

Coalescence in MC generators and implications for cosmic ray studies

Maximilian Horst¹, Chiara Pinto¹, Laura Fabbietti¹, Bhawani Singh¹, Luca Barioglio², Francesca Bellini³, Sushanta Tripathy³

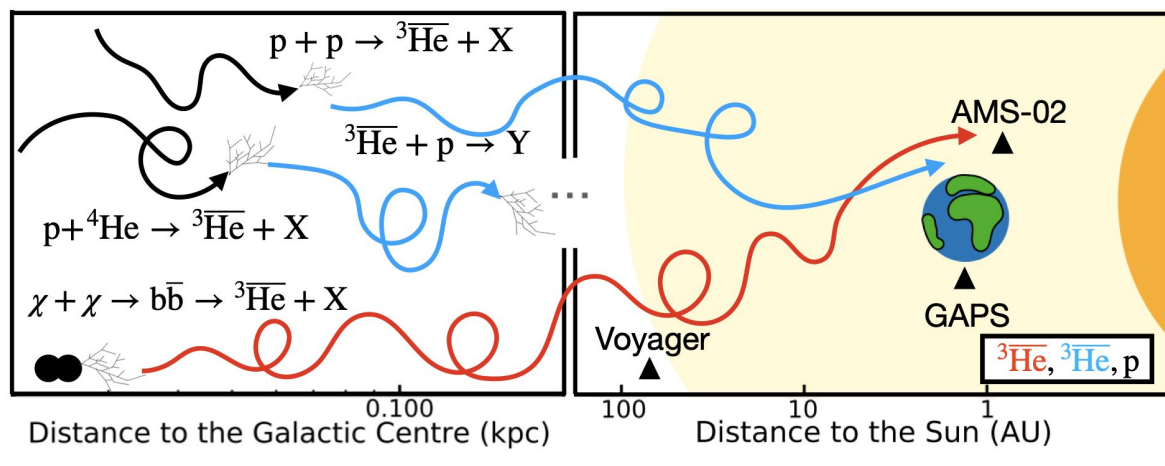
¹Technical University Munich

² INFN Torino

³ INFN and University Bologna

Cosmic Rays

Antinuclei in cosmic rays

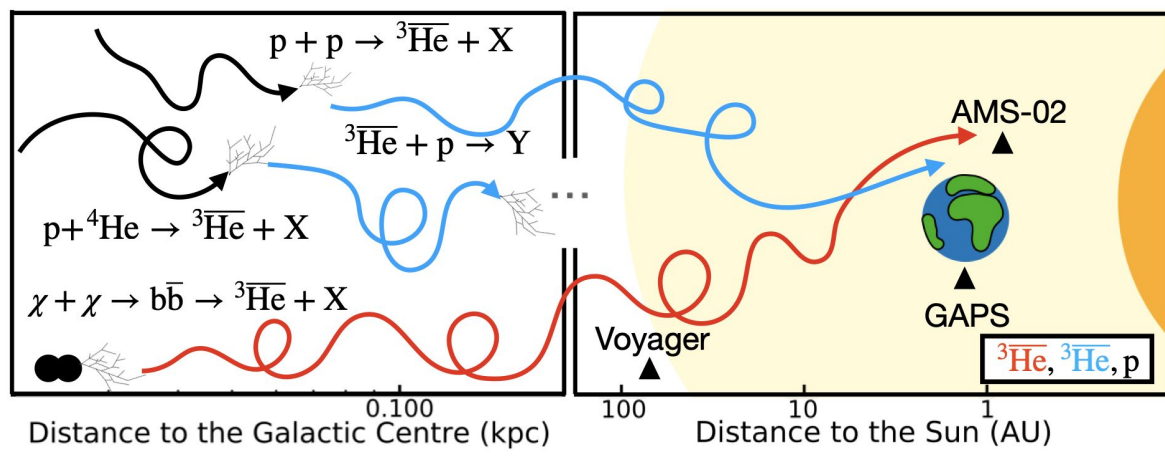


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- pp, p-A and (few) A-A reactions between primary **cosmic rays** and the interstellar medium
- **dark-matter** annihilation processes

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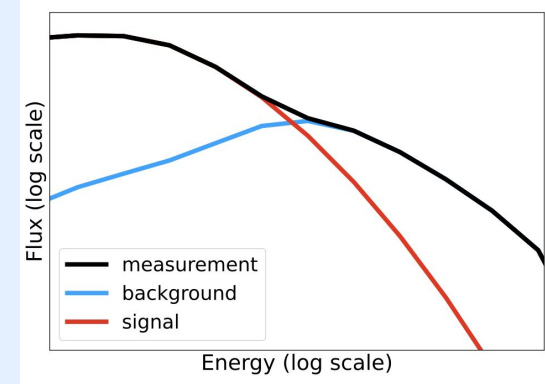


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- High Signal/Noise ratio ($\sim 10^2$ - 10^4) at low E_{kin} expected by models

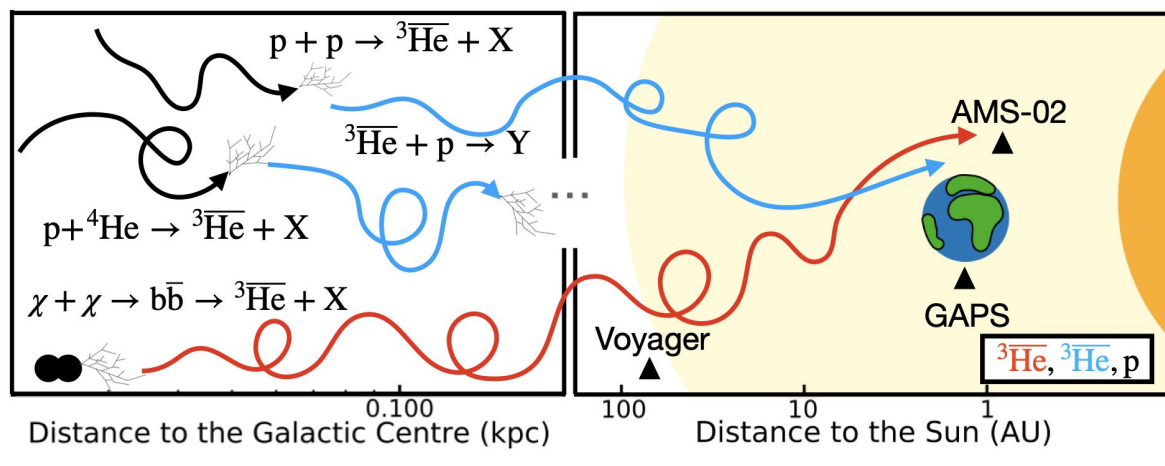
antinuclei cosmic ray flux



ALICE Collab., Nature Phys. (2022)

Cosmic Rays

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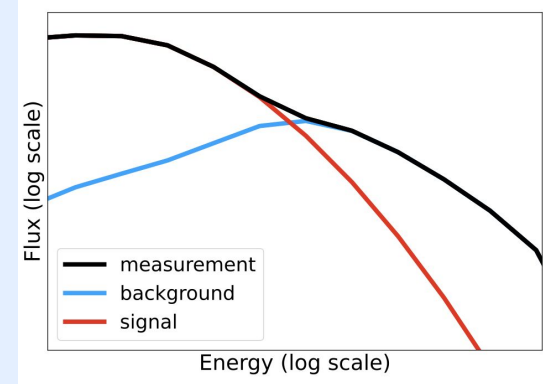


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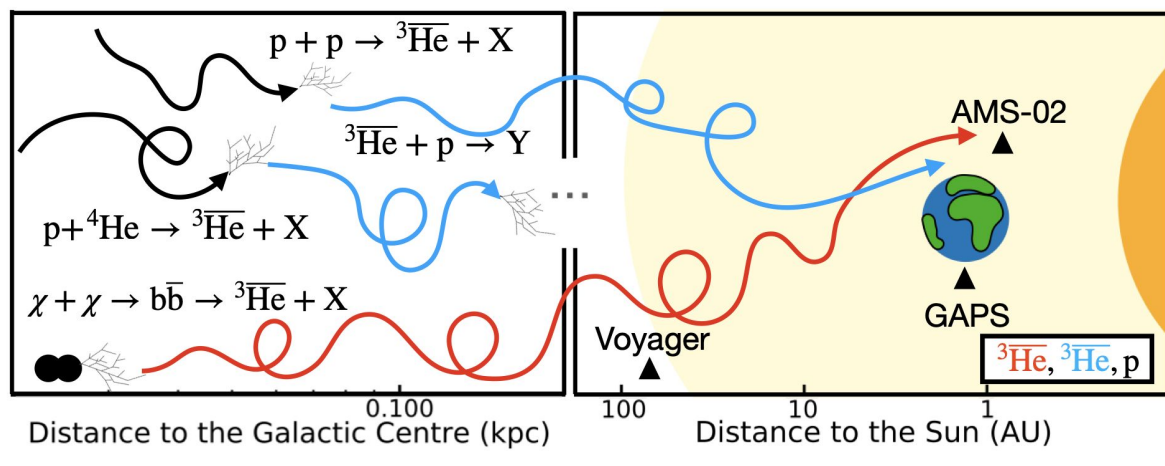
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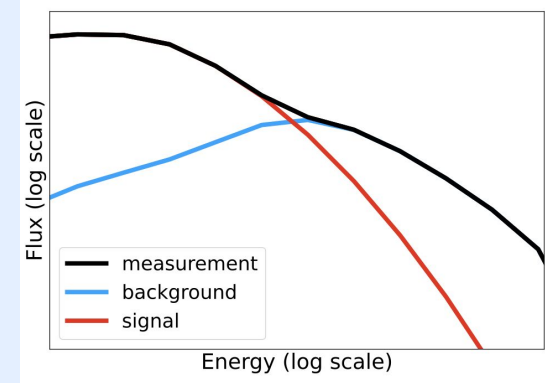


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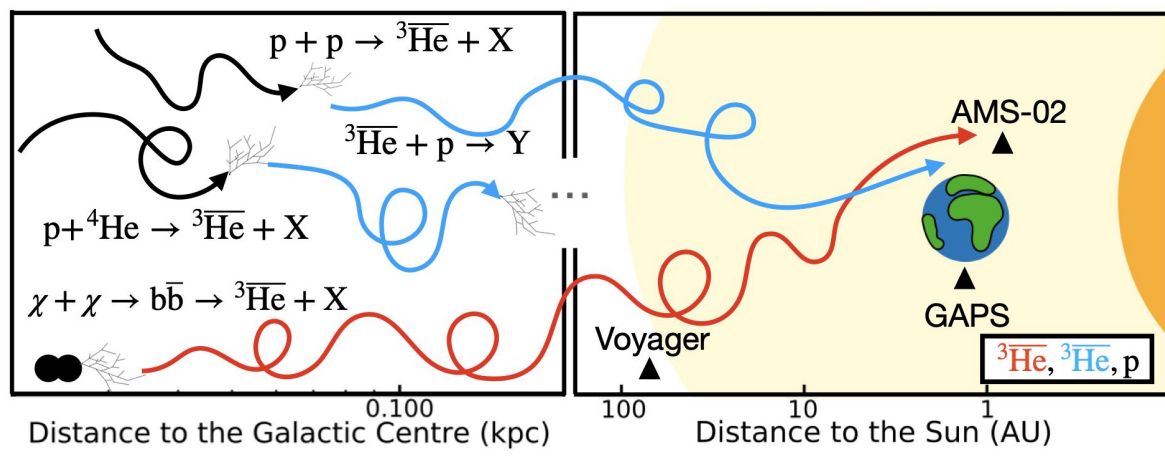
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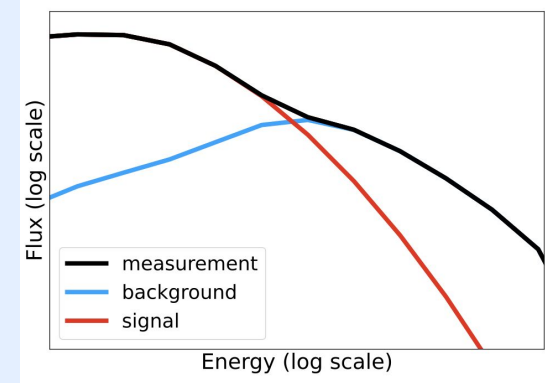


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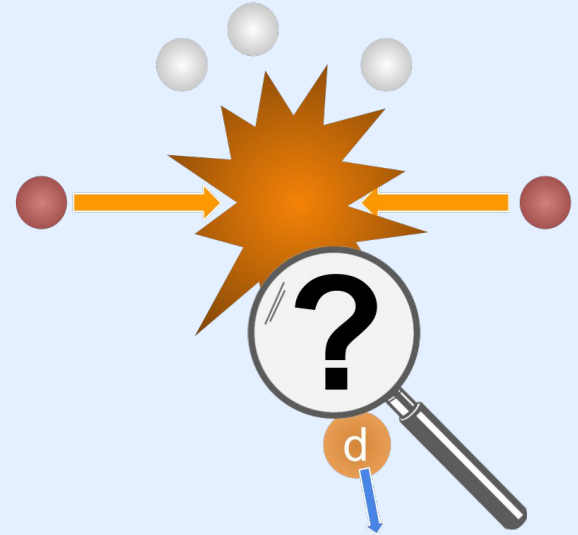


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Modelling (anti)nuclei production

Overview of production models

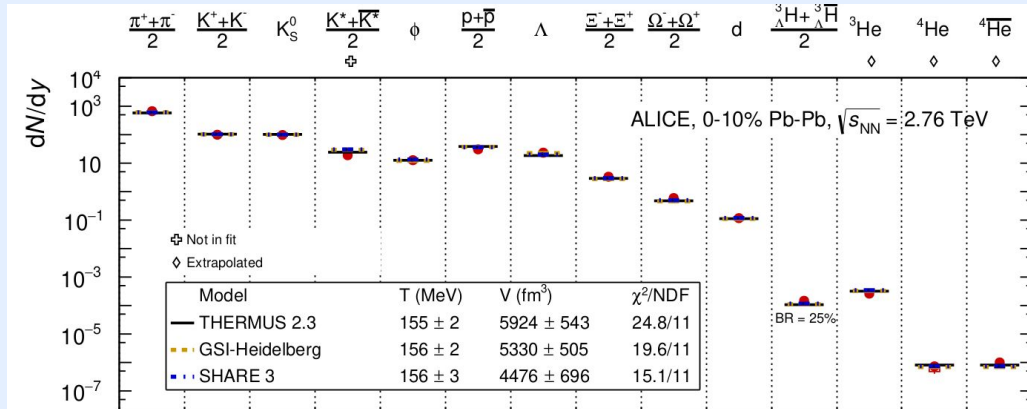
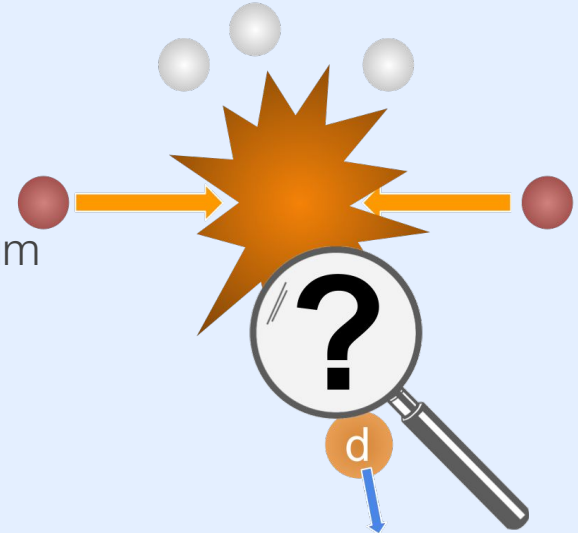
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Statistical hadronization (SHM)

- Particle yields (including nuclei) described by filling the available phase-space after the collision
- Works very well with a common temperature of the medium ($T=154$ MeV)
- No microscopic description of nuclei formation



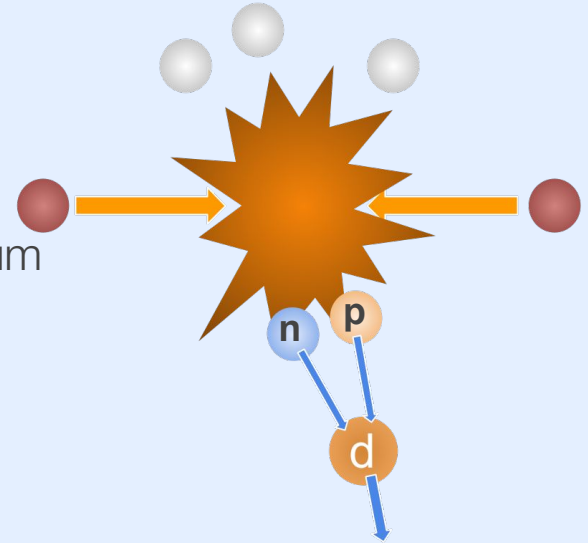
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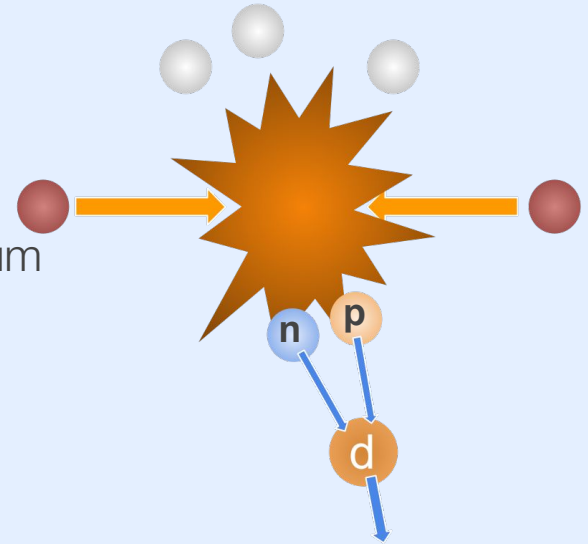
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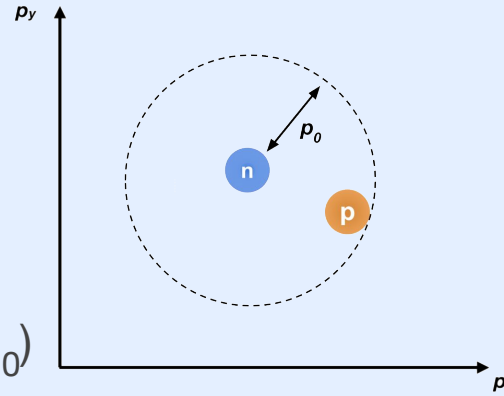
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Coalescence model

- Nucleons bind after chemical freeze-out if they are close in phase-space
- Common implementation:
Spherical Approximation ($\Delta p < p_0$)

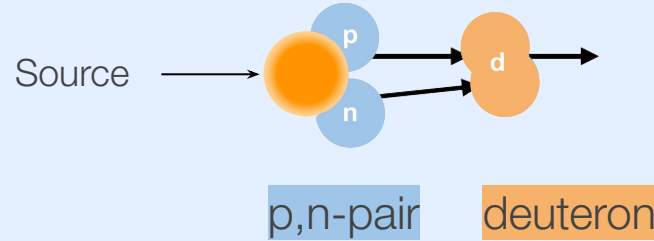


 Butler et al., Phys. Rev. 129 (1963) 836

The coalescence model

Wigner function formalism

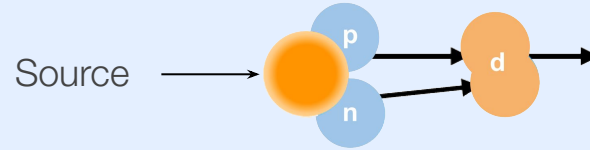
What do we need for coalescence?



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What do we need for coalescence?



p,n-pair deuteron

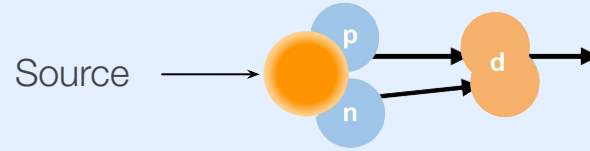
Quantum mechanics:

$$d^3N/dP^3 = \text{Tr}(\rho_d \rho_{\text{Nucl}})$$

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What do we need for coalescence?



p,n-pair

deuteron

$$q = (p_p - p_n)/2$$

$$r = r_p - r_n$$

Quantum mechanics:

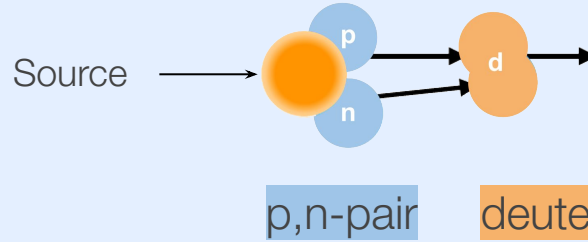
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$$d^3N/dP^3 = \int d^3q \int d^3r_p \int d^3r_n \text{Deuteron Density} \text{Nucleon Density}$$

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$$d^3N/dP^3 = S \int d^3q \int d^3r_p \int d^3r_n W(q,r) W_{pn}(p_p, p_n, r_p, r_n) / (2\pi)^6$$

Spin-Isospin statistics factor
(=3/8 for deuterons)

Wigner function of deuteron

Wigner function of p-n state

Two-nucleon Wigner function

$$W_{np}(\vec{P}/2 + \vec{q}, \vec{P}/2 - \vec{q}, r_n, r_p) = H_{np}(\vec{r}_n, \vec{r}_p) G_{np}(\vec{P}/2 + \vec{q}, \vec{P}/2 - \vec{q})$$

- G_{np} is the momentum distribution of nucleons
- H_{np} is the spatial distribution of nucleons. Assuming a Gaussian source

$$H_{np}(\vec{r}_n, \vec{r}_p) = h(\vec{r}_n)h(\vec{r}_p) = \frac{1}{(2\pi\sigma^2)^3} \exp\left(-\frac{\vec{r}_n^2 + \vec{r}_p^2}{2\sigma^2}\right)$$

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Nucleon momentum phase-space

with

deuteron size (3.2 fm)

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Two-particle emitting source size

The coalescence model

Wigner function formalism

previous similar approaches*
neglected $\pm q$
→ we use event generators!

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* Blum et al. PRC 99 (2019) 4, 044913

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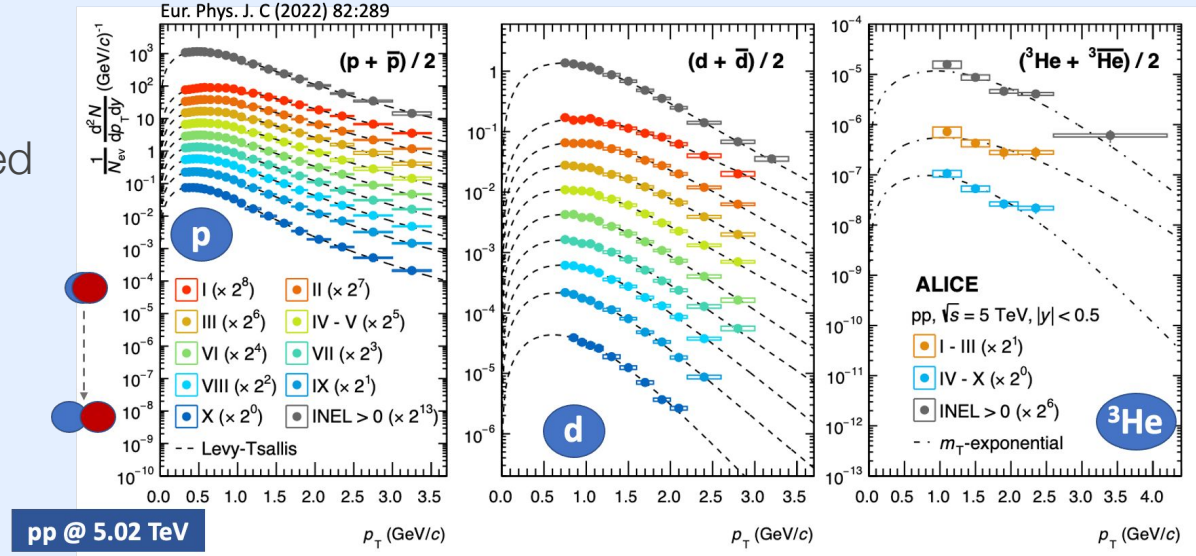
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Light (anti)nuclei measured in ALICE

Transverse momentum spectra

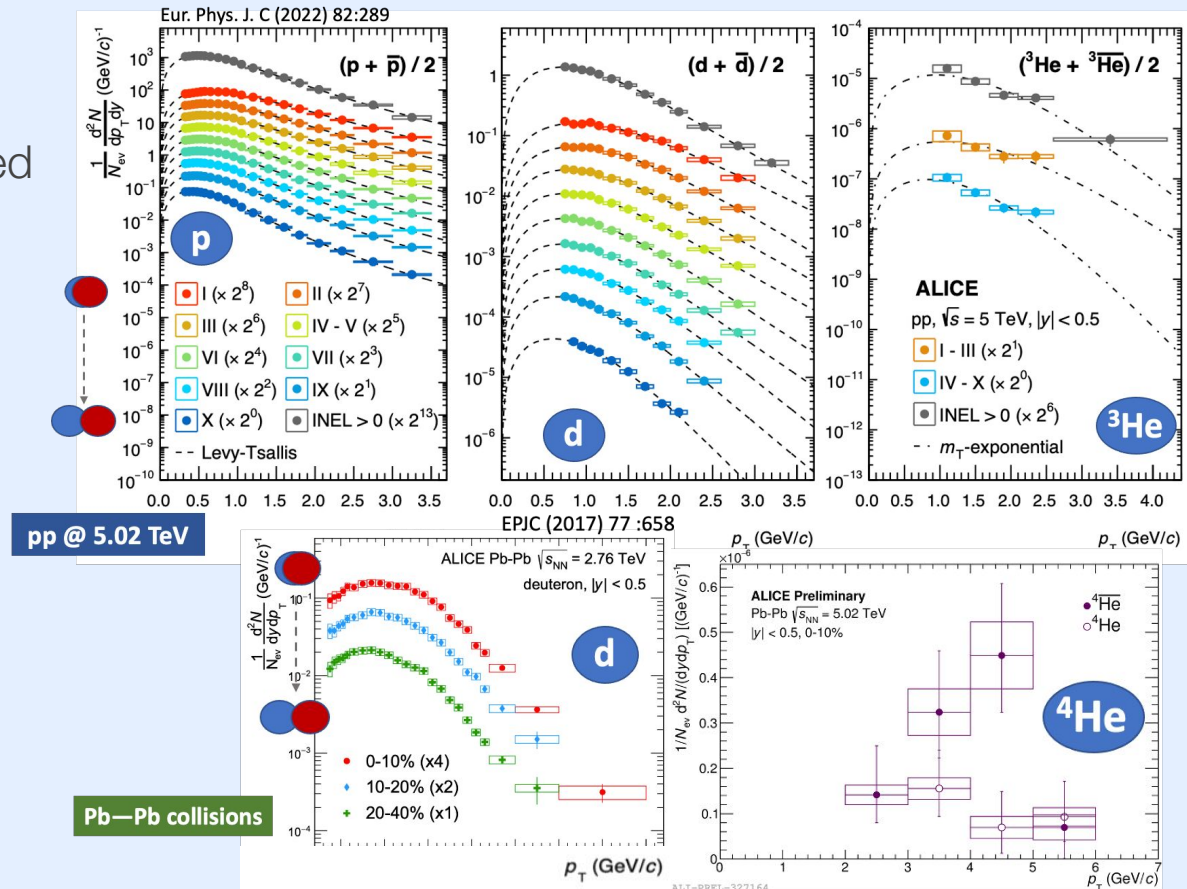
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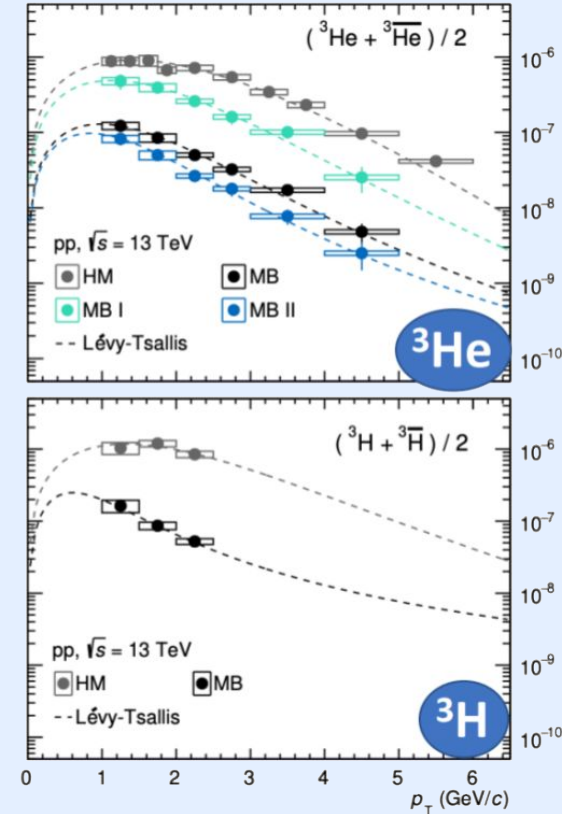
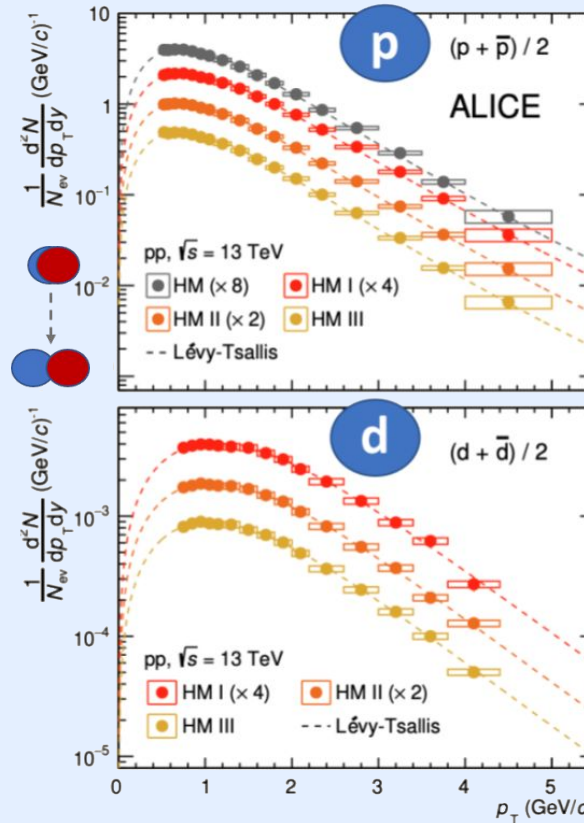


Light (anti)nuclei measured in ALICE

Transverse momentum spectra

HM pp @ 13 TeV

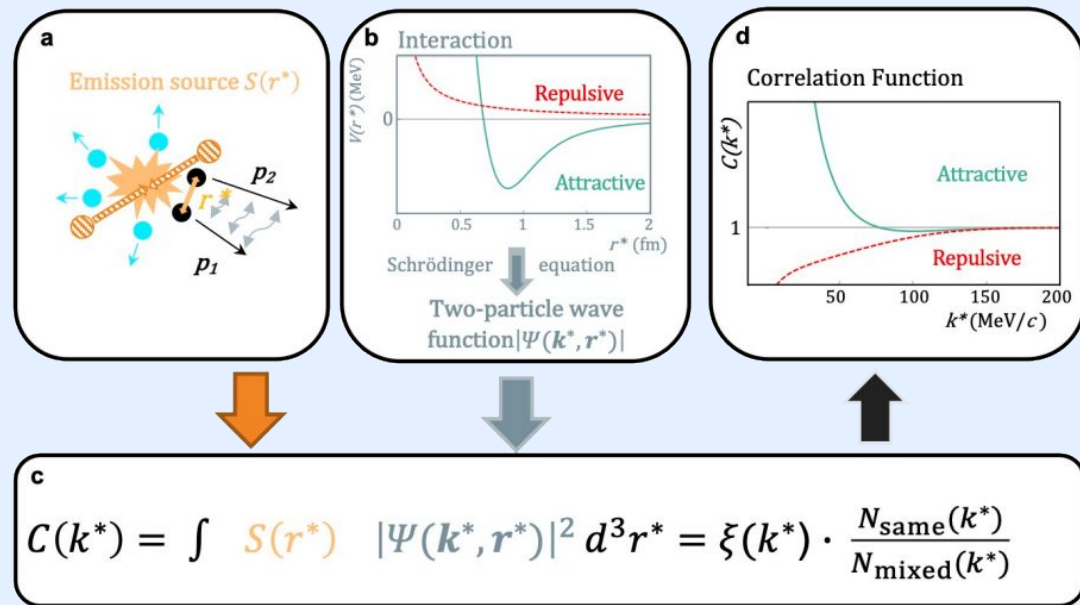
- Comprehensive measurements of light (anti)nuclei have been carried out in ALICE, from pp to Pb–Pb
- From (anti)deuterons to (anti) ^3He and (anti) ^4He
- **High multiplicity (HM)** class in **pp collisions at 13 TeV** (\rightarrow 0-0.17% centrality class)
- In HM class both production spectra and emitting source size measurements available



Emission source size measured in ALICE

Femtoscscopy

- ALICE is pioneering the study of the strong interaction using femtoscopic correlations
- Momentum correlations can be employed to explore two-particle dynamics
- The **correlation function** depends on two ingredients:
 - **Emission source function**
 - Two-particle **wave function** (quantum statistics + Coulomb + strong interaction)

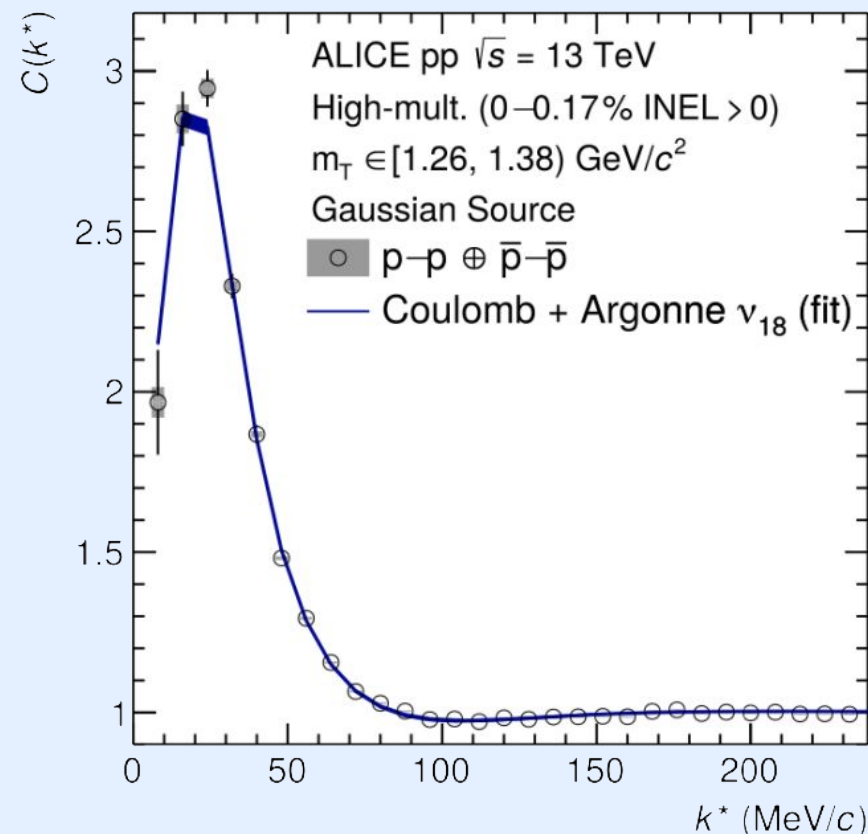


If we measure $C(k^*)$ and use a known **interaction** (e.g. nucleon-nucleon) we can study the **emission source**

Emission source size measured in ALICE TUM

Femtoscopy

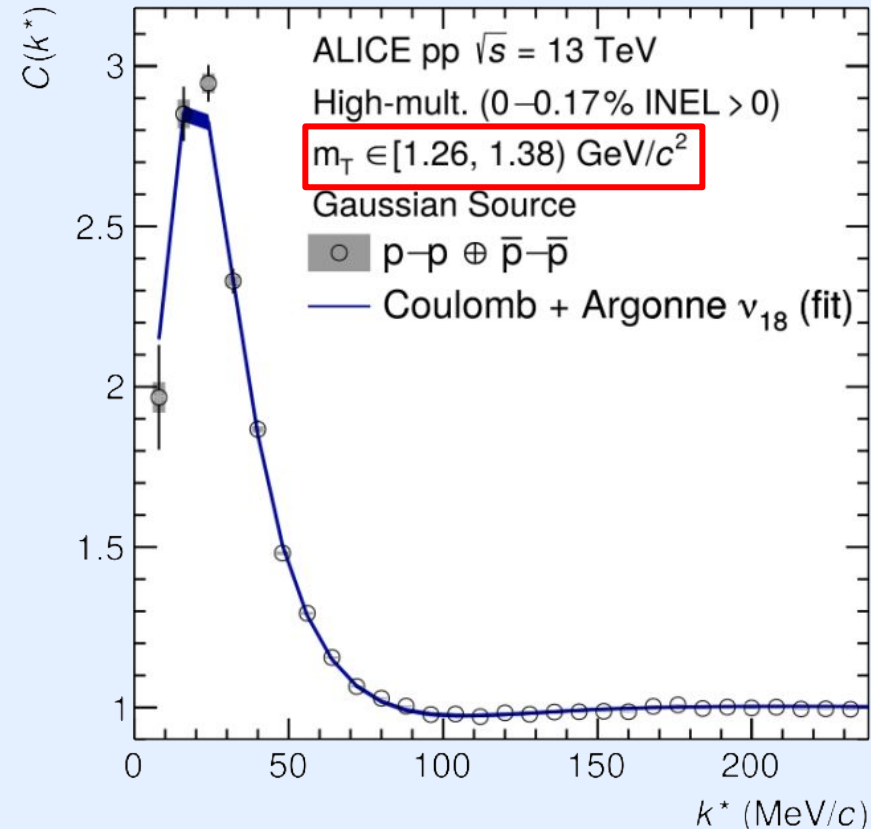
- Good description of the **interaction** with Fermi-Dirac statistics, Coulomb and strong interaction (using v18)
- Only free parameter: the **source size**



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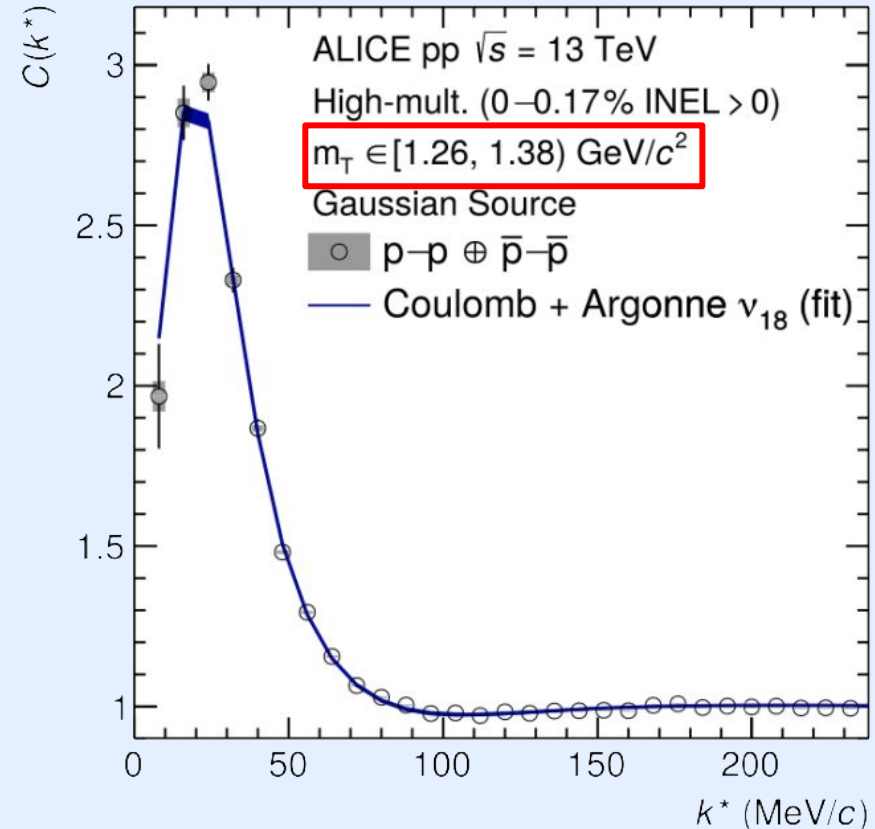
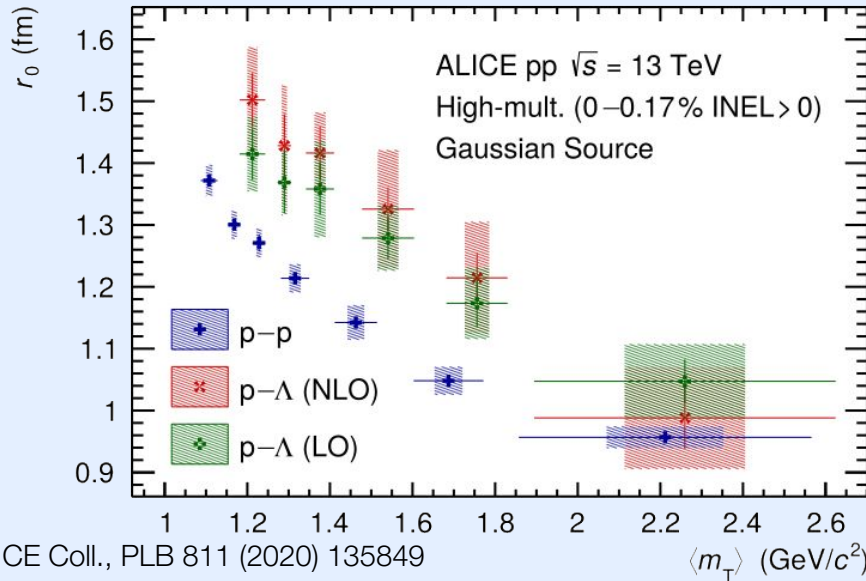
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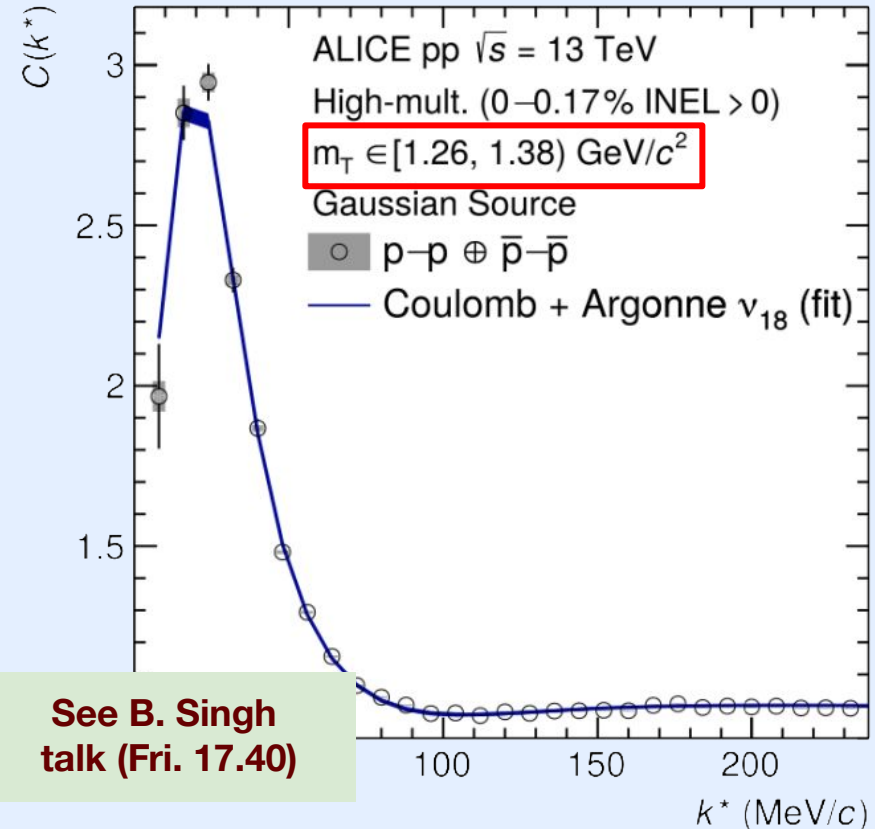
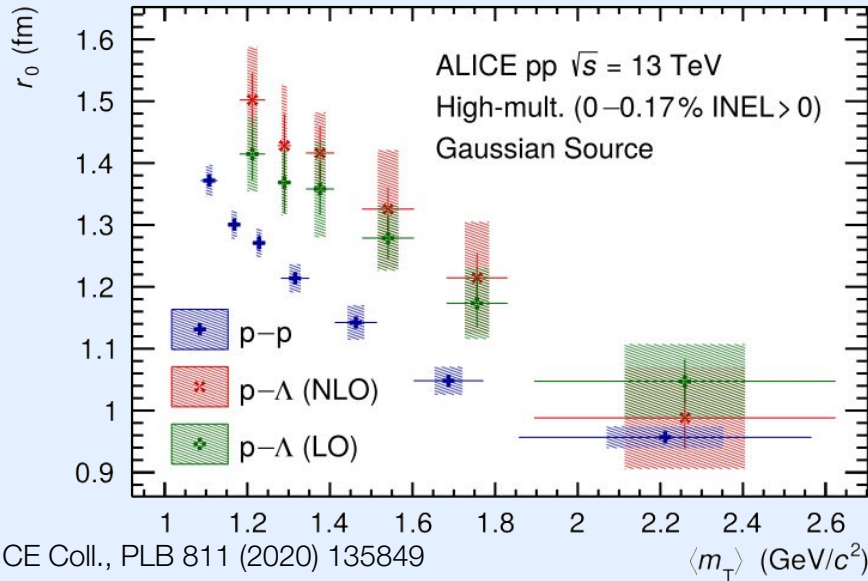
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See B. Singh
talk (Fri. 17.40)

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Constrained
from data!

The coalescence model

Wigner function formalism, tuned to ALICE measurements

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- The term $3\zeta e^{-q^2 d^2}$ can be interpreted as a coalescence probability depending on the relative momentum q and the source size σ
- More in general:

$$p(\sigma, q) = \int d^3 r_p d^3 r_n h(r_n) h(r_p) W(q, r)$$

- This allows us to calculate the coalescence probability for arbitrary Wigner functions

⇒ Probe different hypotheses for the deuteron wave function

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to take into account pn
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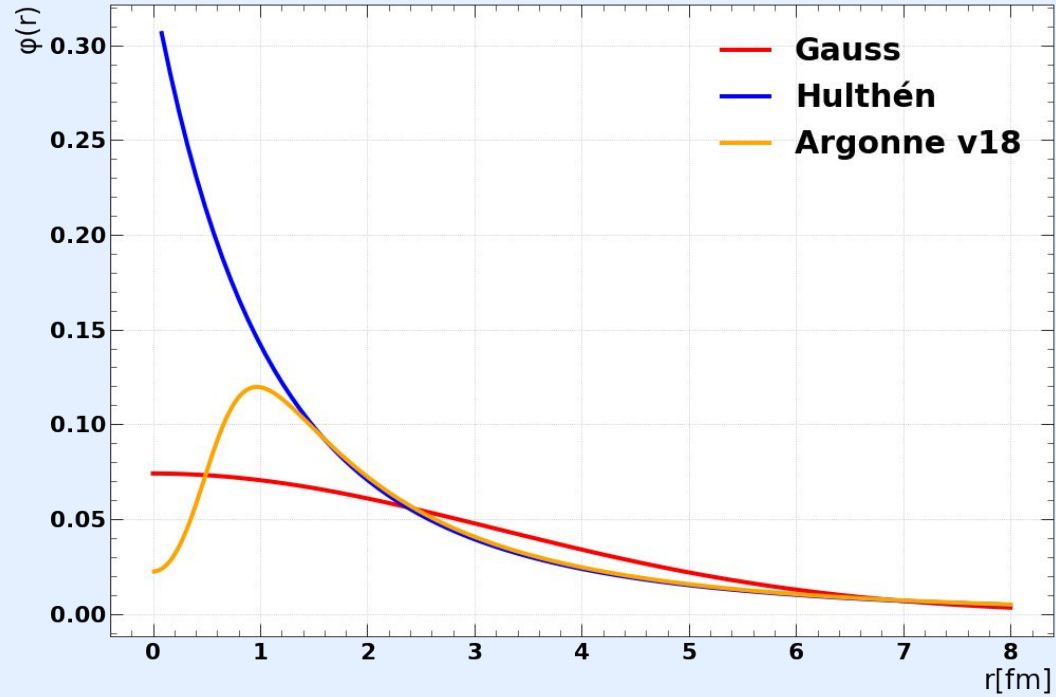
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State of the art coalescence predictions

Wigner function formalism \rightarrow wave functions

There are multiple models for the deuteron wave function

- Simplistic:
Single Gaussian
- From *pion field theory*
(Yukawa-like potential) ('50s)*:
Hulthén
- From pn scattering
measurements**:
Argonne v_{18}



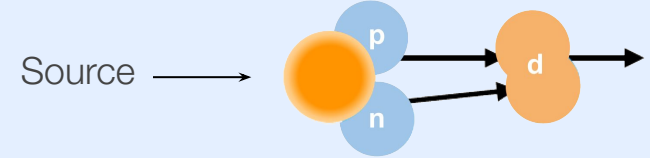
* Scheibl et al., PRC 59 (1999) 1585-1602

** Wiringa et al., PRC 51 (1995) 38-51

State of the art coalescence predictions

Wigner function formalism tuned to ALICE measurements

- Event-by-event coalescence afterburner with Wigner function formalism
- EPOS 3 as event generator



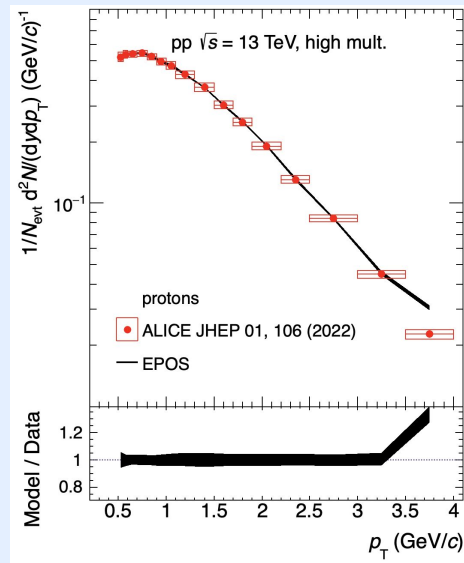
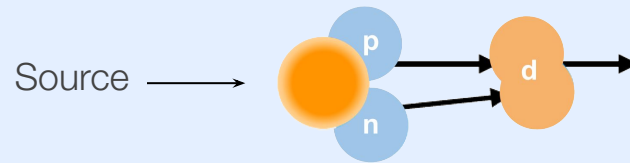
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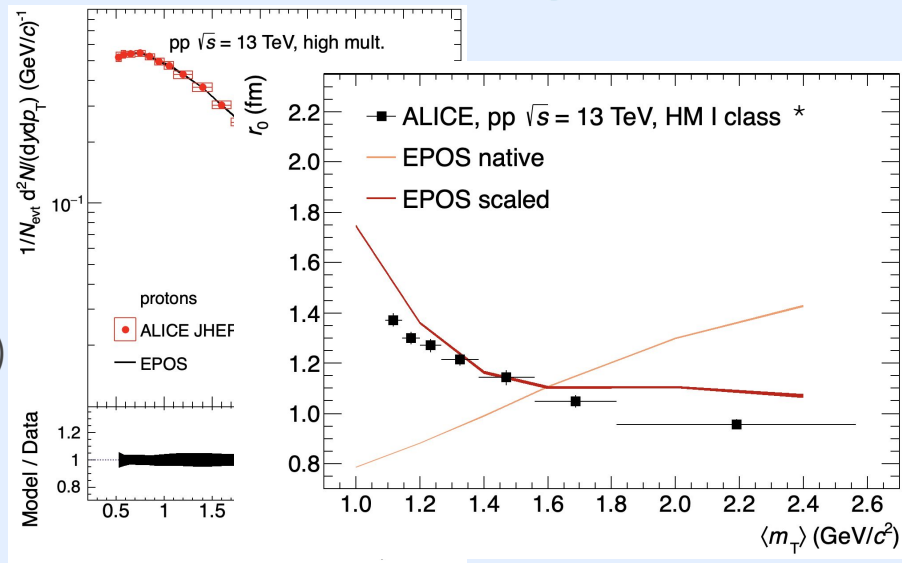
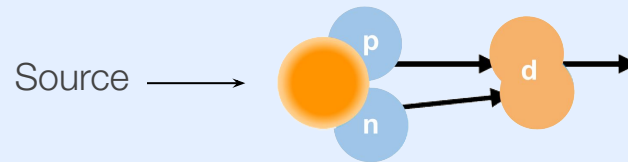
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Ingredients

- Protons (and neutrons) are tuned to p measurements from ALICE
- Event generator is tuned to measurements
 - source size
 - resonance cocktail
 - charged-particle multiplicity (35.8 ± 0.5)



*  ALICE Coll., PLB 811(2020) 135849

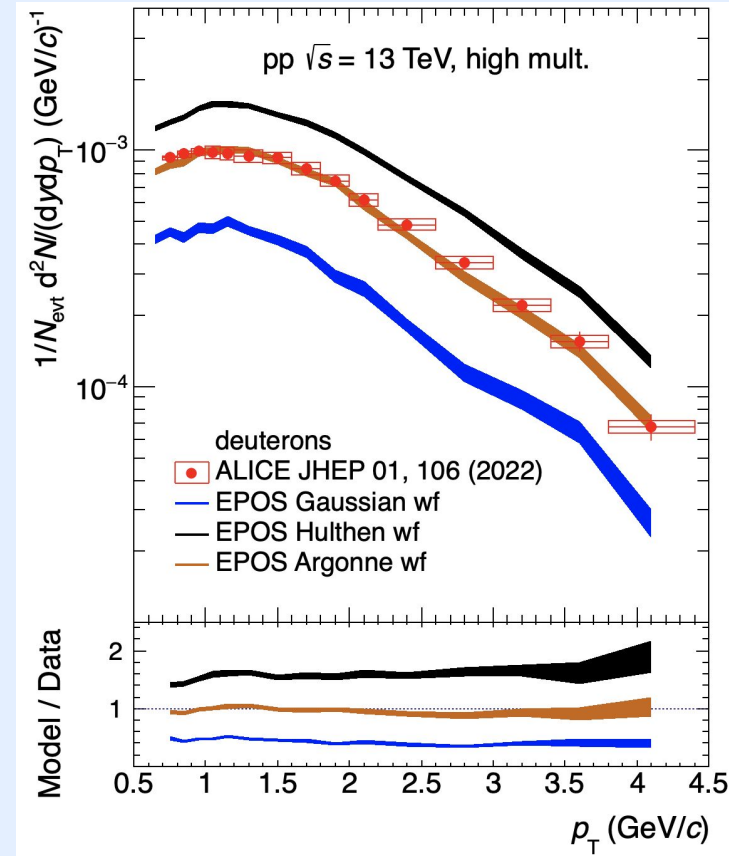
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Wigner function formalism tuned to ALICE measurements

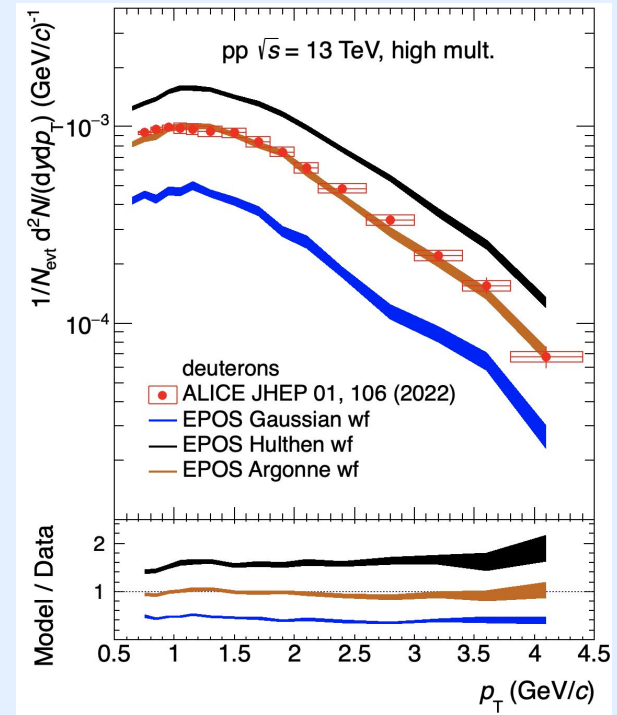
- Event-by-event coalescence afterburner with Wigner function formalism
- EPOS 3 as event generator

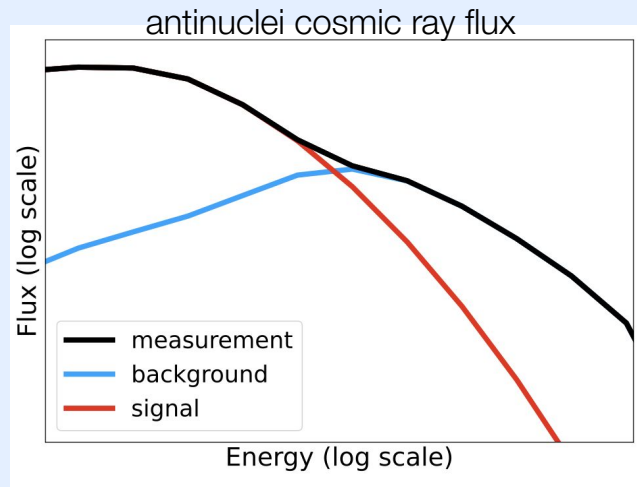
Ingredients 🧑🔬

- Protons (and neutrons) are tuned to p measurements from ALICE
- Event generator is tuned to measurements
 - source size
 - resonance cocktail
 - charged-particle multiplicity (35.8 ± 0.5)
- Compare to measurements by ALICE
- Argonne WF shows the best agreement with measurements

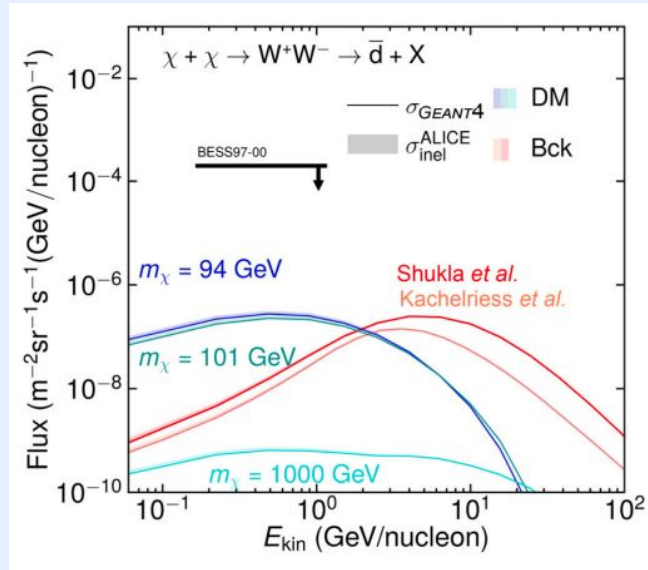


- Novel approach for coalescence based on Wigner function formalism is developed
- Deuteron production in high multiplicity pp collisions $\sqrt{s} = 13$ TeV
- *If* we have control of the underlying physics
 - emission source size
 - (anti)nucleon momentum distributions
 - resonance cocktail
 - charged-particle multiplicity
 - realistic nucleus wavefunction
- Model successfully reproduces data with no free-parameters!

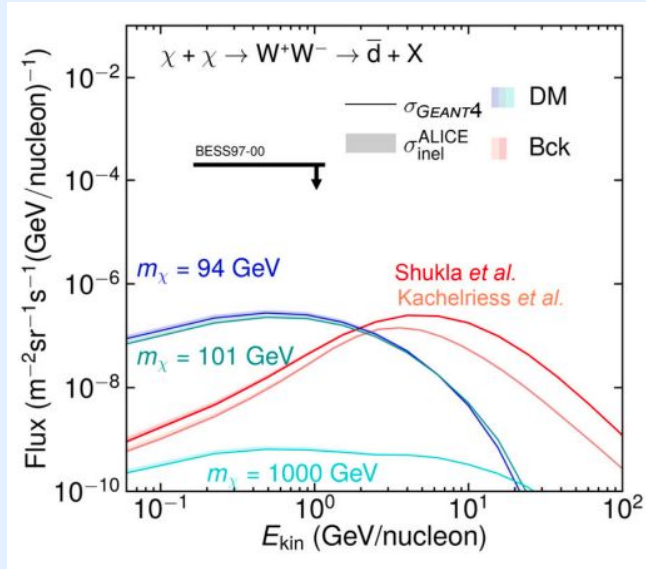




- Modelling antinuclei production is an essential backbone to interpret any future measurement of cosmic ray antinuclear fluxes

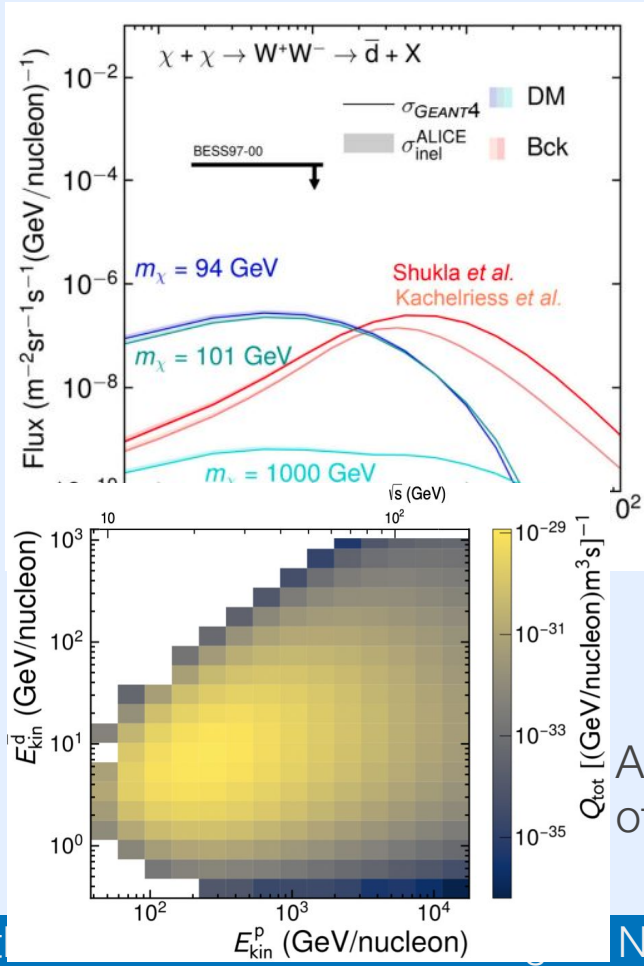


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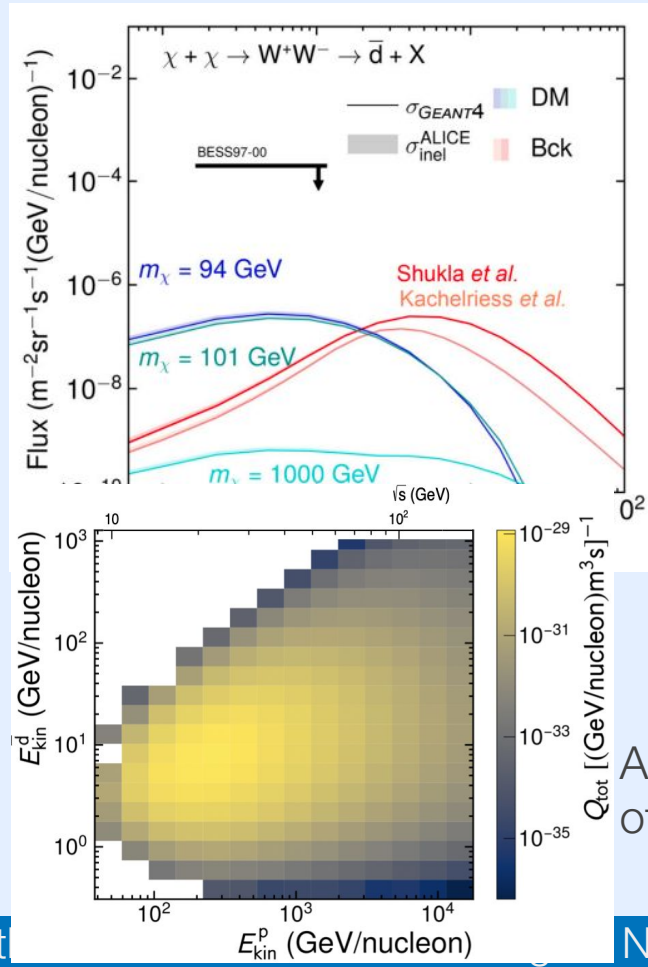


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- Antideuteron production predominantly from collisions of protons of $E_{\text{kin}} \sim 200\text{-}500 \text{ GeV}$ ($\sqrt{s} \sim 19\text{-}30 \text{ GeV}$ for p-H)

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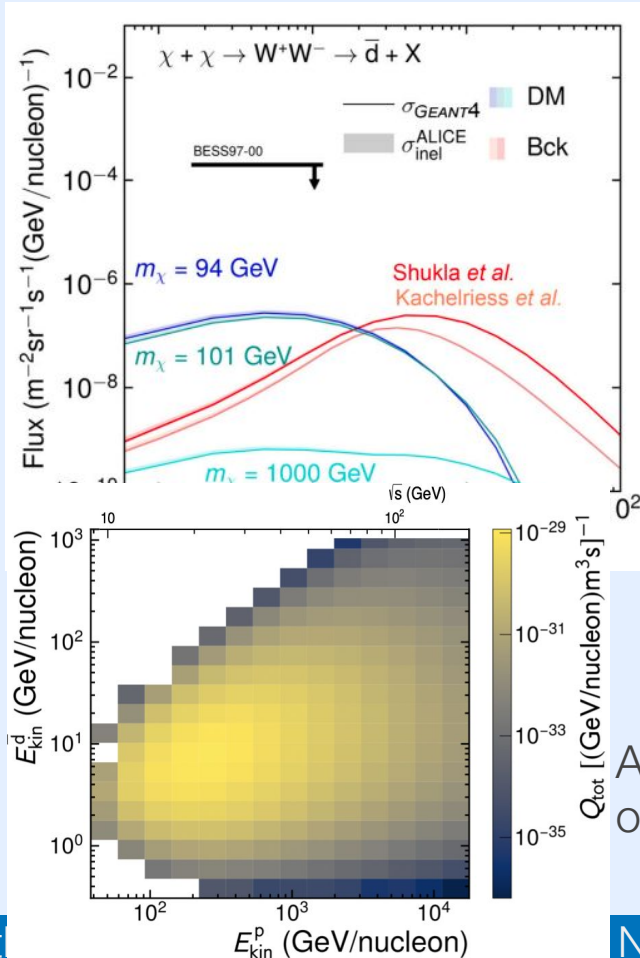


Antideuteron source function as a function of kinetic energy of the incoming proton and produced antideuteron



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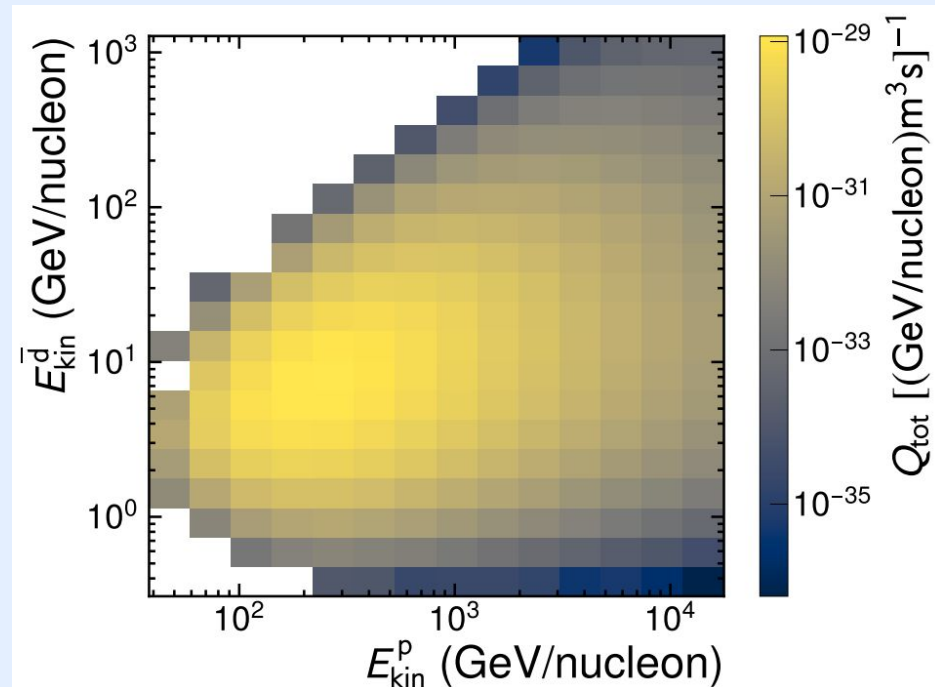
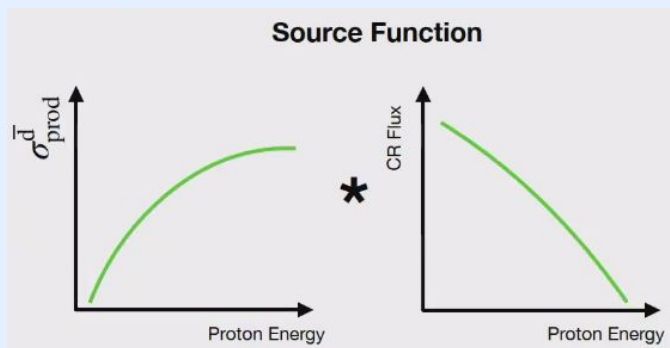


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- Extrapolation in the energy range of interest
- More experimental data at lower energies needed!

Antideuteron source function as a function of kinetic energy of the incoming proton and produced antideuteron

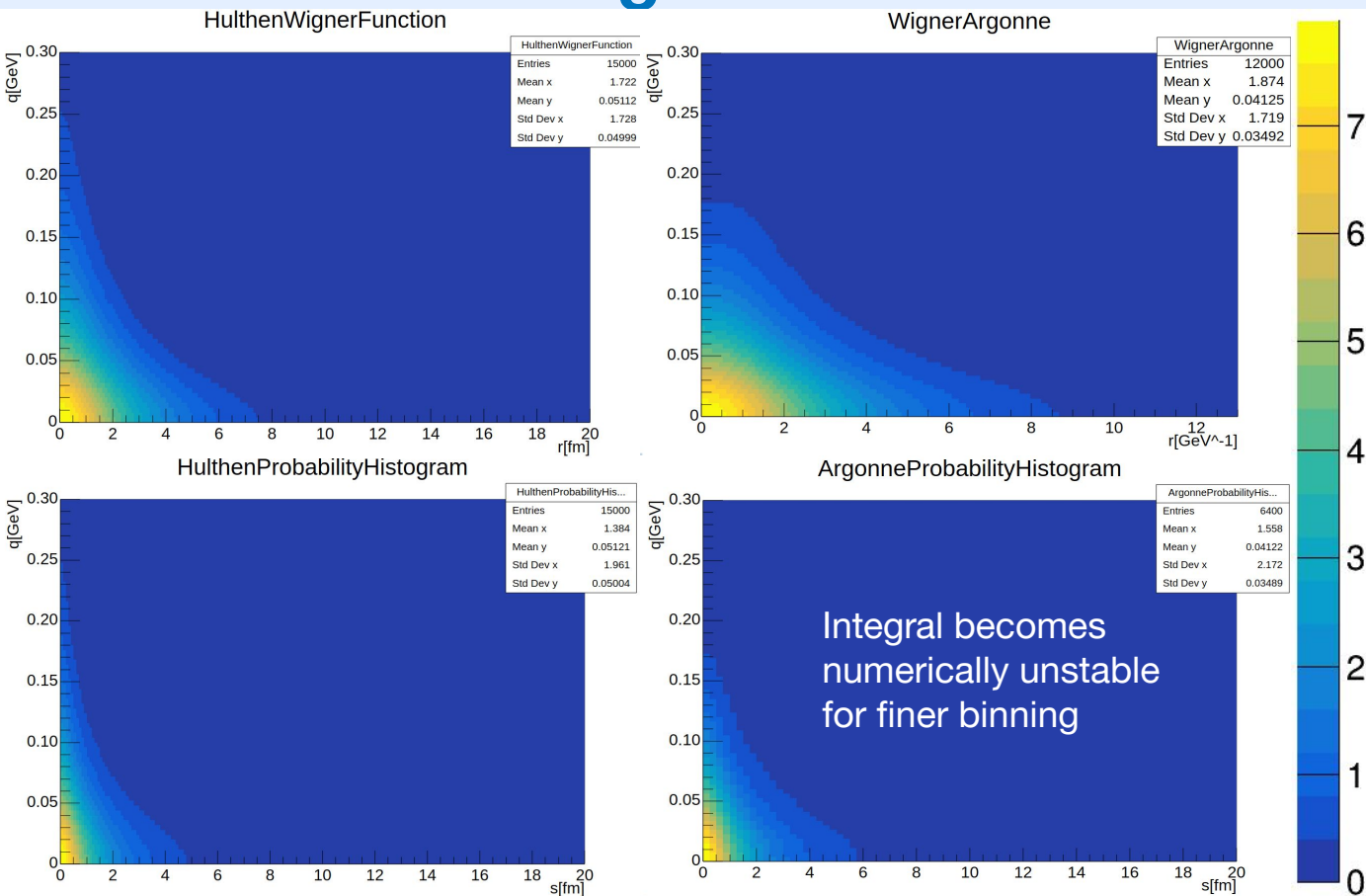
Backup slides

- Antideuteron source function as a function of kinetic energy of the incoming proton and produced antideuteron
- Antideuteron production predominantly for protons of $E_{\text{kin}} \sim 200\text{-}500$ GeV ($\sqrt{s} \sim 19\text{-}30$ GeV for p-H)

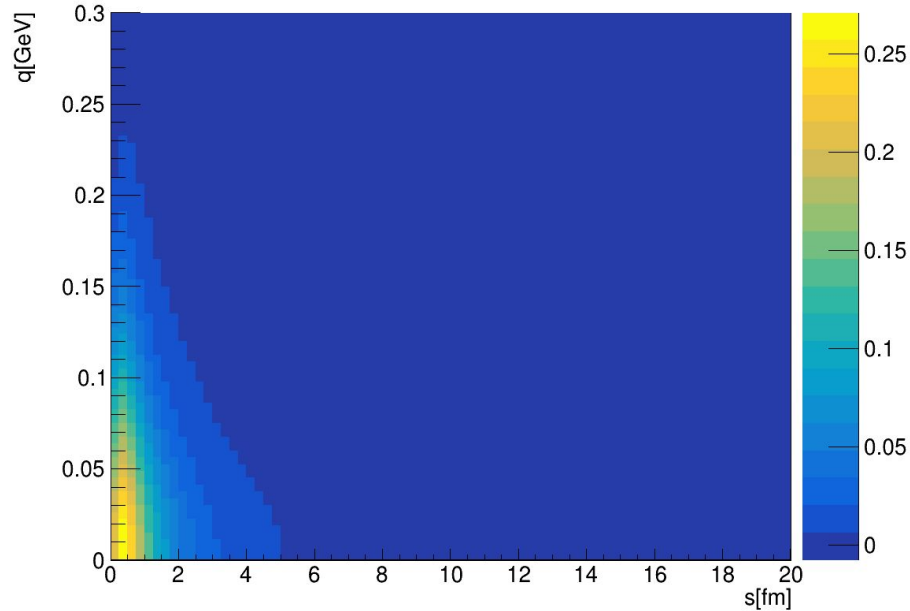


Šerkšnytė, et al. PHYSICAL REVIEW D 105, 083021 (2022)

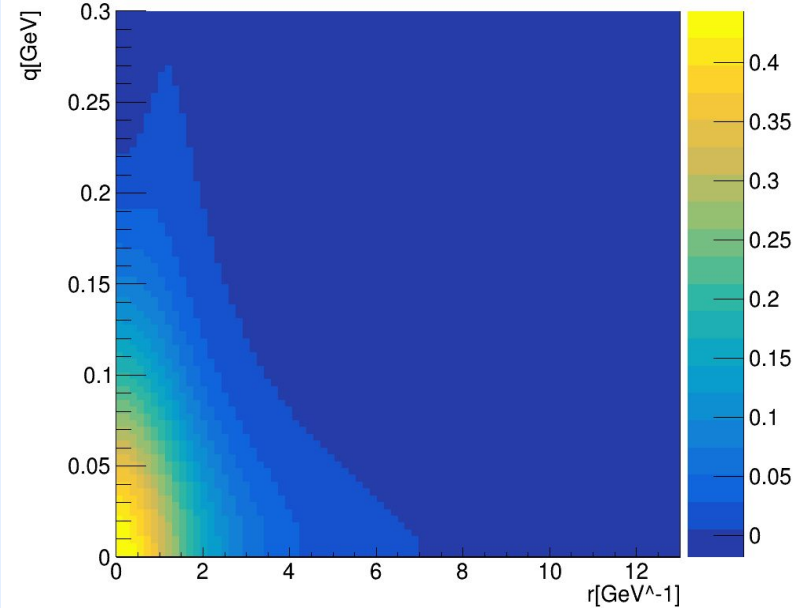
New Wigner functions/Probabilities



ArgonneProbabilityHistogramDWave



WignerArgonne_D_Wave



D-State probability is 6% \rightarrow Maximum $\sim 11\%$ effect

Overview of (anti)nuclei data

(anti)nuclei measurements

- No measurement of antideuteron in the energy region (~19-30 GeV) relevant for astrophysics
- Most measurements are very old (~60s and 70s)
- NA61's energy (17.3 GeV) would be a perfect candidate to study antinuclei for astrophysics

We need precise measurements at the energies of interest to constrain (anti)nuclei production!

Experiment or Laboratory	Collision	p_{lab} (GeV/c)	\sqrt{s} (GeV)
CERN	p + p	19	6.15
CERN	p + p	24	6.8
Serpukhov	p + p	70	11.5
	p + Be		
CERN-SPS	p + Be	200	19.4
	p + Al		
Fermilab	p + Be	300	23.8
CERN-ISR	p + p	1497.8	53
CERN-ALICE	p + p	4.3×10^5	900
CERN-ALICE	p + p	2.6×10^7	7000

■ No antideuteron data!

B_A predictions

- Important observable in accelerator measurements: B_A

$$B_A(p_T^p) = E_A \frac{d^3 N_A}{dp_A^3} \bigg/ \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$

- Theoretical prediction [1]

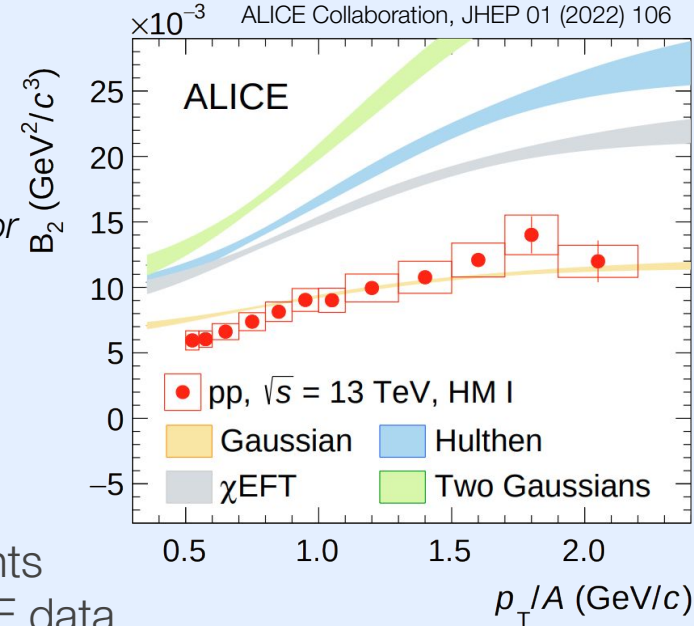
$$B_2(\vec{p}) \approx \frac{3}{2m} \int d^3 q D(\vec{q}) e^{-R^2(p_T) q^2} \text{ later!}$$

$$D(\vec{q}) = \int d^3 r |\phi_d(\vec{r})|^2 e^{-i\vec{q} \cdot \vec{r}}$$

Emission source size

* keep it in mind for

Deuteron wave function

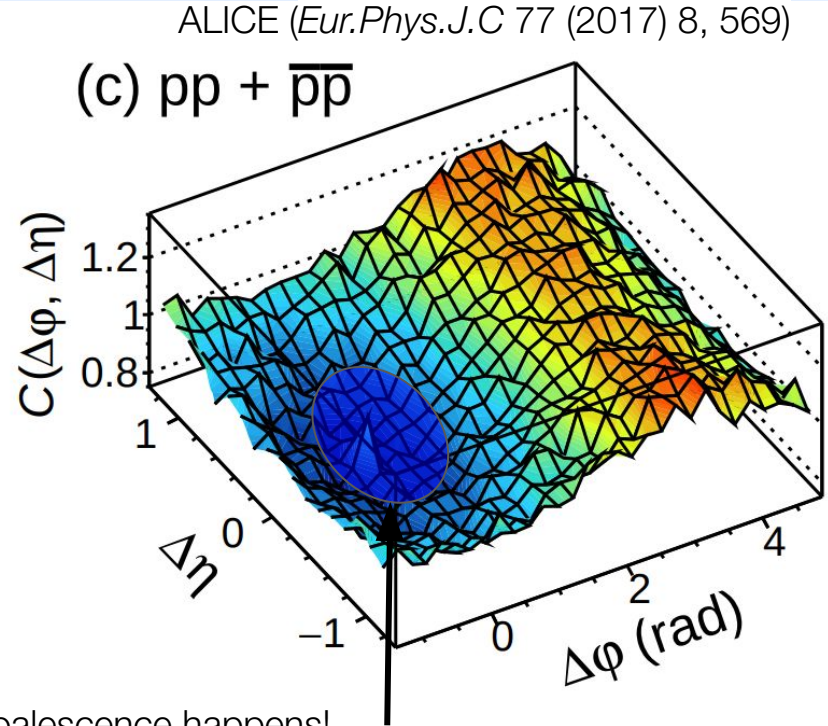
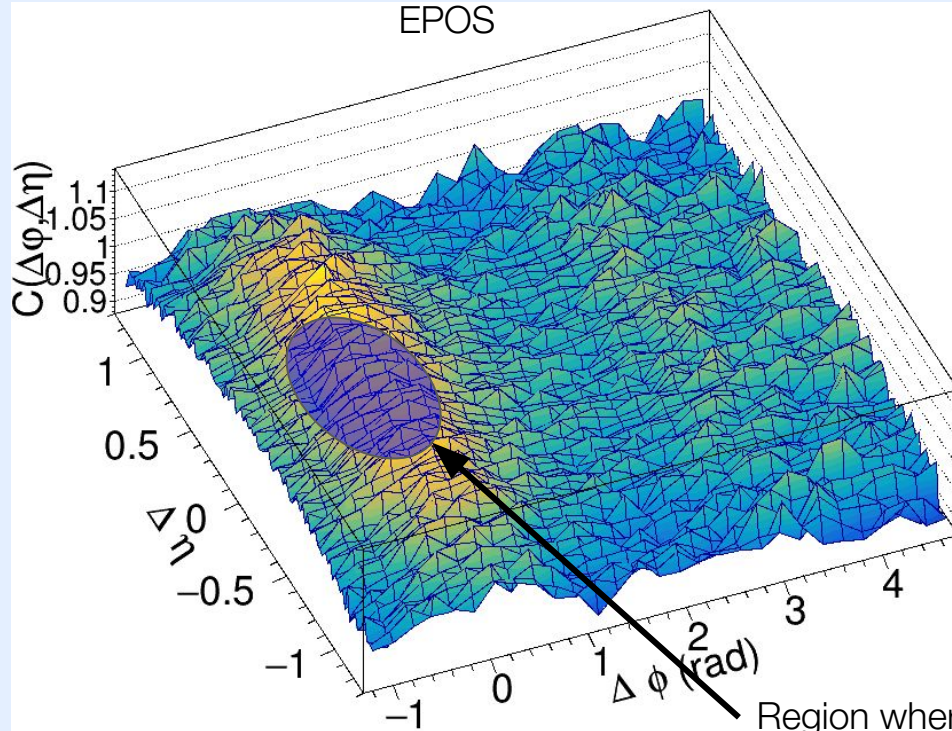


Testing different wave functions:

- **Hulthén:** Favoured by low energy scattering experiments
- **Gaussian:** Best description of currently available ALICE data
- **Two Gaussians:** Approximates Hulthén, easy to use in calculations
- **χ EFT:** Favoured by modern nuclear interaction experiments (e.g. Femtoscopy)

[1] Blum, Takimoto, PRC 99 (2019) 044913

$\Delta\eta$ - $\Delta\phi$ Correlation function



Region where coalescence happens!

Scheme

Propagation scheme:

- We obtain a scaling factor as a function of m_T from the source size measurement
- We move the primordials out radially until we reach the scaled distance
- This distance (\tilde{x}) is the same for both primordials of the pair

