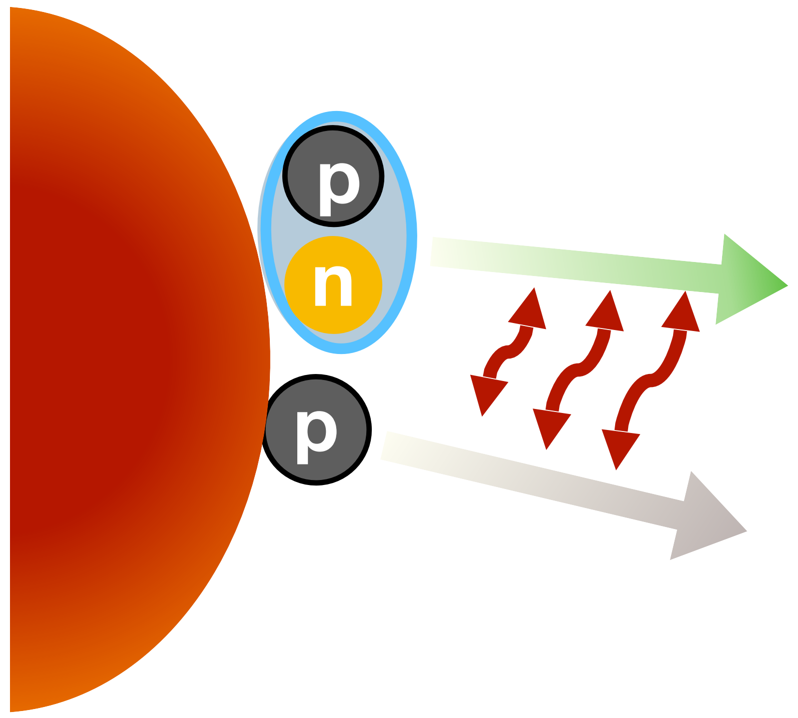
p–d correlation with d as composite object

The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p–d state.

- **Three–body wavefunction for p–d:** $\Psi_{m_2, m_1}(x, y)$ describing three-body dynamics, anchored to p–d scattering observables.
  - x = distance of p-n system within the deuteron
  - y = p-d distance
  - m_2 and m_1 deuteron and proton spin
- $\Psi_{m_2, m_1}(x, y)$ three-nucleon wave function asymptotically behaves as p–d state:



$$\Psi_{m_2,m_1}(\boldsymbol{x},\boldsymbol{y}) = \underbrace{\Psi^{(\text{free})}_{m_2,m_1}}_{\text{Asymptotic form}} + \sum_{LSJ}^{J\leq\bar{J}} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} \underbrace{(1 m_2 \frac{1}{2} m_1 | S J_z)(L0SJ_z|JJ_z)\widetilde{\Psi}_{LSJJ_z}}_{\text{Strong three-body interaction}} .$$



➡ $\tilde{\Psi}_{LSJJ_Z}$ describe the configurations where the three particles are close to each other

➡ $\Psi^{(\text{free})}_{m_1,m_2}$ an asymptotic form of p-d wave function

Kievsky et al, Phys. Rev. C 64 (2001) 024002
Kievsky et al, Phys. Rev. C 69 (2004) 014002
Deltuva et al, Phys. Rev. C71 (2005) 064003

The correlation function

- **Starting with the PPN state that goes into pd state**:
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

$$\boxed{A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int d^3r_1 d^3r_2 d^3r_3\, S_1(r_1)S_1(r_2)S_1(r_3)|\Psi_{m_2,m_1}|^2\,,}$$

Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

The correlation function

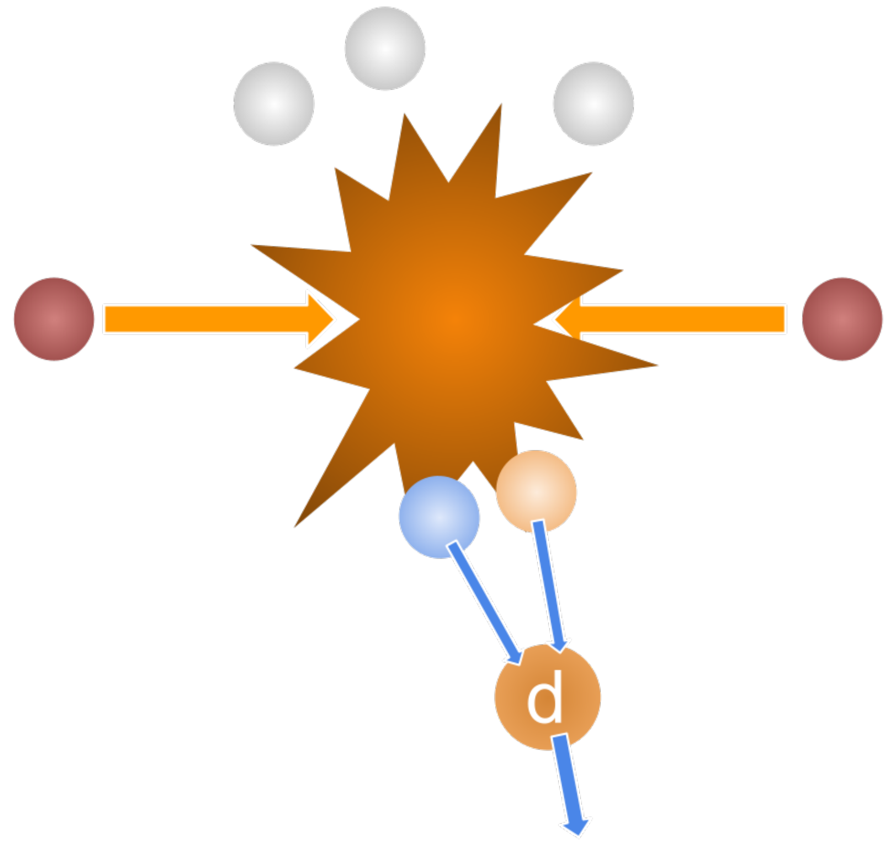
- **Starting with the PPN state that goes into pd state**:
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

$$\boxed{A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2, m_1} \int d^3r_1 d^3r_2 d^3r_3 \, S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2,m_1}|^2 ,}$$

  - Where A_d is the deuteron formation probability using deuteron wavefunction ϕ_{m_2}

$$\boxed{A_d = \frac{1}{3} \sum_{m_2} \int d^3r_1 d^3r_2 \, S_1(r_1) S_1(r_2) |\phi_{m_2}|^2 ,}$$



Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

bhawani.singh@tum.de

The correlation function

- **Starting with the PPN state that goes into pd state**:
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

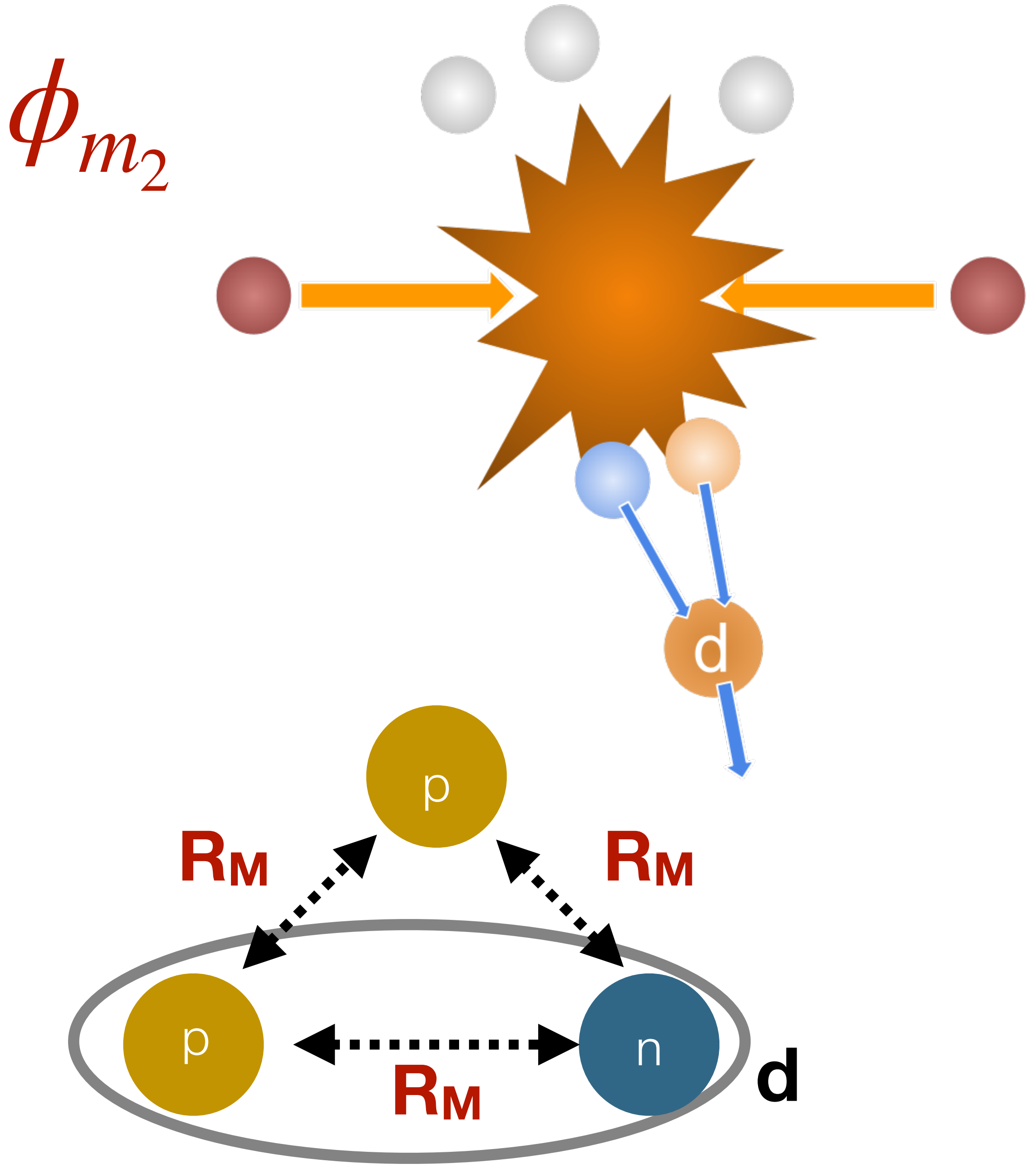
$$A_d C_{pd}(k) \;=\; \frac{1}{6}\sum_{m_2,m_1}\int d^3r_1 d^3r_2 d^3r_3\, S_1(r_1)S_1(r_2)S_1(r_3)|\Psi_{m_2,m_1}|^2\,,$$

  - Where A_d is the deuteron formation probability using deuteron wavefunction ϕ_{m_2}

$$A_d \;=\; \frac{1}{3}\sum_{m_2}\int d^3r_1 d^3r_2\, S_1(r_1)S_1(r_2)|\phi_{m_2}|^2\,,$$

  - Final definition of the correlation with p–p source size R_M :

$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int \rho^5 d\rho d\Omega\, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3}|\Psi_{m_2,m_1}|^2\,.$$

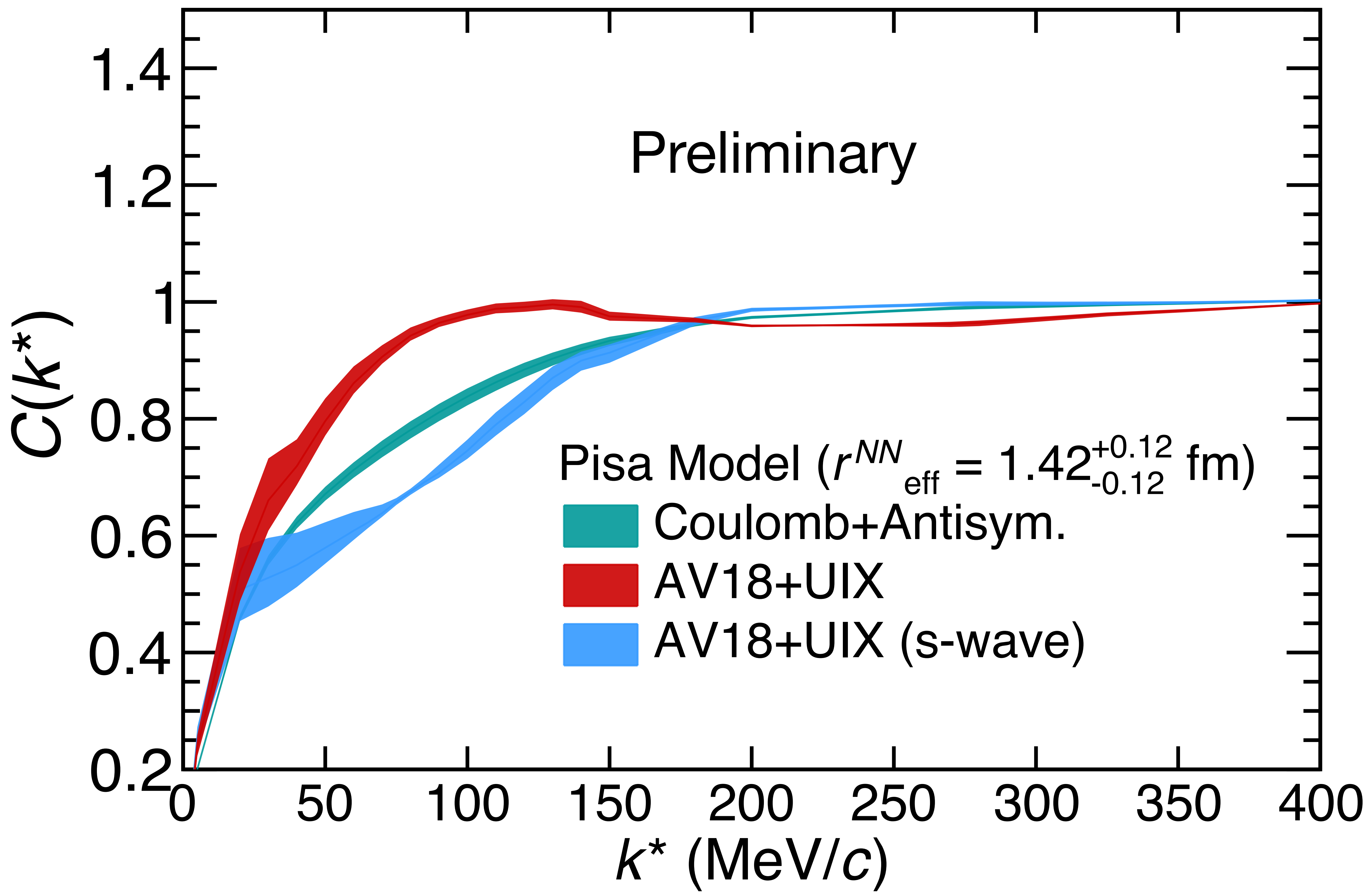


Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

bhawani.singh@tum.de

PISA model: theoretical p-d correlation

Model including NN and NNN interactions in s+d-wave



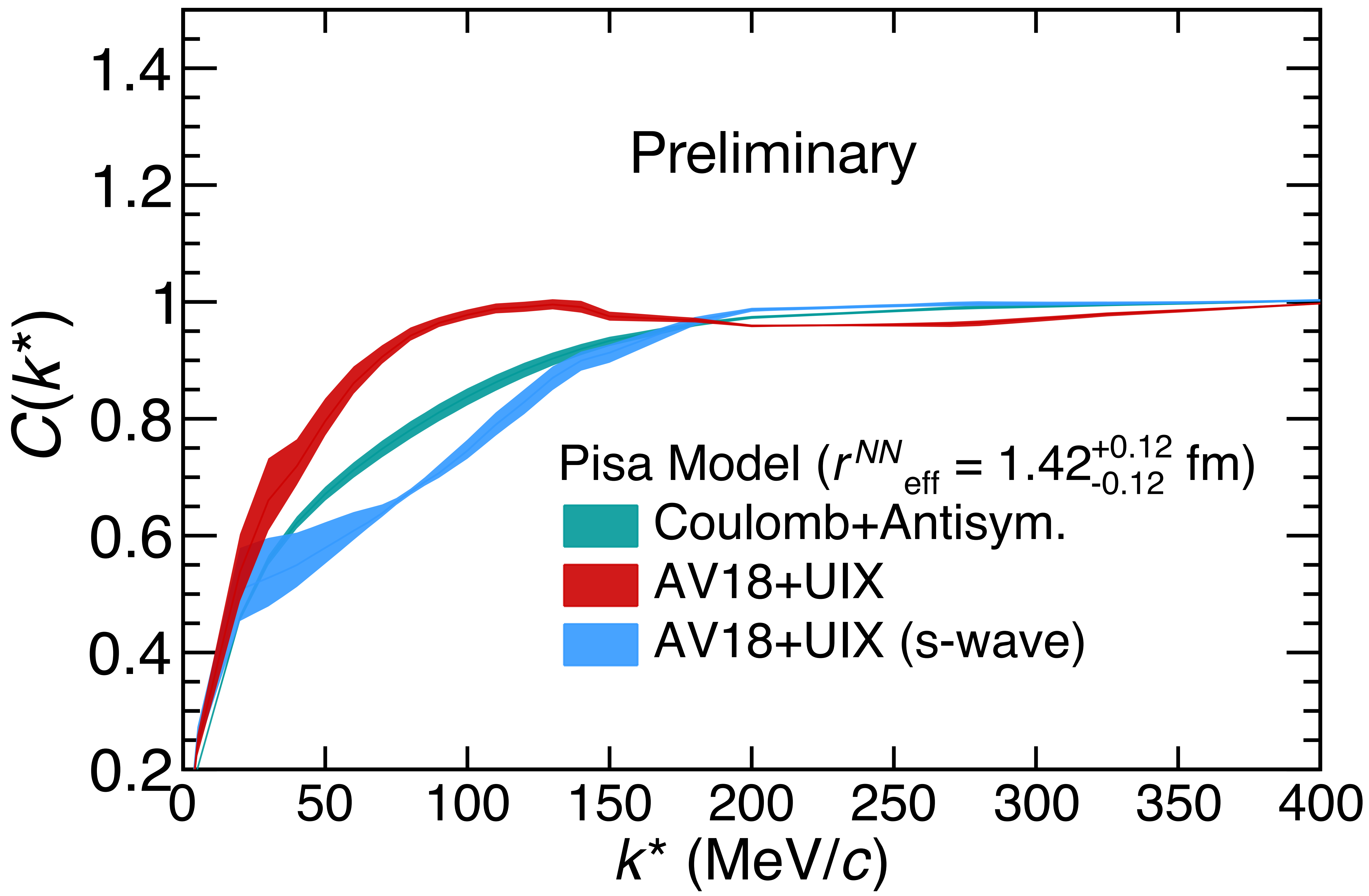
Calculation by Michele Viviani

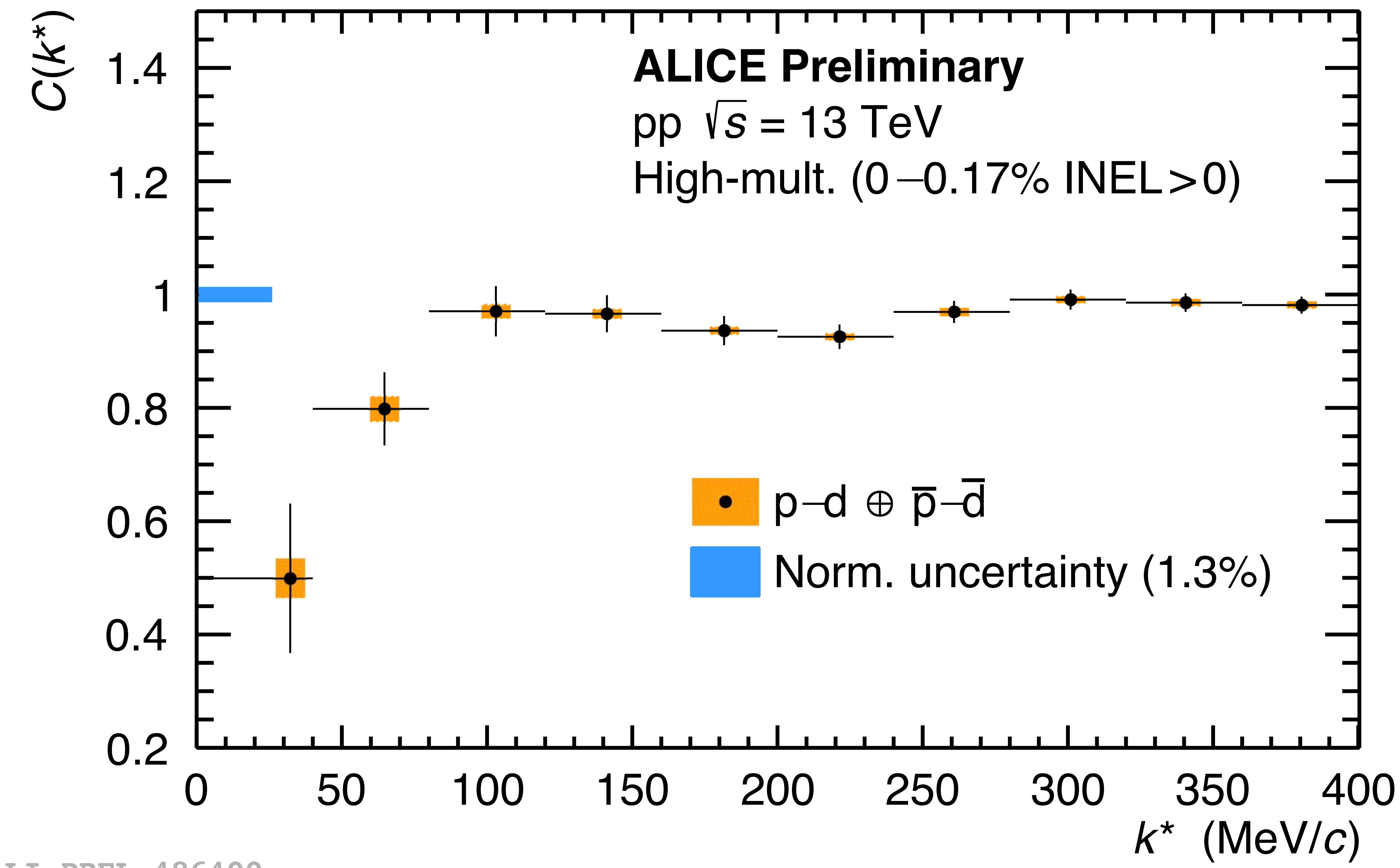
- **Coulomb interaction not enough**
- **Coulomb +s+p+d wave:**

  Source size = 1.08±0.06 fm ⇒ **1.42±0.12 fm** of pp

  source size

PISA model: theoretical p-d correlation





ALI-PREL-486400

- ➡ **Model calculation qualitatively reproduces the data**
- ➡ **The p-d correlation is affected by two + three-body p-p-n interactions!**

Conclusions

Summary:

- Assumption of pointlike and distinguishable particles does not work!
- Model considering p(pn) three-body dynamics reproduces the data for source size of pp/pn extracted from the 'universal' m_T scaling
- Nucleons composing the deuteron are present at the same time as all other unbound nucleons

Outlook:

- Perform a fit using the calculation based on two+three-body interaction
- Deuterons can be combined to other hadrons to study many-body interaction
- Future **m_T differential analysis** would allow to reach lower values: possibility of preparing a three body system with controlled density distributions to investigate the NN and NNN forces.

Conclusions

Summary:

- Assumption of pointlike and distinguishable particles does not work!
- Model considering p(pn) three-body dynamics reproduces the data for source size of pp/pn extracted from the ‘universal’ m_T scaling
- Nucleons composing the deuteron are present at the same time as all other unbound nucleons

Outlook:

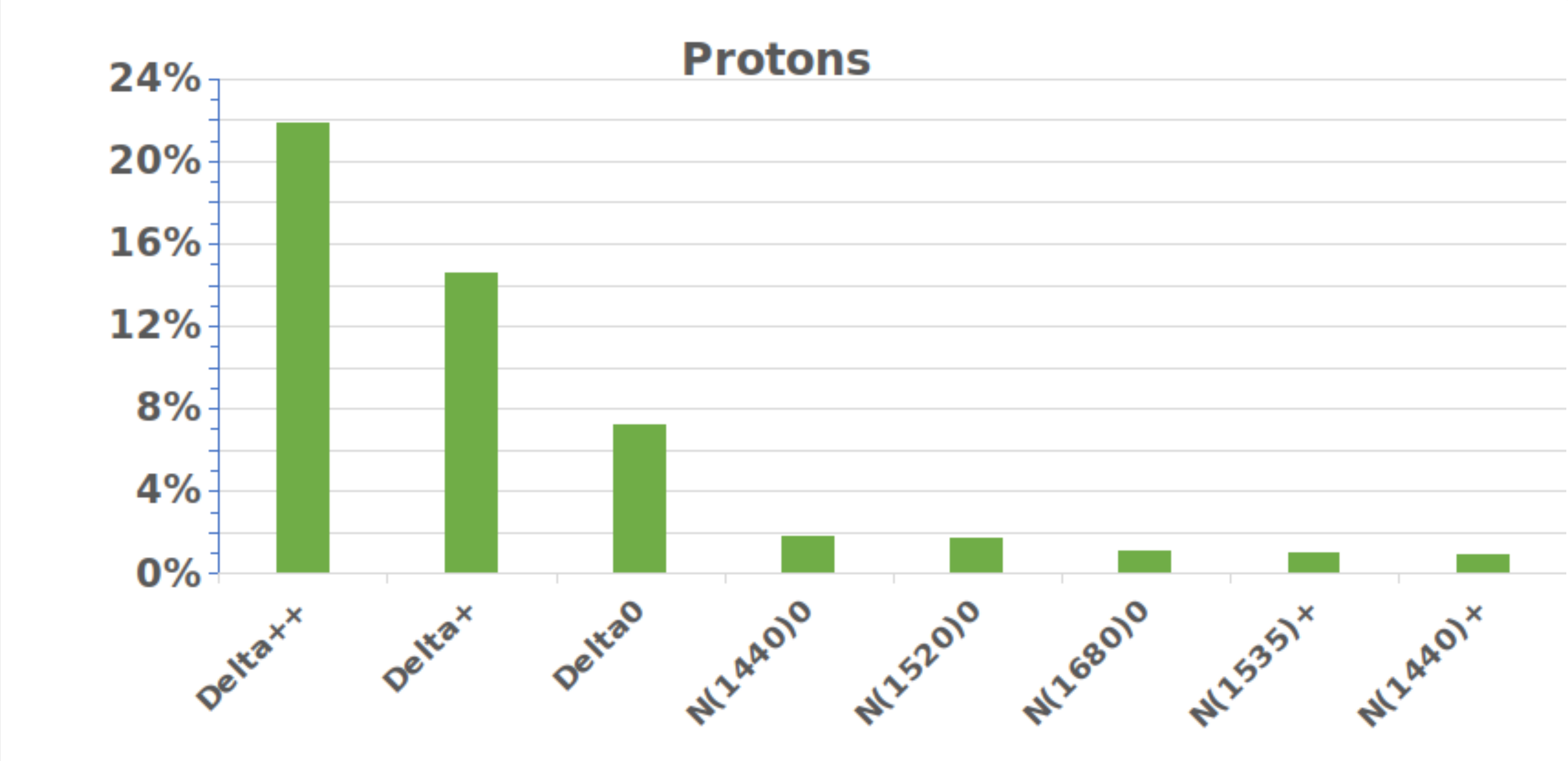
- Perform a fit using the calculation based on two+three-body interaction
- Deuterons can be combined to other hadrons to study many-body interaction
- Future **m_T differential analysis** would allow to reach lower values: possibility of preparing a three body system with controlled density distributions to investigate the NN and NNN forces.

Thanks for your attention!

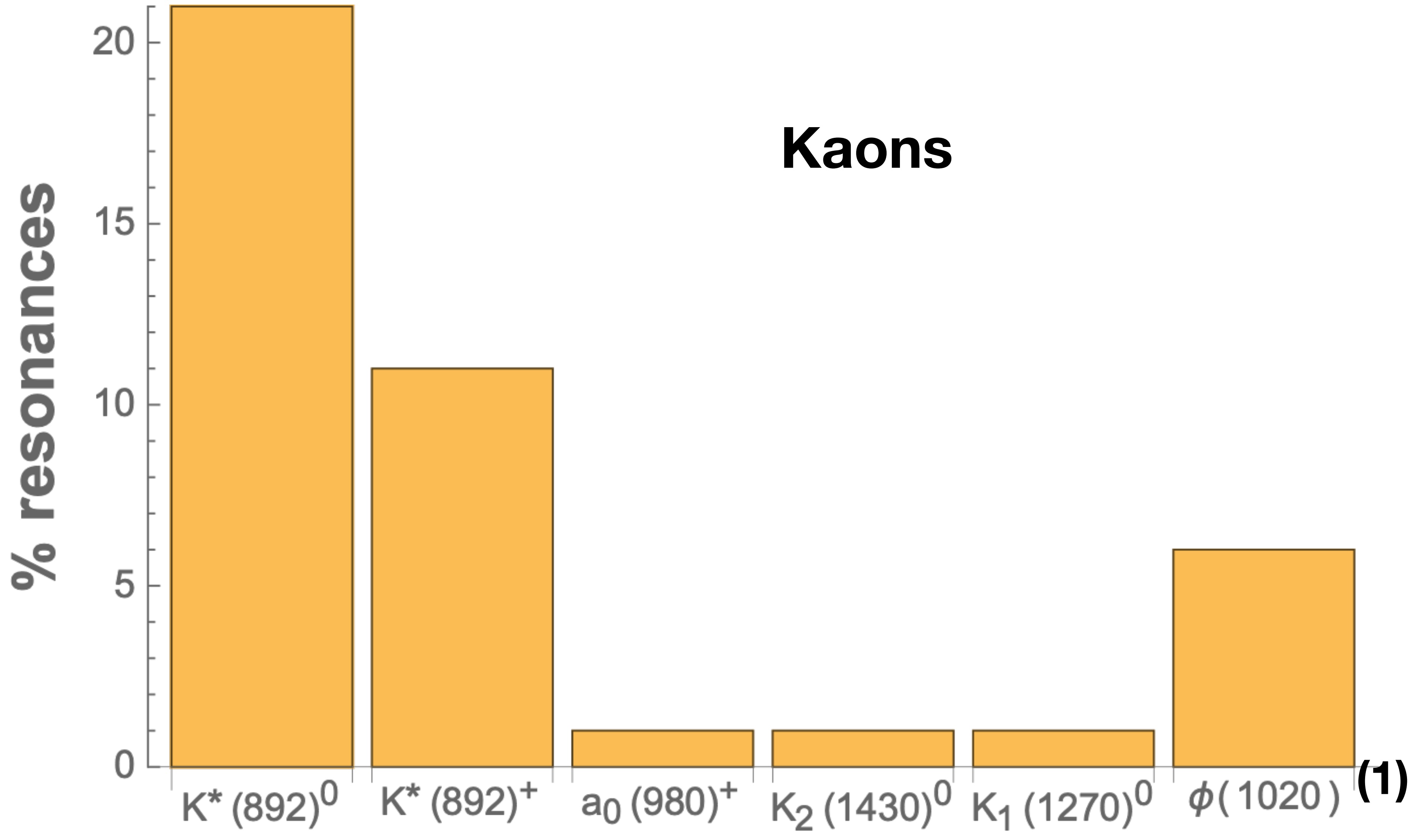
additional slides

The source for pd and K+d

- Short distances in pp and p—Pb collisions
- Particle emission from **Gaussian core** source
- The source radius is effectively increased by **short-lived strongly decaying resonances** ($c\tau \approx r_{core}$) e.g. Δ-resonances in case of protons



Source size	mean value:pd	mean value:k+d
r_{core}	0.99±0.05 fm	1.04±0.04 fm
r_{eff}	1.08±0.06 fm	$1.41^{+0.03}_{-0.09}$ fm



(1) ϕ(1020) corrected as feed-down

Lednicky Model

- For distinguishable particles
  - starting from the scattering parameters ⇒ define the s-wave two-particle relative wave function
  - considers Coulomb effects

- Coulomb-corrected wave function for final-state interactions (Lednicky): arxiv.org/abs/nucl-th/0501065

$$\psi_{-k^*}(r^*) = e^{i\delta_c}\sqrt{A_c(\eta)}\left[e^{-ik^*r^*}F\left(-i\eta, 1, i\zeta\right) + f_c(k^*)\frac{\tilde{G}(\rho,\eta)}{r^*}\right]$$

- f_c : Coulomb normalised scattering amplitude for strong interaction
- $F(-i\eta, 1, i\zeta)$: confluent hypergeometric function
- $\tilde{G}(\rho,\eta)$: combination of singular and regular Coulomb function, describes asymptotic behaviour of wavefunction

⇒ to obtain two-particle correlation we can use Koonin-Pratt formula

How accurate is the theory ?

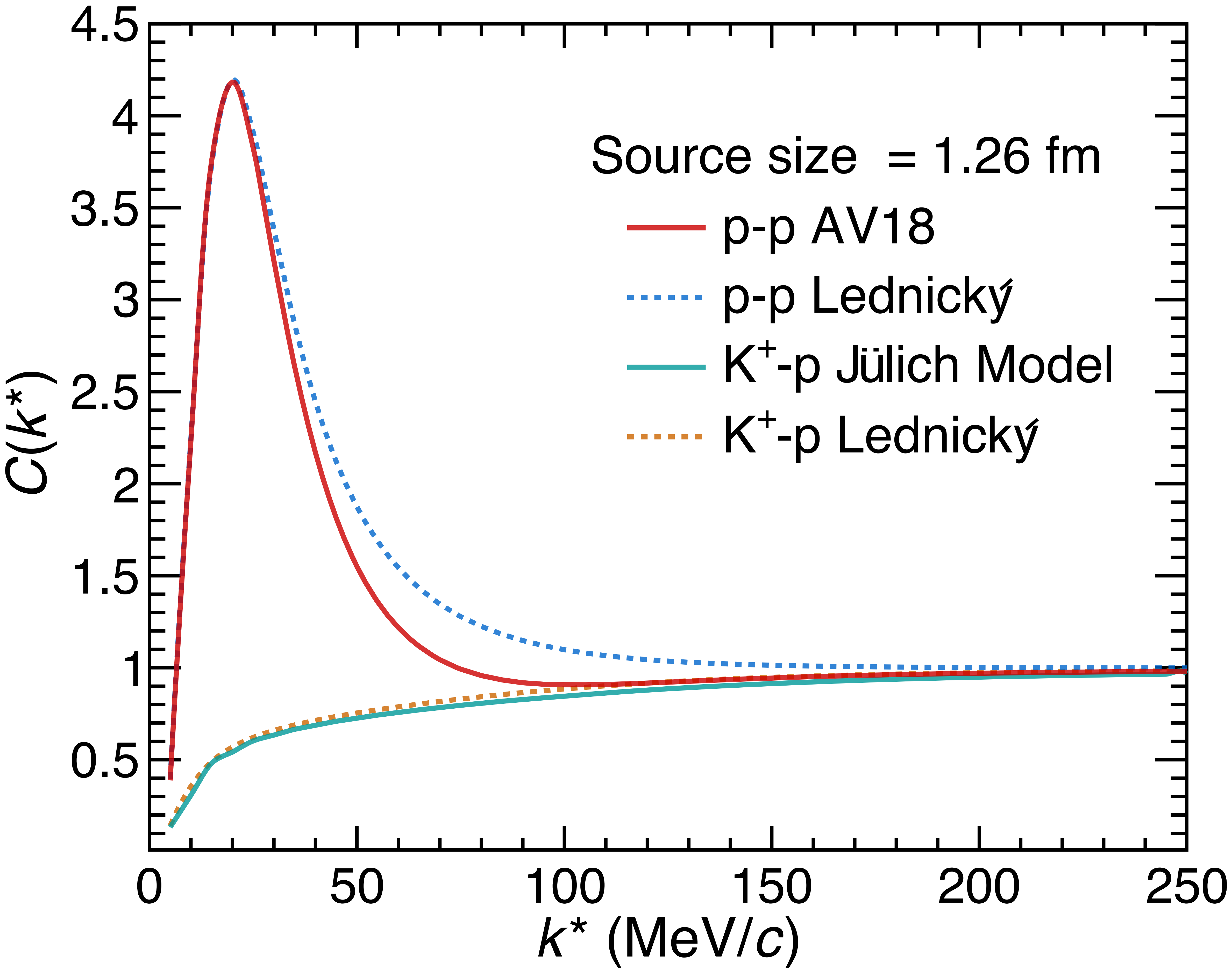
- **Benchmark:** compare correlations with Lednicky model with calculations using

  - pp from AV18 potential

  - K+p from Jülich model

System	f_0(fm)	r_0(fm)	References
p–p (S=0)	7.806	2.788	R. Wiringa et al. [6]
K^+–p (S=1/2)	–0.316	0.373	M. Hoffmann et al. [7]

- **Correlations are well reproduced by Lednicky approach**

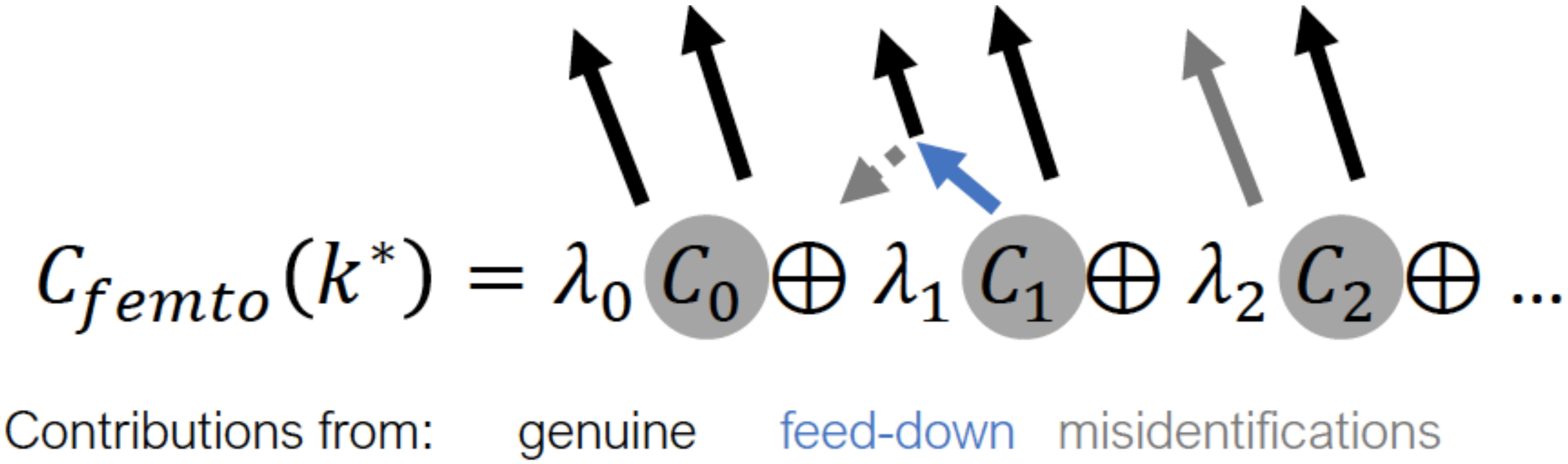


Convention sign: In this presentation positive (negative) f_0 means attractive (repulsive) interaction

bhawani.singh@tum.de

Femtoscopic correlation

- The femtoscopic correlation may have background/contributions from
  - Particles from weak decays
  - Particles from material knock-outs
  - Misidentifications



$$C_{femto}(k^*) = \lambda_0 C_0 \oplus \lambda_1 C_1 \oplus \lambda_2 C_2 \oplus \ldots$$

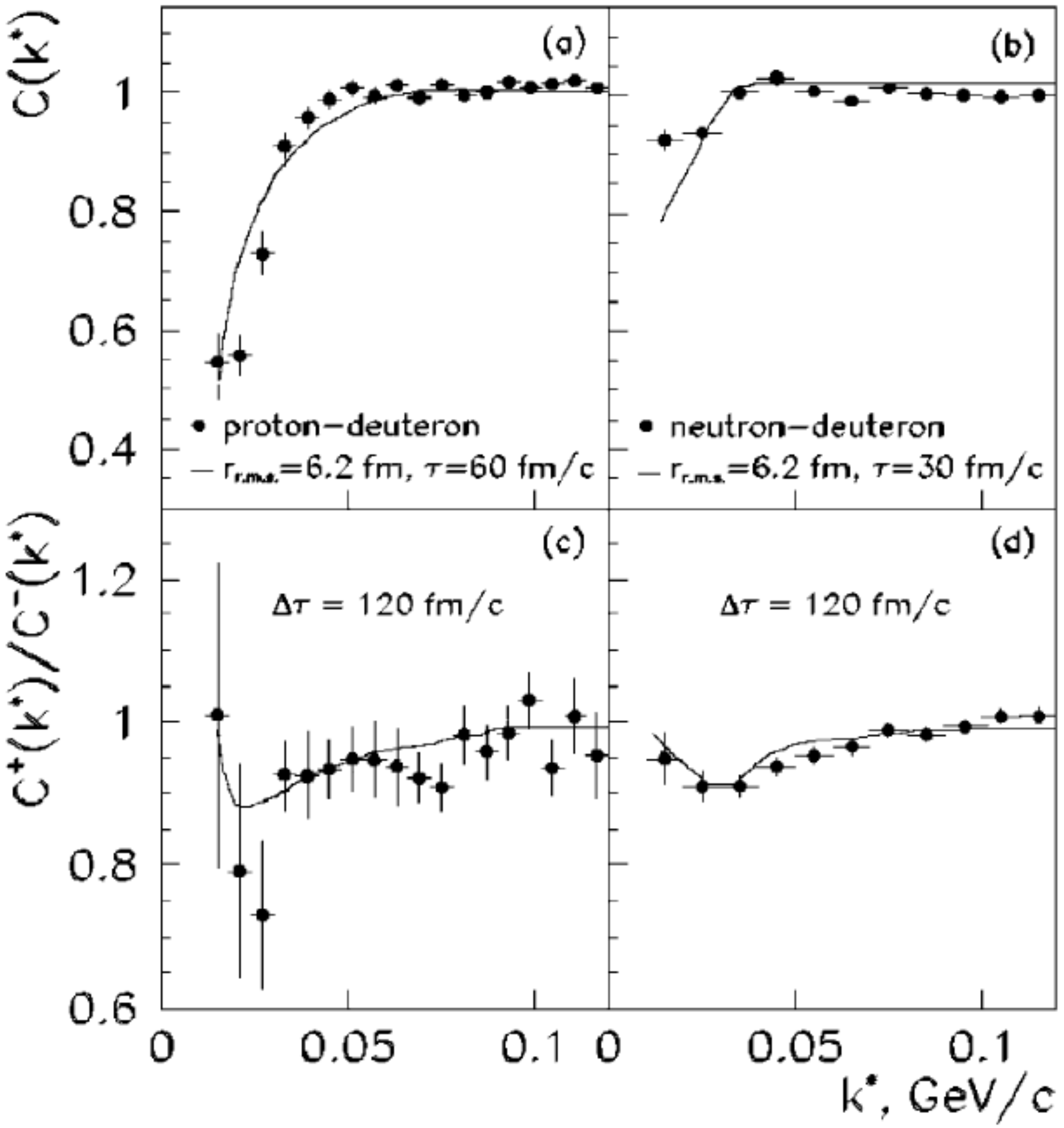
Contributions from: genuine feed-down misidentifications

- Quantification of the contributions to the pairs done by the lambda parameters $\lambda_{ij} = \mathscr{P}_i . f_i \times \mathscr{P}_j . f_j$
  - Purity of the individual particles ($\mathscr{P}_i$)
  - Feed-down fractions (f_i)

bhawani.singh@tum.de

proton-deuteron correlation measurement so far

- **Status**:
  - p-d correlation function from 2006
  - GANIL(Grand Accélérateur National d'Ions Lourds):
    - ^{40}Ar-^{58}Ni reaction at 77 MeV/u
    - Show a clear depletion
    - Only unto 100 MeV/c in relative momentum



[1] Wosińska, K., Pluta, J., Hanappe, F. *et al. Eur. Phys. J. A* 32, 55–59 (2007)

Another calculation at hand

- Hadron-Deuteron Correlations and Production of Light Nuclei in Relativistic Heavy-Ion Collisions: arxiv.org/abs/1904.08320
  - hadron-deuteron correlation function which carries information about the source of the deuterons
  - Allows one to determine whether a deuteron is directly emitted from the fireball or if it is formed afterwards
  - Conclusion:
    - The theoretical p-d correlation function is strongly dependent on the source size

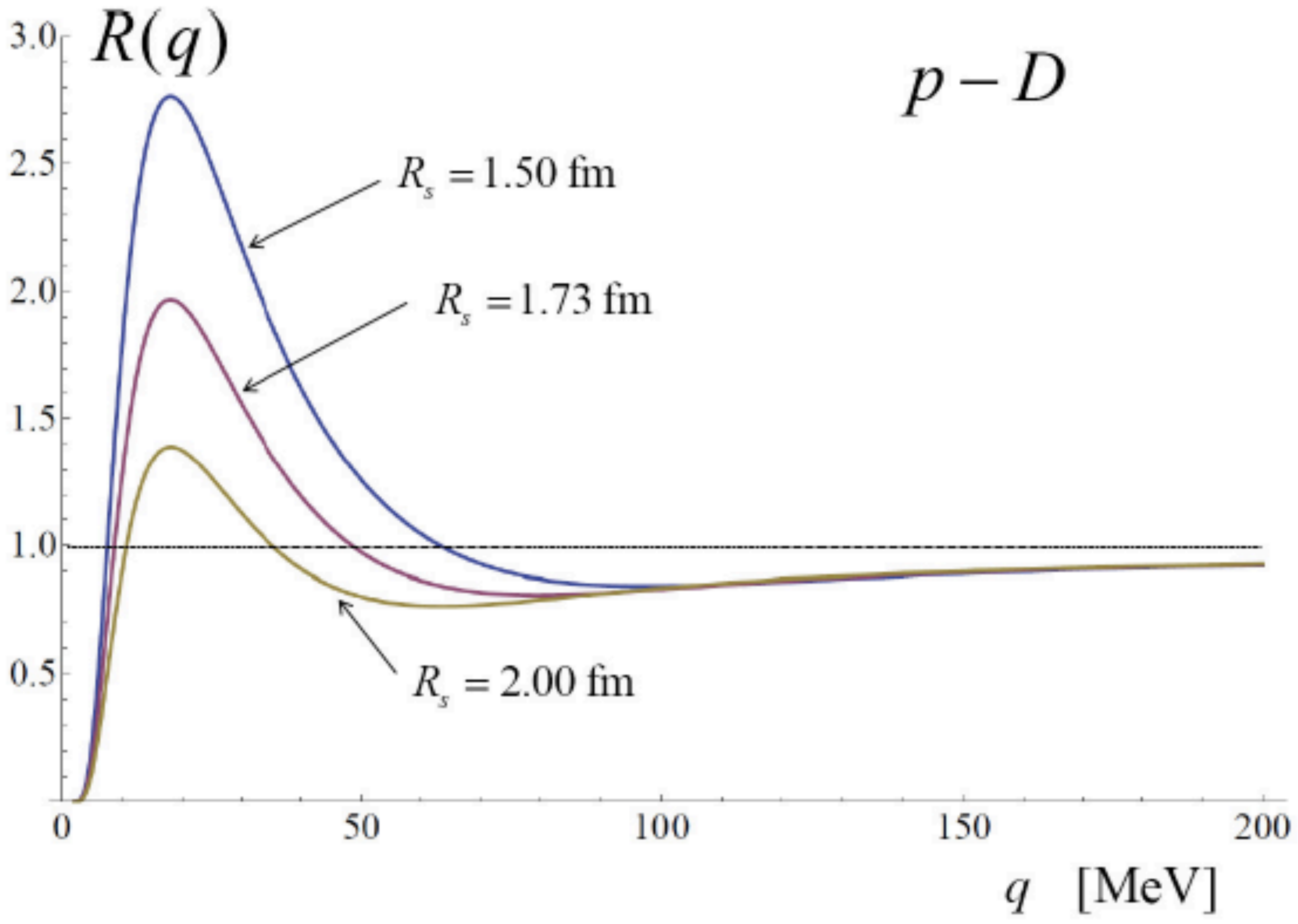


Fig. 2. $p-D$ correlation function