(Towards) Solving the puzzle of high temperature light (anti-)nuclei production in ultra-relativistic heavy-ion collisions

C. Greiner

Bormio 59th Intnl. winter meeting on nuclear physics,
23th -27th january, Bormio, Italy

in collaboration with:

T. Neidig, J. Rais, M. Bleicher, K. Gallmeister, H.v. Hees, J. Schaffner-Bielich and V. Vovchenko

Equilibrium nucleosynthesis (SAHA eq.) in relHIC

off-equilibrium production via a network of master eqs.

quantum mechanical thoughts … and calculations

T. Neidig et al, PL**B** 827 (2022) 136891

J. Rais et al, PRC **106** (2022) 064004

K. Gallmeister, CG, EPJ**A** **57** (2021) 62

V. Vovchenko et al, PL**B** 800 (2020) 135131

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CRC-TR 211 Strong-interaction matter under extreme conditions

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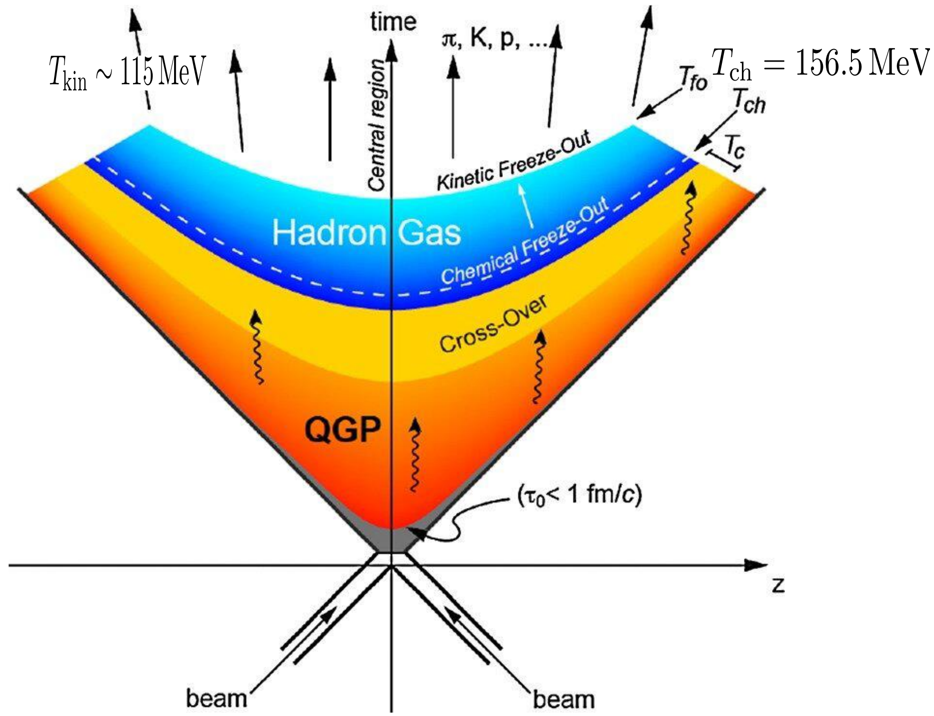
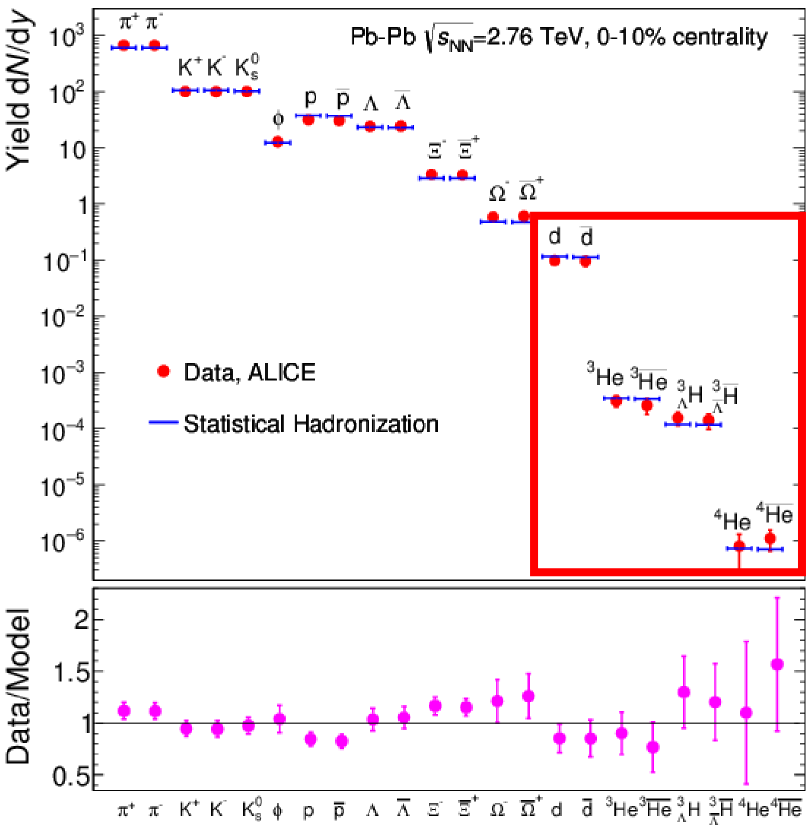
Unterstützt von / Supported by Alexander von Humboldt Stiftung/Foundation

ALICE data – “snowball in hell”





Andronic et al, Nature 561 (2018) 321

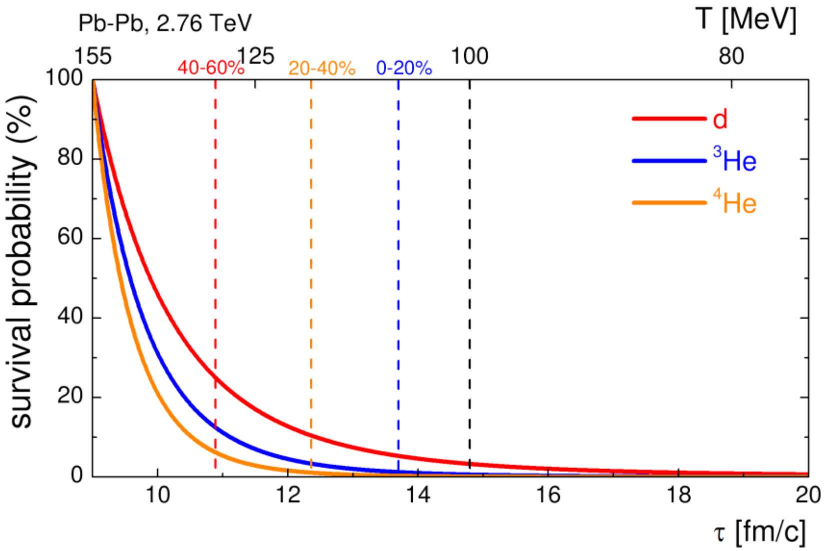
binding energies:

$$ {}^2{\rm H},\ {}^3{\rm He},\ {}^4{\rm He} : 2.22,\ 7.72,\ 28.3\,{\rm MeV} \qquad {}^3_\Lambda{\rm H} : 130\,{\rm keV} \qquad \ll T \sim 150\ {\rm MeV} $$

Can snowball survive hell?

Count only the nuclei produced at "QGP hadronization" (at T_{ch})

$$\frac{dN_A^{qgp}}{d\tau} = -\langle \sigma_{A\pi} v_{rel} \rangle n_\pi^{pce} N_A^{qgp}, \qquad \text{survival probability} = N_A^{qgp}(\tau)/N_A^{qgp}(\tau_{ch})$$



$$\tau_{eq} = \frac{1}{\langle \sigma_{AX}^{in} v_{rel} \rangle n_X}$$

The observed nuclei are unlikely to be (pre-)formed at the "QCD phase boundary" even the "thermal" production mechanism is correct

situation in an expanding hadronic gas

Nuclear Physics B 378 (1992) 95–128
North-Holland

NUCLEAR PHYSICS B

The role of the entropy in an expanding hadronic gas *

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Received 21 May 1991
(Revised 18 November 1991)
Accepted 18 February 1992

The evolution of the hot central region produced in central heavy ion collisions is analyzed. It is shown that the main properties of the final state can be understood as a result of an adiabatic expansion process. The role of the entropy per particle as a global characteristic of the system is emphasized. Our main concern is the evolution of the chemical composition.

1. Introduction

In heavy-ion collisions, a hot central region is formed, which is generally assumed to consist of a quark–gluon plasma if the collision energy is sufficiently high [1]. The minimal energy density necessary for the formation of this plasma is estimated to be around 2 GeV/fm^3. Such densities of energy are presumably attained in present fixed-target collisions with projectile energies of up to 200 GeV/nucleon [2]. As the hot zone expands and cools down, a phase transition to a hadron gas takes place. Soon after the transition, which is estimated to occur at a critical temperature around 200 MeV, the energy density is mostly stored in hadronic resonances (ρ, ω, ...) [3–5]. As the expansion and cooling proceed, these excited states decay, leading to a final state at freeze-out consisting mainly of pions. At presently available energies, central collisions generate final states containing several hundred pions (e.g.: in O + Au (S + S) central collisions at 200 GeV/nucleon ~ 400 (300) pions are observed [6]). Two-pion interferometry indicates that the radius of the "source" as measured with pions close to central rapidity is about 7 fm [6].

The processes which take place during the collision are discussed e.g. in refs. [1,7–9], on the basis of the Landau hydrodynamic model [10]. In the initial phase the hot spot is expected to look like a pancake whose thickness is swelling, an

* Work supported in part by Schweizerischer Nationalfonds.

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The total number of stable hadrons (pions, kaons, nucleons, hyperons) number of stable hadrons (pions, kaons, nucleons, hyperons) of direct hadrons and those of hadronic resonances stay constant.

Fully inelastic reactions (like eg N+N <-> K+Λ+N) ceases below T< 180 MeV.

important note on deuteron production

Nuclear Physics A533 (1991) 712–748
North-Holland

NUCLEAR PHYSICS A

PRODUCTION OF DEUTERONS AND PIONS IN A TRANSPORT MODEL OF ENERGETIC HEAVY-ION REACTIONS

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Received 8 April 1991
(Revised 21 May 1991)

Abstract: Green functions are used to derive transport equations with bound-state production and absorption. The equations are valid in the quasiparticle limit and are used to describe deuteron production in heavy-ion induced reactions. The deuterons are produced in three-nucleon collisions in a process that is inverse to deuteron breakup. We also derive rate equations for pion production by resonance formation and decay. Our equations satisfy detailed balance even in the case of wide resonances, unlike previous formulations. The relation widely employed in the cascade and Boltzmann equation models produces an equilibrium with too many pions. We have solved the equations numerically, finding for a number of cases fair agreement with experimental data. The predicted entropy produced in central Nb+Nb collisions at 650 MeV/nucleon exceeds by half a unit the entropy deduced from data. The predicted pion yields in the cascade limit are much closer to the data measured in central Ar+KCl collisions than was found in earlier treatments.

1. Introduction

A full description of heavy-ion-induced reactions requires other degrees of freedom besides the nucleon single-particle variables. A distinct feature of high-energy reactions is the large number of composites, particularly deuterons, emitted into wide angles [1]). The theory of deuteron production by heavy ions was at first rather crude. Models were based on various assumptions including: chemical equilibrium [2-5]), coalescence in momentum space [6-8]), and coalescence in phase space [9,10]). The first theory based on dynamics was proposed by Remler [11]), and was applied in refs. [12,13]). Dynamic models relying on nucleon degrees of freedom and independent nucleon–nucleon collisions [14-17]) have been quite successful in describing single-particle features such as production of protons. Although the quasiparticle assumption inherent in the models may not be well satisfied in reactions, see e.g. ref. [18]), these models are appealing because they incorporate a range of dynamic effects and are computationally tractable. In this paper we examine the extension of the models beyond the approximation that the equations can be truncated with two-particle collisions and only nucleon degrees of freedom. We

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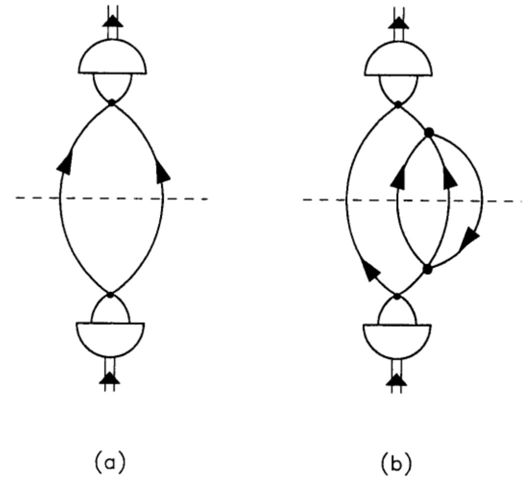
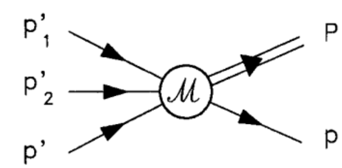


Fig. 3



$D \nleftrightarrow N+N$

Coalescence ??!

N+D <--> D+N+N

Three body reactions

“Bevalac” nucleosynthesis

VOLUME 43, NUMBER 20 PHYSICAL REVIEW LETTERS 12 NOVEMBER 1979

Evidence for a Soft Nuclear-Matter Equation of State

Philip J. Siemens[(a)] and Joseph I. Kapusta
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(Received 3 August 1979)

The entropy of the fireball formed in central collisions of heavy nuclei at center-of-mass kinetic energies of a few hundred MeV per nucleon is estimated from the ratio of deuterons to protons at large transverse momentum. The observed paucity of deuterons suggests that strong attractive forces are present in hot, dense nuclear matter, or that degrees of freedom beyond the nucleon and pion may already be realized at an excitation energy of 100 MeV per baryon.

Because of the reaction $d+N \leftrightarrow p+n+N$, where N is a spectator nucleon or cluster, deuterons will be constantly breaking up and reforming. If collisions are frequent enough, the deuterons will quickly reach an equilibrium concentration determined by detailed balancing[4]:

$$\exp(-\mu_d/T)d_d(\vec{R},\vec{P},S_z)=\sum\nolimits_{s_z} d_p(\vec{R},\vec{P}/2,s_z)d_n(\vec{R},\vec{P}/2,S_z-s_z)\exp[-(\mu_n+\mu_p)/T],$$

partial chemical equilibrium (PCE)

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B ’92

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through pseudo-elastic resonance reactions $\pi\pi \leftrightarrow \rho$, $\pi\mathrm{K} \leftrightarrow \mathrm{K}^*, \pi N \leftrightarrow \Delta$, etc.

E.g.: $\pi + 2\rho + 3\omega + \cdots = const, \quad \mathrm{K} + \mathrm{K}^* + \cdots = const, \quad N + \Delta + N^* + \cdots = const,$

Effective chemical potentials:

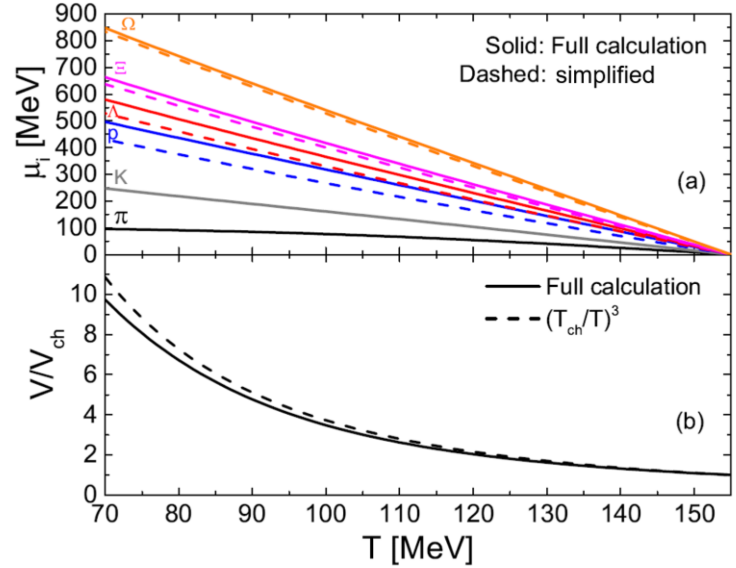
$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \, \mu_i, \qquad \langle n_i \rangle_j$ – mean number of hadron i from decays of hadron j, $\qquad j \in \text{HRG}$

Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \, V = N_i(T_{\text{ch}}), \quad i \in \text{stable}$$

$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) \, V = S(T_{\text{ch}})$$

numerical solution $\{\mu_i(T)\}, V(T)$



Numerical implementation within **Thermal-FIST** package

[**V. Vovchenko**, H. Stoecker, *Comput. Phys. Commun.* ***244,*** *295 (2019)*]

open source: https://github.com/vlvovch/Thermal-FIST

Light nuclei: Saha equation

Detailed balance for nuclear reactions

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\mathrm{eq}}}{\prod_i n_{A_i}^{\mathrm{eq}}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.} \quad \mu_d = \mu_p + \mu_n, \; \mu_{^3\mathrm{He}} = 2\mu_p + \mu_n, \; \ldots$$

Saha equation

Kinetic theory example: deuteron number evolution through $p + n + X \leftrightarrow d + X$ reactions

gain **loss**

$$\frac{dN_d}{d\tau} = \langle \sigma_{dX} v_{rel} \rangle N_d^0 n_x^0 e^{\mu_p/T} e^{\mu_n/T} e^{\mu_X/T} - \langle \sigma_{dX} v_{rel} \rangle N_d^0 n_x^0 e^{\mu_d/T} e^{\mu_X/T}$$

small ***big*** ***big***

gain $\approx$ loss $\quad \rightarrow \quad \mu_d \approx \mu_p + \mu_n$

Saha equation

= detailed balance = law of mass action

Early Universe:

$$X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

E. Kolb, M. Turner, "The Early Universe" (1990)

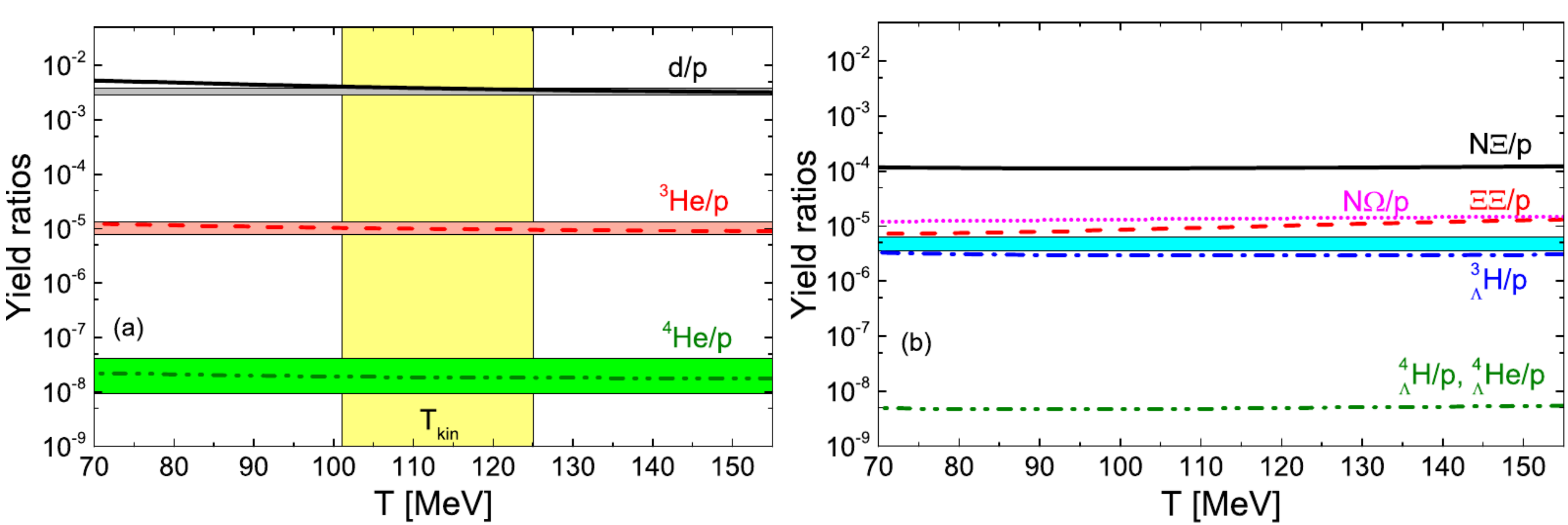
this work:

$$\frac{N_A(T)}{N_p} = \frac{g_A}{g_M^{1-A}} \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{1+A}{2}}\right] A^{\frac{3}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \times \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$$\eta = \frac{n_B}{n_\gamma} \sim 6 \cdot 10^{-10} \quad \rightarrow \quad \eta_B = \frac{N_N}{N_M} \sim 0.03$$

V. Vovchenko et al, PL**B** 800 (2020) 135131

Saha equation for light (hyper-) nuclei at the LHC

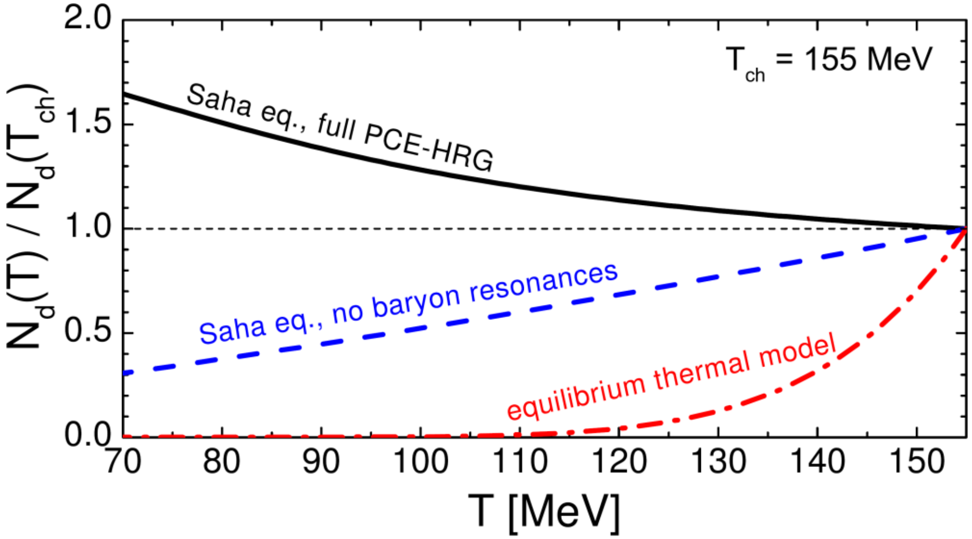


Deviations from thermal model predictions are moderate despite significant cooling and dilution. Is this the reason for why thermal model works so well?

Echoes earlier transport model conclusions for d D. Oliinychenko, et al., PRC 99, 044907 (2019)

Hypernuclei stay close to the thermal model prediction. An exception is a hypothetical ΞΞ state ← *planned measurement in Runs 3 & 4 at the LHC*

Closer look to the yields



Saha equation (no resonances):

$$\frac{N_A(T)}{N_A(T_{ch})} \simeq \left(\frac{T}{T_{ch}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T}-\frac{1}{T_{ch}}\right)\right]$$

Thermal model:

$$\left[\frac{N_A(T)}{N_A(T_{ch})}\right]_{eq.} \simeq \left(\frac{T}{T_{ch}}\right)^{-\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T}-\frac{1}{T_{ch}}\right)\right]$$

$$m_A \gg T$$

Nuclei yields are *not constant* in the Saha equation approach but the strong exponential dependence on the temperature is eliminated

Quantitative outcome is sensitive to the feeding from baryonic resonances

Light nuclei off-equilibrium production with rate equations

T. Neidig et al, PL**B** 827 (2022) 136891

gain **loss**

$$\underbrace{\frac{dN_d}{d\tau}}_{\cancel{\text{small}}} = \underbrace{\langle \sigma_{dX} v_{rel} \rangle N_d^0 n_x^0 e^{\mu_p/T} e^{\mu_n/T} e^{\mu_X/T}}_{\cancel{\text{big}}} - \underbrace{\langle \sigma_{dX} v_{rel} \rangle N_d^0 n_x^0 e^{\mu_d/T} e^{\mu_X/T}}_{\cancel{\text{big}}}$$

~~gain ≈ loss → $\mu_d \approx \mu_p + \mu_n$~~

~~**Saha equation**~~
~~= detailed balance~~
~~= law of mass action~~

Relax the assumption of equilibrium for $AX \leftrightarrow \sum_i A_i X$ reactions

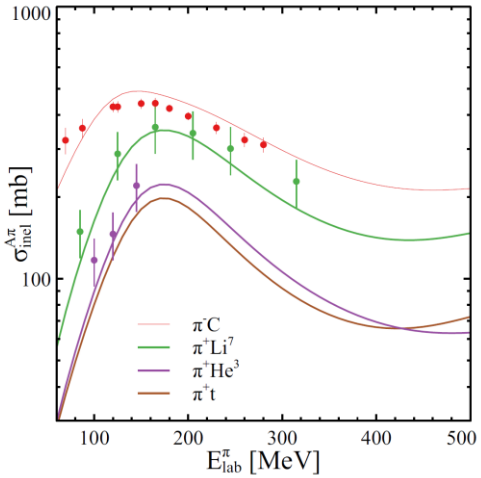
Catalyzed light nuclei reactions. Destruction through $AX \rightarrow \sum_i A_i X$ and creation through $\sum_i A_i X \rightarrow AX$. Detailed balance principle respected but *relative chemical equilibrium not enforced*

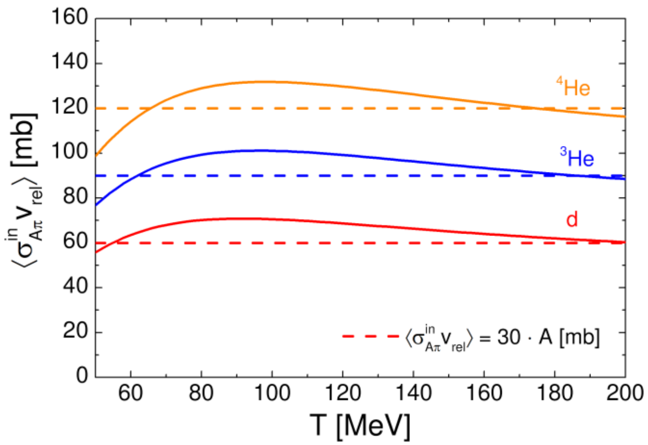
$$\frac{dN_A}{d\tau} = \langle \sigma_{AX}^{\text{in}} v_{\text{rel}} \rangle \, n_X \, (N_A^{\text{saha}} - N_A)$$

some details …

Rates: Use guidance from kinetic theory (and full hadronic transport models)

Optical model for $\sigma_{A\pi}^{\text{in}}$ [J. Eisenberg, D.S. Koltun, '80]





Expansion (both transverse and longitudinal)

$$\frac{V}{V_{\text{ch}}} = \frac{\tau}{\tau_{\text{ch}}} \frac{\tau_\perp^2 + \tau^2}{\tau_\perp^2 + \tau_{\text{ch}}^2}, \qquad \tau_{\text{ch}} = 9 \text{ fm}, \qquad \tau_\perp = 6.5 \text{ fm}$$

… and the full hadronic network

$$
\begin{aligned}
\frac{dN_N}{dt} &= 2\tilde{\alpha}_{D+\pi\to 2N+\pi} N_\pi (N_d - c_d^{N^2}\cdot N_N^2) + 3\tilde{\alpha}_{T+\pi\to 3N+\pi} N_\pi (N_t - c_t^{N^3}\cdot N_N^3) + 3\tilde{\alpha}_{He^3+\pi\to 3N+\pi} N_\pi (N_{He^3} - c_{He^3}^{N^3}\cdot N_N^3) \\
&+ 4\tilde{\alpha}_{He^4+\pi\to 4N+\pi} N_\pi (N_{He^4} - c_{He^4}^{N^4}\cdot N_N^4) + \tilde{\alpha}_{\Delta\to N+\pi} (N_\Delta - c_\Delta^{N\pi}\cdot N_N N_\pi) + 2\tilde{\alpha}_{D+K/\overline{K}\to 2N+K/\overline{K}} N_{K/\overline{K}} (N_d - c_d^{N^2}\cdot N_N^2) \\
&+ 3\tilde{\alpha}_{T+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (N_t - c_t^{N^3}\cdot N_N^3) + 3\tilde{\alpha}_{He^3+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (N_{He^3} - c_{He^3}^{N^3}\cdot N_N^3) \\
&+ 4\tilde{\alpha}_{He^4+K/\overline{K}\to 4N+K/\overline{K}} N_{K/\overline{K}} (N_{He^4} - c_{He^4}^{N^4}\cdot N_N^4) \\
\frac{dN_d}{dt} &= \tilde{\alpha}_{D+\pi\to 2N+\pi} N_\pi (-N_d + c_d^{N^2}\cdot N_N^2) + \tilde{\alpha}_{D+K/\overline{K}\to 2N+K/\overline{K}} N_{K/\overline{K}} (-N_d + c_d^{N^2}\cdot N_N^2) \\
\frac{dN_t}{dt} &= \tilde{\alpha}_{T+\pi\to 3N+\pi} N_\pi (-N_t + c_t^{N^3}\cdot N_N^3) + \tilde{\alpha}_{T+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (-N_t + c_t^{N^3}\cdot N_N^3) \\
\frac{dN_{He^3}}{dt} &= \tilde{\alpha}_{He^3+\pi\to 3N+\pi} N_\pi (-N_{He^3} + c_{He^3}^{N^3}\cdot N_N^3) + \tilde{\alpha}_{He^3+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (-N_{He^3} + c_{He^3}^{N^3}\cdot N_N^3) \\
\frac{dN_{He^4}}{dt} &= \tilde{\alpha}_{He^4+\pi\to 4N+\pi} N_\pi (-N_{He^4} + c_{He^4}^{N^4}\cdot N_N^4) + \tilde{\alpha}_{He^4+K/\overline{K}\to 4N+K/\overline{K}} N_{K/\overline{K}} (-N_{He^4} + c_{He^4}^{N^4}\cdot N_N^4) \\
\frac{dN_{\overline{N}}}{dt} &= 2\tilde{\alpha}_{\overline{d}+\pi\to 2\overline{N}+\pi} N_\pi (N_{\overline{d}} - c_{\overline{d}}^{\overline{N}^2}\cdot N_{\overline{N}}^2) + 3\tilde{\alpha}_{\overline{t}+\pi\to 3\overline{N}+\pi} N_\pi (N_{\overline{t}} - c_{\overline{t}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) + 3\tilde{\alpha}_{\overline{He^3}+\pi\to 3\overline{N}+\pi} N_\pi (N_{\overline{He^3}} - c_{\overline{He^3}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) \\
&+ 4\tilde{\alpha}_{\overline{He^4}+\pi\to 4\overline{N}+\pi} N_\pi (N_{\overline{He^4}} - c_{\overline{He^4}}^{\overline{N}^3}\cdot N_{\overline{N}}^4) + \tilde{\alpha}_{\overline{\Delta}\to \overline{N}+\pi} (N_{\overline{\Delta}} - c_{\overline{\Delta}}^{\overline{N}+\pi}\cdot N_{\overline{N}} N_\pi) + 2\tilde{\alpha}_{\overline{d}+K/\overline{K}\to 2\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{d}} - c_{\overline{d}}^{\overline{N}^2}\cdot N_{\overline{N}}^2) \\
&+ 3\tilde{\alpha}_{\overline{t}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{t}} - c_{\overline{t}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) + 3\tilde{\alpha}_{\overline{He^3}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{He^3}} - c_{\overline{He^3}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) \\
&+ 4\tilde{\alpha}_{\overline{He^4}+K/\overline{K}\to 4\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{He^4}} - c_{\overline{He^4}}^{\overline{N}^3}\cdot N_{\overline{N}}^4) \\
\frac{dN_{\overline{d}}}{dt} &= \tilde{\alpha}_{\overline{d}+\pi\to 2\overline{N}+\pi} N_\pi (-N_{\overline{d}} + c_{\overline{d}}^{\overline{N}^2}\cdot N_{\overline{N}}^2) + \tilde{\alpha}_{\overline{d}+K/\overline{K}\to 2\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{d}} + c_{\overline{d}}^{\overline{N}^2}\cdot N_{\overline{N}}^2) \\
\frac{dN_{\overline{t}}}{dt} &= \tilde{\alpha}_{\overline{t}+\pi\to 3\overline{N}+\pi} N_\pi (-N_{\overline{t}} + c_{\overline{t}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) + \tilde{\alpha}_{\overline{t}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{t}} + c_{\overline{t}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) \\
\frac{dN_{\overline{He^3}}}{dt} &= \tilde{\alpha}_{\overline{He^3}+\pi\to 3\overline{N}+\pi} N_\pi (-N_{\overline{He^3}} + c_{\overline{He^3}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) + \tilde{\alpha}_{\overline{He^3}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{He^3}} + c_{\overline{He^3}}^{\overline{N}^3}\cdot N_{\overline{N}}^3) \\
\frac{dN_{\overline{He^4}}}{dt} &= \tilde{\alpha}_{\overline{He^4}+\pi\to 4\overline{N}+\pi} N_\pi (-N_{\overline{He^4}} + c_{\overline{He^4}}^{\overline{N}^3}\cdot N_{\overline{N}}^4) + \tilde{\alpha}_{\overline{He^4}+K/\overline{K}\to 4\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{He^4}} + c_{\overline{He^4}}^{\overline{N}^3}\cdot N_{\overline{N}}^4) \\
\frac{dN_\Delta}{dt} &= \tilde{\alpha}_{\Delta\to N+\pi} (-N_\Delta + c_\Delta^{N\pi}\cdot N_N N_\pi) \\
\frac{dN_{\overline{\Delta}}}{dt} &= \tilde{\alpha}_{\overline{\Delta}\to \overline{N}+\pi} (-N_{\overline{\Delta}} + c_{\overline{\Delta}}^{\overline{N}+\pi}\cdot N_{\overline{N}} N_\pi) \\
\frac{dN_\pi}{dt} &= \tilde{\alpha}_{\Delta\to N+\pi} (N_\Delta - c_\Delta^{N\pi}\cdot N_N N_\pi) + \tilde{\alpha}_{\overline{\Delta}\to \overline{N}+\pi} (N_{\overline{\Delta}} - c_{\overline{\Delta}}^{\overline{N}+\pi}\cdot N_{\overline{N}} N_\pi) + 2\tilde{\alpha}_{\rho\to 2\pi} (N_\rho - c_\rho^{\pi^2}\cdot N_\pi^2) + 3\tilde{\alpha}_{\omega\to 3\pi} (N_\omega - c_\omega^{\pi^3}\cdot N_\pi^3) \\
\frac{dN_\rho}{dt} &= \tilde{\alpha}_{\rho\to 2\pi} (-N_\rho + c_\rho^{\pi^2}\cdot N_\pi^2) \\
\frac{dN_\omega}{dt} &= \tilde{\alpha}_{\omega\to 3\pi} (-N_\omega + c_\omega^{\pi^3}\cdot N_\pi^3) \\
\frac{dN_K}{dt} &= \tilde{\alpha}_{K^*\to K+\pi} (N_{K^*} - c_{K^*}^{\pi K}\cdot N_\pi N_K) \\
\frac{dN_{\overline{K}}}{dt} &= \tilde{\alpha}_{\overline{K^*}\to \overline{K}+\pi} (N_{\overline{K^*}} - c_{\overline{K^*}}^{\pi\overline{K}}\cdot N_\pi N_{\overline{K}}) \\
\frac{dN_{K^*}}{dt} &= \tilde{\alpha}_{K^*\to K+\pi} (-N_{K^*} + c_{K^*}^{\pi K}\cdot N_\pi N_K) \\
\frac{dN_{\overline{K^*}}}{dt} &= \tilde{\alpha}_{\overline{K^*}\to \overline{K}+\pi} (-N_{\overline{K^*}} + c_{\overline{K^*}}^{\pi\overline{K}}\cdot N_\pi N_{\overline{K}})
\end{aligned}
$$

$$c_{AB\ldots}^{XY\ldots} := \frac{N_A^{\mathrm{eq}} N_B^{\mathrm{eq}} \ldots}{N_X^{\mathrm{eq}} N_Y^{\mathrm{eq}} \ldots}$$

Mesons: pions, kaons, ρ-meson, K*, ω-meson

Baryons: nucleons, light nuclei , Δ(1232), N*

$$R \rightleftharpoons X + Y$$

$$\frac{\mathrm{d}N_R}{\mathrm{d}t} = -\alpha_{R\rightleftharpoons X+Y} (N_R - c_R^{XY} N_X N_Y)$$

$$
\begin{aligned}
\frac{\mathrm{d}N_\pi}{\mathrm{d}t} = &\ \tilde{\alpha}_{\Delta\rightleftharpoons N\pi} (N_\Delta - c_\Delta^{N\pi} N_N N_\pi) \\
&+ \tilde{\alpha}_{\overline{\Delta}\rightleftharpoons \overline{N}\pi} (N_{\overline{\Delta}} - c_{\overline{\Delta}}^{\overline{N}\pi} N_{\overline{N}} N_\pi) \\
&+ 2\tilde{\alpha}_{\rho\rightleftharpoons 2\pi} (N_\rho - c_\rho^{\pi^2} N_\pi^2) \\
&+ 3\tilde{\alpha}_{\omega\rightleftharpoons 3\pi} (N_\omega - c_\omega^{\pi^3} N_\pi^3)\ .
\end{aligned}
$$

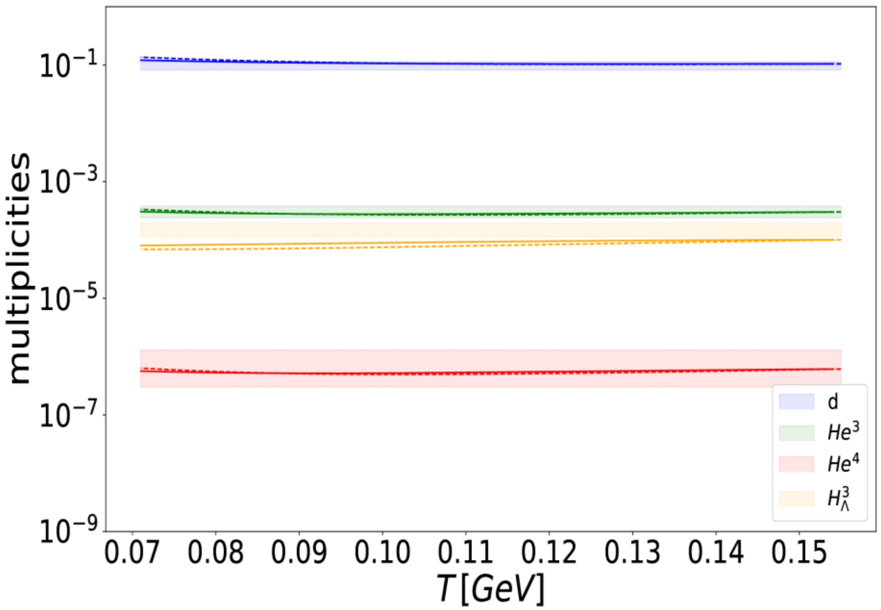
$$A + X \rightleftharpoons a \cdot N + X$$

$$\frac{\mathrm{d}N_\mathrm{d}}{\mathrm{d}t} = -\sum_{x=\pi,K,\overline{K}} \tilde{\alpha}_{\mathrm{d}+x\rightleftharpoons 2N+x} N_x (N_\mathrm{d} - c_\mathrm{d}^{N^2} N_N^2)$$

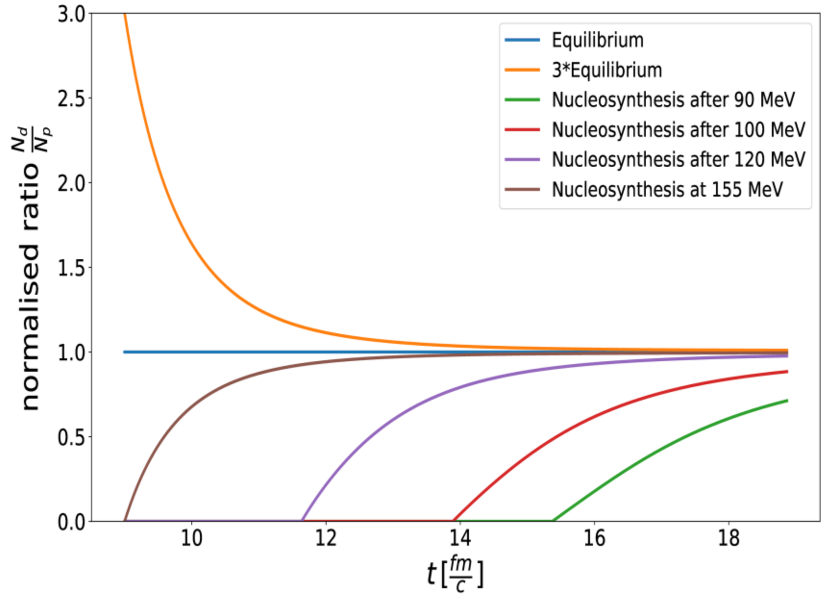
$$
\begin{aligned}
\alpha_{A+X\to aN+X} &= \frac{\langle \sigma_{A+X\to aN+X} v_{\mathrm{rel}} \rangle}{V} N_X \\
&= \tilde{\alpha}_{A+X\to aN+X} N_X\ .
\end{aligned}
$$

Rate equations results at LHC

starting at equilibrium:



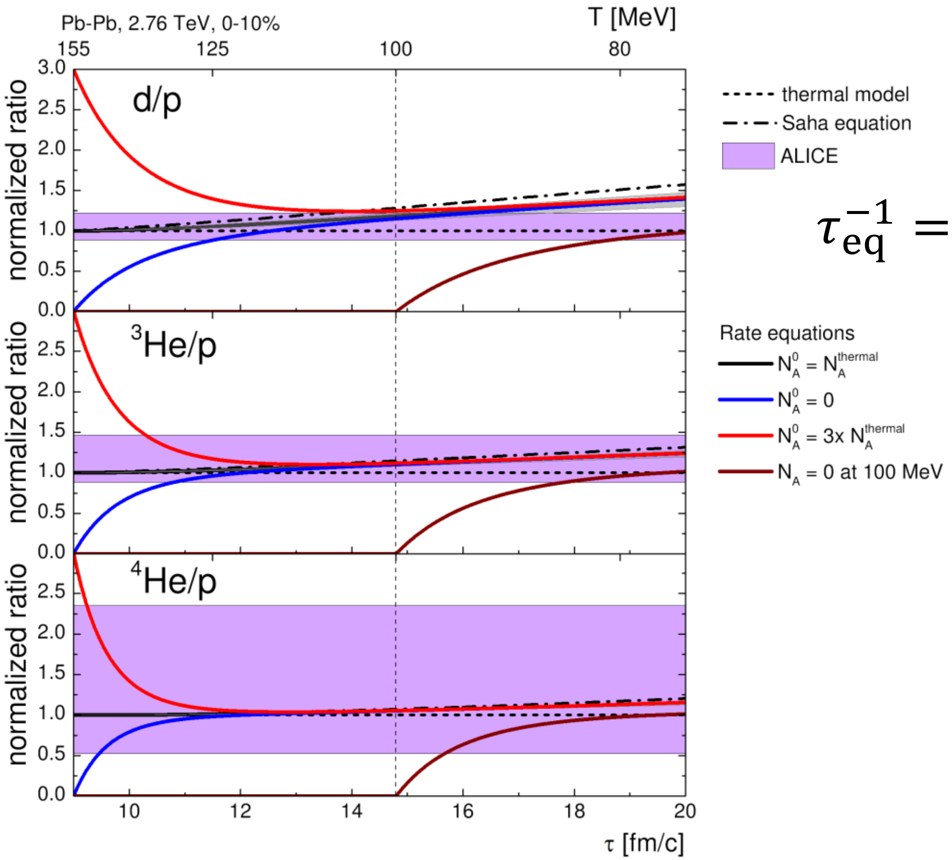
starting out of equilibrium:



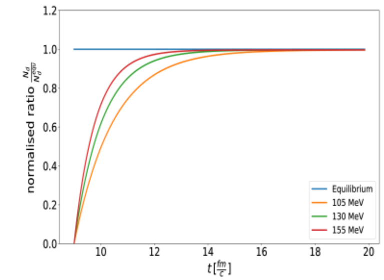
Solid lines represent the results of the rate equations, while dashed curves show the result of the HRG in PCE (i.e. SAHA). The colored bands represent the experimental data (ALICE).

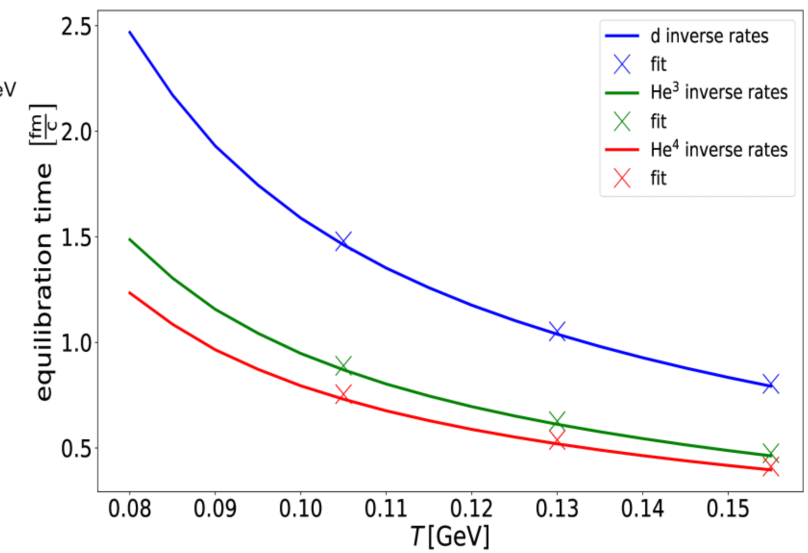
The ratio of deuterons to protons normalized to the same ratio at equilibrium for different initial conditions.

Rate equations results at LHC



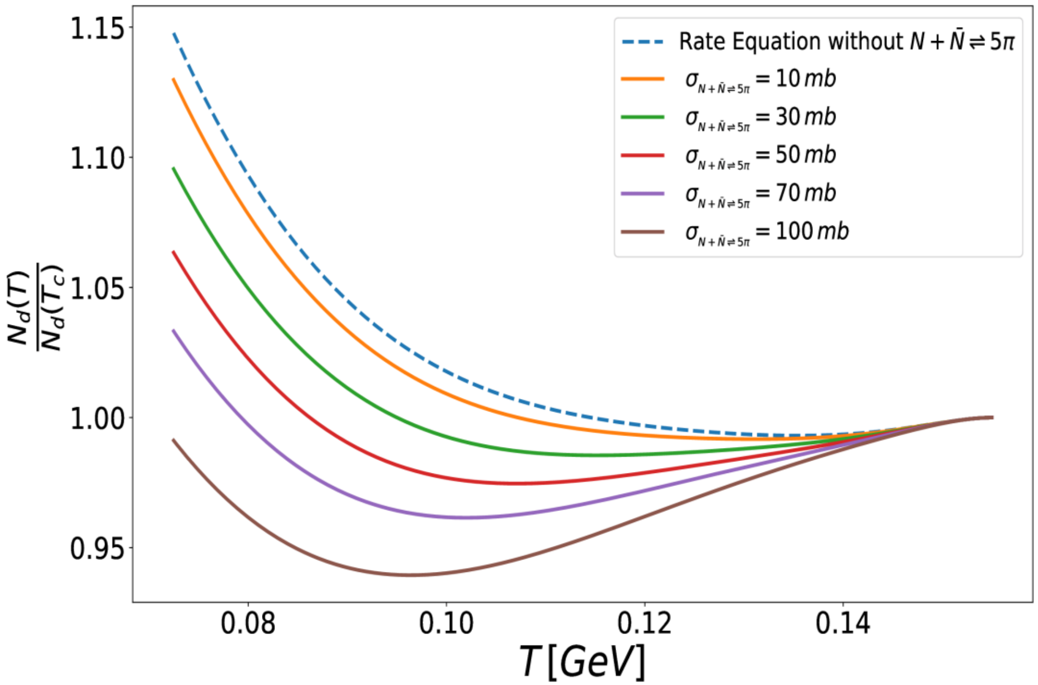
$$\tau_{\text{eq}}^{-1} = \langle \sigma_{A\pi}^{\text{in}} v_{rel} \rangle \, n_\pi^{\text{pce}}$$





- Local equilibration times remain small
- $\tau_A^{\text{eq}} \ll B_A^{-1}$ meaning light nuclei are not fully formed
- (**gain** + **loss**) $\gg$ |**gain** − **loss**| → **Saha equation at work**

Effect of baryon-antibaryon annihilations



$$\frac{\mathrm{d}N_N}{\mathrm{d}t} += \tilde{\alpha}_{N+\overline{N}\rightleftharpoons 5\pi}\left(-N_N N_{\overline{N}} + c^{\pi^5}_{N\overline{N}} N_\pi^5\right)$$

$$\frac{\mathrm{d}N_{\overline{N}}}{\mathrm{d}t} += \tilde{\alpha}_{N+\overline{N}\rightleftharpoons 5\pi}\left(-N_N N_{\overline{N}} + c^{\pi^5}_{N\overline{N}} N_\pi^5\right)$$

$$\frac{\mathrm{d}N_\pi}{\mathrm{d}t} += 5\tilde{\alpha}_{N+\overline{N}\rightleftharpoons 5\pi}\left(N_N N_{\overline{N}} - c^{\pi^5}_{N\overline{N}} N_\pi^5\right).$$

Annihilations decrease light nuclei yields

Stronger effect (up to 25%) for heavier nuclei ^{3}He, ^{4}He

(intermediate) summary and conclusions

Saha equation is an extended thermal model framework for light nuclei production

- results agree with the thermal model but essentially *any* $T < T_{ch}$ permitted
- quantitative predictions are sensitive to baryon resonance feeddowns

Rate equations validate the framework when using nuclei break-up cross sections based on kinetic theory

- nuclei (pre-)formed at “QCD phase boundary” do not survive the hadronic phase
- baryon annihilations may suppress the nuclei yields on ~5-20% level
- the same procedure could be done for RHIC or SPS energies

Outlook: quantum mechanical formation of bound states

- Schrödinger equation, quantum Langevin(-Schrödinger) equation
- Bound state formation in Open Quantum Systems

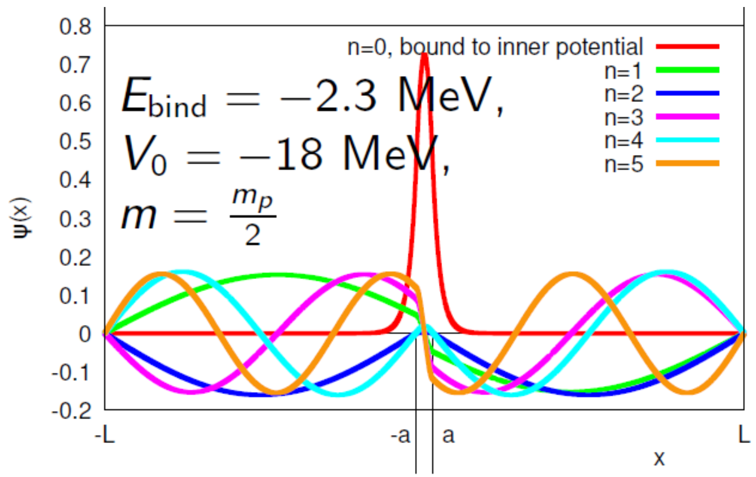
Formation of Bound State (and Continuum) with Time Dependent Potential

J. Rais et al, PRC **106** (2022) 064004

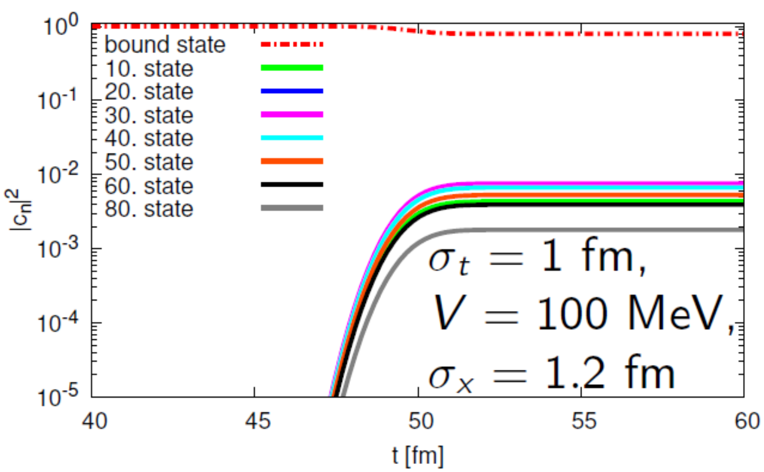
Wavefunction expressed by

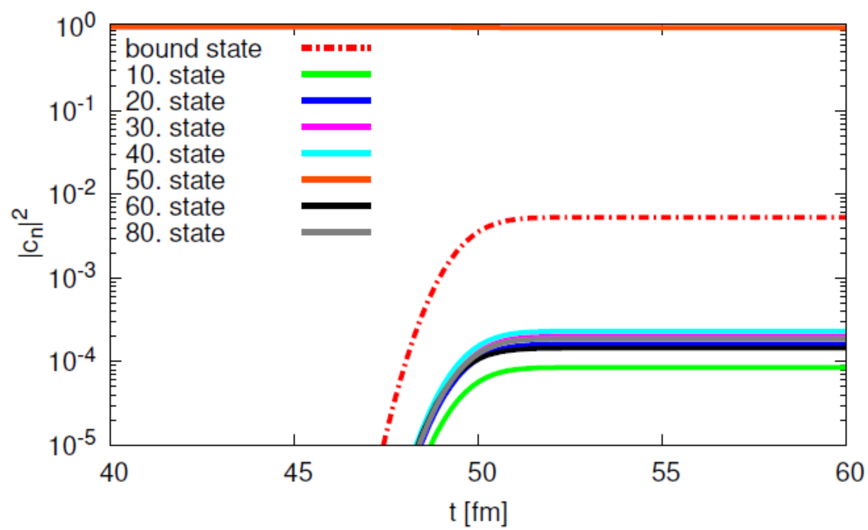
$\psi(x,t) = \sum_n c_n \psi_n(x)$ to solve $i\frac{dc_j(t)}{dt} = \sum_n V_{jn}(t) e^{i(E_j - E_n)t} c_n(t)$

from time dependent Schrödinger equation

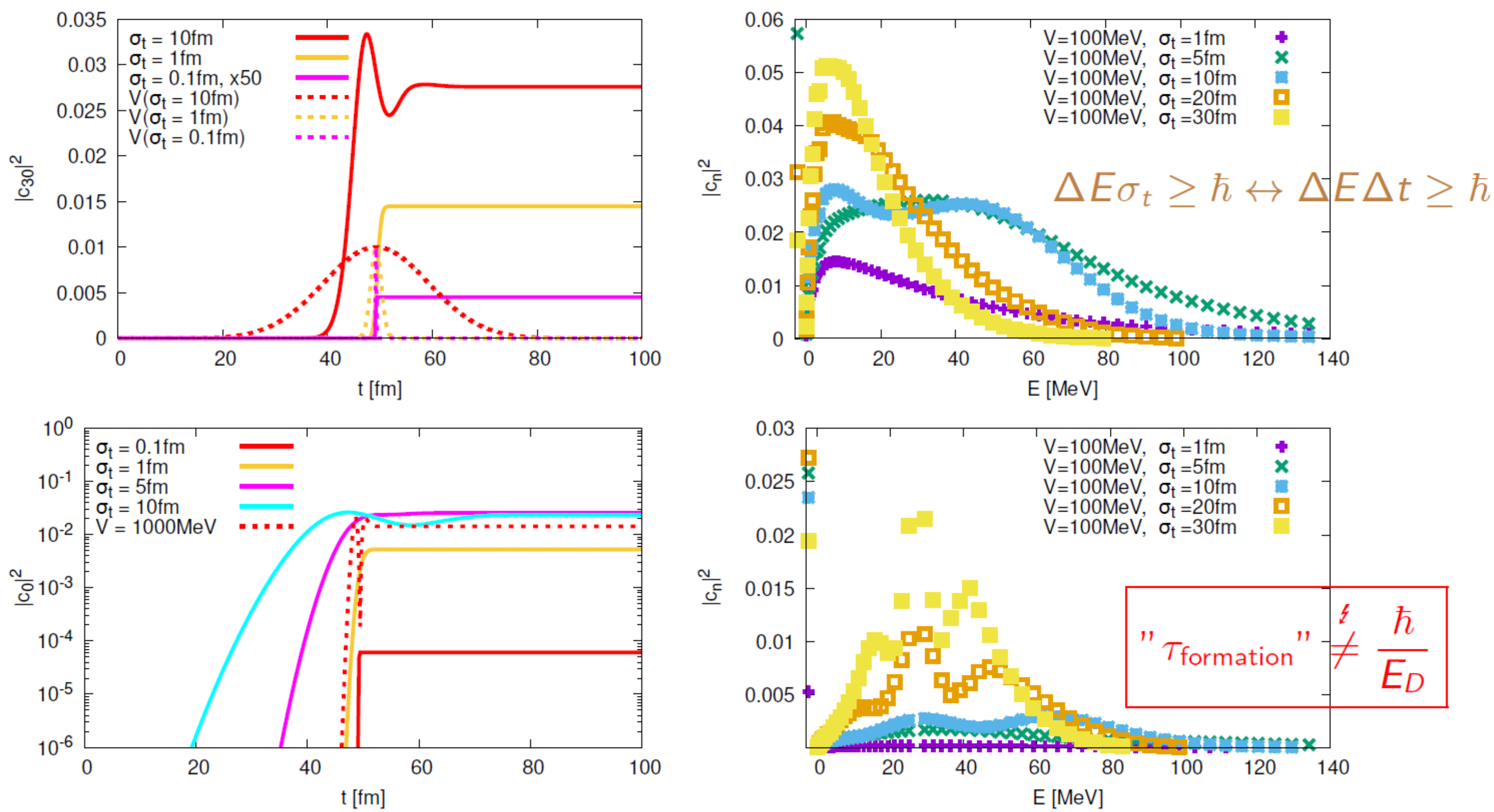


- calculate energy spectrum of system
- interacts with time dependent pulse
  $V(t) = V_0 \exp\left[-\frac{x^2}{2\sigma_t^2} - \frac{x^2}{2\sigma_x^2}\right]$
- originally one bound state i.e. $c_0(t=0) = 1$ or $c_{50}(t=0) = 1$





formation time



- bound states are formed during the interaction
- Heisenberg's uncertainty relation obtained in the distribution of states
- Formation of states instantaneously and simultaneously to time-dep. potential impact
- perturbation theory is applicable for $\mathcal{O}(\sigma_t) \sim 1$ fm and $\mathcal{O}(V) \sim 100$ MeV

perspectives: bound state formation in open quantum systems

Tim Neidig, Jan Rais

Particle interacting with th. heat bath $\underbrace{[H_A + H_B + V_{A,B}]}_{H}\psi_{A,B} = -\frac{\partial}{\partial t}\psi_{A,B}$

Caldeira-Leggett master equation starting with

$$H = \frac{p^2}{2M} + V(x) + \sum_{j=1}^{N}\left[\frac{p_j^2}{2m_j} + \frac{1}{2}m_j\omega_j\left(x_j - \frac{C_j}{m_j\omega_j^2}x\right)^2\right]$$

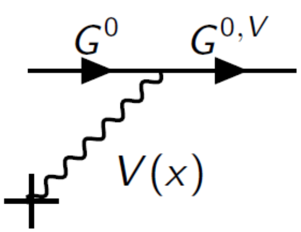
or Dyson-Schwinger equations ($\rightarrow$ Kadanoff-Baym with Gaussian potential)

$$G(1,1') = G^{0,V}(1,1') + \int d2 \int d3\, G^{0,V}(1,2)\Sigma(2,3)G(3,1'),$$

$$G(1,1') = G^{0,V}(1,1') + \int d2 \int d3\, G(1,2)\Sigma(2,3)G^{0,V}(3,1'),$$

$\Sigma =$

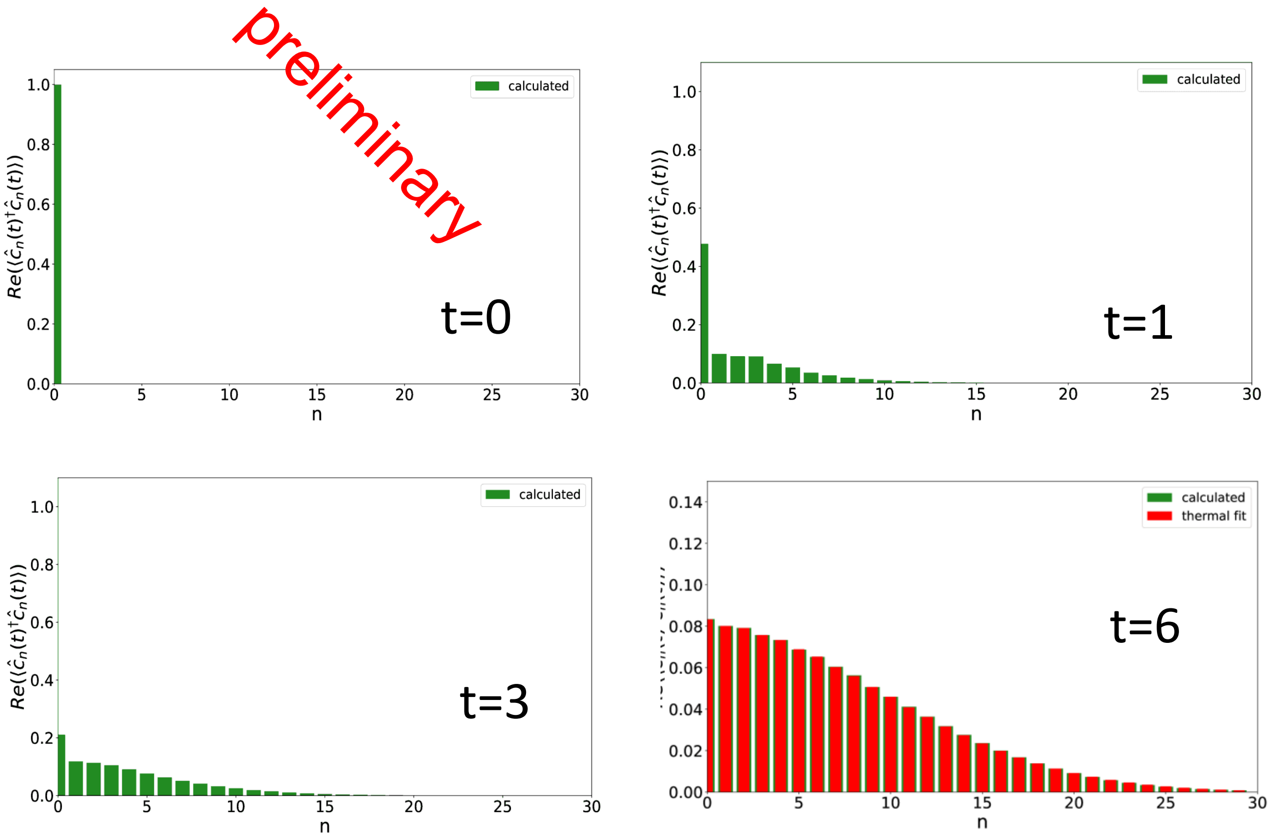
$G^{0,V} =$



- Direct Born Diagram

Kadanoff-Baym solutions …

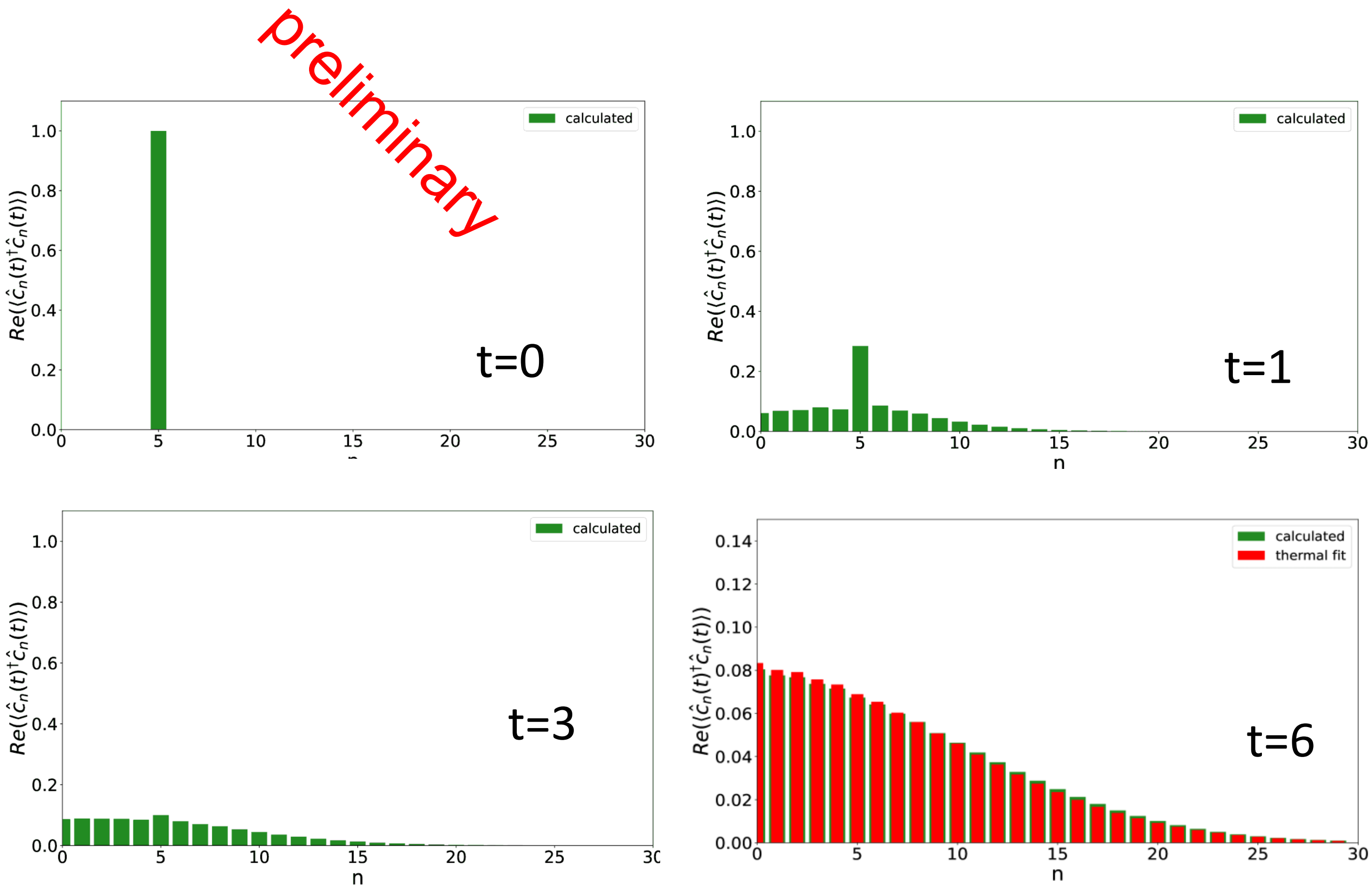
Initial state: bound state fully occupied



Thermalistion with the (bosonic) bath degrees of freedom

Kadanoff-Baym solutions …

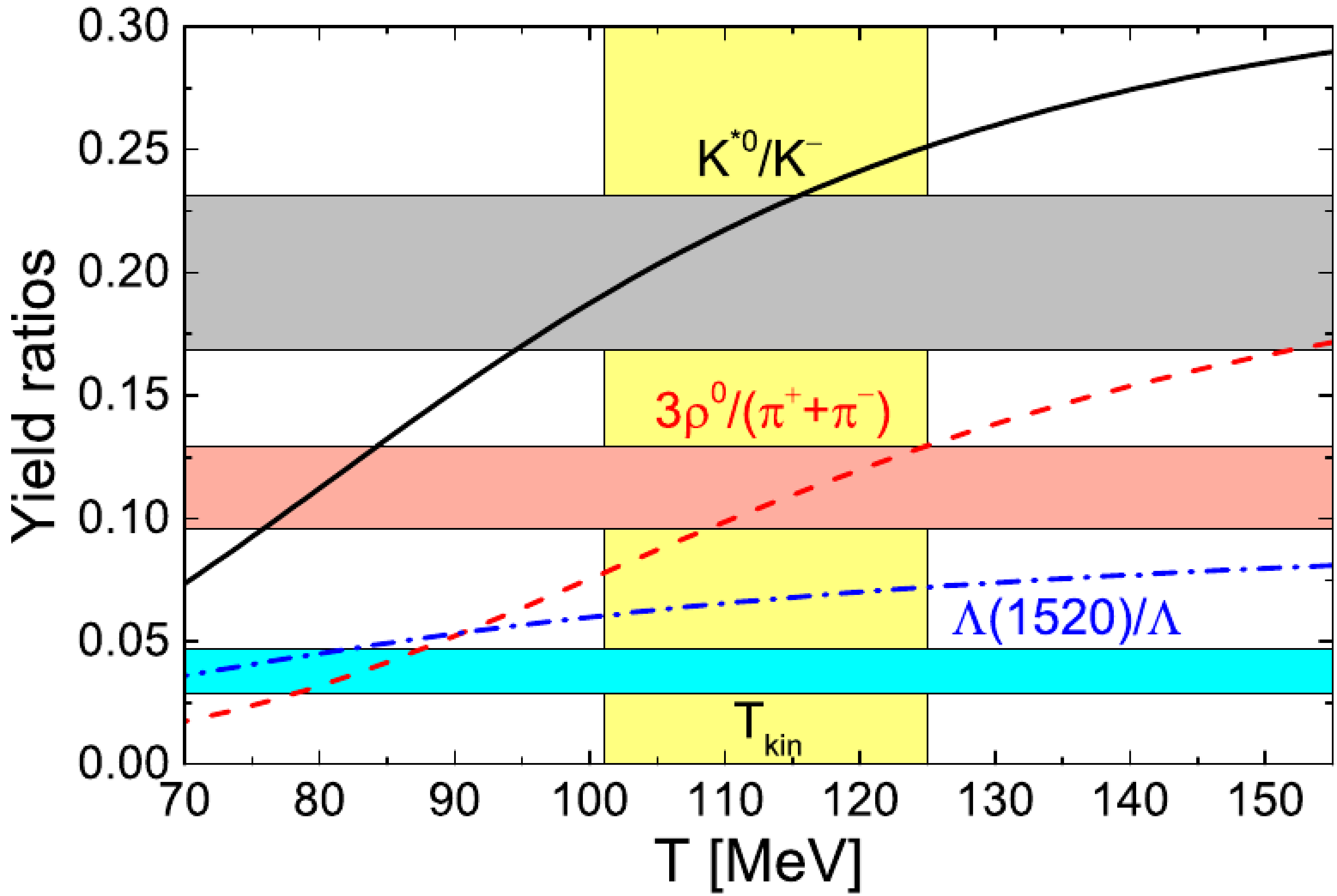
Initial state: single free quantum level fully occupied



Thermalistion with the (bosonic) bath degrees of freedom

Supplemental:

Full calculation: results for resonances



LHC nucleosythesis

PHYSICAL REVIEW C **99**, 044907 (2019)

Editors' Suggestion   Featured in Physics

Microscopic study of deuteron production in PbPb collisions at $\sqrt{s} = 2.76$ TeV via hydrodynamics and a hadronic afterburner

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[5] *GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany*

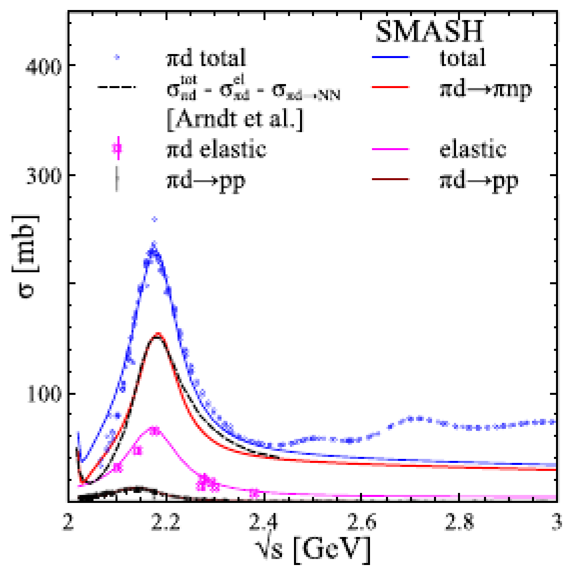

FIG. 1. Deuteron-pion interaction cross sections from SAID database [40] and partial wave analysis [41] are compared to our parametrizations (Tables II and III in the Appendix). Inelastic $d\pi \leftrightarrow$

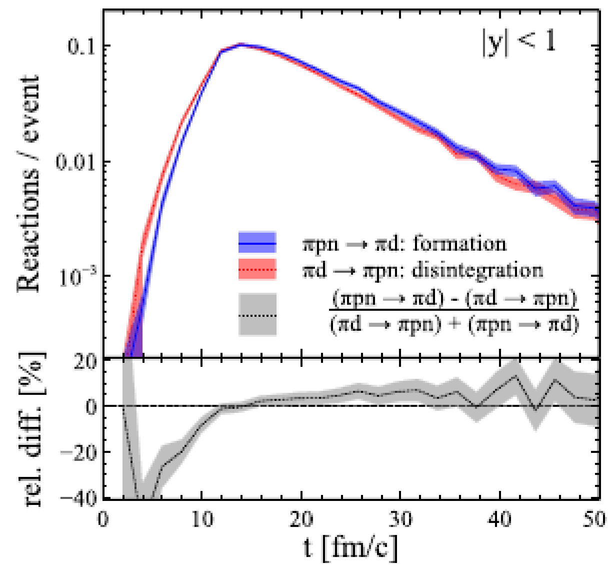

FIG. 5. Reaction rates of the most important $\pi d \leftrightarrow \pi pn$ reaction in forward and reverse direction.

law of mass action at work

some details …

$$\frac{\mathrm{d}N_\pi}{\mathrm{d}t} = \tilde{\alpha}_{\Delta \rightleftharpoons N\pi}\,(N_\Delta - c_\Delta^{N\pi}\,N_N N_\pi)$$
$$+\,\tilde{\alpha}_{\overline{\Delta} \rightleftharpoons \overline{N}\pi}\,(N_{\overline{\Delta}} - c_{\overline{\Delta}}^{\overline{N}\pi}\,N_{\overline{N}} N_\pi)$$
$$+\,2\tilde{\alpha}_{\rho \rightleftharpoons 2\pi}\,(N_\rho - c_\rho^{\pi^2}\,N_\pi^2)$$
$$+\,3\tilde{\alpha}_{\omega \rightleftharpoons 3\pi}\,(N_\omega - c_\omega^{\pi^3}\,N_\pi^3)\;.$$

$$\frac{\mathrm{d}N_N}{\mathrm{d}t} = \tilde{\alpha}_{\Delta \rightleftharpoons N+\pi}\,(N_\Delta - c_\Delta^{N\pi}\,N_N N_\pi) + \sum_{X=\mathrm{d,t,He^3,He^4}} A_X$$
$$\times \sum_{x=\pi,K,\overline{K}} \tilde{\alpha}_{X+x \rightleftharpoons A_X+x} N_x (N_X - c_X^{N^{A_X}}\,N_N^{A_X})$$
$$+ \sum_{x=\pi,K,\overline{K}} 2\tilde{\alpha}_{\mathrm{H}^3_\Lambda+x \rightleftharpoons NN\Lambda+x} N_x (N_{\mathrm{H}^3_\Lambda} - c_{\mathrm{H}^3_\Lambda}^{N^2\Lambda}\,N_N^2 N_\Lambda).$$

$$\frac{1}{\tau_A^{eq}} = \sum_{x=\pi,K,\overline{K}} \tilde{\alpha}_{\mathrm{A}+x \rightleftharpoons aN+x} N_x = \alpha_A$$

primordial nucleosynthesis: network

Deuterium

$$p(n,\gamma)\mathrm{D}$$

$$p + n \to \mathrm{D} + \gamma$$
$$\mathrm{D} + \gamma \to p + n$$

Helium

$$\mathrm{D}(\mathrm{D},\gamma)^4\mathrm{He} \qquad {}^3\mathrm{He}(n,\gamma)^4\mathrm{He}$$

$$\mathrm{T}(p,\gamma)^4\mathrm{He} \qquad {}^3\mathrm{He}(\mathrm{D},p)^4\mathrm{He}$$

$$\mathrm{T}(\mathrm{D},n)^4\mathrm{He} \qquad {}^3\mathrm{He}({}^3\mathrm{He},2p)^4\mathrm{He}$$

here now:

$$\pi + A \leftrightarrow \pi + A' + m_1 p + n_1 n$$

e.g.

$$\pi + \mathrm{D} \leftrightarrow \pi + p + n$$