New ways to search for right-handed current in $B \to \rho \ell \bar{\nu}$ decay

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Work in collaboration with F. Bernlochner and Z. Ligeti

[arXiv:1408.2516]

Heavy Quarks & Leptons 2014
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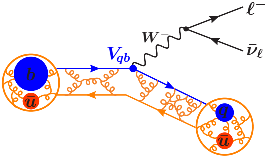
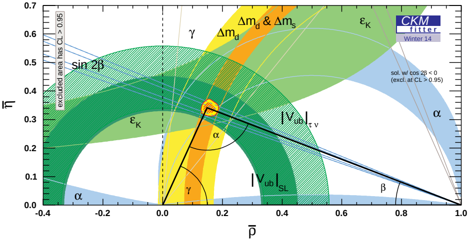
Outline

Unitarity Triangle and $|V_{ub}|$ Measurements





$$\color{red}{V_{\rm ub}^*}\color{black}{V_{\rm ud} + V_{\rm tb}^* V_{\rm td} + V_{\rm cb}^* V_{\rm cd} = 0}$$

Decay	$\lvert V_{ub}\rvert \times 10^3$
$B \to \pi\,\ell\bar{\nu}$	3.23 ± 0.30
$B \to X_u\ell\bar{\nu}$	4.39 ± 0.21
$B \to \tau\,\bar{\nu}_\tau$	4.32 ± 0.42

[HFAG]

$$\mathrm{BR}(\bar{B} \to \rho^+\ell^-\bar{\nu}_\ell) = (3.34 \pm 0.23_{\rm tot}) \cdot 10^{-4}$$

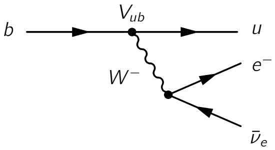
[arXiv:1306.2781]

Beyond Standard Model Contribution to Ease Tension

- Assumption in the following:

  Additional right-handed admixture

  [0807.0896,0907.2461,1003.4022,1105.3679]



Effective Lagrangian Ansatz

$$\mathcal{L}^{(\mathrm{Eff.})} = -\frac{4G_F}{\sqrt{2}} \left[V_{ub}^L\right] \bar{\nu}_\ell \gamma^\mu \frac{1-\gamma^5}{2} \ell$$
$$\times \frac{1}{2}\left[(1+\epsilon_R)\,\bar{u}\gamma_\mu b - (1-\epsilon_R)\,\bar{u}\gamma_\mu\gamma^5 b\right] + \mathrm{h.c.}$$

Typical Observables

- (Partially) integrated branching fraction
- Differential rate as $q^2 \to q_0^2$

$\Rightarrow$ All diluted by extracting $|V_{ub}^L \epsilon_R|$

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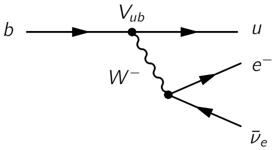

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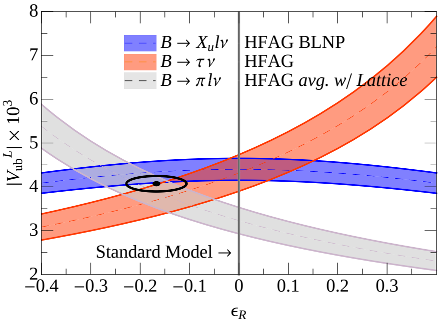
Combined Fit for New Physics Parameter



[F. Bernlochner, ST, CKM2012],

see also [Crivellin et. al., arXiv:0907.2461]

[Buras. et. al., arXiv:1007.1993]

- Current bounds weak
- Strong correlation

$$V_{ub} - \epsilon_R$$

- Can we derive an "orthogonal" bound?

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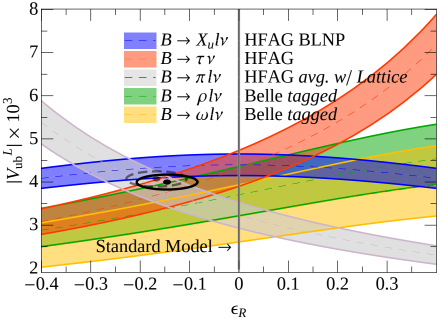

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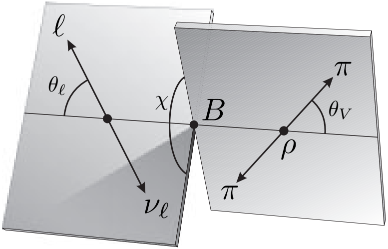
The Angular Distribution



- q^2: Momentum transfer
- θ_V: Angle between π in ρ restframe and moving direction of ρ in B restframe
- θ_l: Angle between ℓ in $W(\equiv \rho)$ restframe and moving direction of W in B restframe
- χ: Angle of decay planes

Possible Observables

- Consider 1D and 2D Asymmetries
- Full Angular Analysis
  - Very difficult

$\Rightarrow$ Consider asymmetries in three angles

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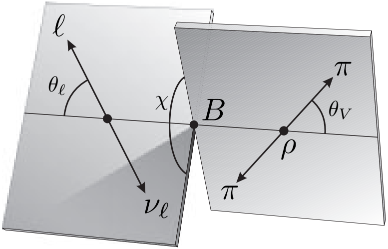

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Decay Distribution (Analogy to $B \to K^* \ell^+ \ell^-$)

$$\frac{d\Gamma}{dq^2\, d\cos\theta_V\, d\cos\theta_\ell\, d\chi} = \frac{G_F^2 |V_{ub}^L|^2 m_B^3}{2\pi^4}$$

$$\times \Big\{ J_{1s} \sin^2\theta_V + J_{1c} \cos^2\theta_V + (J_{2s} \sin^2\theta_V + J_{2c} \cos^2\theta_V) \cos 2\theta_\ell$$

$$+ J_3 \sin^2\theta_V \sin^2\theta_\ell \cos 2\chi + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos\chi + J_5 \sin 2\theta_V \sin\theta_\ell \cos\chi$$

$$+ (J_{6s} \sin^2\theta_V + J_{6c} \cos^2\theta_V) \cos\theta_\ell$$

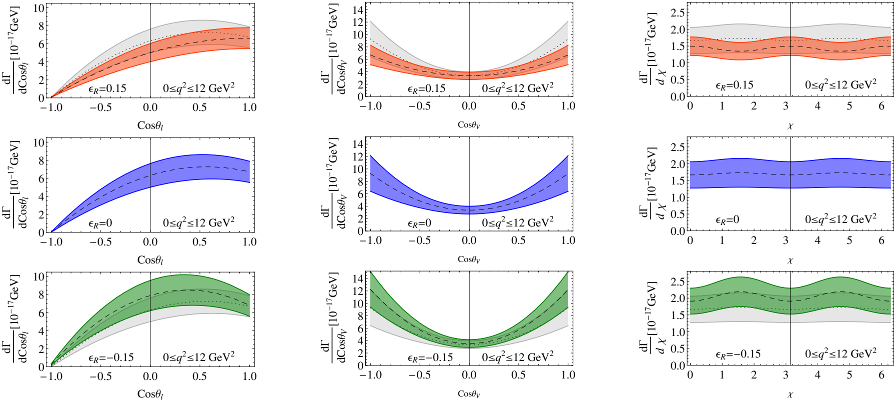
$$+ J_7 \sin 2\theta_V \sin\theta_\ell \sin\chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin\chi + J_9 \sin^2\theta_V \sin^2\theta_\ell \sin 2\chi \Big\} .$$

Simplifications

- $J_{1s} = 3J_{2s}\,, \;\; J_{1c} = 3J_{2c}\,, \;\; J_{6c} = J_7 = 0$
- For real ϵ_R we have $J_8 = J_9 = 0$
- Full angular analysis currently(?) not possible

- Balance between theo. uncertainty and statistics: $0 \leq q^2 \leq 12\,\mathrm{GeV}^2$

Angular Distributions



One- and generalized two-dimensional asymmetries

Sensitivities

- Antisymmetric in $\cos\theta_\ell$
- Symmetric in $\cos\theta_V$ and χ

$\Rightarrow$ Best sensitivity by integrating out χ

Forward Backward Asymmetry

$$A_{\rm FB} = \frac{\int_{-1}^{0} \mathrm{d}\cos\theta_\ell (\mathrm{d}\Gamma/\mathrm{d}\cos\theta_\ell) - \int_{0}^{1} \mathrm{d}\cos\theta_\ell (\mathrm{d}\Gamma/\mathrm{d}\cos\theta_\ell)}{\int_{-1}^{1} \mathrm{d}\cos\theta_\ell\, (\mathrm{d}\Gamma/\mathrm{d}\cos\theta_\ell)} .$$

Generalized Two Dimensional Asymmetries

- Optimize sensitivity to ϵ_R by defining two regions $[A, B]$ in the $(\cos\theta_\ell, \cos\theta_V)$ regions

$$S = \frac{A-B}{A+B}$$

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Binned Measurement of the J_i

J_i	η_i^{χ}	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+, a, a, +\}$	$\{-, c, c, -\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+, a, a, +\}$	$\{+, d, d, +\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-, b, b, -\}$	$\{-, c, c, -\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-, b, b, -\}$	$\{+, d, d, +\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+, -, -, +, +, -, -, +\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+, +, -, -, -, -, +, +\}$	$\{+, +, -, -\}$	$\{+, +, -, -\}$	$4(4/3)^2$
J_5	$\{+, +, -, -, -, -, +, +\}$	$\{+\}$	$\{+, +, -, -\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+, +, -, -\}$	$\{-, c, c, -\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+, +, -, -\}$	$\{+, d, d, +\}$	$2\pi(1)(2/5)$
J_7	$\{+, +, +, +, -, -, -, -\}$	$\{+\}$	$\{+, +, -, -\}$	$4(\pi/2)(4/3)$
J_8	$\{+, +, +, +, -, -, -, -\}$	$\{+, +, -, -\}$	$\{+, +, -, -\}$	$4(4/3)^2$
J_9	$\{+, +, -, -, +, +, -, -\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$

$a = 1 - 1/\sqrt{2}$, $b = a\sqrt{2}$, $c = 2\sqrt{2} - 1$, and $d = 1 - 4\sqrt{2}/5$

$$J_i = \frac{1}{N_i} \sum_{j=1}^{8} \sum_{k,l=1}^{4} \eta_{i,j}^{\chi}\, \eta_{i,k}^{\theta_\ell}\, \eta_{i,l}^{\theta_V} \left[\chi^{(j)} \otimes \theta_\ell^{(j)} \otimes \theta_V^{(k)}\right]$$

Simple Observables I

Analogy to "clean observables" in $B \to K^* \ell^+ \ell^-$ [1303.5794]

$$\langle P_1 \rangle_{\text{bin}} = \frac{1}{2} \frac{\int_{\Delta q^2} \mathrm{d}q^2 J_3}{\int_{\Delta q^2} \mathrm{d}q^2 J_{2s}}$$

$$\langle P_4' \rangle_{\text{bin}} = \frac{\int_{\Delta q^2} \mathrm{d}q^2 J_4}{\sqrt{- \int_{\Delta q^2} \mathrm{d}q^2 J_{2s} \, \int_{\Delta q^2} \mathrm{d}q^2 J_{2c}}}$$

$$\langle P_5' \rangle_{\text{bin}} = \frac{1}{2} \frac{\int_{\Delta q^2} \mathrm{d}q^2 J_5}{\sqrt{- \int_{\Delta q^2} \mathrm{d}q^2 J_{2s} \, \int_{\Delta q^2} \mathrm{d}q^2 J_{2c}}}$$

Simple Observables II

Simple Ratios

$$\langle P_{i,j} \rangle_{\text{bin}} = \frac{\int_{\Delta q^2} \mathrm{d}q^2 J_i}{\int_{\Delta q^2} \mathrm{d}q^2 J_j}$$

Sensitivity to Real Part

- Best sensitivity for $\langle P_{3,4} \rangle$, $\langle P_{3,5} \rangle$, and $\langle P_{5,4} \rangle$

Sensitivity to Imaginary Part

- Starting linear in $\text{Im}\,\epsilon_R$ and quadratic in $\text{Re}\,\epsilon_R$

$$\langle P_{8,5} \rangle \quad , \quad \langle P_{9,5} \rangle$$

- Linear in $\text{Im}\,\epsilon_R$ and $\text{Re}\,\epsilon_R$, but large slope with respect to ϵ_R

$$\langle P_{8,3} \rangle$$

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The Strategy for the J_i Integration

Our Input

1. List of form factor values und uncertainties
   1. Light-Cone Sum Rules for small q^2 [hep-ph/0412079]
   2. Lattice QCD not (yet) reliable for high q^2 [hep-lat/0402023]
2. Use correlation between points at different q^2 [1004.3249]
3. Use correlation matrix between different form factors (Estimate!)

Our Fit to obtain Form factors

- Assume z parametrisation (Series Expansion)
- Perform combined fit to all form factor points
- Resulting Output
  1. Fit parameters for each form factor $\hat{=}$ expansion coefficient
  2. Uncertainty for each of these parameters
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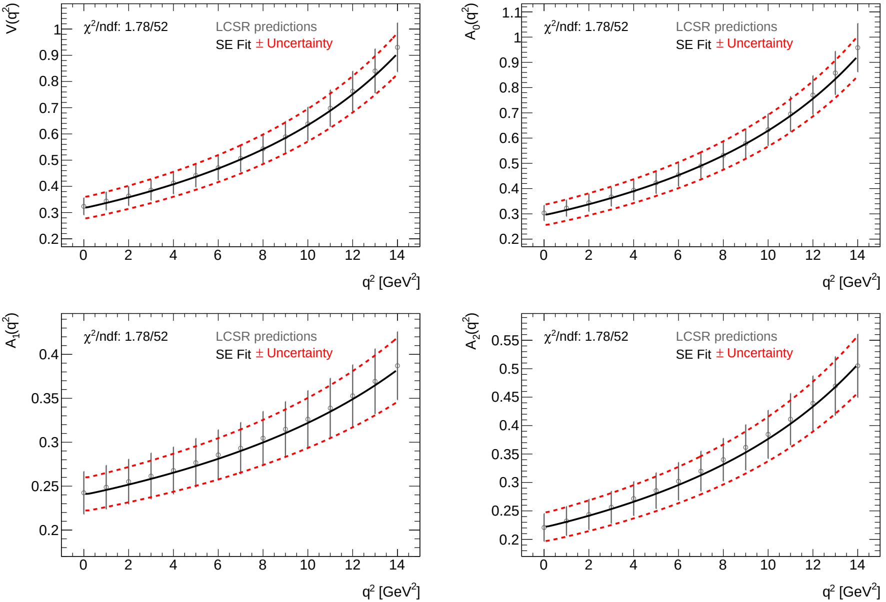
V(q²)
χ²/ndf: 1.78/52
LCSR predictions
SE Fit ± Uncertainty
q² [GeV²]
A₀(q²)
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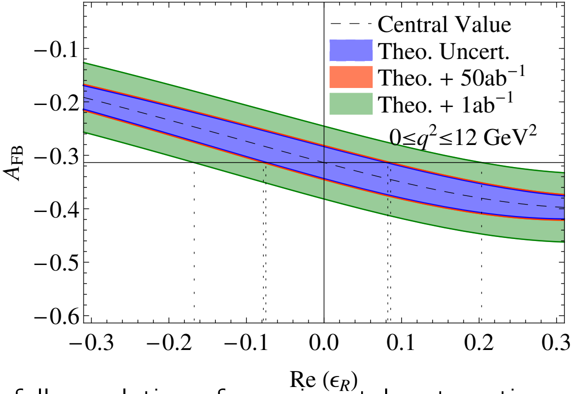
Forward Backward Asymmetry

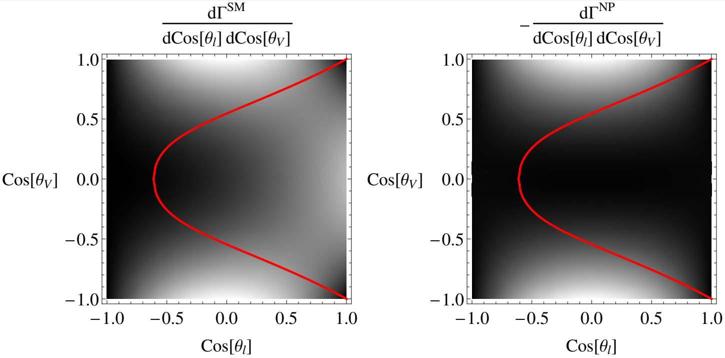


- Assumes full correlation of experimental systematic uncertainties in disjoint regions of phase-space [1306.2781]
- Improvement of factor 3 for systematic uncertainties at 50 ab^{-1}, motivated by $B \to X_u \ell \bar{\nu}$ [1002.5012]

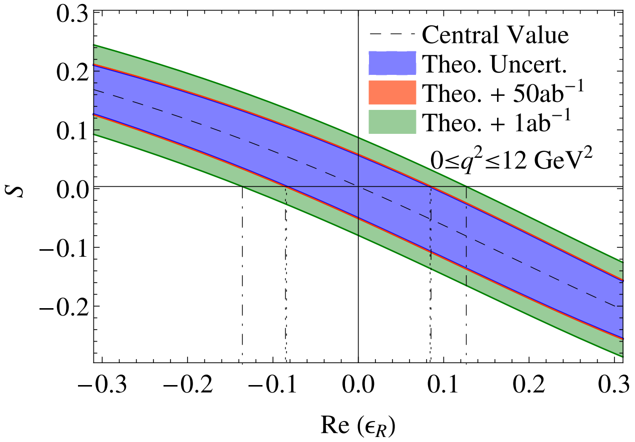
Define the Two Regions for the 2D Asymmetry

Preliminaries

1. Differential rate behaves smoothly $\Rightarrow d\Gamma_{SM} = \kappa d\Gamma^{(1)}_{NP}$
2. SM and NP are separately symmetric in $\cos\theta_V$
3. SM and NP are asymmetric in $\cos\theta_L$ to each other

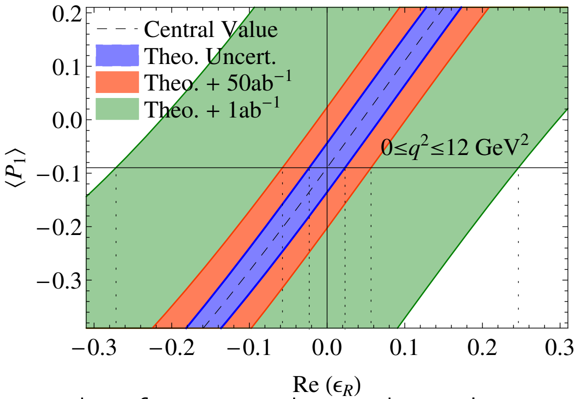


Generalized 2D Asymmetry Result



- Same experimental assumptions than for A_{FB}

Simple Ratio Observables for Real Part



- Same assumptions for systematic experimental uncertainties
- Statistical correlations between bins are estimated using Monte Carlo Methods
- Background influence neglected

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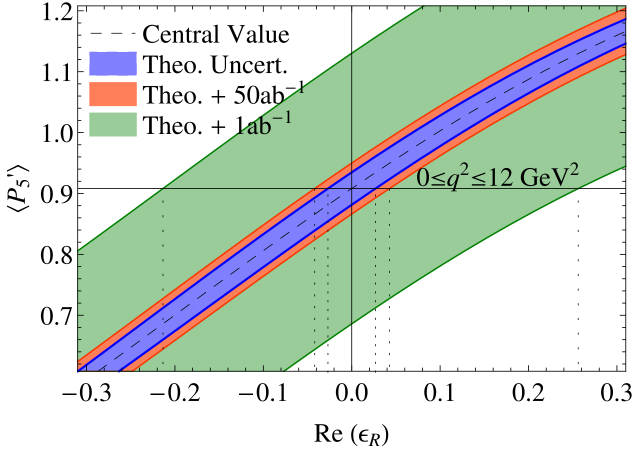

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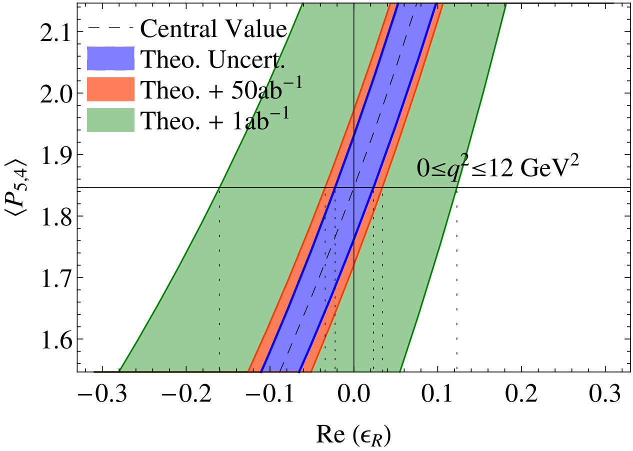
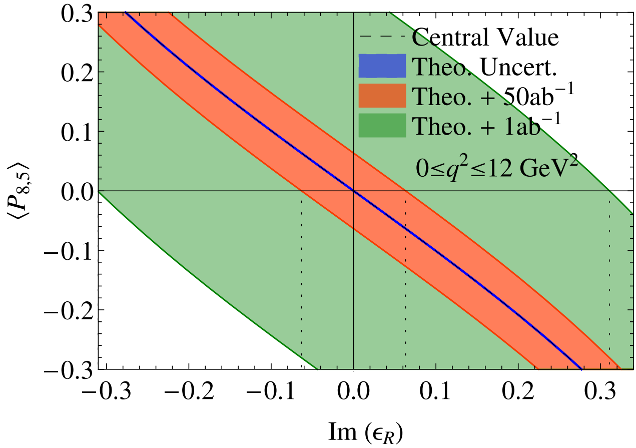


Simple Ratio Observables for Imaginary Part



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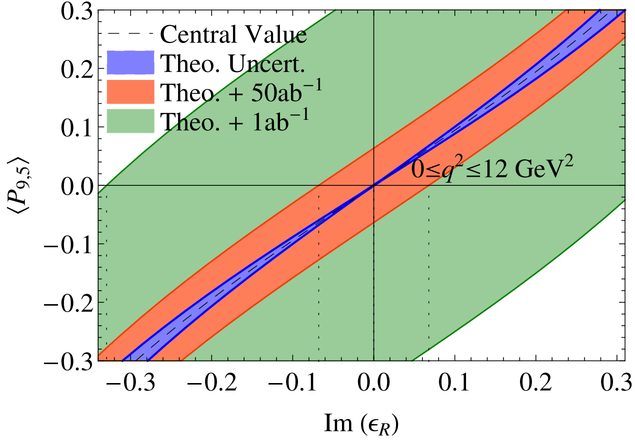

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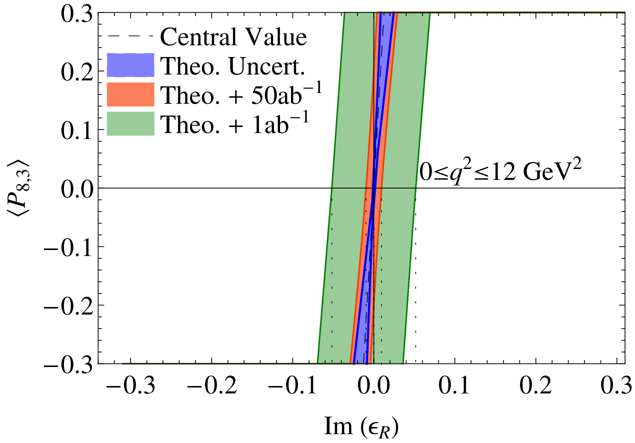
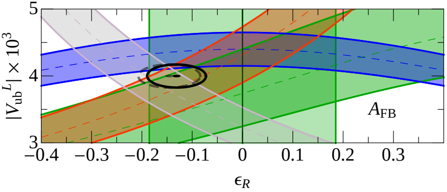
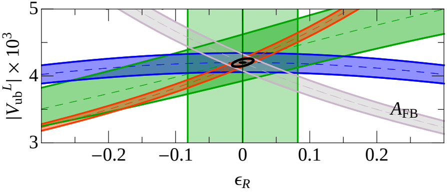


Global demo fit for $|V_{ub}^L|$ and ϵ_R with A_{FB}



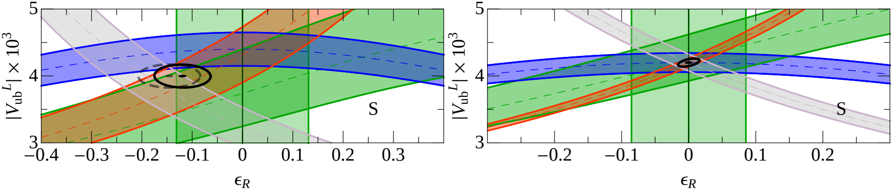


- Fixed SM $V_{ub} = 4.2 \cdot 10^{-3}$ for extrapolation
- Est. uncert. $\Delta\left(\left|V_{ub}^L\right| \times 10^3, \Delta\epsilon_R\right) = (0.18, 0.061)$ and $(0.06, 0.016)$
- Expected relative reduction of the uncertainties

$$1\,\text{ab}^{-1}: \qquad \delta\left(\left|V_{ub}^L\right|\right) = -0.3\% \qquad \delta\left(\epsilon_R\right) = -5\%$$

$$50\,\text{ab}^{-1}: \qquad \delta\left(\left|V_{ub}^L\right|\right) = -0.4\% \qquad \delta\left(\epsilon_R\right) = -2\%$$

Global demo fit for $|V_{ub}^L|$ and ϵ_R with S



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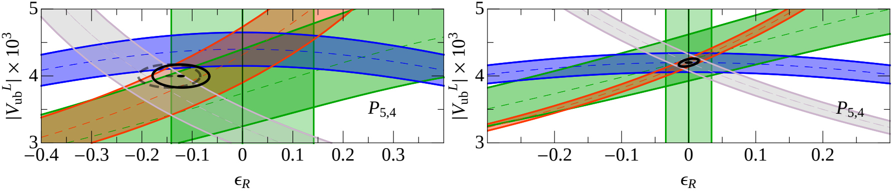
$$\begin{aligned} 1\,\mathrm{ab}^{-1}: &\quad \delta\left(\left|V_{ub}^L\right|\right) = -0.5\% &\quad \delta\left(\epsilon_R\right) = -9\% \\ 50\,\mathrm{ab}^{-1}: &\quad \delta\left(\left|V_{ub}^L\right|\right) = -0.5\% &\quad \delta\left(\epsilon_R\right) = -2\% \end{aligned}$$

Global demo fit for $|V_{ub}^L|$ and ϵ_R with $P_{5,4}$



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$1\,\mathrm{ab}^{-1}$:	$\delta\left(\left|V_{ub}^L\right|\right) = -0.5\%$	$\delta(\epsilon_R) = -8\%$
$50\,\mathrm{ab}^{-1}$:	$\delta\left(\left|V_{ub}^L\right|\right) = -3\%$	$\delta(\epsilon_R) = -10\%$

Summary and Comments

Summary

- Considered 1D,2D and 3D Asymmetries to constrain right-handed currents in $B \to \rho\ell\bar{\nu}_\ell$
    - Independent of $|V_{ub}|$
    - Best observable need to be determined: Balance between experimental and theoretical uncertainties
    - Sensitivity estimate
- Binned analysis of "clean observables" also possible in $B \to K^*\ell^+\ell^-$ to cross-check angular folding technique
- Fit program for form factor parametrisations
    1. Capable of LCSR and Lattice simultaneous input
    2. Inputs: Uncertainties, correlations among FF and different q^2 points

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  - Best observable need to be determined: Balance between experimental and theoretical uncertainties
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- Binned analysis of "clean observables" also possible in $B \to K^*\ell^+\ell^-$ to cross-check angular folding technique
- Fit program for form factor parametrisations
  1. Capable of LCSR and Lattice simultaneous input
  2. Inputs: Uncertainties, correlations among FF and different q^2 points

Summary and Comments

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Wish List from Theory

- Form factor input: Correlation matrix!
- Lattice QCD for high q^2

$\Rightarrow$ Reliable full q^2 spectrum fit

$\Rightarrow$ Consistency Checks

Wish List for Experiment

- Try to measure S with current data

$\Rightarrow$ Give a Limit on ϵ_R (Might give hint where to look first)

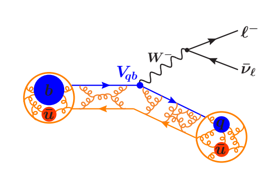
- In Future: Attempt to obtain full information and determine best and consistent variable
- Possible to do at LHCb?
  - Much better statistics?

  $\Rightarrow$ But reconstruction of neutrino momentum unambigiously possible?

Backup Slides

Hadronic Part: Form Factors

Form Factor definitions



$$\langle V(p_V)|\bar{q}\gamma_\mu b|B(p_B)\rangle = \epsilon_{\mu\nu\rho\sigma}\epsilon^{*,\nu} p_B^\rho p_V^\sigma \frac{2V(q^2)}{m_B + m_V}$$

$$\langle V(p_V)|\bar{q}\gamma_\mu\gamma_5 b|B(p_B)\rangle = i\epsilon^*_\mu (m_B + m_V) A_1(q^2) - i(p_B + p_V)_\mu(\epsilon^* \cdot q)\frac{A_2(q^2)}{m_B + m_V}$$

$$-iq_\mu(\epsilon^* q)\frac{2m_V}{q^2}\left(\frac{m_B + m_V}{2m_V}A_1(q^2) - \frac{m_B - m_V}{2m_V}A_2(q^2) - A_0(q^2)\right)$$

- Include new physics corresponds to the replacement

$$V \to (1 + \epsilon_R)V, \qquad A_i \to (1 - \epsilon_R)A_i$$

Several Possible Parametrisations of q^2 dependence

- Pole parametrisation
- Here: Make use of unitarity ⇒ Series Expansion
- Compute values by non-perturbative methods

⇒ Combined fit to parametrisation

Analytic Parametrisation: The Series Expansion

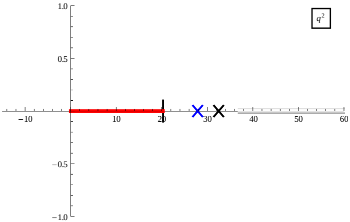
- Map $t \equiv q^2 \to z(t, t_0)$, $t_\pm = (m_B \pm m_\rho)^2$

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

- t_0: Minimize in physical region

$$t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$$

- Unitarity constraints $\Rightarrow \Phi_F(t)$
- Expand in residual dependence



- Blaschke factor

$$B_F(t) \equiv \sum_R z(t, M_{R_F}^2)$$

$$F(t) = \frac{1}{B_F(t)\Phi_F(t)} \sum_{k=0}^{K} \alpha_k^F z(t)^k$$

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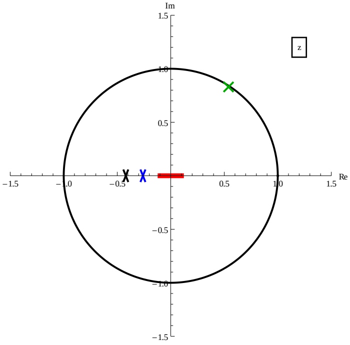

$$\boxed{F(t) = \frac{1}{B_F(t)\Phi_F(t)} \sum_{k=0}^{K} \alpha_k^F z(t)^k}$$

Sources of Uncertainties

Uncertainties in the Calculation ≡ Input Uncertainties

- Used input: not under our control
- Value for each form factor at several values of q^2
- For each value we have only a combined uncertainty
- No information on correlation

Parametrisation Uncertainties ≡ Fit Uncertainties

- Shape constraints
  1. Remnant expansion: Small uncertainty
  2. Resonances with $t_- < M_R^2 < t_+ \;\Rightarrow$ Blaschke factors
- Assumptions on derivation are obstructed by
  1. Branch cuts below threshold of light hadrons: see next slides
  2. Broad width of ρ? Experimental identification of ρ?

Light Cone Sum Rules Input [low q^2]

[hep-ph/0412079]

- Input given by LCSR

Form Factor F	$F(q^2=0)$	Δ_{7P}	Δ_{m_b}	Δ_L	Δ_T
$V(0)$	0.323	0.025	0.007	0.005	0.013
$A_0(0)$	0.303	0.026	0.004	0.009	0.006
$A_1(0)$	0.242	0.020	0.007	0.004	0.010
$A_2(0)$	0.221	0.018	0.008	0.002	0.011

- Extract values $q^2 > 0$ from plot and add $\sim 1\%$ uncertainty

Lattice QCD Input [high q^2]

[hep-lat/0402023]

- Huge uncertainties and two inconsistent(?) sets
- Recent discussion: Timescale for reliable uncertainties: $\gtrsim 5$ yrs
- Desired to do simultaneous fit to LCSR and Lattice data

Belle 2 timeline!

Estimating Correlation between Form Factors

Uncertainty Sources (RP) of [hep-ph/0412079]

1. Δ_{mb}: Variation due to m_b, s_0 and M according to table 4
2. Δ_{7p}: combination of 7 parameters, described in the text on page 24
3. Δ_L: variation due to vector coupling in table 3
4. Δ_T: variation due to tensor coupling in table 3

(Conservative) Postulate, see also [hep-ph/0412079,0811.1214,1308.4379]

- Correlation among Axialvector form factors

$$\{\rho^{A_i}_{7P}, \rho^{A_i}_{m_b}, \rho^{A_i}_{L}, \rho^{A_i}_{T}\} = \{0.6, 1., 1., 1.\}$$

- Correlation between Axialvector and Vector Form Factors

$$\{\rho^{V,A_i}_{7P}, \rho^{V,A_i}_{m_b}, \rho^{V,A_i}_{L}, \rho^{V,A_i}_{T}\} = \{0.6, 1., 1., 1.\}$$

- Resulting form

$$C = C_{7P} + C_{m_b} + C_L + C_T$$

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- Resulting form

$$C = C_{7P} + C_{m_b} + C_L + C_T$$

Resulting Form Factor Correlation

For each uncertainty parameter the correlation matrix is given as

$$C_i = \begin{pmatrix} \left(\Delta_j^V\right)^2 & \rho_j^{V,A_i}\Delta_j^V\Delta_j^{A_0} & \rho_j^{V,A_i}\Delta_j^V\Delta_j^{A_1} & \rho_j^{V,A_i}\Delta_j^V\Delta_j^{A_2} \\ \rho_j^{V,A_i}\Delta_j^V\Delta_j^{A_0} & \left(\Delta_j^{A_0}\right)^2 & \rho_j^{A_i}\Delta_j^{A_0}\Delta_j^{A_1} & \rho_j^{A_i}\Delta_j^{A_0}\Delta_j^{A_2} \\ \rho_j^{V,A_i}\Delta_j^V\Delta_j^{A_1} & \rho_j^{A_i}\Delta_j^{A_0}\Delta_j^{A_1} & \left(\Delta_j^{A_1}\right)^2 & \rho_j^{A_i}\Delta_j^{A_1}\Delta_j^{A_2} \\ \rho_j^{V,A_i}\Delta_j^V\Delta_j^{A_2} & \rho_j^{A_i}\Delta_j^{A_0}\Delta_j^{A_2} & \rho_j^{A_i}\Delta_j^{A_1}\Delta_j^{A_2} & \left(\Delta_j^{A_2}\right)^2 \end{pmatrix}$$

Resulting correlation matrix

$$C = \begin{pmatrix} 1. & 0.65 & 0.71 & 0.72 \\ 0.65 & 1. & 0.64 & 0.62 \\ 0.71 & 0.64 & 1. & 0.72 \\ 0.72 & 0.62 & 0.72 & 1. \end{pmatrix}$$

Verification Test of the Fit

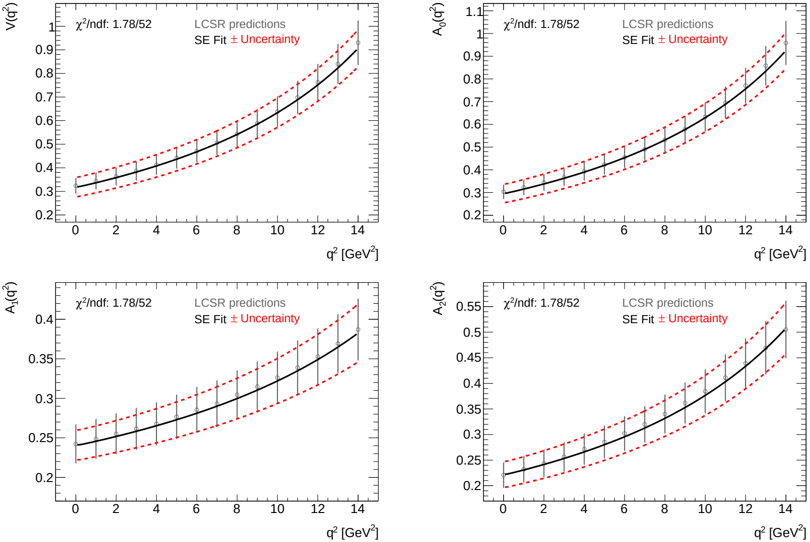
Fit procedure

- Simultaneous χ^2 fit to all points and form factors

$$\chi^2 = (\mathbf{x}_{\text{Theo}} - \mathbf{x}_{\text{LCSR}})\text{Cov}^{-1}(\mathbf{x}_{\text{Theo}} - \mathbf{x}_{\text{LCSR}})$$

- Validation of the results through ensamble of pseudo-experiments
  1. Errors
  2. Correlations
  3. Central values

Fitted Form Factors: SE linear expansion



Sample Numerical Results for Form Factors

FF	Param.	Value	Uncertainty
A_0	a_0	$-0.351 \cdot 10^{-3}$	$0.032 \cdot 10^{-3}$
	a_1	$1.250 \cdot 10^{-3}$	$0.147 \cdot 10^{-3}$
A_1	a_0	$-0.111 \cdot 10^{-3}$	$0.010 \cdot 10^{-3}$
	a_1	$-0.208 \cdot 10^{-3}$	$0.042 \cdot 10^{-3}$
A_2	a_0	$-0.138 \cdot 10^{-3}$	$0.014 \cdot 10^{-3}$
	a_1	$0.170 \cdot 10^{-3}$	$0.049 \cdot 10^{-3}$
V	a_0	$-0.366 \cdot 10^{-3}$	$0.034 \cdot 10^{-3}$
	a_1	$1.148 \cdot 10^{-3}$	$0.145 \cdot 10^{-3}$

Comments to three parameter fit

- Each a_0 and a_1 compatible within uncertainty
- Fitted a_2 compatible with zero within uncertainty

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Sample Numerical Results for Form Factors

$\text{corr}(A_0, A_1, A_2, V) \approx$

$$\begin{pmatrix} 1.00 & -0.86 & 0.77 & 0.35 & 0.74 & -0.26 & 0.78 & -0.57 \\ -0.86 & 1.00 & -0.60 & -0.27 & -0.58 & 0.20 & -0.61 & 0.44 \\ 0.77 & -0.60 & 1.00 & 0.31 & 0.86 & -0.31 & 0.85 & -0.62 \\ 0.35 & -0.27 & 0.31 & 1.00 & 0.39 & -0.14 & 0.39 & -0.28 \\ 0.74 & -0.58 & 0.86 & 0.39 & 1.00 & -0.49 & 0.86 & -0.63 \\ -0.26 & 0.20 & -0.31 & -0.14 & -0.49 & 1.00 & -0.31 & 0.22 \\ 0.78 & -0.61 & 0.85 & 0.39 & 0.86 & -0.31 & 1.00 & -0.82 \\ -0.57 & 0.44 & -0.62 & -0.28 & -0.63 & 0.22 & -0.82 & 1.00 \end{pmatrix}$$

Comments to three parameter fit

- Correlations between a_0 and a_1 same sign
- Value is shifted $\rightarrow$ OK

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1.00 & -0.86 & 0.77 & 0.35 & 0.74 & -0.26 & 0.78 & -0.57 \\
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0.77 & -0.60 & 1.00 & 0.31 & 0.86 & -0.31 & 0.85 & -0.62 \\
0.35 & -0.27 & 0.31 & 1.00 & 0.39 & -0.14 & 0.39 & -0.28 \\
0.74 & -0.58 & 0.86 & 0.39 & 1.00 & -0.49 & 0.86 & -0.63 \\
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\end{pmatrix}$$

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$$\mathrm{corr}(J_i) \approx$$

$$\begin{pmatrix} 100. & 71.6 & 100. & -71.6 & -34.5 & 88.6 & 94.2 & 100. \\ 71.6 & 100. & 71.6 & -100. & -68. & 95. & 89.8 & 70.5 \\ 100. & 71.6 & 100. & -71.6 & -34.5 & 88.6 & 94.2 & 100. \\ -71.6 & -100. & -71.6 & 100. & 68. & -95. & -89.8 & -70.5 \\ -34.5 & -68. & -34.5 & 68. & 100. & -65.3 & -46.2 & -32.3 \\ 88.6 & 95. & 88.6 & -95. & -65.3 & 100. & 97.3 & 87.7 \\ 94.2 & 89.8 & 94.2 & -89.8 & -46.2 & 97.3 & 100. & 93.9 \\ 100. & 70.5 & 100. & -70.5 & -32.3 & 87.7 & 93.9 & 100. \end{pmatrix}$$

- $0 \leq q^2 \leq 12\ \mathrm{GeV}^2$
- $\epsilon_R \equiv 0$
- Only J_{1s} to J_{6s}