ChPT tests at NA48 and NA62 experiments at CERN

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on behalf of the **NA48/2** and **NA62** collaborations

Mainz, August 27, 2014

Kaon decays provide an important laboratory for the investigations at «intensity frontier»: rare decays and high statistics precision measurements.

NA62 NA48, NA48/2 NA62(R_K phase)

- SM tests: CKM ($\pi\nu\bar{\nu}$), CPV ($\pi\pi$, $\pi\pi\pi$), Lepton universality ($R_K = \Gamma(e\nu)/\Gamma(\mu\nu)$)

- ChPT development — low energy strong interaction parameters, form factors *etc* (needed for some SM tests and important for particle physics in general) :

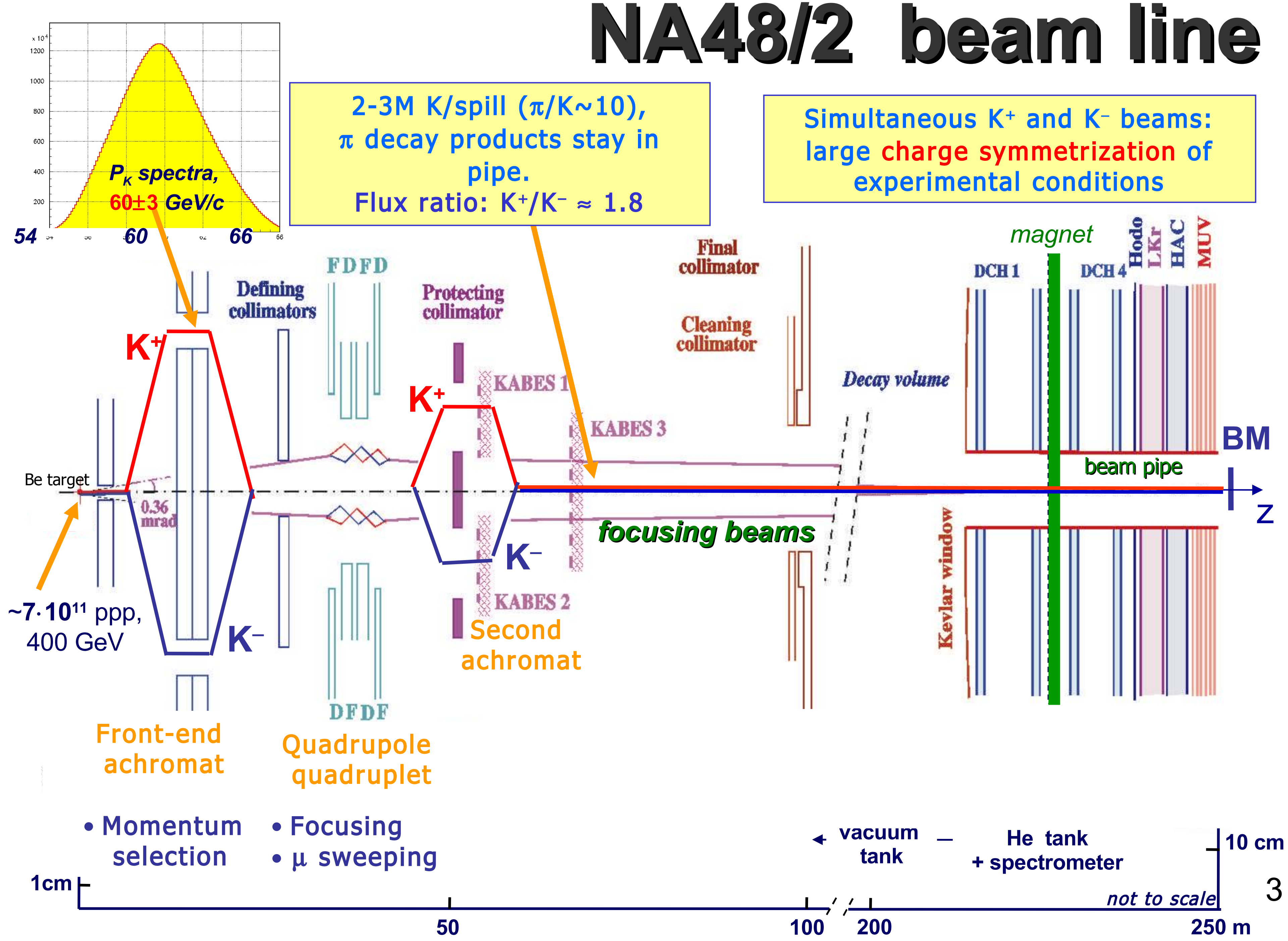
Outline

- NA48/2 experiment
- Ke4 : introduction
- NA48/2: $\mathbf{K^\pm \to \pi^+\pi^-e^\pm\nu}$
- NA48/2: $\mathbf{K^\pm \to \pi^0\pi^0e^\pm\nu}$
- NA48/2 and NA62 (R_K phase): $\mathbf{K^\pm \to \pi^+\gamma\gamma}$
- Summary

NA48/2 beam line

2-3M K/spill (π/K~10),
π decay products stay in pipe.
Flux ratio: $K^+/K^- \approx 1.8$

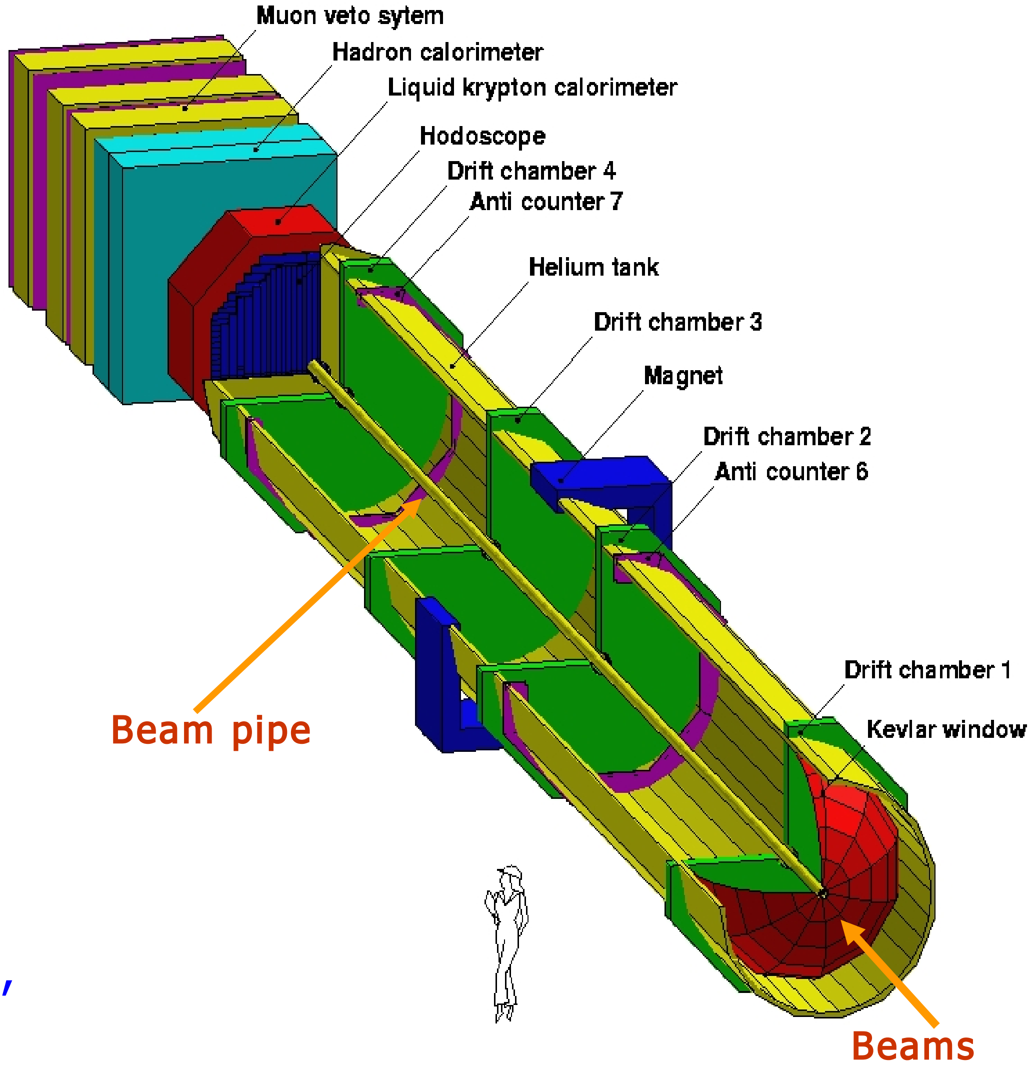
Simultaneous K^+ and K^- beams:
large charge symmetrization of experimental conditions



The NA48 detector

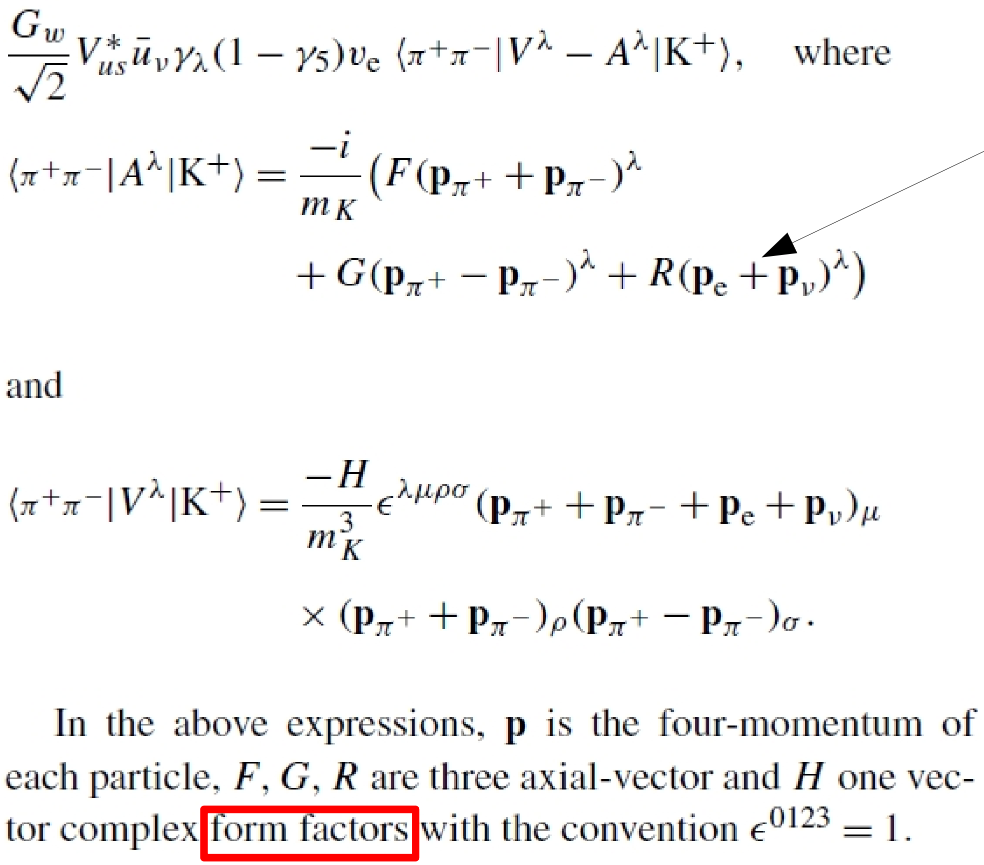
Main detector components:

- Magnetic spectrometer (4 DCHs):
  4 views/DCH:
  redundancy ⇒ efficiency;
  used in trigger logic;
  $\Delta p/p = 1.0\% \oplus 0.044\%*p$
  [p in GeV/c].
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogeneous
  $\sigma_E/E = 3.2\%/E^{1/2} \oplus 9\%/E \oplus 0.42\%$
  $\sigma_x=\sigma_y=(0.42/E^{1/2} \oplus 0.06)$cm
  [E in GeV]. (0.15cm@10GeV).
- Hadron calorimeter, muon veto counters, photon vetoes.



Introduction: K_{e4} amplitude

$K^\pm \to \pi^+\pi^- e^\pm \nu$ amplitude
is a product of weak leptonic current and (V-A) hadronic current:



$$\frac{G_w}{\sqrt{2}} V_{us}^* \bar{u}_\nu \gamma_\lambda (1-\gamma_5) v_e \, \langle \pi^+\pi^- | V^\lambda - A^\lambda | \mathrm{K}^+ \rangle, \quad \text{where}$$

$$\langle \pi^+\pi^- | A^\lambda | \mathrm{K}^+ \rangle = \frac{-i}{m_K} \big(F (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})^\lambda + G (\mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^-})^\lambda + R (\mathbf{p}_e + \mathbf{p}_\nu)^\lambda \big)$$

and

$$\langle \pi^+\pi^- | V^\lambda | \mathrm{K}^+ \rangle = \frac{-H}{m_K^3} \epsilon^{\lambda\mu\rho\sigma} (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-} + \mathbf{p}_e + \mathbf{p}_\nu)_\mu \times (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})_\rho (\mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^-})_\sigma .$$

In the above expressions, **p** is the four-momentum of each particle, F, G, R are three axial-vector and H one vector complex form factors with the convention $\epsilon^{0123} = 1$.

R enters in the decay rate multiplied by lepton mass squared => this term is negligible for K_{e4}

F,G,R,H form factors (FF) depend on decay Lorentz invariants, so their parameterisation (or some tabulation) is needed to describe data.

Ke4 decays : formalism of ($\pi^+\pi^-$) and ($\pi^0\pi^0$) modes

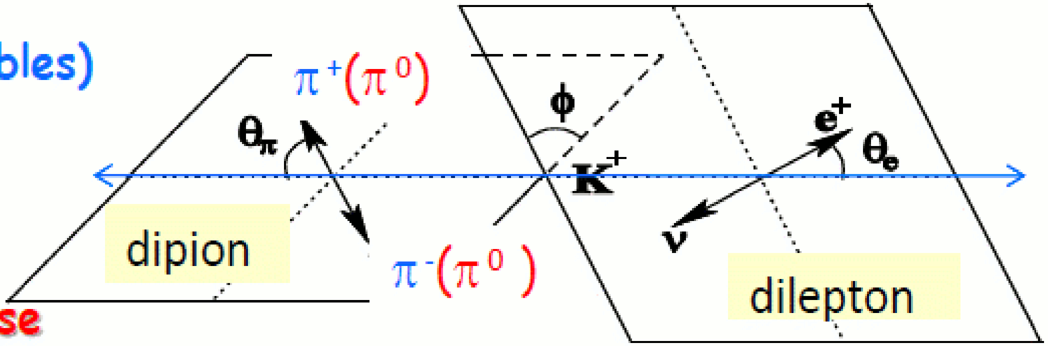
Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$), cosθ_π, cosθ_e and ϕ

Reduce to 3 variables in the ($\pi^0\pi^0$) case

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$), cosθ_e



Partial Wave expansion of the amplitude
into s and p waves (Pais-Treiman 1968)
+ Watson theorem (T-invariance) for δ_l^I

$\delta_0^0 \equiv \delta_s$ and $\delta_1^1 \equiv \delta_p$

F, G = 2 complex Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{id\delta p} \cos(\theta_\pi)$

$G = G_p e^{i\delta g}$

H = 1 complex Vector Form Factor

$H = H_p e^{i\delta h}$

Reduces to the single Fs Form Factor

Map the distributions of the Ca.Ma. variables in the five-dimensional space with 4 real Form factors and only one phase shift , assuming identical phases for the p-wave Form Factors F_p, G_p, H_p

Dalitz plot density proportional to Fs^2

The fit parameters (real) are :

F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

reduce to the only Fs

Ke4(+-) : $K^{\pm}\rightarrow\pi^{+}\pi^{-}e^{\pm}\nu$

Signal (π^{+} π^{-} $e^{\pm}$ ν) topology:

- 3 charged tracks, good vertex
- Opposite sign 2π («Right Sign»)
- 1 electron (E_{LKr} /P_{DCH} ~ 1)

Main background sources: $K\rightarrow3\pi$

Case of K^{+} :

a $K^{+} \rightarrow [\pi^{+}$ misident. as $e^{+}]\pi^{+}\pi^{-}$

$K^{+} \rightarrow [\pi^{+} \rightarrow e^{+}\nu]\pi^{+}\pi^{-}$

contributes twice more to «Right Sign» events:

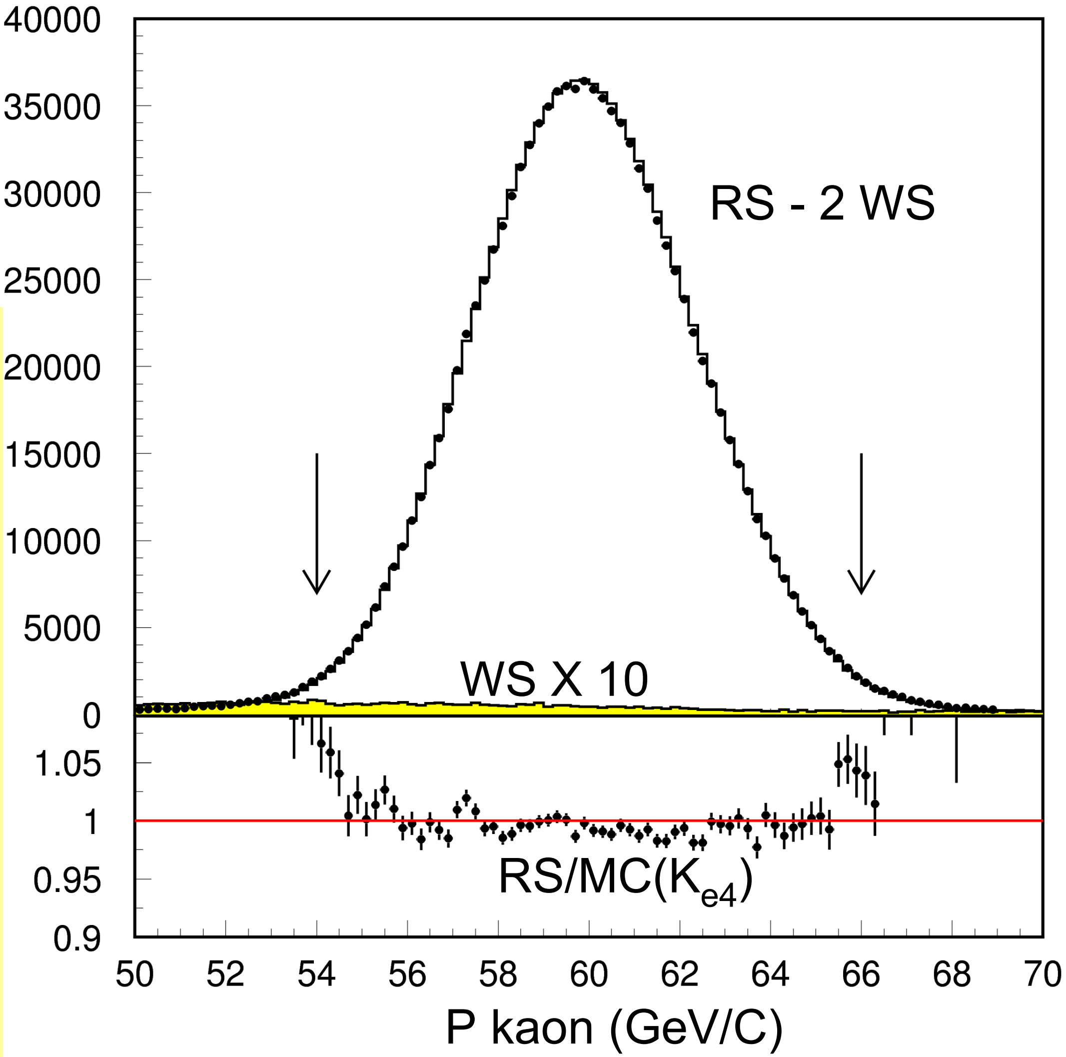
(RS = $e^{+}\pi^{+}\pi^{-}$, 2 π^{+} can decay)

than to «Wrong Sign» ones:

(WS = $e^{-}\pi^{+}\pi^{+}$, 1 π^{-} can decay).

b $K^{+} \rightarrow [\pi^{0} \rightarrow \overset{\text{misident.}}{e^{+}}e^{-}\ \overset{\text{lost}}{\gamma}]\ \pi^{0}\ \pi^{+}$

almost negligible



Total background is below 1% ,

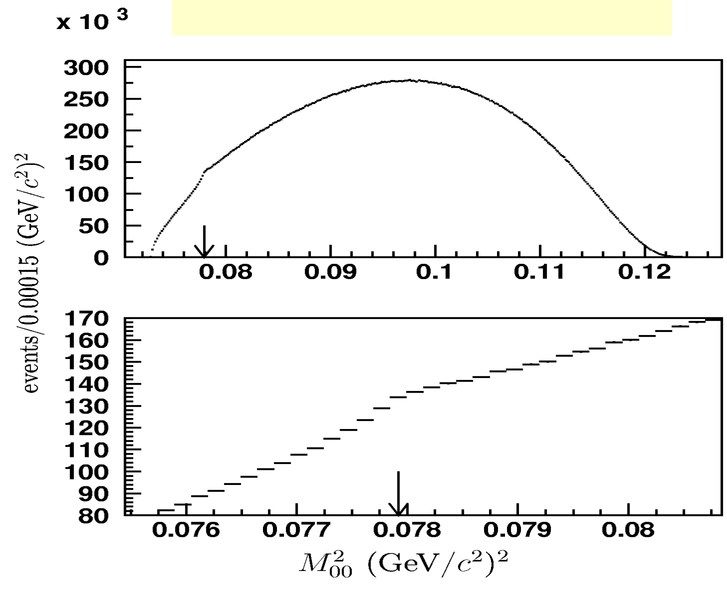
*estimated from WS events (contribution **a** is dominant) and checked by MC*

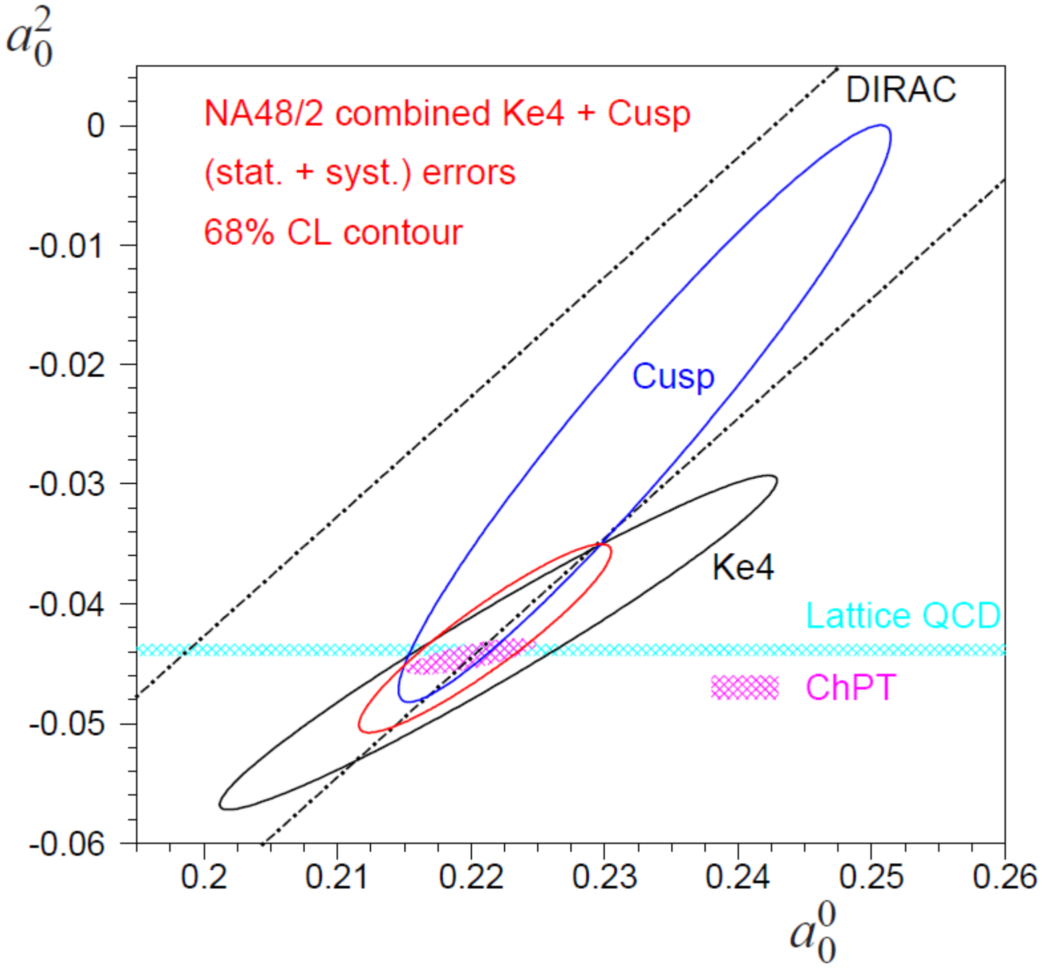
$\pi\pi$ scattering lengths measurement from phase shift $\delta\,(M_{\pi\pi}) = \delta_s - \delta_p$

[Eur.Phys. C70 (2010) 635]

Cusp in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

Published in EPJ C64(2009)589





combined $\pi\pi$ scattering lengths result:

$$a_0^0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm syst},$$

$$a_0^2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm syst},$$

$$a_0^0 - a_0^2 = 0.2639 \pm 0.0020_{\rm stat} \pm 0.0015_{\rm syst}.$$

Form factors (normalized to f_s)

[Eur.Phys. C70 (2010) 635]

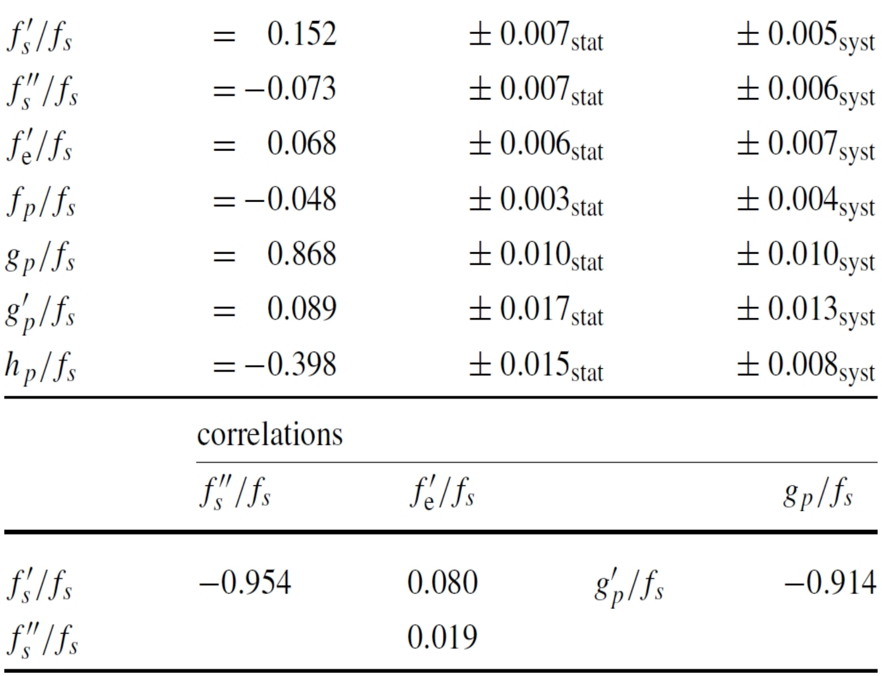
K_{e4} formfactors: fit results

Series expansion with:

- $q^2 = S_\pi/(4m_\pi^2) — 1$
- $S_e/(4m_\pi^2)$

$$F_s^2 = f_s^2(1+f'_s/f_s q^2+f''_s/f_s q^4+f'_e/f_s S_e/4m_\pi^2)^2$$

$$\boxed{G_p=f_s(g_p/f_s + g'_p/f_s q^2)}$$



	value	stat	syst
f'_s/f_s	$= 0.152$	$\pm 0.007_{\text{stat}}$	$\pm 0.005_{\text{syst}}$
f''_s/f_s	$= -0.073$	$\pm 0.007_{\text{stat}}$	$\pm 0.006_{\text{syst}}$
f'_e/f_s	$= 0.068$	$\pm 0.006_{\text{stat}}$	$\pm 0.007_{\text{syst}}$
f_p/f_s	$= -0.048$	$\pm 0.003_{\text{stat}}$	$\pm 0.004_{\text{syst}}$
g_p/f_s	$= 0.868$	$\pm 0.010_{\text{stat}}$	$\pm 0.010_{\text{syst}}$
g'_p/f_s	$= 0.089$	$\pm 0.017_{\text{stat}}$	$\pm 0.013_{\text{syst}}$
h_p/f_s	$= -0.398$	$\pm 0.015_{\text{stat}}$	$\pm 0.008_{\text{syst}}$

	correlations			
	f''_s/f_s	f'_e/f_s		g_p/f_s
f'_s/f_s	-0.954	0.080	g'_p/f_s	-0.914
f''_s/f_s		0.019		

$K_{e4}(+ -)$ branching fraction measurement [PLB 715 (2012) 105]

$$K^\pm \rightarrow \pi^+\pi^-\nu e^\pm \ / \ K^\pm \rightarrow \pi^+\pi^-\pi^\pm$$

$Br(K_{e4}(+-)) = (4.257 \pm 0.004_{stat} \pm 0.016_{syst} \pm 0.031_{ext}) \times 10^{-5}$

(PDG 2012: $(4.09 \pm 0.10_{tot}) \times 10^{-5}$)

Absolute form factor value (for $|V_{us}| = 0.2252 \pm 0.0009$ from PDG 2012) :

$F_s(q2=0,Se=0) = 5.705 \pm 0.003_{stat} \pm 0.017_{syst} \pm 0.031_{ext}$

Ke4(00) : $K^{\pm} \rightarrow \pi^0 \pi^0 \nu e^{\pm}$

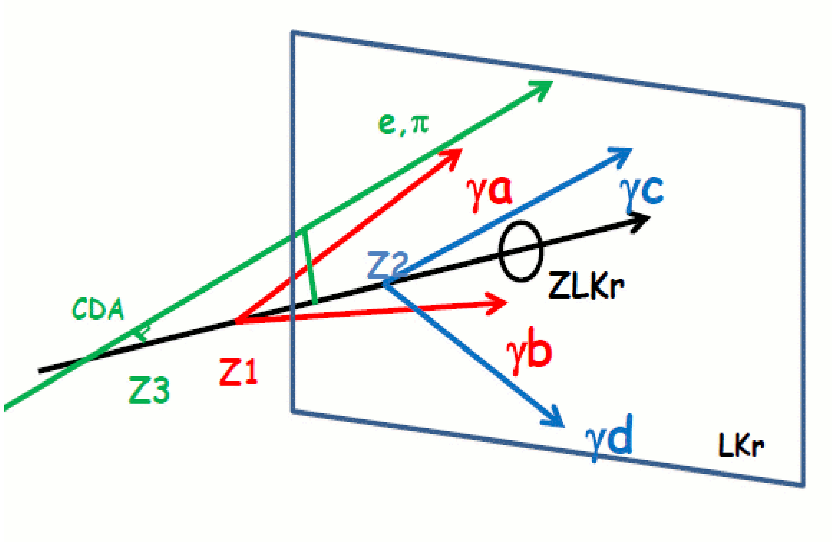
CERN-PH-EP-2014-145,
Accepted by JHEP

$K^{\pm} \rightarrow \pi^0 \pi^0 \nu e^{\pm}$ relative to $K^{\pm} \rightarrow \pi^0 \pi^0 \pi^{\pm}$ with Br=(1.761±0.022)%

Common part of event reconstruction:

Find LKr γ-cluster pairs (ab) and (cd)
in-time (±2.5 ns), E>3GeV

- Decay positions Z_1 and Z_2 assuming $\pi^0 \rightarrow \gamma\gamma$.
- $Zn = (Z_1+Z_2)/2$ within (-16,+90) m
- $D_{Zn} = |Z_1-Z_2| < 500$ cm



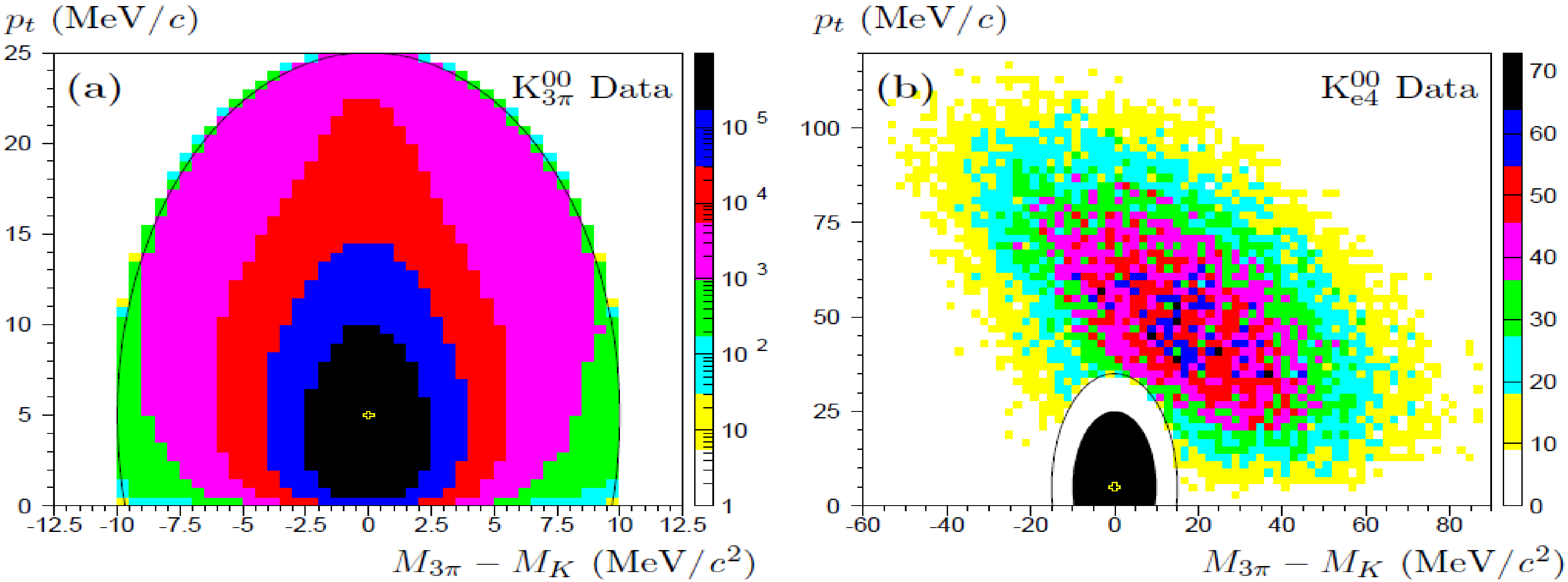
Combined with charged track (Z_3 at CDA to beam line) if:

- $D_Z = |Z_3-Z_n| < 800$ cm

K_{e4}(00) signal selection

- Assign m_π to the charged track, plot P_t (to beam) vs invariant mass
- Cut $K_{3\pi}$ events with a small P_t and ~ kaon PDG mass
- Cut $S_e < 0.25$ (GeV/c^2)^2 ,rejects 0.5% candidates (mis-reconstructed tracks in fake electrons and accidentals)
- No extra close cluster E > 3 GeV

Elliptic cuts separate ~93 x 10^6 $K_{3\pi}$ from ~65000 K_{e4} candidates



K_{e4}(00) background rejection

Electron identification:

- LKr cluster associated to track is in-time (10 ns) with track and $2\pi^0$
- E(LKr)/P(DCH) ~ 1 [0.9-1.1]
- Extra rejection using a dedicated discriminating variable. It is a linear combination of variables related to shower properties and trained on real and fake electrons from data.

• Fake-electron background ($K\rightarrow\pi^0\pi^0\pi^+$)	0.65 %
• Decay electron background ($K\rightarrow\pi^0\pi^0\pi^+$; $\pi\rightarrow e\nu$)	0.12 %
• Accidental track or photon	0.23 %
Total	**1.00 %**

Kaon momentum reconstruction imposing energy-momentum conservation and zero neutrino mass.

Form Factor measurement

Because of two identical particles in the final state, the $\pi^0\,\pi^0$ system cannot be in a l=1 state and only the S-wave term contributes to the partial wave expansion of the form factors (F_s).

The differential rate depends only on 3 kinematic variables:

$$d^3\Gamma = \frac{G_F^2|V_{us}|^2}{2(4\pi)^6 m_K^5}\ \rho(S_\pi, S_\mathrm{e})\ J_3(S_\pi, S_\mathrm{e}, \cos\theta_\mathrm{e}) \times\ dS_\pi\ dS_\mathrm{e}\ d\cos\theta_\mathrm{e}$$

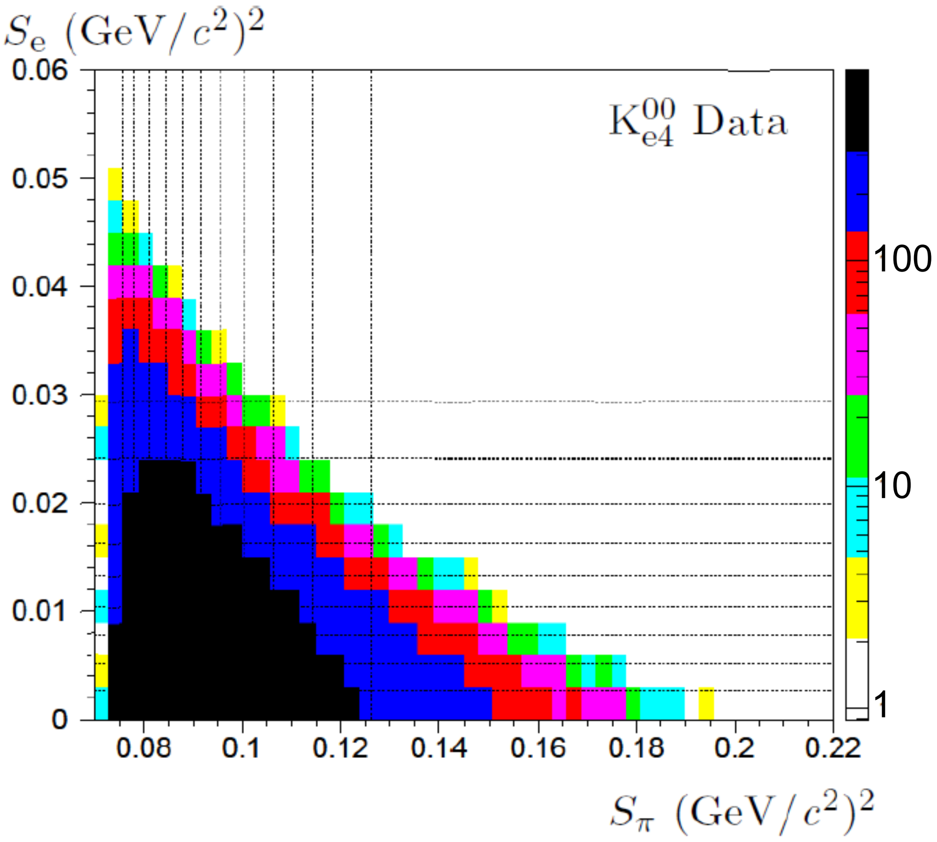
$$J_3 = |XF_s|^2(1 -\ \cos 2\theta_\mathrm{e}) = 2|XF_s|^2 \sin^2\theta_\mathrm{e}$$

where $\rho(S_\pi, S_\mathrm{e})$ is the phase space factor $X\sigma_\pi\,(1 - z_\mathrm{e})$, with $X = \frac{1}{2}\lambda^{1/2}(m_K^2, S_\pi, S_\mathrm{e})$, $\sigma_\pi =$ $(1 - 4m_\pi^2/S_\pi)^{1/2}$, $z_\mathrm{e} = m_\mathrm{e}^2/S_\mathrm{e}$, and $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$.

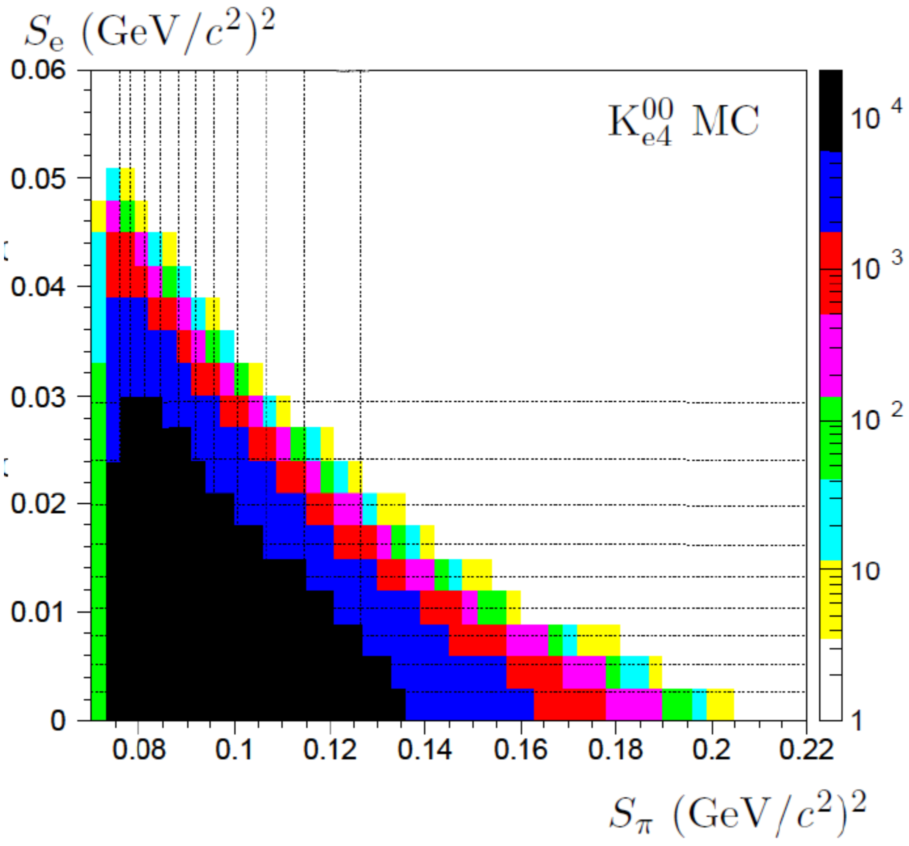
No F_s dependence with θ_e angle, only to be studied in the (Sπ,Se) plane

- Density of events is proportional to $|F_s|^2$
- Subtract background in the 2d-plane
- Compare to the same distribution from simulation including acceptance, resolution, trigger efficiency, radiative corrections and kinematic factors but using a constant form factor.
- Define a grid of 10 equal population bins in S_π above the $2m_{\pi+}$ threshold and two equal population bins below (10 bins with 6000 events each, 2 bins with 3000 events each), 10 bins in S_e (300 or 600 events in 2d-bins)

Form Factor measurement: 2d plot (S_π, S_e)

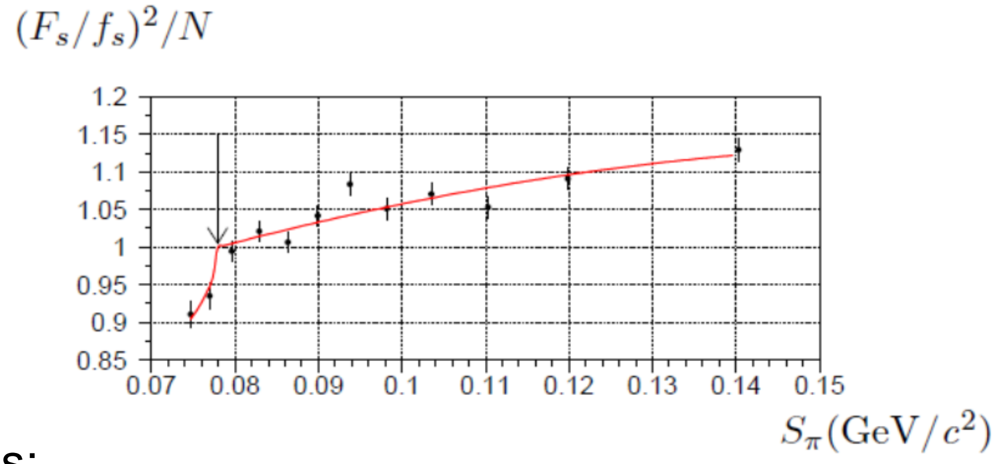


K_{e4} candidates — background
~65000 events



K_{e4} simulated with constant F_s
100 millions simulated

Fit procedure



We observe the cusp-like behavior of Form Factor s_π dependence with a threshold at $4m^2_{\pi+}$

Define the dimensionless variables:

$X = q^2 = S_\pi/(4m^2_{\pi+}) — 1$
$Y = S_e/(4m_\pi^2)$

And 2d fit function:

$G = N\,(1 + aX + bX^2 + cY)^2 \qquad X > 0$
$G = N\,(1 + d(|X|/(1+X))^{1/2} + cY)^2 \qquad X < 0$

To minimize:

$$\chi^2 = \sum_{i=1}^{12}\sum_{j=1}^{10}((n_{ij}/m_{ij} - G(X_i, Y_j, \hat{p}))/\sigma_{ij})^2$$

(Data-bkg)

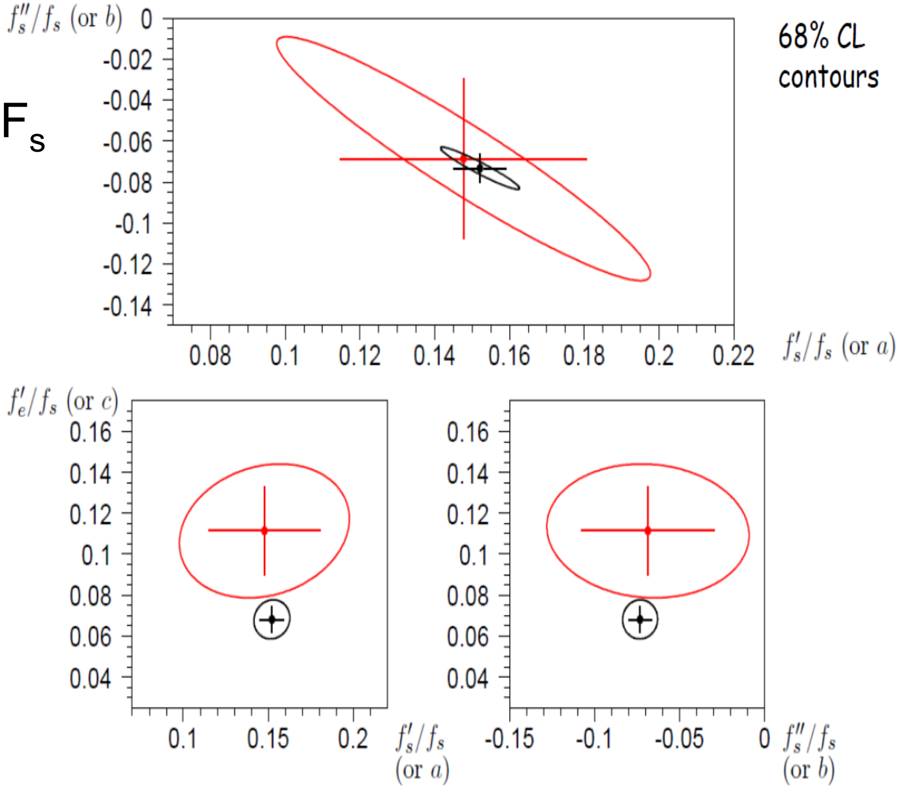
MC with F_s=1

Fit parameters

X_i, Y_j are calculated in the corresponding bin centers.

Form Factor measurement

Consistent with K_{e4}(+/-) F_s for $q^2 > 0$



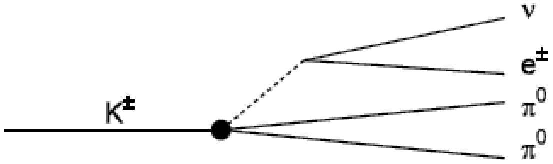
a = 0.149 ± 0.033;
b = −0.070 ± 0.039;
c = 0.113 ± 0.022;
d = −0.256 ± 0.049

χ^2/ndf =101.4/107: 63% probability

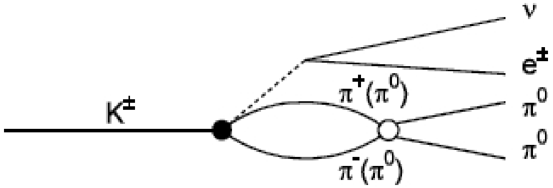
Error	δa	δb	δc	δd
Trigger simulation	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Background	0.0140	0.0122	0.0062	0.0164
Radat. simulation	0.0037	0.0035	0.0033	0.0013
Systematic error	0.014	0.013	0.007	0.016

K_{e4}(00) Form Factor interpretation by analogy

1-loop calculation for 3π decays: Cabibbo, PRL 93(2004)121801



Tree level M0



1-loop M1

Above threshold: $|M|^2 = |M0 + i\,M1|^2 = M0^2 + M1^2$

Below threshold : $|M|^2 = |M0 + M1|^2 = M0^2 + M1^2 + 2\,M0\,M1$

$q2 = S\pi/4m\pi+^2 - 1 \quad \sigma\pi = \sqrt{(4m\pi+^2/S\pi - 1)} = \sqrt{(|q2|/(1+q2))}$

M0 = unperturbed amplitude: $Fs = fs\,(1 + a\,q2 + b\,q4 + c\,Se/4m\pi+^2)$

M1 = scattering amplitude: $-\,2/3\,(a0 - a2)\,fs\,\sqrt{(|q2|/(1+q2))}$

We don't plan to extract a0 and a2 from K_{e4}(00) data,
so precise interpretation is not necessary to finish the work.

$K_{e4}(00)$: Br measurement

Br is measured in independent subsamples and then combined.

N(K_{e4} candidates) = 65210 N(bkg) = 651
N($K_{3\pi}$ candidates) = 93.54 M

Acceptances: A(K_{e4}) = 1.926(1)% A($K_{3\pi}$) = 4.052(2)%
Trigger efficiency $\varepsilon(K_{e4})$ = 96.06(3)%, $\varepsilon(K_{3\pi})$ = 97.42(0)%
Normalization: Br ($K_{3\pi}$) = (1.761 ± 0.022)% - source of external error

Systematic Uncertainty (% to Br value)

Acceptance	0.16
Form Factor	0.17
Background	0.25
Trigger cut	0.04
Rad. Corr.	0.19
Simulation stat	0.07
Trigger efficiency	0.03
Total	**0.40**

$\mathrm{Br}(K_{e4}{}^{00})$ =

$(2.552 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}} \pm 0.032_{\text{ext}})10^{-5}$

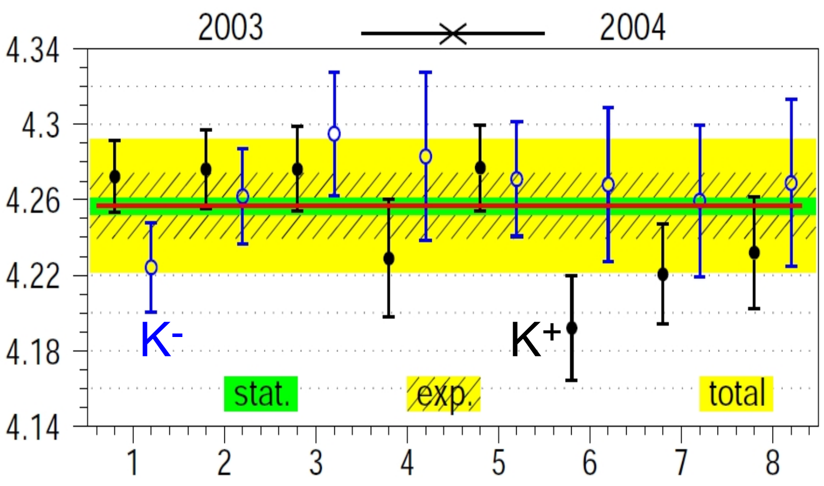
PDG 2012 : $(2.2 \pm 0.4)\ 10^{-5}$

Absolute form factor value
(no radiative corrections,
for $|V_{us}| = 0.2252 \pm 0.0009$ from PDG 2012) :

$(1+\delta_{EM})\ F_s(q2=0,Se=0)$
$= 6.079 \pm 0.012_{\text{stat}} \pm 0.027_{\text{syst}} \pm 0.046_{\text{ext}}$

Ke4 Br measurement in statistically independent subsamples

all in units of 10^{-5}

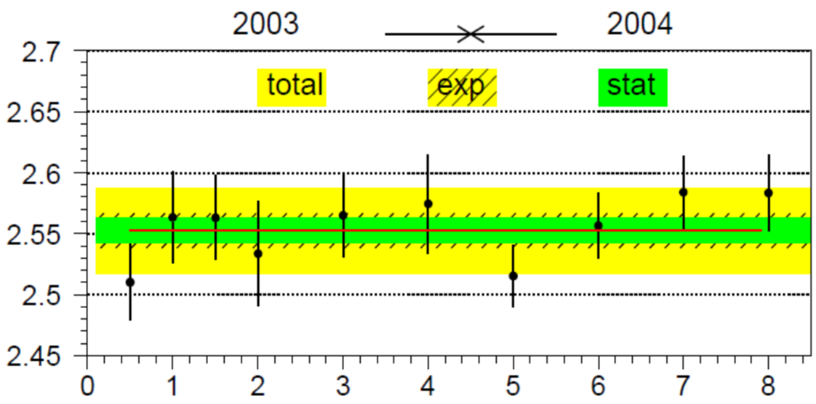


Phys.Lett. B715 (2012) 105:

K_{e4}(+-) normalized to $K_{3\pi}$(+-)

(4.257 ± 0.004 ± 0.016 ± 0.031)=
Stat Syst Ext

(4.257 ± 0.035) 0.8% rel.err.



Final:

K_{e4}(00) normalized to $K_{3\pi}$(00)

(2.552 ± 0.010 ± 0.010 ± 0.032)=
Stat Syst Ext

(2.552 ± 0.035) 1.4% rel.err.

ChPT: $K^{\pm}\rightarrow\pi^{\pm}\gamma\gamma$

- Dependence on a single parameter **ĉ** at $O(p^4)$ and $O(p^6)$

$$\frac{\partial\Gamma}{\partial y\partial z}(\hat{c},y,z)=\frac{m_K}{2^9\pi^3}\left[z^2\left(\underbrace{|A(\hat{c},z,y^2)}_{\text{loop contribution}}+B(z)|^2+\underbrace{|C(z)|^2}_{\text{pole contribution}}\right)+\left(y^2-\frac{1}{4}\lambda(1,r_\pi^2,z)\right)^2\underbrace{|B(z)|^2}_{\text{loop } O(p^6)}\right]$$

where $z=\left(\frac{m_{\gamma\gamma}}{m_K}\right)^2$, $y=\frac{p(q_1-q_2)}{m_K^2}$

p,q_1,q_2 : $K^{\pm},\gamma,\gamma$ momenta

G.D'Ambrosio, J.Portoles.
PLB 386 (1996) 403

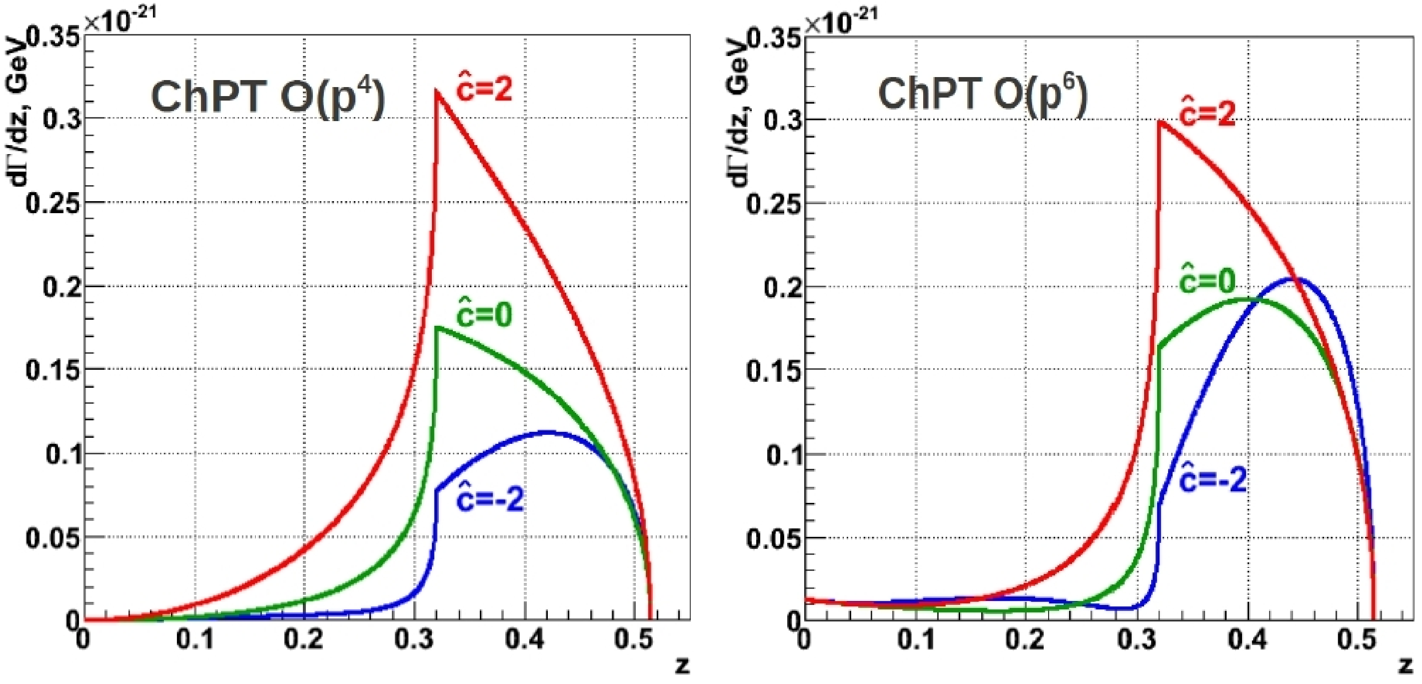
Experimental status befefore present work:

E787
[PRL 79 (1997) 4079]:

$Br=(1.1\pm0.3)10^{-6}$

31 cand. , 5 est. bkg.

$\hat{c}$ = 1.1±0.6 for $O(p^4)$
$\hat{c}$ = 1.8±0.6 for $O(p^6)$

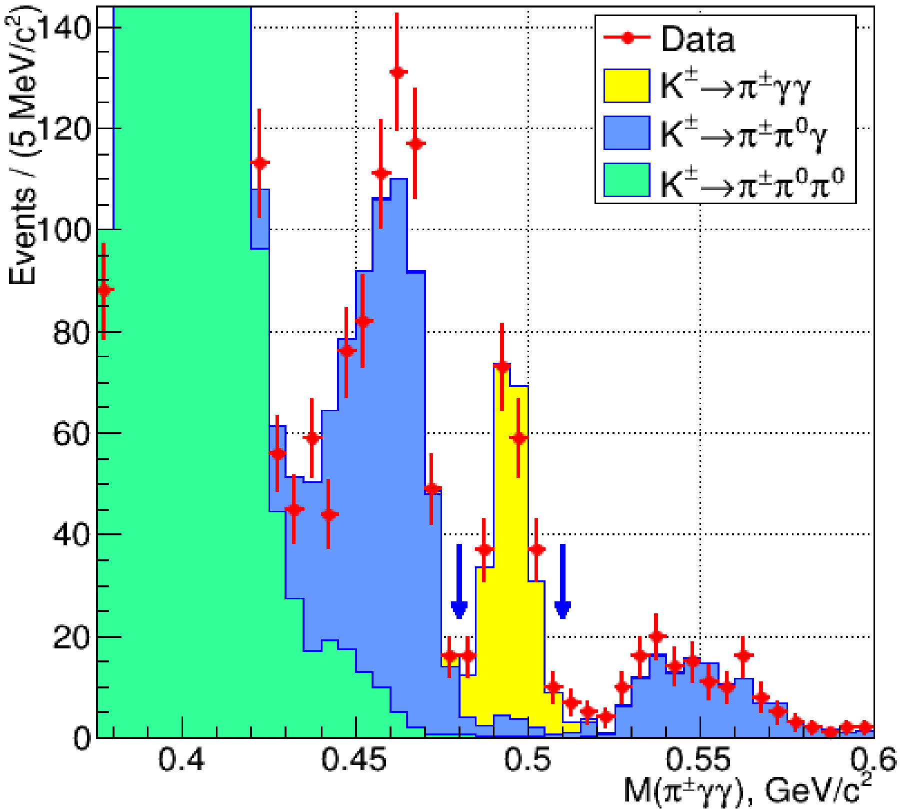


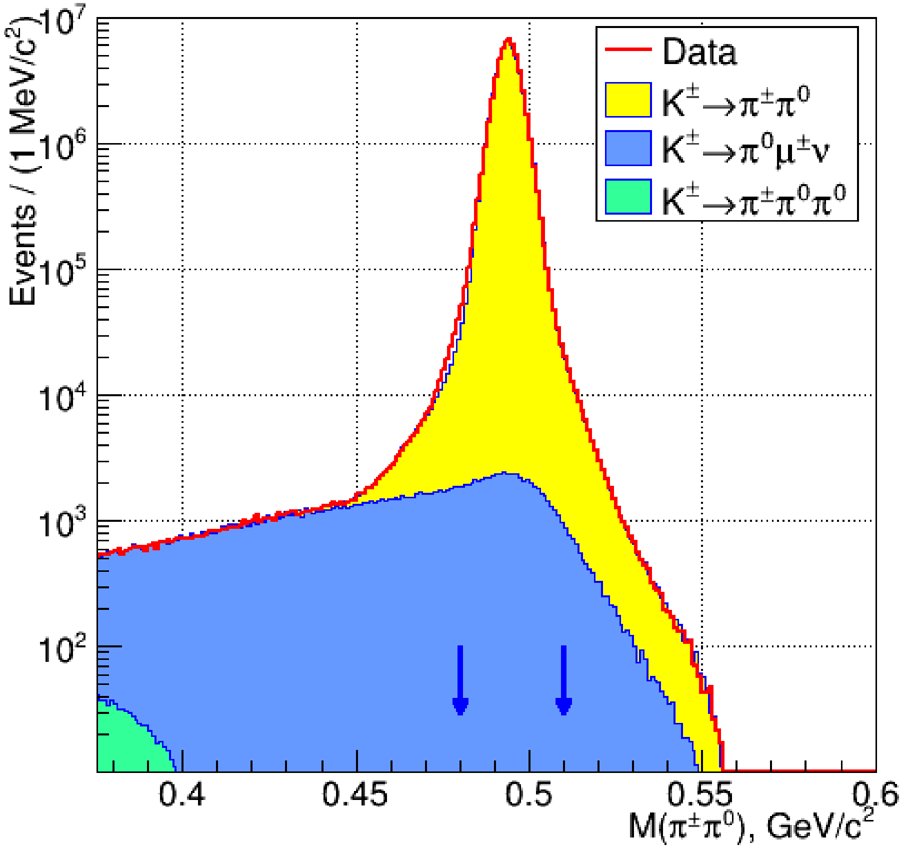
Cusp at $m_{\gamma\gamma}=2m_\pi$+ threshold

$K^\pm \to \pi^\pm \gamma\gamma$

[PLB 730 (2014) 141, PLB 732(2014) 65]

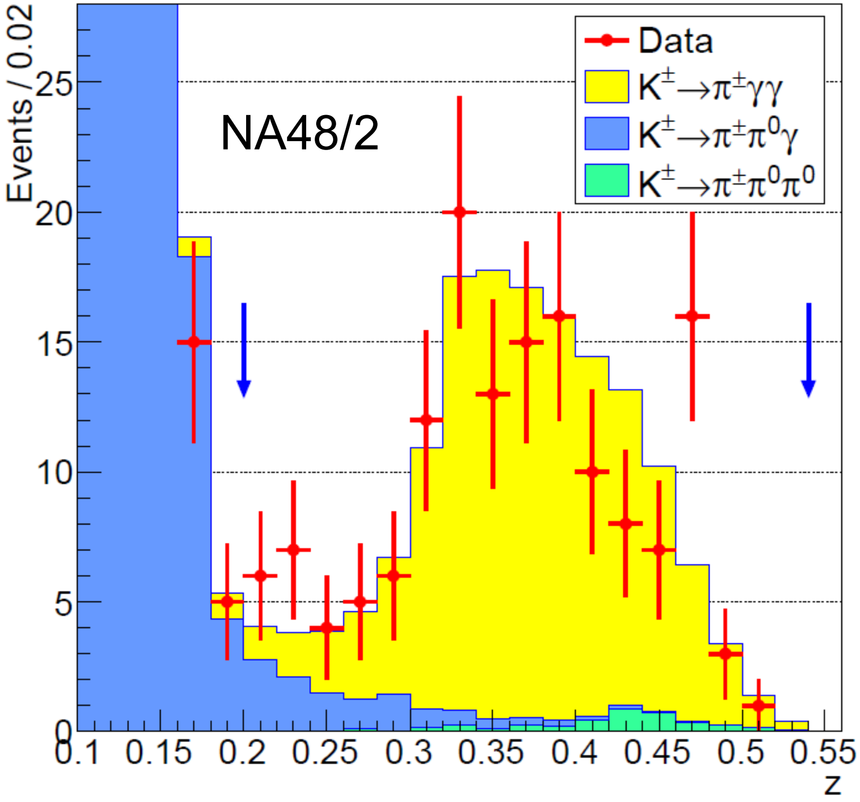
- NA48/2 2004 data (3 days special minimum bias run)
- NA62 (R_K phase) 2007 data (3 month control min. bias trigger downscaled by 20)

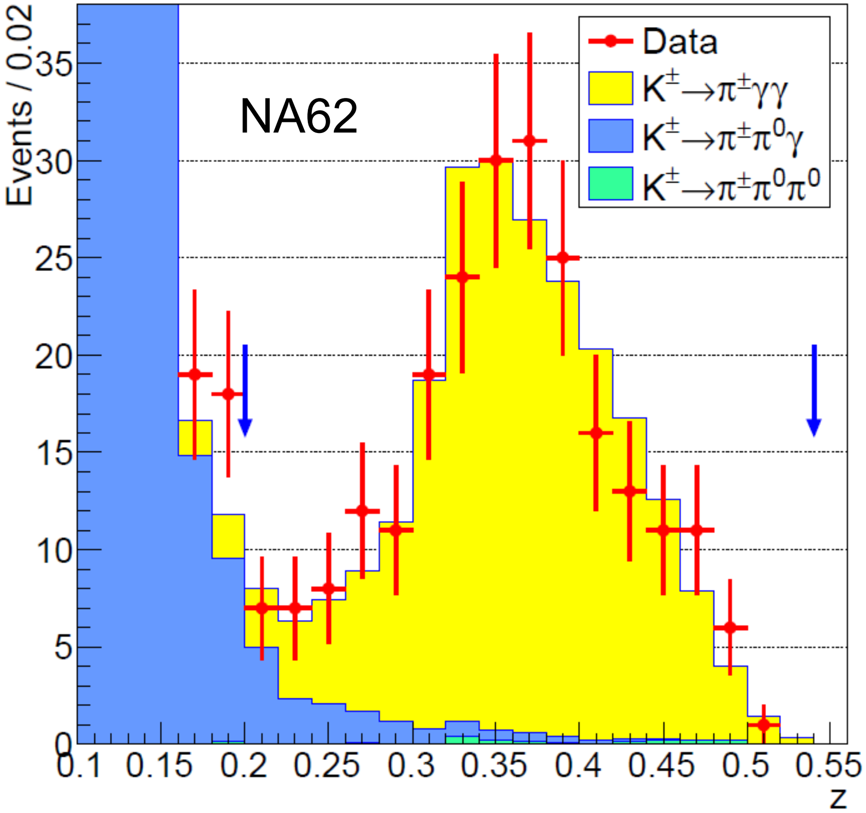




Signal and reference channel invariant mass plots for NA62 run (2007)

$K^{\pm} \to \pi^{\pm}\gamma\gamma$: *z* spectrum Data/MC



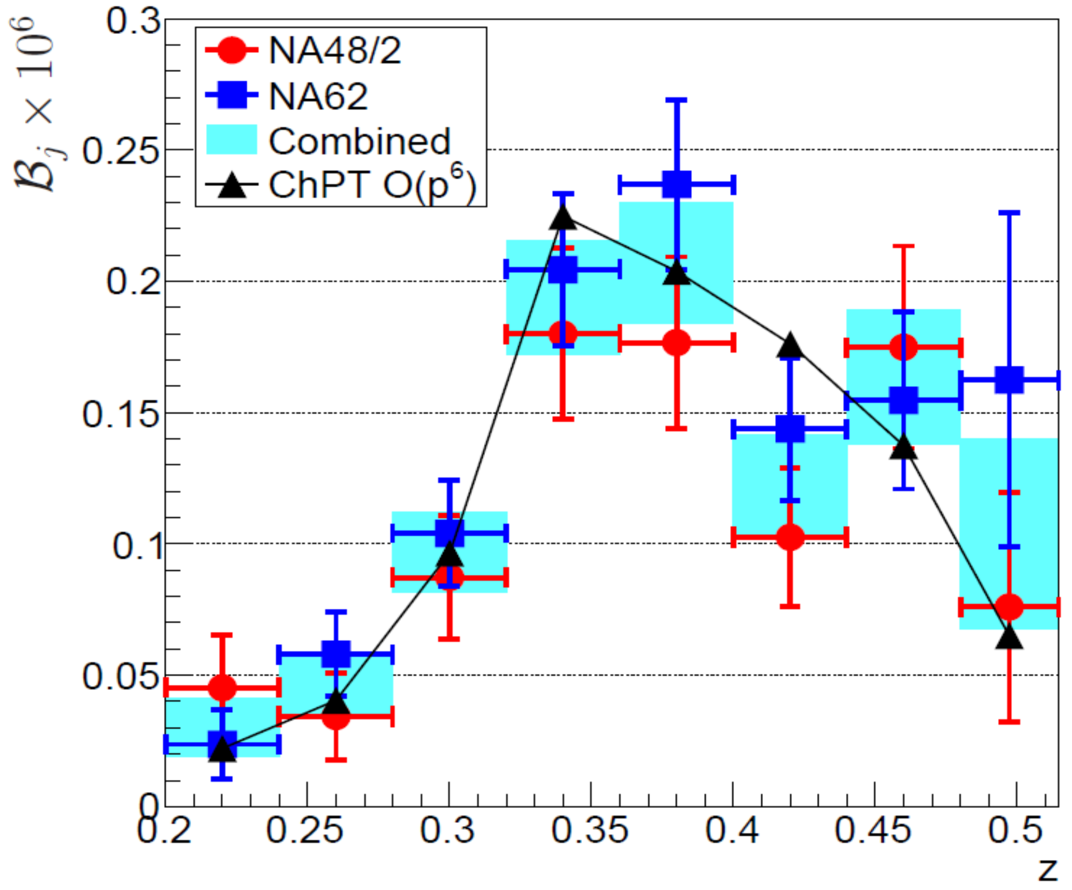


$\pi^{\pm}\gamma\gamma$ cand.	**149**
Bkg $\pi^{\pm}\pi^{0}\gamma$	11.4±0.6
Bkg $\pi^{\pm}\pi^{0}\pi^{0}$	4.1±0.4

$\pi^{\pm}\gamma\gamma$ cand.	**232**
Bkg $\pi^{\pm}\pi^{0}\gamma$	15.3±1.1
Bkg $\pi^{\pm}\pi^{0}\pi^{0}$	2.1±0.3

Model-independent measurement of branching ratios in *z* bins

Statistical errors are shown



Model-independent branching ratio:

$$B_{MI}(z > 0.2) = (0.965 \pm 0.061_{stat} \pm 0.014_{syst}) \times 10^{-6}$$

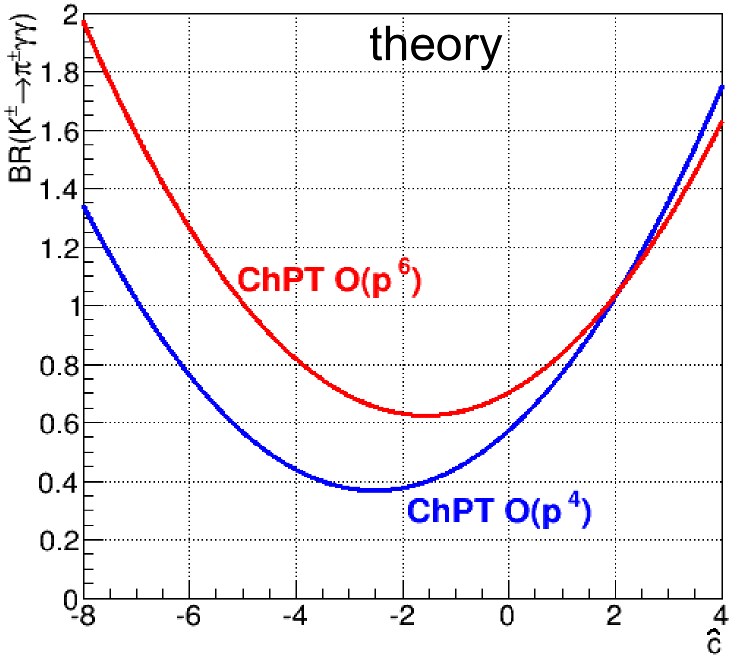
Charge asymmetry: $(B_{MI}^{+}-B_{MI}^{-})/(B_{MI}^{+}+B_{MI}^{-}) = -0.03 \pm 0.07$

$K^{\pm} \rightarrow \pi^{\pm}\gamma\gamma$

Combined NA48/2 and NA62 final result,
based on 381 events
(10 times the old world sample):

$\hat{c}$ for $O(p^4)$ = $\mathbf{1.72 \pm 0.20_{stat} \pm 0.06_{syst}}$

$\hat{c}$ for $O(p^6)$ = $\mathbf{1.86 \pm 0.23_{stat} \pm 0.11_{syst}}$



Both formulations for this ***z*** spectrum predict very similar Br values.

Measured Br for $O(p^6)$ = $(1.003 \pm 0.051_{stat} \pm 0.024_{syst}) \cdot 10^{-6}$

Summary

- **1.11 millons of reconstructed $K^{\pm}\rightarrow\pi^{+}\pi^{-}\nu e^{\pm}$ ($K_{e4}(+-)$) and ~65000 of $K^{\pm}\rightarrow\pi^{0}\pi^{0}\nu e^{\pm}$ ($K_{e4}(00)$) decays (2003+2004 data).**
- **Improved branching fractions:**

  Br $K_{e4}(+-) = (4.257 \pm 0.035)\ 10^{-5}$ [Phys.Lett. B715 (2012) 105] (3 times better/PDG)

  Br $K_{e4}(00) = (2.552 \pm 0.035)\ 10^{-5}$ [CERN-PH-EP-2014-145, Accepted by JHEP]

  (10 times better/PDG)
- **$K_{e4}(00)$ F_s form factor is compatible with the $K_{e4}(+-)$ one above $2m_{\pi+}$ threshold. Deficit below can be due to $\pi\pi$ final state charge exchange scattering.**
- **Final results for $\pi^{\pm}\gamma\gamma$** **based on 381 events**

  $\hat{c}$ for $O(p^4) = 1.72 \pm 0.20_{stat} \pm 0.06_{syst}$

  $\hat{c}$ for $O(p^6) = 1.86 \pm 0.23_{stat} \pm 0.11_{syst}$

  $Br(\pi^{\pm}\gamma\gamma)$ ($z > 0.2$) = $(0.965 \pm 0.063) \cdot 10^{-6}$, charge asymmetry: -0.03 ± 0.07

  $Br(\pi^{\pm}\gamma\gamma)$ for $O(p^6) = (1.003 \pm 0.056) \cdot 10^{-6}$