Recent results on semileptonic B and B_s decays from Belle

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(On behalf of the Belle collaboration)

Outline of this talk

Semileptonic B decays

- $B \to D\ell\nu$ exclusive — *New prelim. results*
- $B \to X_u\ell\nu$ exclusive — *PRD 88, 032005 (2013)*

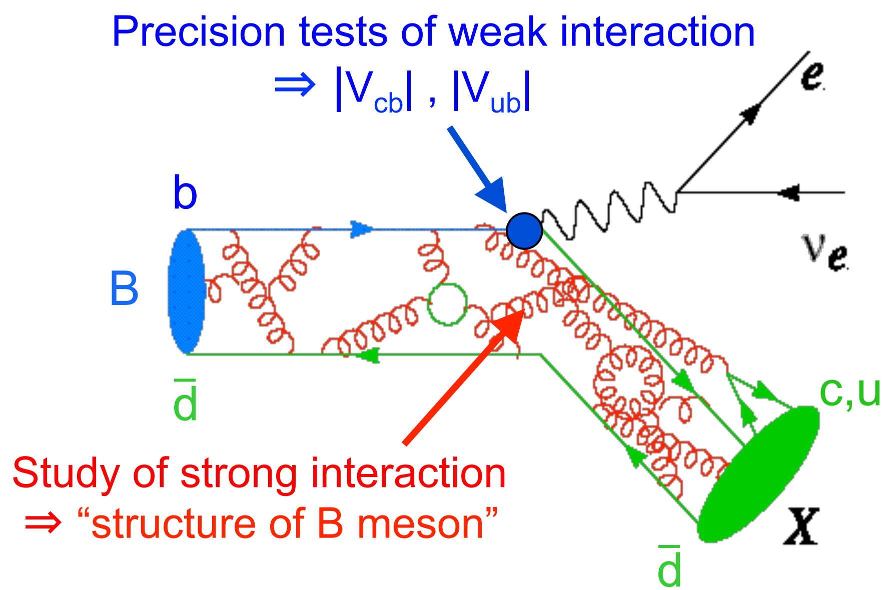
Semileptonic B_s decays

- $B_s \to X\ell\nu$ inclusive — *PRD 87, 072008 (2013)*
- $B_s \to D_s^{(*)}X\ell\nu$ semi-exclusive — *New prelim. results*

Semileptonic B decays to baryons

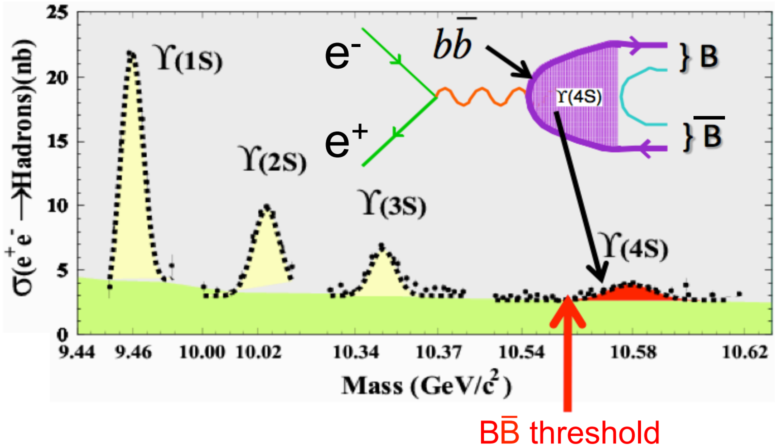
- $B \to p\bar{p}\ell\nu$ — *PRD 89, 011101 (2013)*

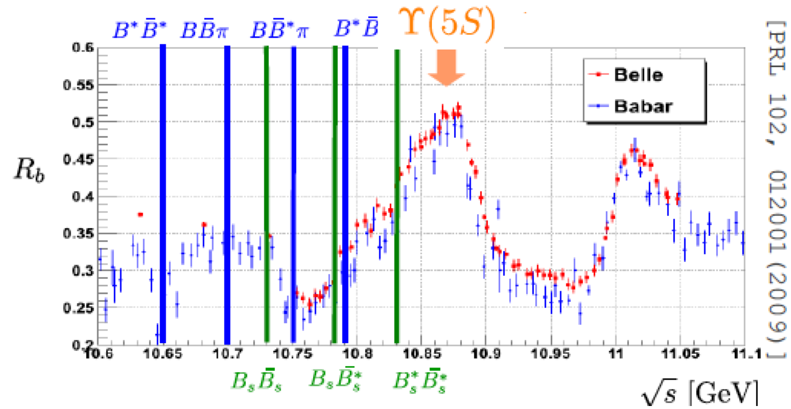
Motivation: Semileptonic B/B_s decays at Belle

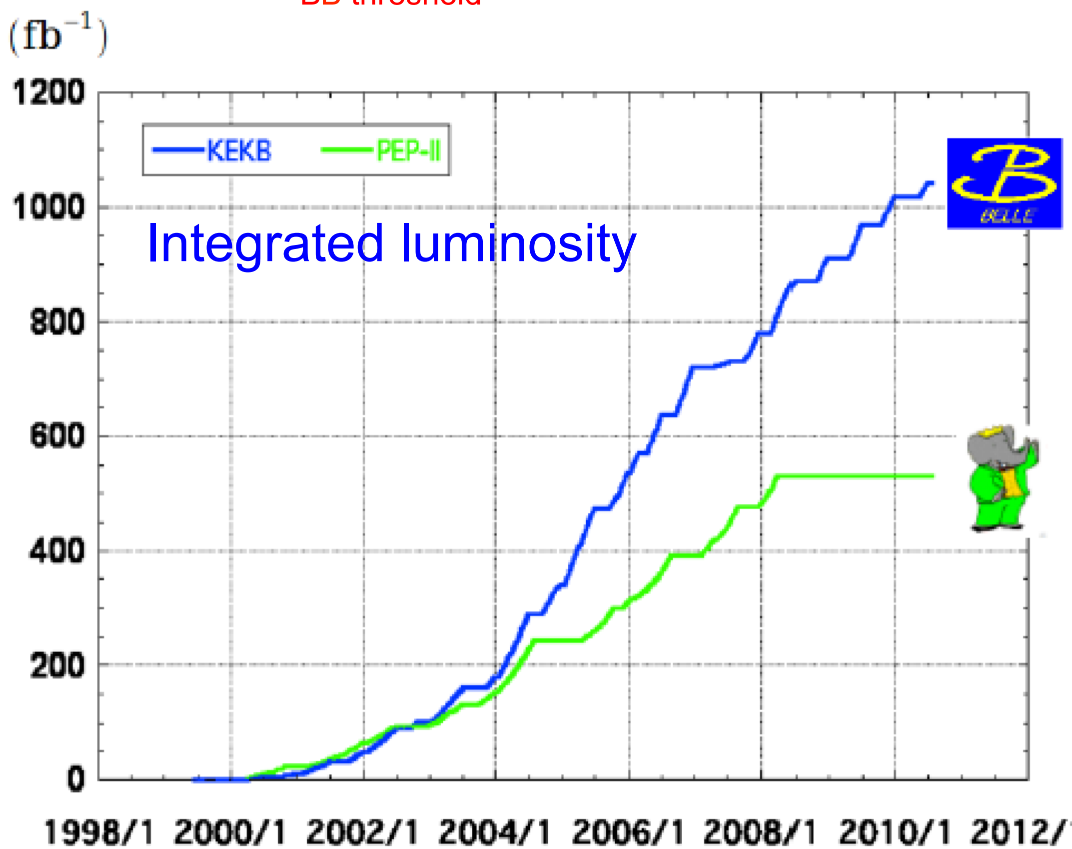


- Semileptonic B decays are ideally suited to
  - determine $|V_{cb}|$ and $|V_{ub}|$
  - test theoretical tools to describe hadronic effects in B decays
- Semileptonic B_s decays provide
  - possibility for independent cross-check of $|V_{cb}|$, $|V_{ub}|$
  - important input to determine B_s production at B factories and LHC
  - possibility to test SU(3) flavor symmetry: $\frac{\Gamma(B_s^0 \to X\ell\nu)}{\Gamma(B^0 \to X\ell\nu)} \approx 0.99$ *[JHEP1109,012(2011)]*

Belle Υ(4S) and Υ(5S) datasets







> 1 ab^{-1}

On resonance:

- Υ(5S): 121 fb^{-1}
- Υ(4S): 711 fb^{-1}
- Υ(3S): 3 fb^{-1}
- Υ(2S): 25 fb^{-1}
- Υ(1S): 6 fb^{-1}

Off reson./scan

~ 100 fb^{-1}

~ 550 fb^{-1}

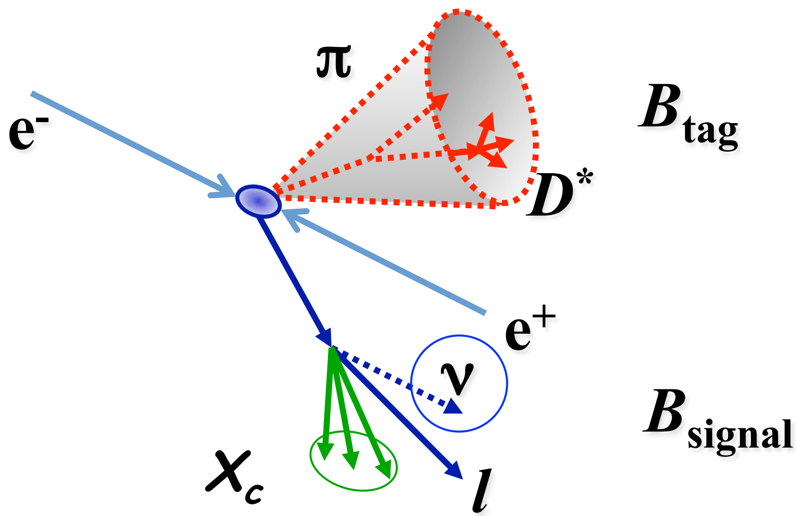
On resonance:

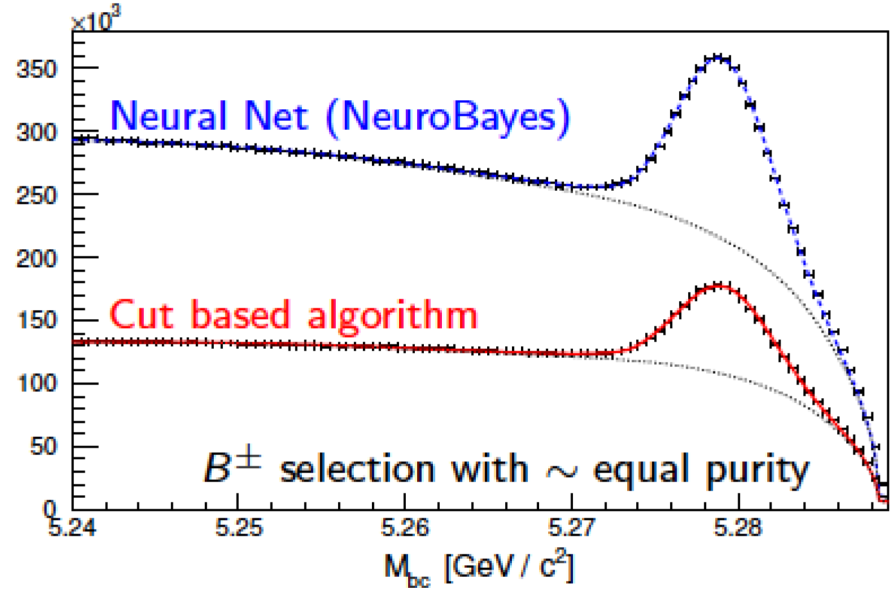
- Υ(4S): 433 fb^{-1}
- Υ(3S): 30 fb^{-1}
- Υ(2S): 14 fb^{-1}

Off resonance:

~ 54 fb^{-1}

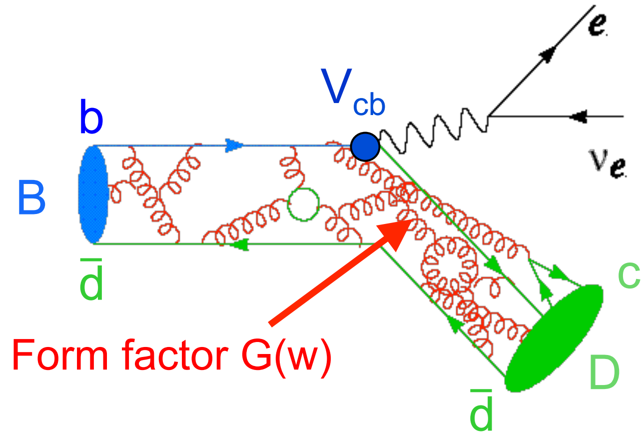
Experimental tool: Hadronic B tagging





- B-tagging algorithm based on NeuroBayes Neural Network package
- B_{tag} fully reconstructed in more than 1100 hadronic decay modes
- Increase of ~ factor 2 in tagging efficiency at ~ same purity
- Calibrated with semileptonic decays (uncertainty 4-5%)

$B \to D\ell\nu$



$$q^2 = (p_\ell + p_\nu)^2 = (p_B - p_D)^2$$

$$w = v_B v_D = \frac{m_B^2 + m_D^2 - q^2}{2 m_B m_D}$$

- Decay rate:

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 G^2(w)$$

Parametrization by Caprini et al. [Nucl.Phys.B380,376]

$$G(w) = G(1)(1 - 8\rho^2 z + (51\rho^2 - 10) z^2 - (252\rho^2 - 84) z^3)$$

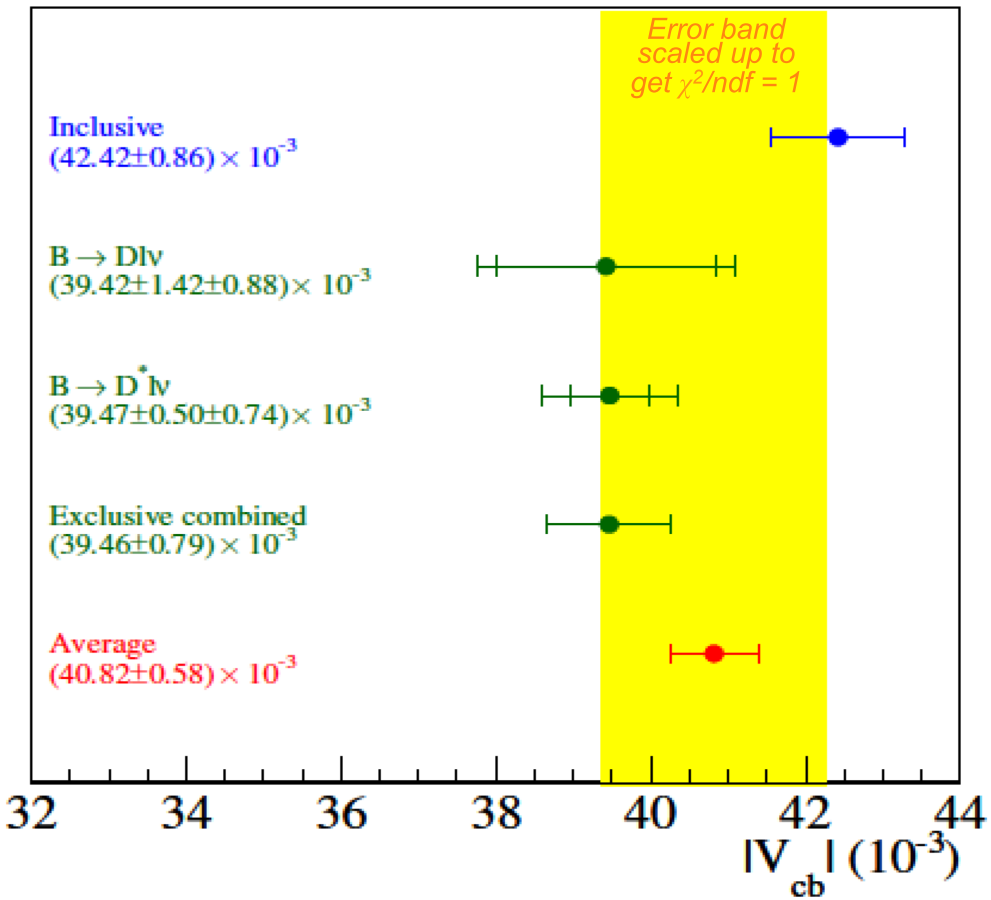
$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Measure: $|V_{cb}|G(1)$ and ρ^2

- 3 previous B-factory measurements:

Belle untagged (10 fb^{-1}), BaBar untagged (207 fb^{-1}), BaBar hadronic-tag (417 fb^{-1})

Status of $|V_{cb}|$ determinations



Gambino, Schwanda
[Phys.Rev.D89,014022 (2014)]

G(1) = 1.074 ± 0.024
[arXiv:hep-lat/0409116 (2005)]

F(1) = 0.902 ± 0.017
[LATTICE2010, 311 (2010)]

- Current disagreement between $|V_{cb}|$ from incl. and excl. decays: 2.5σ
- New $B \to D^*$ LQCD value F(1) = 0.906 ± 0.013 *[Phys.Rev.D89, 114504 (2014)]* increases disagreement

$B \to D\ell\nu$: New preliminary results

- Hadronic-tag measurement (711 fb^{-1})

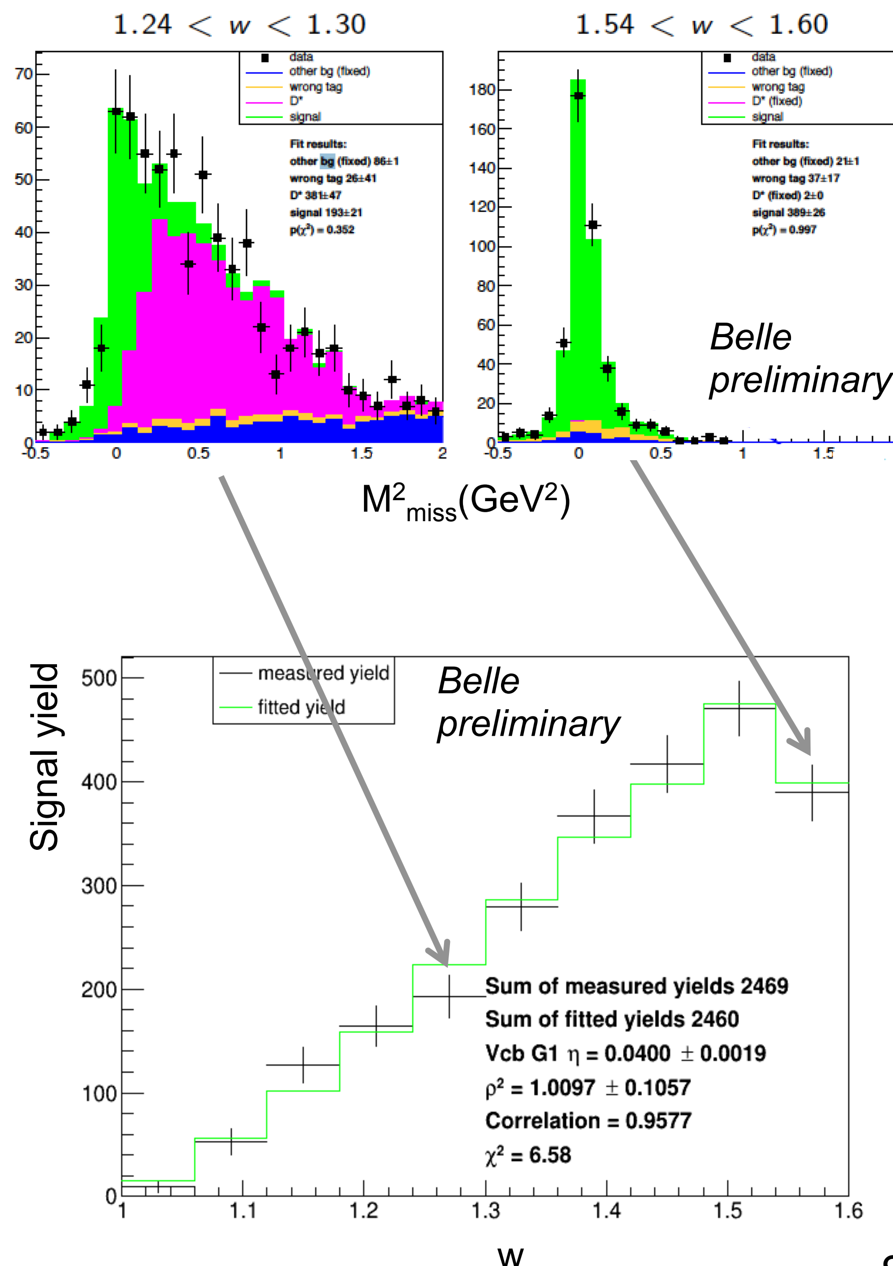
- Four signal samples:
  $B^0 \to D^+\ell^-\nu$ and $B^- \to D^0 e^-\nu$ for $\ell = e,\mu$

- D^0, D^+ recon. in many hadronic decay modes, then paired with a lepton (e,μ)

- Signal extracted with fit to M^2_{miss} in 10 bins of w

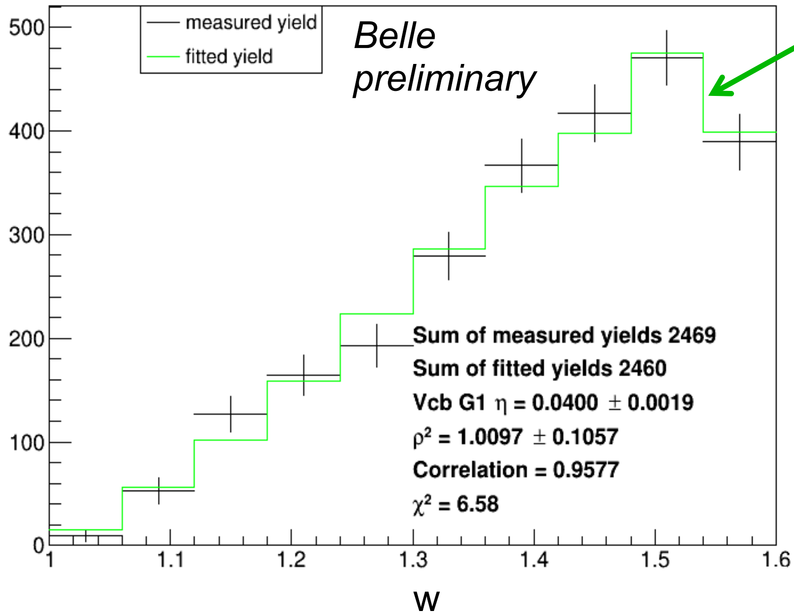
$$M^2_{\text{miss}} = (p_{\text{beam}} - p_{B_{tag}} - p_D - p_\ell)^2$$

- Free fit parameters: signal, D* and wrong-tag background



$B \to D\ell\nu$: $|V_{cb}|G(1)$ and ρ^2

- Determine $|V_{cb}|G(1)$ and ρ^2 from χ^2 fit of predicted to measured yields:



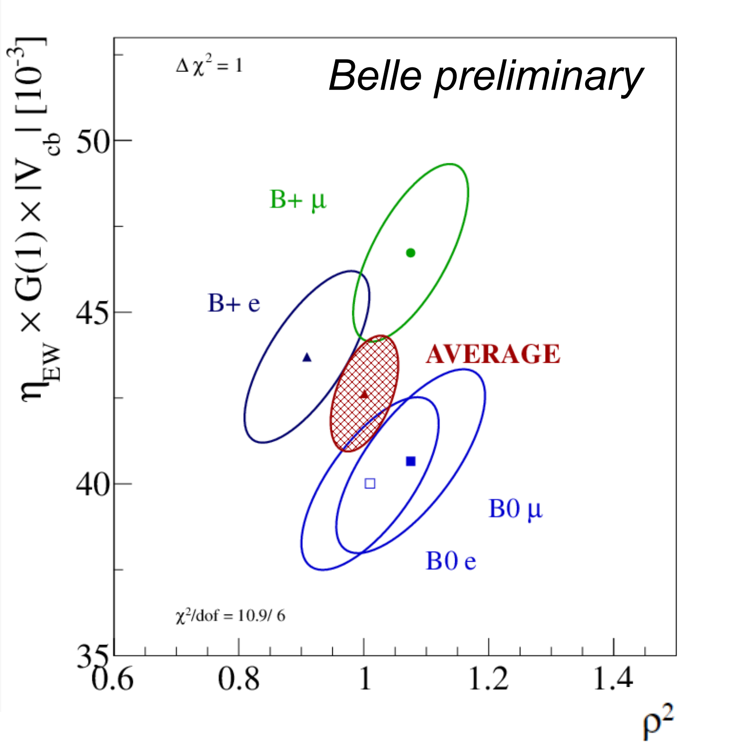
Branching fraction results:

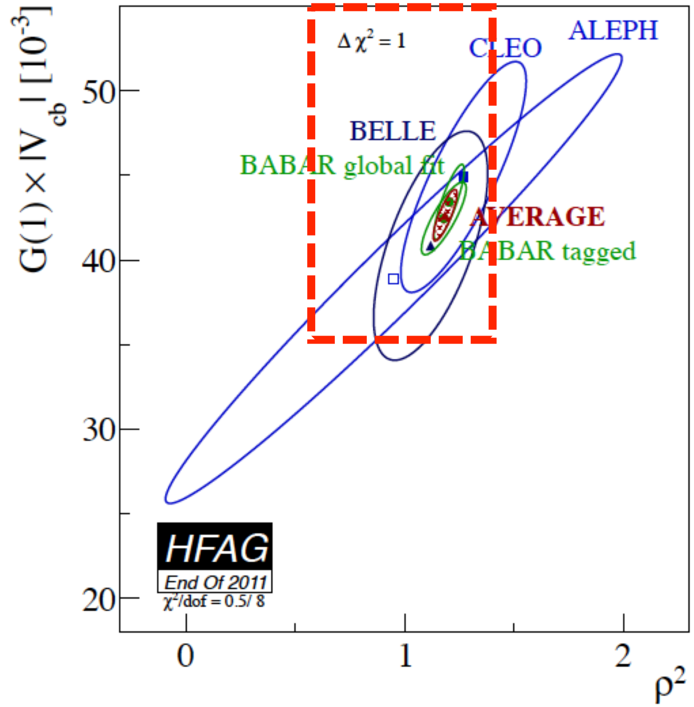
$$\mathcal{B}(\bar{B}^0 \to D^+\ell^-\bar{\nu}_\ell) = [2.49 \pm 0.17]\%$$
$$\mathcal{B}(B^- \to \bar{D}^0\ell^-\bar{\nu}_\ell) = [2.70 \pm 0.19]\%$$

Dominant syst. error: B_{tag} calibration

Sample	$\eta_{EW}\mathcal{G}(1)\lvert V_{cb}\rvert$ $[10^{-3}]$	ρ^2	correlation
$\bar{B}^0 \to D^+e^-\bar{\nu}_\ell$	$40.01 \pm 1.89(\text{stat}) \pm 1.66(\text{syst})$	$1.010 \pm 0.106(\text{stat}) \pm 0.029(\text{syst})$	0.692
$\bar{B}^0 \to D^+\mu^-\bar{\nu}_\ell$	$40.66 \pm 2.07(\text{stat}) \pm 1.70(\text{syst})$	$1.075 \pm 0.115(\text{stat}) \pm 0.031(\text{syst})$	0.713
$B^- \to \bar{D}^0e^-\bar{\nu}_\ell$	$43.70 \pm 1.86(\text{stat}) \pm 1.67(\text{syst})$	$0.909 \pm 0.099(\text{stat}) \pm 0.014(\text{syst})$	0.711
$B^- \to \bar{D}^0\mu^-\bar{\nu}_\ell$	$46.73 \pm 1.87(\text{stat}) \pm 1.79(\text{syst})$	$1.075 \pm 0.091(\text{stat}) \pm 0.014(\text{syst})$	0.680
Average	$42.63 \pm 0.96(\text{stat}) \pm 1.39(\text{syst})$	$1.001 \pm 0.051(\text{stat}) \pm 0.018(\text{syst})$	0.494

$B \to D\ell\nu$: Comparison with previous results

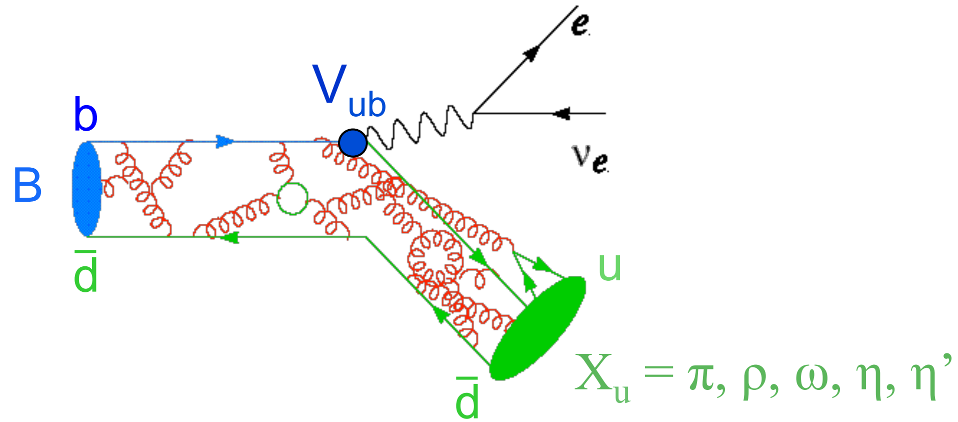


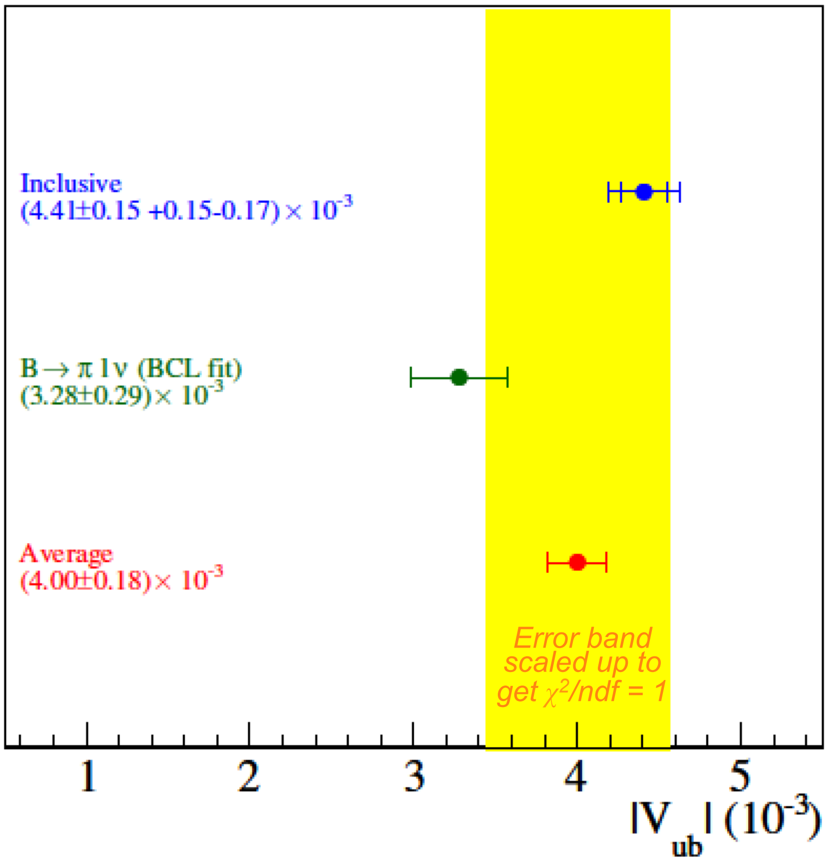


	$\eta_{EW} G(1)\|V_{cb}\|$ [10^{-3}]	ρ^2
Belle had-tag prelim.	$42.63 \pm 0.96 \pm 1.39$	$1.001 \pm 0.051 \pm 0.018$
BaBar had-tag	$42.45 \pm 1.88 \pm 1.05$	$1.180 \pm 0.089 \pm 0.051$
HFAG average	$42.64 \pm 0.72 \pm 1.35$	$1.186 \pm 0.036 \pm 0.041$

- Most precise single measurement of $B \to D\ell\nu$ (preliminary)
- ρ^2 value 2.4σ smaller than HFAG average

Determination of $|V_{ub}|$





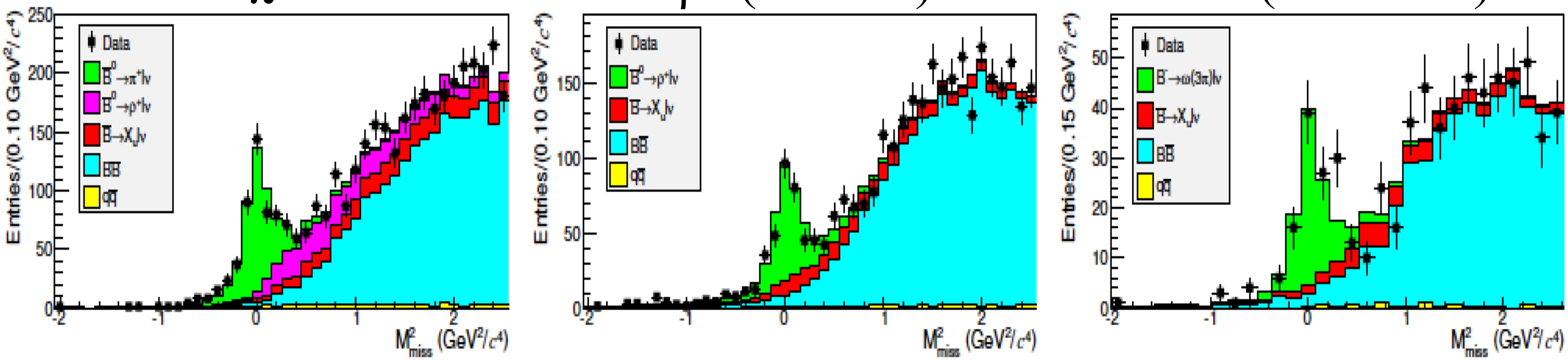
- Current disagreement between $|V_{ub}|$ from inclusive and exclusive ($B \to \pi \ell \nu$) decays: 3.0σ

- Interesting to determine $|V_{ub}|$ from other exclusive $B \to X_u \ell \nu$ decay modes
  - ⇒ clean hadronic-tag measurement particularly promising

Exclusive hadronic-tag $B \to X_u \ell\nu$ *[PRD 88, 032005 (2013)]*

- Hadronic-tag measurement (711 fb^{-1}) ⇒ low background, high purity
- Five $B \to X_u \ell\nu$ decay modes studied: $X_u = \pi^+, \pi^0, \rho^+, \rho^0, \omega$
- Hadronic tag allows for precise reconstruction of neutrino kinematics
- Signal yields extracted from fit to M^2_{miss} in bins of q^2

π^+ $\quad$ $\rho^+ (\to \pi^+\pi^0)$ $\quad$ $\omega (\to \pi^+\pi^-\pi^0)$



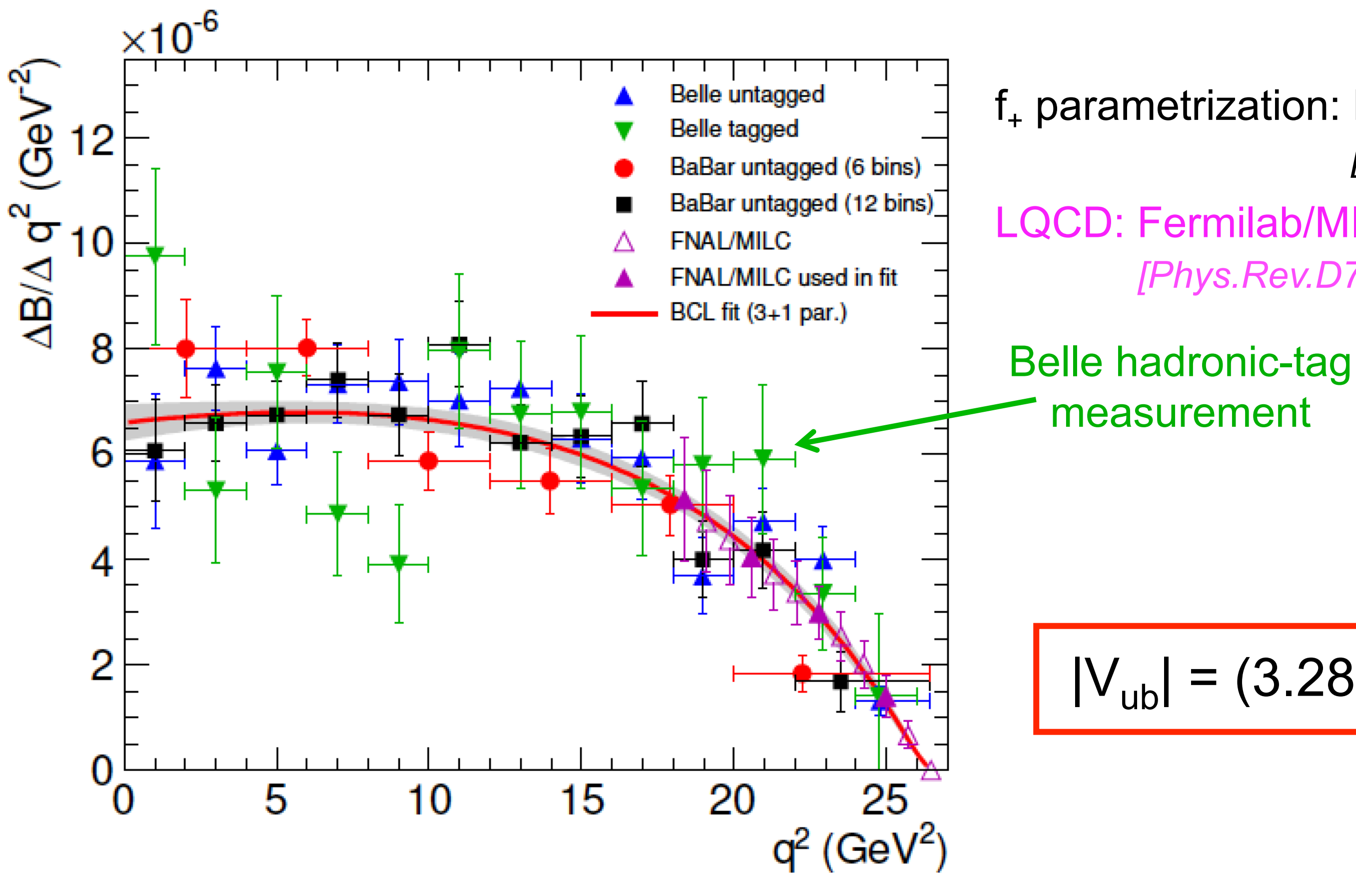
X_u	N_{sig}	$\mathcal{B} \times 10^4$
π^+	463 ± 28	$1.49 \pm 0.09 \pm 0.07$
π^0	232 ± 23	$0.80 \pm 0.08 \pm 0.04$
ρ^+	343 ± 28	$3.22 \pm 0.27 \pm 0.24$
ρ^0	622 ± 35	$1.83 \pm 0.10 \pm 0.10$
ω	106 ± 15	$1.07 \pm 0.15 \pm 0.07$

$|V_{ub}|$ from $B \to \pi\ell\nu$

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^- \ell^+ \nu) = \frac{G_F}{24\pi^3} p_\pi^3 \, |V_{ub}|^2 \, |f_+(q^2)|^2$$

Simultaneous fit of f_+ parametrization to $B \to \pi\ell\nu$ data and LQCD points

$\Rightarrow$ f_+ shape from data + LQCD, normalization from LQCD

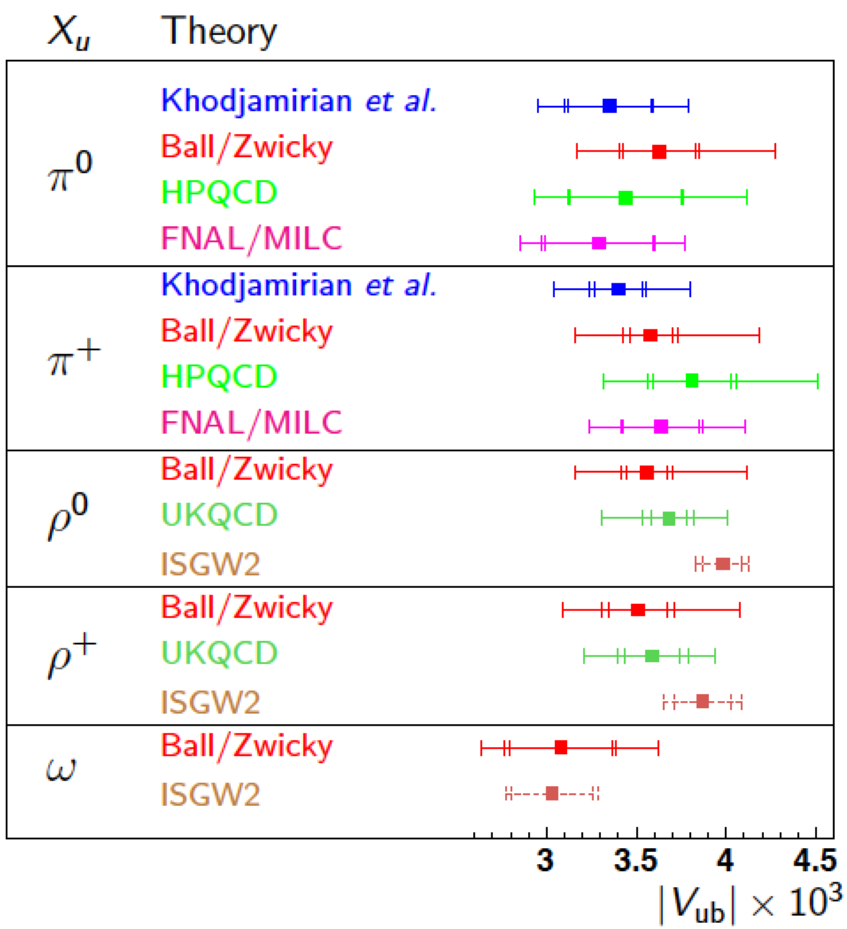


f_+ parametrization: Bourrely et al. (BCL)
[Phys.Rev.D79,013008]

LQCD: Fermilab/MILC (2009)
[Phys.Rev.D79,054507]

$|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3}$

$|V_{ub}|$ from other exclusive decays *[PRD 88, 032005 (2013)]*



Khodjamirian *et al.*
PRD **83**, 094031 (2011)

Ball/Zwicky
PRD **71**, 014015 (2005)
PRD **71**, 014029 (2005)

HPQCD
PRD **73**, 074502 (2006)

FNAL/MILC
PRD **79**, 054507 (2009)

UKQCD
PLB **416**, 392 (1998)

ISGW2
PRD **52**, 2783 (1995)
Theory error is not available.

- $|V_{ub}|$ values from $B \to \rho \ell \nu$ and $B \to \omega \ell \nu$ combined with UKQCD or sum rules lie in the range $(3.1 – 3.7) \times 10^{-3}$, consistent with $B \to \pi \ell \nu$

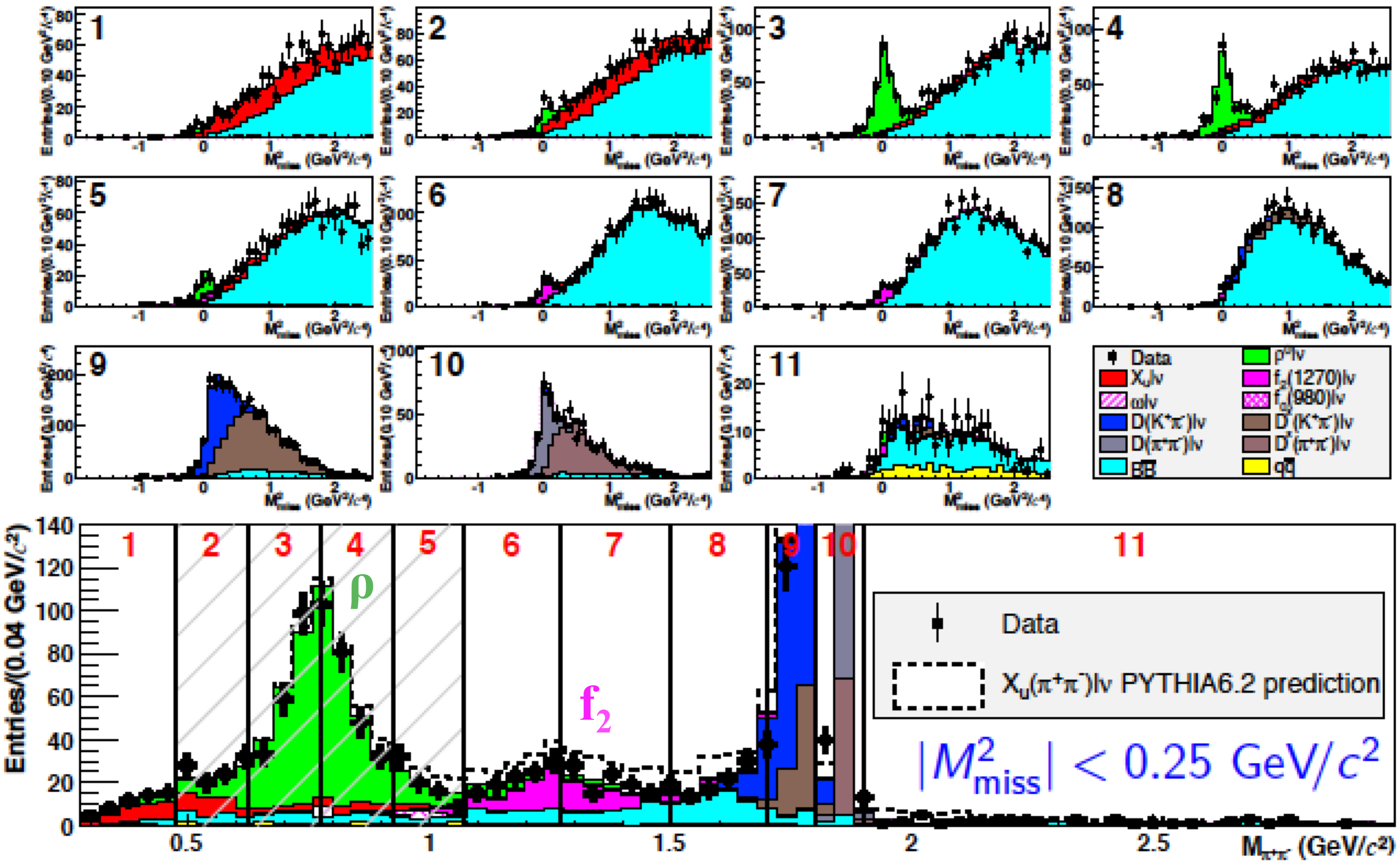
- Results rely on $B \to \rho,\omega$ FF's from sum rules/quenched LQCD/quark models
  ⇒ would be nice to have unquenched LQCD calculations

$B \to X_u(\to\pi\pi)\ell\nu$

[PRD 88, 032005 (2013)]

Non-resonant decays with $X_u = \pi\pi$ are background to $B \to \rho(\to \pi\pi)\ell\nu$

⇒ more detailed study by performing fit to M^2_{miss} in bins of $M_{\pi\pi}$



- $B \to \pi\pi\ell\nu$: No sign of non-resonant decays: $N_{fit} = 46 \pm 46$, $N_{PYTHIA6.2} = 335$
- $B^- \to f_2(1270)\ell^-\nu$: Fitted yield 2-3 times higher than ISGW2 model prediction: $N_{fit} = 154 \pm 22$, $N_{ISGW2} = 58$

Inclusive $B_s \to X\ell\nu$ [PRD 87, 072008 (2013)]

- Tag $B_s^{(*)}\bar{B}_s^{(*)}$ event by reconstructing $D_s^+ \to \phi\pi^+$
- Select same-sign $D_s^+\ell^+$ pairs ($\Rightarrow$ different B_s)
- Measure ratio

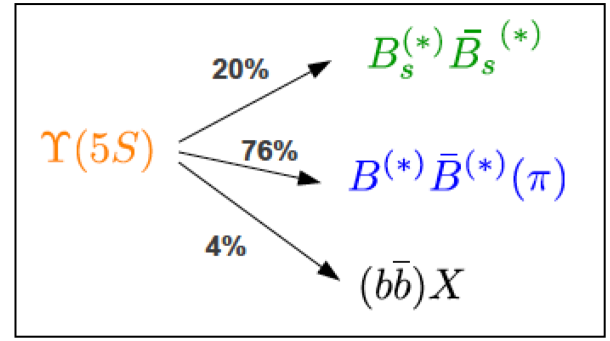
$$\mathcal{R} = \frac{\mathcal{N}(D_s^+\ell^+)}{\mathcal{N}(D_s^+)} \propto \frac{\mathcal{N}(B_s^0 \to \ell)}{\mathcal{N}(B_s^0)} = \mathcal{B}(B_s^0 \to X\ell\nu)$$

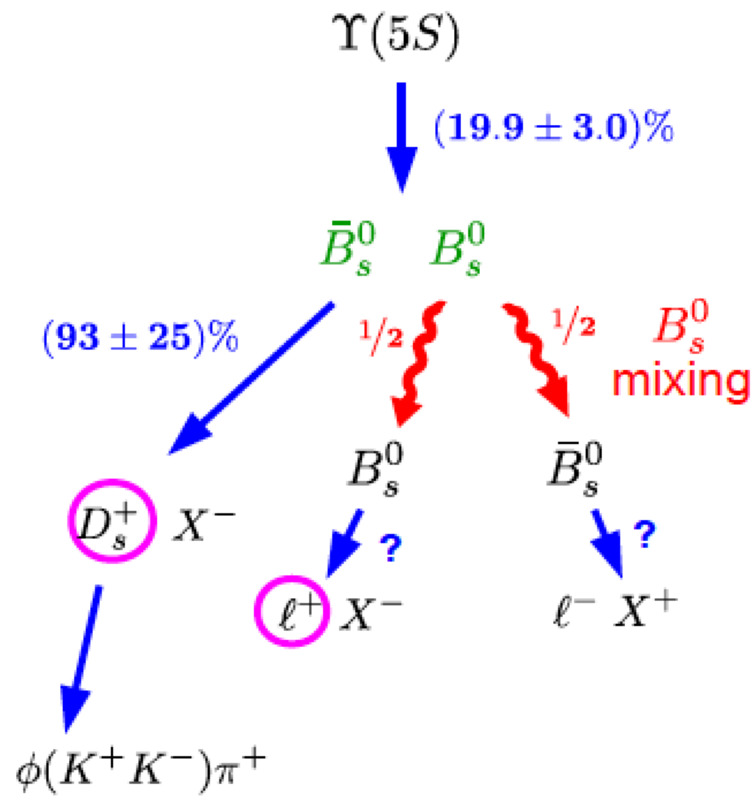
- Selected sample contains background from $B_{u/d}$ decays:

$$\frac{\mathcal{N}(D_s^+\ell^+)}{\mathcal{N}(D_s^+)} = \frac{\mathcal{N}_s(D_s^+\ell^+) + \mathcal{N}_{u,d}(D_s^+\ell^+)}{\mathcal{N}_s(D_s^+) + \mathcal{N}_{u,d}(D_s^+)}$$

measured

external parameters
(B and B_s prod. fraction,
BF's, mixing probabilities, …)





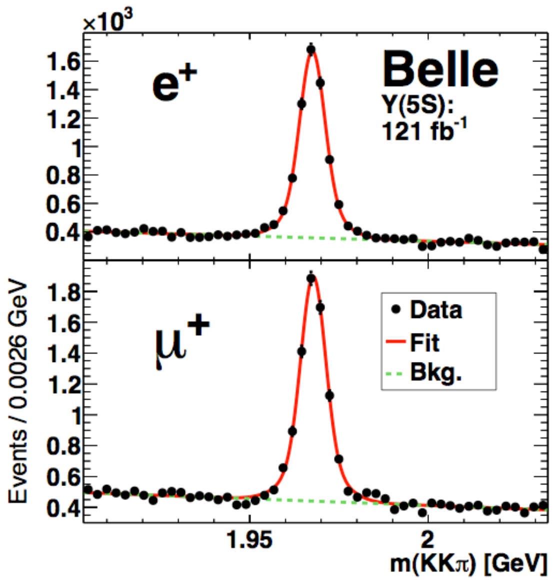
Inclusive $B_s \to X\ell\nu$

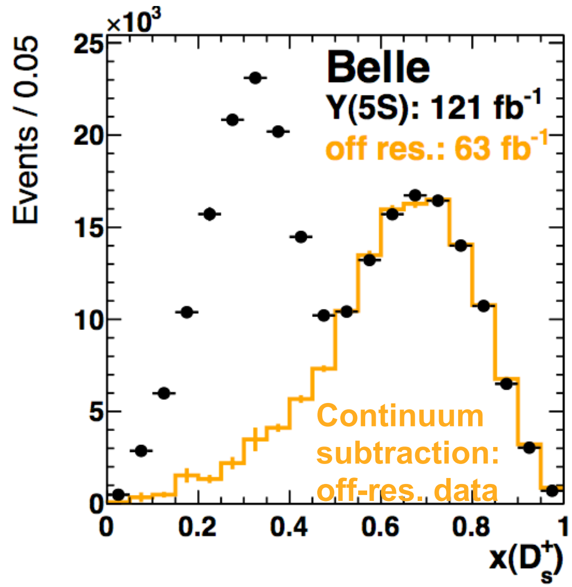
[PRD 87, 072008 (2013)]

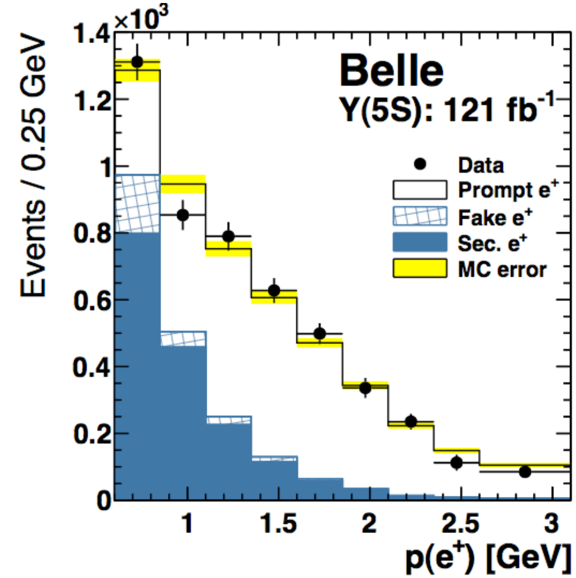
$B_s^0 \to D_s^- X$, $D_s^- \to \phi\, \pi^-$, $\phi \to K^+K^-$

$B_s^0 \to D_s^- X$

$\bar{B}_s^0 \xrightarrow{?} \ell^- X$







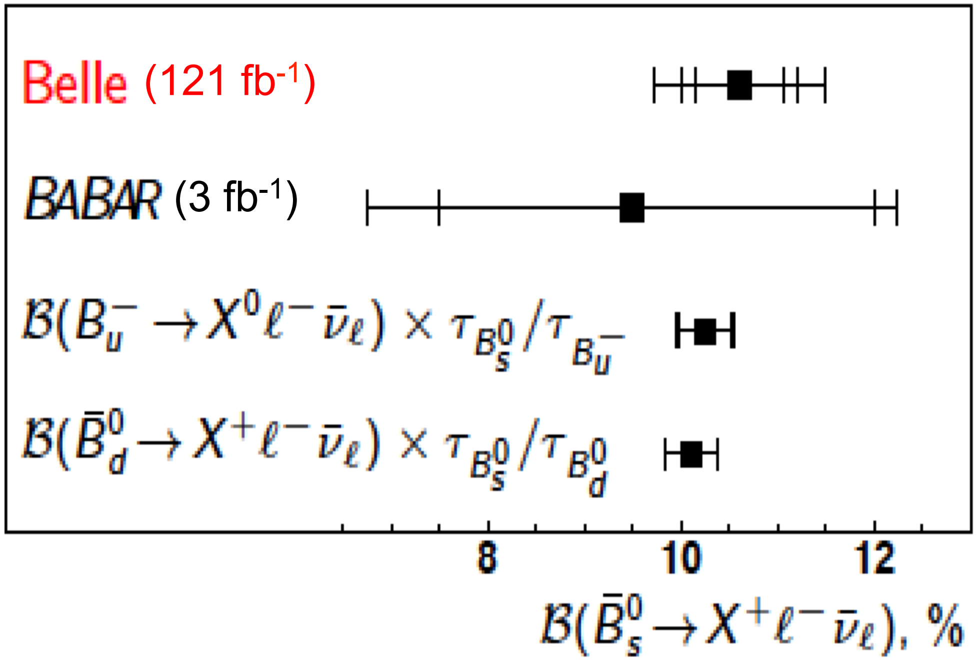
KKπ mass fits
Performed in bins of $x(D_s)$ and $p(\ell)$

$$x(D_s^+) = \frac{p^*(D_s^+)}{p^*_{\max}(D_s^+)} < 0.5$$

Fit to $p(\ell)$ spectrum
- prompt ℓ
- secondary + fake ℓ

Inclusive $B_s \to X\ell\nu$: Branching fraction result

$$\mathcal{B}(B_s^0 \to X^- \ell^+ \nu_\ell) = [10.6 \pm 0.5(\text{stat}) \pm 0.4(\text{syst}) \pm 0.6(\text{ext})]\%$$



Syst. BF uncertainties [%]

Detector effects	1.2
Fitting procedure	2.4
Background modelling	1.8
Signal modelling	1.4
External parameters	6.0
Total systematic	7.0
Statistical	4.2

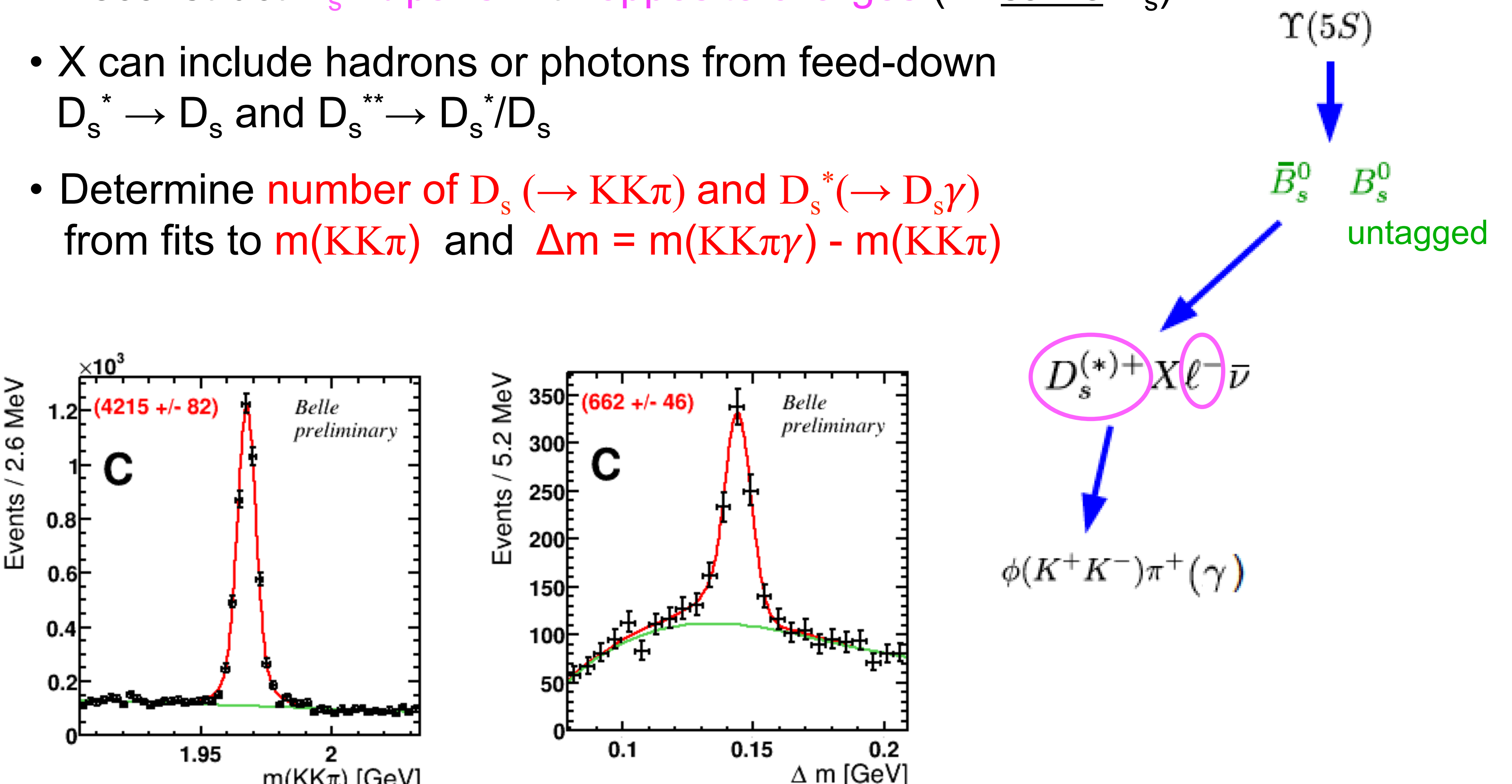
Dominant: B_s production at $\Upsilon(5S)$

Test of SU(3) flavor symmetry: $\Gamma_{\text{SL}}(B_s^0) = (1.04 \pm 0.09) \cdot \Gamma_{\text{SL}}(B^0)$

⇒ with ~9% precision no evidence for SU(3) flavor symmetry breaking

Semi-exclusive $B_s \to D_s^{(*)} X \ell \nu$

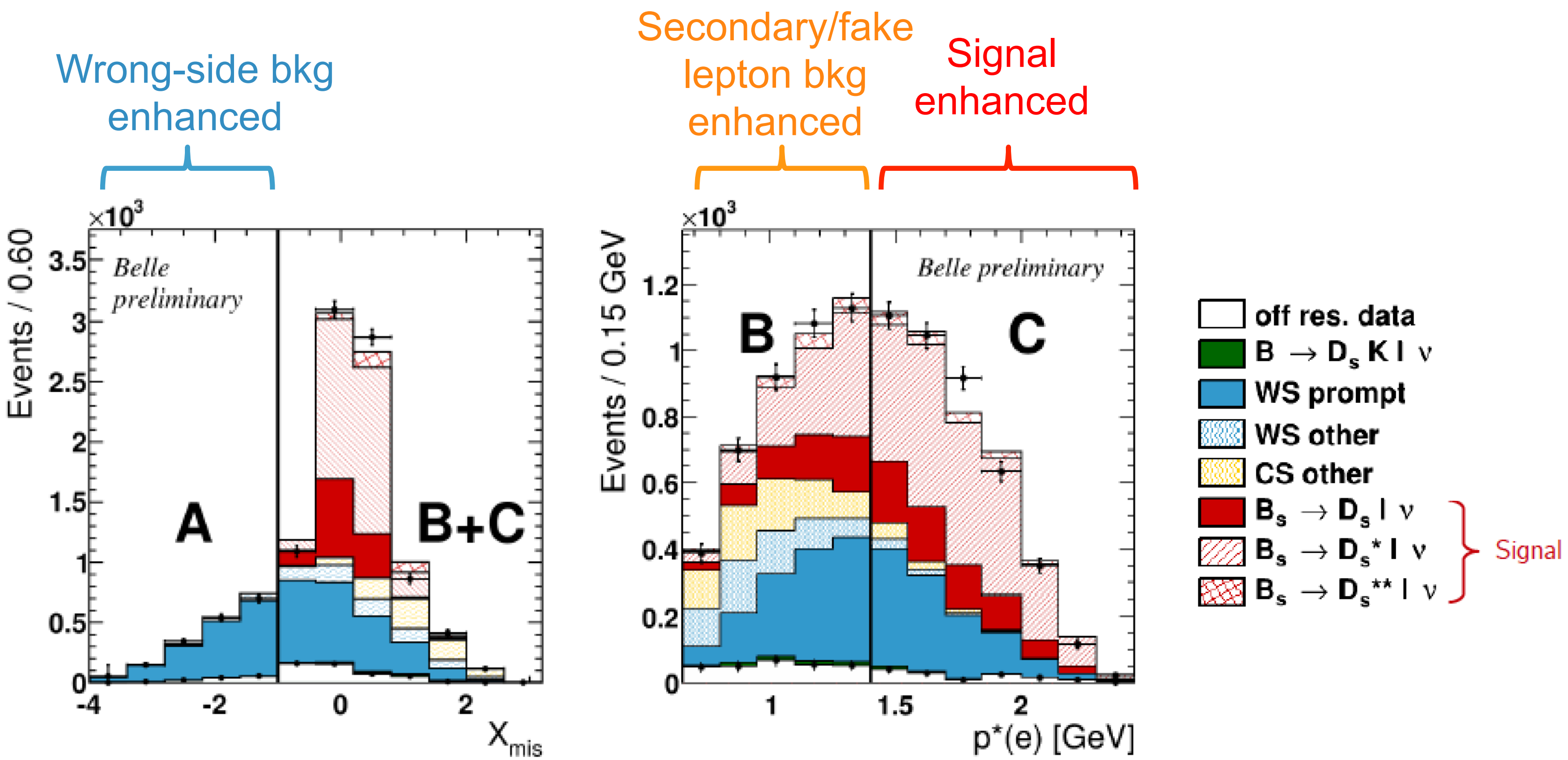
- Untagged analysis (121 fb^{-1})
- Reconstruct $D_s^{(*)}\ell$ pairs with opposite charges (⇒ <u>same</u> B_s)
- X can include hadrons or photons from feed-down $D_s^* \to D_s$ and $D_s^{**} \to D_s^*/D_s$
- Determine number of D_s ($\to KK\pi$) and D_s^*($\to D_s\gamma$) from fits to $m(KK\pi)$ and $\Delta m = m(KK\pi\gamma) - m(KK\pi)$



Semi-exclusive $B_s \to D_s^{(*)} X \ell \nu$

- Determine signal yields from three regions in X_{mis} and $p^*(\ell)$:

$$X_{\text{mis}} = \frac{(E_B^* - E_{\text{vis}}^*) - p_{\text{vis}}^*}{p_B^*}$$



- Event counting in regions A,B,C ⇒ independent of detailed modeling of X_{mis} shape (relative D_s, D_s^*,D_s^{**} composition, B_s^* mass, …)

$B_s \to D_s^{(*)}X\ell\nu$: Branching fraction results

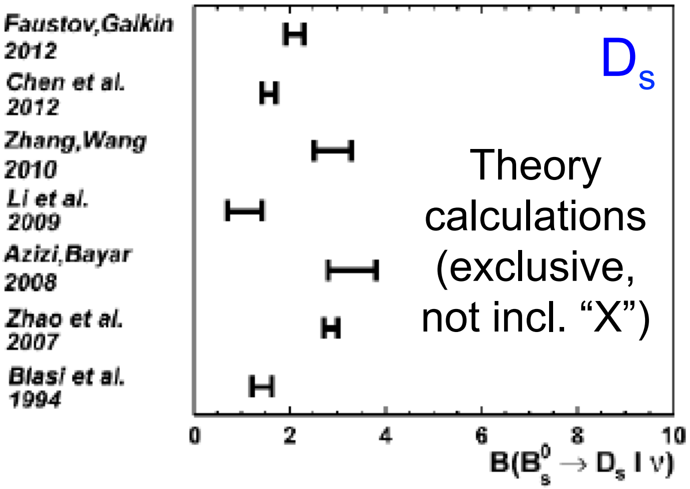
Belle preliminary (121 fb^{-1}):

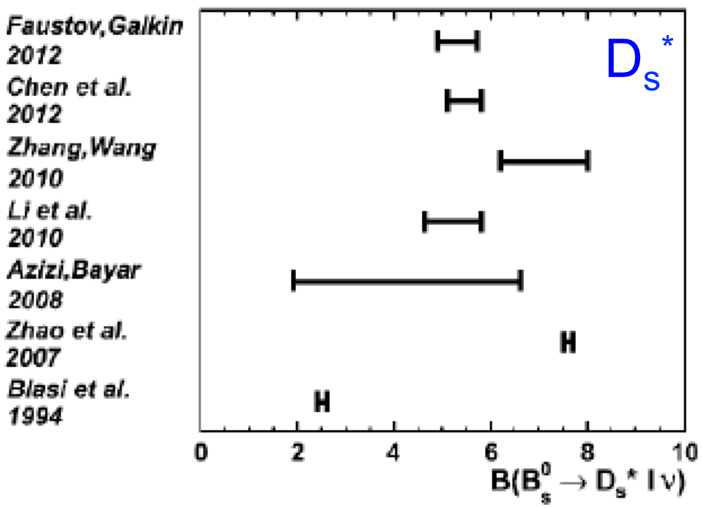
$$\mathcal{B}(B_s \to D_s X e\nu) = (8.2 \pm 0.3 \pm 0.7 \pm 1.5)\%$$
$$\mathcal{B}(B_s \to D_s X \mu\nu) = (8.3 \pm 0.3 \pm 0.8 \pm 1.5)\%$$
$$\mathcal{B}(B_s \to D_s^* X e\nu) = (5.2 \pm 0.6 \pm 0.5 \pm 0.9)\%$$
$$\mathcal{B}(B_s \to D_s^* X \mu\nu) = (5.8 \pm 0.6 \pm 0.6 \pm 1.0)\%$$

Combined:

$$\mathcal{B}(B_s \to D_s X \ell\nu) = (8.2 \pm 0.2 \pm 0.8 \pm 1.5)\%$$
$$\mathcal{B}(B_s \to D_s^* X \ell\nu) = (5.4 \pm 0.4 \pm 0.5 \pm 1.0)\%$$

stat *syst* N_{Bs}





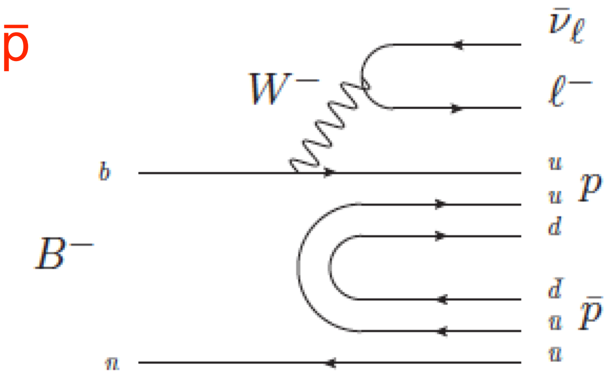
- First step towards understanding composition of semileptonic B_s rate at Belle
- Next step: More detailed investigation of exclusive components

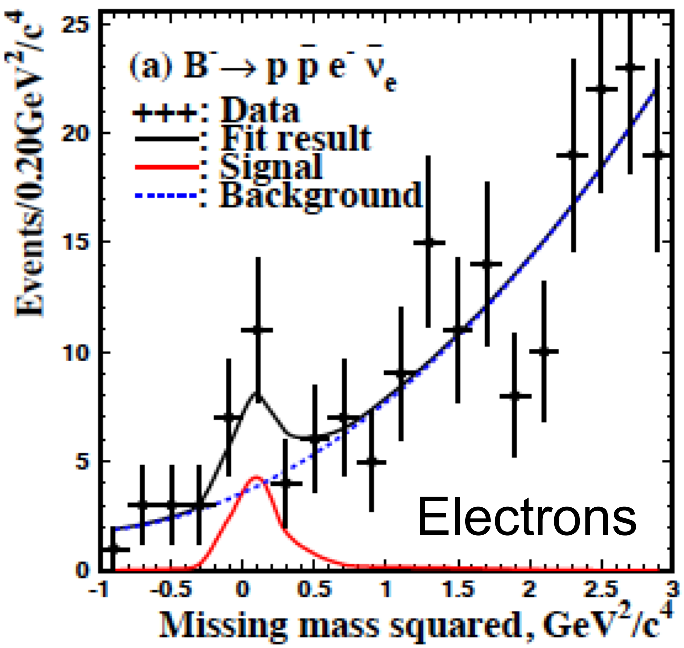
B decay into baryons: $B \to p\bar{p}\ell\nu$ [PRD 89, 011101 (2013)]

- First measurement of semileptonic B decay into $p\bar{p}$
- CLEO set upper limit:

  $\mathcal{B}(B^- \to p\bar{p}\ell^-\nu) < 5.2 \times 10^{-3}$ [PRD 68, 012004 (2003)]
- Recent theory prediction surprisingly large:

  $\mathcal{B}(B^- \to p\bar{p}\ell^-\nu) = (1.04 \pm 0.83) \times 10^{-4}$ [PLB 704, 495 (2011)]





- Belle hadronic-tag analysis (711 fb^{-1})
- Signal extracted from unbinned fit to M^2_{miss}

Mode	$\mathcal{B}$ (10^{-6})	U.L. (10^{-6})
$B^- \to p\bar{p}e^-\bar{\nu}_e$	$8.2\,^{+3.7}_{-3.2} \pm 0.6$	13.8
$B^- \to p\bar{p}\mu^-\bar{\nu}_\mu$	$3.1\,^{+3.1}_{-2.4} \pm 0.7$	8.5
Combined sample	$5.8\,^{+2.4}_{-2.1} \pm 0.9$	9.6

Signal significance: 3.2σ

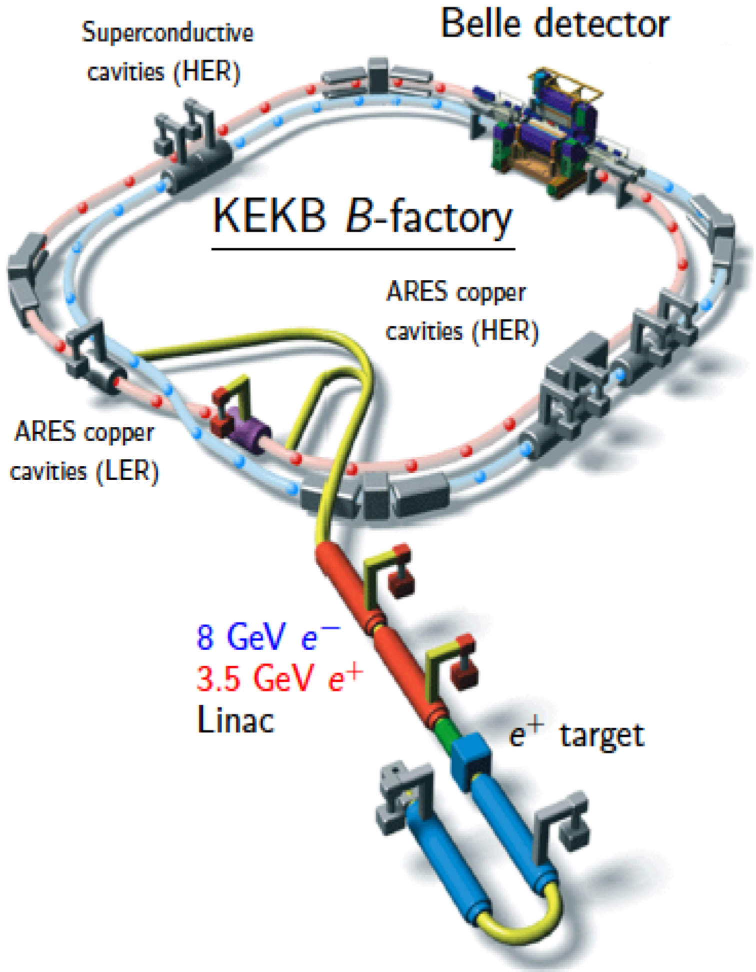
Summary

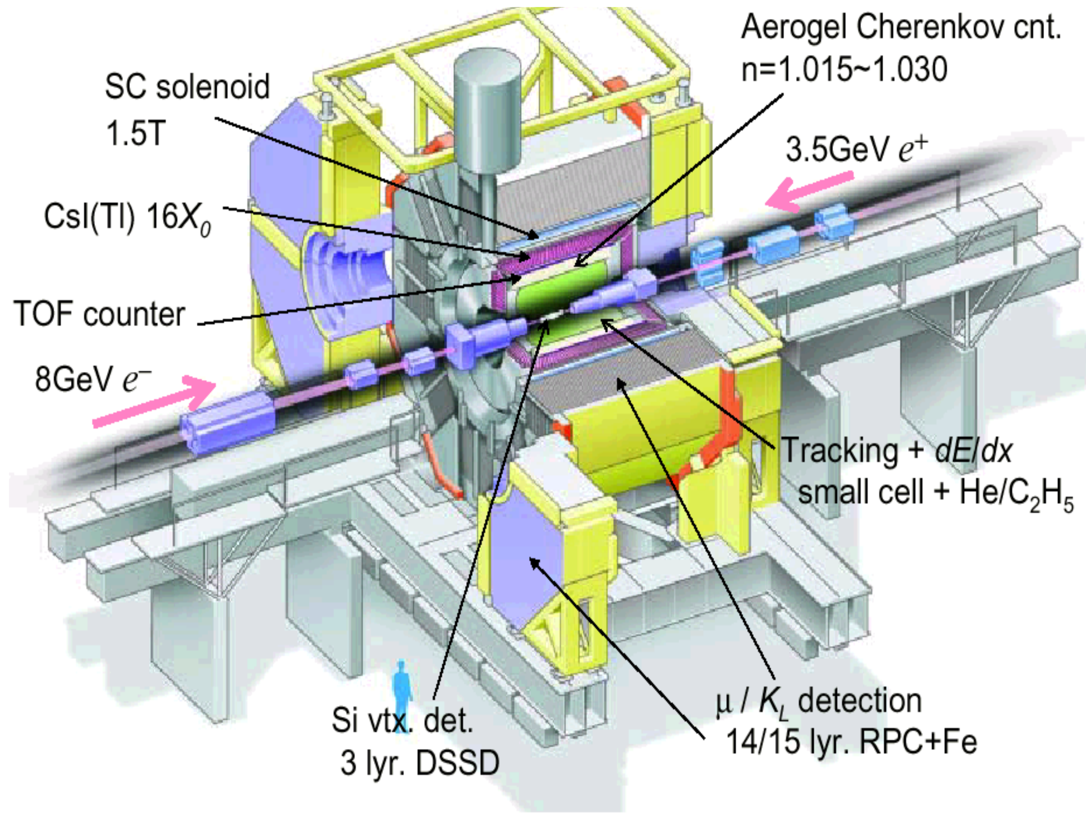
- After end of data-taking in 2010, Belle has continued to produce very interesting physics results
- Persisting difference between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$:
  - New analysis of $B \to D\ell\nu$
    - ⇒ Most precise single measurement
  - Hadronic-tag exclusive analysis of $B \to X_u\ell\nu$
    - ⇒ BF and $|V_{ub}|$ from $B \to \pi\ell\nu$ and higher-mass charmless states
- Most precise measurement of inclusive semileptonic B_s decays and first analysis of semi-exclusive semileptonic B_s decays at Belle
- Observed first evidence of semileptonic B decay into baryons: $B \to p\bar{p}\ell\nu$

Stay tuned for more interesting results from Belle (and Belle II in the future)!

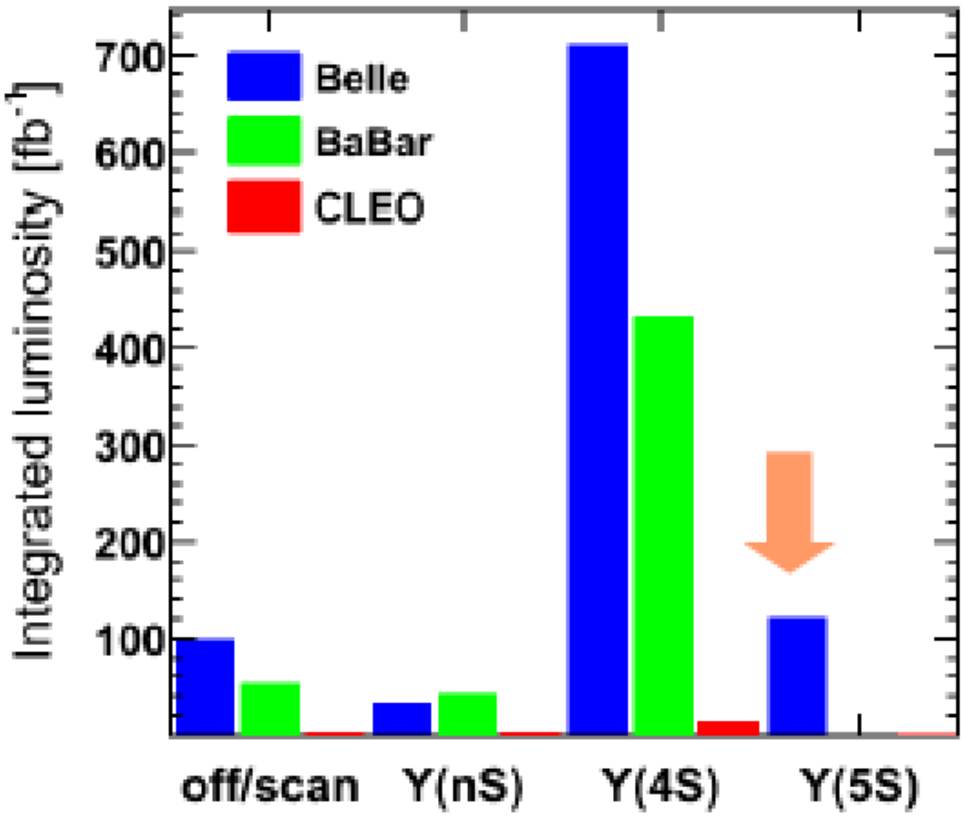
Backup slides

KEKB and Belle





Datasets of Belle, BaBar and CLEO



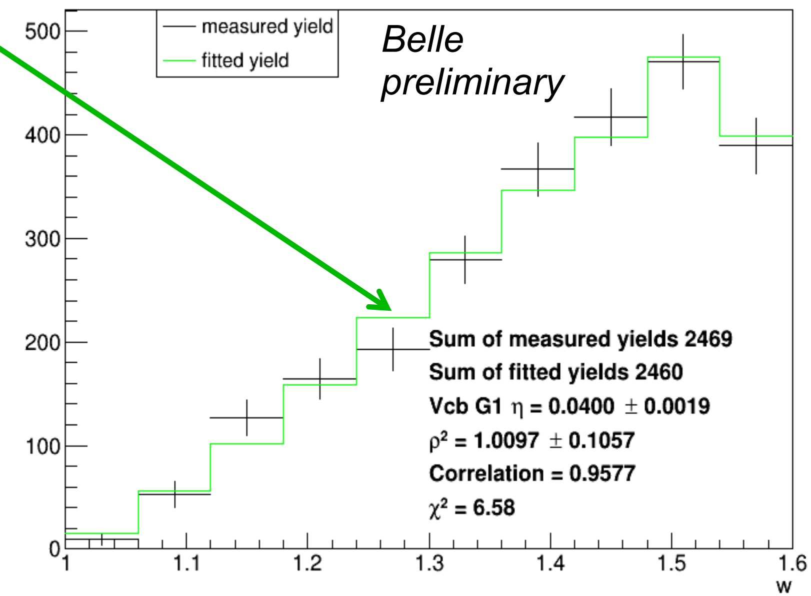
$B \to D\ell\nu$: $|V_{cb}|G(1)$ and ρ^2

- Determine $|V_{cb}|G(1)$ and ρ^2 from χ^2 fit of predicted to measured yields:

$$N_i = \frac{\int_{w_i}^{w_i+\Delta w} \frac{d\Gamma(\eta_{EW} G(1)|V_{cb}|,\rho^2)}{dw} dw}{\int_{w_i}^{w_i+\Delta w} \frac{d\Gamma_{MC}}{dw} dw} \mathrm{N}_{\mathrm{MC},i}$$

Predicted signal in bin i

differential decay width using MC parameters



- Branching fraction results:

$$\mathcal{B}(\bar{B}^0 \to D^+\ell^-\bar{\nu}_\ell) = [2.49 \pm 0.17]\%$$
$$\mathcal{B}(B^- \to \bar{D}^0\ell^-\bar{\nu}_\ell) = [2.70 \pm 0.19]\%$$

Sample	$\eta_{EW}\mathcal{G}(1)\|V_{cb}\|$ $[10^{-3}]$	ρ^2	correlation
$\bar{B}^0 \to D^+e^-\bar{\nu}_\ell$	$40.01 \pm 1.89(\mathrm{stat}) \pm 1.66(\mathrm{syst})$	$1.010 \pm 0.106(\mathrm{stat}) \pm 0.029(\mathrm{syst})$	0.692
$\bar{B}^0 \to D^+\mu^-\bar{\nu}_\ell$	$40.66 \pm 2.07(\mathrm{stat}) \pm 1.70(\mathrm{syst})$	$1.075 \pm 0.115(\mathrm{stat}) \pm 0.031(\mathrm{syst})$	0.713
$B^- \to \bar{D}^0e^-\bar{\nu}_\ell$	$43.70 \pm 1.86(\mathrm{stat}) \pm 1.67(\mathrm{syst})$	$0.909 \pm 0.099(\mathrm{stat}) \pm 0.014(\mathrm{syst})$	0.711
$B^- \to \bar{D}^0\mu^-\bar{\nu}_\ell$	$46.73 \pm 1.87(\mathrm{stat}) \pm 1.79(\mathrm{syst})$	$1.075 \pm 0.091(\mathrm{stat}) \pm 0.014(\mathrm{syst})$	0.680
Average	$42.63 \pm 0.96(\mathrm{stat}) \pm 1.39(\mathrm{syst})$	$1.001 \pm 0.051(\mathrm{stat}) \pm 0.018(\mathrm{syst})$	0.494

Semileptonic B_s decays: Estimation of D_s and $D_s\ell$ yields

$B_s^{0(*)}\bar{B}_s^{0(*)}$

$$N_s(D_s^+)/N_{b\bar{b}} = 2 \cdot f_s \cdot \mathcal{B}(B_s^0 \to D_s^\pm X)$$

$$N(D_s^+\ell^+)/N_{b\bar{b}} = 2 \cdot f_s \cdot \mathcal{B}(B_s^0 \to X^-\ell^+\nu) \cdot (1-\chi_s) \cdot \mathcal{B}(B_s^0 \to D_s^\pm X)$$

$B_{u,d}^{(*)}\bar{B}_{u,d}^{(*)}(\pi)$

$$N_{ud}(D_s^+)/N_{b\bar{b}} = 2 \cdot f_d \cdot \mathcal{B}(B^0 \to D_s^\pm X) + 2 \cdot f_u \cdot \mathcal{B}(B^+ \to D_s^\pm X)$$

$$N_{ud}(D_s^+\ell^+)/N_{b\bar{b}} =$$

$$2 \cdot \frac{f_d}{f_{ud}} \cdot \underbrace{\begin{array}{l}\left[F_{B\bar{B}} + F_{B^*\bar{B}^*} + \frac{1}{3}(f_{ud}-F_2)\cdot(F'_{B\bar{B}\pi} + F'_{B^*\bar{B}^*\pi}) + (f_{ud}-F_2)\cdot(1-F'_3)\right] \cdot \\ \left\{\chi_d^{(-)} \cdot \mathcal{B}(B^0 \to D_s^+ X) + \left(1-\chi_d^{(-)}\right) \cdot \mathcal{B}(B^0 \to D_s^- X)\right\} \cdot \mathcal{B}(B^0 \to X^-\ell^+\nu_\ell)\end{array}}_{B^{0(*)}\bar{B}^{0(*)}\text{ pairs, } \mathcal{C}\text{ even}}$$

$$+\ 2 \cdot \frac{f_d}{f_{ud}} \cdot \underbrace{\begin{array}{l}\left[F_{B^*B} + \frac{1}{3}(f_{ud}-F_2)\cdot F'_{B^*\bar{B}\pi})\right] \cdot \\ \left\{\chi_d^{(+)} \cdot \mathcal{B}(B^0 \to D_s^+ X) + \left(1-\chi_d^{(+)}\right) \cdot \mathcal{B}(B^0 \to D_s^- X)\right\} \cdot \mathcal{B}(B^0 \to X^-\ell^+\nu_\ell)\end{array}}_{B^0\bar{B}^{0*}\text{ pairs, } \mathcal{C}\text{ odd}}$$

$$+\ 2 \cdot \frac{f_u}{f_{ud}} \cdot \underbrace{\left[F_2 + \frac{1}{3}(f_{ud}-F_2)\cdot F'_3 + (f_{ud}-F_2)\cdot(1-F'_3))\right] \cdot \mathcal{B}(B^+ \to D_s^- X) \cdot \mathcal{B}(B^+ \to X\ell^+\nu_\ell)}_{B^{+(*)}B^{-(*)}\text{ pairs}}$$

$$+\ \underbrace{\begin{array}{l}\left[\frac{2}{3}\cdot(f_{ud}-F_2)\cdot F'_3\right] \cdot \\ \left(\left\{\chi_d^{(-)} \cdot \mathcal{B}(B^0 \to D_s^+ X) + \left(1-\chi_d^{(-)}\right) \cdot \mathcal{B}(B^0 \to D_s^- X)\right\} \cdot \mathcal{B}(B^+ \to X\ell^+\nu_\ell) + \right. \\ \left.\left\{\chi_d^{(-)} \cdot \mathcal{B}(B^+ \to D_s^+ X) + \left(1-\chi_d^{(-)}\right) \cdot \mathcal{B}(B^+ \to D_s^- X)\right\} \cdot \mathcal{B}(B^0 \to X^-\ell^+\nu_\ell)\right)\ .\end{array}}_{B^{+(*)}B^{0(*)}\text{ and } B^{-(*)}B^{0(*)}\text{ pairs}}$$

Semi-exclusive $B_s \to D_s^{(*)} X \ell \nu$