Theory of Kaon Physics

Giancarlo D'Ambrosio
INFN-Sezione di Napoli

Mainz, 27th August 2014

Outlook

- Chiral perturbation theory (CHPT)
- CHPT tests $\pi\pi$ scattering lenghts, K_{l4}
- CHPT tests $K_S \longrightarrow \gamma\gamma$: no CT's $\Longrightarrow$ only chiral loop
- CHPT tests $K_L \longrightarrow \pi^0\gamma\gamma$ VMD effects and importance to control long distance effects for exact short distance effects
- CHPT tests $K^\pm \longrightarrow \pi^\pm\gamma\gamma$: $\Delta S = 1$ -tests
- CP violation in $K \longrightarrow \pi\pi\gamma$, $K \longrightarrow \pi\pi ee$ Cappiello, Cata,G.D.,Gao
- Interplay SD and LD: $K_{S,L} \longrightarrow \mu^+\mu^-$
- Short distance dominated $K \longrightarrow \pi\bar{\nu}\nu$
- Conclusions

Chiral Perturbation Theory

χPT effective field theory based on the two assumptions

- π's are the Goldstone boson of $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$
- *(chiral) power counting* i.e. the theory has a small expansion parameter: $p^2/\ \Lambda^2_{\chi SB}$: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2$ GeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{F_\pi{}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} + \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

Fantastic chiral prediction $A_{\pi\pi} \sim (s - m^2_\pi)/F_\pi{}^2$ Weinberg, Colangelo *et al*

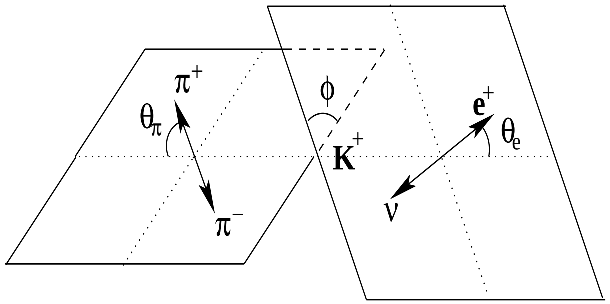
$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\, K\to\pi l^+ l^-} + \cdots$$

K_{l4} and $\pi\pi$ strong phases $\delta_I^l(s)$

Cabibbo Maksymowicz

$$\frac{G_F}{\sqrt{2}} V_{us}\ \bar{e}\gamma^\mu(1-\gamma^5)\nu\ H_\mu(p_1, p_2, q)$$

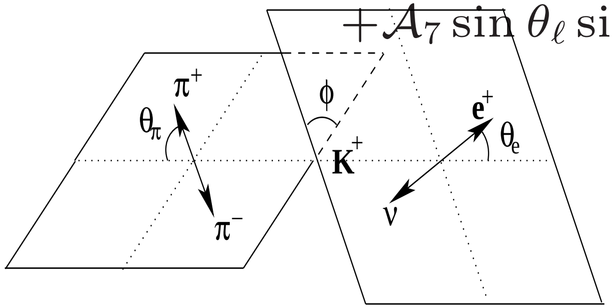
$$H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta. \qquad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$



several form factors (F, G, H) with kin $(q^2, \cos\theta_\pi)$ and final state $(\delta_0^0, \delta_1^1,)$ dependence

K_{l4} and $\pi\pi$ strong phases $\delta^l_I(s)$ Colangelo,.. Descotes,

$$\frac{d^5\Gamma}{dE^*_\gamma dT^*_c dq^2 d\cos\theta_\ell d\phi} = \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi$$
$$+\mathcal{A}_4 \sin 2\theta_\ell \cos\phi + \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell$$
$$+\mathcal{A}_7 \sin\theta_\ell \sin\phi + \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi$$



- crucial to measure $\sin\delta \Longrightarrow$ interf F_3
- Look angular plane asymmetry

Latest K_{e4} results from NA48/2

$$B(K_{e4}^{+-}) = (4.257 \pm 0.004 \pm 0.016 \pm 0.031_{ext})10^{-5}$$

is 3 times more precise than available PDG value

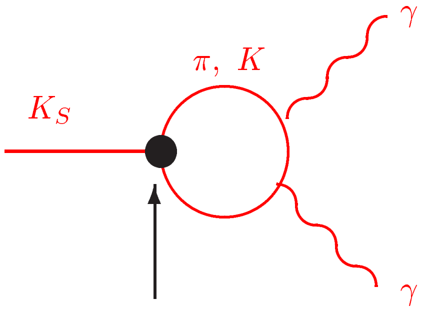
Preliminary

$$B(K_{e4}^{00}) = (2.552 \pm 0.010 \pm 0.010 \pm 0.032_{ext})10^{-5}$$

is 10 times more precise than available PDG value

$K_S \to \gamma\gamma$

- No short distance contributions, No $O(p^2)$
- Neutral particles $(K_S) \Rightarrow$ No $O(p^4)$ CT : $F_{\mu\nu}F^{\mu\nu}\langle\lambda_6 QU^+QU\rangle$
- Loop contribution finite scale independent and unambigous χPT prediction



G_8 from $K_S \to \pi\pi$

$\mathrm{Br}_{\chi\mathrm{PT}}(K_S \to \gamma\gamma)$=$2.1\cdot10^{-6}$

(G.D. and Espriu 86, Goity 87)

$(2.78 \pm 0.072) \cdot 10^{-6}$ (NA48 '02)

- $O(p^6)_{\rm CT}$ $\qquad F^{\mu\nu}F_{\mu\nu}\langle\lambda_6 Q^2 \mu M U^+\rangle$

  No VMD $\qquad \Rightarrow \dfrac{A^{(6)}}{A^{(4)}} \sim \dfrac{m_K^2}{(4\pi F_\pi)^2} \sim 0.2$

  (No terms $\sim \dfrac{m_K^2}{m_\rho^2} \sim 0.4$)

- $\mathrm{NA48} \Rightarrow \dfrac{A^{(6)}}{A^{(4)}} \sim 15\%$

  $\Downarrow$

- The error in the amplitude, is smaller than the naive expectation, 20-30%

$K_L(p) \rightarrow \pi^0(p_3)\gamma(q_1)\gamma(q_2)$

Two types of amplitudes A and B: important to show that

- B important since establish the size of the CP conserving contribution to $K_L \rightarrow \pi^0 ee$, generated by two photon exchange,
- data show that B is small (from low diphoton invariant mass) and
- higher order contributions from VMD are important

$K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$

Lorentz + gauge invariance $\Rightarrow$ $M \sim$ $A(y,z)$ $B(y,z)$

$\gamma\gamma$ $\gamma\gamma$

$y{=}p{\cdot}(q_1-q_2)/m_K^2, \quad z{=}(q_1+q_2)^2/m_K^2$ $J=0$ D − wave too

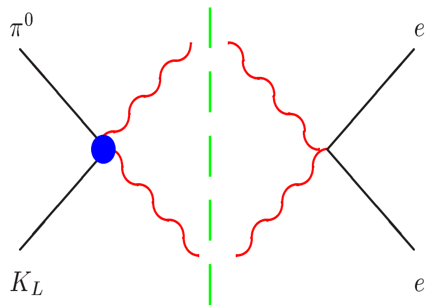
$r_\pi{=}m_\pi/m_K$ $F^{\mu\nu}F_{\mu\nu}$ $F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0$

- $\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2$ S, B
- Different gauge structure $\Rightarrow$ B$\neq$0 at $z \to 0$ (collinear photons).

Crucial role in $K_L \to \pi^0 e^+ e^-$

A suppressed by m_e/m_K

B is not



Morozumi et al, Flynn Randall

Sehgal Heiliger, Ecker et al., Donoghue et al.

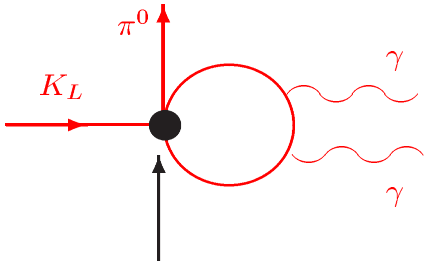
$K_L \to \pi^0 \gamma\gamma$

- $O(p^4)$

Ecker, Pich, de Rafael; Cappiello,G.D

CT Loop

0



only A **But** $\frac{\Gamma(K_L \to \pi^0 \gamma\gamma)_{p_4}}{\Gamma(K_L \to \pi^0 \gamma\gamma)_{\exp}} \sim \frac{1}{2.5}$

G_8 from $K_S \to 2\pi$

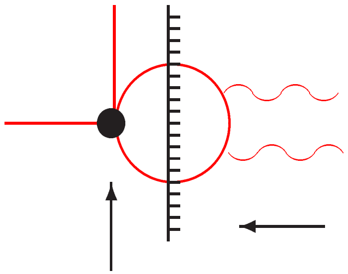
- $O(p^6)$ A, B from:

3 CT's

$F_{\mu\nu} F^{\mu\alpha} \partial_\alpha K_L \partial^\nu \pi^0$

$F^2 \ \partial K_L \partial \pi^0$

$F^2 \ m_K^2 K_L \pi^0$



Cappiello, G.D., Miragliulo
Cohen, Ecker, Pich

← Full description of unitarity cut

$A(K \to 3\pi) = a + b\ Y + c\ Y^2 + d\ X^2$

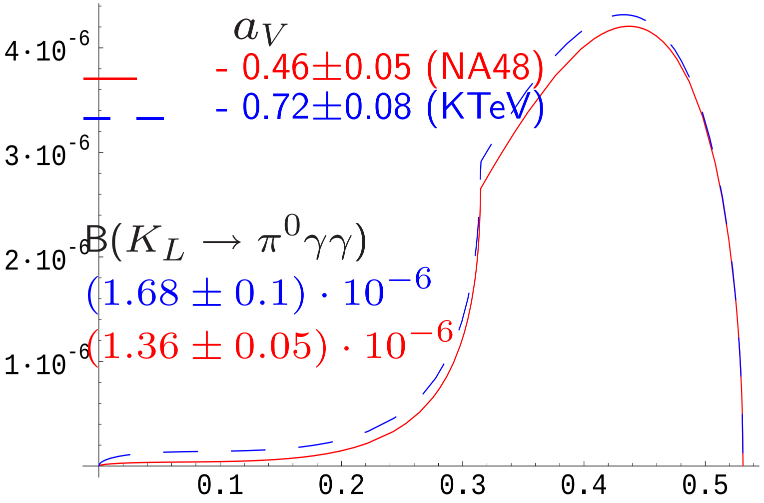
$$A_{\rm CT} = \alpha_1(z - r_\pi^2) + \alpha_2$$
$$B_{\rm CT} = \beta$$

VMD $\Rightarrow$ 1 coupling a_V ($\sim$−0.6 G.D., Portoles)

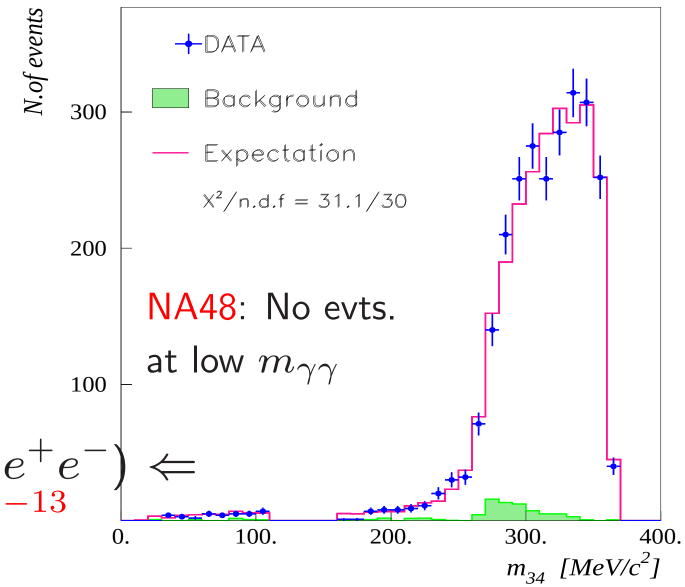
(Ecker, Pich, de Rafael;Sehgal et al.)

$$\alpha_1 = \frac{\beta}{2} = -\frac{\alpha_2}{3} = -4a_V \sim 2 \qquad \text{n.d.a.}\sim 0.2$$

- KTeV and NA48: 1 parameter fit (a_V) with all the unitarity corrections

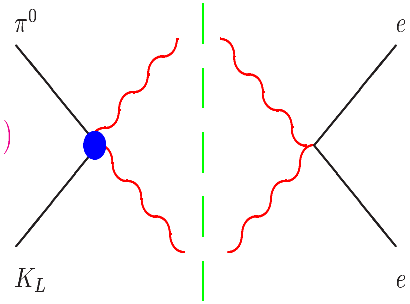


$B(K_L \to \pi^0 e^+ e^-) \Leftarrow$
$< 5 \cdot 10^{-13}$



$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10}$ at 90% CL KTeV

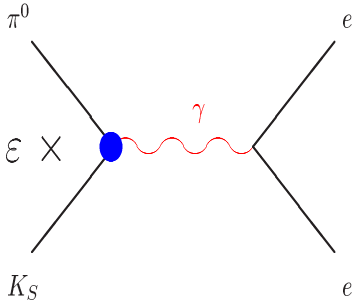
1) 

CP conserving NA48

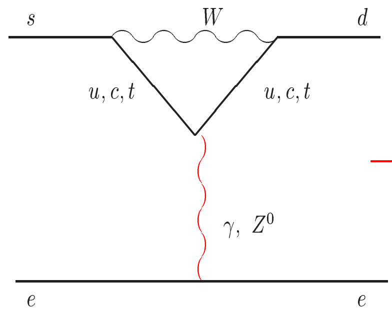
$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP

2) $\varepsilon\times$



$+$ 3)



$\longrightarrow$ Im λ_t
$\lambda_t = V_{td}V^*_{ts}$

$\uparrow$ $\mathrm{B}(K_S \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\ a_S^2 - 6.8\frac{Im\lambda_t}{10^{-4}}\ a_S + 2.8\left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right]\cdot 10^{-12}$$

$$[17.7 \pm \qquad 9.5 + \qquad 4.7] \quad \cdot 10^{-12}$$

$K^+ \to \pi^+\gamma\gamma$

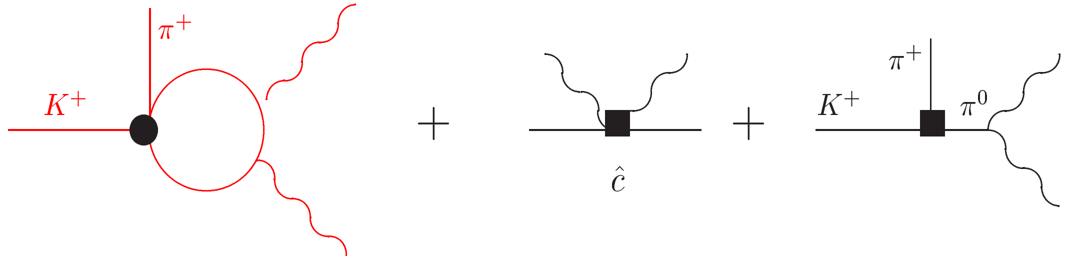
$$\gamma\gamma \quad \text{in} \quad \overbrace{\underset{\substack{P=+1\\ A}}{F_{\mu\nu}F^{\mu\nu}} \qquad \underset{\substack{P=-1\\ C}}{F\tilde{F}}}^{J=0} \qquad \underset{B}{J=2} \quad + \ \ldots$$

Lorentz + gauge invariance

$$\frac{d^2\Gamma}{dydz} \sim \left[z^2(|A+B|^2+|C|^2) + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2\right]$$

$K^+ \to \pi^+\gamma\gamma$

- $O(p^4)$



Ecker, Pich, de Rafael

In factorization $\hat{c} = \frac{128\pi^2}{3}\left[3(L_9 + L_{10}) + N_{14} - N_{15} - 2N_{18})\right] = 2.3(1 - 2k_f)$

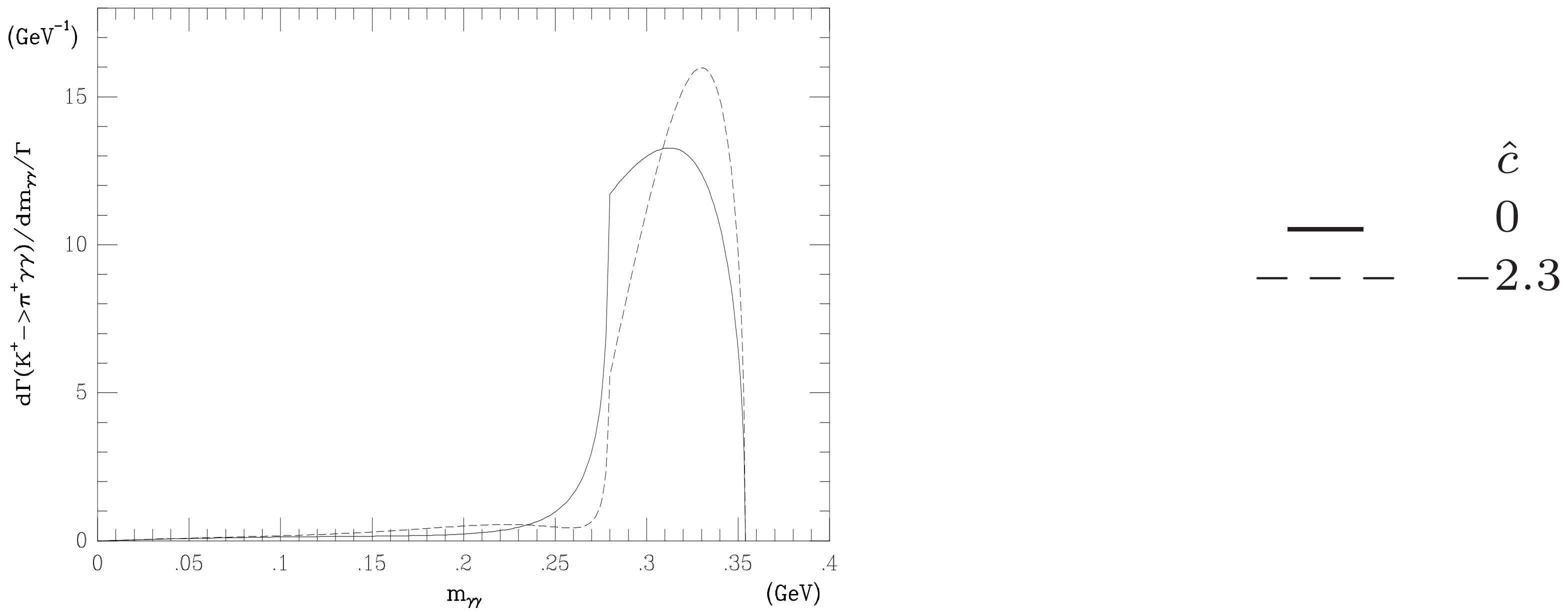
spin-1 contributions (axials) to $\hat{c}$

- $O(p^6)$ G.D., Portoles 96

Unitarity corrections: 30%-40%

a_{V^+} negligible

$K^+ \to \pi^+\gamma\gamma$



BNL 787 (96) got 31 events Br$\sim$ $(6 \pm 1.6)\cdot 10^{-7}$ $\hat{c} = 1.8 \pm 0.6$ E949 no events at low $m_{\gamma\gamma}$:
NA48 $K^+ \to \pi^+ e^+ e^- \gamma$ $\hat{c} = 0.90 \pm 0.45$

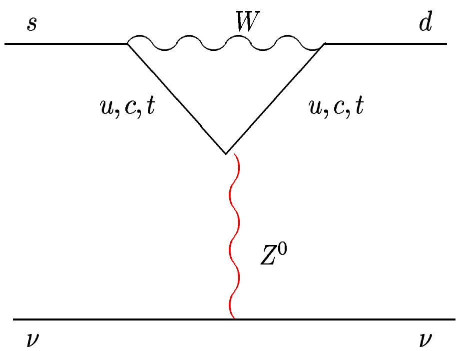
Final NA48/2 + NA62 $B = (1.003 \pm 0.056)\cdot 10^{-6}$, $\hat{c} = 1.86 \pm 0.26$

$K_S \to \mu\overline{\mu}$ **interesting bound from LHCB**

- LHCB $B(K_S \to \mu\overline{\mu}) < 11 \times 10^{-9}$ at 95% CL after 40 years
- Long Distance (LD) dominated, Short distance (SD) small **ONLY CP Violating** small BUT VERY INTERESTING
- SM SD $B_{SD} = 1 \times 10^{-5} |\Im(V_{ts}^{*} V_{td})|^2 \sim 10^{-13}$ SMALL; NP could be larger and so EXPERIMENTS ARE RELEVANT $\sim 10^{-11}$ allowed;

$K \to \pi\nu\overline{\nu}$

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\, m_q^2\right]$$



$$\sim \qquad \left[A^2\lambda^5\,(1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM: $\underbrace{V-A \otimes V-A}_{\Downarrow}$ Littenberg

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow \; J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

$K^+ \to \pi^+ \nu \overline{\nu}$

Brod,CKM2010, Straub, Gorbhan

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_\mathrm{c}}{\lambda} (P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 \right]$$

- κ_+ from K_{l3}
- P_c: SD charm quark contribution (30%±2.5% to BR)
  LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^\pm) = (8.22 \pm 0.27 \pm 0.29) \times 10^{-11}$ first error parametric (V_{cb}), second non-pert. QCD
- E949 $B(K^\pm) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

K_L

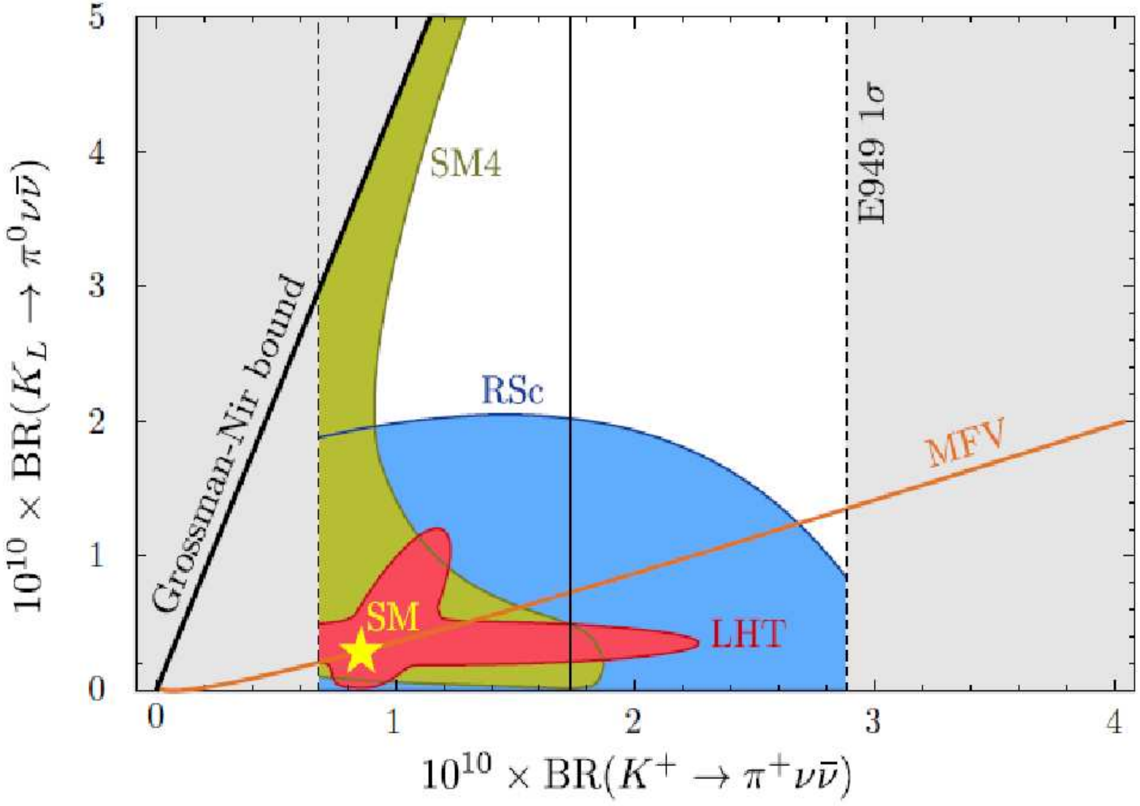
$B(K_L) = (2.43 \pm 0.25 \pm 0.06) \times 10^{-11}$ vs

E391a $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L.

K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\overline{s}d\overline{\nu}\nu$ Grossman-Nir

$$B(K_L) \le \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\mathrm{E949}} \le 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

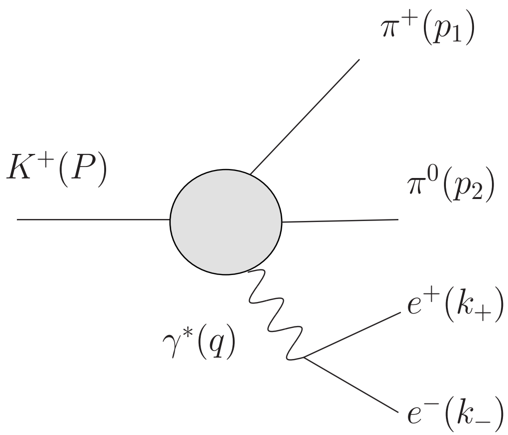
NA62 , KOTO



Straub, CKM 2010 workshop (arXiv:1012.3893v2)

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$

Sehgal et al; Savage,Wise et al

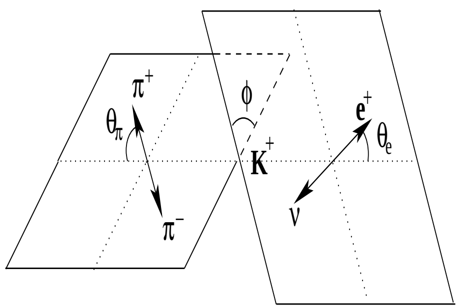


- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\; e\; H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\; \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

Interference $E \quad M$: CP effect related to $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$ **similar to** K_{e4}

$$\frac{d^5\Gamma}{dE^*_\gamma dT^*_c dq^2 d\cos\theta_\ell d\phi} = \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi$$
$$+\mathcal{A}_4 \sin 2\theta_\ell \cos\phi + \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell$$
$$+\mathcal{A}_7 \sin\theta_\ell \sin\phi + \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi$$



angular asymm. useful to disentangle various P and CP violating effects

$K^+ \to \pi^+\pi^0\gamma^* \to \pi^+\pi^0 e^+e^-$

Cappiello, Cata,G.D. and Gao,

- Fight against a large Bremss
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6} \ \sim \ 50 \ B(K^+)_M$
- Short distance info without having simultaneously K^+ and K^- by just asymm. in phase space, (P-violation) interesting! No ϵ-contamination
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B
- at $q^2 = 50\mathrm{MeV}$ IB only 10 times larger than DE

Conclusions

- KOTO and NA62 will do very important physics research
- BUT ALSO alternative channels (to $K \to \pi\nu\nu$) may give important results