Deblurring 3D Characteristics of Heavy-Ion Collisions

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59 Int. Winter Meeting on Nuclear Physics
Bormio, Italy

January 23-27, 2023

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane*
from Theory and Experiment

no control over plane

What is it?!

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n

Still not clear what the system is. . .

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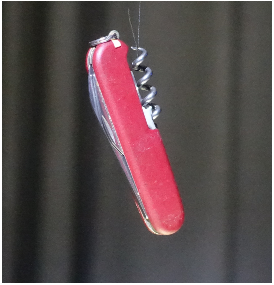
Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n



full control, $\frac{d^3N}{dp^3}$

Claim: You can go from center to right panel through deblurring

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Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car

Deblurred

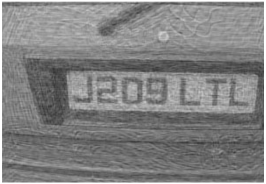
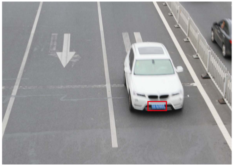


Photo of Parked Car

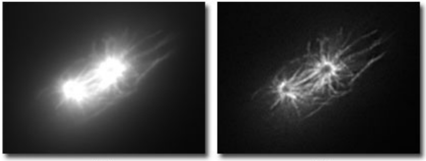
Fast Moving



Lu *et al.*, IEEE Trans Image Processing 25, 2311 (2016)

Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



(a) (b)

Figure 1

https://micro.magnet.fsu.edu/primer/digitalimaging/deconvolution

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Deblurring in Astronomy

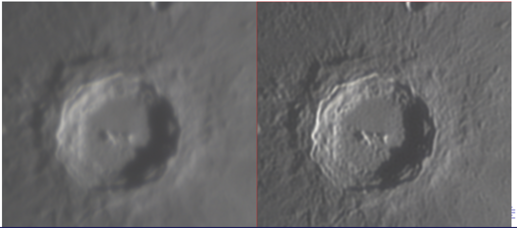
Much advancement in astronomy, in particular spurred by the Hubble Space Telescope (HST) flaw

Carasso, NISTIR 7632 (2009)

Original HST NGC1309 **Linnik deblur**

Before and after deblurring of image of lunar crater Copernicus

https://en.wikipedia.org/wiki/Deconvolution



Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon}\, n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\text{prod}} \simeq E_{\text{det}} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int \text{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ. Optical terminology: P - blurring or transfer function.

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Bayesian Deblurring

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - complementary probability that ptcle has characteristic ξ while measured at ζ - unknown.

Bayesian relation: number of times ptcle has characteristic in dξ while measured in dζ is

$$P(\zeta|\xi) \, N(\xi) \, \mathrm{d}\xi \, \mathrm{d}\zeta = Q(\xi|\zeta) \, n(\zeta) \, \mathrm{d}\xi \, \mathrm{d}\zeta$$

Hence $N(\xi) = \dfrac{\int \mathrm{d}\zeta \, Q(\xi|\zeta) \, n(\zeta)}{\int \mathrm{d}\zeta' \, P(\zeta'|\xi)}$, $\quad Q(\xi|\zeta) = \dfrac{P(\zeta|\xi) \, N(\xi)}{\int \mathrm{d}\xi' \, P(\zeta|\xi') \, N(\xi')}$

Richardson-Lucy method solves eqs iteratively till stabilization

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Richardson-Lucy (RL) Method from Astronomy

Iterative method, r - iteration index

$$n^{(r)}(\zeta) = \int \mathrm{d}\xi \, P^{(r)}(\zeta|\xi) \, N^{(r)}(\xi)$$

$$A^{(r)}(\xi) = \frac{\int \mathrm{d}\zeta \, \frac{n(\zeta)}{n^{(r)}(\zeta)} \, P^{(r)}(\zeta|\xi)}{\int \mathrm{d}\zeta' \, P^{(r)}(\zeta'|\xi)}$$

$$N^{(r+1)}(\xi) = A^{(r)}(\xi) \, N^{(r)}(\xi)$$

ξ & ζ are binned (pixelated), n & N are arrays and P transformation (transfer) matrix from the method/apparatus. Deblurring amounts to iterative multiplication of arrays by matrices + matrix reconstruction. Typical start: $N^{(1)}(\xi) = n(\xi)$

Richardson JOSA 62(1972)55 ; Lucy AJ 79(1974)745
D'Agostini NIMPRA362(1995)487
https://en.wikipedia.org/wiki/Richardson-Lucy_deconvolution

PD&Kurata-Nishimura PRC105(2022)034608

Other methods include Fourier transformation

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3D Nature of Collisions of Heavy Nuclei

Transport simulation of 2 GeV/nucl Au + Au at $b = 6\,\text{fm}$

PD *et al.* Science298(2002)1592 z - beam, x - reaction plane

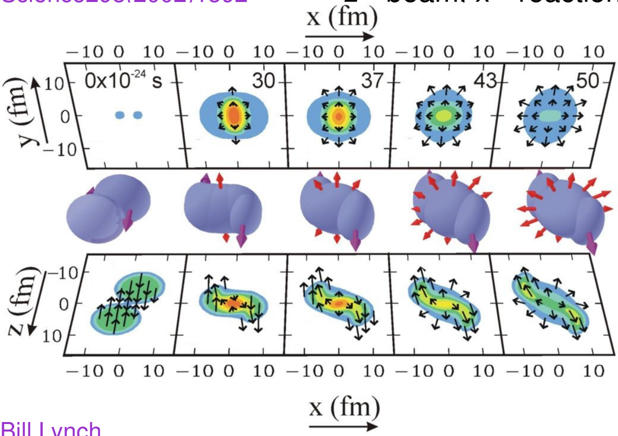
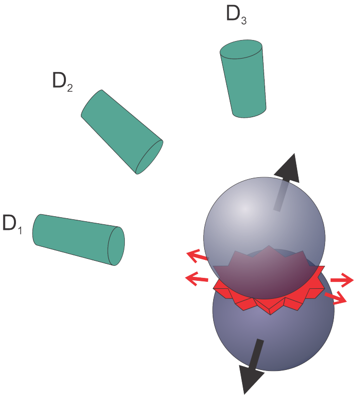


figure by Bill Lynch

Rich 3D structure, but no control over the reaction-plane direction in experiment

Estimating Reaction-Plane Direction



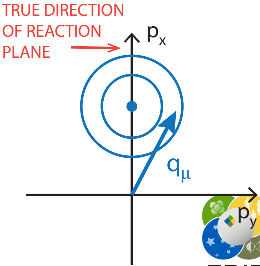
Plane direction estimated with

$$\mathbf{q}_\mu = \frac{1}{N} \sum_{\nu \neq \mu} \omega_\nu \, \mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec
PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction



Any direct record of 3D characteristics will be blurred!

Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

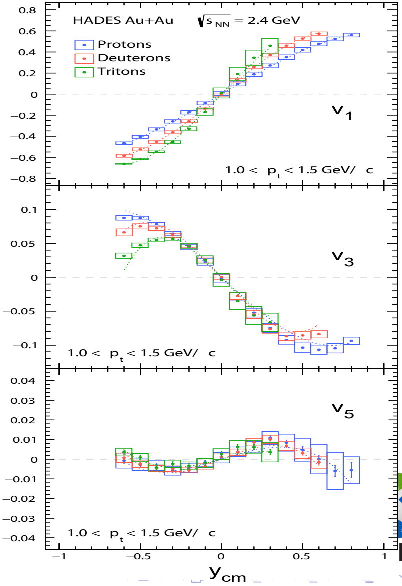
$$\langle \mathbf{p}^{\perp}_{\mu} \cdot \mathbf{q}_{\mu} \rangle \simeq p^{\perp} \langle q^{x} \rangle \langle \cos \phi \rangle$$

for different $p^{\perp}$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301



Schematic 1D Model

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~v_n~~?

First 1D deblurring test.

Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like $\mathrm{d}N/\mathrm{d}v$ in projectile cm. Task: Measuring ptcles in lab, determine $\mathrm{d}N/\mathrm{d}v$. Cm velocity V' estimated from remaining ptcles, so V' & $\mathrm{d}N/\mathrm{d}v'$ smeared:

$$\frac{\mathrm{d}N}{\mathrm{d}v'} = \int \mathrm{d}V' \, \frac{\mathrm{d}P}{\mathrm{d}V'} \, \frac{\mathrm{d}N}{\mathrm{d}v}$$

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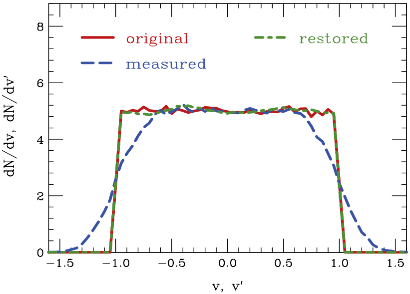
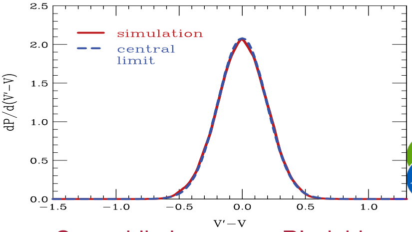


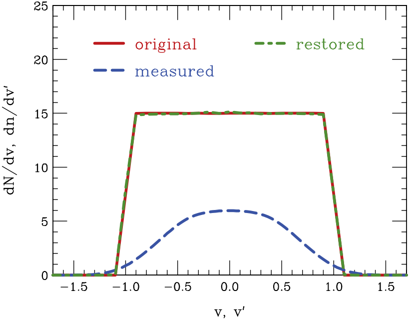


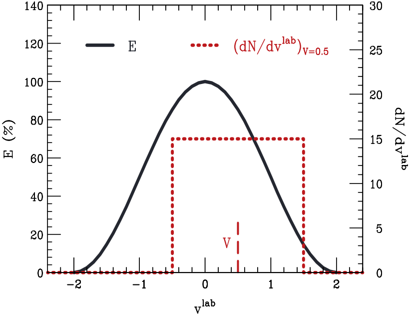
→Central-limit smear + RL deblur

1D Model with Detector Inefficiencies

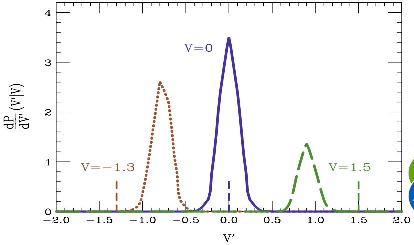
Again projectile at unknown velocity *V* deexcites emitting *N* particles, but now measured w/detector of strongly changing efficiency *E*. Find dN/dv.

$N = 30$ unknown.





"keyhole view"



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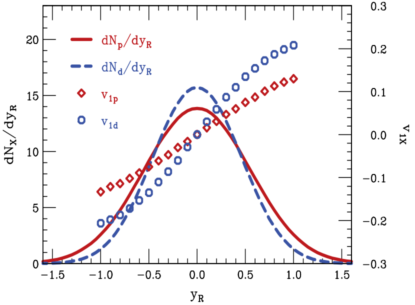
⇒ RL deblur w/s-consistent transfer mtx

3D Model for Collisions

Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$
Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl

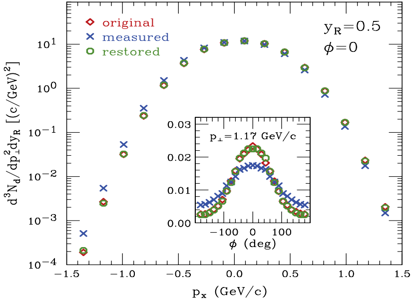


$$\frac{dN}{d\phi'} = \int d\Phi' \, \frac{dP}{d\Phi'} \, \frac{dN}{d\phi}$$

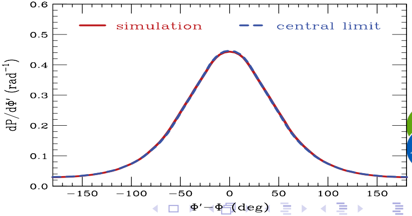
$$\phi' + \Phi' = \phi + \Phi$$

RL deblur + central-limit
Strong anisotropies restored!

Triple differential spectrum in reaction plane:



Uncertainty in reaction plane:



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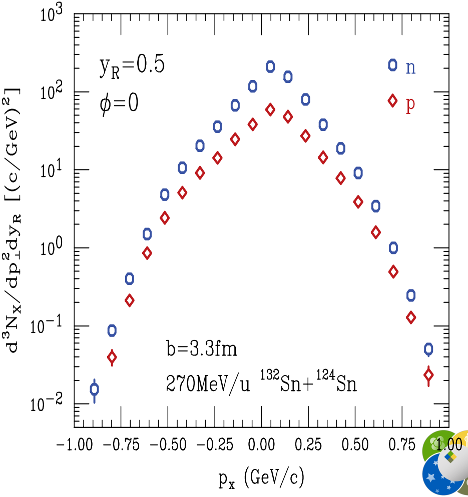
Why 3D Characteristics?

Transport-model simulation of 270 MeV/nucl ^{132}Sn + ^{124}Sn collision at $b = 3.3$ fm

3-differential spectrum for same conditions as in thermal model, but looks very different. Not parabolic, i.e., Gaussian, cusps, different left-right slopes, knees. Steeper slope on spectator side, softer on participant. Physics??

Averaged over ϕ, the spectrum would look thermal and no obvious sign in v_n...

HADES at $p^{\perp}/A \lesssim 100$ MeV/c *blind*, to miss physics ☹?



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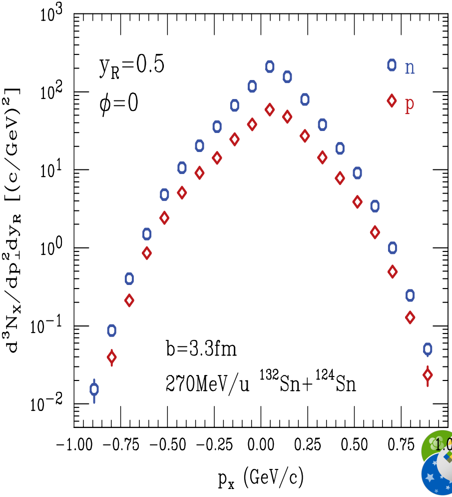

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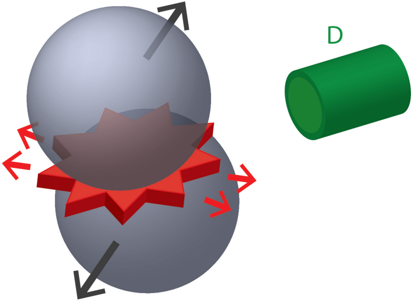
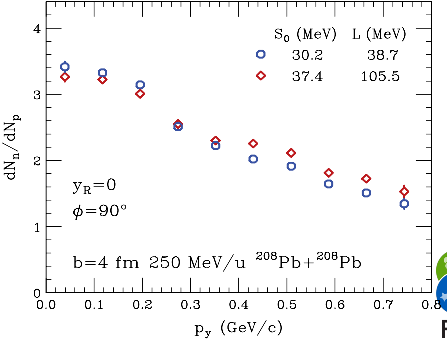
Symmetry energy at $\rho > \rho_0$?

Deblurring allows to effectively look into the heart of matter, unobscured high-density central region in the collisions

Transport-model simulations of 250 MeV/nucl ^{208}Pb + ^{208}Pb collisions w/medium-soft & stiff symmetry energy. n/p yield ratio at $\phi = 90^\circ$, perp to reaction plane, and $y_{cm} = 0$.





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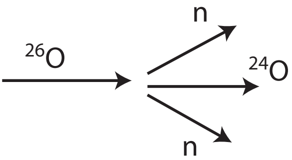
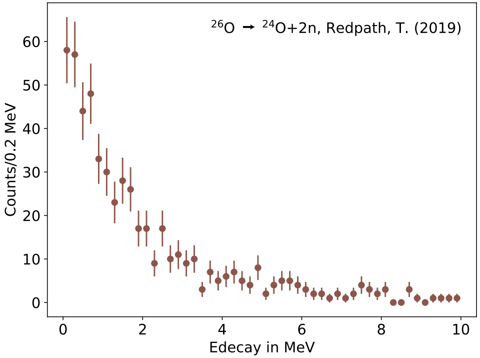
Deblurring of Decay Spectrum

MoNA study: ^{27}F (-p) $\rightarrow$ ^{26}O $\rightarrow$ ^{24}O + 2n



Thomas Redpath PhD Thesis MSU (2019)
Redpath *et al.* NIMPRA977(2020)164284

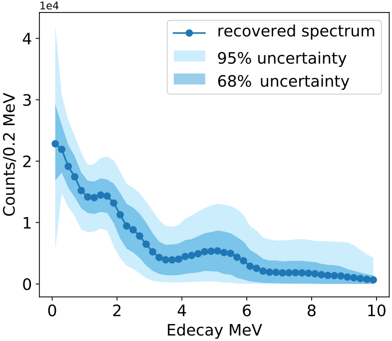


Distribution of events in decay energy

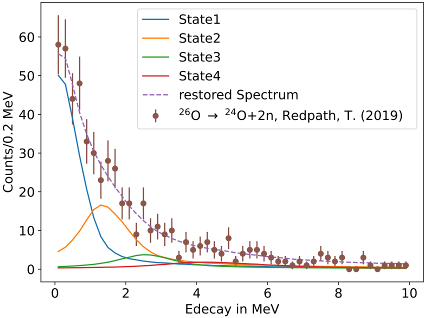
Deblurred Decay Spectrum of ^{26}O

Deblurring: Pierre Nzabahimana; Transfer matrix: Thomas Redpath

arXiv:2210.00157



Deblurred



Original - decomposed

Bumps indicative of energy levels in ^{26}O:
$\sim 0\,\text{MeV}$ - 0^+, $\sim 1.3\,\text{MeV}$ - 2^+, etc.

Recall registration plate

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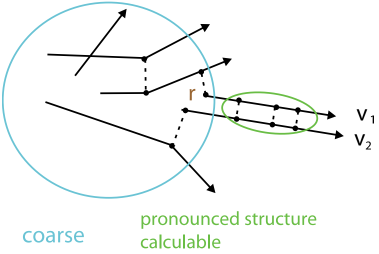
Imaging Source from Correlation (HBT)

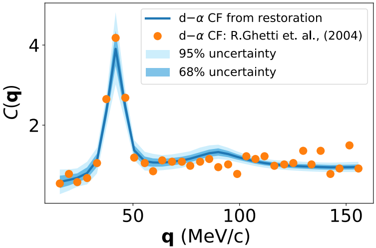


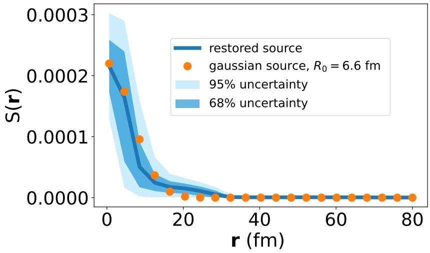
2-ptcle correlation at low $|\mathbf{v}_1 - \mathbf{v}_2|$

$$C(\mathbf{v}_1 - \mathbf{v}_2) = \frac{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{v}_1\, d\mathbf{v}_2}}{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{v}_1}\,\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{v}_2}}$$

$$= \int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{v}_1-\mathbf{v}_2}(\mathbf{r})|^2$$

Koonin-Pratt eq: measured C as convolution over unknown source S





Source imaging as R-L deblurring: Pierre Nzabahimana

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Conclusions

- Deblurring: strong record in optics & fields that heavily rely on optics: forensic science, astronomy & microscopy
- Deblurring can expand the reach of measurement ahead of any comparison to theory
- No reason for deblurring to confine to photons and not extend to other particles - its domain are probabilities
- Deblurring should effectively allow to control reaction plane in energetic heavy-ion collisions, hopefully expand horizons
- Nuclear problems where deblurring started producing results: $^{26}\mathrm{O} \rightarrow ^{24}\mathrm{O} + n + n$ decay, source-imaging from 2-particle correlations in heavy-ion collisions, triple-differential distributions in heavy-ion collisions

PD&Kurata-Nishimura PRC105(2022)034608 Berkowitz Physics 15(2022)s26
https://www.energy.gov/science/np/articles/deblurring-can-reveal-3d-features-heavy-ion-collisions
Supported by US Department of Energy under Grant US DE-SC0019209

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