Fluid-dynamic approach to heavy-quark diffusion in the quark-gluon plasma

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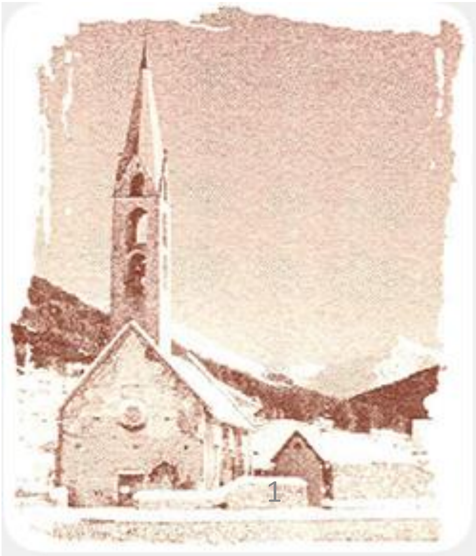
Based on
Capellino, Beraudo, Dubla, Floerchinger, Masciocchi, Pawlowski, Selyuzhenkov (Phys. Rev. D 106, 034021 (2022))
Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi (in preparation)

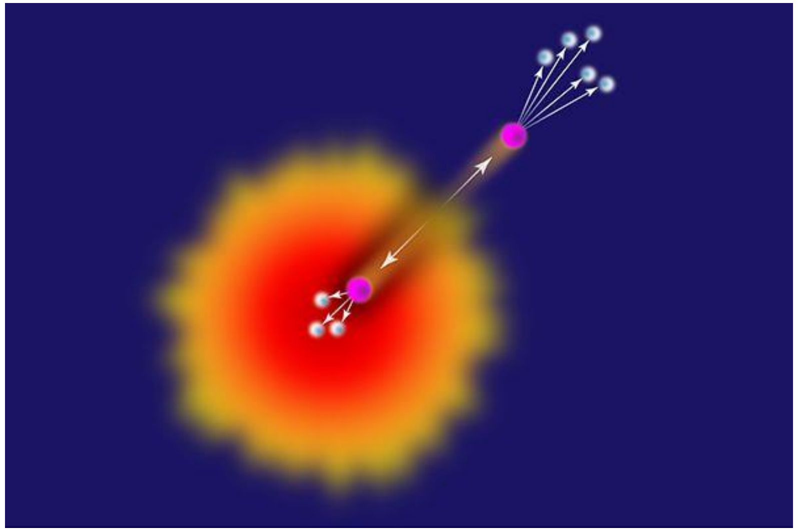


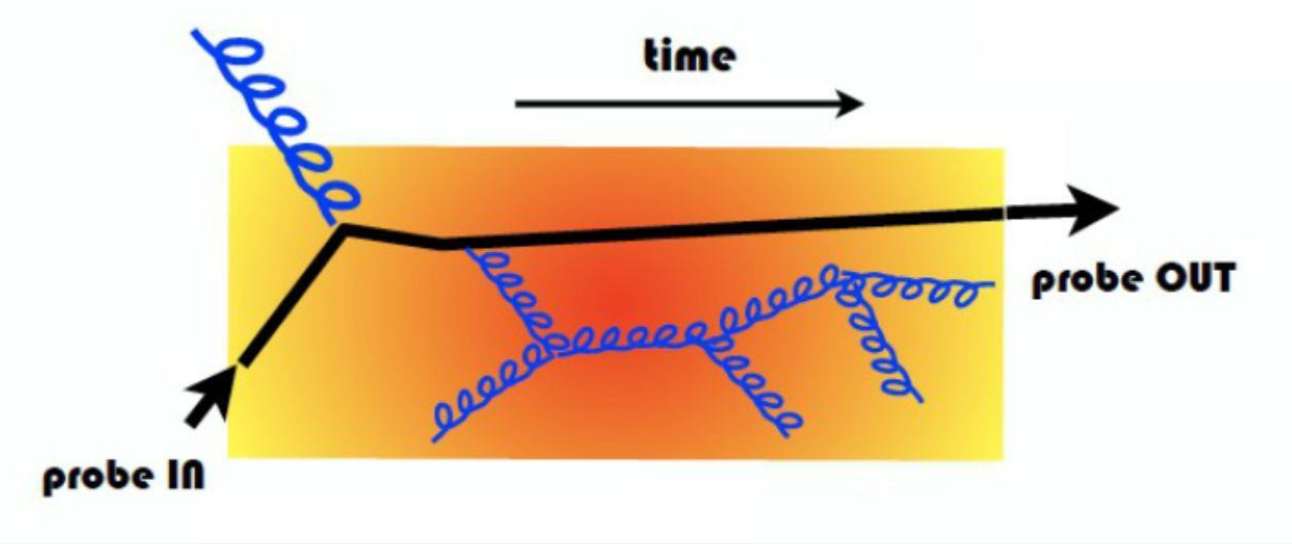
Heavy quarks as hard probes of the QGP

Heavy quarks (charm and bottom) in heavy-ion collisions

- Are produced via hard scatterings at the beginning of the collision: they go through all the stages of the expanding fireball

- They interact with light quarks and gluons in the QGP via elastic and radiative processes → they can lead to a better understanding of the parton-medium interaction





- They help us in the **characterization of the QGP**

- In the low p_T region they provide a window to study **equilibration processes**

Thermalization

If particles have enough time to interact with each other they will eventually relax to (at least local) thermal equilibrium.

- **Chemical equilibrium:**

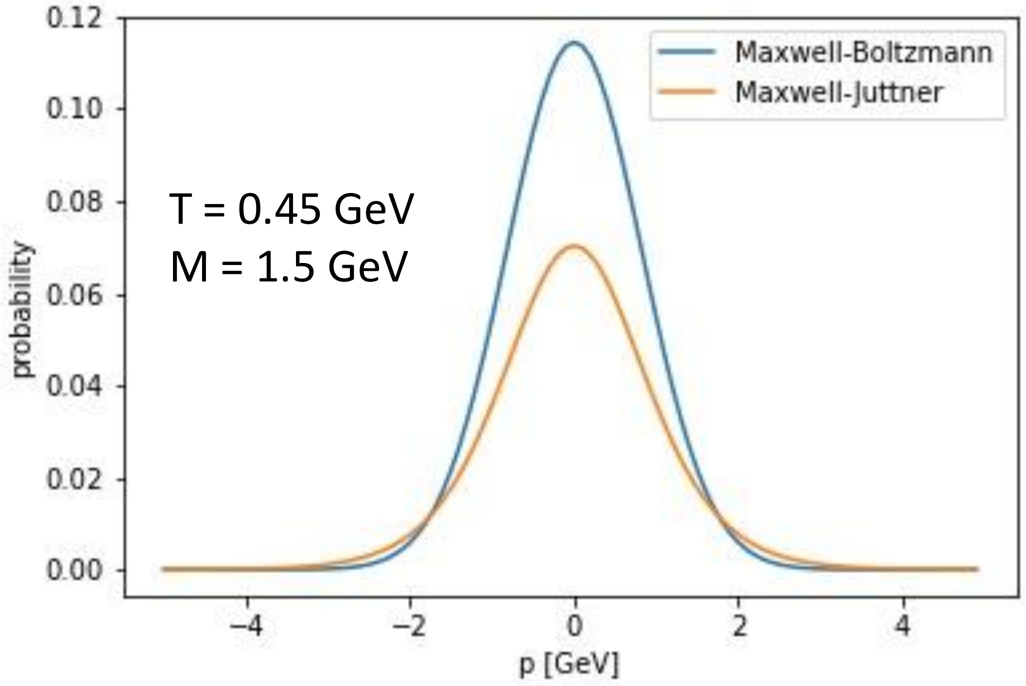
the particle multiplicity is given by a thermal distribution at chemical potential μ = 0
For HQs: initial hard production very far from chemical equilibrium! **Fugacity factor** needed.

- **Kinetic equilibrium:**

the momentum distribution of the particles approaches a Maxwell-Boltzmann (Maxwell-Juttner) distribution
For HQs: possibly get quite close to local kinetic equilibrium within the lifetime of the fireball .

fugacity factor

$$f(E,x) \sim \underbrace{e^{-E/T(x)} e^{\mu(x)/T(x)}}_{\text{in kinetic equilibrium}} + \underbrace{\delta f(E,x)}_{\text{out of kinetic equilibrium}}$$



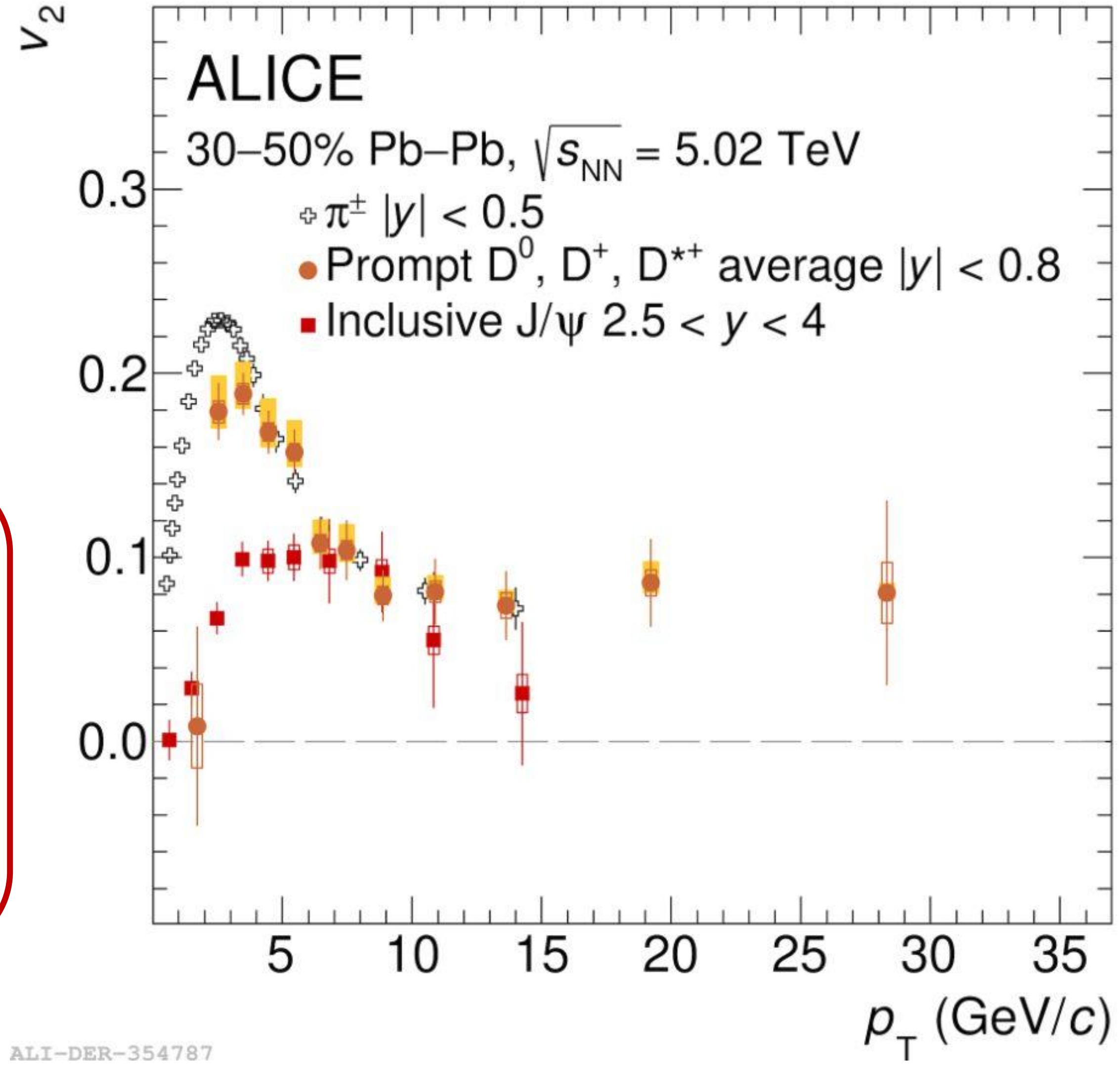
Hints of thermalization

PLB 813 (2021) 136054 ALICE coll.

Significant measurements of J/Ψ and D mesons elliptic flow

- How strong are charm quarks interacting with the partons in the QGP?

- Do they interact long enough with the medium to be considered part of the medium itself?



Driven by the experimental data, we propose a new way of studying the heavy-quark dynamics in the QGP.

We assume heavy quarks interacted long enough with the medium to approach local kinetic equilibrium

→ we treat them with a **fluid-dynamic approach**!

Heavy-quark conserved current

We write down a current associated to the conservation of QQbar pairs in the QGP medium

$$N^\mu = n\, u^\mu + \nu^\mu$$

HQ density

HQ diffusion current

1) conservation law

$$\partial_\mu N^\mu = 0$$

2) Equation of motion for the diffusion current

gives rise to the fugacity factor

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \left(\frac{\mu}{T} \right)$$

Relaxation time

HQ diffusion coefficient

Charm quark relaxation time

We computed the **relaxation time** and **diffusion coefficient** associated to charm quarks by integrating the first moment of the Fokker-Planck equation

$$p^{\mu}\partial_{\mu} f(\mathbf{p},\mathbf{x},\mathrm{t}) = \frac{\partial}{\partial p^i}\left[\underbrace{A(p)}_{\text{Drag coefficient}} p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}\underbrace{D(p)}_{\text{Momentum-diffusion coefficient}} f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

Charm quark relaxation time

We computed the **relaxation time** and **diffusion coefficient** associated to charm quarks by integrating the first moment of the Fokker-Planck equation.

$$\tau_n \propto D_s$$

Where the **spatial diffusion coefficient** is defined as

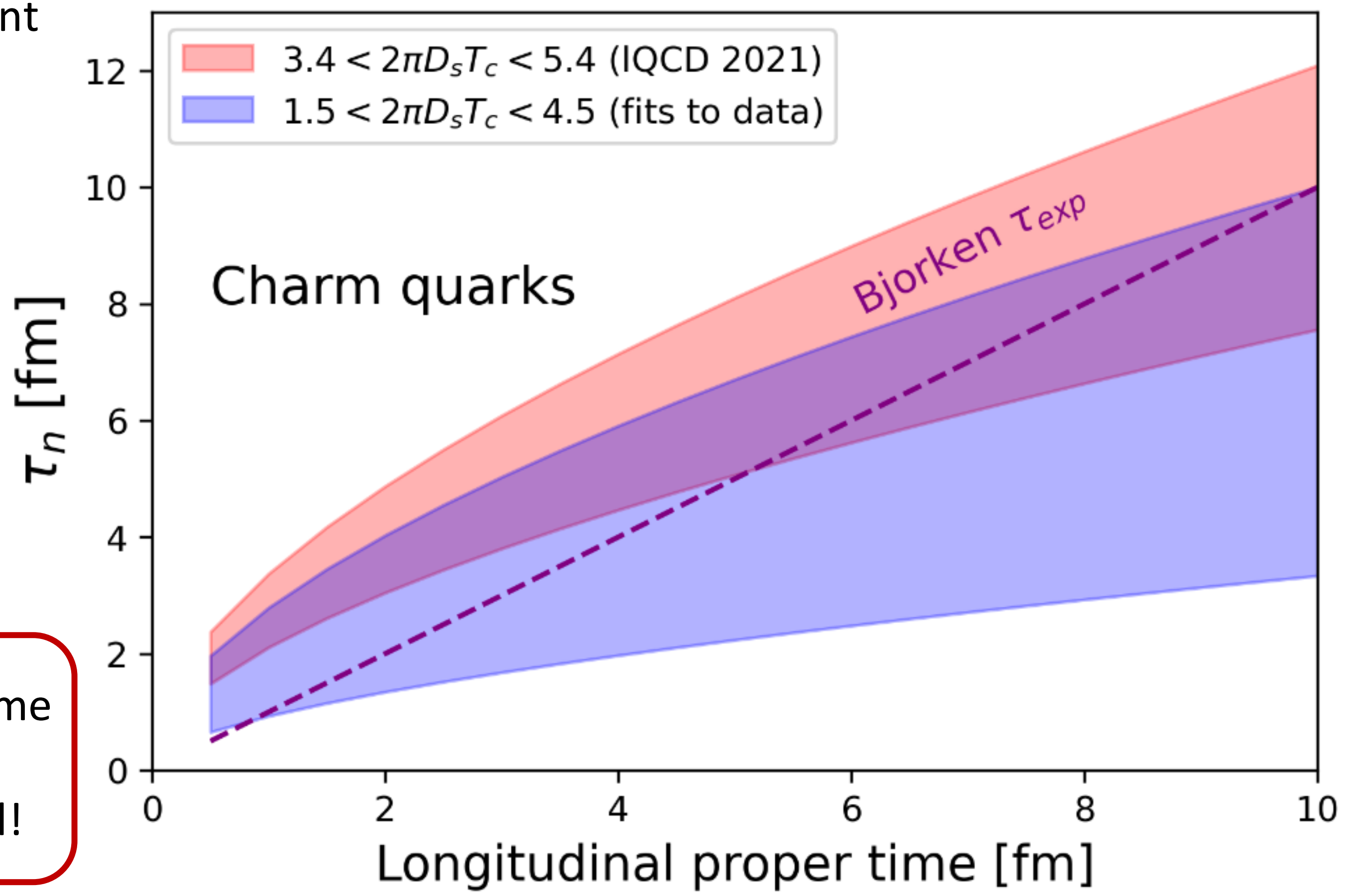
$$D_s = \lim_{k \to 0} \frac{T}{MA(k)}$$

Relaxation time is much shorter than typical expansion time of the QGP in Bjorken flow

→ Fluid-dynamic description of charm looks meaningful!

PRD 106 (2022) 034021

Bjorken flow: $v_x = v_y = 0 \quad v_z = z/t$



lQCD 2021: PRD 103 (2021) 014511 Altenkort et al.
ALICE fits to data: JHEP 01 (2022) 174 ALICE coll.

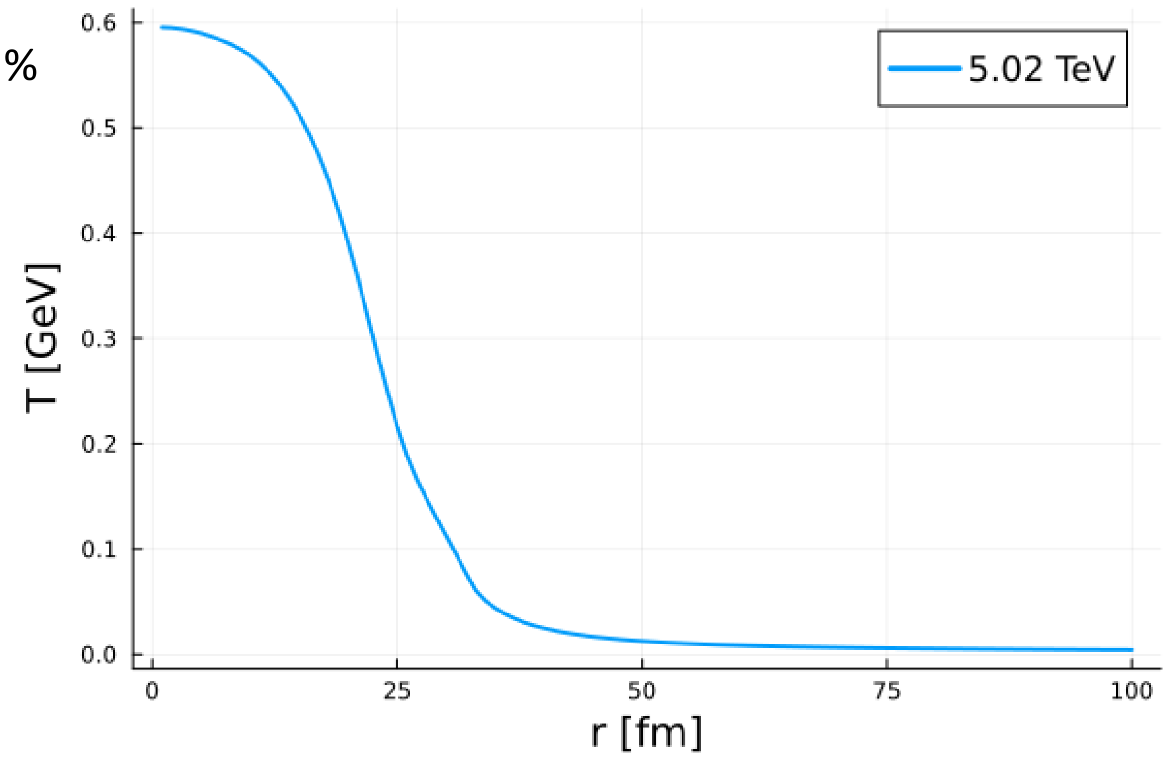
Calculations for charmed hadrons

We employ our fluid-dynamic framework **Fluid*u*M** to compute momentum distributions for charmed hadrons.

For details on Fluid*u*M
PRC 100, 014905 (2019) Floerchinger et al.
JHEP 06 (2020) 044 10.1007 Devetak et al.

Initial conditions

- TRENTO profile for PbPb at 5.02 TeV used as initial entropy density
- Initial time τ_0 = 0.4 fm
- Initial normalization such that the number of protons in the 0-5 % centrality class is reproduced ($\sim$75 protons ALICE coll. PRC 101, 044907 (2020)).
- Bulk viscosity = 0
- Shear viscosity over entropy = 0.2



A **uniform fugacity factor** is assumed at the freeze-out surface i.e. charm density is assumed uniform in the whole fireball!

Notice: similar procedure to SHMc, uniform fugacity + hydro parameters fitted to light flavours in the core.

SHMc JHEP07(2021)035 10.1007 Andronic et al.

Integrated yields for charmed hadrons

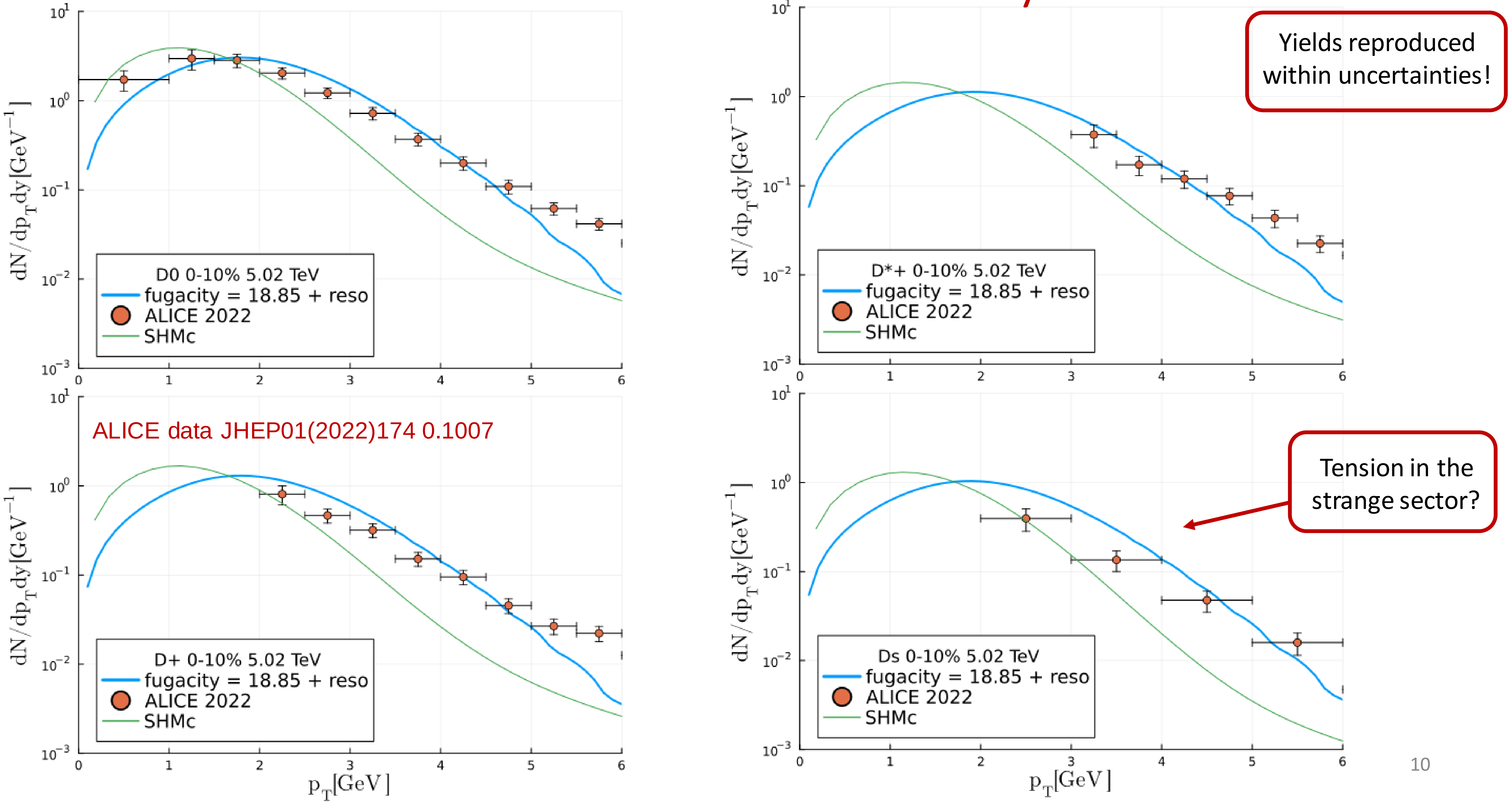
Our results fo integrated yields show good agreement with SHMc estimates and ALICE measurements of D mesons.
Main differences:

- J/Ψ yield: hydro is not enough to describe all of them?
- Lc+: without charmed baryon-enhancement, also SHMc underpredicts Lc

Species	Thermal yield	With resonance decays	SHMc	ALICE
J/Ψ	0.07		0.127	0.115 ± 0.007 (stat) ± 0.012 (syst)
D0	1.65	6.81	6.81	6.819 ± 0.457 (stat) ± 0.9 (syst)
D+	1.61	2.95	3.02	3.041 ± 0.073 (stat) ± 0.155 (syst)
D*+	2.17	2.67	2.67	3.803 ± 0.037 (stat) ± 0.085 (syst)
D_s	0.92	2.42	2.36	1.89 ± 0.007 (stat) ± 0.55 (syst)
Lc	0.296	1.46	1.64	3.27 ± 0.42 (stat) ± 0.45(syst)

Yields dN/dy in 0-10 %
ALICE Data: ALICE coll. D mesons JHEP 01 (2022) 174, Ds PLB 807 (2022) 136986,J/Ψ arxiv:2211.04384, Lc arxiv: arXiv:2112.08156
SHMc: arxiv:2104.12754

Results for D mesons in 0-10% centrality class



Results for J/Ψ in 0-10% centrality class

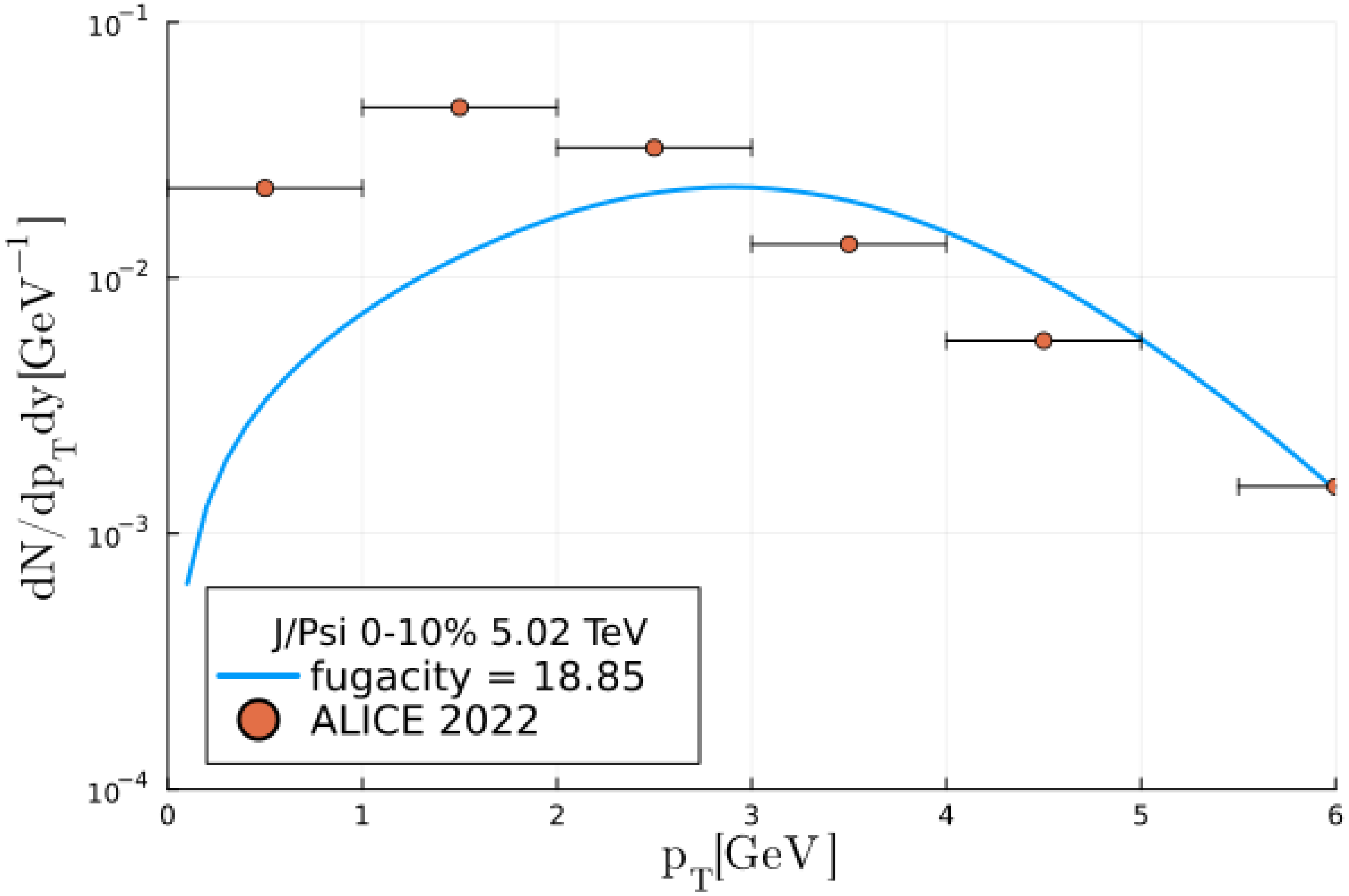
The yield of J/Psi is underestimated by our hydro prediction by almost 40%.

Possible explanations:

- Contribution from regeneration mechanisms could enhance the low transverse momentum region

- Non-uniform fugacity might make a difference
- Different hydro parameter could move the peak of radial flow to lower momenta

- Radial flow of charm is not the same as the rest of the fluid (in D mesons we don't see it since charm is dragged by a light quark)



...under investigation...

Summary and outlook

Summary

- We computed particle spectra for charmed hadrons assuming thermalized charm quarks uniformly distributed in the fireball
    - Good agreement for all D mesons
    - Underestimation of charmed baryons
    - Underestimation of J/Ψ al low p_T

Outlook

- Build a model for the initial HQ density and study the effect of having a non-uniform fugacity at the freeze-out surface
- Investigate effect of recombination mechanisms on J/Ψ at low p_T
- Fluid-parameter optimization in PbPb at 5.02 TeV not yet done: might e.g. lead to different radial flow which would move the peak of the final momentum distributions

Thank you for your attention!

Back up

HQ distribution function

Heavy quarks are initially produced **out of kinetic equilibrium**: they cannot be described by a thermal Boltzmann distribution at the same temperature of the QGP

$$f(E,x) \sim e^{-E/T(x)} \underbrace{e^{\mu(x)/T(x)}}_{\text{Fugacity: needed to match the number of QQbar pairs}} + \underbrace{\delta f(E,x)}_{\text{Out-of-equilibrium component}}$$

The **diffusion current** is given by the first moment of the out-of-equilibrium distribution function and the **density** is given by integrating the equilibrium part, in the Landau frame.

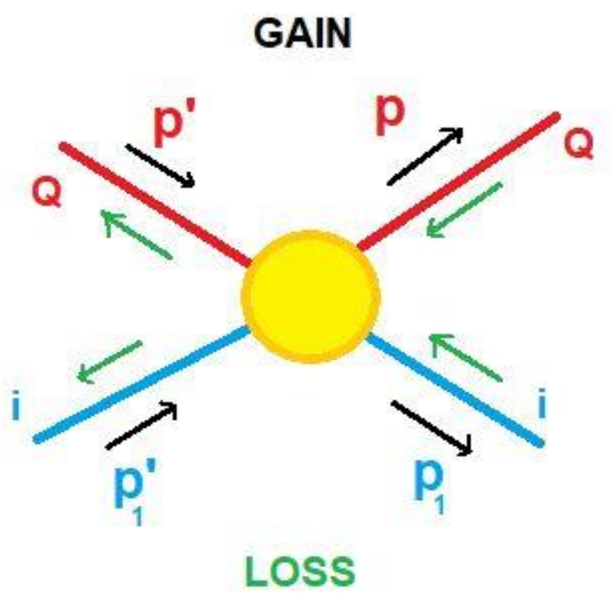
PRD 85 (2012) 114047 Denicol et al.

Boltzmann equation

We start with the relativistic Boltzmann equation, which studies how the distribution function of the heavy quark varies in time

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = C[f(\mathbf{p},\mathbf{x},\mathrm{t})]$$

where the **collision integral** reads $$C[f(\mathbf{p},\mathbf{x},\mathrm{t})] = \int dP_1 dP_1' dP' W_{p',p_1' \to p,p_1} \left(f(\mathbf{p}') f_i(\mathbf{p}_1') - f(\mathbf{p}) f_i(\mathbf{p_1}) \right)$$



Heavy quarks

- treated like Brownian particles
- asymptotically reach thermal equilibrium

Great effort was put to determine the value of the **transport coefficients** that encode the interaction between the heavy quark and the partons from the medium.

Equation of motion for the HQ diffusion current

We derive hydrodynamic equations of motion from kinetic theory (Boltzmann) in a **Fokker-Planck approximation**

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

Drag coefficient - inverse relaxation time

Momentum-diffusion coefficient

The **spatial diffusion coefficient** is defined as $D_s = \lim_{k\to 0}\frac{T}{MA(k)}$

Equation of motion for the HQ diffusion current

We derive hydrodynamic equations of motion from kinetic theory (Boltzmann) in a **Fokker-Planck approximation**

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

By integrating the first moment of the equation

$$\int dP p^\nu p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \int dP p^\nu \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

We obtain a relaxation-type equation for the diffusion current

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i\left(\frac{\mu}{T}\right)$$

Relaxation time

HQ diffusion coefficient

$$\tau_n = \frac{D_s I_{31}}{T P_o}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Fluid-dynamic transport coefficients

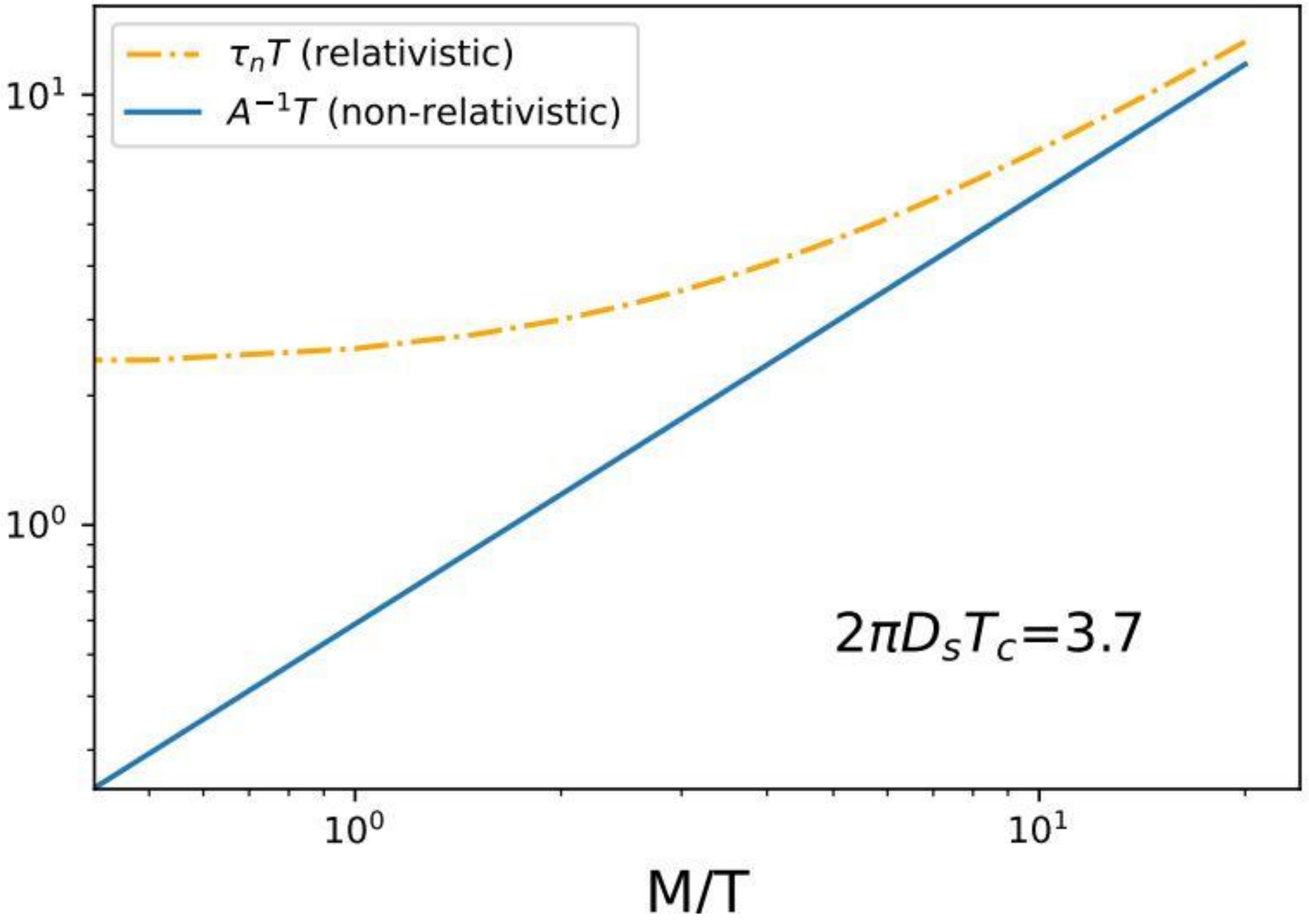
$$\boxed{\begin{aligned} \tau_n &= \frac{D_s I_{31}}{T P_o} \\ \kappa_n &= \frac{T^2}{D} n = D_s n \end{aligned}}$$

$$I_{31} = \frac{1}{3} \int dP p^0 p^2 f_0(p)$$

$$p^0 \sim M \qquad I_{31} \sim M P_0$$

$$\tau_n \sim \frac{D_s M P_0}{T P_0} = D_s \frac{M}{T}$$

$$\boxed{\tau_n T \sim (2\pi D_s T) \frac{1}{2\pi} \frac{M}{T} = A^{-1} T}$$

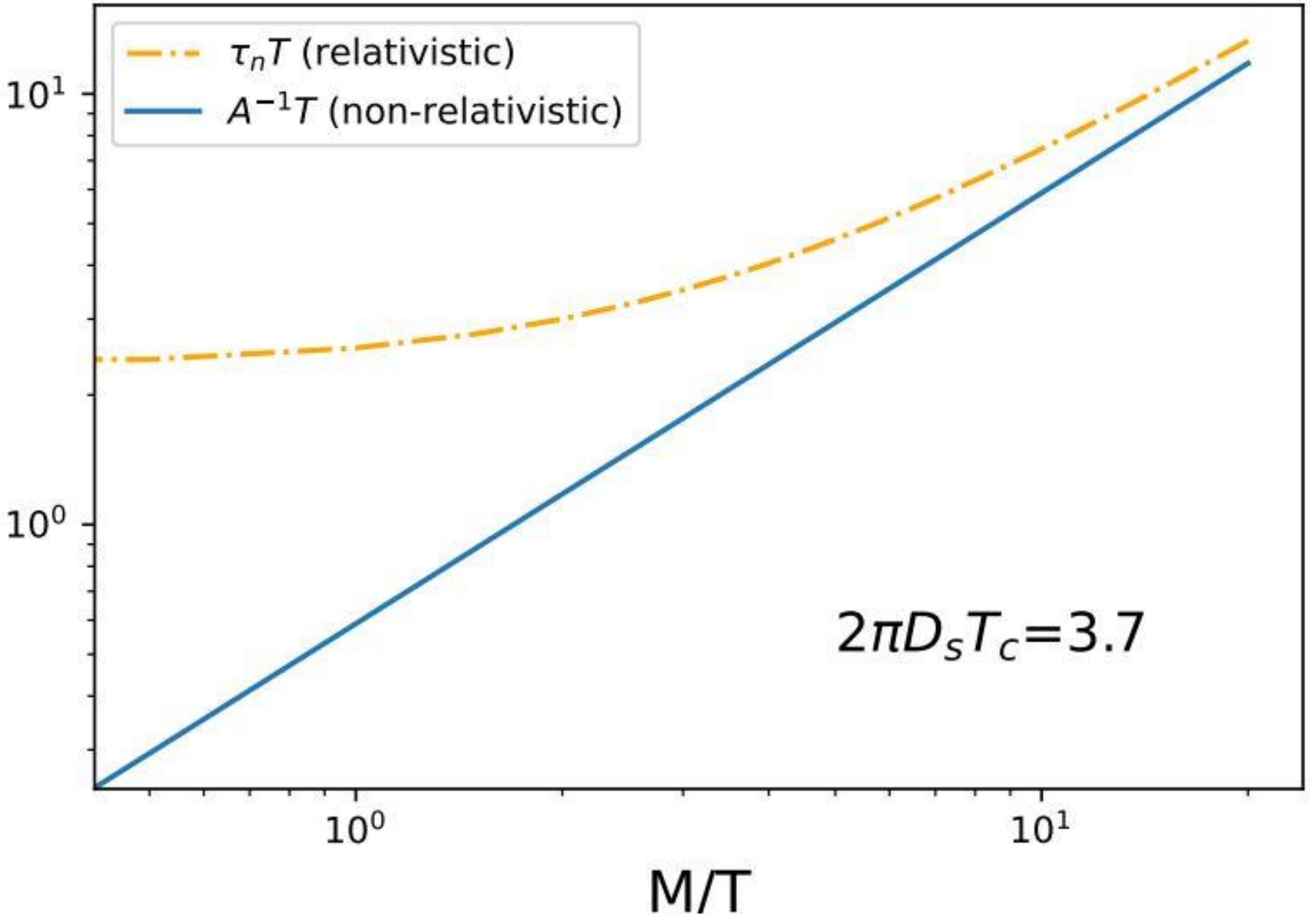


Important check: the hydrodynamic **relaxation time** is consistent with the relaxation time found within the Fokker-Planck approach in the non-relativistic limit.

Fluid-dynamic transport coefficients

$$\tau_n = \frac{D_s I_{31}}{T P_o}$$
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

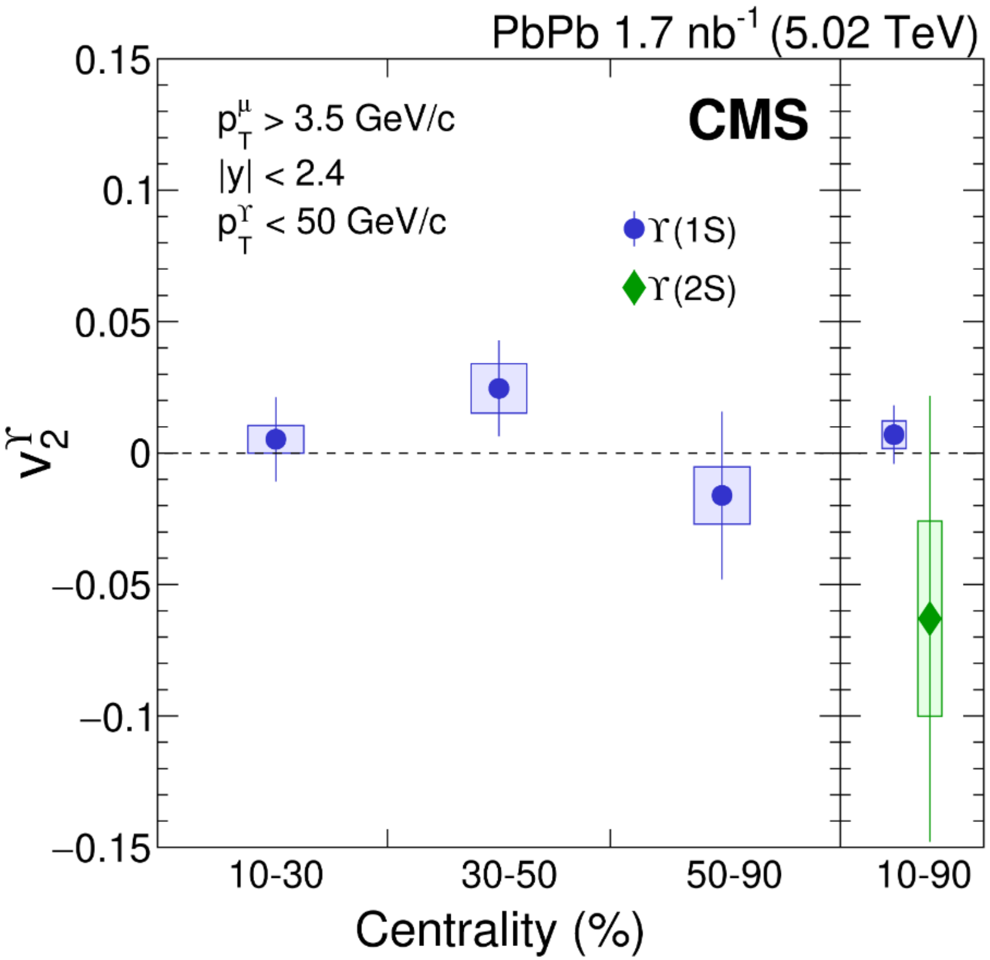
The relation between **spatial diffusion coefficient** and **momentum-diffusion coefficient** - usually found in a non-relativistic setup - arises naturally also in a relativistic framework.



This work, PRD 106 (2022) 034021

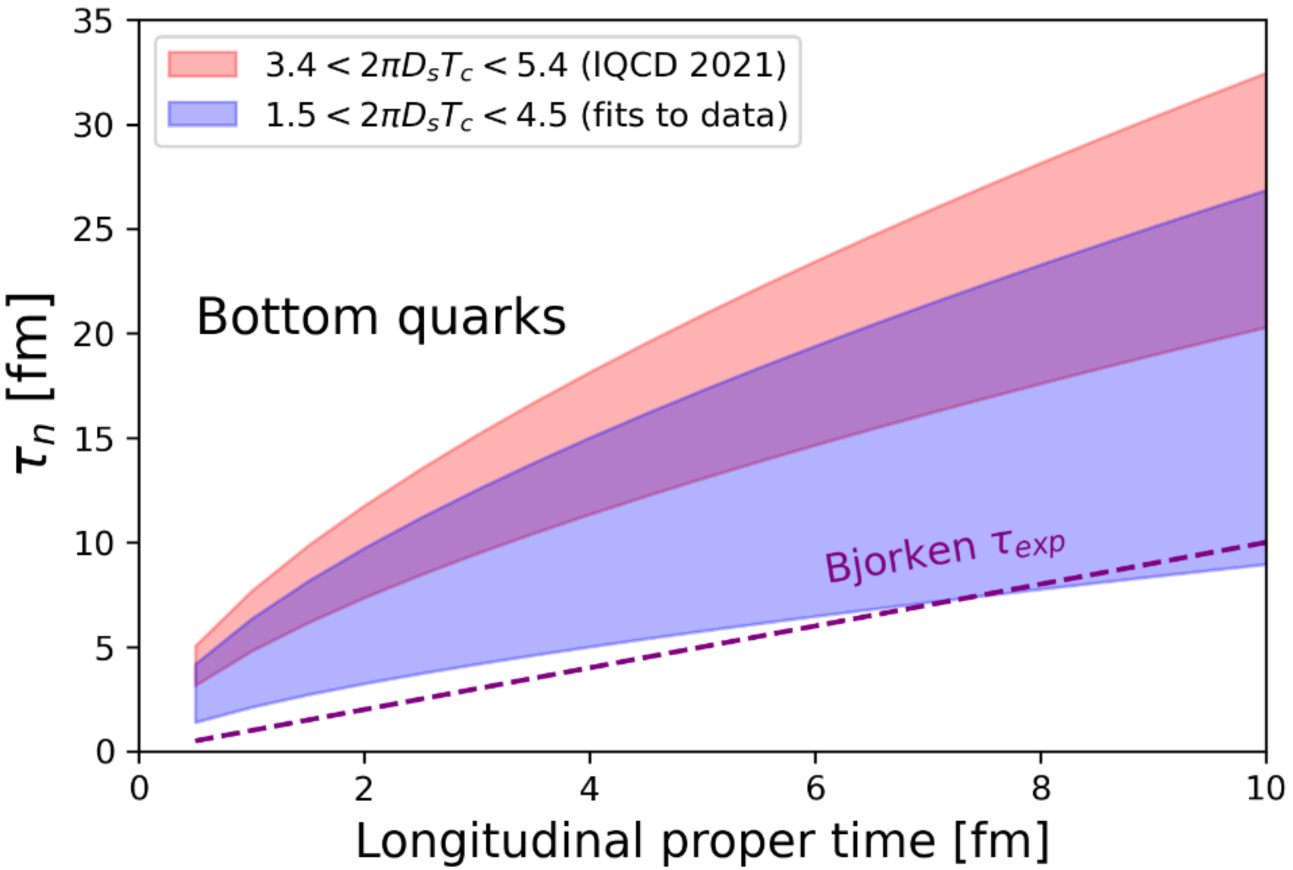
Results for bottom quarks

- From new data at low p_T, can we see partial thermalization of beauty quarks in the QGP?
- Can we constrain estimates for the transport coefficients by systematic comparison with data?

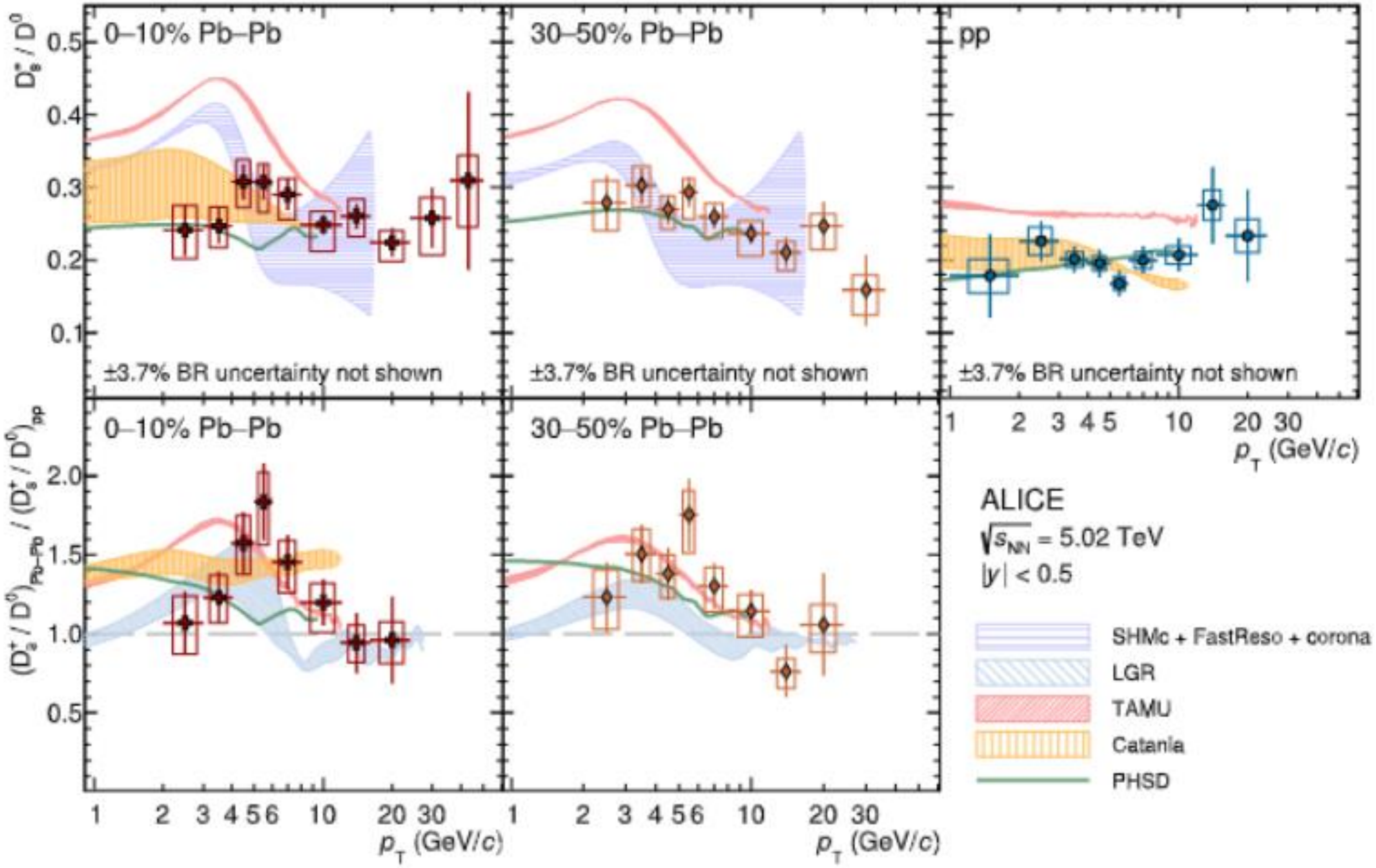


PLB 819 (2021) 136385 CMS coll.

This work, PRD 106 (2022) 034021

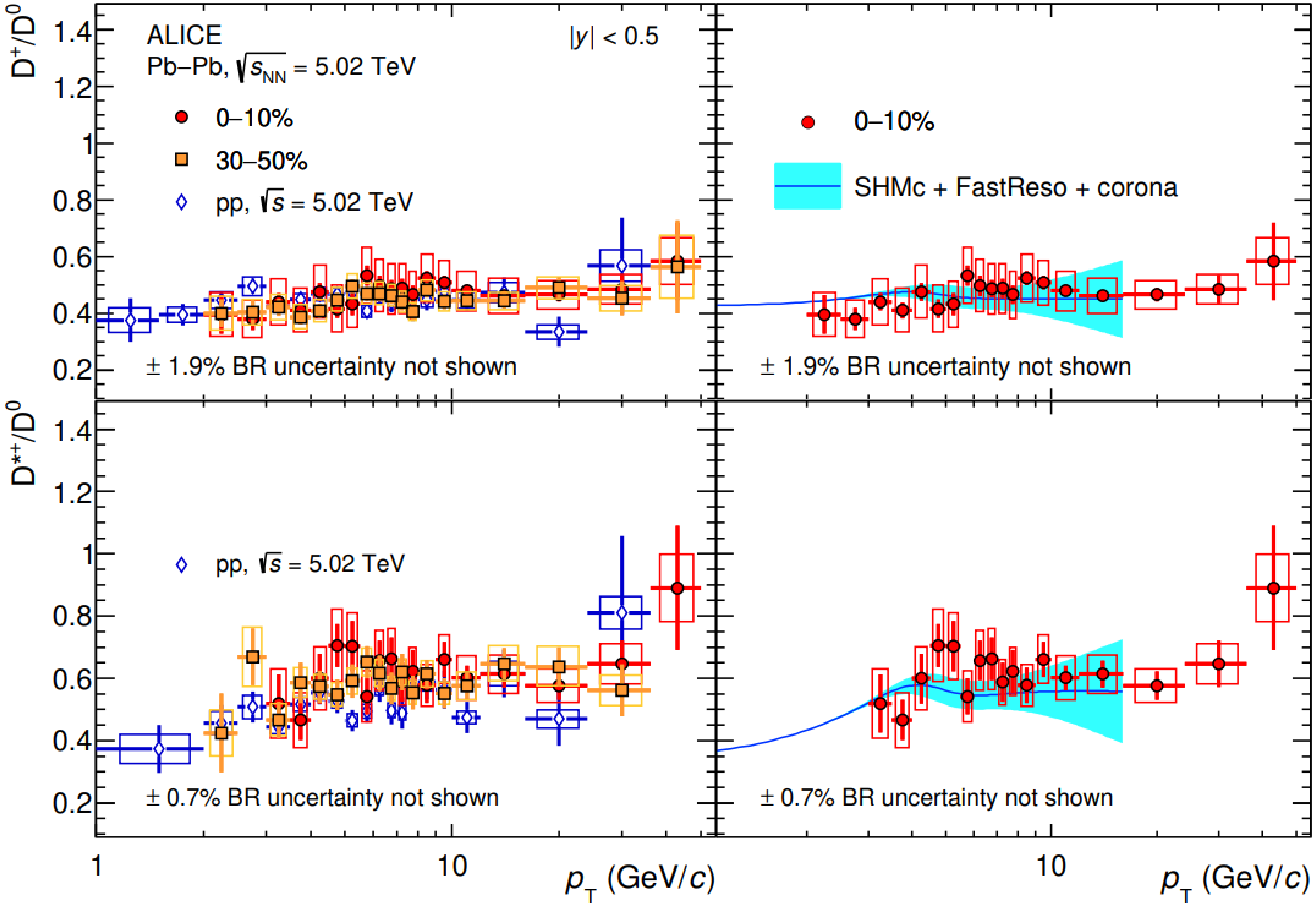


Ds/D0 ratio



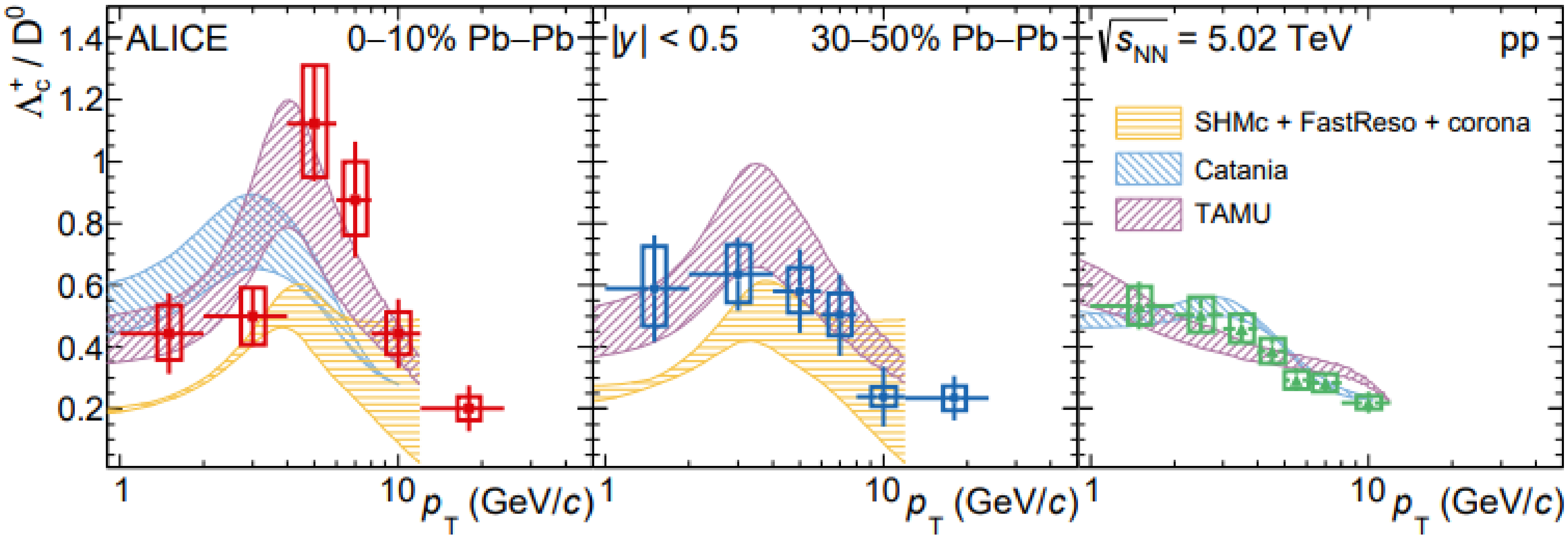
0–10% Pb–Pb
30–50% Pb–Pb
pp
±3.7% BR uncertainty not shown
±3.7% BR uncertainty not shown
±3.7% BR uncertainty not shown
0–10% Pb–Pb
30–50% Pb–Pb
ALICE
$\sqrt{s_{NN}}$ = 5.02 TeV
|y| < 0.5
SHMc + FastReso + corona
LGR
TAMU
Catania
PHSD
p_T (GeV/c)
D_s^+ / D^0
$(D_s^+/D^0)_{Pb-Pb} / (D_s^+/D^0)_{pp}$

D+/D0 ratio



ALICE
Pb–Pb, $\sqrt{s_{NN}}$ = 5.02 TeV
|y| < 0.5
0–10%
30–50%
pp, $\sqrt{s}$ = 5.02 TeV
± 1.9% BR uncertainty not shown
SHMc + FastReso + corona
± 0.7% BR uncertainty not shown
D^+/D^0
D^{*+}/D^0
p_T (GeV/c)
1
10
0.2
0.4
0.6
0.8
1
1.2
1.4

Lc/D0 ratio



ALICE
0–10% Pb–Pb
|y| < 0.5
30–50% Pb–Pb
$\sqrt{s_{NN}}$ = 5.02 TeV
pp
SHMc + FastReso + corona
Catania
TAMU
Λ_c^+ / D^0
1.4
1.2
1
0.8
0.6
0.4
0.2
1
10
p_T (GeV/c)

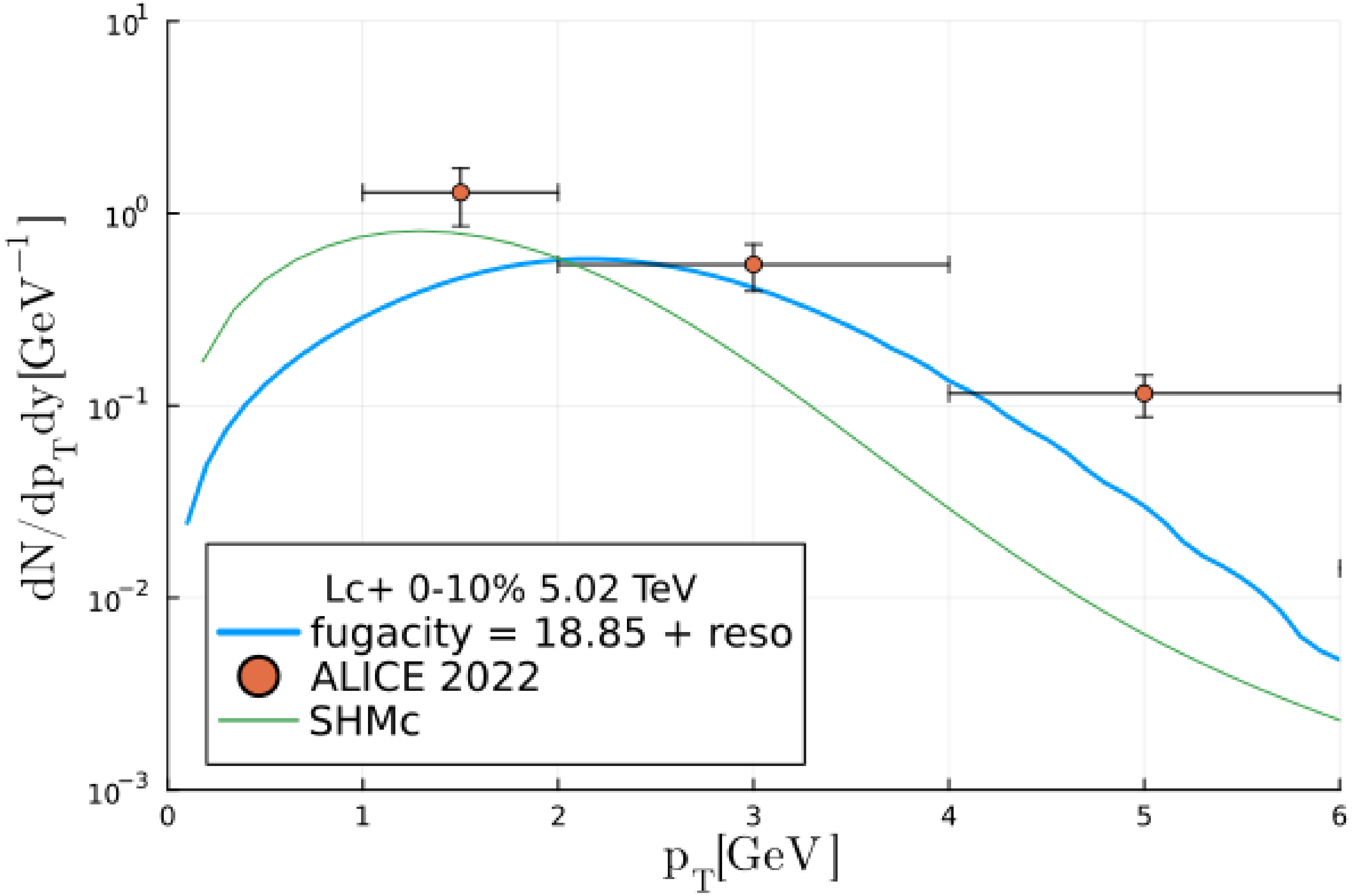
Results for Λc baryons in 0-10% centrality class

The yield of Λc baryons is underestimated by our hydro prediction and by the SHMc.

Possible explanations:

- Missing charm resonances?
- Coalescence mechanisms?

Problem under debate in the community.



…under investigation…