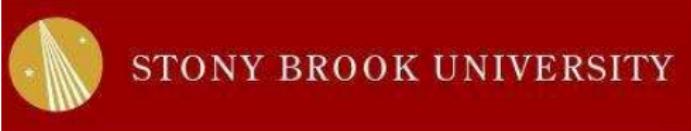


# Tight Symmetry Energy Parameter Constraints from Neutron Skin Measurements

J. M. Lattimer

Department of Physics & Astronomy



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## Recent Collaborators:

Duncan Brown & Soumi De (Syracuse), Christian Drischler, Madappa Prakash & Tianqi Zhao (Ohio), Sophia Han (TDLI), Evgeni Kolomeitsev (Matej Bei, Slovakia), Akira Ohnishi (YITP, Kyoto), Sanjay Reddy (INT), Achim Schwenk (Darmstadt), Andrew Steiner (Tennessee) & Ingo Tews (LANL)

# Nuclear Symmetry Energy and Pressure

The symmetry energy is the difference between the energies of pure neutron matter ( $x = 0$ ) and symmetric ( $x = 1/2$ ) nuclear matter:

$$S(n) = E(n, x=0) - E(n, x=1/2)$$

Usually approximated as an expansion around the saturation density ( $n_s$ ) and isospin symmetry ( $x = 1/2$ ):

$$E(n, x) = E(n, 1/2) + (1-2x)^2 S_2(n) + \dots$$

$$S_2(n) = S_v + \frac{L}{3} \frac{n - n_s}{n_s} + \dots$$

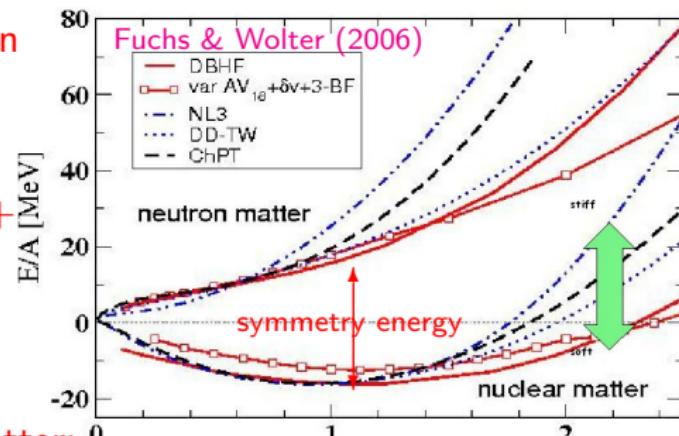
$$S_v \simeq 31 \text{ MeV}, \quad L \simeq 50 \text{ MeV}$$

Extrapolated to pure neutron matter:

$$E_N = E(n_s, 0) \approx S_v + E(n_s, 1/2) \equiv S_v - B, \quad P_N = P(n_s, 0) = L n_s / 3$$

Neutron star matter (beta equilibrium) is nearly neutron matter:

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad P_{NSM}(n_s) \simeq \frac{L n_s}{3} \left[ 1 - \left( \frac{4 S_v}{\hbar c} \right)^3 \frac{4 - 3 S_v / L}{3 \pi^2 n_s} \right]$$

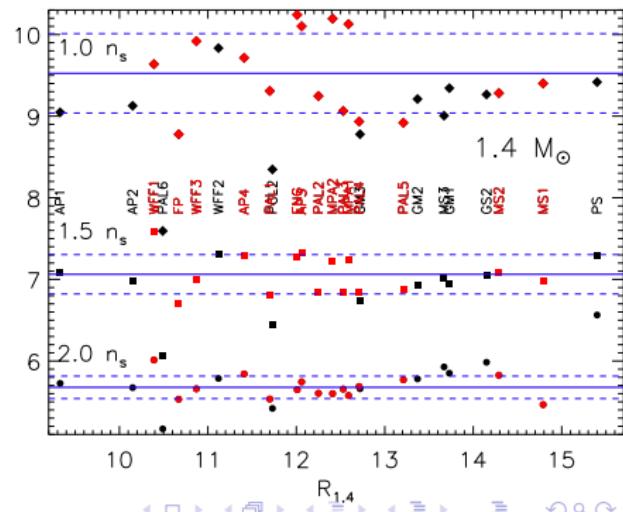
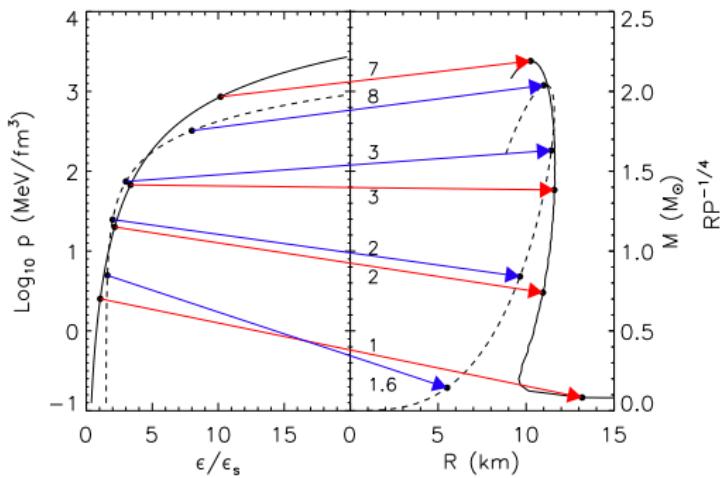


# Why is the Symmetry Energy Important?

The equation of state in a neutron star depends strongly on the density dependence of the symmetry energy ( $u = n_B/n_s$ ):

$$P_{NSM}(u) \simeq n_s u^2 \left[ \frac{L}{3} + \frac{K_N}{9}(u-1) + \frac{Q_N}{54}(u-1)^2 + \dots \right].$$

A strong correlation exists between radii and  $P_{NSM}$  near  $n_s$ :  
 $R_{1.4} \sim P_{NSM}(n_B)^{1/4}$ .



# Symmetry Parameter Correlation from Masses

Liquid drop model approximately valid

$$E_{\text{sym}}(N, Z) = (S_V A - S_s A^{2/3}) I^2$$

$$\chi^2 = \frac{1}{N\sigma_D^2} \sum_{i=1}^N (E_{\text{ex},i} - E_{\text{sym},i})^2$$

$$\chi_{vv} = \frac{2}{N\sigma_D^2} \sum_{i=1}^N I_i^4 A_i^2 \simeq 61.6 \sigma_D^{-2}$$

$$\chi_{ss} = \frac{2}{N\sigma_D^2} \sum_{i=1}^N I_i^4 A_i^{4/3} \simeq 1.87 \sigma_D^{-2}$$

$$\chi_{vs} = -\frac{2}{N\sigma_D^2} \sum_{i=1}^N I_i^4 A_i^{5/3} \simeq -10.7 \sigma_D^{-2}$$

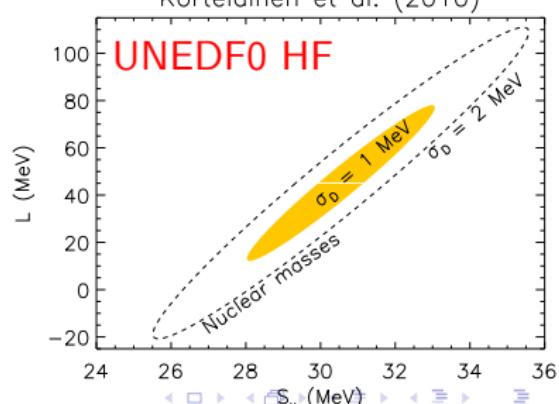
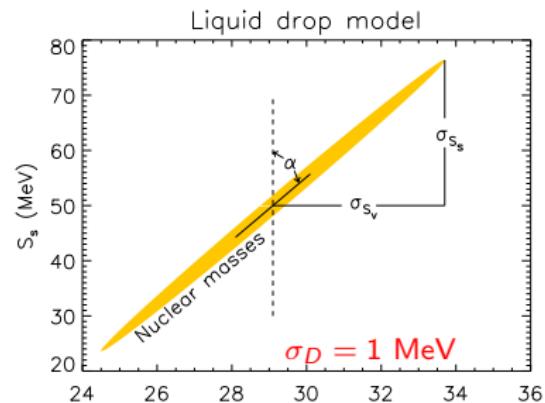
$$\sigma_{S_V} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 2.3 \sigma_D$$

$$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 13.2 \sigma_D$$

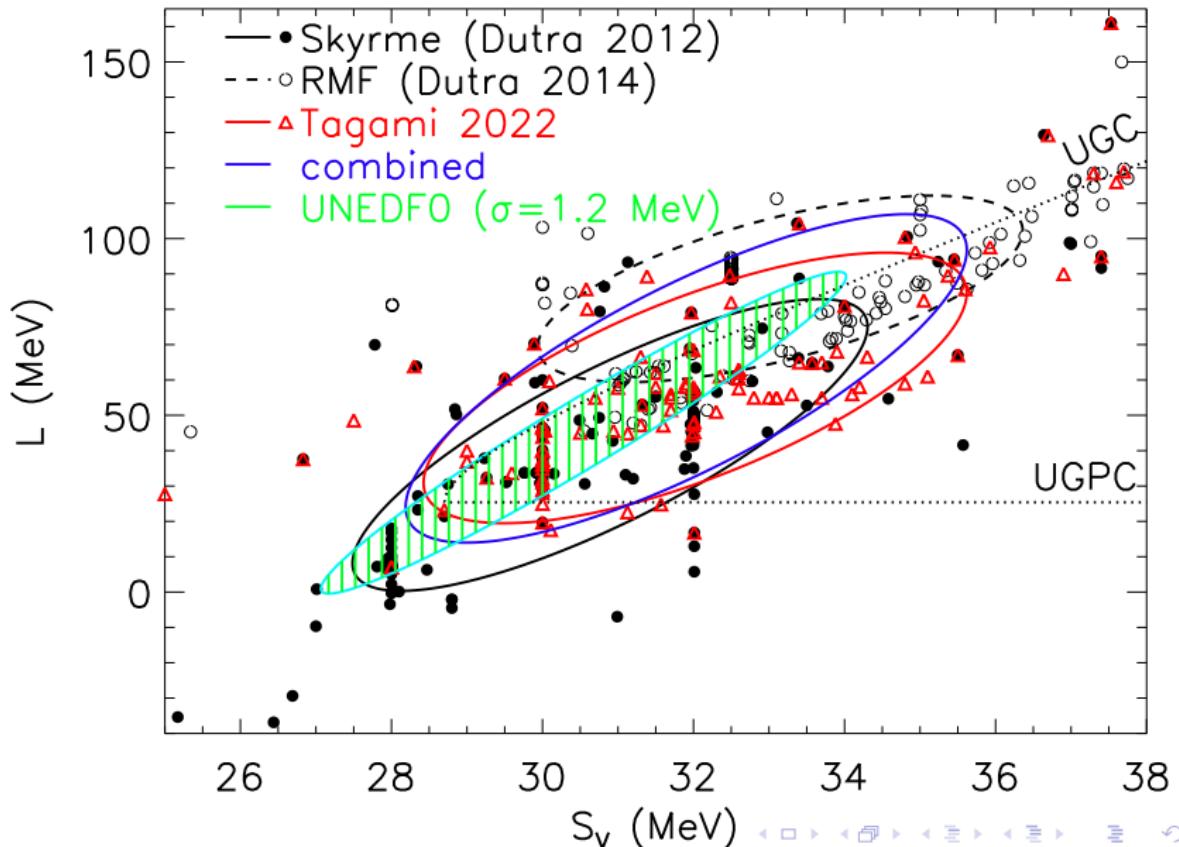
$$\alpha = \frac{1}{2} \tan^{-1} \frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}} \simeq 9.8^\circ$$

$$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}} \simeq 0.997$$

$S_s/S_V$  is linearly correlated with  $L$ .



# Fitting Nuclear Binding Energies



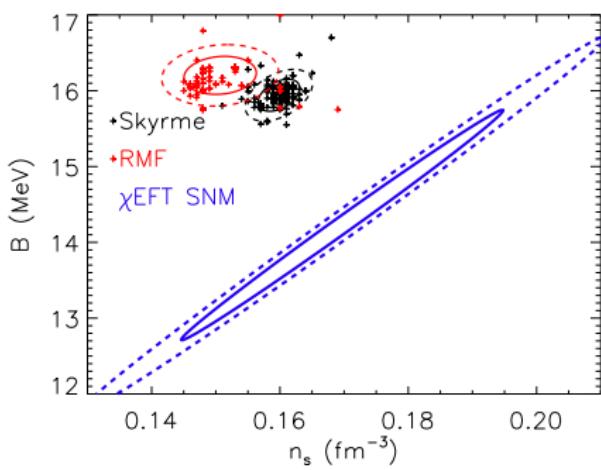
# Saturation Properties of Nuclear Interactions

## Empirical Saturation Window

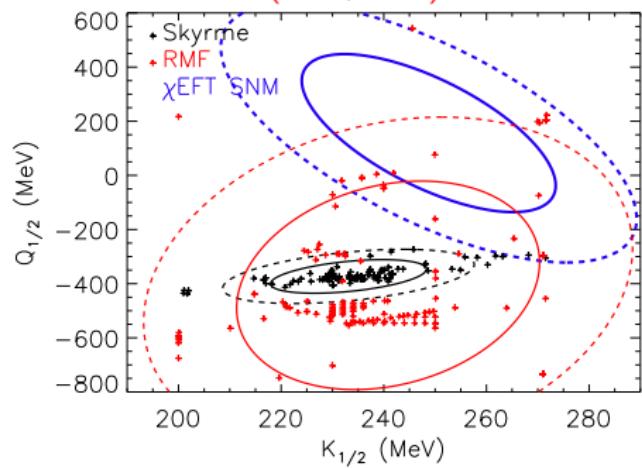
$$B = 16.06 \pm 0.20 \text{ MeV}$$

$$n_s = 0.1558 \pm 0.0054 \text{ fm}^{-3}$$

$$K_{1/2} = 236.5 \pm 15.4 \text{ MeV}$$

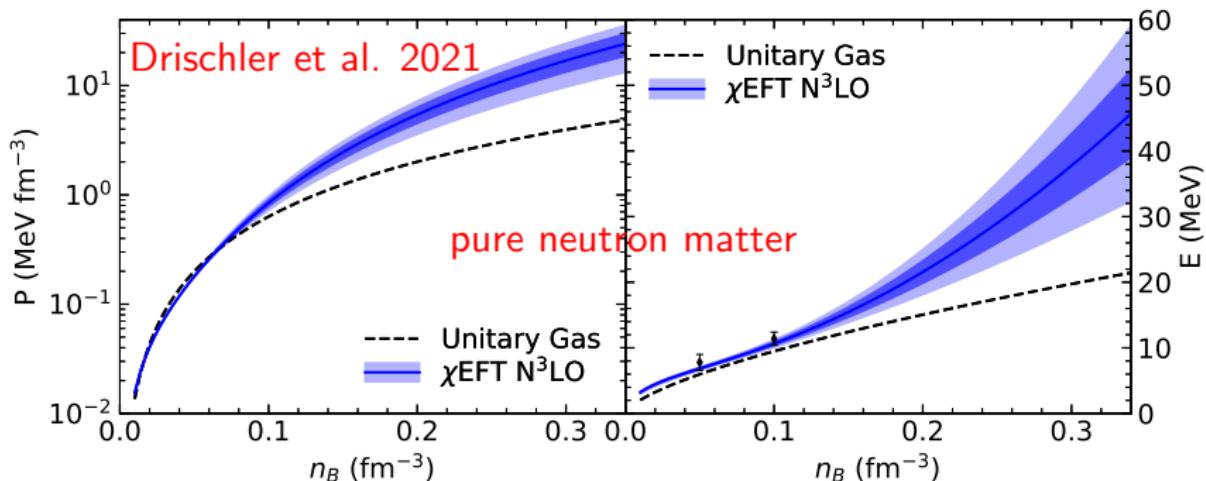


Data from Dutra (2012, 2014)



# Theoretical Neutron Matter Studies

Recently developed chiral effective field theory allows a systematic expansion of nuclear forces at low energies based on the symmetries of quantum chromodynamics. It exploits the gap between the pion mass (the pseudo-Goldstone boson of chiral symmetry-breaking) and the energy scale of short-range nuclear interactions established from experimental phase shifts. It provides the only known consistent framework for estimating energy uncertainties.

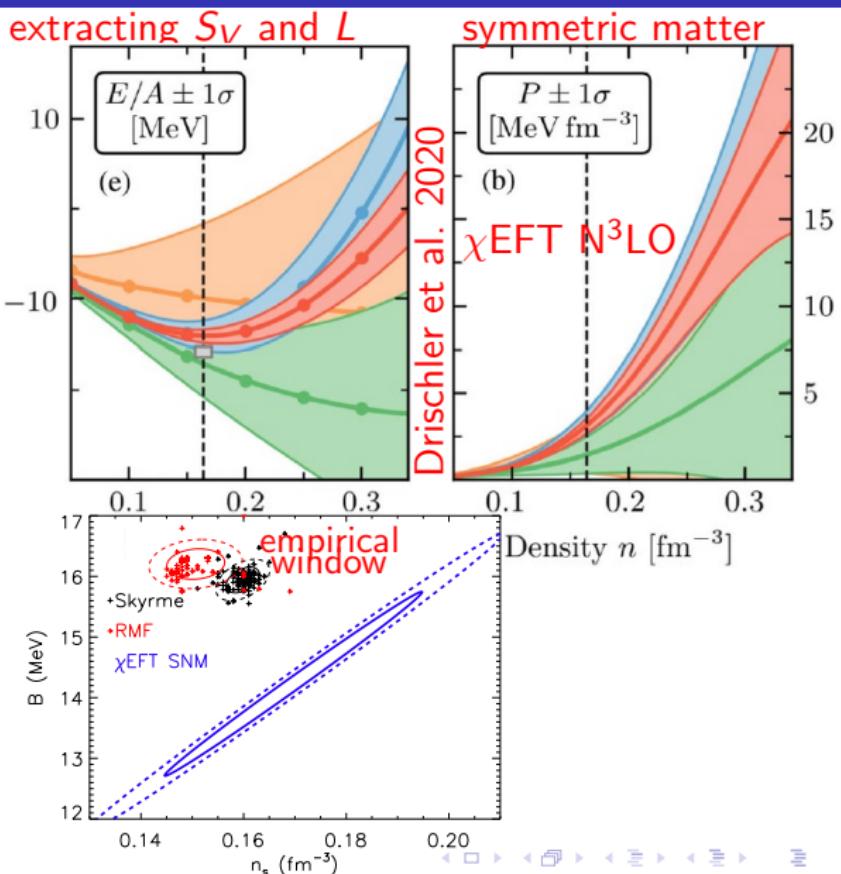


# Symmetry Parameters From Chiral EFT

Two approaches to extracting  $S_V$  and  $L$

1. Take the difference between pure neutron and symmetric matter energies and pressures at the calculated saturation density.
2. Use pure neutron matter energy and pressure with the empirical saturation window from nuclear mass fits.

$$S_V = E_N(n_s) + B,$$
$$L = 3P_N(n_s)/n_s.$$



# Symmetry Parameters From Neutron Matter

Pure neutron matter calculations are more reliable than symmetric matter calculations.

Symmetric matter emerges from a delicate cancellation sensitive to short- and intermediate-range three-body interactions at  $N^2LO$  that are Pauli-blocked in pure neutron matter.

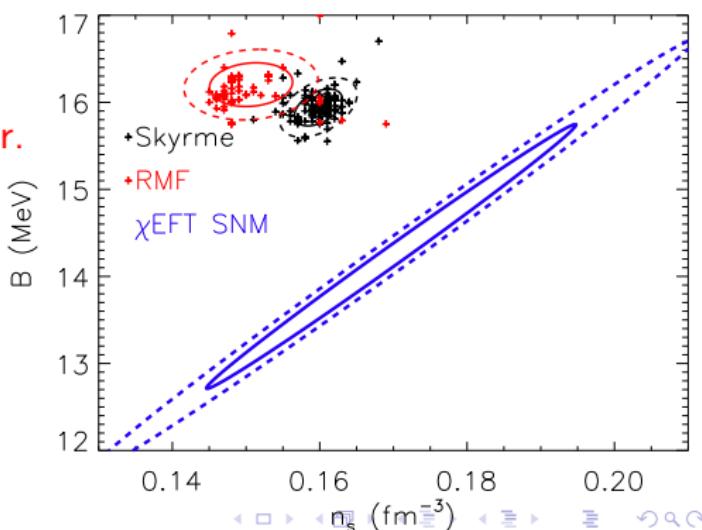
$N^3LO$  symmetric matter calculations don't saturate within empirical ranges for  $n_s$  and  $B$ , and introduce spurious correlations in symmetric matter.

We infer symmetry parameters from  $E_N(n_s)$  and  $P_N(n_s)$  using

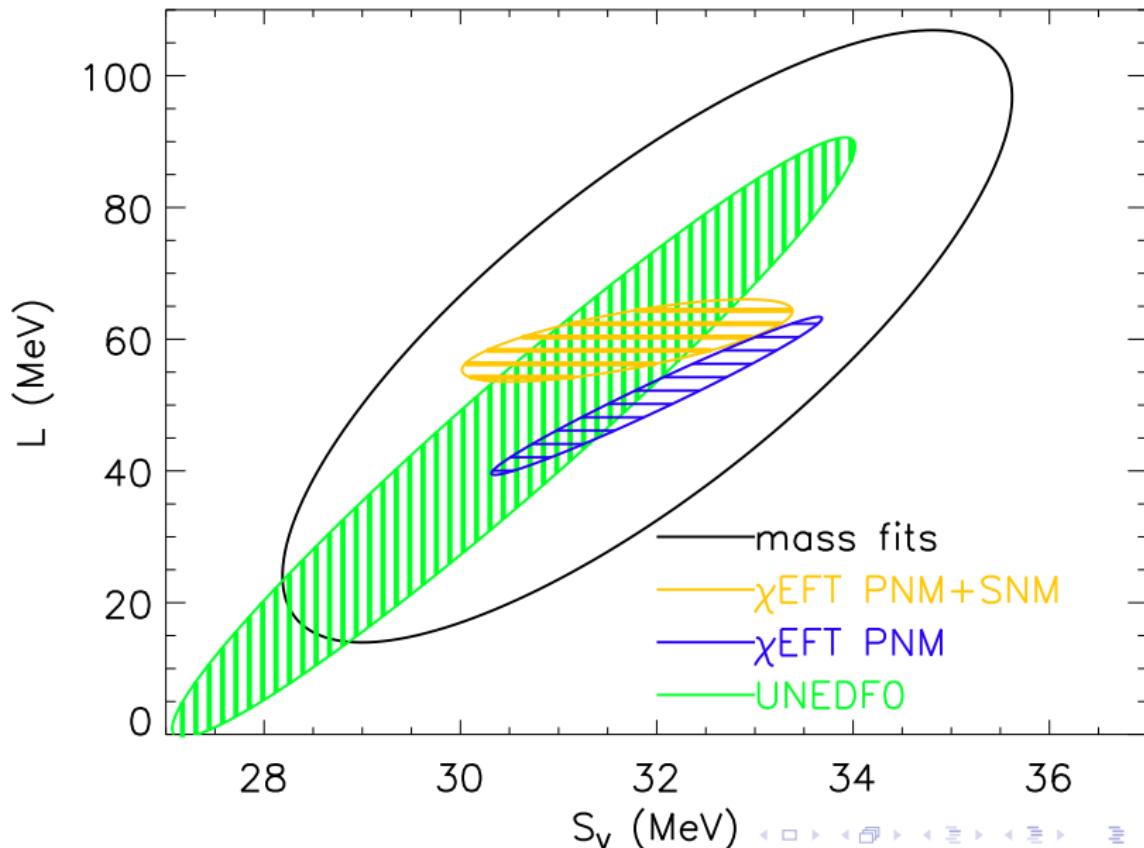
$$S_V = E_N(n_s) + B$$

$$L = 3P_N(n_s)/n_s$$

and include uncertainties in  $E_N$ ,  $P_N$ ,  $n_s$  and  $B$ .



# Correlations From Chiral EFT



# Bounds From The Unitary Gas Conjecture

## The Conjecture (UGC):

Neutron matter energy always larger than unitary gas energy.

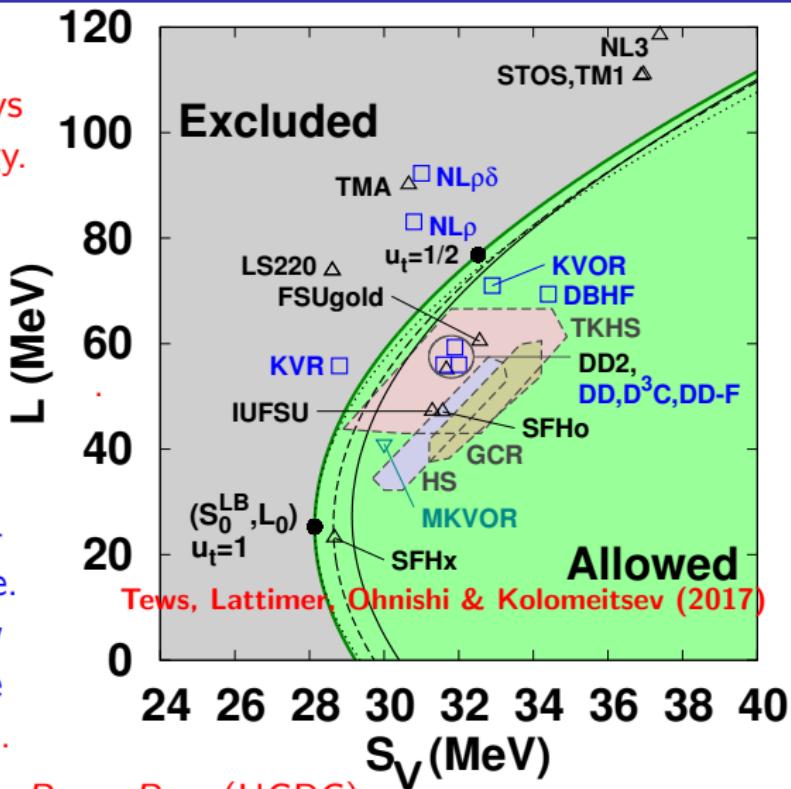
$$E_{UG} = \xi_0(3/5)E_F, \text{ or}$$

$$E_{UG} \simeq 12.6 \left( \frac{n}{n_s} \right)^{2/3} \text{ MeV.}$$

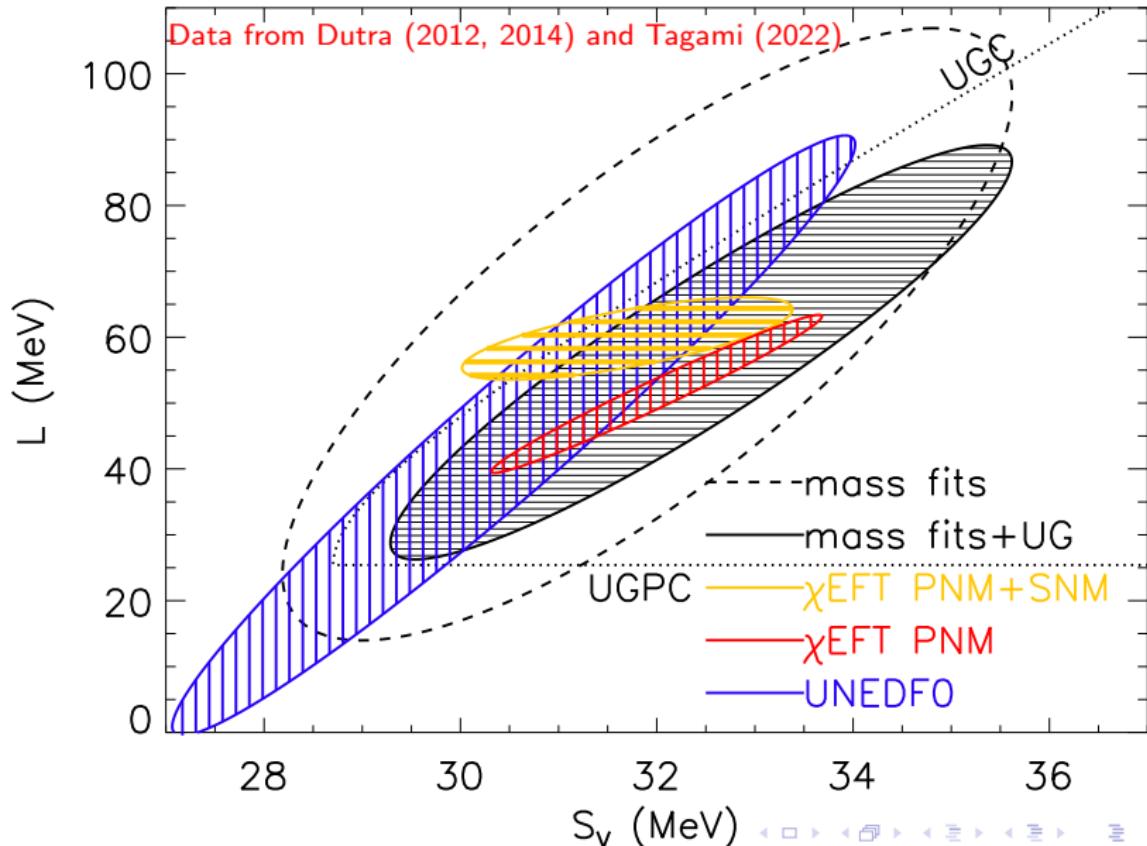
The unitary gas consists of fermions interacting via a pairwise short-range s-wave interaction with infinite scattering length and zero range. Cold atom experiments show a universal behavior with the Bertsch parameter  $\xi_0 \simeq 0.37$ .

For  $n \geq n_s$ , one also observes  $P_N > P_{UG}$  (UGPC).

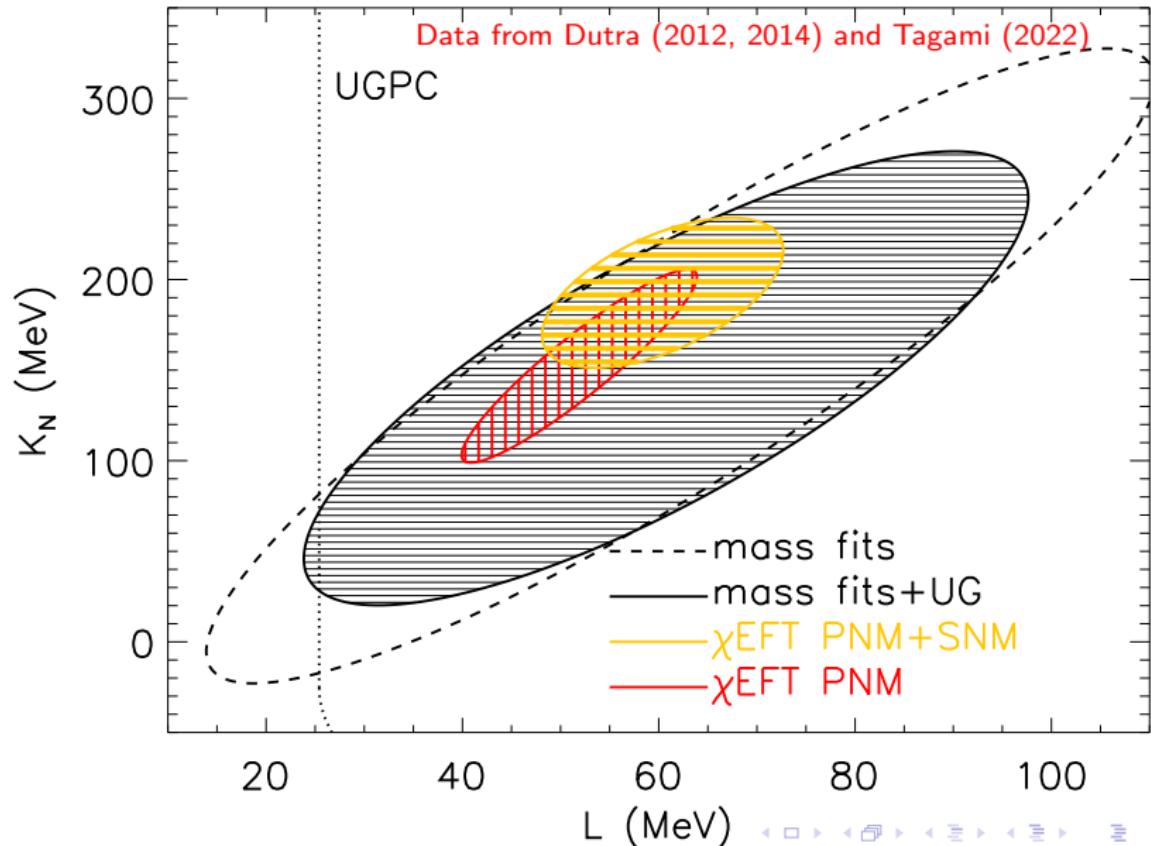
$S_v \geq 28.6 \text{ MeV}; L \geq 25.3 \text{ MeV}; P_N(n_s) \geq 1.35 \text{ MeV fm}^{-3}; R_{1.4} \geq 9.7 \text{ km}$



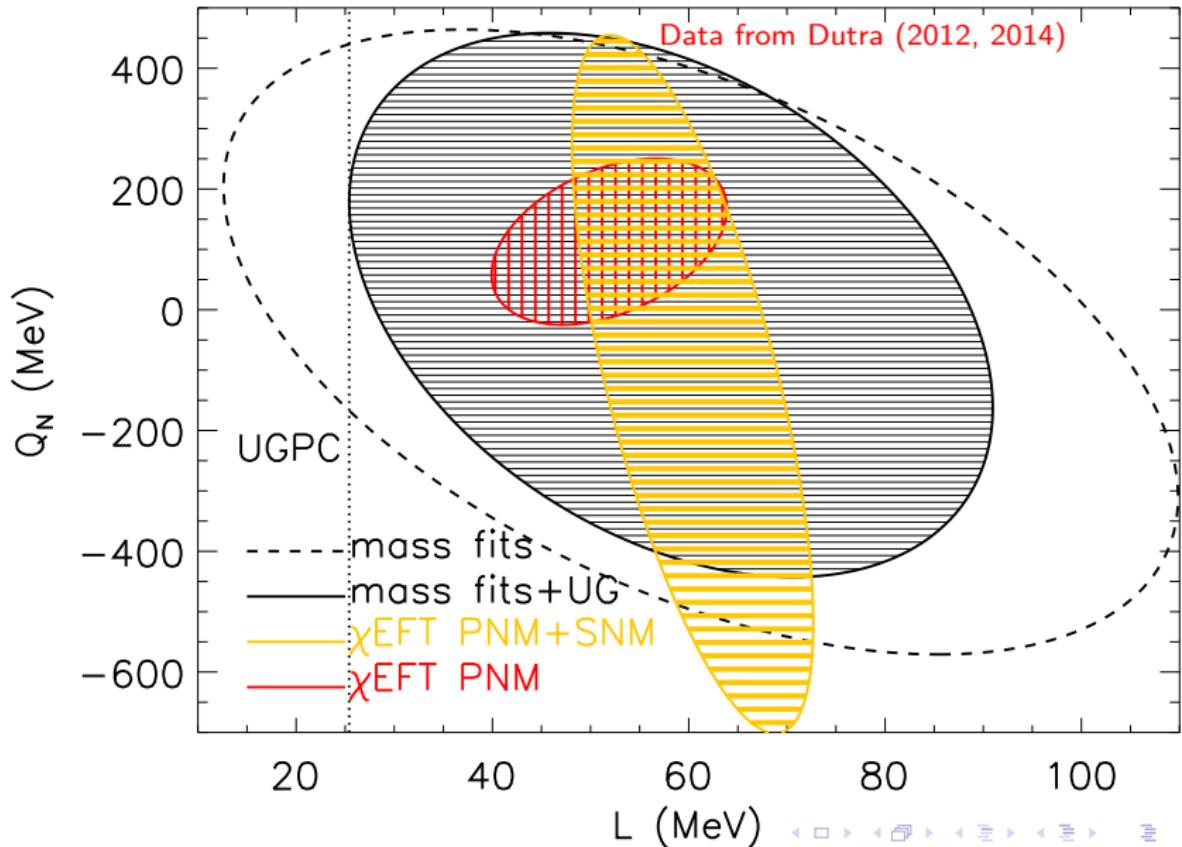
# Applying Unitary Gas Constraints



# $K_N - L$ Correlations



# $Q_N - L$ Correlations



# Neutron Skin Thickness

The difference between the mean neutron and proton radii in the liquid droplet model is

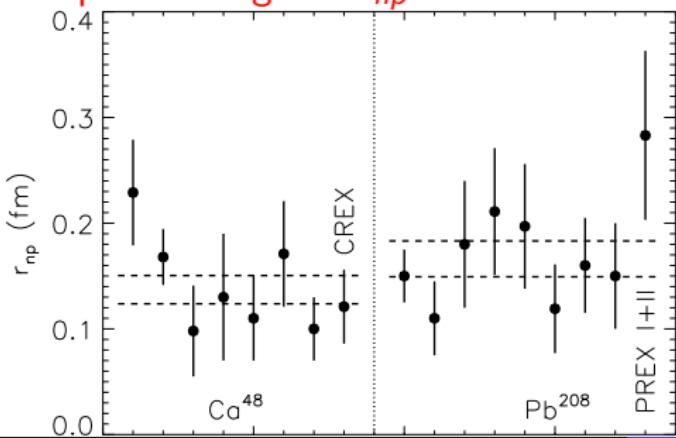
$$t_{np} = R_n - R_p.$$

The mean square difference is

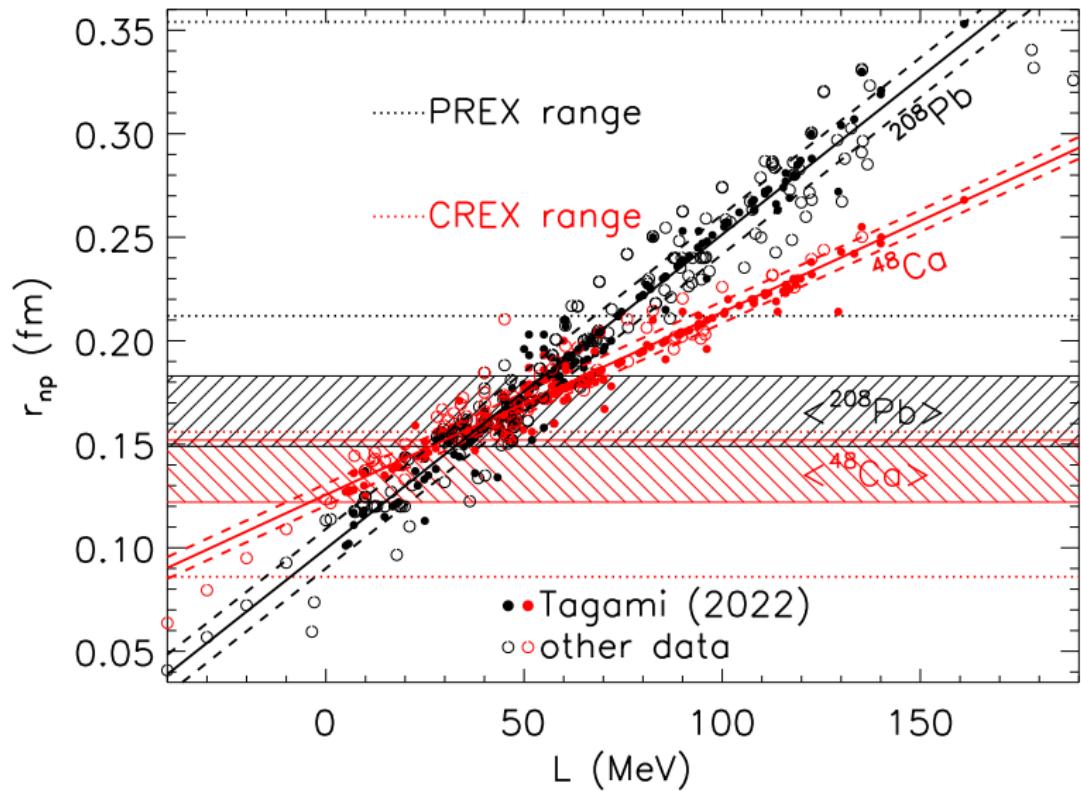
$$r_{np}^2 = \langle R_n \rangle^2 - \langle R_p \rangle^2.$$

$$r_{np} = \frac{2r_o I}{3} \frac{S_s}{S_V} \left[ 1 + S_s A^{-1/3} / S_V \right]^{-1}$$

Implies strong  $L - r_{np}$  correlation.



# Calculated $L - r_{np}$ Correlations



# Implied $L$ Values

Historical experimental weighted average  $^{208}\text{Pb}$

$r_{np}^{208} = 0.166 \pm 0.017$  fm, implying  $L = 45 \pm 13$  MeV.

Historical experimental weighted average  $^{48}\text{Ca}$

$r_{np}^{48} = 0.137 \pm 0.015$  fm, implying  $L = 14 \pm 21$  MeV.

Combined  $L = 36 \pm 11$  MeV.

Parity-violating electron scattering measurements at JLab:

PREX I+II  $^{208}\text{Pb}$  (Adhikari et al. 2021):

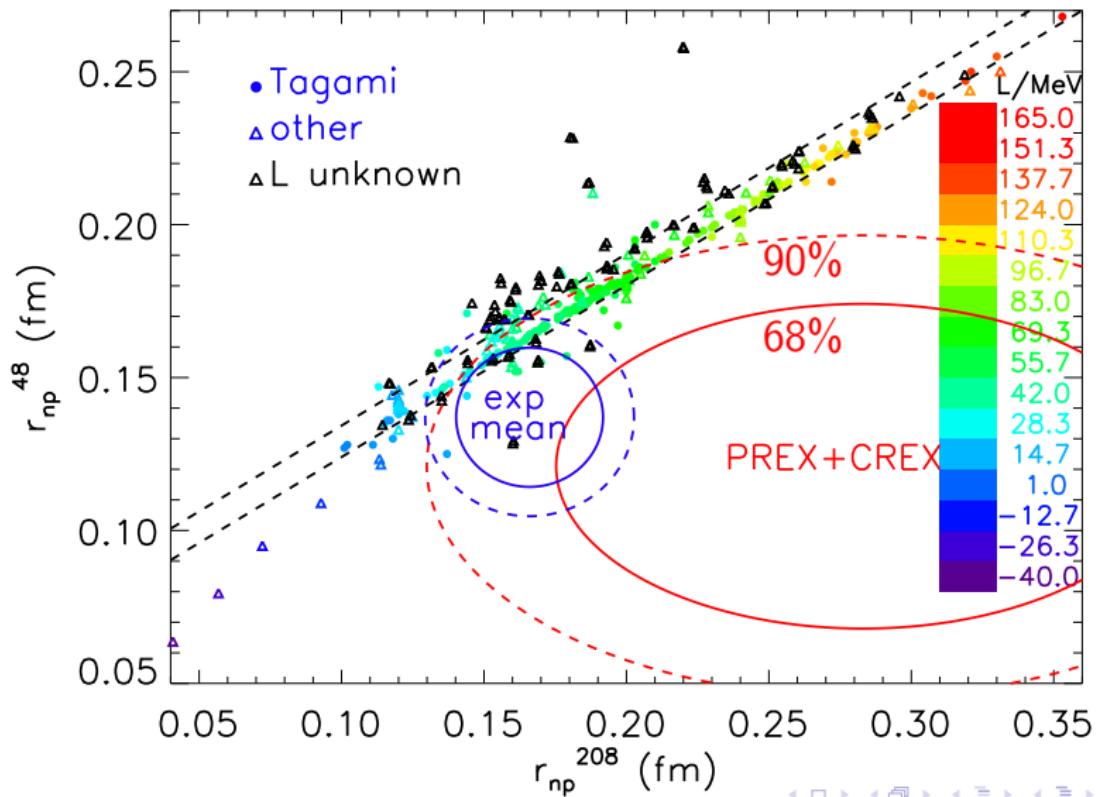
$r_{np}^{208} = 0.283 \pm 0.071$  fm, implying  $L = 119 \pm 46$  MeV.

CREX  $^{48}\text{Ca}$  (Adhikari et al. 2022):

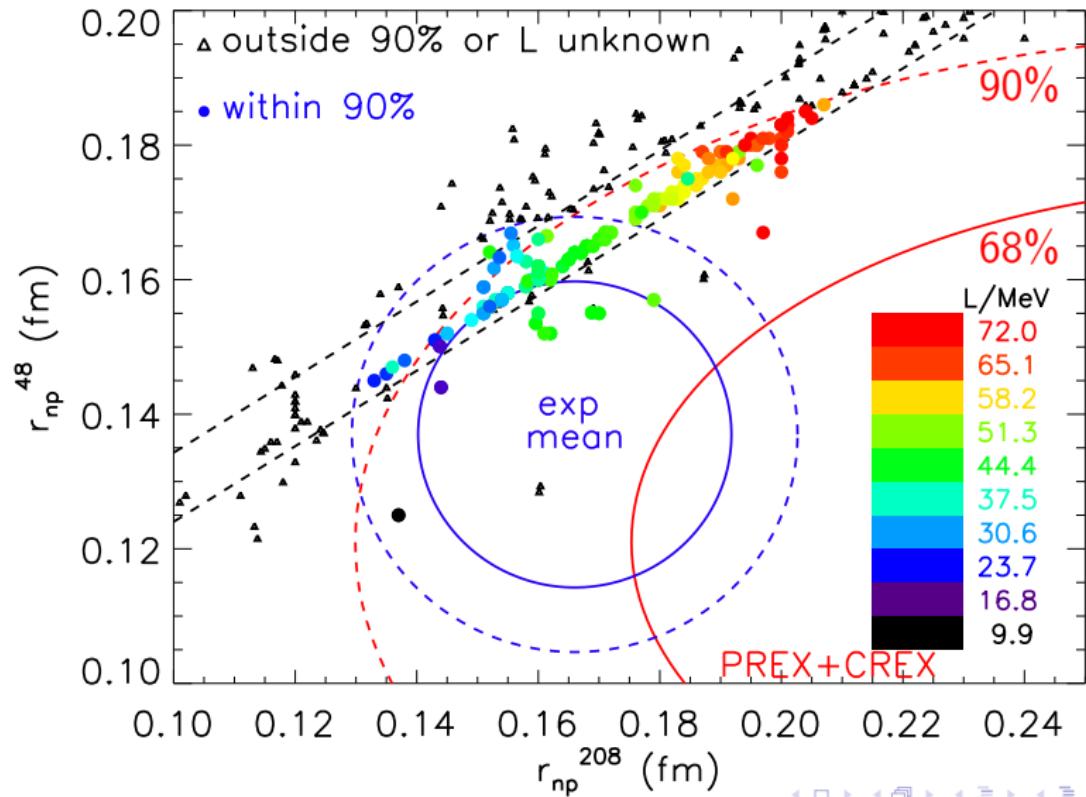
$r_{np}^{48} = 0.121 \pm 0.035$  fm, implying  $L = -5 \pm 42$  MeV.

Combined  $L = 51 \pm 31$  MeV.

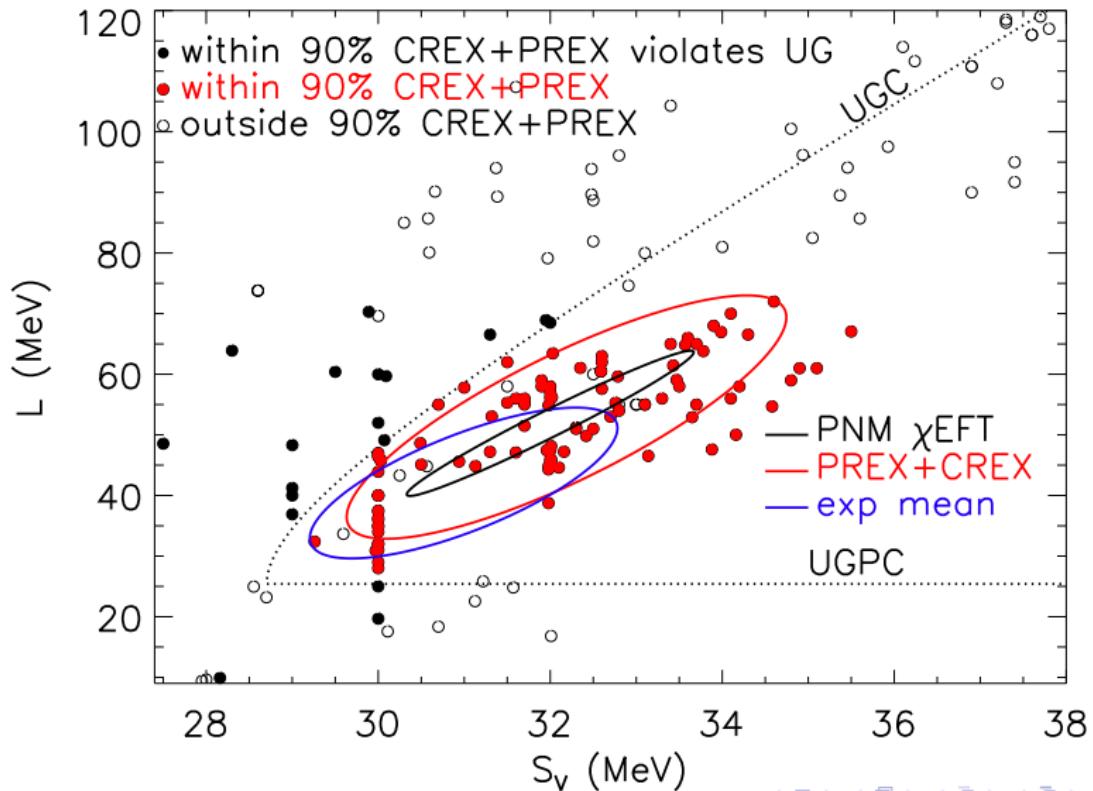
# $r_{np}^{208} - r_{np}^{48}$ Linear Correlation



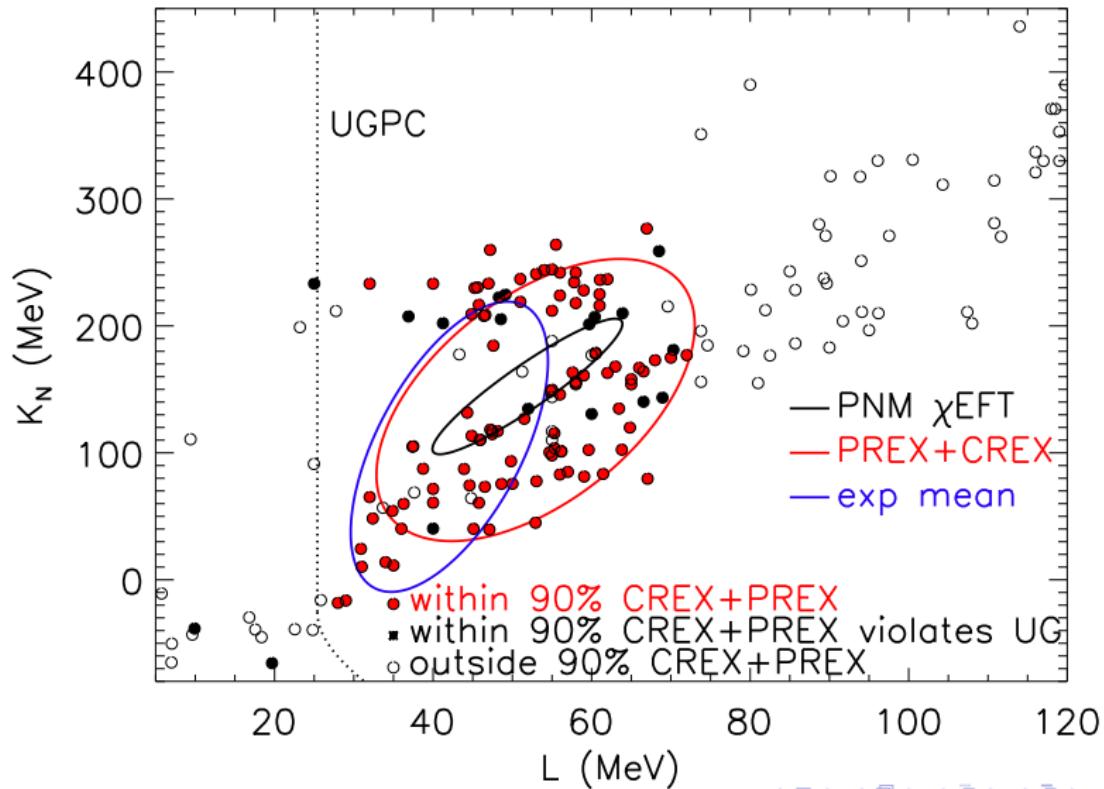
# Detail



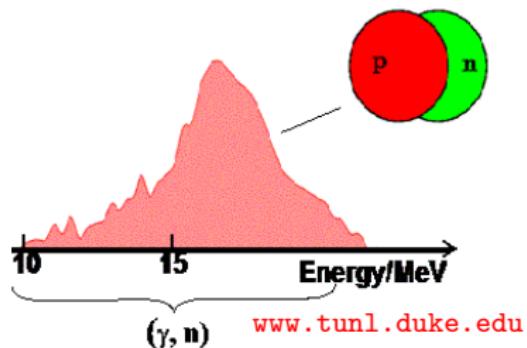
# Implied $S_V - L$



# Implied $K_N - L$



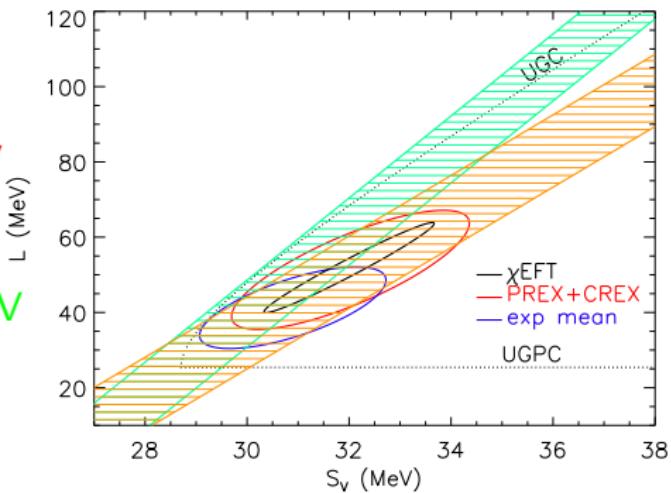
# Giant Dipole Resonances



$S(0.1 \text{ fm}^{-3}) = 24.1 \pm 0.8 \text{ MeV}$   
Trippa et al. (2008)

$S(0.05 \text{ fm}^{-3}) = 16.5 \pm 1.0 \text{ MeV}$   
Zhang et al. (2015)

$$E_{-1} \propto S_V \left( S_V + \frac{5S_s}{3A^{1/3}} \right)^{-1/2}$$



# Dipole Polarizability

The linear response, or dynamic polarizability, of a nucleus excited from its ground state to an excited state due to an external oscillating dipolar field. In the liquid droplet model

$$\alpha_D S_V = \frac{AR^2}{20} \left( 1 + \frac{5}{3} \frac{S_s}{S_v} A^{-1/3} \right)$$

Hashimoto (2015) Sn:  $\alpha_D^{120} = (8.59 \pm 0.37) \text{ fm}^3$

Tamii (2012) Pb:  $\alpha_D^{208} = (19.6 \pm 0.6) \text{ fm}^3$

Birkhan (2017) Ca:  $\alpha_D^{48} = (2.07 \pm 0.22) \text{ fm}^3$

Roca-Maza et al. (2015):

$$\begin{aligned}\alpha_D^{48} &= (0.10 \pm 0.01) \alpha_D^{208} + (0.36 \pm 0.07) \text{ fm}^3, \\ \alpha_D^{208} &= (2.2 \pm 0.1) \alpha_D^{120} + (0.1 \pm 0.5) \text{ fm}^3.\end{aligned}$$

Implies  $\alpha_D^{48} = 2.32 \pm 0.21 \text{ fm}^3$  and  $\alpha_D^{208} = 19.0 \pm 1.2 \text{ fm}^3$ ,  
consistent with measurements.

# Dipole Polarizability Predictions for Skin Thickness

Roca-Maza et al. (2015), Piekarewicz (2021):

$$\alpha_D^{48} S_V = (355 \pm 44) (r_{np}^{48}/\text{fm}) + (12 \pm 19) \text{ MeV fm}^3,$$

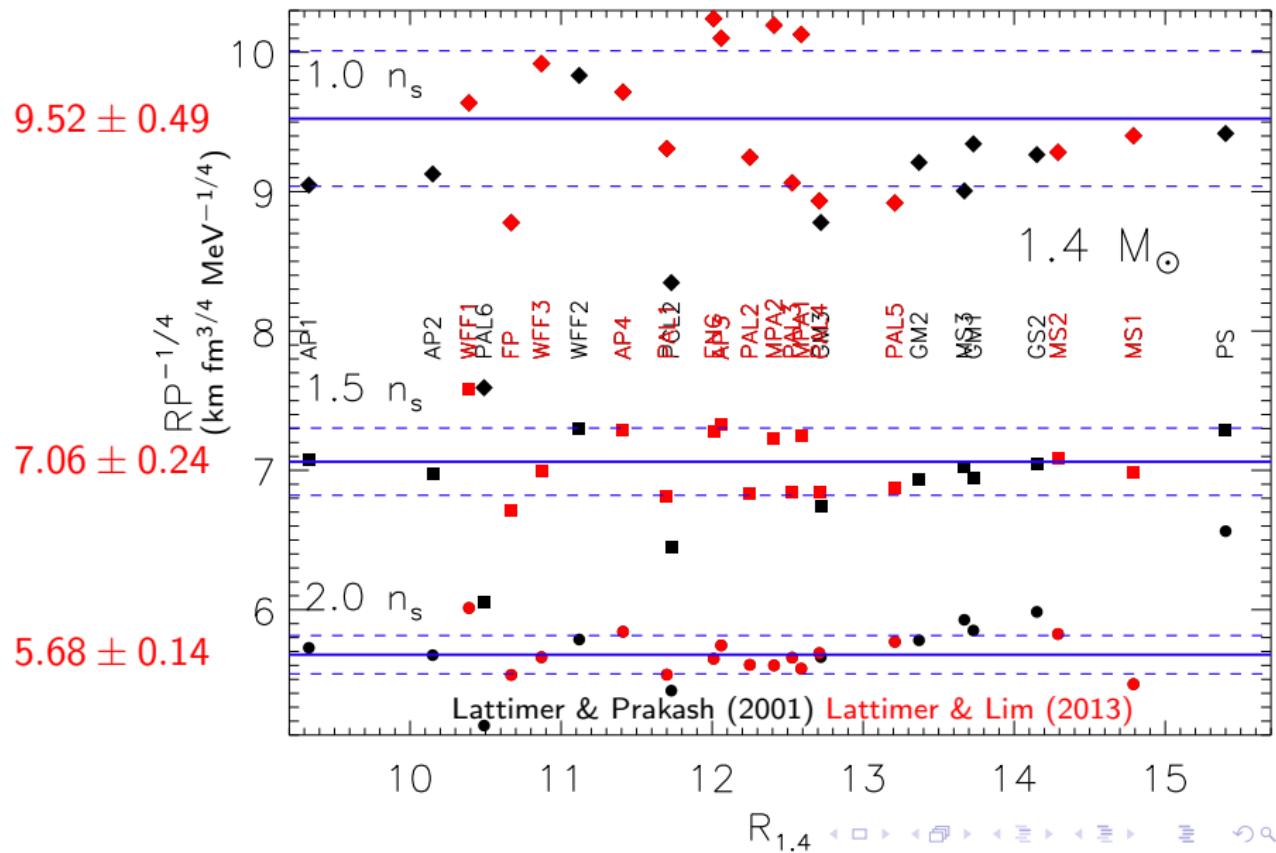
$$\alpha_D^{120} S_V = (1234 \pm 93) (r_{np}^{120}/\text{fm}) + (115 \pm 36) \text{ MeV fm}^3,$$

$$\alpha_D^{208} S_V = (1922 \pm 73) (r_{np}^{208}/\text{fm}) + (301 \pm 32) \text{ MeV fm}^3,$$

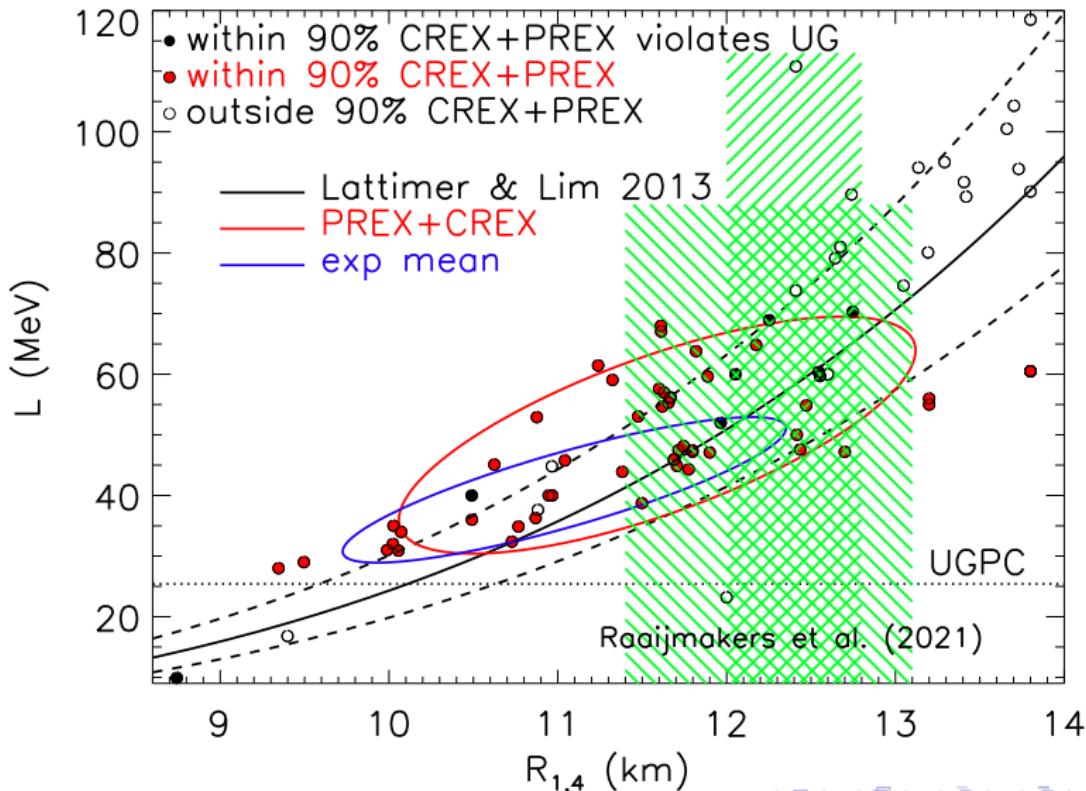
Measured  $\alpha_D^{48}$  and  $\alpha_D^{208}$  and the average experimental value for  $r_{np}^{48}$  predict  $r_{np}^{208} = (0.171 \pm 0.015) \text{ fm}$ , almost exactly the average experimental value but significantly smaller than the PREX I+II measurement.

Conversely, the predicted value for  $r_{np}^{48}$  is consistent with its average experimental value but slightly larger than the CREX measurement.

# The Radius – Pressure Correlation



# Implied $R_{1.4} - L$

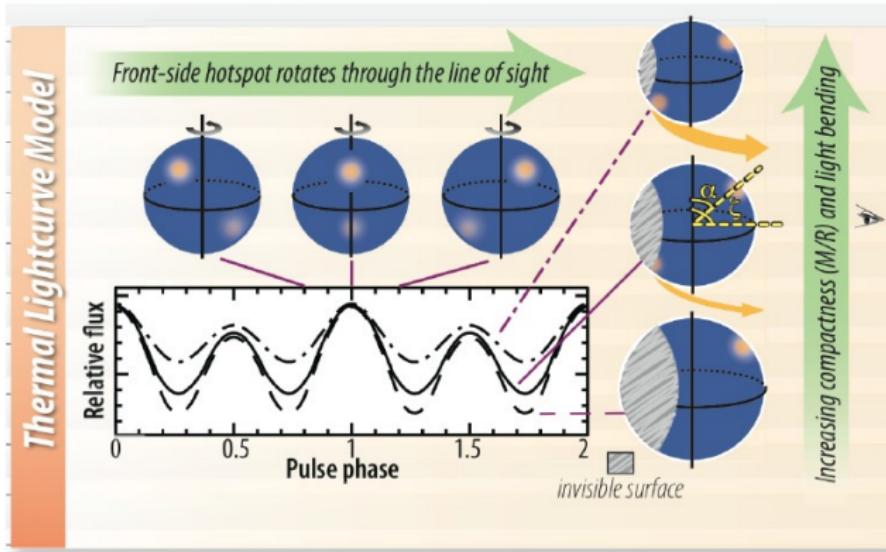


# Neutron Star Interior Composition ExploreR (NICER)

## Science Measurements

NICER

Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches

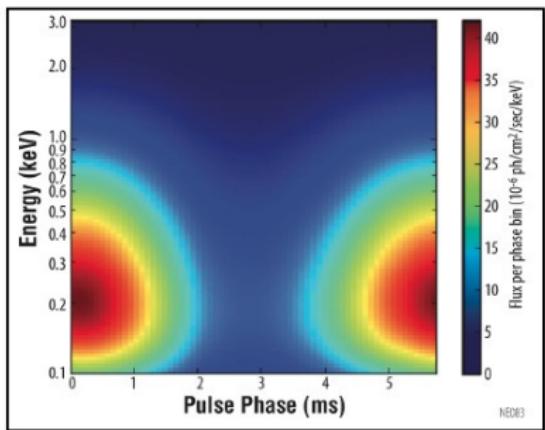


**Lightcurve modeling** constrains the compactness ( $M/R$ ) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending...

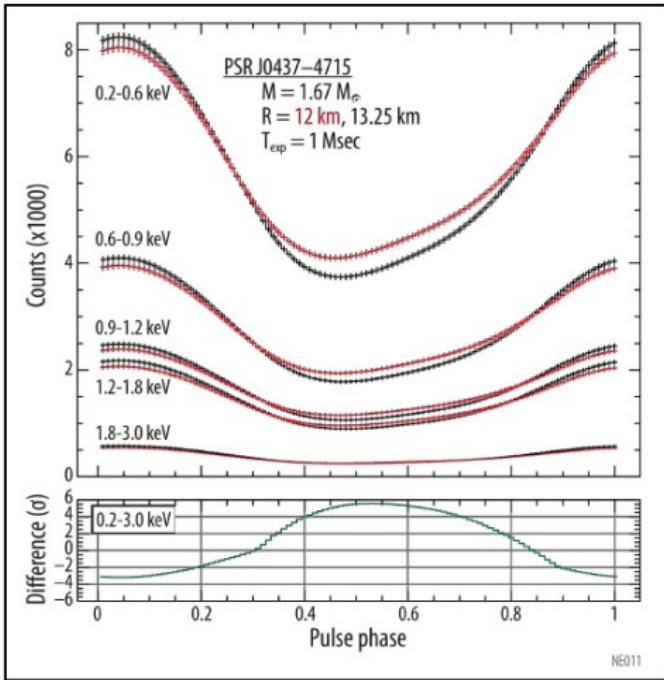


# Science Measurements (cont.)

NICER

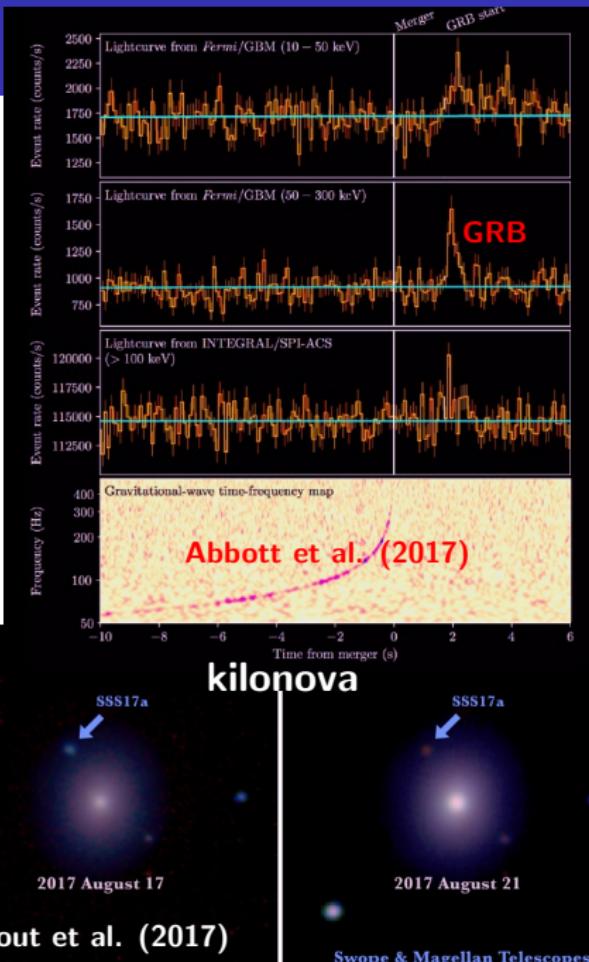


... while phase-resolved spectroscopy promises a direct constraint of radius  $R$ .



# GW170817

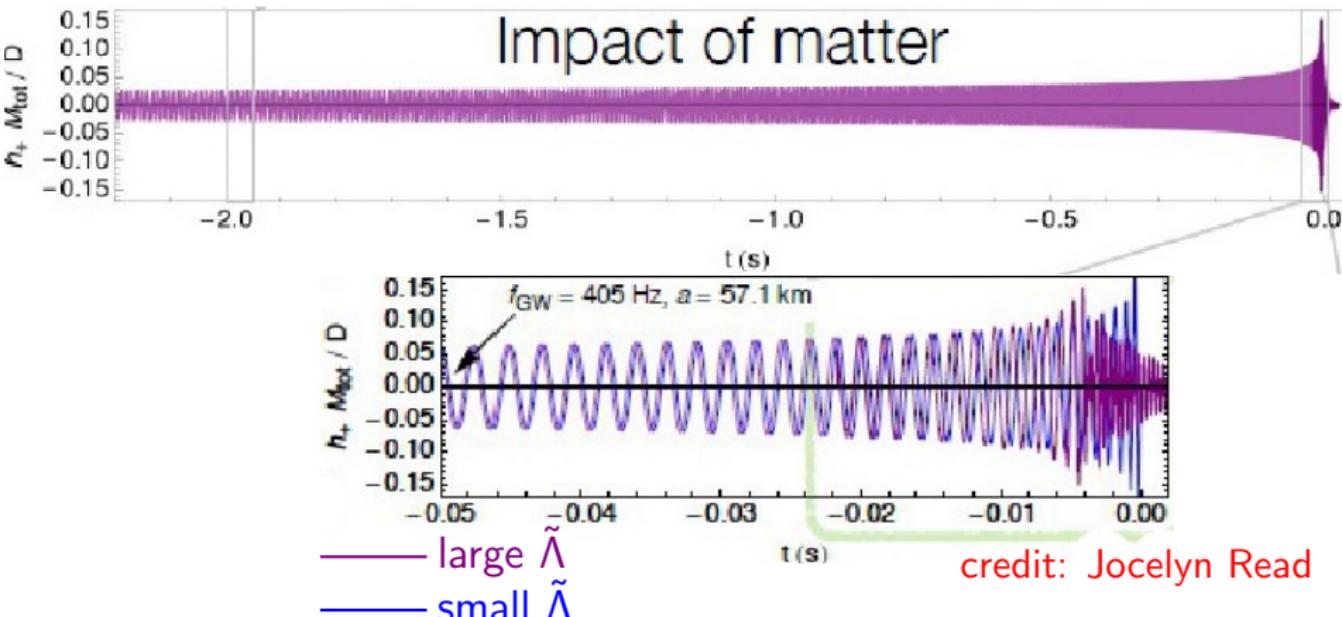
- ▶ LVC detected a signal consistent with a BNS merger, followed 1.7 s later by a weak gamma-ray burst.
- ▶  $\simeq 10100$  orbits observed over 317 s.
- ▶  $\mathcal{M} = 1.186 \pm 0.001 M_{\odot}$
- ▶  $M_{T,\min} = 2^{6/5}\mathcal{M} = 2.725 M_{\odot}$
- ▶  $E_{\text{GW}} > 0.025 M_{\odot}c^2$
- ▶  $D_L = 40^{+8}_{-14} \text{ Mpc}$
- ▶  $75 < \tilde{\Lambda} < 560$  (90%)
- ▶  $M_{\text{ejecta}} \sim 0.06 \pm 0.02 M_{\odot}$
- ▶ Blue ejecta:  $\sim 0.01 M_{\odot}$
- ▶ Red ejecta:  $\sim 0.05 M_{\odot}$
- ▶ Possible r-process production
- ▶ Ejecta + GRB:  $M_{\max} \lesssim 2.22 M_{\odot}$



# The Effect of Tides

Tides accelerate the inspiral and produce a phase shift compared to the case of two point masses.

## Impact of matter



$$\delta\Phi_t = -\frac{117}{256} \frac{(1+q)^4}{q^2} \left( \frac{\pi f_{\text{GW}} G M}{c^3} \right)^{5/3} \tilde{\Lambda} + \dots$$

# Tidal Deformability

The tidal deformability  $\lambda$  is the ratio of the induced dipole moment  $Q_{ij}$  to the external tidal field  $E_{ij}$ ,  $Q_{ij} \equiv -\lambda E_{ij}$ .

Use  $\beta = GM/Rc^2$  and

$$\Lambda = \frac{\lambda c^{10}}{G^4 M^5} \equiv \frac{2}{3} k_2 \beta^{-5}.$$

$k_2 \propto \beta^{-2}$  is

dimensionless Love

number, so  $\Lambda \simeq a \beta^{-6}$ .

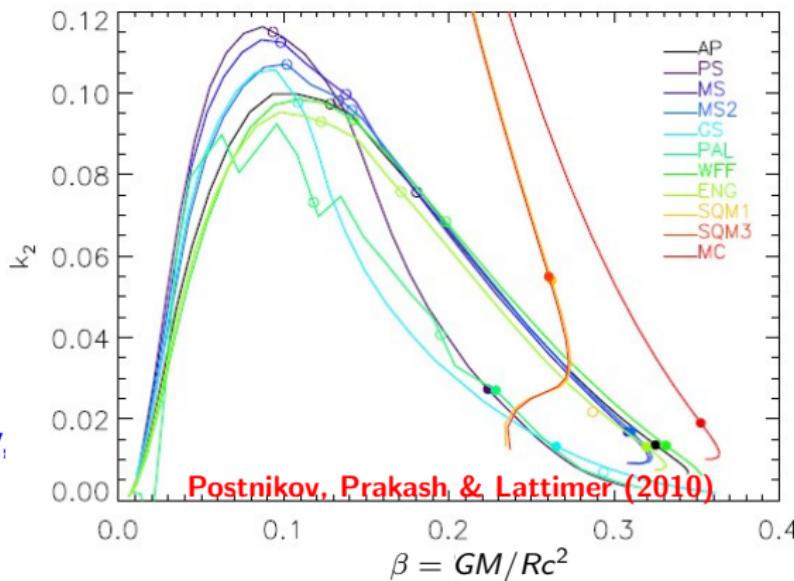
For  $1 < M/M_\odot < 1.6$ ,

$a = 0.0093 \pm 0.0007$ .

For a neutron star binary,  
the mass-weighted  $\tilde{\Lambda}$  is  
the relevant observable:

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5},$$

$$q = M_2/M_1 \leq 1$$



# Binary Deformability and the Radius

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1+12q)\Lambda_1 + q^4(12+q)\Lambda_2}{(1+q)^5} \simeq \frac{16a}{13} \left( \frac{R_{1.4}c^2}{GM} \right)^6 \frac{q^{8/5}(12-11q+12q^2)}{(1+q)^{26/5}}.$$

This is very insensitive to  $q$  for  $q > 0.5$ , so

$$\tilde{\Lambda} \simeq a' \left( \frac{R_{1.4}c}{GM} \right)^6.$$

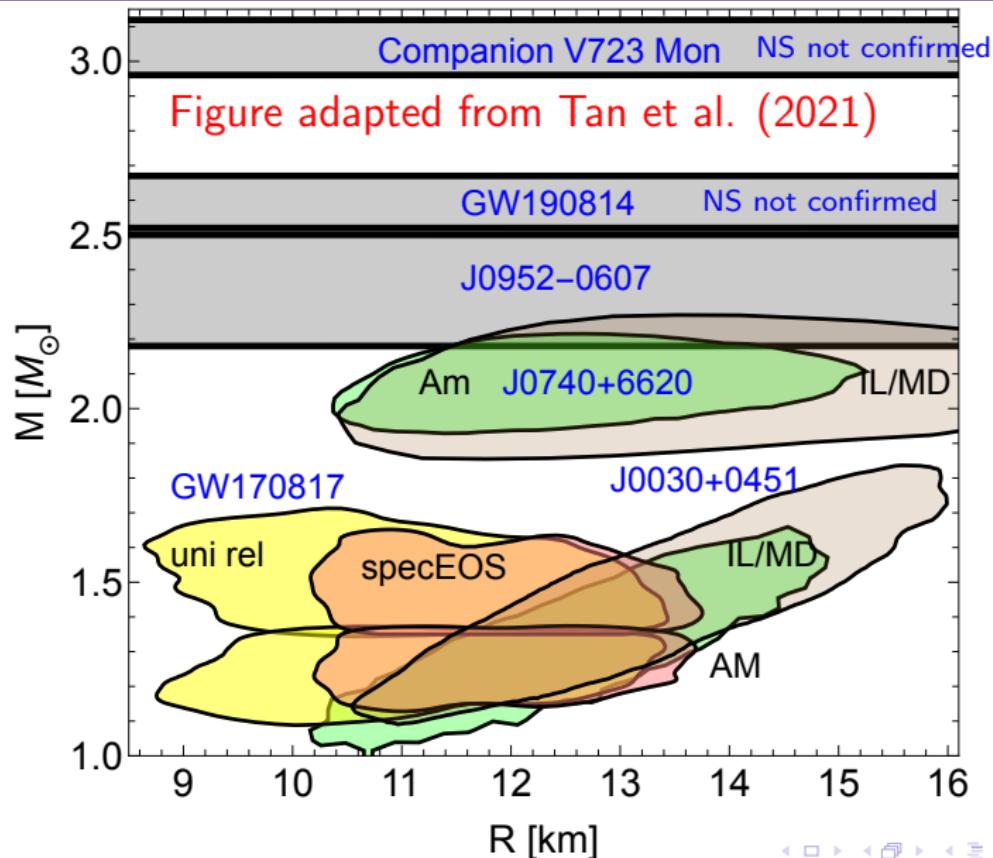
For  $M = (1.2 \pm 0.2) M_\odot$ ,  $a' = 0.0035 \pm 0.0006$ ,

$$R_{1.4} = (11.5 \pm 0.3) \frac{M}{M_\odot} \left( \frac{\tilde{\Lambda}}{800} \right)^{1/6} \text{ km.}$$

For GW170817,  $M = 1.186 M_\odot$ ,  $a' = 0.00375 \pm 0.00025$ ,

$$R_{1.4} = (13.4 \pm 0.1) \left( \frac{\tilde{\Lambda}}{800} \right)^{1/6} \text{ km.}$$

# Summary of Astrophysical Observations



# Model-Dependence of $R_{1.4} - L$ Relation

Many studies show that  $R_{1.4}$  is most sensitive to the pressure at about  $2n_s$ .

The usual energy expansion for dense matter ( $u = n/n_s, x = n_p/n$ ) is

$$E(u, x \simeq 0) = -B + \frac{K_0}{18}(u-1)^2 + (1-2x)^2 \left[ S_V + \frac{L}{3}(u-1) + \frac{K_{\text{sym}}}{18}(u-1)^2 + \dots \right],$$

In  $\beta$  equilibrium,  $x \ll 0.5$ , so the pressure of neutron star matter is

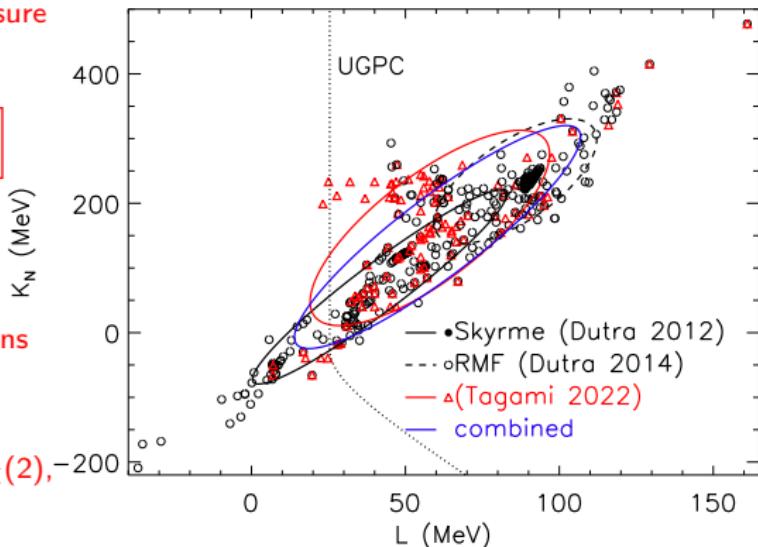
$$P_{\text{NSM}}(u) \simeq n_s u^2 \left[ \frac{L}{3} + \frac{K_N}{9}(u-1) \right]$$

$$P_{\text{NSM}}(2) \simeq \frac{4n_s}{3} \left[ L + \frac{K_N}{3} \right]$$

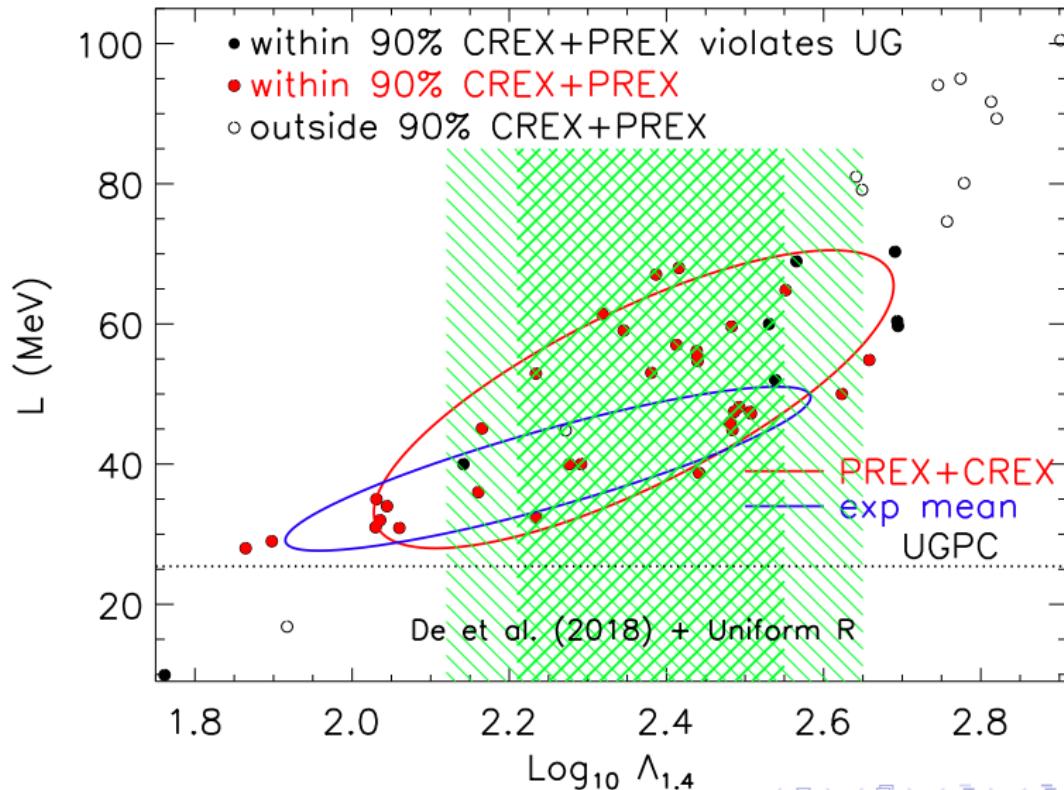
Around  $L \sim 60$  MeV, Skyrme interactions average  $K_N \sim 100$  MeV, but RMF interactions average  $K_N \sim 200$  MeV.

This produces  $\simeq 40\%$  increase in  $P_{\text{NSM}}(2)$ , and 10% larger  $R_{1.4}$ , for the same  $L$ .

For  $L \gtrsim 70$  MeV, values of  $K_N$  for Skyrme and RMF are similar, but these  $L$  values are disfavored by nuclear systematics (mass fits, neutron matter theory, neutron skin and dipole polarizability measurements).



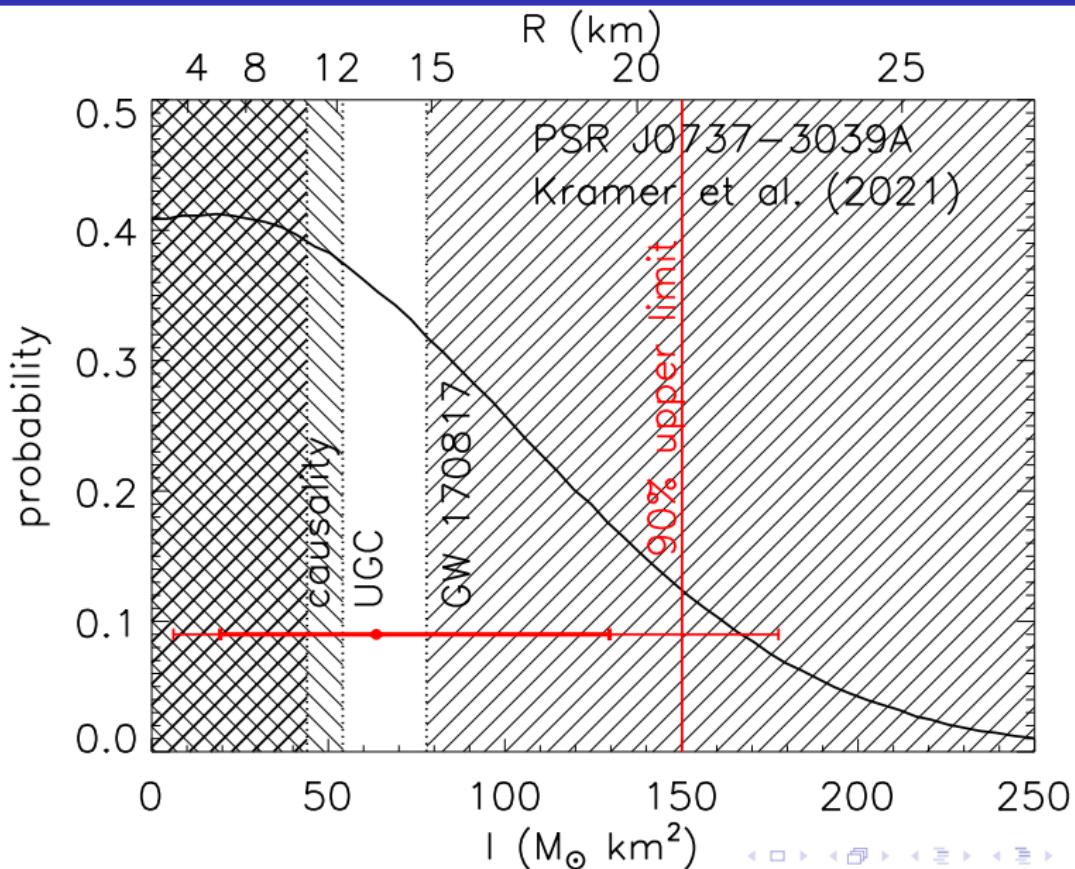
# Implied $\Lambda_{1.4} - L$



# Moment of Inertia

- ▶ Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- ▶ Precession alters inclination angle (observable if system is face-on) and periastron advance (observable if system is edge-on)
- ▶ More EOS sensitive than  $R$ :  $I \propto MR^2$
- ▶ Requires system to be extremely relativistic to measure
- ▶ Double pulsar PSR J0737-3037 is an edge-on candidate;  
 $M_A = 1.3382M_\odot$
- ▶ Even more relativistic systems are likely to be found, based on faintness and nearness of PSR J0737-3037

# Recent Moment of Inertia Measurement



# Conclusions

Nuclear experiments and theory, including EDF fits to nuclear binding energies, chiral EFT calculations, and neutron skin and dipole polarizability measurements of  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$ , consistently predict narrow ranges for the symmetry energy parameters without any astrophysical inputs:

$$S_V = (32 \pm 2) \text{ MeV}, \quad L = (50 \pm 10) \text{ MeV}, \quad K_N = (140 \pm 70) \text{ MeV}.$$

Neutron star radius predictions are about  $R_{1.4} = (11.5 \pm 1.0)$  km.

This is consistent with inferences from GW170817, NICER X-ray timing measurements and X-ray observations of quiescent thermal and photospheric radius expansion burst sources.

We eagerly anticipate new results of tidal deformabilities from neutron star mergers observed with LIGO/Virgo/Kagra, new mass and moment of inertia measurements from radio pulsar timing, and additional observations of rapidly rotating sources with NICER and planned X-ray telescopes.