JLU NEUE WEGE. SEIT 1607.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

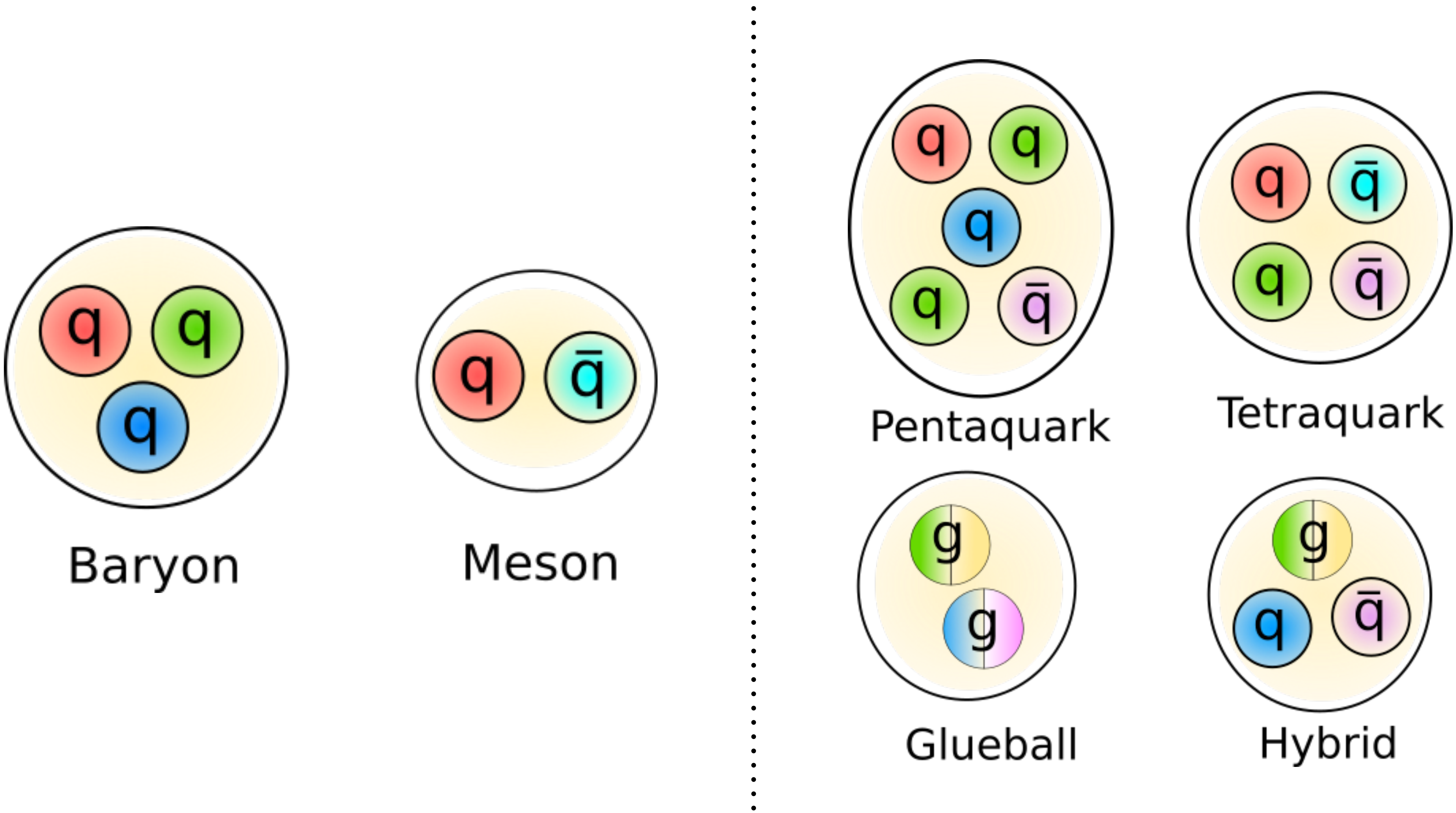
HFHF Helmholtz Forschungsakademie Hessen für FAIR

Winter Meeting
Bormio 2023

Exotic mesons with functional methods

with Markus Huber, Gernot Eichmann, Joshua Hoffer, Nico Santowski, Paul Wallbott

Hadrons: baryons, mesons and ... exotics !

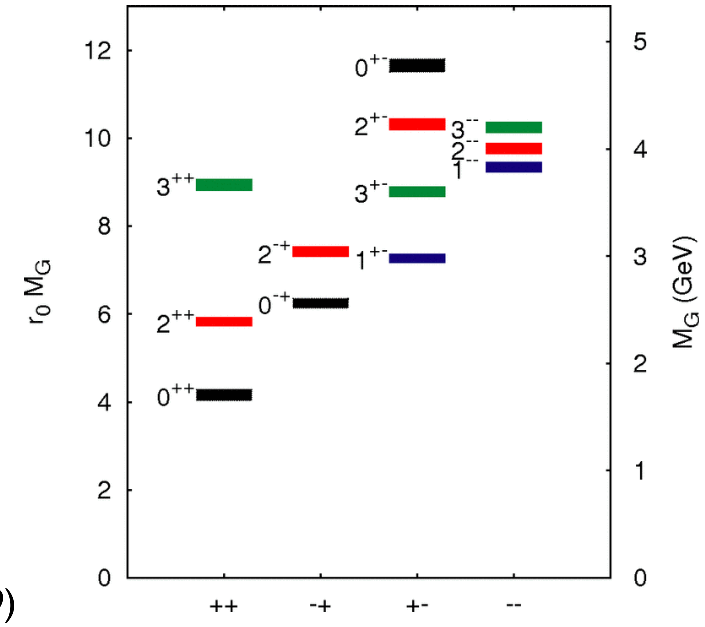


ordinary hadrons

'exotic' hadrons

Glueballs

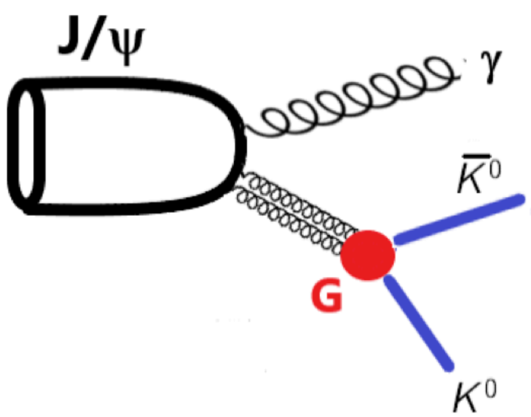
Theory:

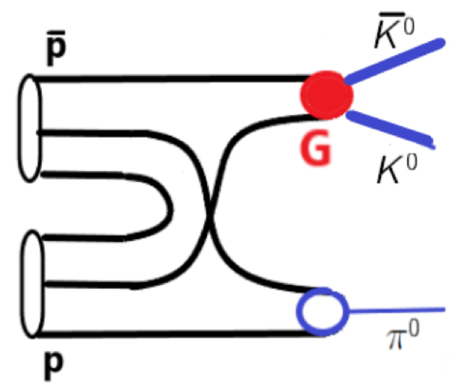


Morningstar and Peardon, PRD 60 (1999)

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \begin{pmatrix} |n\bar{n}\rangle \\ |s\bar{s}\rangle \\ |gg\rangle \end{pmatrix}$$

Experiment:





Klempt, arXiv:2211.12901

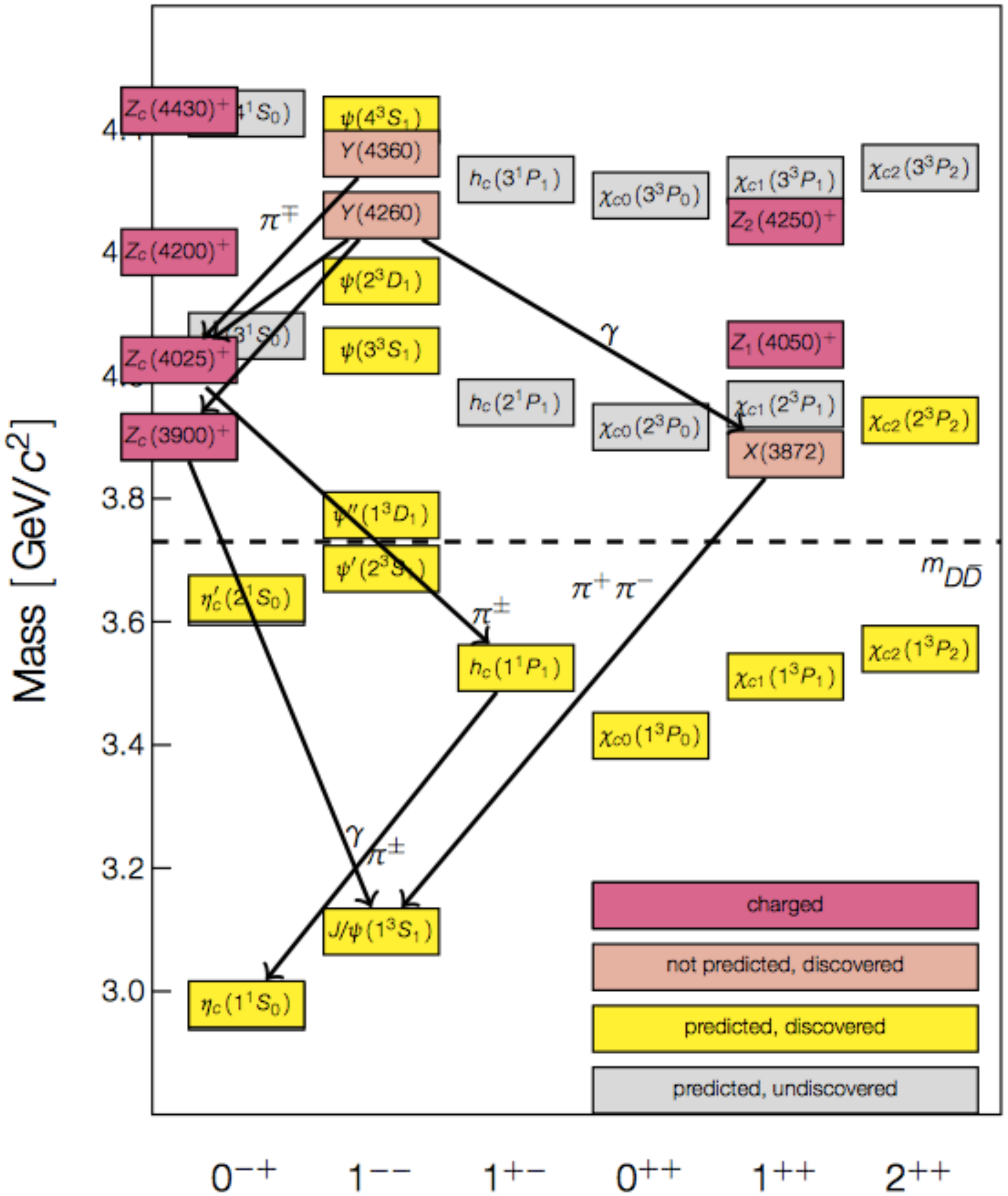
$$M_{0^{++}} = 1865 \pm 25^{+10}_{-30}$$

Sarantsev et al., PLB 816 (2021) 136227

$$M_{0^{++}} \to f_0(1710)$$

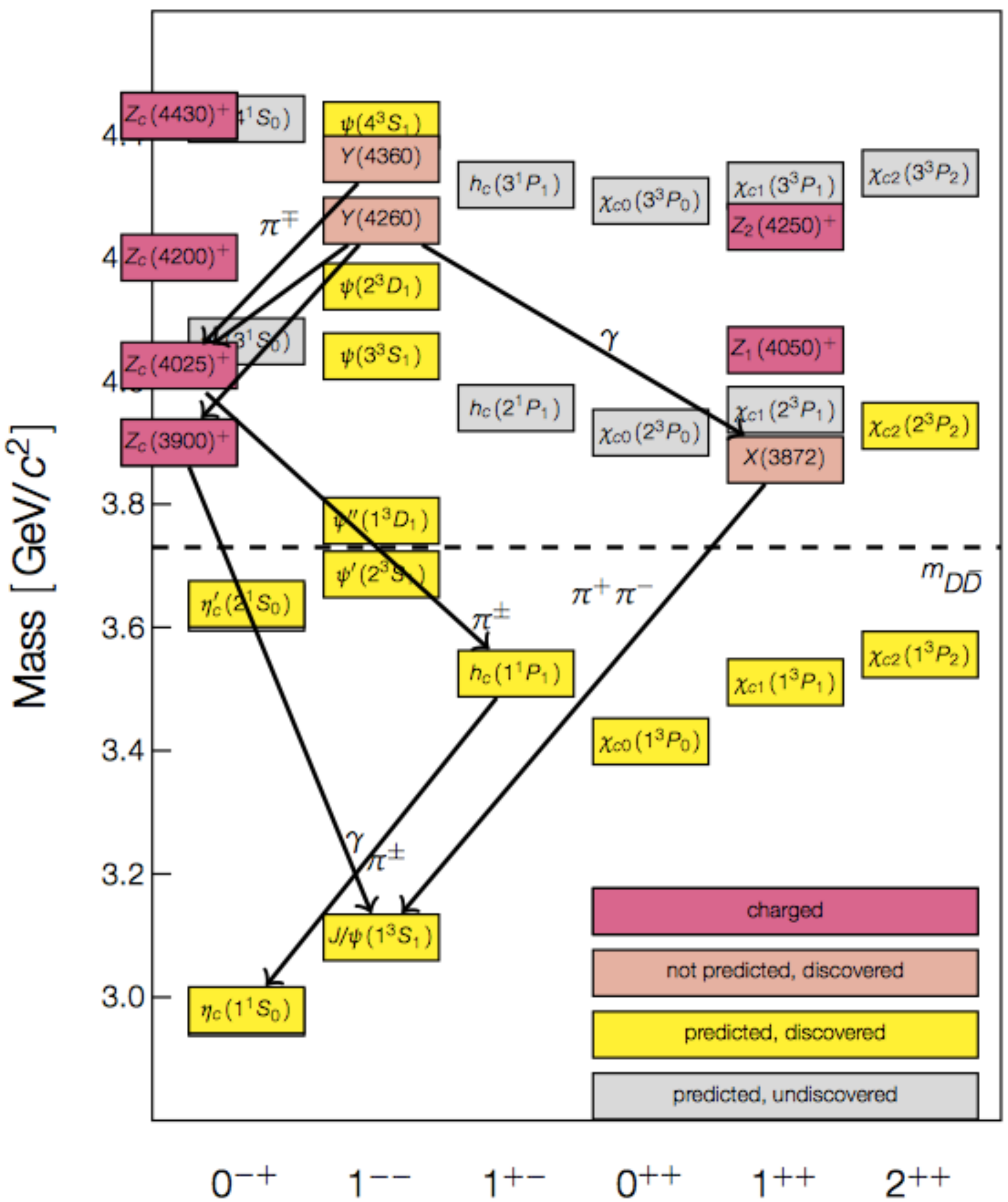
Rodas et al. (JPAC), EPJC 82 (2022) 1, 80

Tetraquark candidates with $cq\bar{q}\bar{c}$-content



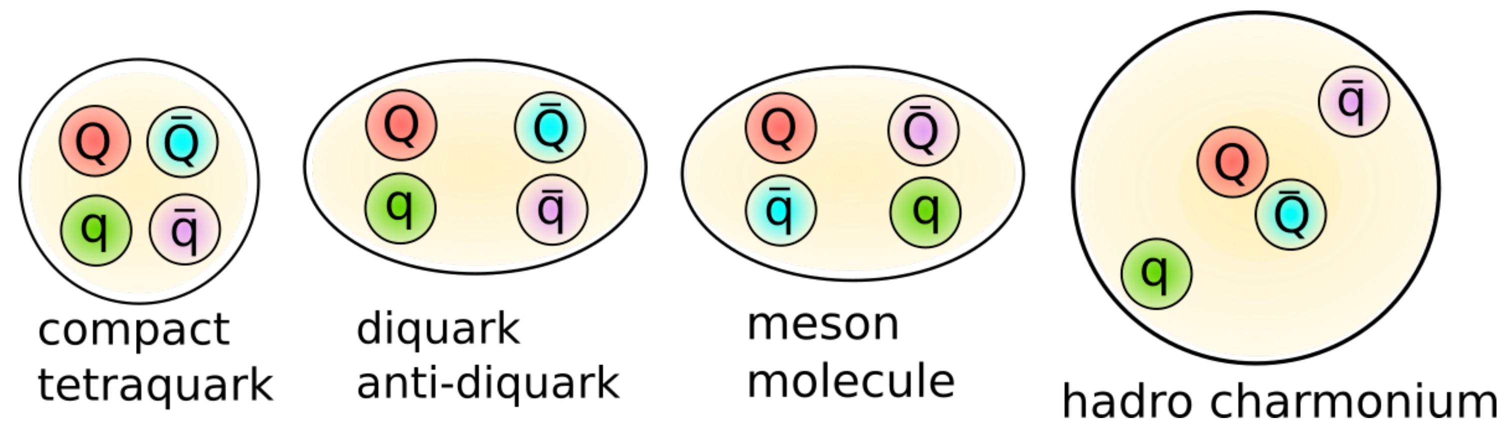
Many new unexpected states found: Belle, BABAR, BES, LHCb …

Tetraquark candidates with $cq\bar{q}\bar{c}$-content



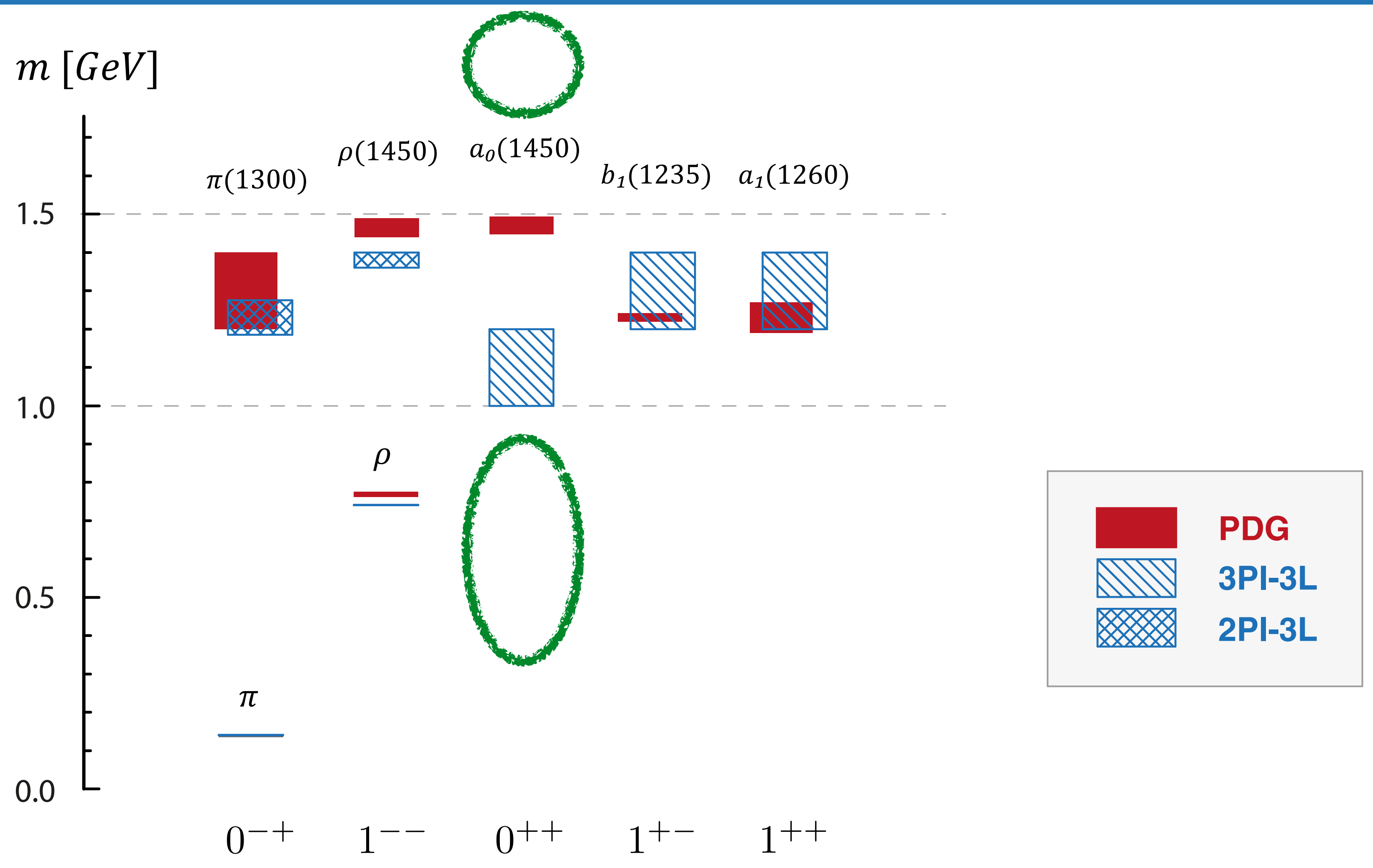
Many new unexpected states found: Belle, BABAR, BES, LHCb …

Internal structure ??



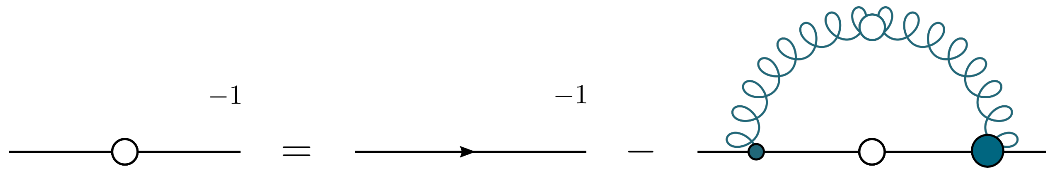
Related to details of underlying QCD forces between quarks and gluons

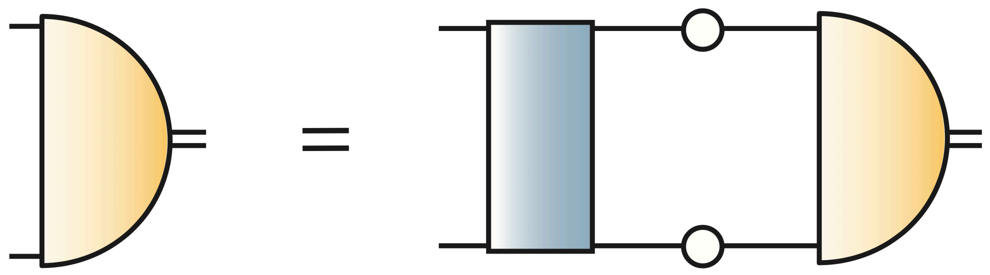
Overview

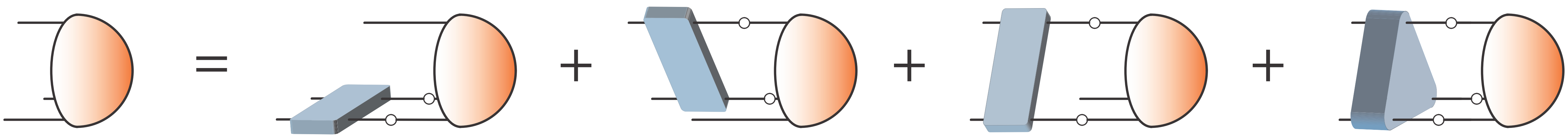


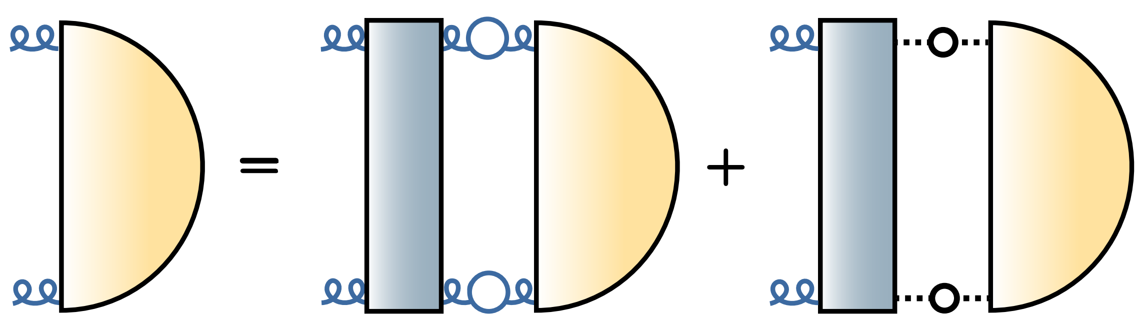
1. Conventional mesons
2. Glueballs: pure Yang-Mills
3. Four-quark states: mixing and heavy-light states

BSEs:







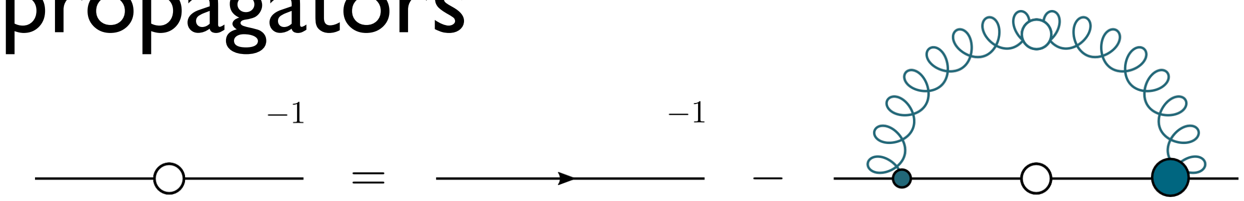


Eigenvalue equations: masses and wave functions

Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{- \int d^4x \left(\bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2\right)\right\}$$

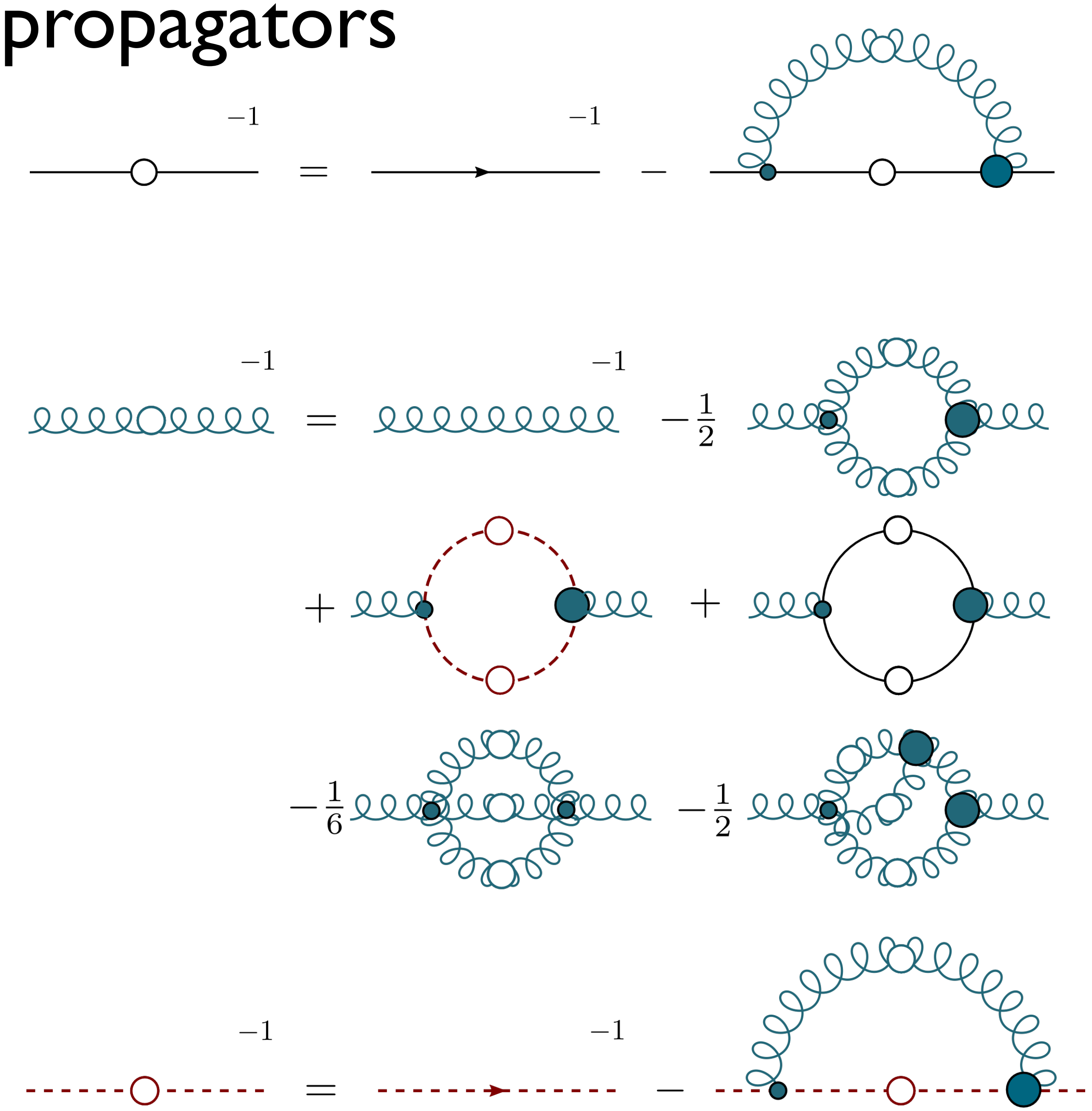
propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{-\int d^4x \left(\bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2\right)\right\}$$

propagators

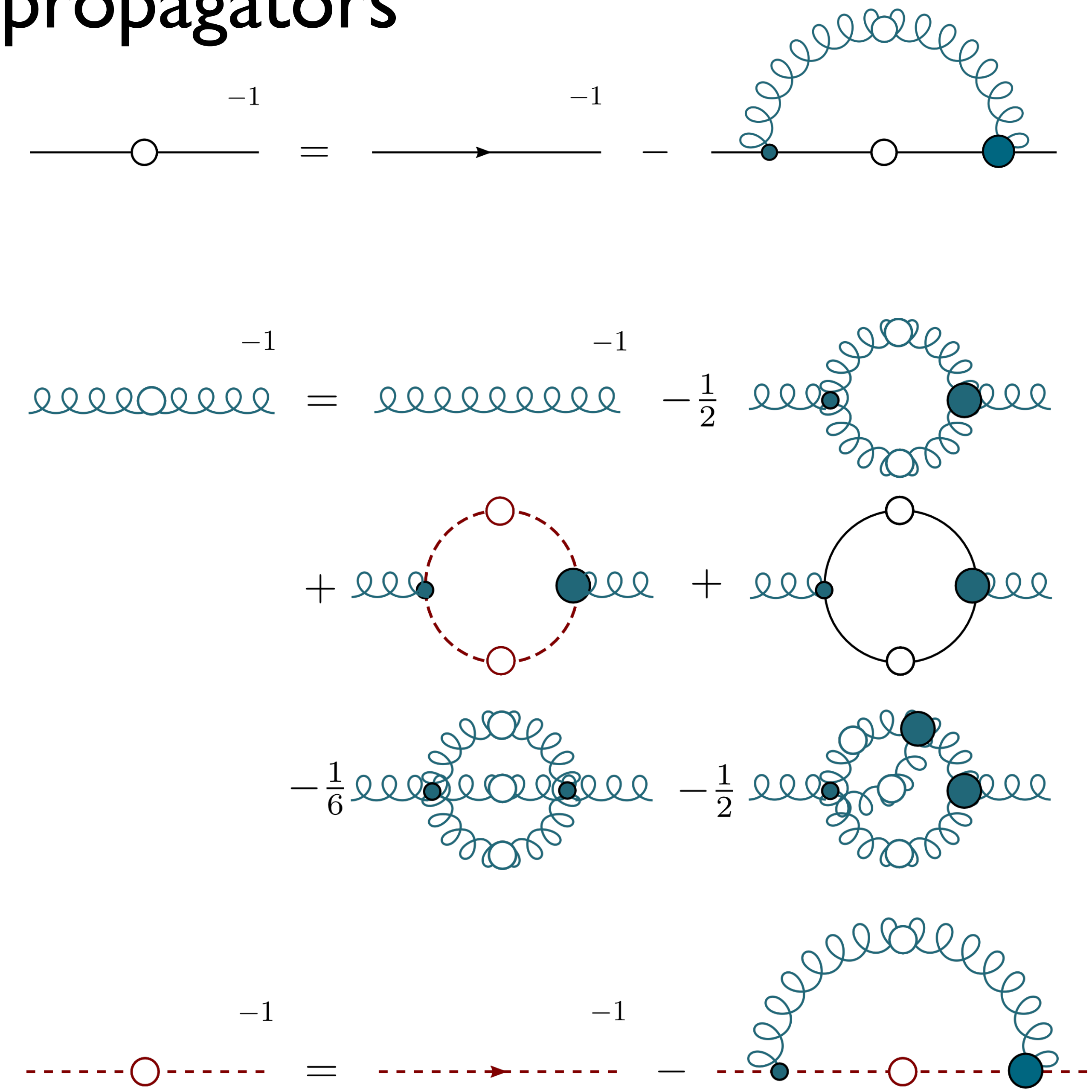


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations - "3PI vs RL"

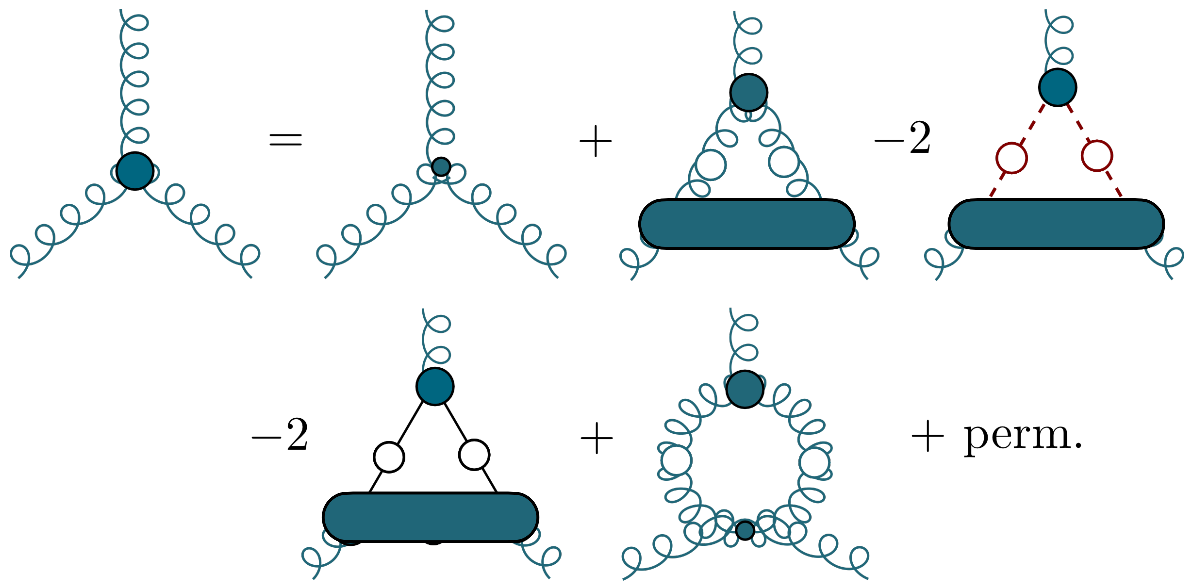
$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{-\int d^4x \left(\bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2\right)\right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

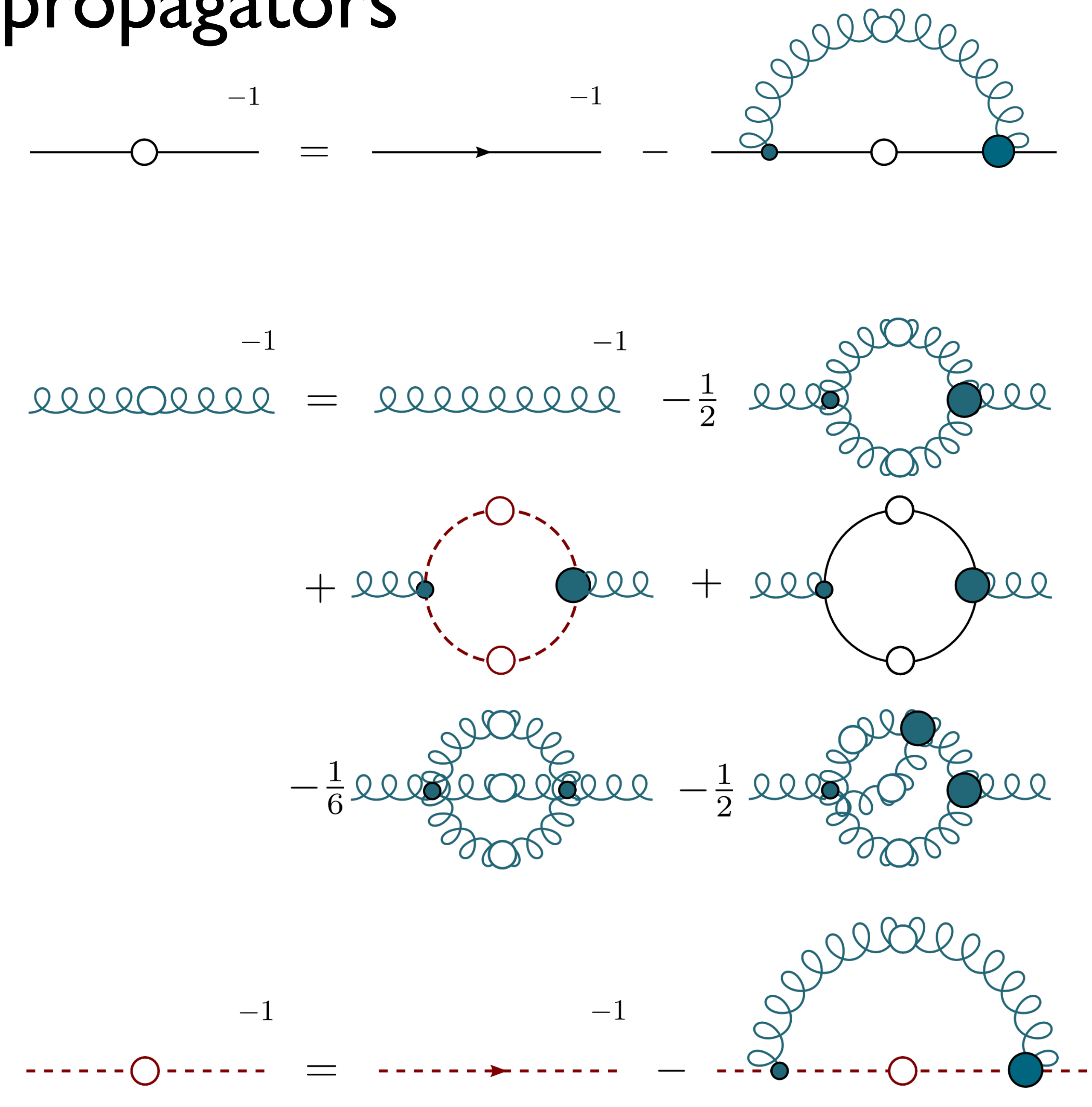
vertices



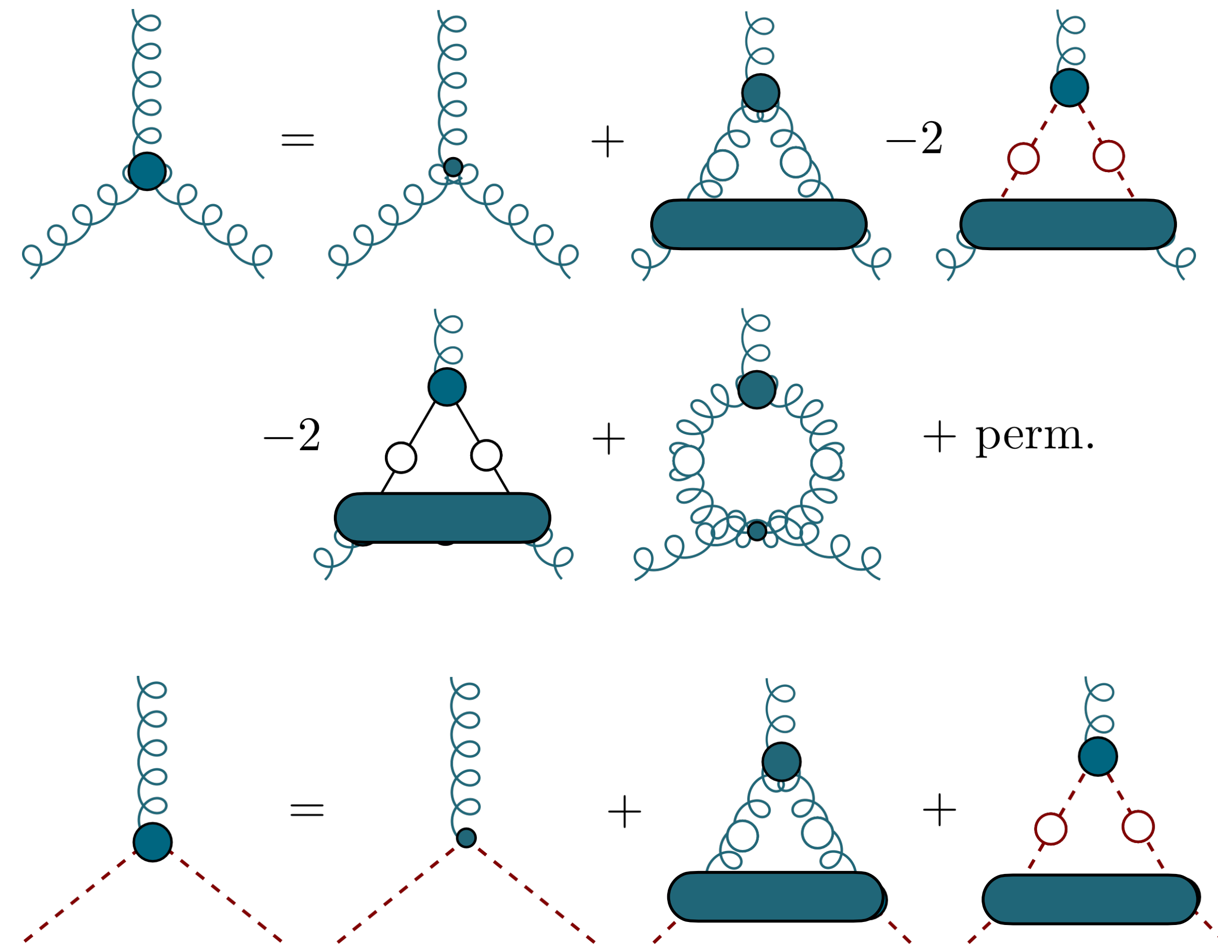
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



vertices

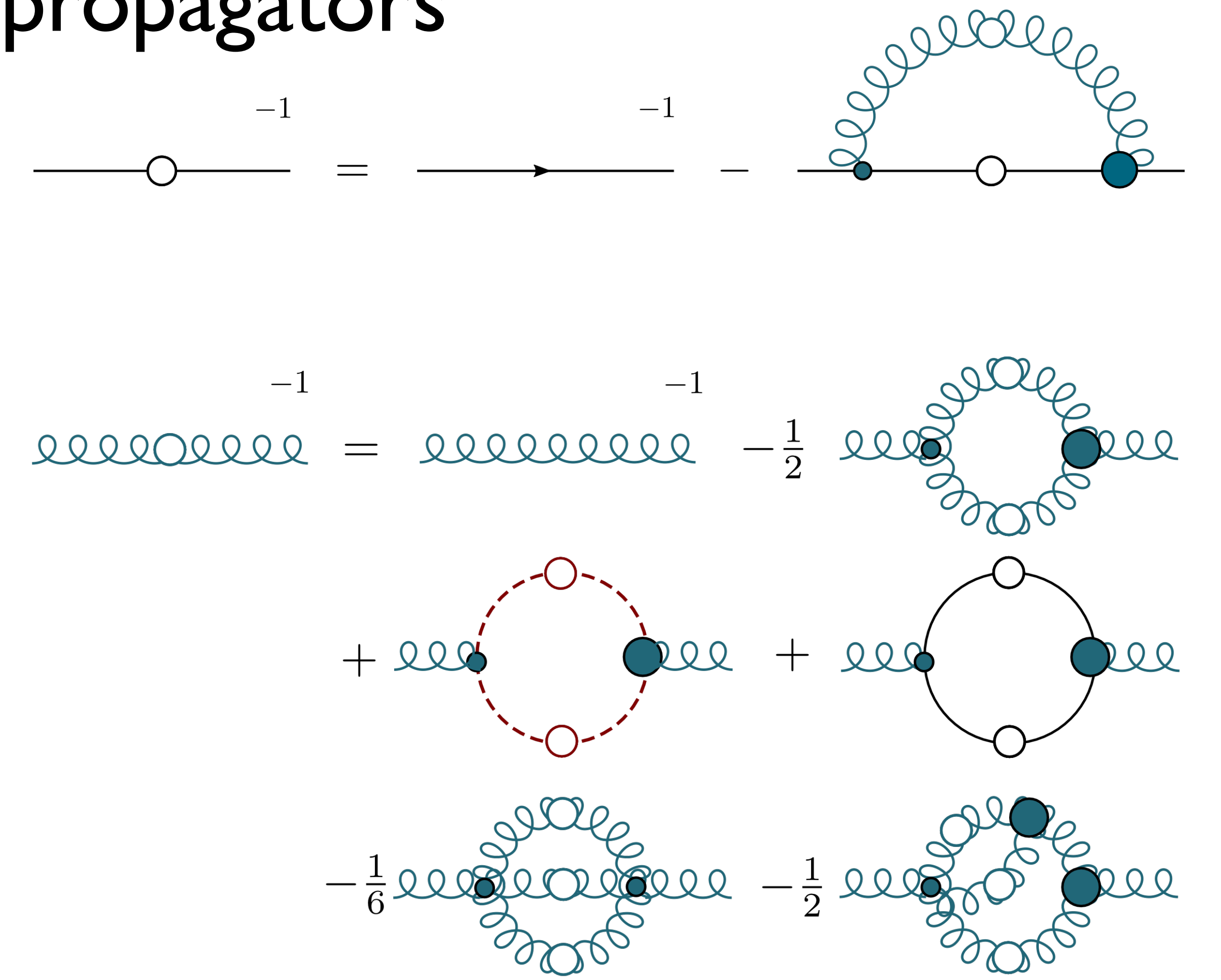


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations - "3PI vs RL"

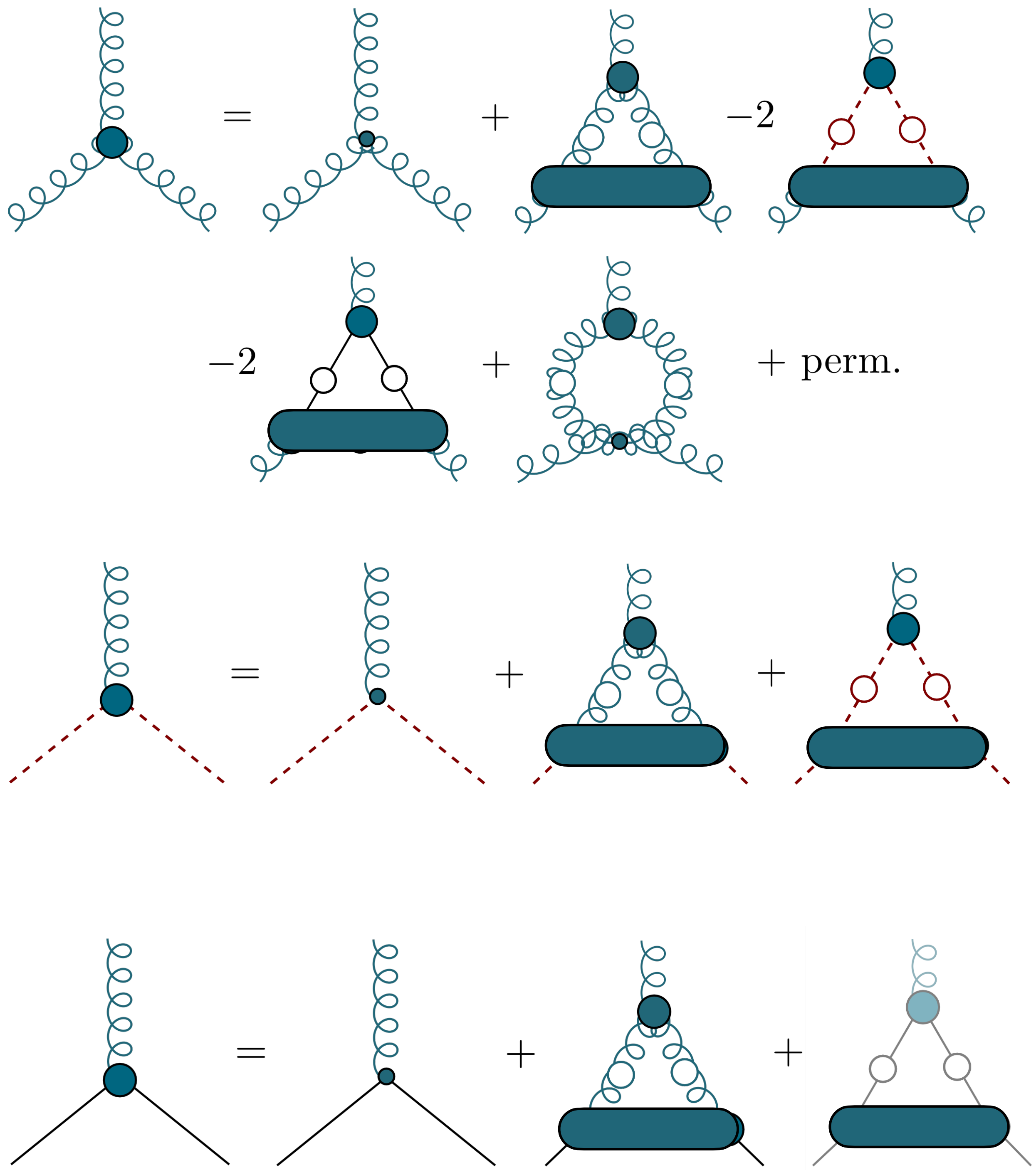
$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ -\int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 \right) \right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

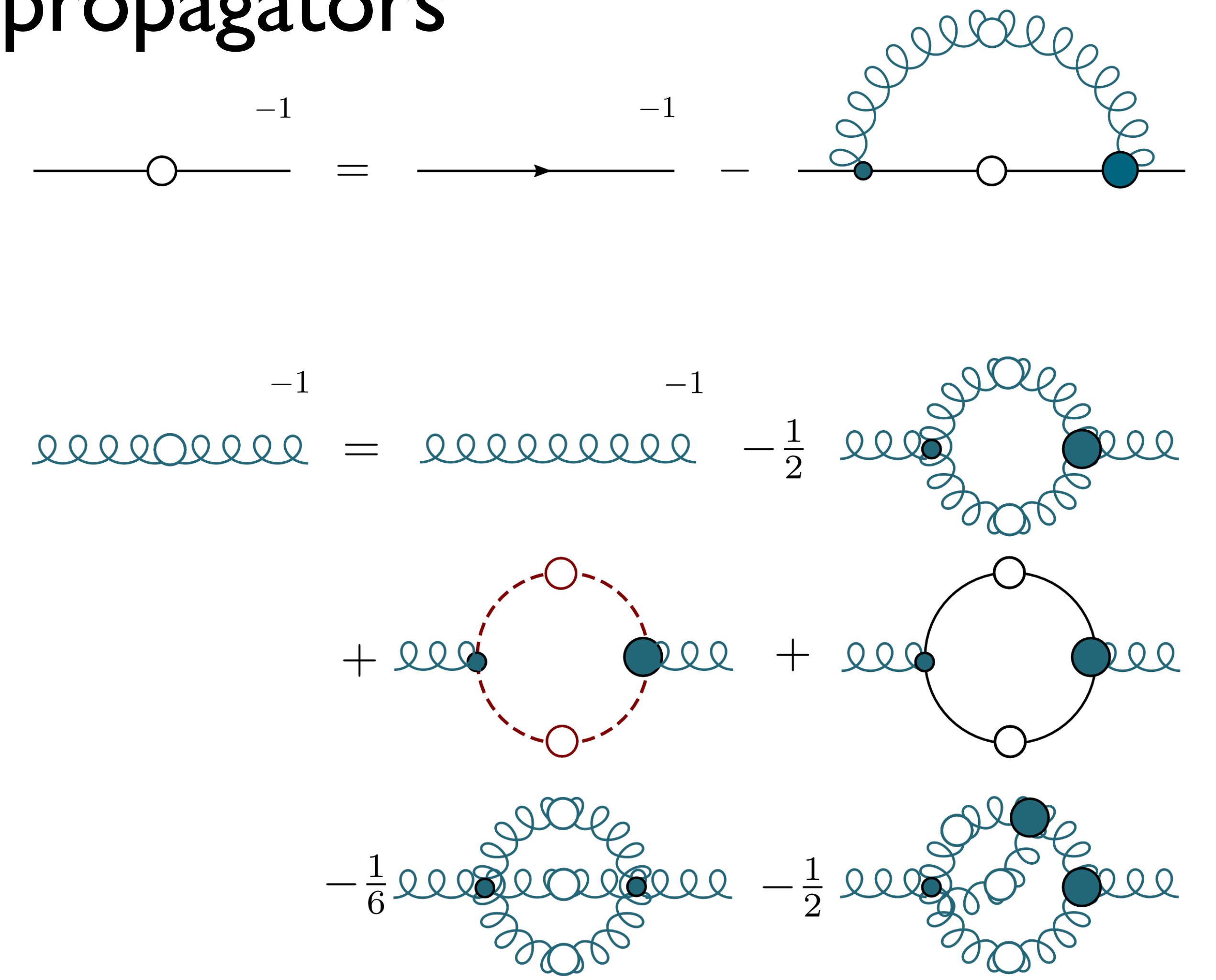
vertices



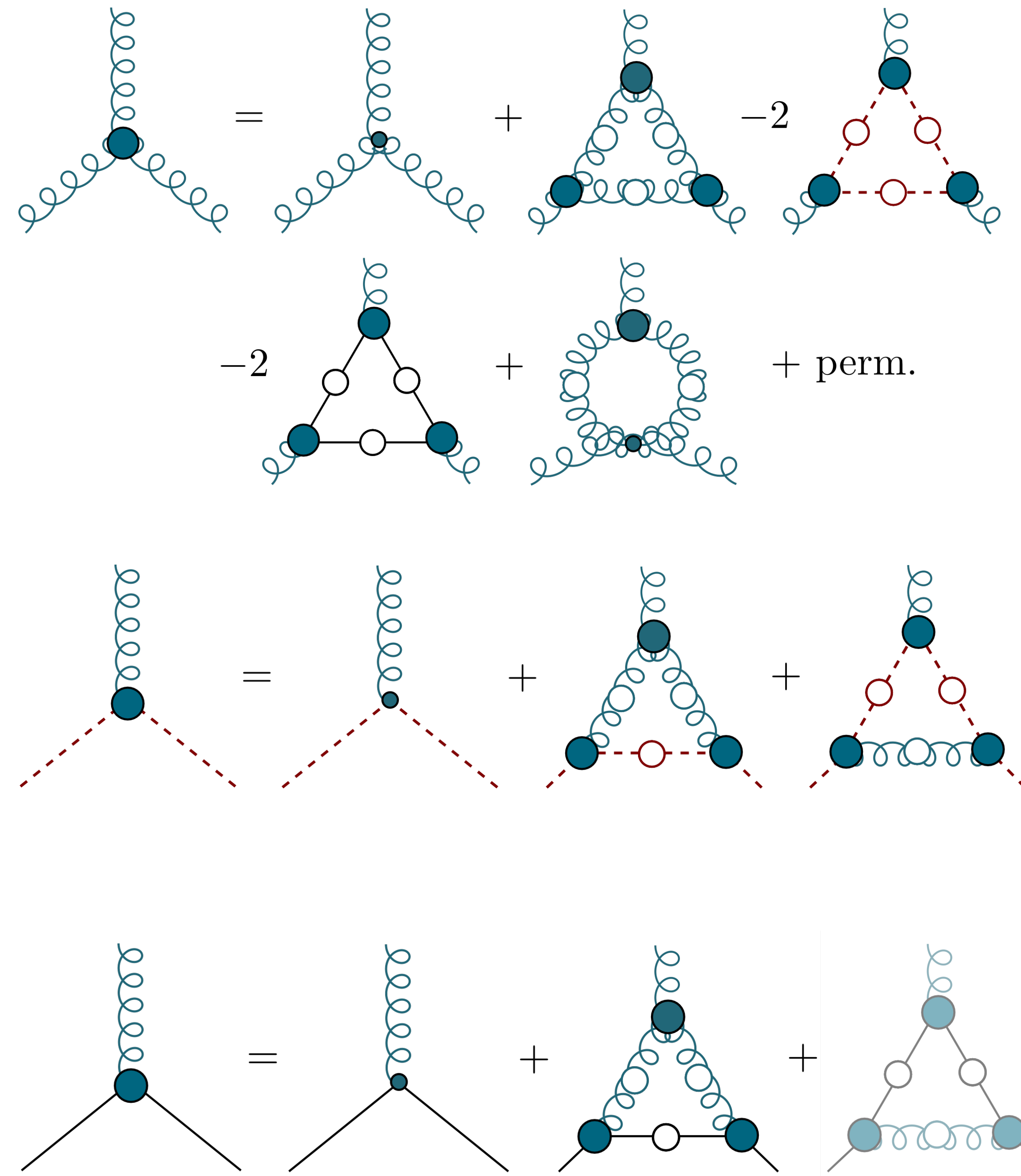
Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 \right) \right\}$$

propagators



vertices

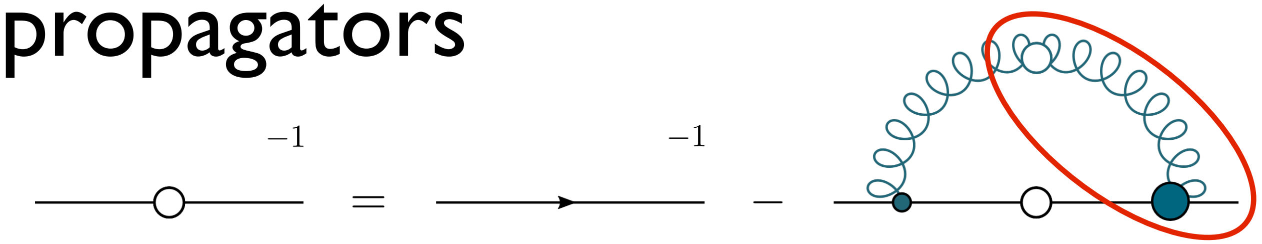


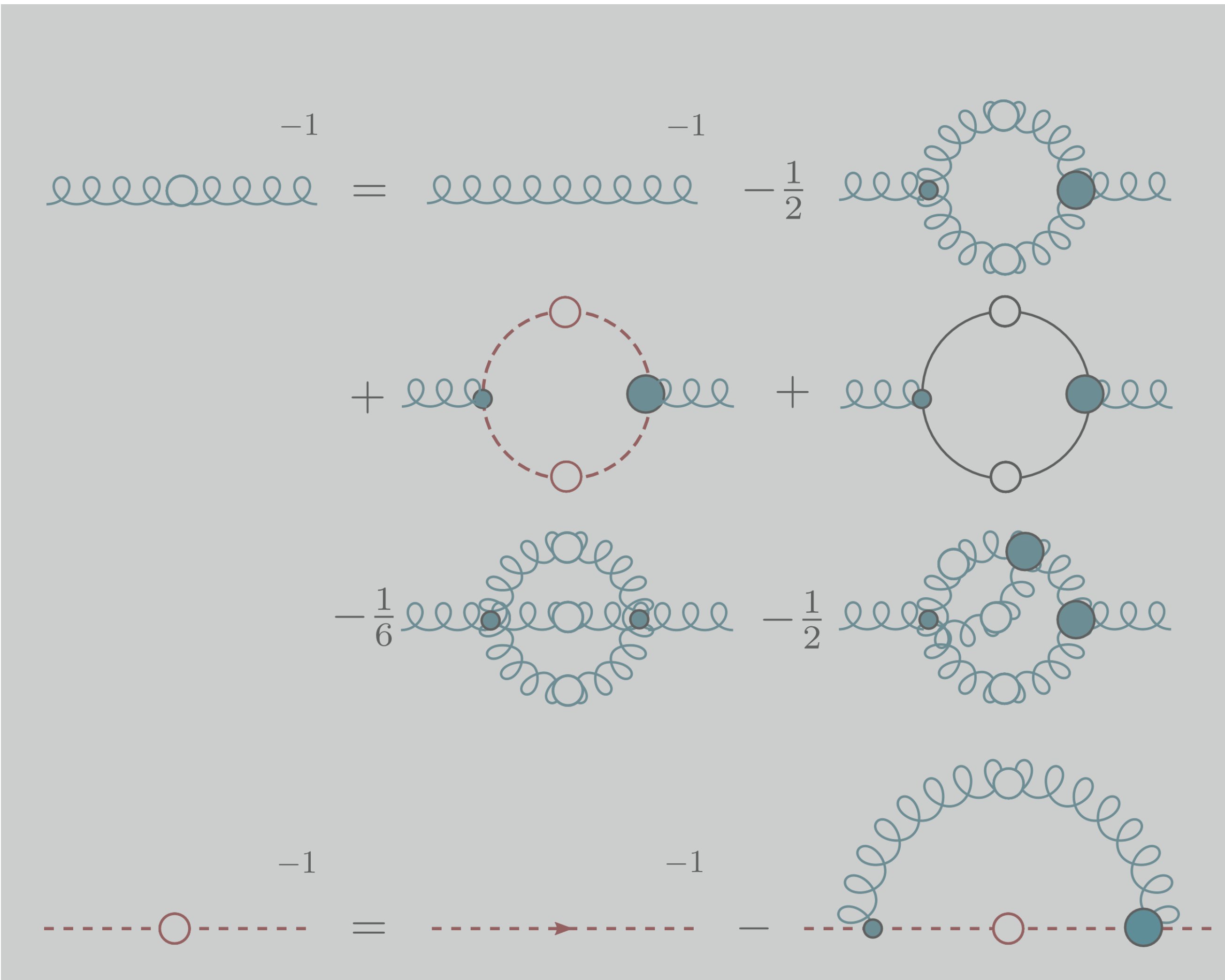
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

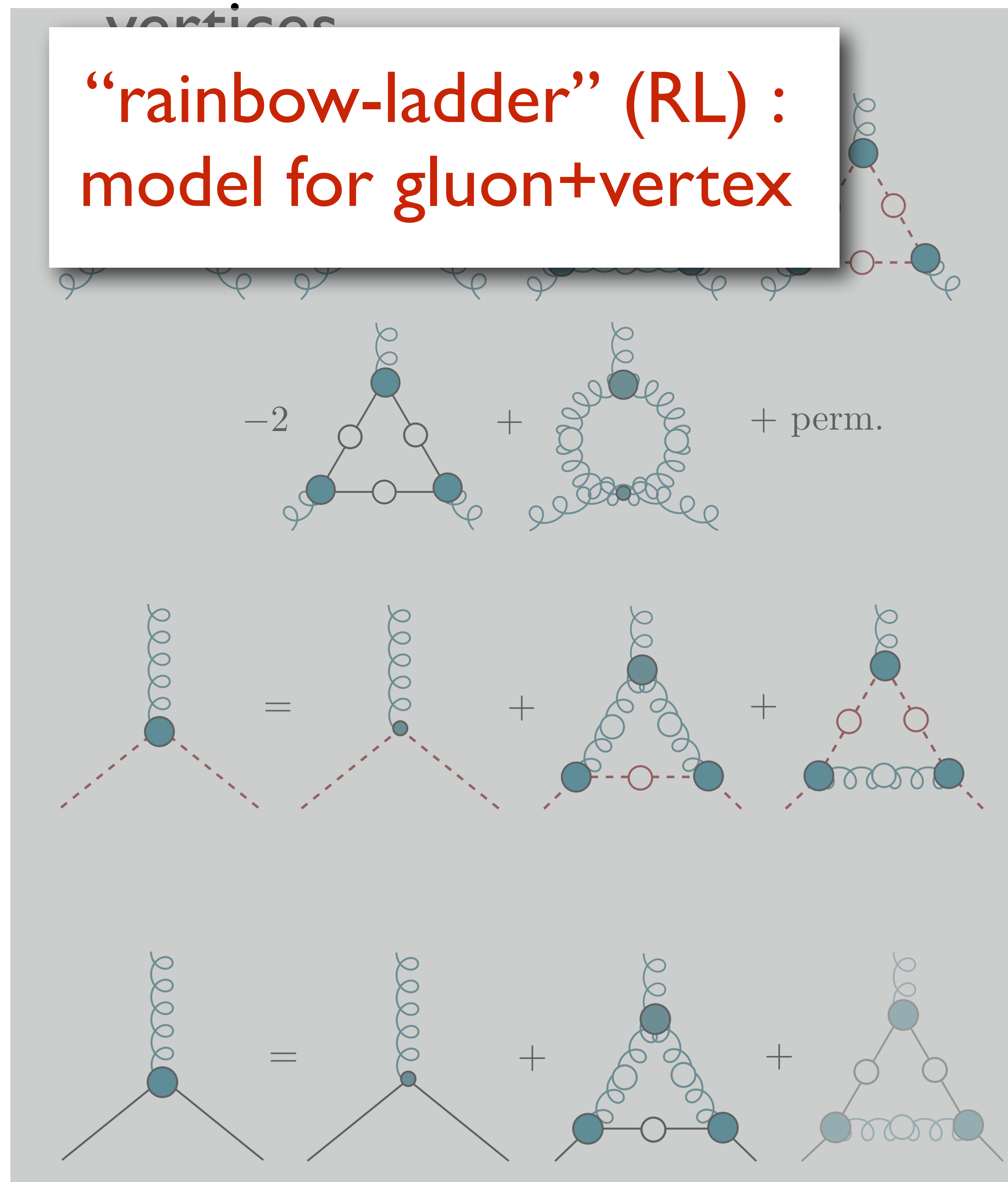
propagators





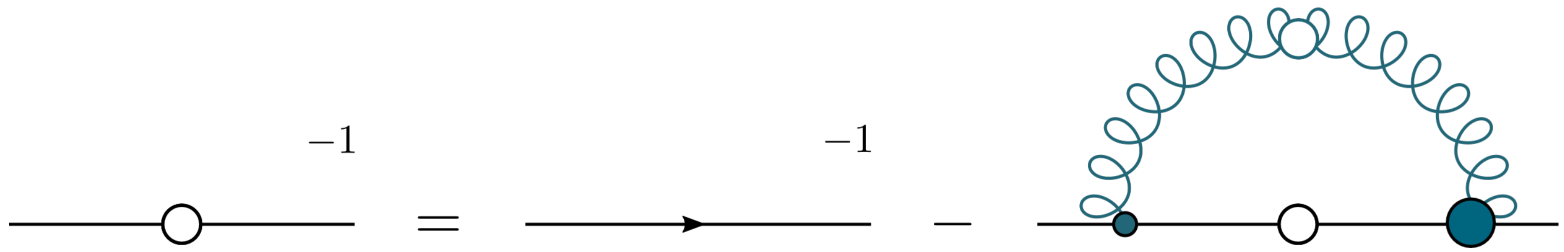
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

vertices



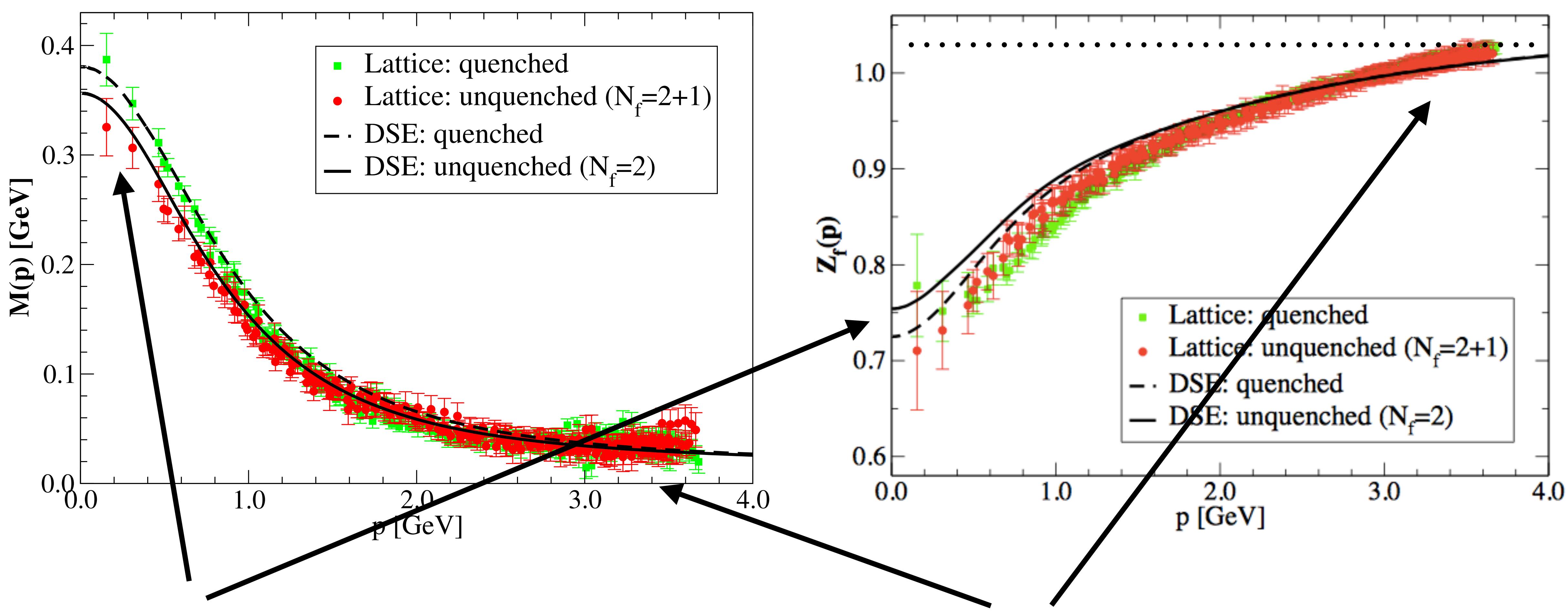
“rainbow-ladder” (RL) :
model for gluon+vertex

Quarks: mass from interaction



$$S(p) = Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

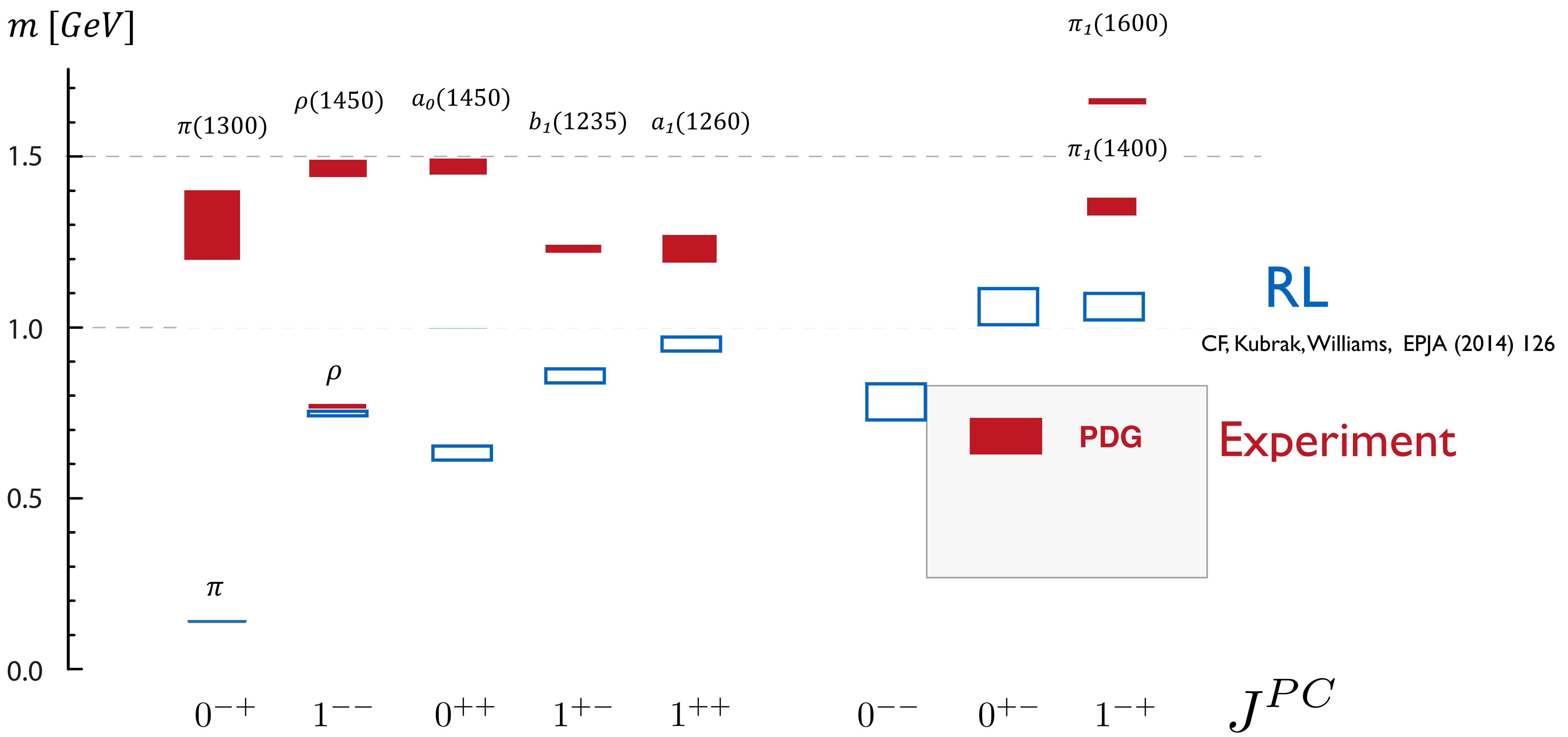
DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507



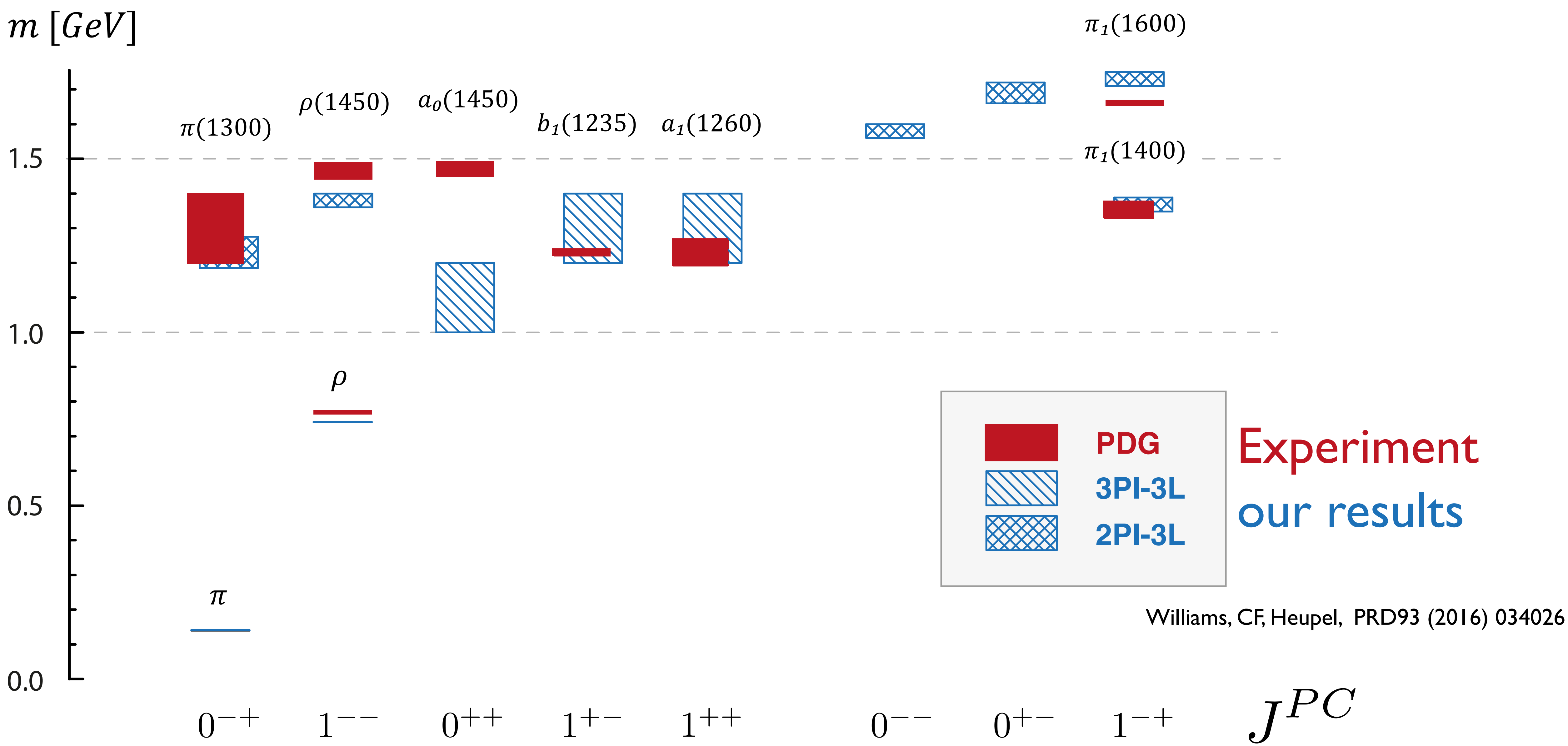
'constituent quark':
large mass; very composite

'current quark':
- small mass; non-composite

Light meson spectrum - full 3PI-calculation



Light meson spectrum - full 3PI-calculation

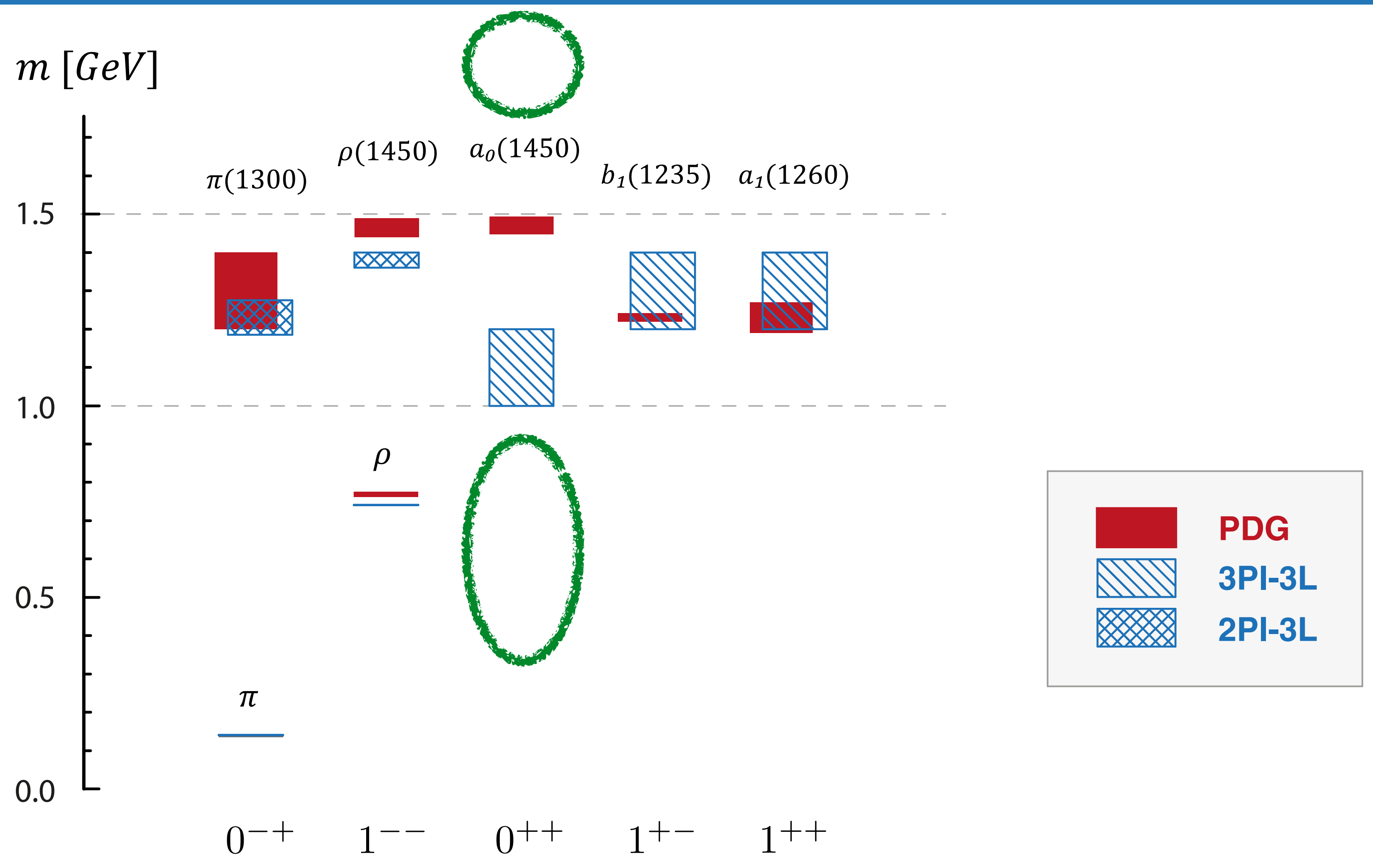


- good agreement with experiment in most channels
- special channels:

  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons

  scalar 0^{++} : complicated channel…

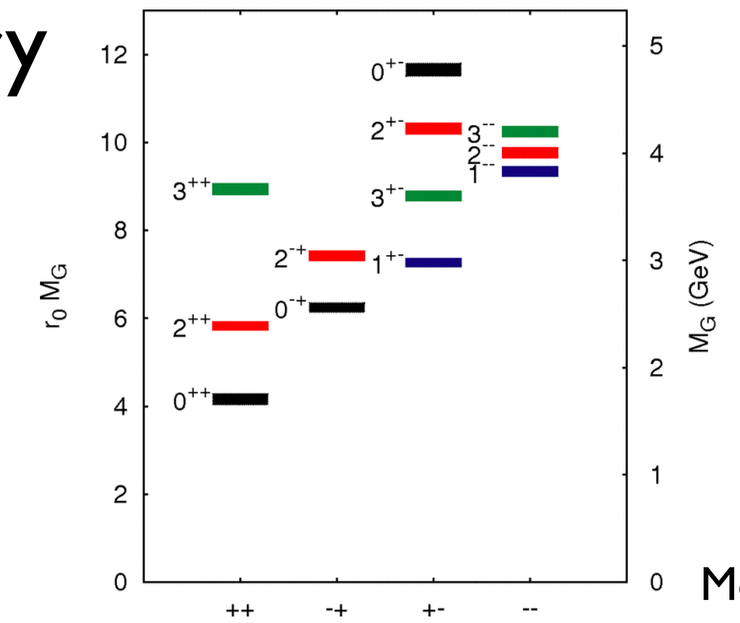
Overview



1. Conventional mesons
2. Glueballs: pure Yang-Mills
3. Four-quark states: mixing and heavy-light states

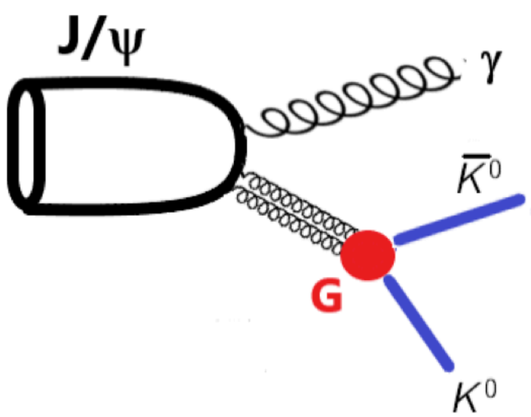
Glueballs

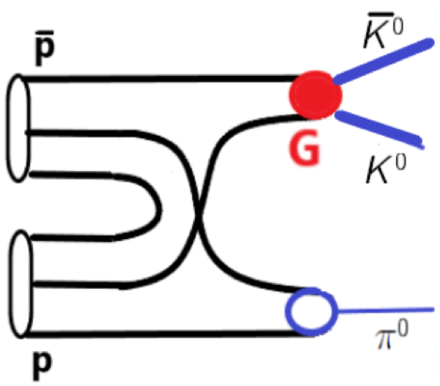
Lattice YM-theory



Morningstar and Peardon, PRD 60 (1999)

Experiment:





Klempt, arXiv:2211.12901

$$M_{0^{++}} = 1865 \pm 25^{+10}_{-30}$$

Sarantsev et al., PLB 816 (2021) 136227

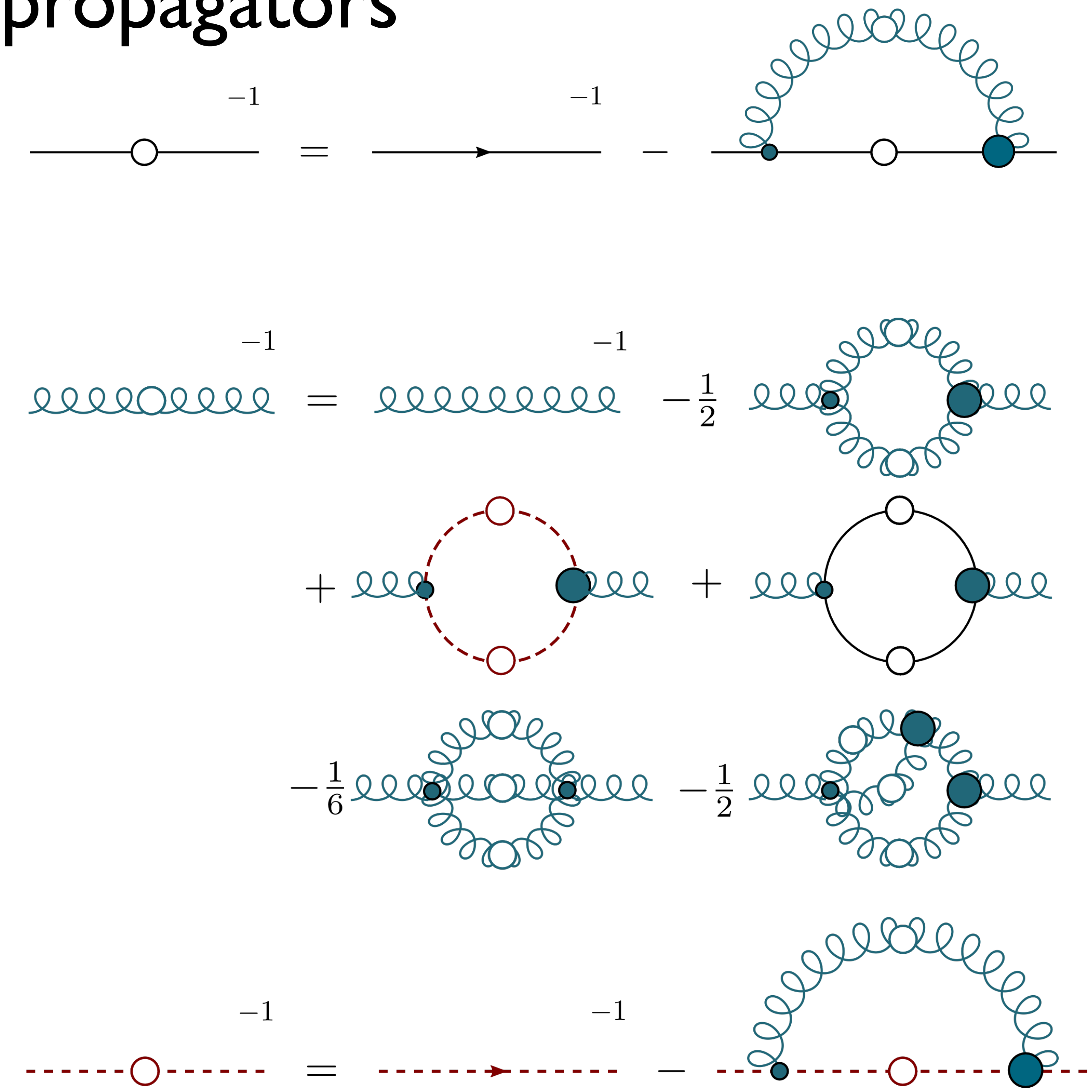
$$M_{0^{++}} = f_0(1710)$$

Rodas et al. (JPAC), EPJC 82 (2022) 1, 80

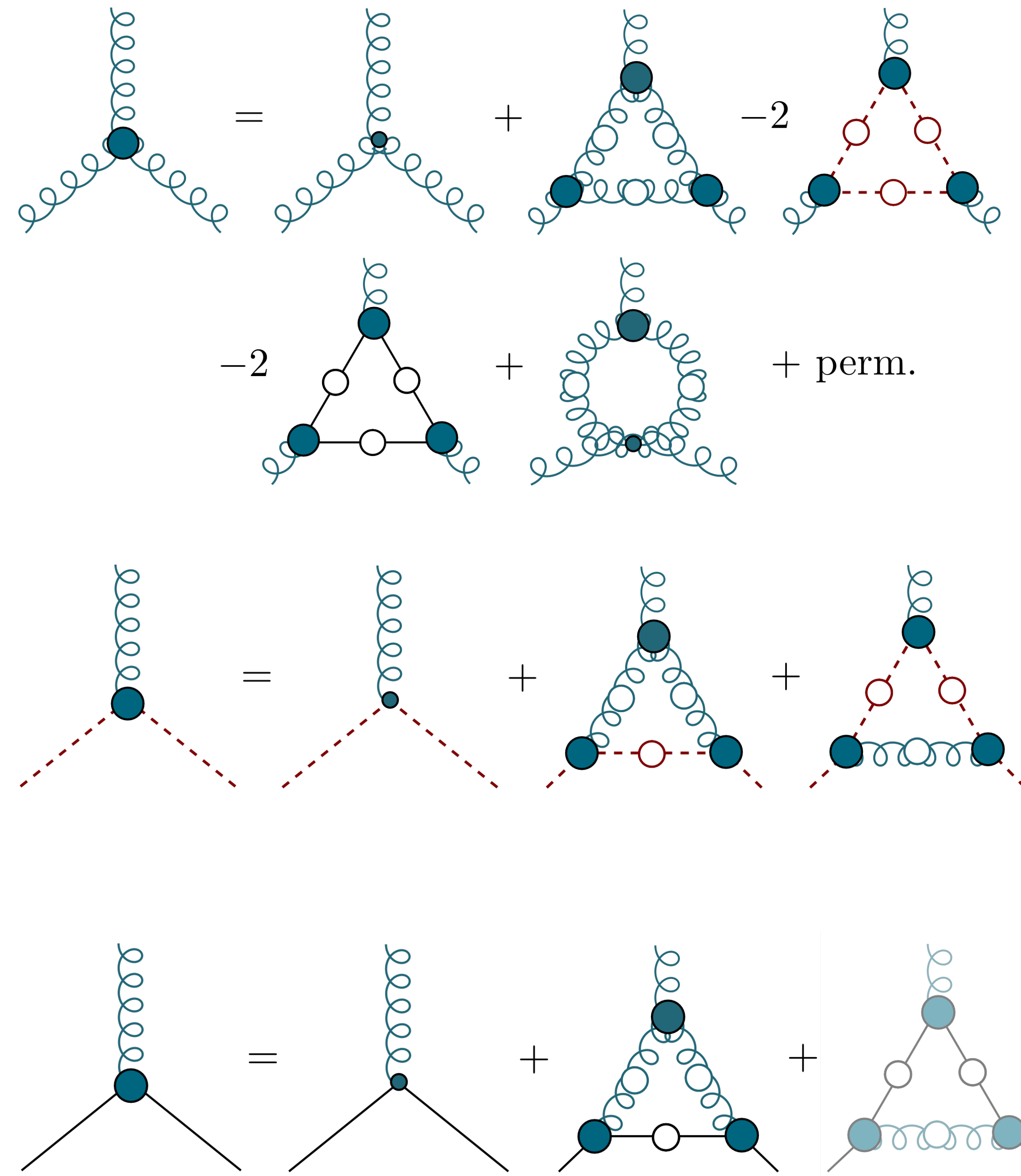
Dyson-Schwinger equations -

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\Big\{ - \int d^4x \Big(\bar{\Psi} \, (i$$

propagators



vertices



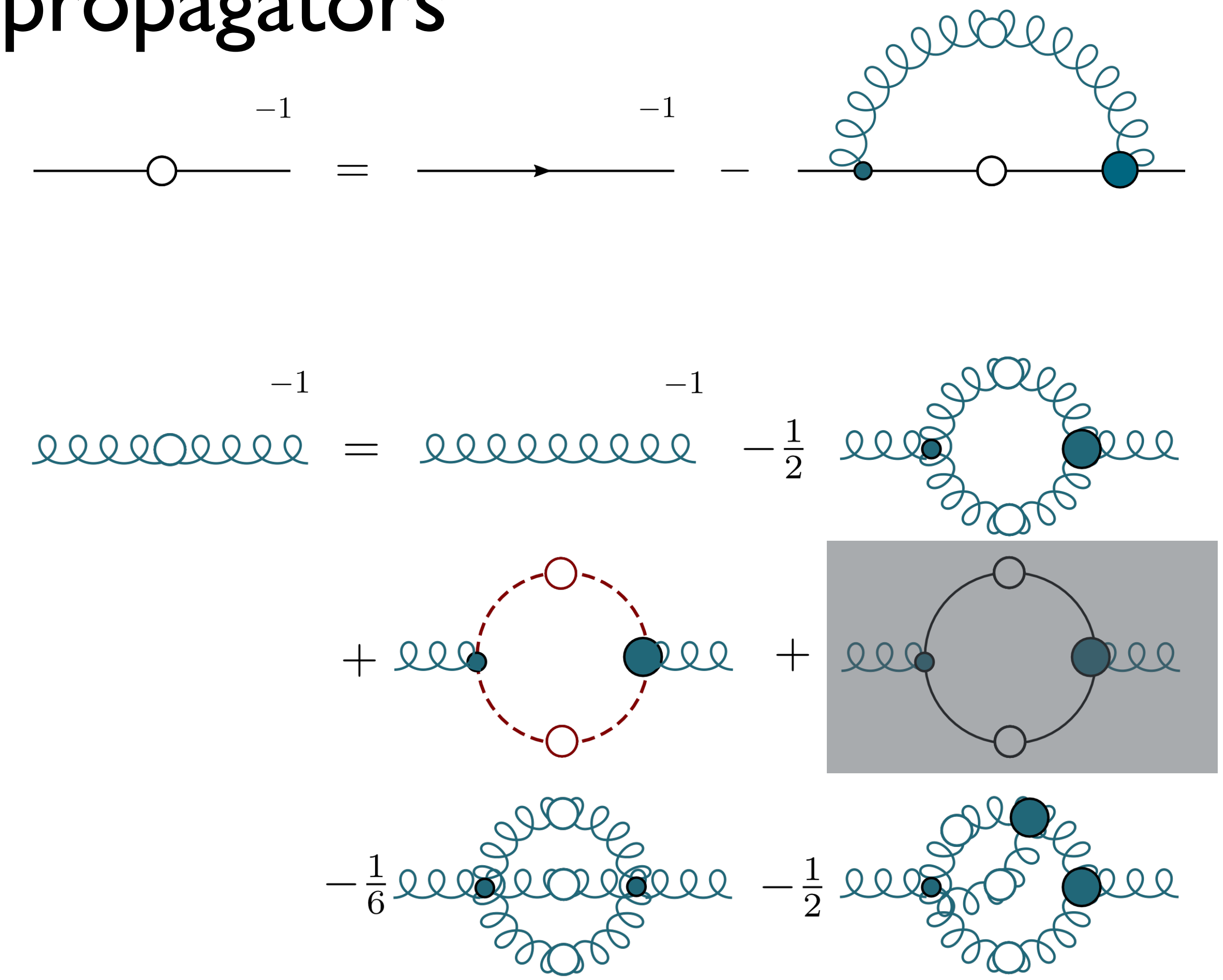
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations -

“quenched”

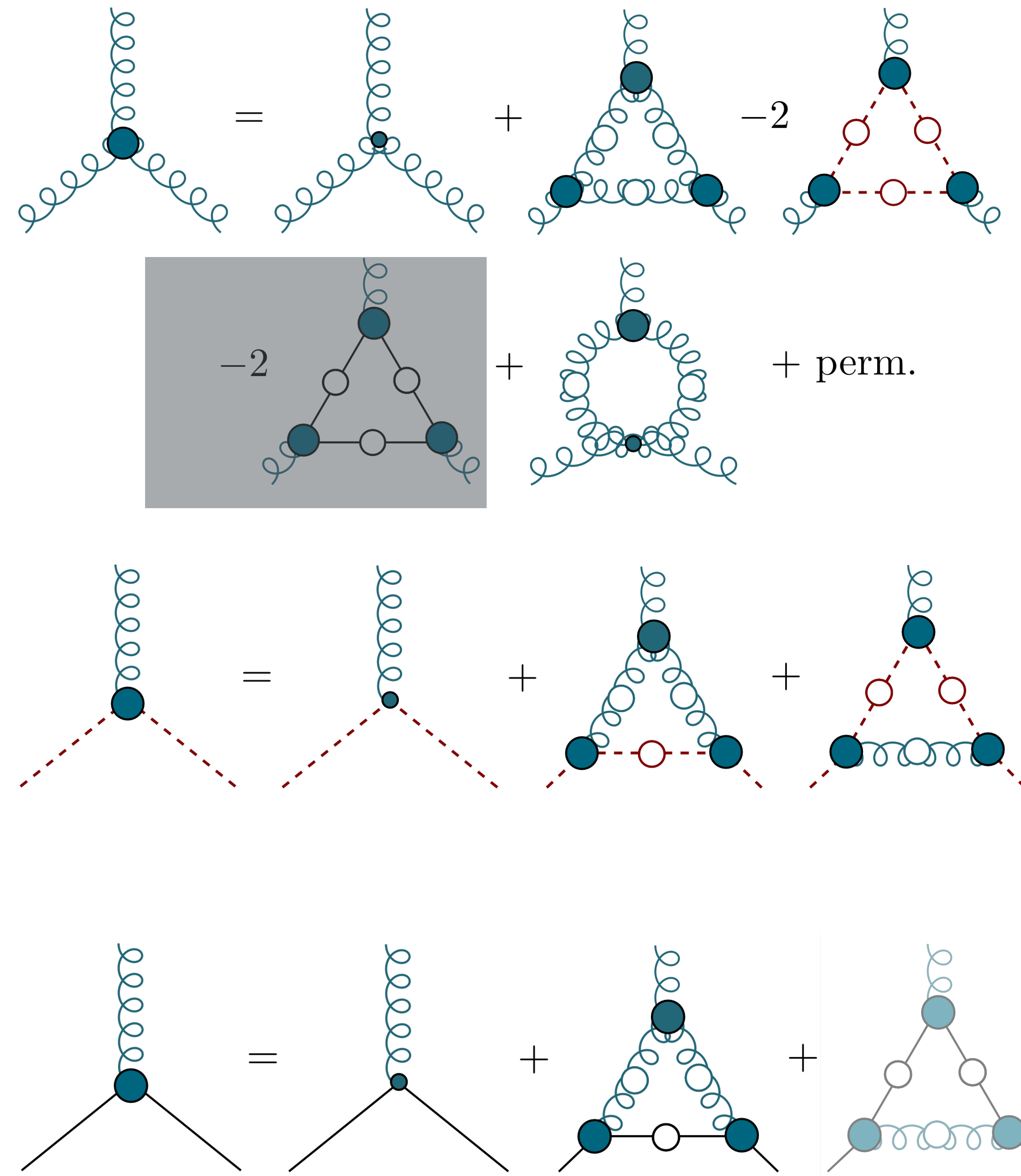
$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi}\, (i \right.\right.$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

vertices

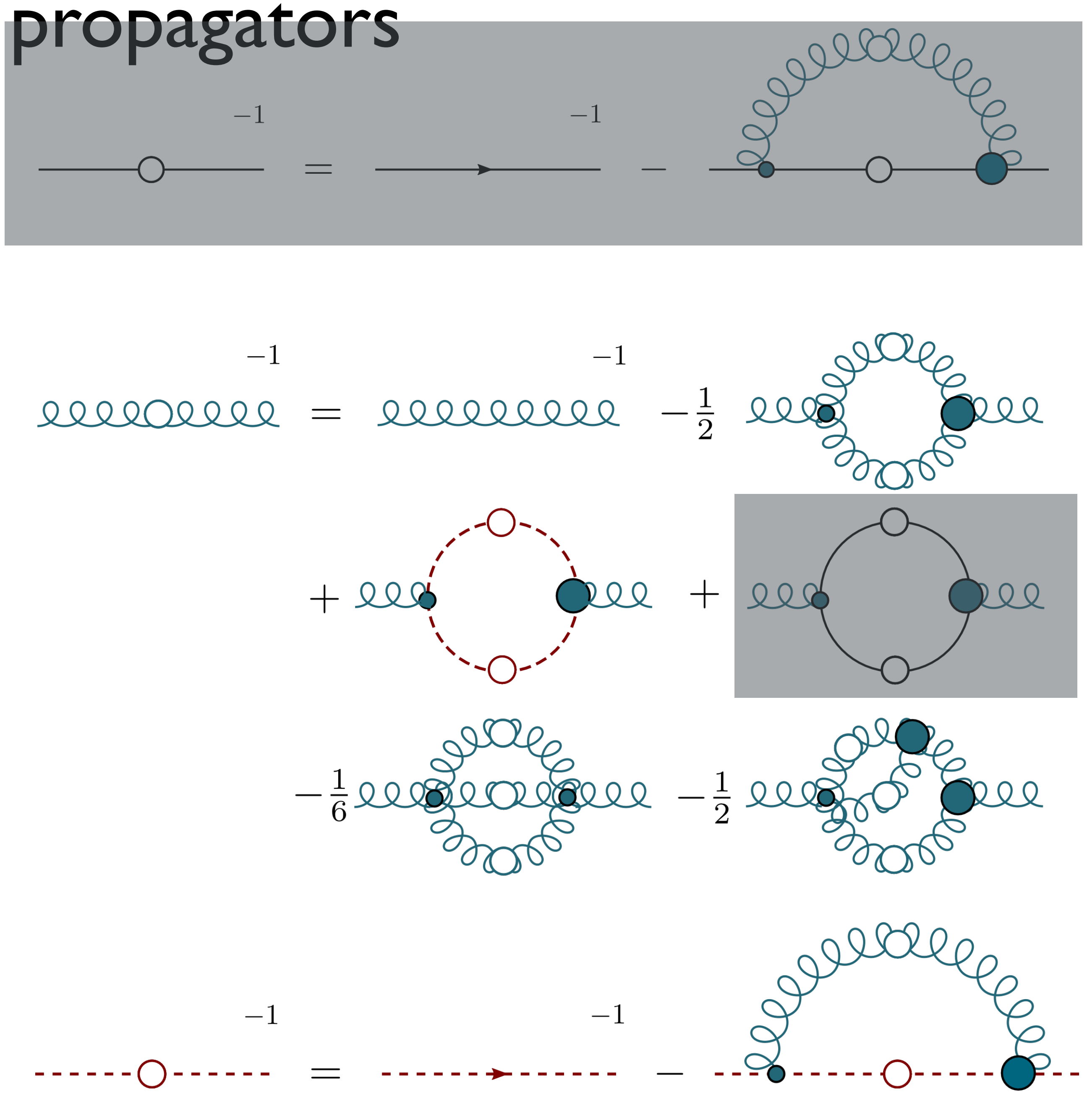


Dyson-Schwinger equations -

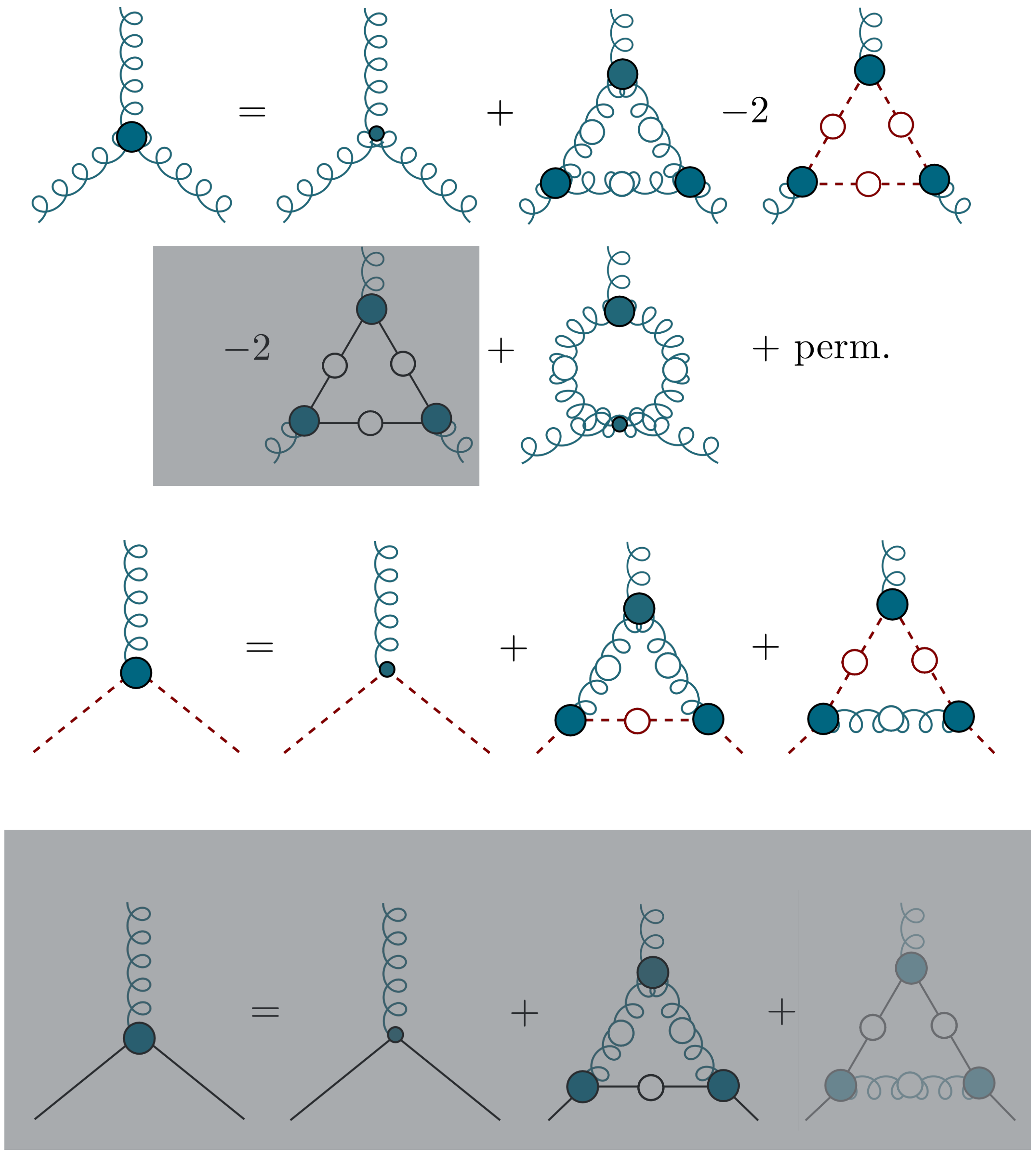
$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \, (i \right.\right.$$

pure YM-Theory

propagators



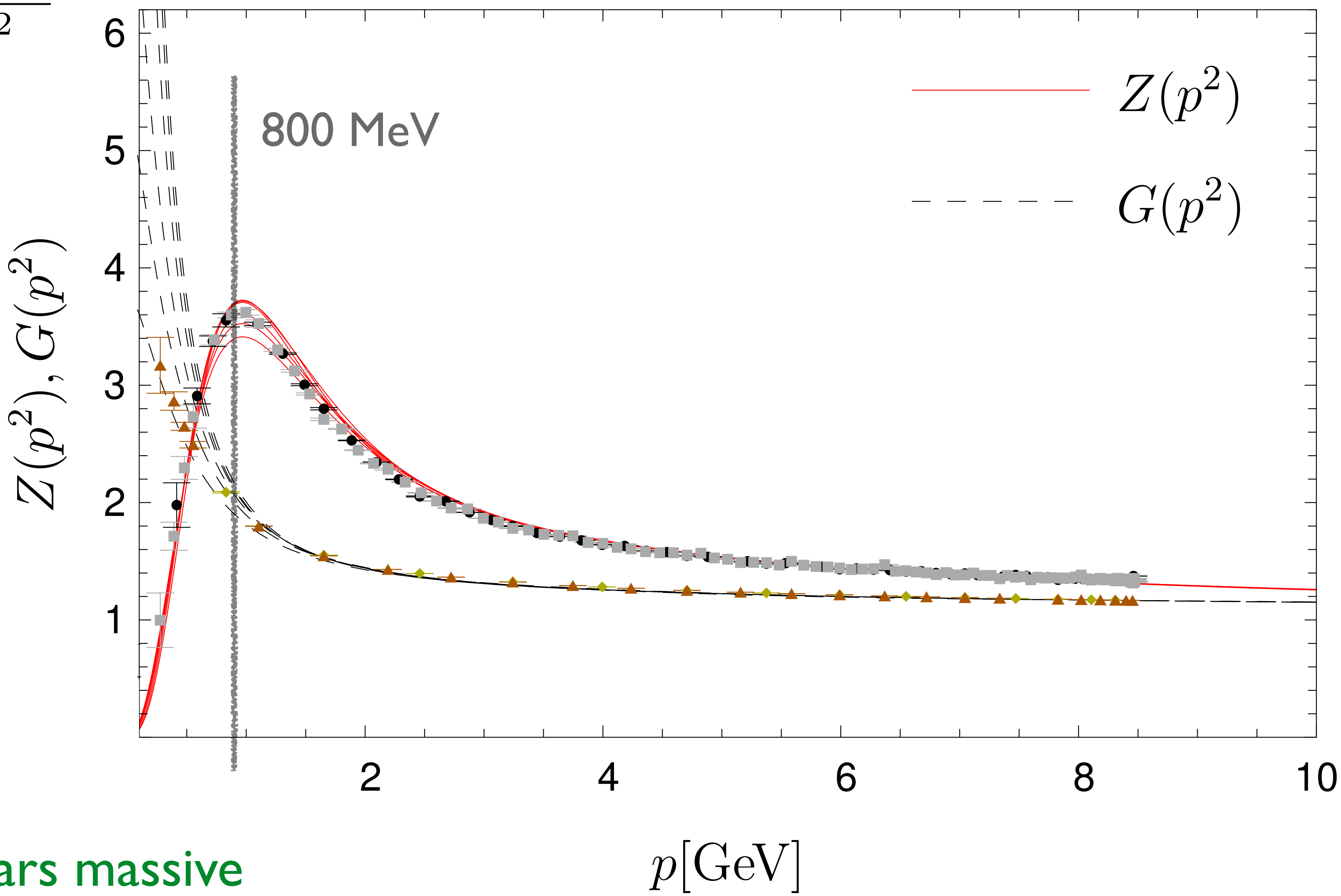
vertices



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Landau gauge gluon propagator

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

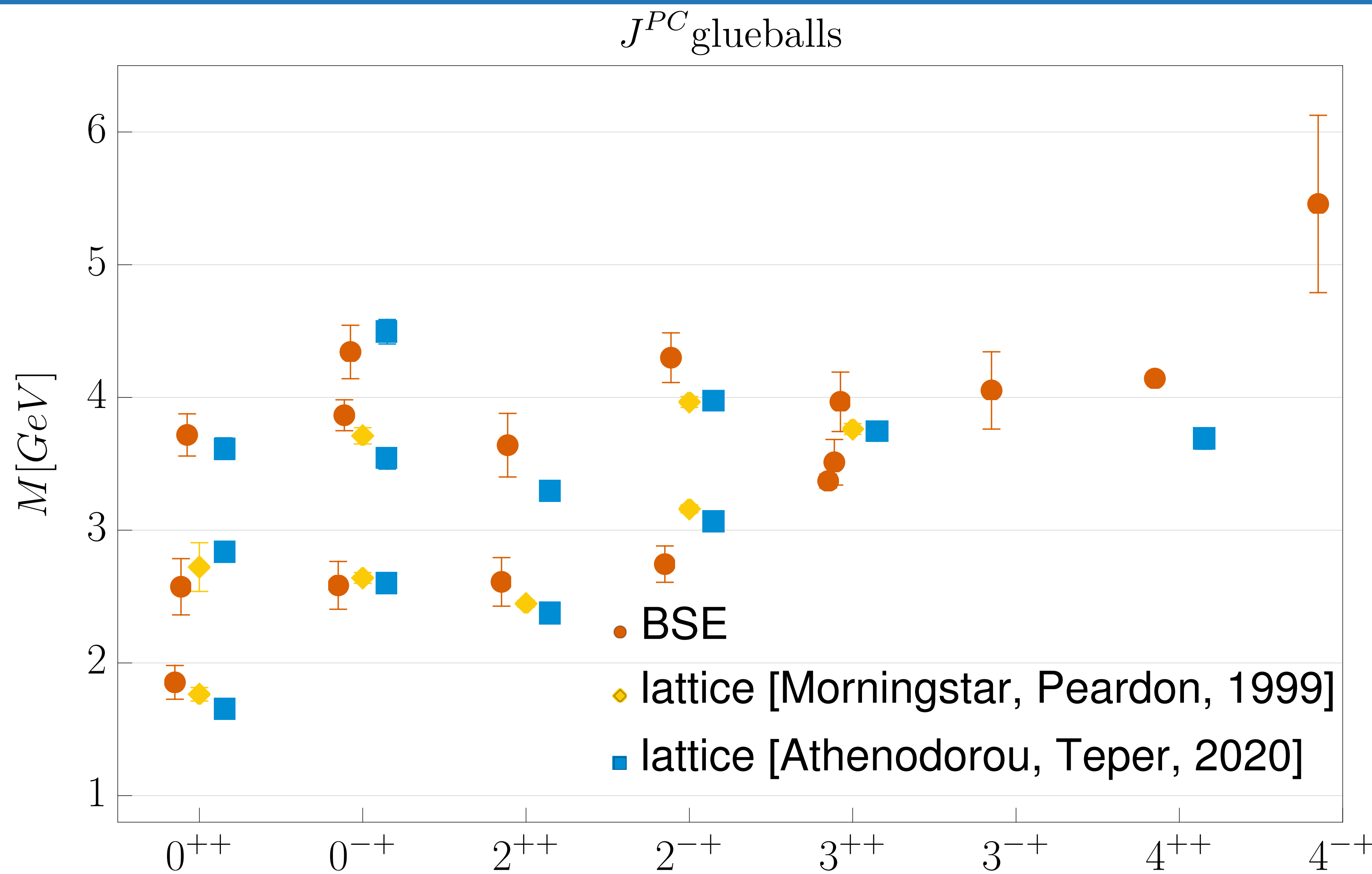


- fully dressed gluon appears massive

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

Glueballs: results



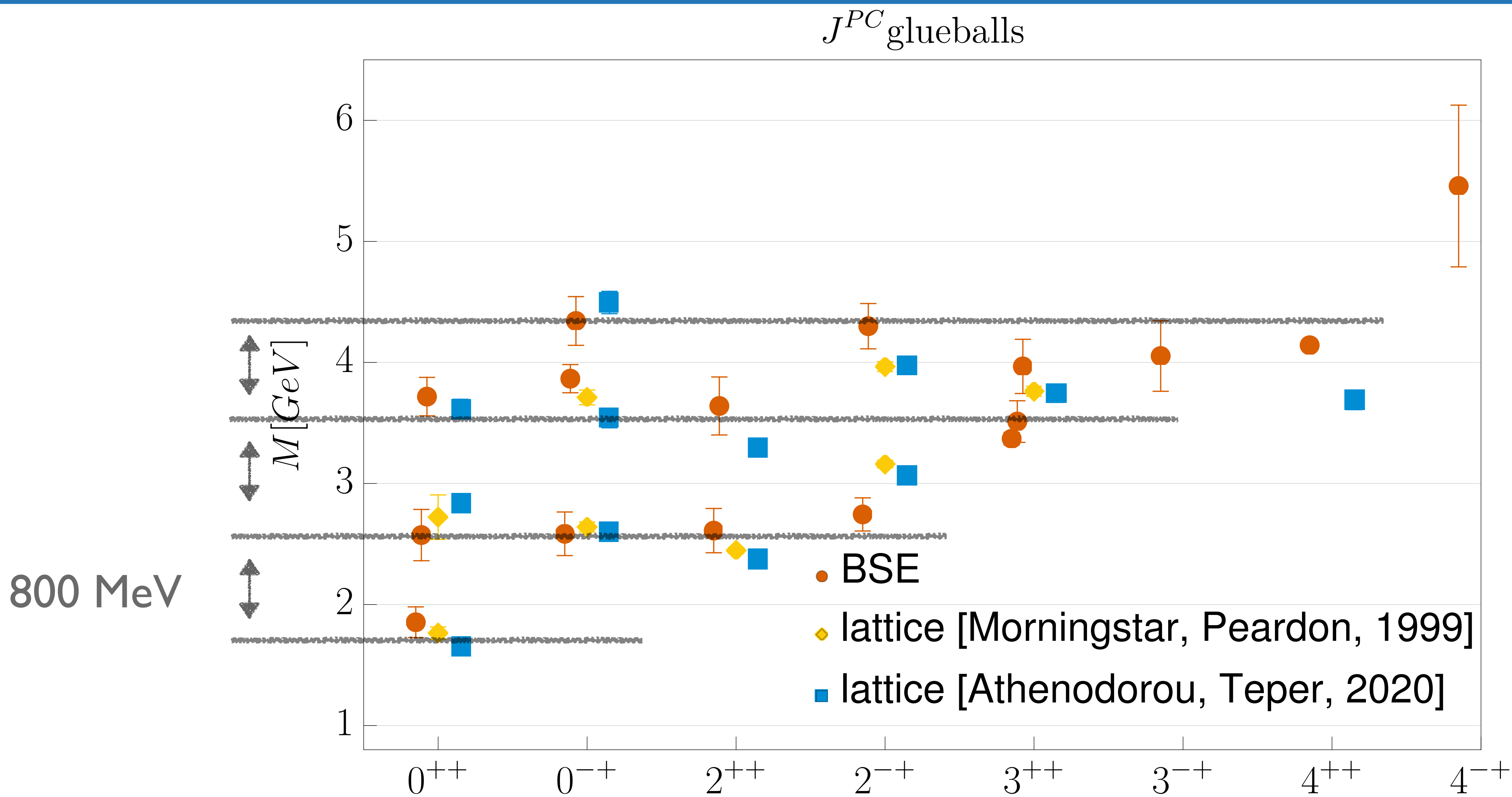
- confirmation of results from lattice YM-theory
- predictions for some channels

To do:

chart the mixing of glueballs with conventional meson states…

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

Glueballs: results

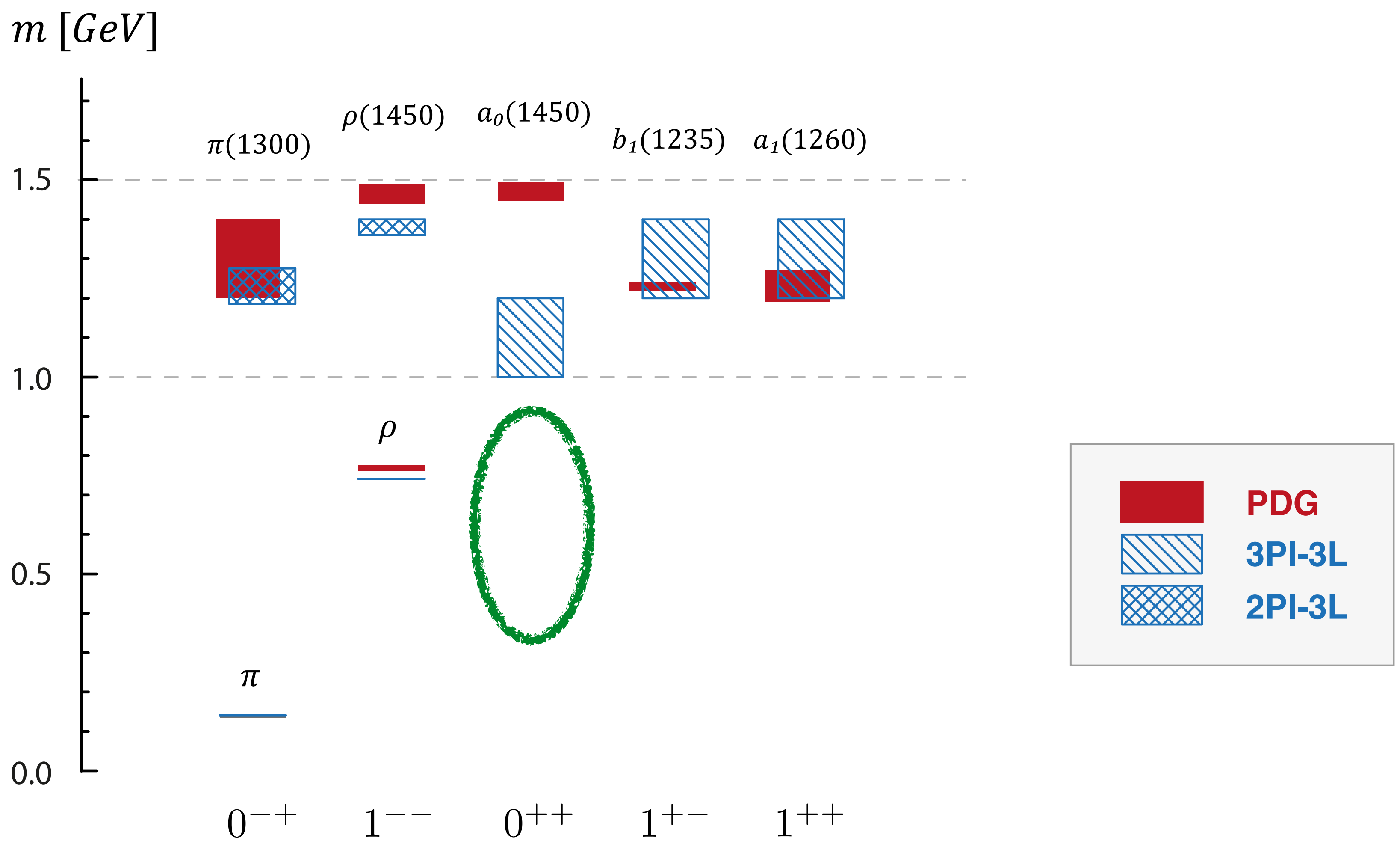


- confirmation of results from lattice YM-theory
- predictions for some channels

To do:
chart the mixing of glueballs with conventional meson states…

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

Overview

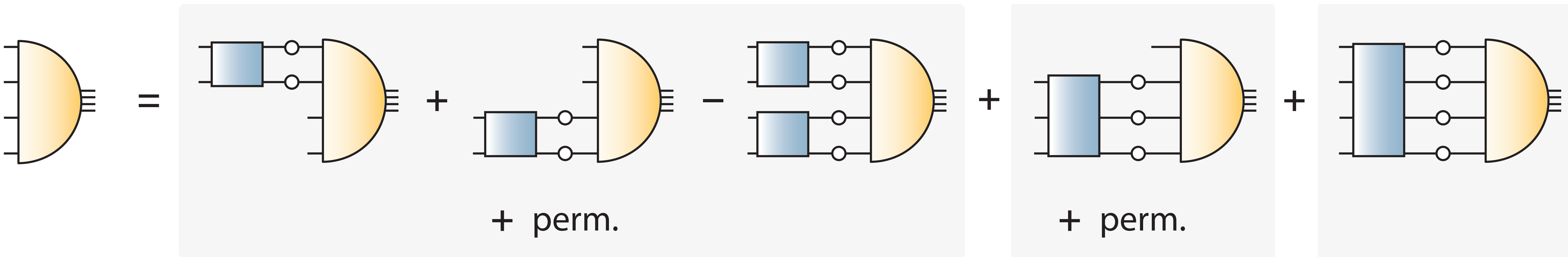


1. Conventional mesons
2. Glueballs: pure Yang-Mills
3. Four-quark states: mixing and heavy-light states

Four-quark states from the four-body equation

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Exact equation:



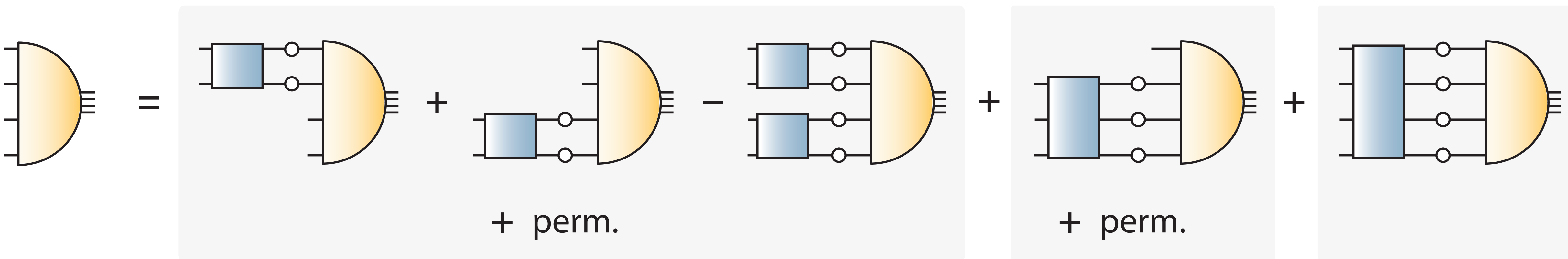
Two-body interactions

Three- and four-body interactions

Four-quark states from the four-body equation

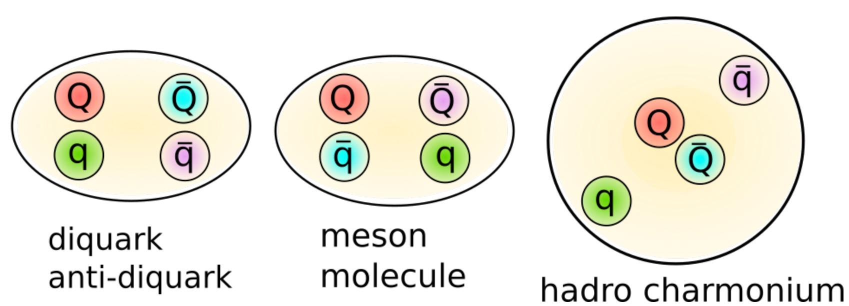
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

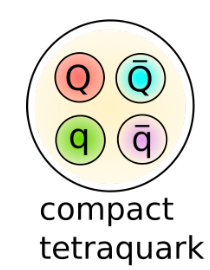
Exact equation:



Two-body interactions

Three- and four-body interactions

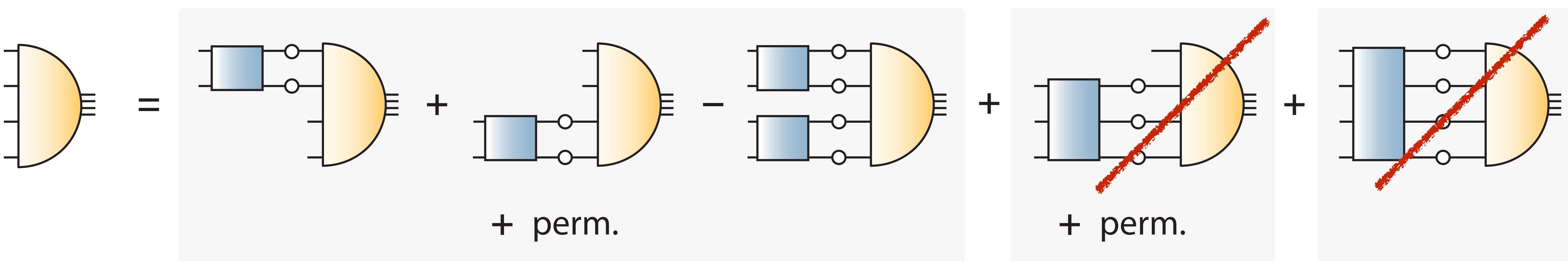




Four-quark states from the four-body equation

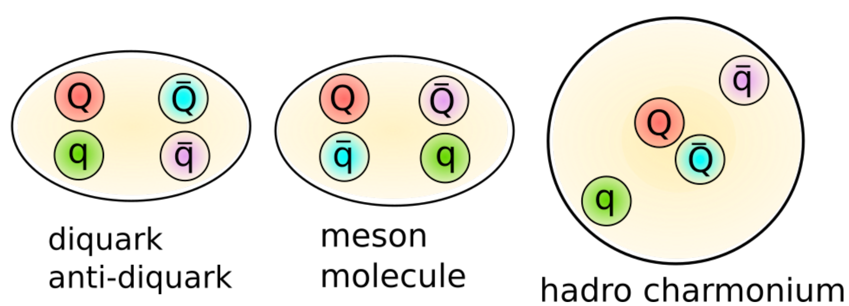
Exact equation:

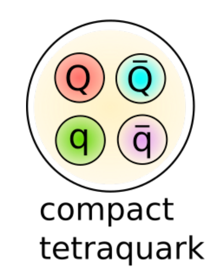
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions

Three- and four-body interactions

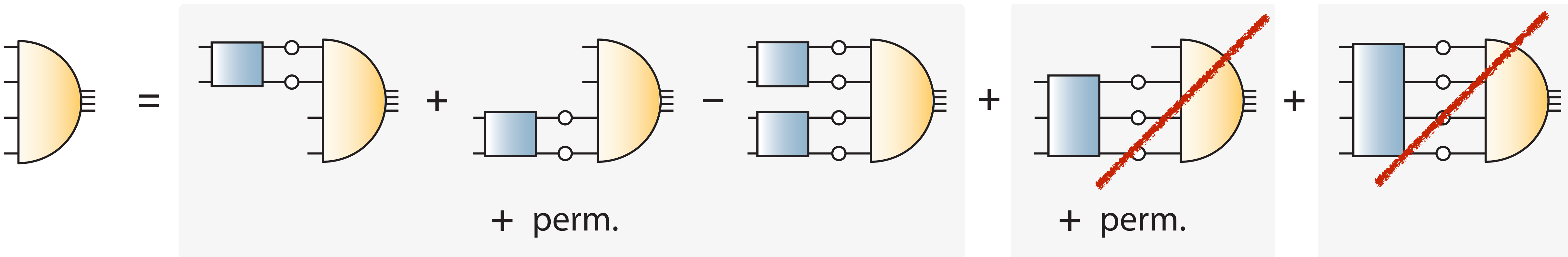




Four-quark states from the four-body equation

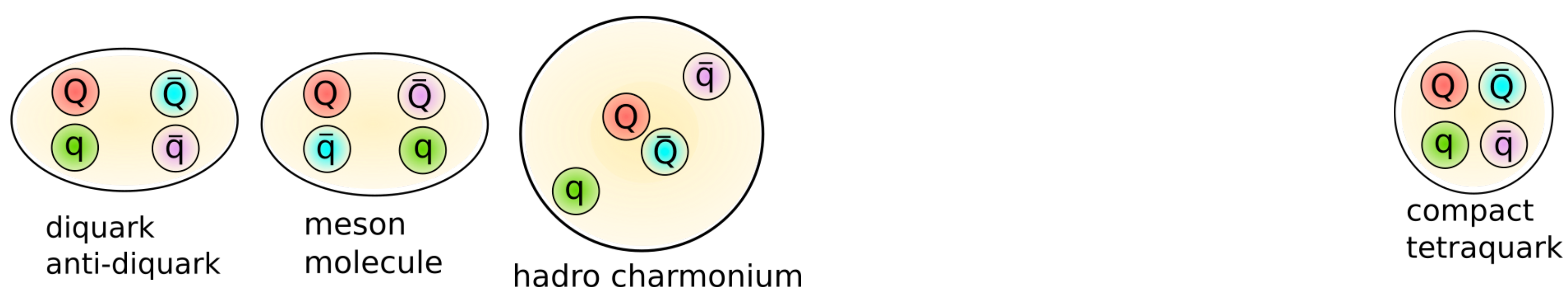
Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions

Three- and four-body interactions



$$f_0(500) : \pi\pi - \text{component dominates!}$$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Mass evolution of four-quark state: 0++

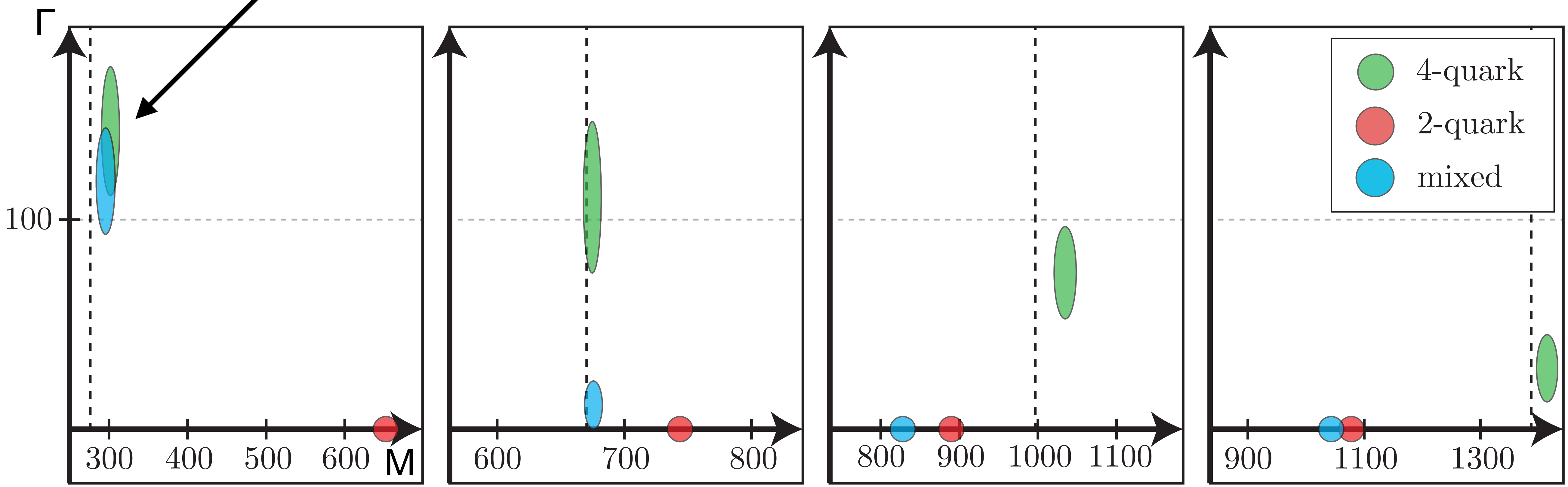
$f_0(500) : \pi\pi - \text{component dominates!}$

$m_q = 3.8\,\text{MeV}$

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

Mass evolution of four-quark state: 0++

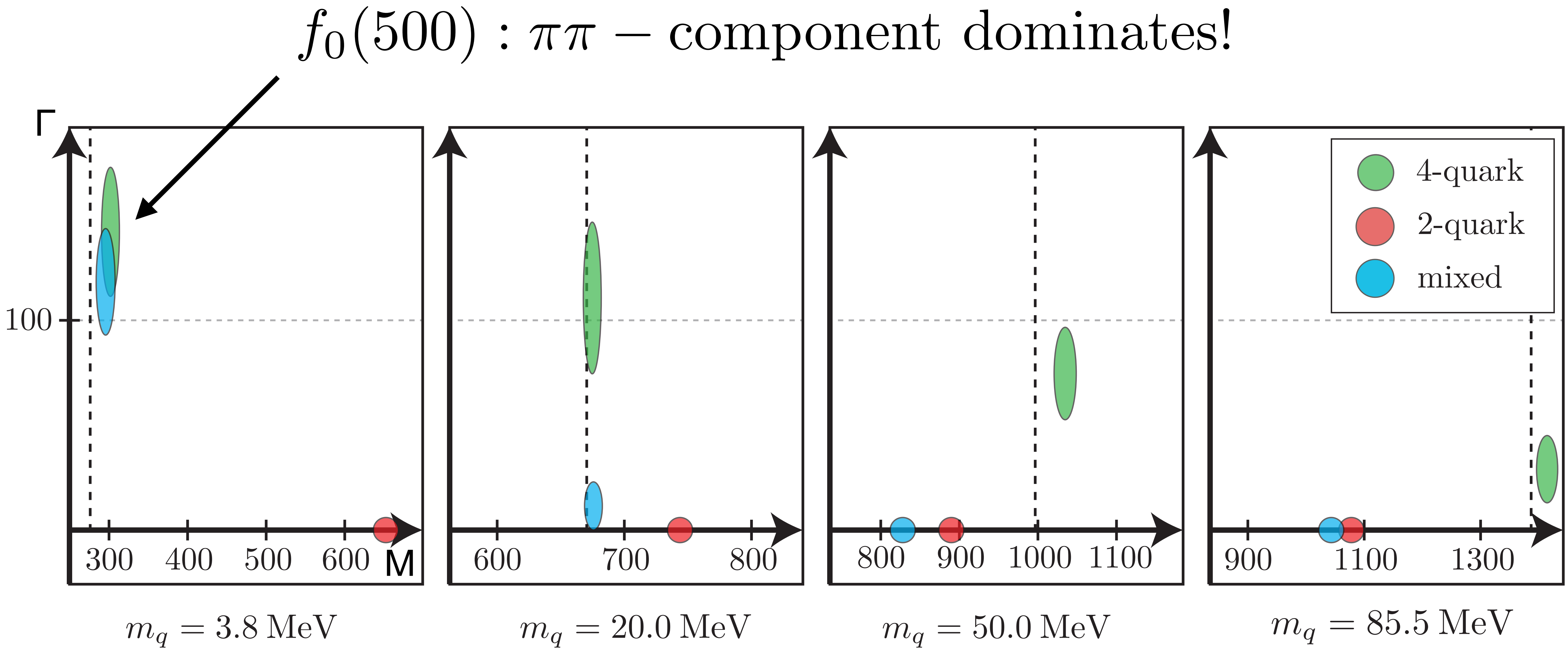
$f_0(500)$: $\pi\pi$ − component dominates!



$m_q = 3.8$ MeV $m_q = 20.0$ MeV $m_q = 50.0$ MeV $m_q = 85.5$ MeV

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

Mass evolution of four-quark state: 0++

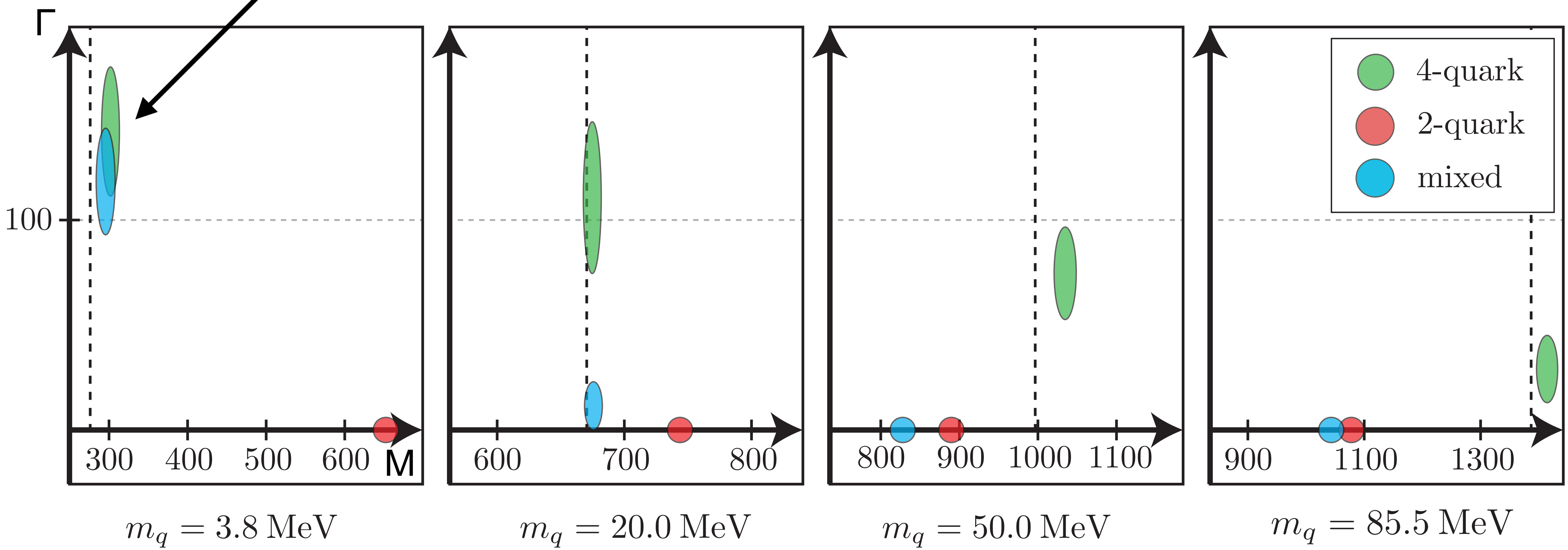


Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

- mixed state becomes qq-dominated for large m_q
- dynamical decision !

Mass evolution of four-quark state: 0++

$f_0(500) : \pi\pi$ – component dominates!



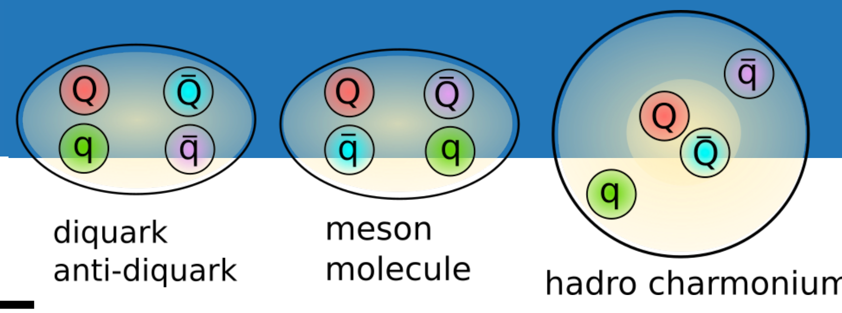
Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

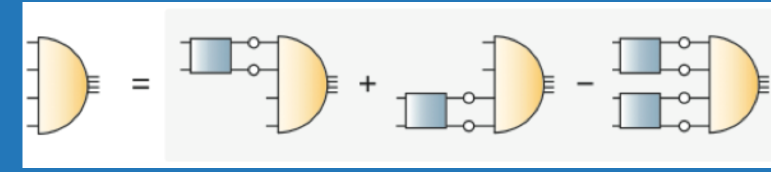
- mixed state becomes qq-dominated for large m_q
- dynamical decision !

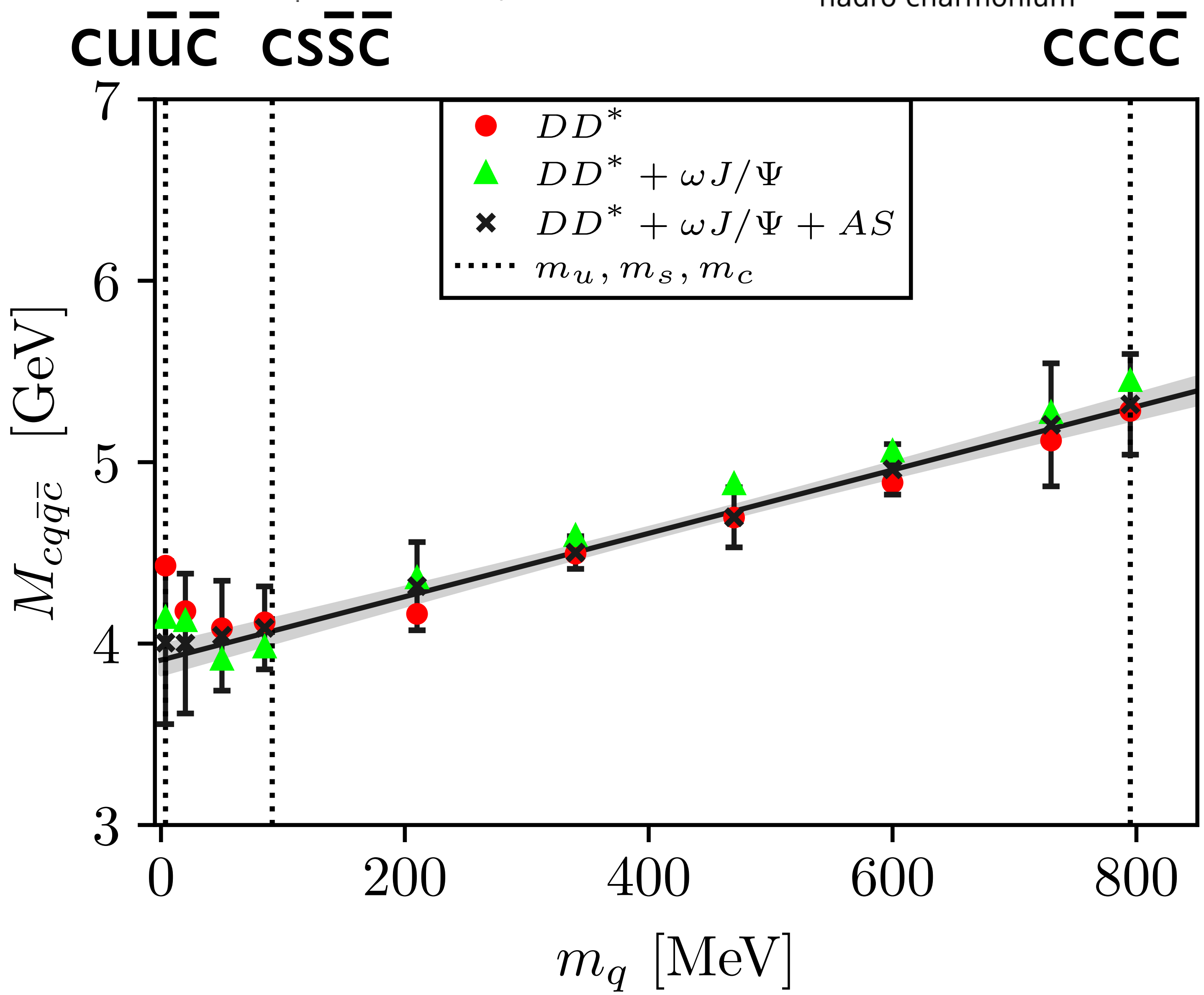
⟶ consequences for ccqq, ccss, bbq, bbss, bbcc ?

work to be done!

$J^{PC} = 1^{++}$







m_c fixed

m_q varied

- DD* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74)\,\text{MeV} \longrightarrow X(3872)$$

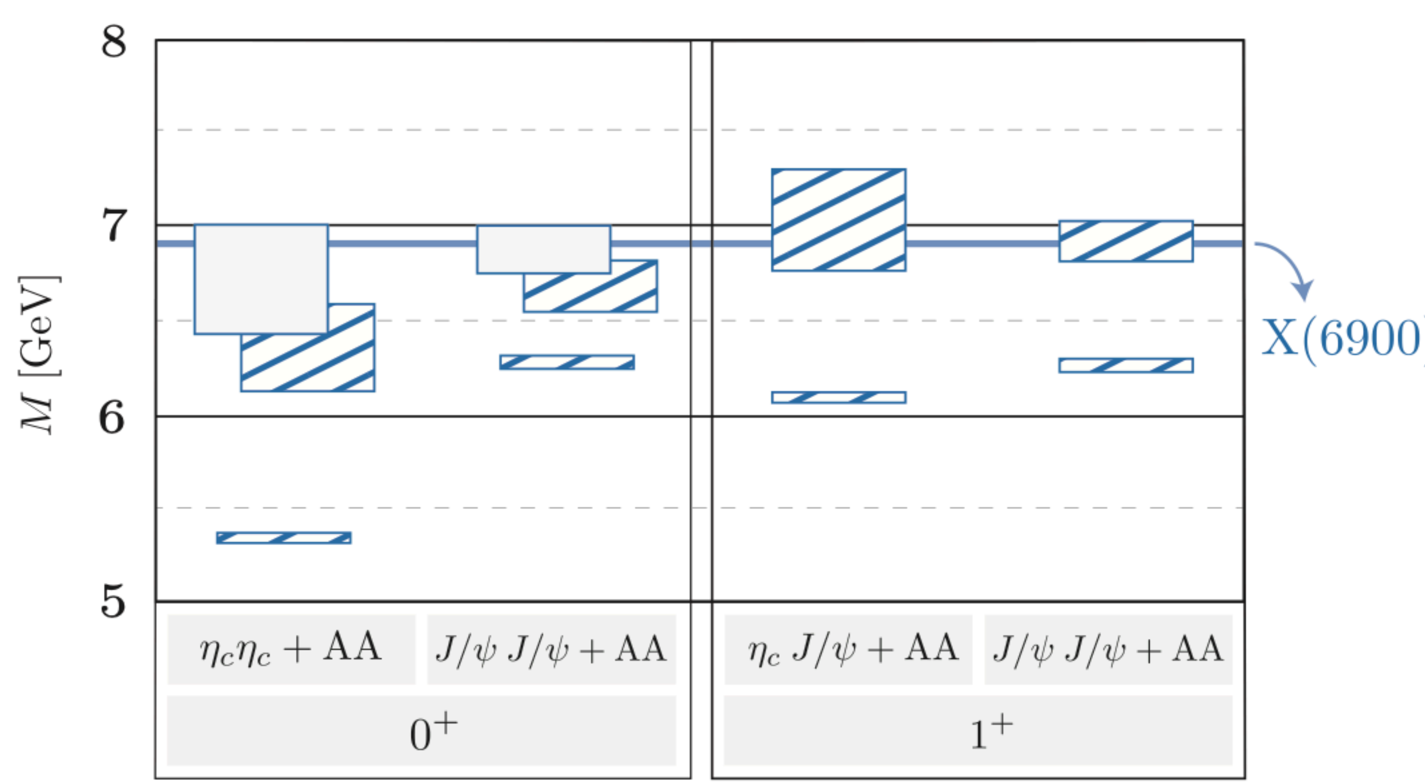
Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

Heavy four-quark states from DSE/BSEs

	$I(J^{PC})$	dominant	4-body	eff. 2-body	Exp.
hidden charm	$0(0^{++})$	DD	3.20 (11)	3.49 (25)	
$(c\bar{c}q\bar{q})$	$0(1^{++})$	DD*	3.92 (7)	3.85 (18)	$X(3872)$
	$1(1^{+-})$	DD*	3.74 (9)	3.79 (31)	$Z_c(3900)$
	$1(0^{++})$	DD		3.20 (31)	
open charm	$0(1^{+})$	DD*	3.90 (8)	3.49 (48)	$T_{cc}(3875)$
$(cc\bar{q}\bar{q})$	$1(0^{+})$	DD+AA	3.80 (10)	3.21 (2)	
	$1(1^{+})$	DD*+AA	4.22 (44)	3.47 (24)	

all charm

$(\bar{c}\bar{c}cc)$



Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

Wallbott, Eichmann and CF, PRD102 (2020), 051501, [2003.12407]

Santowsky, CF, EPJC 82 (2022) 4, 313

Summary

Internal dynamics very important !!

Glueballs:

- First quantitatively reliable results using very involved truncation

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

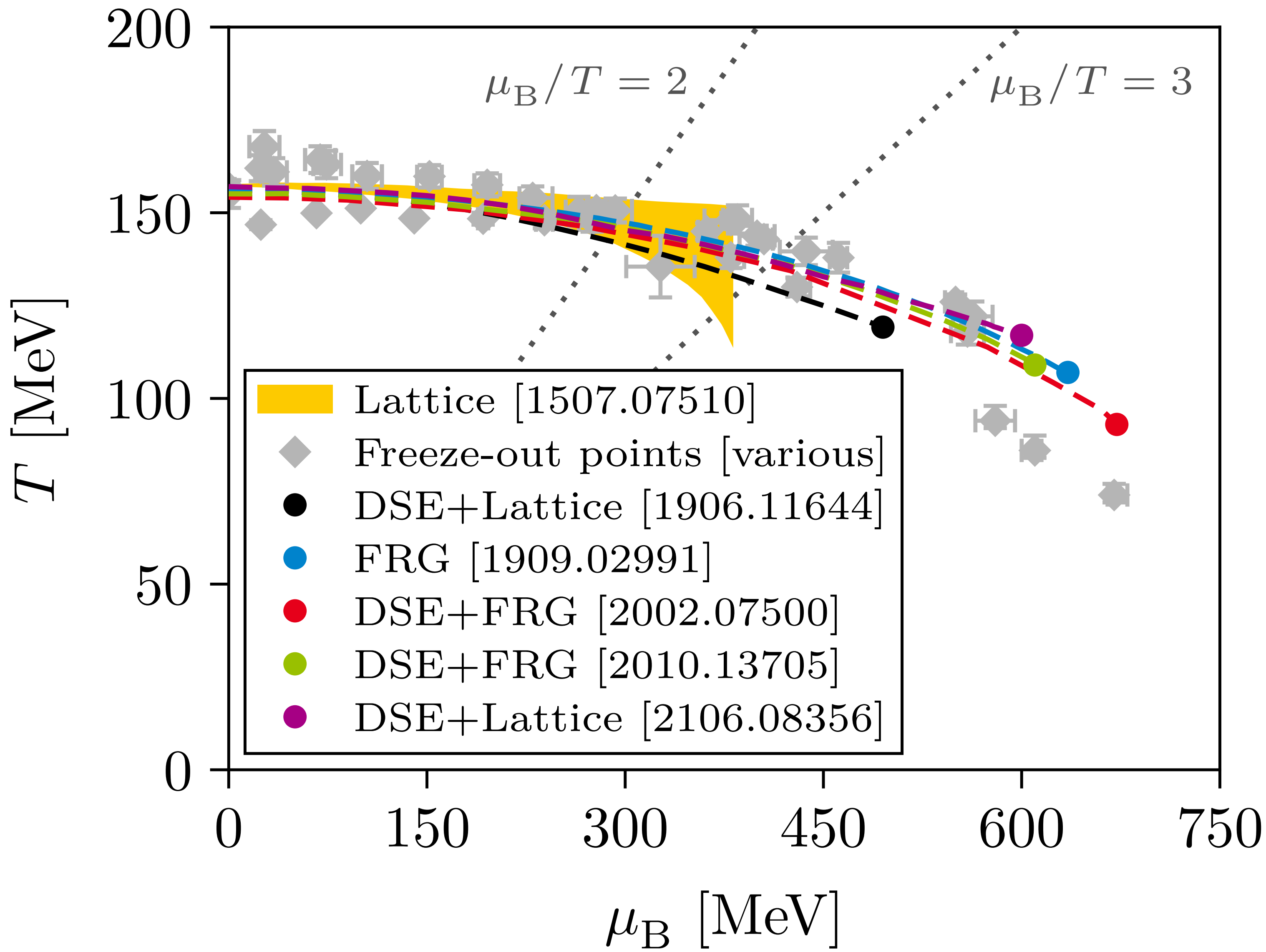
Four-quark states:

- Dynamical description of σ: π-π resonance — Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Dynamical description of X(3872) and Z(3900): DD* dominated
- First results in open charm channels — Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]; Wallbott, Eichmann and CF, PRD102 (2020) 051501, [2003.12407]
- Mixing with $q\bar{q}$ studied for light mesons — Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, [2007.06495].

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38, [2008.10240]

Backup Slides

QCD phase diagram and heavy ion collisions



Bernhardt, CF, Isserstedt, 2208.01981

heavy ion collisions

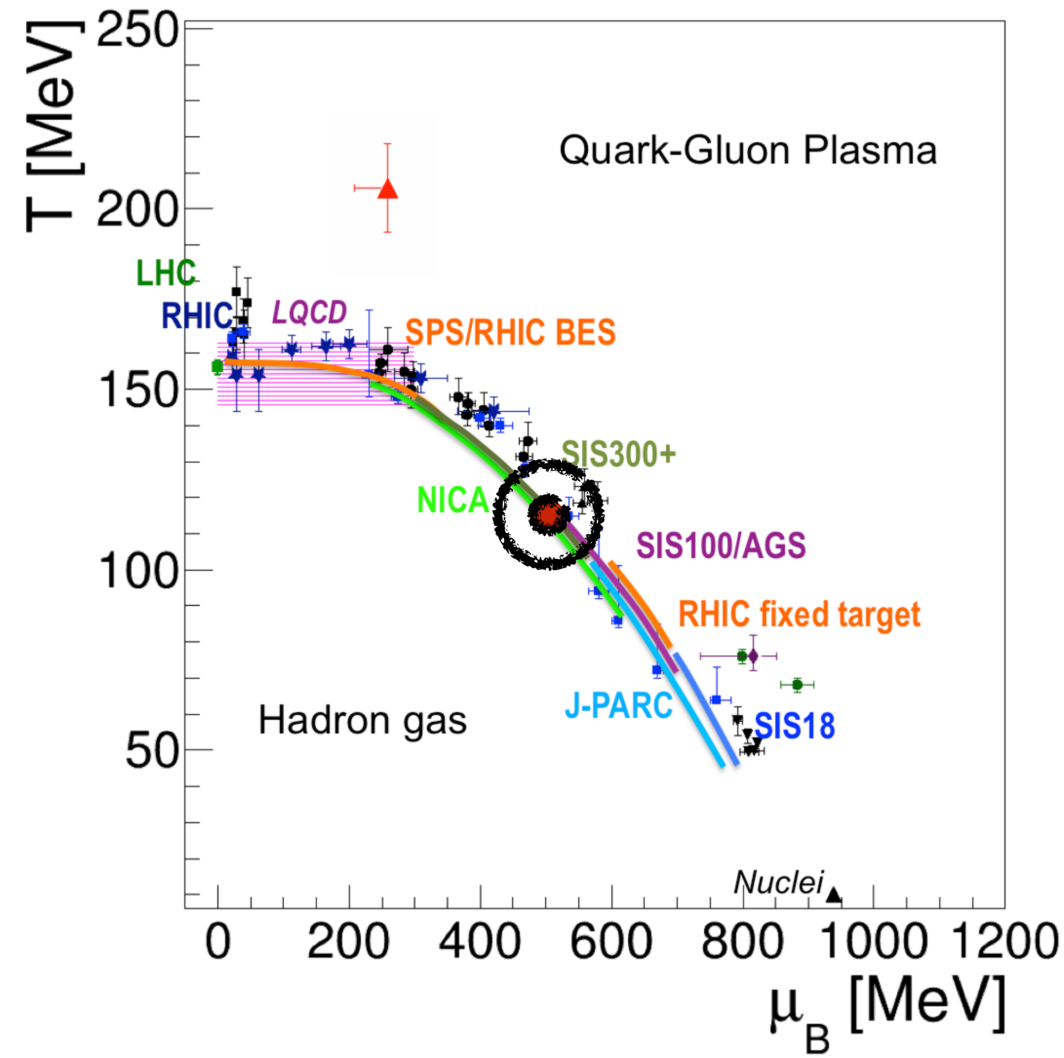
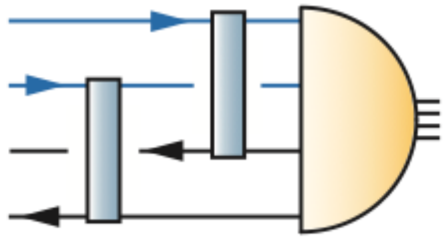


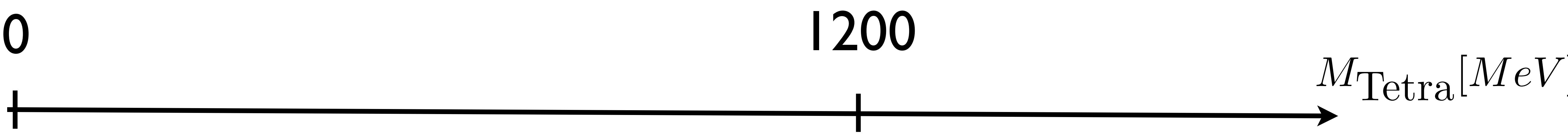
Figure adapted from talk of T. Galatyuk, Erice 2016

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \cancel{s, a, \ldots})$$

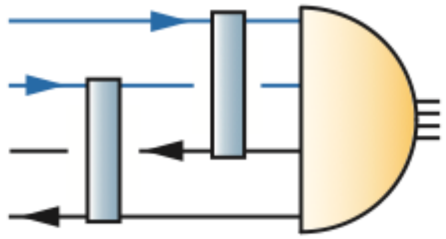
without twobody-clustering



Bound state of
four massive quarks

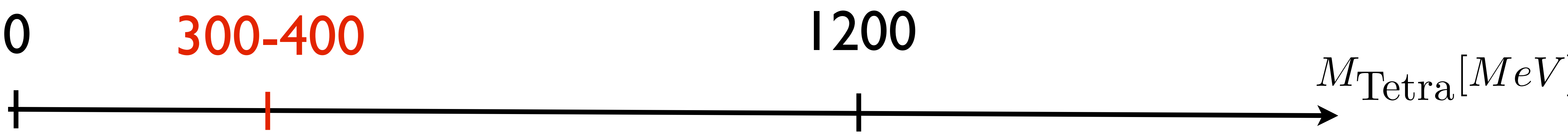
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$\Gamma(S_0, s, a, \ldots)$

without twobody-clustering



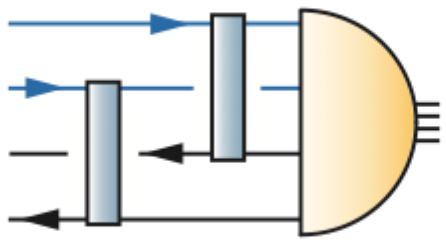
with π-clustering

Two-pion resonance

Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$\Gamma(S_0, s, a, \ldots)$

without twobody-clustering

0   300-400   1200 $M_{\text{Tetra}}[MeV]$

with π-clustering

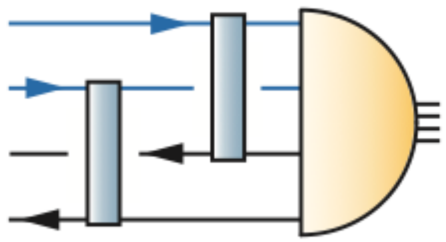
Two-pion resonance

Bound state of
four massive quarks

⟶ identify with $f_0(500)$ ('σ-meson')

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$\Gamma(S_0, s, a, \ldots)$

without twobody-clustering

0 — 300-400 — 1200 — $M_{\text{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

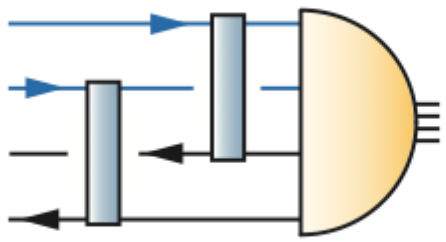
Bound state of
four massive quarks

⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $m(a_0, f_0) \approx 1GeV$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$\Gamma(S_0, s, a, \ldots)$

without twobody-clustering

0 — 300-400 — 1200 — $M_{\mathrm{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

Bound state of four massive quarks

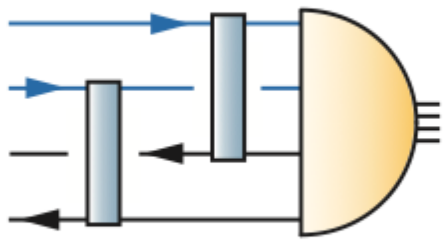
⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $m(a_0, f_0) \approx 1 GeV$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Meson-meson components dominate over diquarks !

Bound state vs resonance: scalar four-quark states



$\Gamma(S_0, s, a, ...)$

without twobody-clustering

0 — 300-400 — 1200 — $M_{\mathrm{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

Bound state of
four massive quarks

⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $m(a_0, f_0) \approx 1 GeV$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

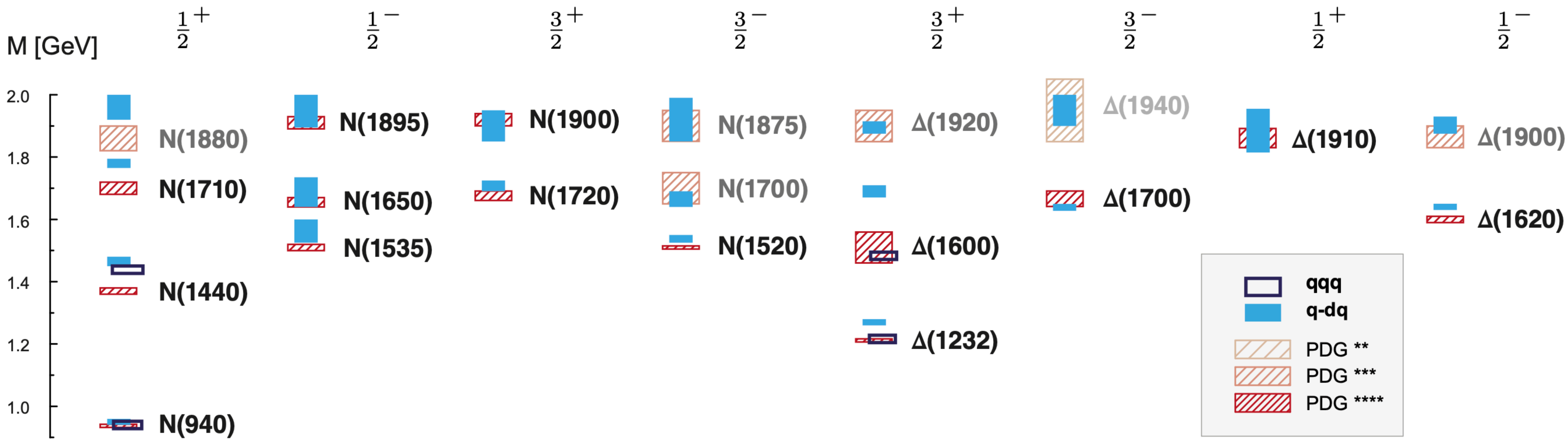
Meson-meson components dominate over diquarks !

Mixing with $q\bar{q}$: small effect

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014
Santowsky, CF, PRD 105 (2022) 4,313

Light baryon spectrum: DSE-RL

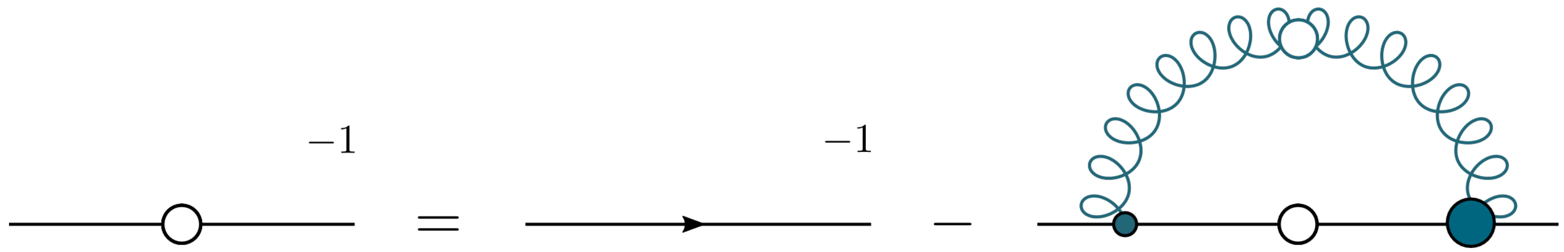
3 parameters + $m_{u,d,s}$



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
Eichmann, Few Body Syst. 63 (2022) no.3,

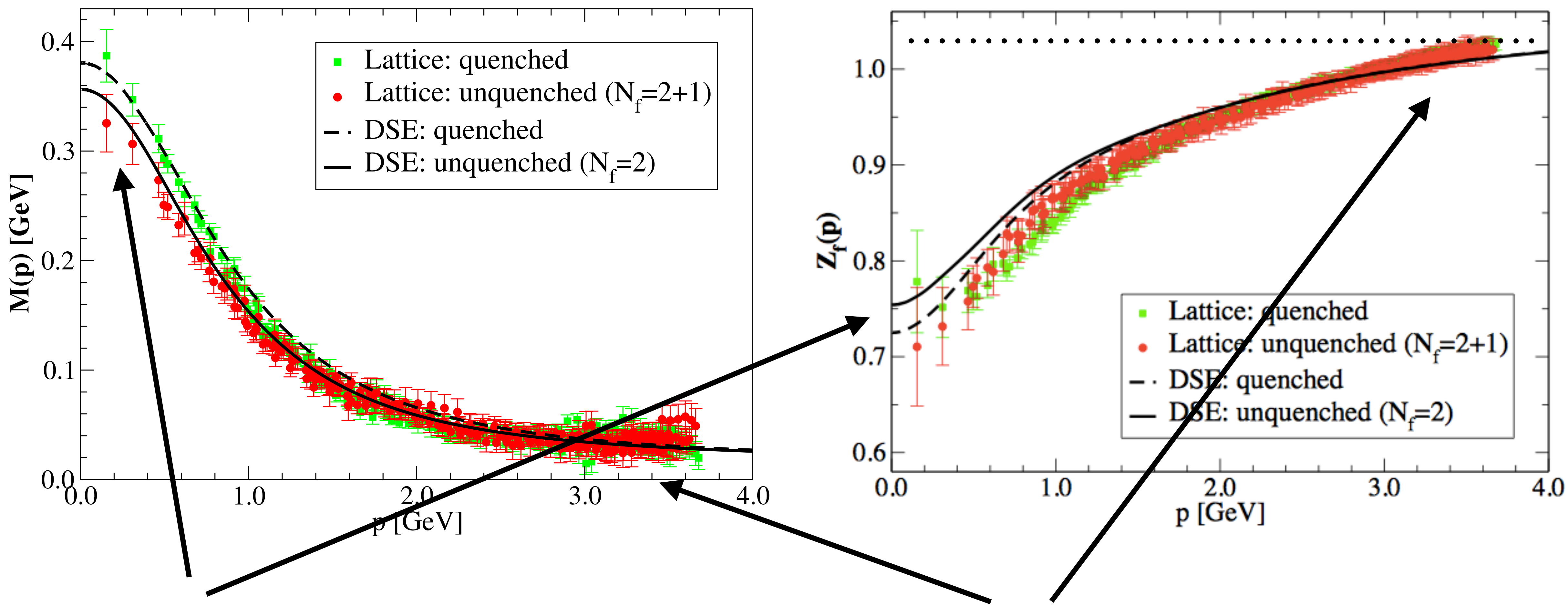
- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- three-body agrees with diquark-quark where applicable

Quarks: mass from interaction



$$S(p) = Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

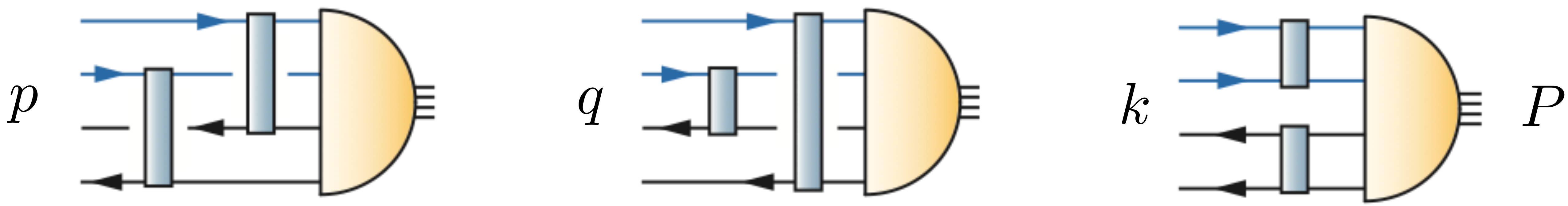


‘constituent quark’:
large mass; very composite

‘current quark’:
- small mass; non-composite

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

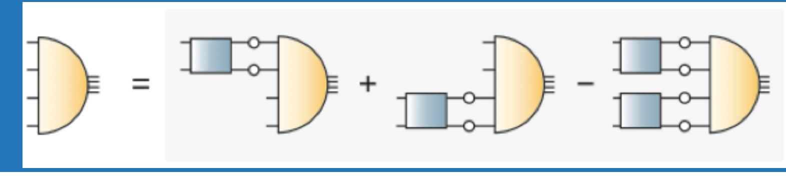
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

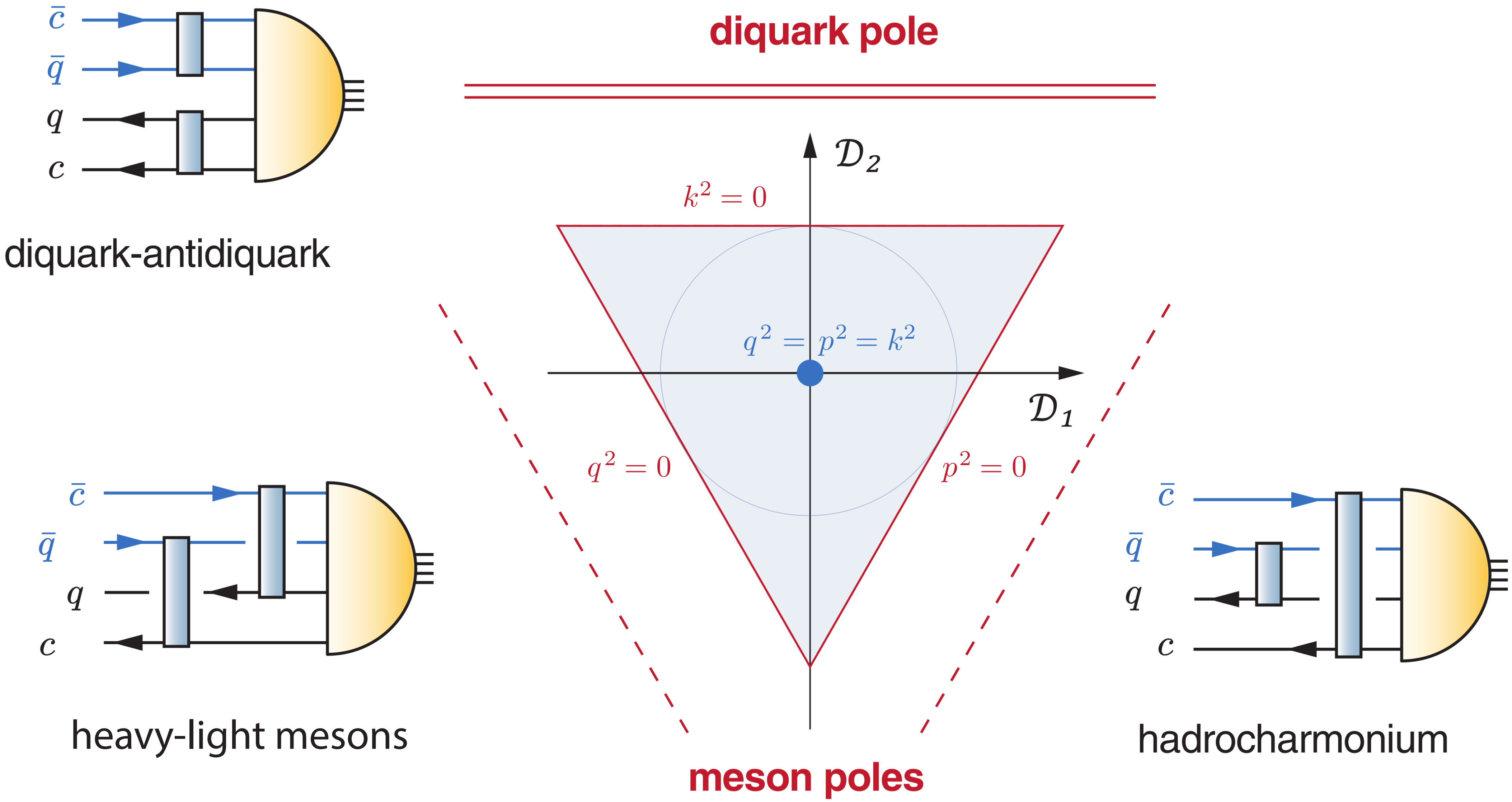
- reduce # tensor structures guided by physics: → ~20 tensor structures

Four-body equation: permutations

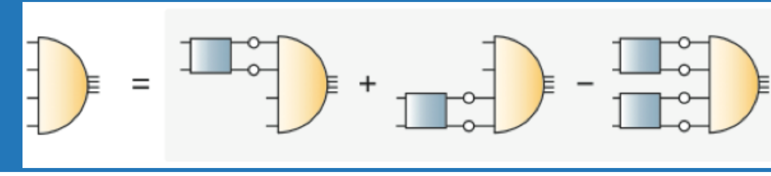


- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ $\quad p, q, k$: relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

  $\mathcal{D}_2 \sim q^2 - p^2$

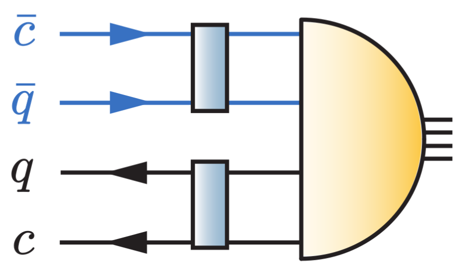


Four-body equation: permutations

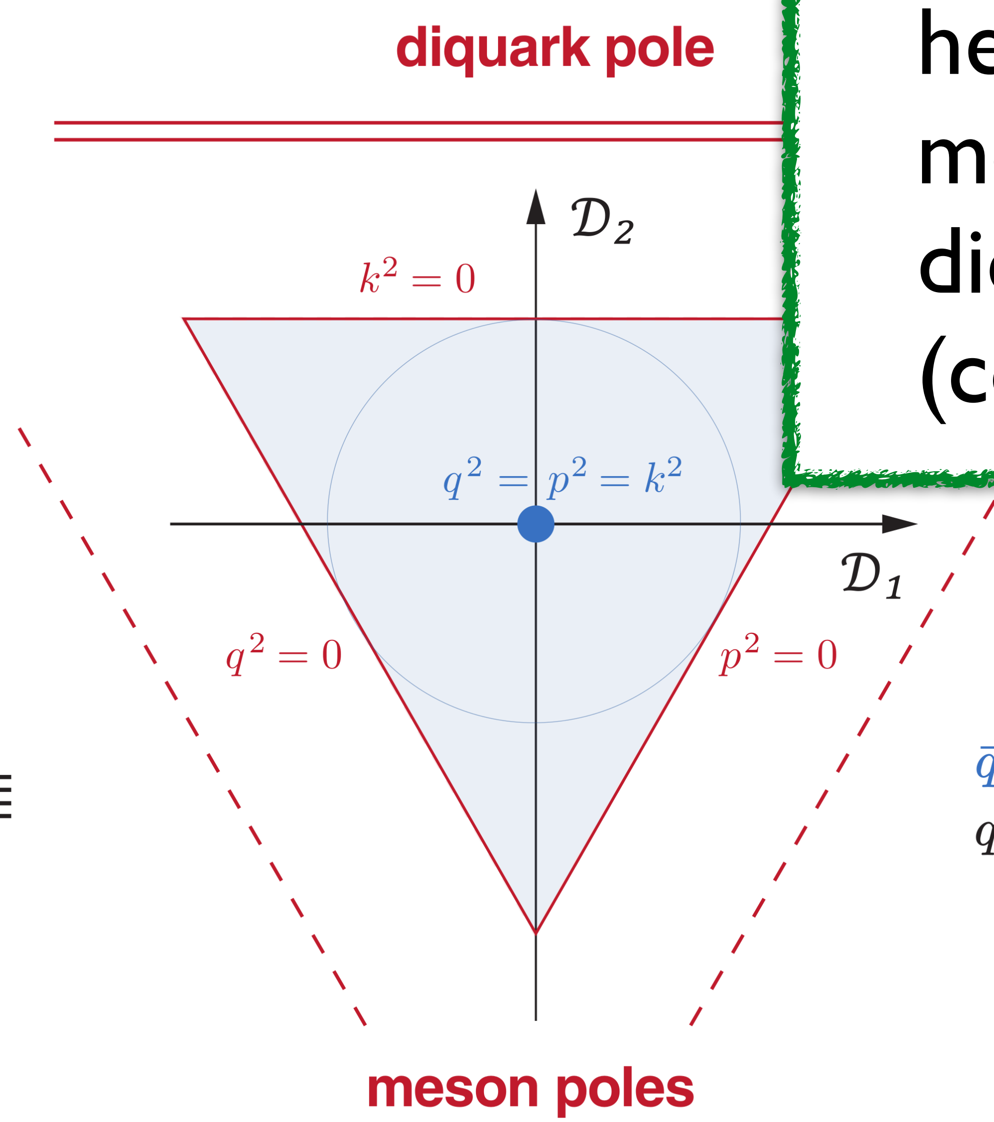


- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ $\qquad p, q, k$: relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

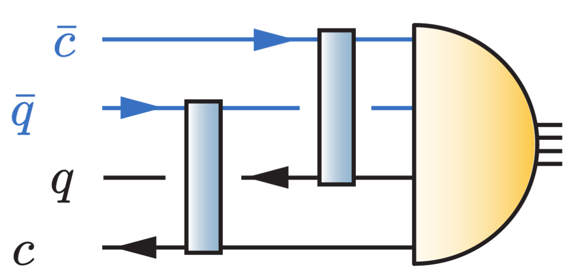
  $\mathcal{D}_2 \sim q^2 - p^2$



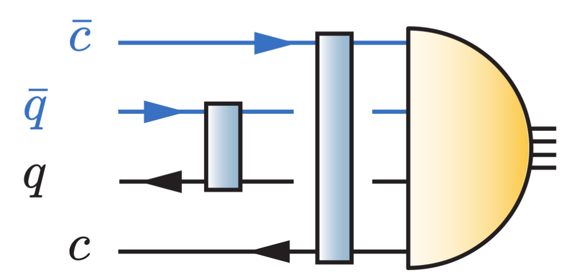
diquark-antidiquark



- model independent: heavy-light meson poles more important than diquark poles (color factor !)

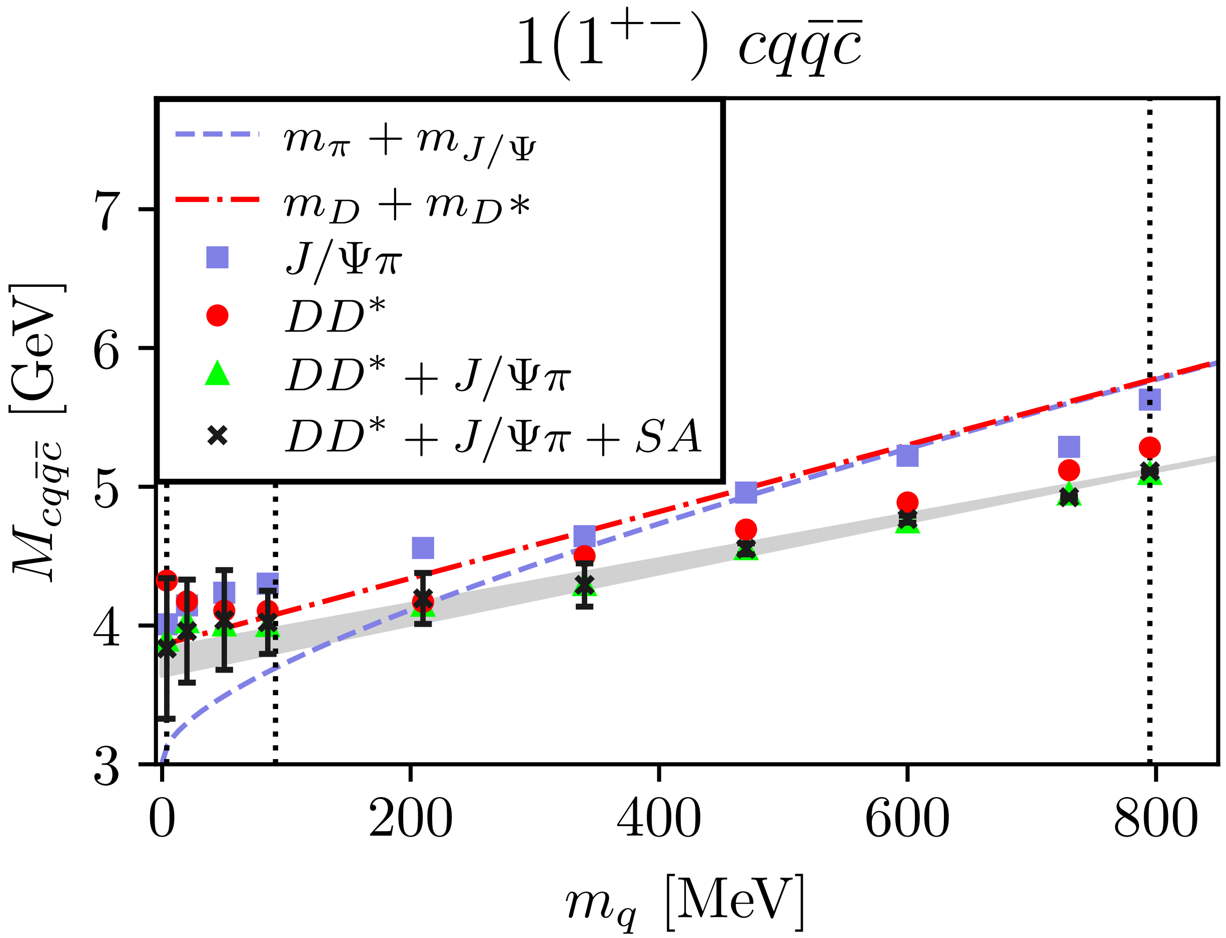


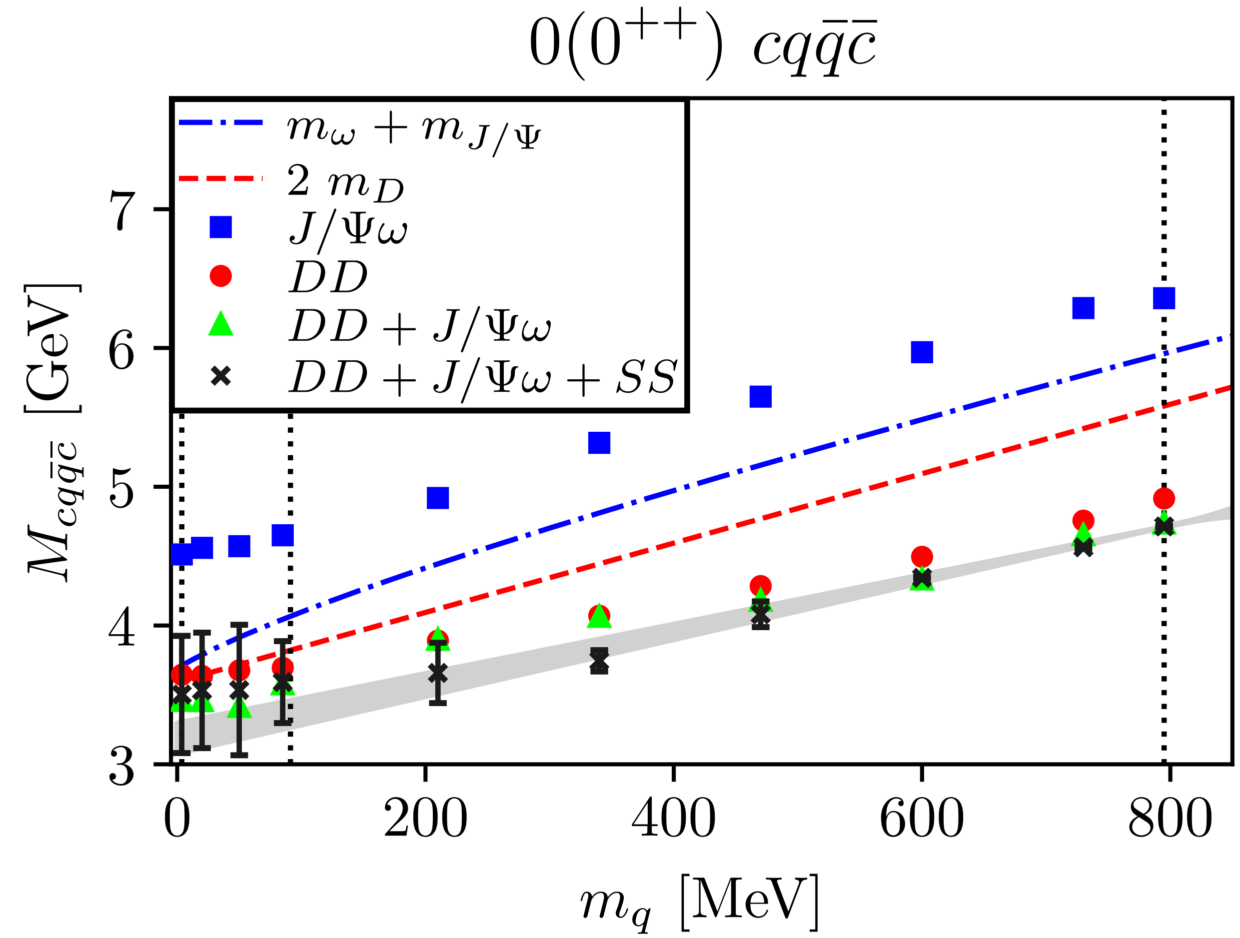
heavy-light mesons



hadrocharmonium

$J^{PC} = 1^{+-}$ and 0^{++}



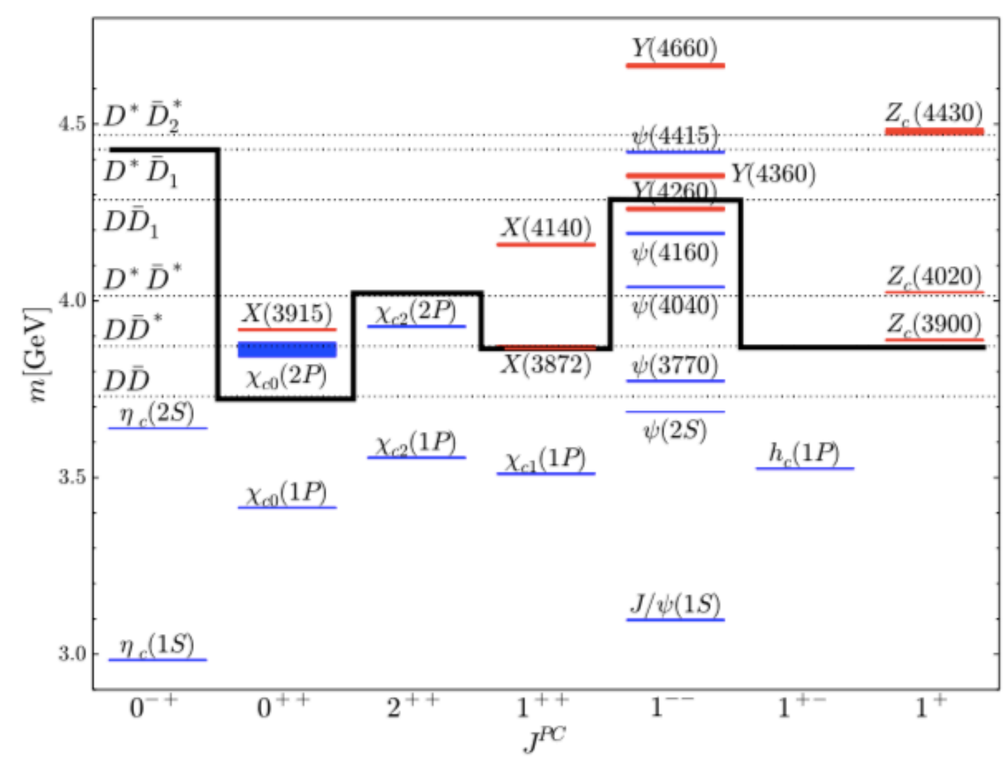


Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

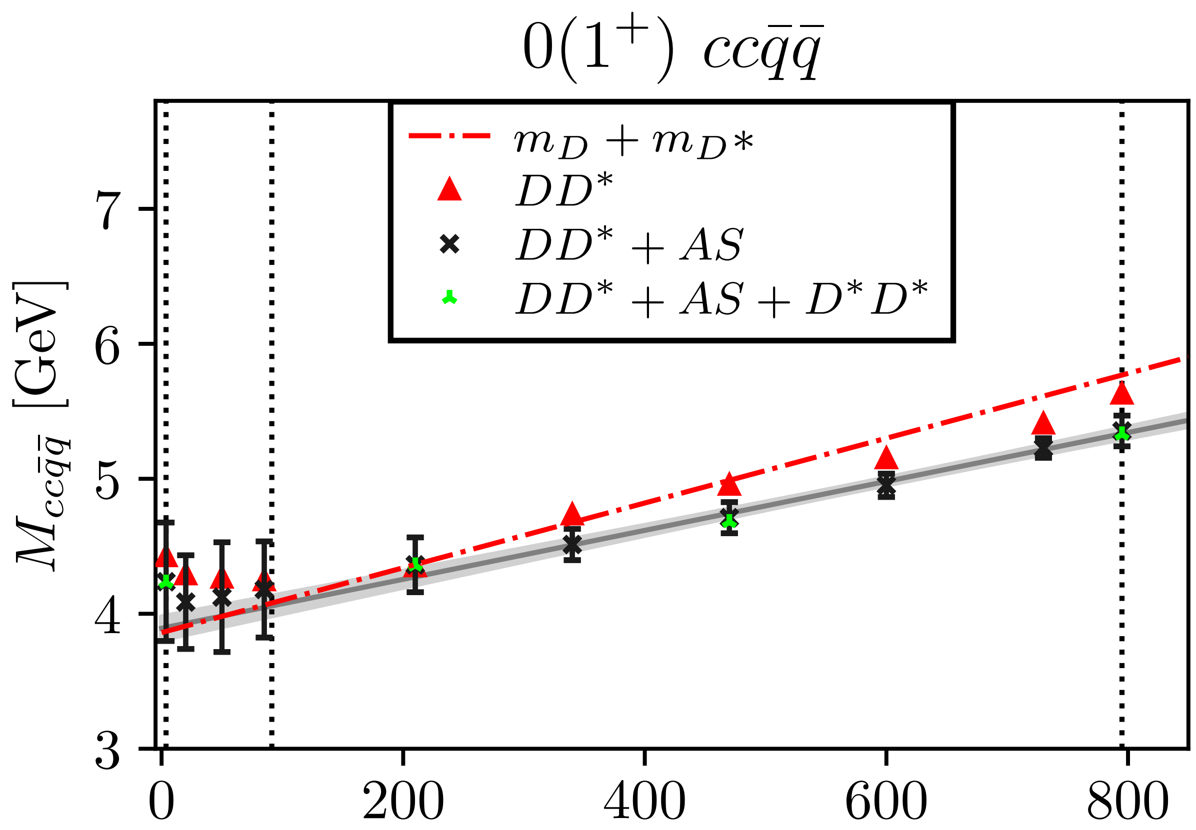
$$M^{cq\bar{q}\bar{c}}_{1^{+-}} = 3741(91) \ \rightarrow \ Z(3900)$$

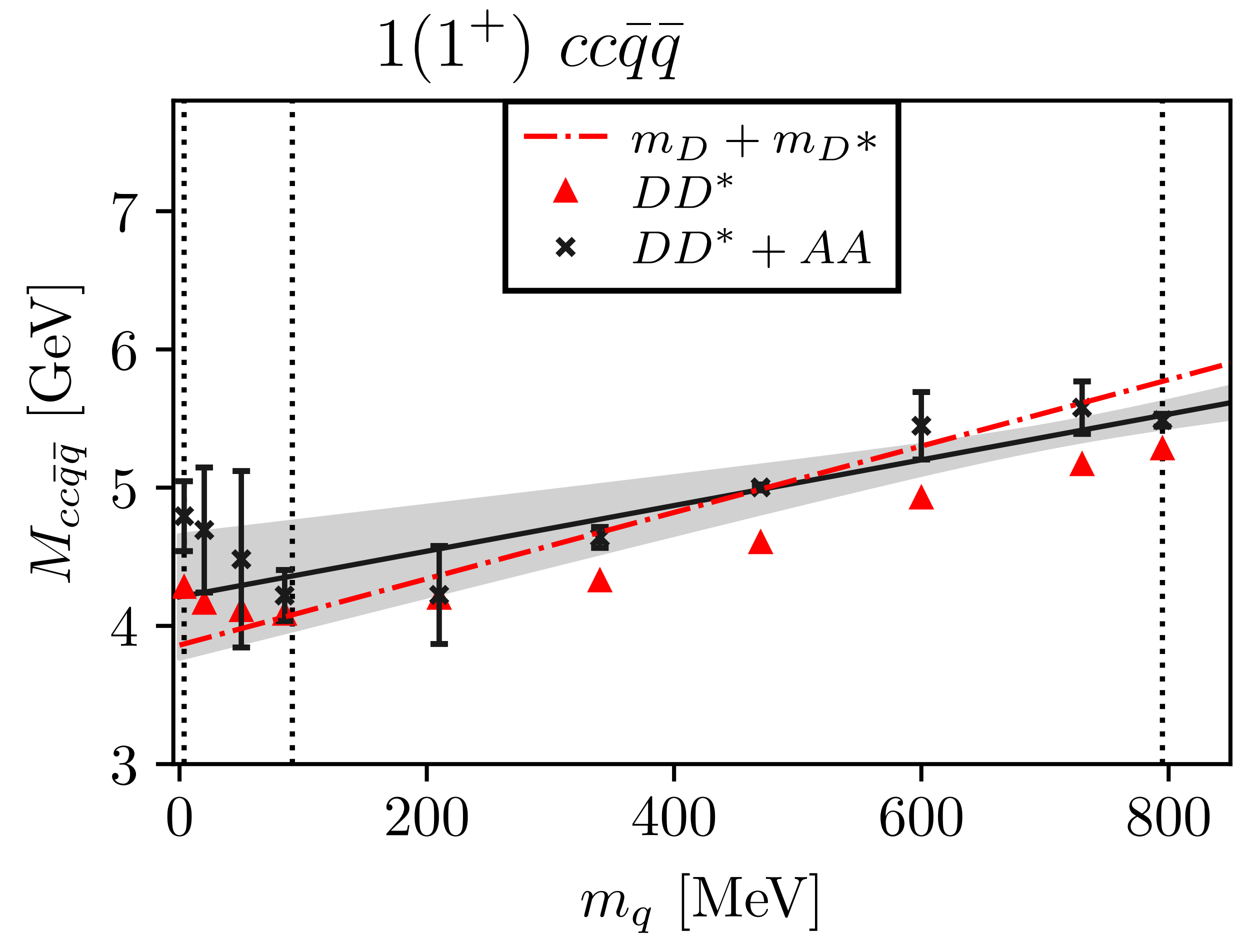
$$M^{cq\bar{q}\bar{c}}_{0^{++}} = 3195(107) \ \rightarrow ?$$

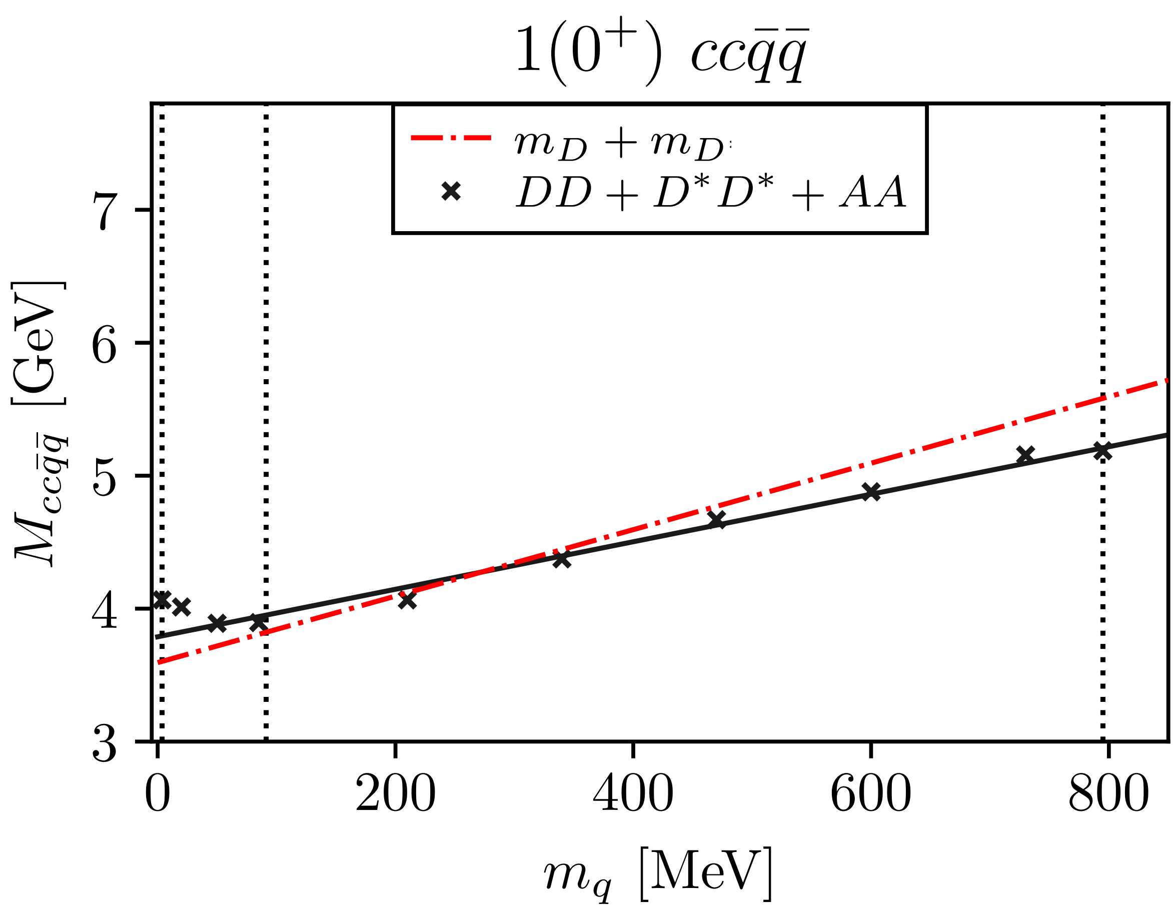
mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states



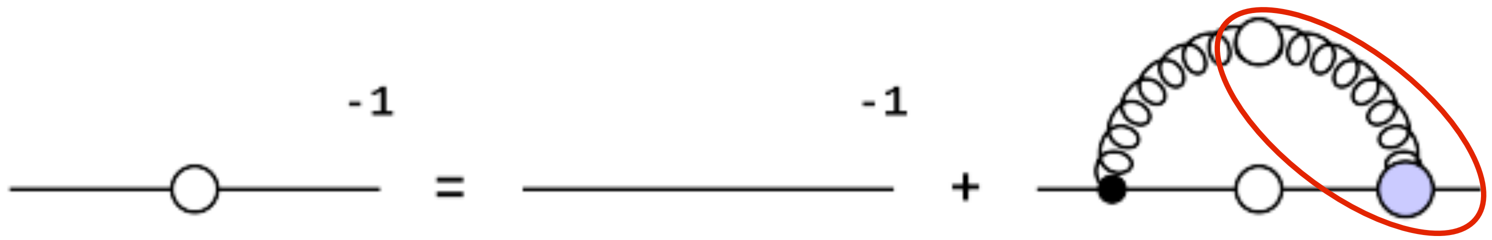




- DD(*) and diquarks important!

Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

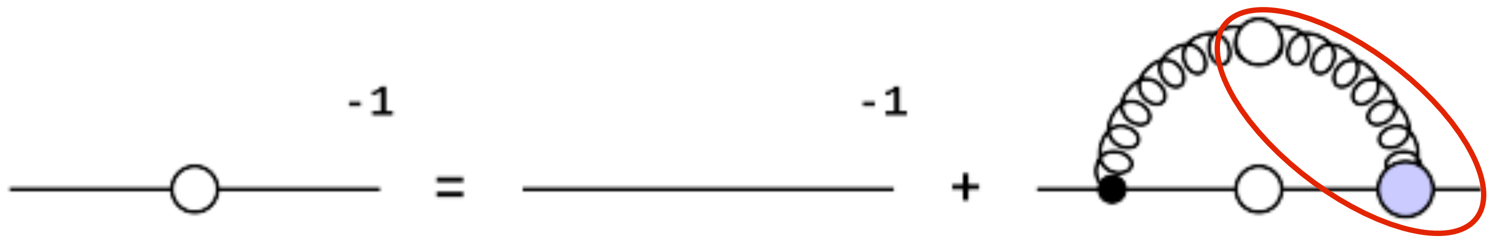
$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

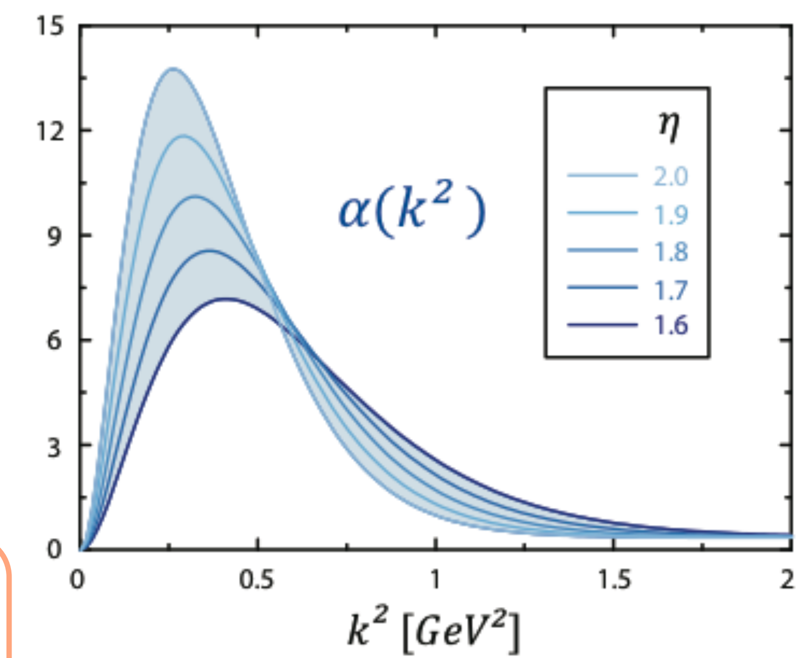
$$\sim \gamma^\mu \tau(k^2)$$ "approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi}\tau(k^2)Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein