The Higgs Boson and Electroweak Symmetry Breaking

2. Decays of the Higgs Boson

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September 2014

Now that we have discovered the Higgs boson, we have a chance to study its properties.

In my opinion, the most interesting measurable quantities associated with the Higgs boson are its partial widths to its various decay modes.

In this lecture, I will discuss these quantities one by one, giving some basic orientation, then the state of the SM theory, and then examples of new physics effects.

The partial widths are the simplest quantities to analyze theoretically, but they are not directly observable. Rather,

$$\Gamma(h \to A\overline{A}) = \Gamma_h \cdot BR(h \to A\overline{A})$$

where Γ_h is the Higgs total width. I will discuss at the end of the lecture how to resolve this ambiguity.

The dominant SM Higgs decay is $h \to b\bar{b}$. Consider more generally $h \to f\overline{f}$.

It is easy to work out the tree-level decay width. The result is:

$$\Gamma(h \to f\overline{f}) = \frac{\alpha_w}{8} m_h \frac{m_f^2}{m_W^2} \left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2}$$

Now, for decays to leptons, we are essentially done.

$$\Gamma(h \to \tau^+\tau^-) = 260 \text{ keV}$$

$$\Gamma(h \to \mu^+\mu^-) = 9 \text{ keV}$$

For quarks, we also need to address the question of what value of the quark mass to use in evaluating this formula.

Quark masses in QCD run with energy scale, according to

$$m_f(Q) = m_f(m_f)\left[\frac{\alpha_s(Q)}{\alpha_s(m_f)}\right]^{4/b_0} \qquad b_0 = (33 - 2n_f)/3$$

(to leading order). To evaluate the Higgs decay widths, we should use $\overline{MS}$ masses evaluated at a scale close to m_h .

These are not your familiar quark mass values. Instead,

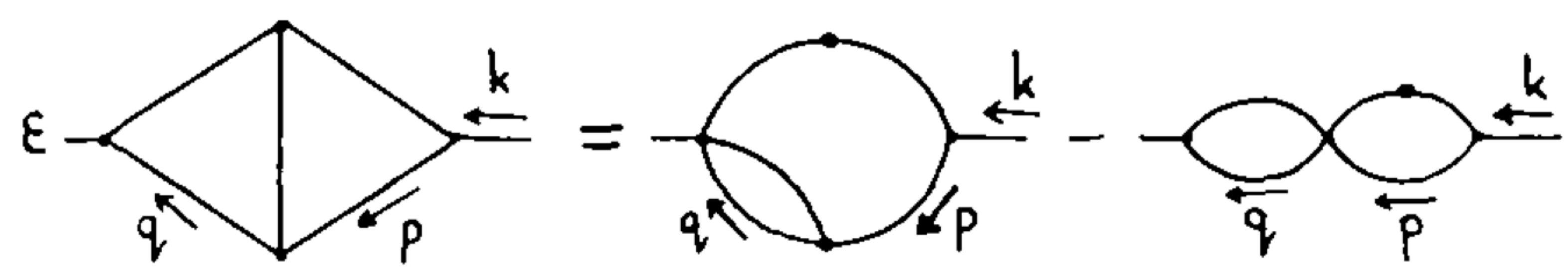
m_u	m_d	m_s	m_c	m_b
1.5	3	60	750	3000

In addition, there is a large QCD correction

$$\left[1 + \frac{17}{3\pi}\alpha_s(m_h) + \cdots\right]$$

Much work has been done, mainly here in Germany, to bring these predictions to a high level of precision.

For QCD, what is required is the evaulation of a 2-pt function.   There is a technique of integration by parts, invented by Chetyrkin and Tkachov, that allows any QCD 2-pt function with massless internal legs to be evaluated analytically.   A simple example is



The Karlsruhe group (Baikov, Chetyrkin, Kuhn) have brought this technology to a high art.

For top quark loops, they expand in $m_h^2/4m_t^2$. This expansion can be carried out to high order.

Currently, the QCD corrections to $\Gamma(h \to q\bar{q})$ have been worked out to order α_s^4 .

Let $a = \alpha_s(m_h)/\pi$. Then (Baikov, Chetyrkin, Kuhn)

$$
\begin{aligned}
\tilde{R} &= 1 + 5.667a + 29.15a^2 + 41.76a^3 - 825.7a^4 \\
&= 1 + 0.2037 + 0.0377 + 0.0019 - 0.0013 \ ,
\end{aligned}
$$

The electroweak corrections are known at leading order, and the leading 2-loop corrections have been computed.

$$
\delta\Gamma = 0.3\% - 0.02\% + 0.05\%
$$

$\mathcal{O}(\alpha a m_t^2/m_h^2)$ $\mathcal{O}(\alpha^2 m_t^4/m_h^4)$

Kwiatkowski-Steinhauser Butenschoen-Fugel-Kniehl

Kniehl-Spira

It would be an excellent Ph.D. thesis (maybe two) to complete the 2-loop electroweak corrections and bring this theory to the 0.1% level.

In models of the Higgs sector more general than the Standard Model, these couplings can be modified. The modifications might be either at the tree or loop level.

In a model with two Higgs doublets, the physical states are mixtures of the two fields

mixing angle $\begin{array}{lll} \alpha : & h^0, H^0 & \\ \beta : & \pi^0, A^0 & \pi^\pm, H^\pm \end{array}$ $\tan\beta = v_u/v_d$

Then the coupling modifications are

$$g(b\overline{b}) = -\frac{\sin\alpha}{\cos\beta}\frac{m_b}{v} \qquad g(c\overline{c}) = \frac{\cos\alpha}{\sin\beta}\frac{m_c}{v}$$

Unfortunately, in full models such as SUSY, the two angles are not independent. In fact, typically,

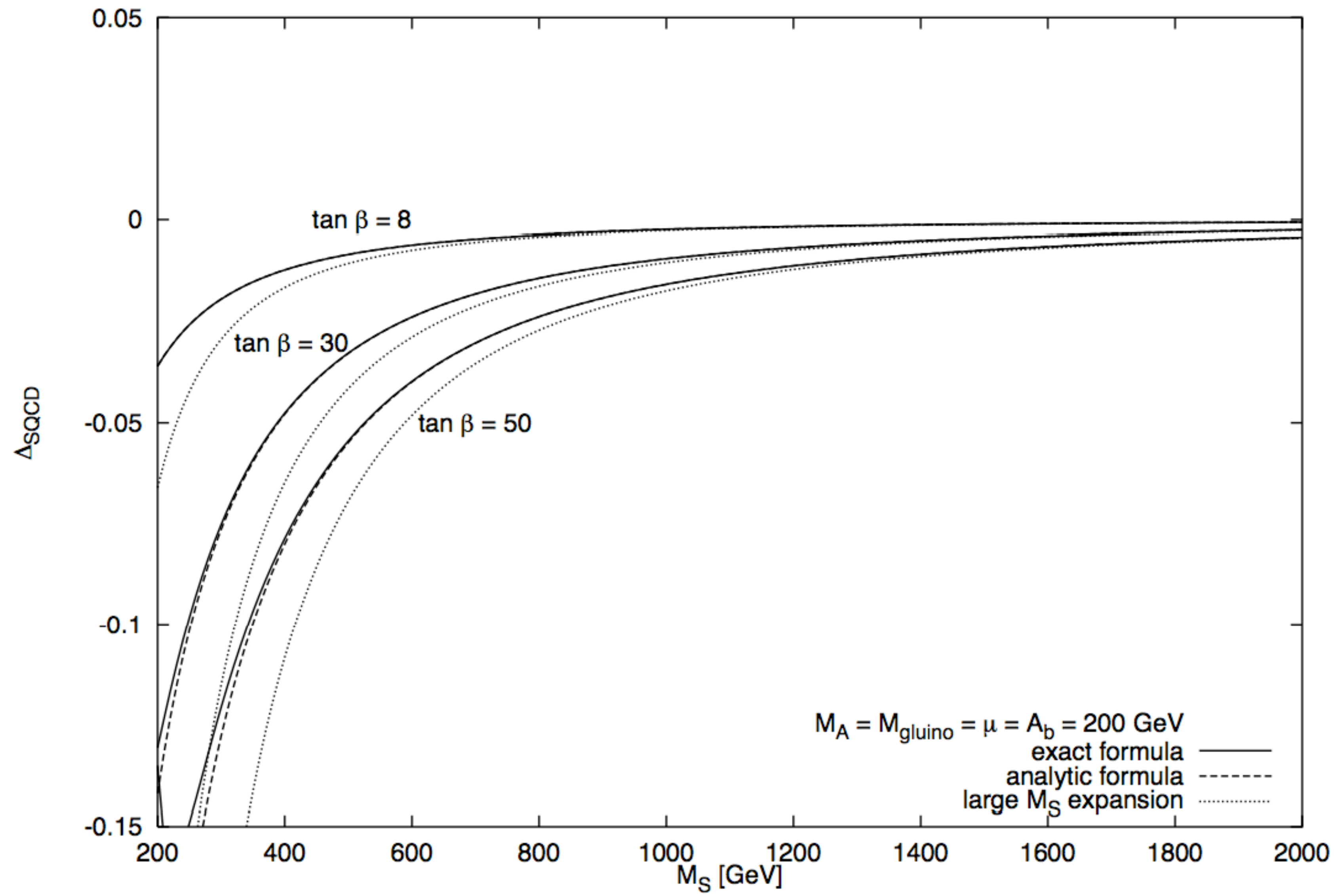
$$-\frac{\sin\alpha}{\cos\beta} = 1 + \mathcal{O}(\frac{m_Z^2}{m_A^2})$$

Then, typically, the corrections decrease as the SUSY mass scale becomes larger, for example

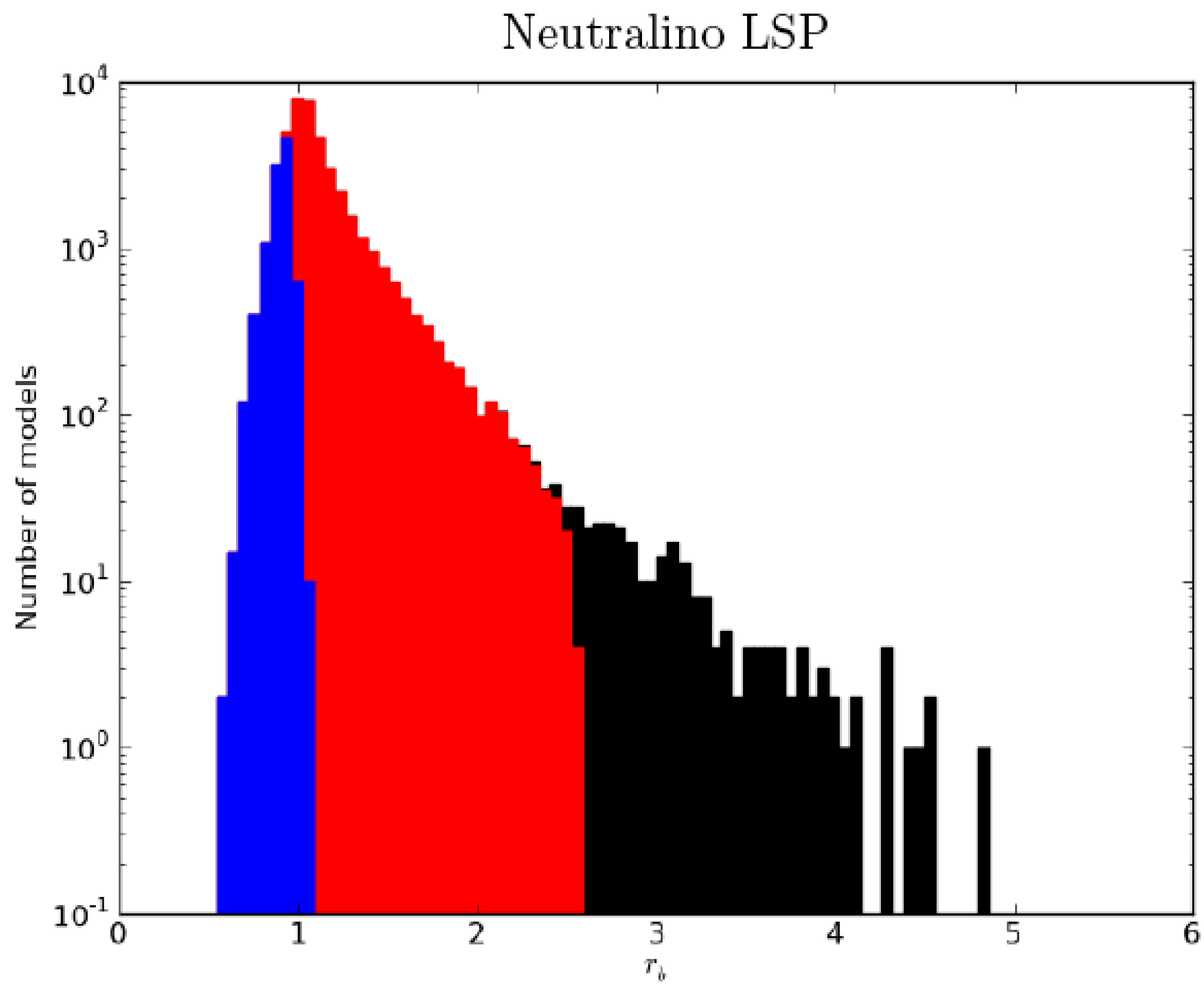
$$\frac{g_{hbb}}{g_{h_{\rm SM}bb}} = \frac{g_{h\tau\tau}}{g_{h_{\rm SM}\tau\tau}} \simeq 1 + 40\% \left(\frac{200\ \text{GeV}}{m_A}\right)^2$$

Loop with b,t squarks and gluinos can also modify this vertex, especially at large tan β.

Haber, Herrero, Logan, Penaranda, Rigolin, Temes



In a large survey of SUSY parameter points: (pMSSM) by Cahill-Rowley, Hewett, Ismail, Rizzo:



In fact, the small size of corrections to the Higgs couplings is general. It is explained by the "Decoupling Theorem" of Howard Haber:

Consider a model consisting of the Standard Model plus new particles at a mass scale M, with

$$M \gg m_h, m_t$$

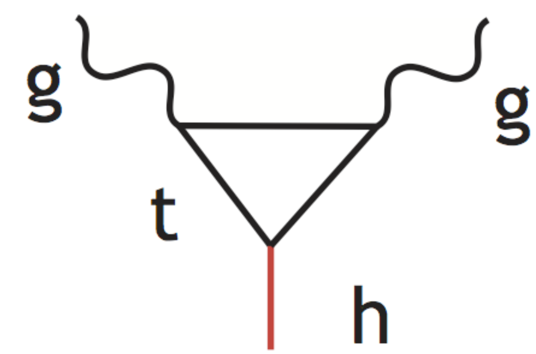
Then we can integrate out the heavy particles and represent their effects by higher-dimension operators in an effective theory.

The leading term in this theory is the SM result. This is fixed by the value of m_h .

The corrections to this result are of order m_h^2/M^2.

Next, discuss the loop-induced decays: $h \to gg, \gamma\gamma, \gamma Z$

Begin with the decay to gluons. In the SM, this goes through the diagrams



$+(g_1 \leftrightarrow g_2)$

By gauge invariance, this must be of the form

$$A(k_1 \cdot k_2 g^{\mu\nu} - k_1^{\nu} k_2^{\mu})\delta^{ab}$$

To work out A, consider the limit

$$m_h^2 \to 0$$

Then this diagram becomes a vacuum polarization diagram that corrects the QCD coupling.

The contribution of one quark to the QCD coupling renormalization is

$$(k^2 g^{\mu\nu} - k^\mu k^\nu) \cdot \frac{\alpha_s}{6\pi} \delta^{ab} \log \frac{\Lambda^2}{m_t^2}$$

Now, $m_t^2 \sim v^2 \to v^2(1 + 2h/v)$

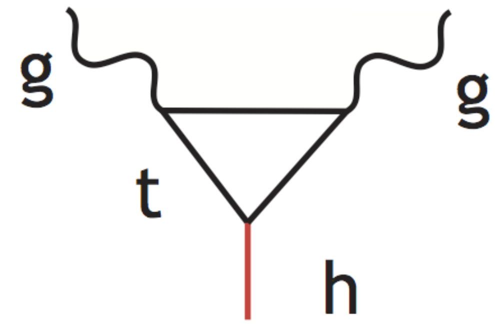
and we can read off:

$$A = \frac{\alpha_s}{3\pi} \frac{1}{v}$$

This gives for the partial width, in the limit $m_h^2 \ll 4m_t^2$.

$$\Gamma(h \to gg) = \frac{\alpha_w \alpha_s^2}{72\pi^2} \frac{m_h^3}{m_W^2}$$

It is odd that this answer is independent of the mass of the top quark. But, actually, it is not so hard to understand. In



the Higgs vertex is proportional to m_t, and the loop diagram has dimensions

$$\mathrm{mass}^{-1} \sim 1/m_t$$

For the loop contribution from a light quark, the Higgs coupling is proportional to the quark mass, and the loop diagram is of order $1/m_h$, thus the amplitude is of order

$$m_q/m_h$$

So only quarks heavier than the Higgs contribute to this process. The observed rate of Higgs production already tells us that there are no 4th generation quarks with chiral couplings.

It is straightforward to include the full dependence on $m_h/4m_t^2$. It is not so straightforward to compute higher orders in α_s, but this has been done. The result, due to Schreck and Steinhauser, Baikov and Chetyrkin, and Moch and Vogt, is

$$\begin{aligned}\frac{\Gamma}{\Gamma_0} &= 1.0671 + 19.306a + 172.76a^2 + 467.68a^3 \\ &= 1.0671 + 0.6942 + 0.2234 + 0.0217\end{aligned}$$

The electroweak correction is +5%, due to Actis, Passarino, Sturm, and Uccirati; this is already a 2-loop analysis.

The $h \to gg$ amplitude can receive corrections from any heavier colored particle that receives mass from the Higgs boson.

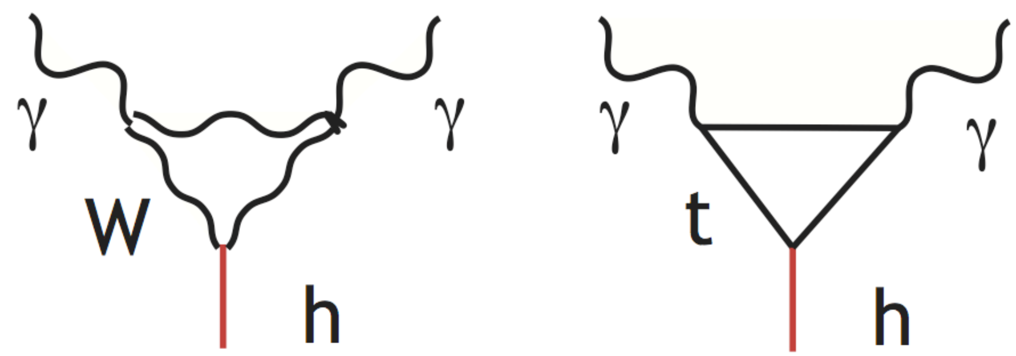
We have seen that a 4th generation gives effects that are already too large.

However, models with extra dimensions or Higgs compositeness contain new heavy quarks that couple in a vectorlike way to the SM interactions. These quarks obtain some contribution to their mass through mixing with the t and b quarks. One estimate of the effect gives

$$\frac{g_{hgg}}{g_{h_{\rm SM}gg}} \simeq 1 + 2.9\% \left(\frac{1\ \mathrm{TeV}}{m_T}\right)^2$$

Again, we see the effect of decoupling.

The theory of the decay $h \to \gamma\gamma$ can be worked out in the same way. The leading order diagrams are



Again, we can evaluate the diagrams in the limit $m_h \to 0$
The contribution to the electromagnetic coupling constant renormalization from the top quark is

$$(k^2 g^{\mu\nu} - k^\mu k^\nu) \cdot \frac{3Q_f^2 \alpha}{3\pi} \delta^{ab} \log \frac{\Lambda^2}{m_t^2}$$

The W contribution has the opposite sign. It is proportional to the β function coefficient in SU(2). In all

$$(k^2 g^{\mu\nu} - k^\mu k^\nu) \cdot \frac{\alpha}{3\pi} \left[\frac{22}{4} - \frac{1}{4} - 3(\frac{2}{3})^2 \right] \delta^{ab} \log \frac{\Lambda^2}{v^2}$$

The corrections to the leading result again require 2-loop diagrams. The relative corrections are

$$-1.6\% + 1.8\% + 0.08\%$$

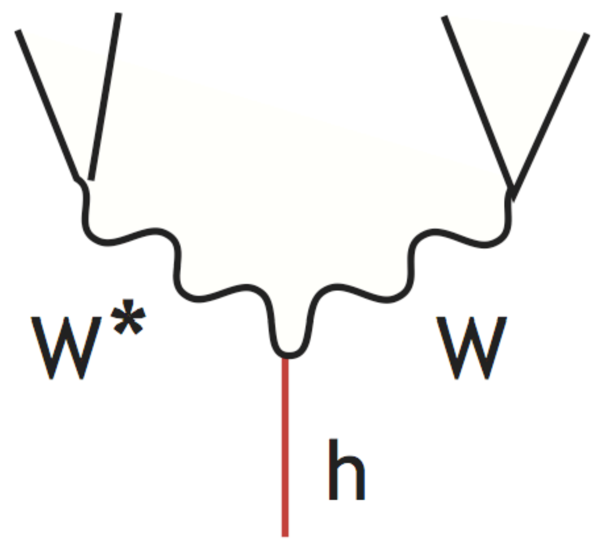
Passarino, Sturm, Uccirati

Maierhofer and Marquard

for the electroweak and QCD corrections, respectively.

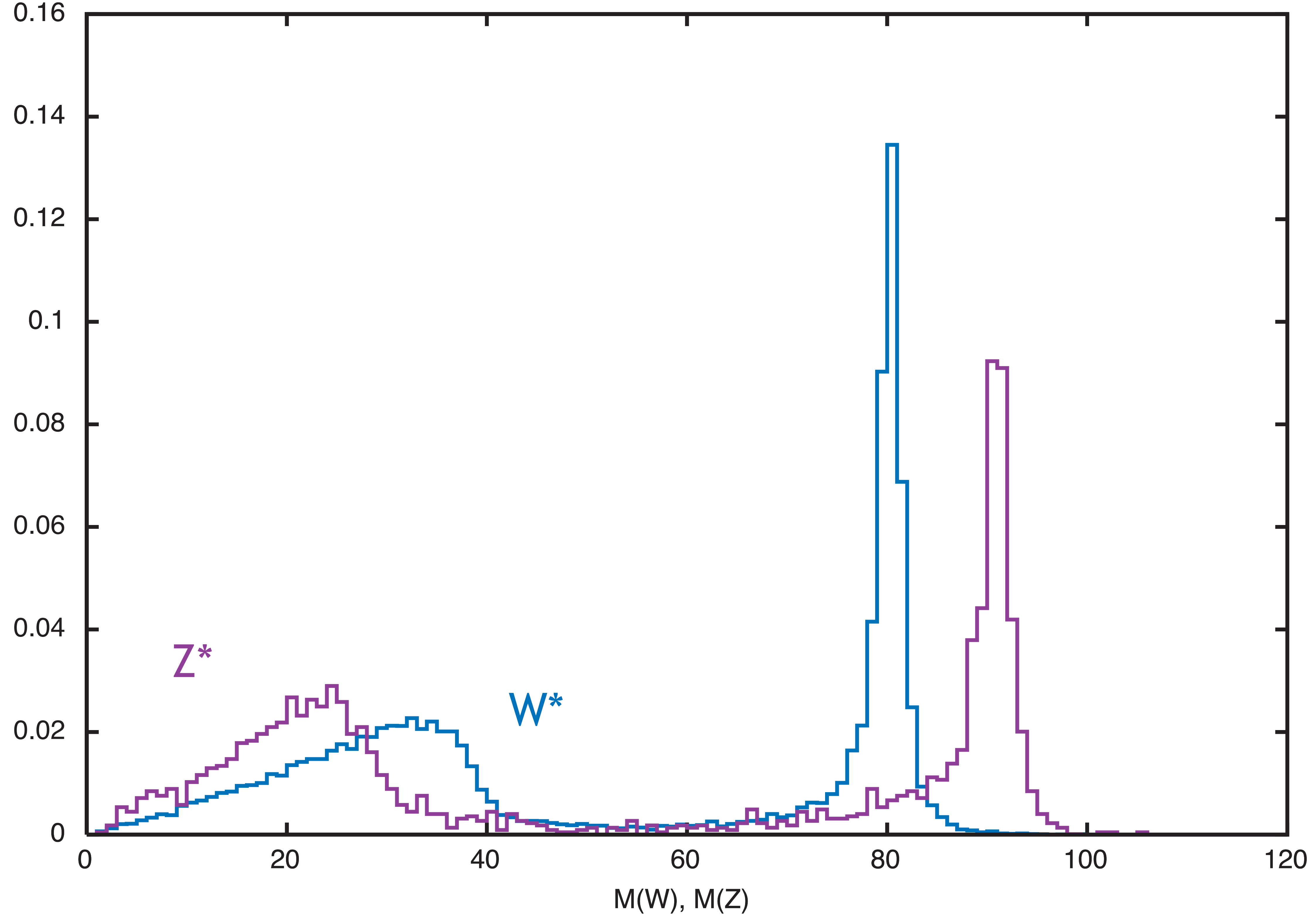
Beyond the SM, the coupling can receive corrections from heavy vectorlike quarks and leptons. These corrections are the few % level for new fermions with masses of order 1 TeV.

Finally, though the Higgs boson cannot decay to on-shell W and Z bosons, it can decay to off-shell W and Z bosons which then materialize as quark or lepton pairs



The rate depends strongly on the mass of the Higgs boson. At 126 GeV, the decay to WW* is competitive with the decay to $b\overline{b}$, while the decay to ZZ* is a factor of 10 smaller.

2-jet mass distributions in h→WW*, ZZ* decays
m(h) = 120 GeV



The 1-loop corrections to these decays have been computed by Bredenstein, Denner, Dittmaier, and Weber and incorporated into an event generator prophecy4f.

The corrections can be large. However, there is an approximate description called the Improved Born Approximation (IBA), which includes

- QCD corrections separately for the two Ws or Zs
- loop corrections to the hWW, hZZ vertex
- Coulomb corrections for the WW system

The full result agrees with this to 1%. The IBA result has been evaluated to 2 loops by Kniehl and Veretin.

It seems very challenging to improve this theory to 0.1%. The needs a full 2-loop analysis. However, it will be very important to do so.

These decay widths also depend strongly on the mass of Higgs boson:

$$\delta_W = 6.9 \cdot \delta m_h \ , \quad \delta_Z = 7.7 \cdot \delta m_h$$

This is a 0.2% uncertainty for $\Delta m_h = 30 \ \mathrm{MeV}$.

This is the primary motivation (in my opinion) for a very accurate Higgs mass measurement.

The hWW and hZZ coupling can obtain corrections from a number of sources outside the SM.

Mixing of the Higgs with a singlet gives corrections

$$g(hVV) \sim \cos\phi \sim (1 - \phi^2/2)$$

These might be most visible in the hVV couplings. Similarly, field strength renormalization of the Higgs can give 1% level corrections (Craig and McCullough).

If the Higgs is a composite Goldstone boson, these couplings are corrected by (f ~ 1 TeV)

$$g(hVV) = (1 - v^2/f^2)^{1/2} \approx 1 - v^2/2f^2 \approx 1 - 3\%$$

Corrections due to an extended Higgs sector are very suppressed:

$$g(hVV) = 1 + \mathcal{O}(\frac{m_Z^4}{m_A^4})$$

There is one more question to ask about the possibility of precision SM predictions for the Higgs couplings. Most of these couplings depend on SM inputs such as m_b and α_s . The most important dependences are: ($\delta_A = \Delta\Gamma(A)/\Gamma(A)$)

$$\delta_b = 1. \cdot \delta m_b(10) \oplus (-0.28) \cdot \delta\alpha_s(m_Z)$$

$$\delta_c = 1. \cdot \delta m_c(3) \oplus (-0.80) \cdot \delta\alpha_s(m_Z)$$

$$\delta_g = 1.2 \cdot \delta\alpha_s(m_Z)$$

Thus, we also need to know these quantities to the 0.1% level. Is it possible.

Many of the best determinations of these quantities now come from Lattice QCD. Mackenzie, Lepage, and I projected the errors from Lattice QCD ten years into the future and estimated:

	$\delta m_b(10)$	$\delta\alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g
current errors [10]	0.70	0.63	0.61	0.77	0.89	0.78
+ PT	0.69	0.40	0.34	0.74	0.57	0.49
+ LS	0.30	0.53	0.53	0.38	0.74	0.65
+ LS^2	0.14	0.35	0.53	0.20	0.65	0.43
+ PT + LS	0.28	0.17	0.21	0.30	0.27	0.21
+ PT + LS^2	0.12	0.14	0.20	0.13	0.24	0.17
+ PT + LS^2 + ST	0.09	0.08	0.20	0.10	0.22	0.09
ILC goal				0.30	0.70	0.60

relative errors in percent

I have now explained that we are on the road to a precise SM theory of the Higgs decay widths, and that many models predict deviations from the SM decay pattern at the few-percent level. Can we possibly test these predictions experimentally?

The LHC will dramatically improve our knowledge of Higgs couplings over the next 15 years, especially in its high luminosity phase.   However, I feel that the real goal is beyond the capability of the LHC.   The LHC errors are project to reach 3%-10%.   This is ~1 σ for the effects that I have described.

To do better, we need to study the Higgs boson in e^+e^- collisions.   An accelerator proposed to do this is the International Linear Collider, which would operate at 250 GeV, 500 GeV, and, eventually, 1 TeV.

To measure the Higgs couplings with precision, we need two things:

1. We must measure Higgs BR's very accurately. In e^+e^- collisions, Higgs production rates are 1% of the total cross section and 90% of typical selected event samples. In contrast, at the LHC, Higgs production rates are 10^{-10} of the total cross section and 10% of typical selected event samples. This makes it possible to reduce systematic errors to the 0.1% level.

2. We must determine the Higgs boson width in a model-independent way. Since the Higgs width is about 4 MeV, it cannot be measured directly at colliders.

Fortunately, some sets of measurements give us absolutely normalized coupling strengths. If one coupling can be determined, the rest can be worked out from the BR's.

Some sets of measurements that determine the Higgs total width through

$$\Gamma_h = \Gamma(h \to A\overline{A})/BR(h \to A\overline{A})$$

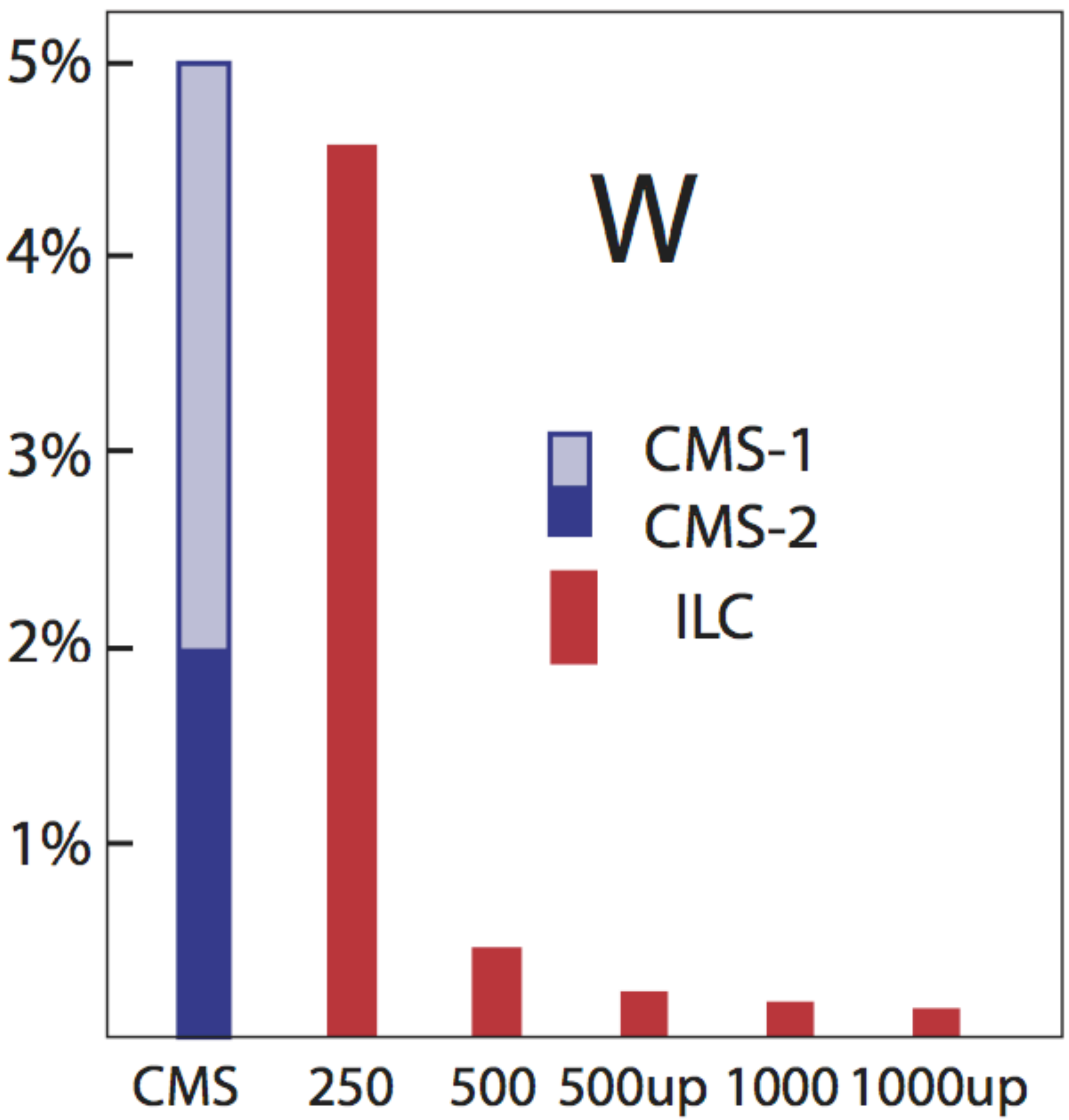
are

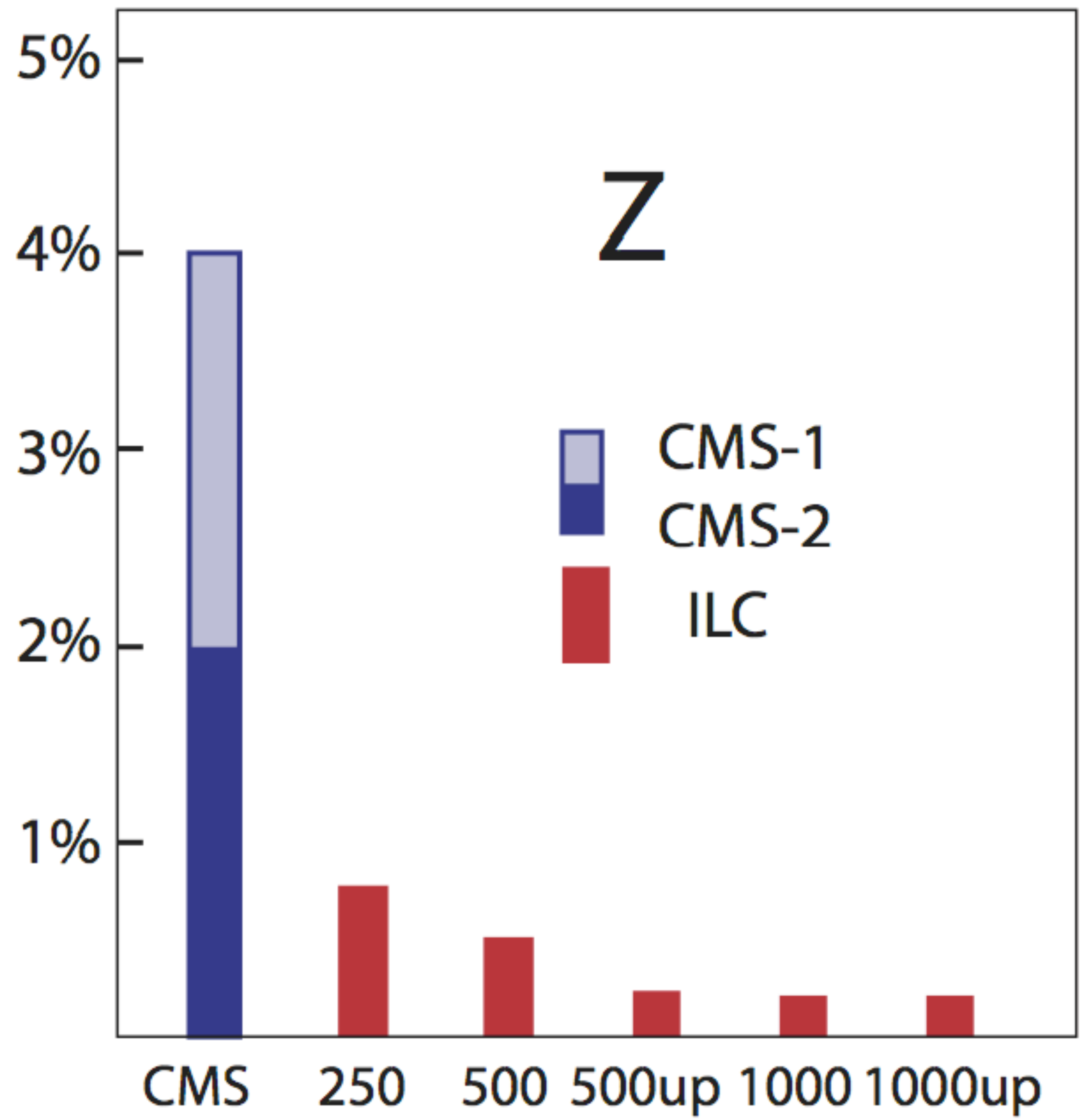
$$BR(h \to ZZ^*) \ , \ \sigma(e^+e^- \to Zh)$$

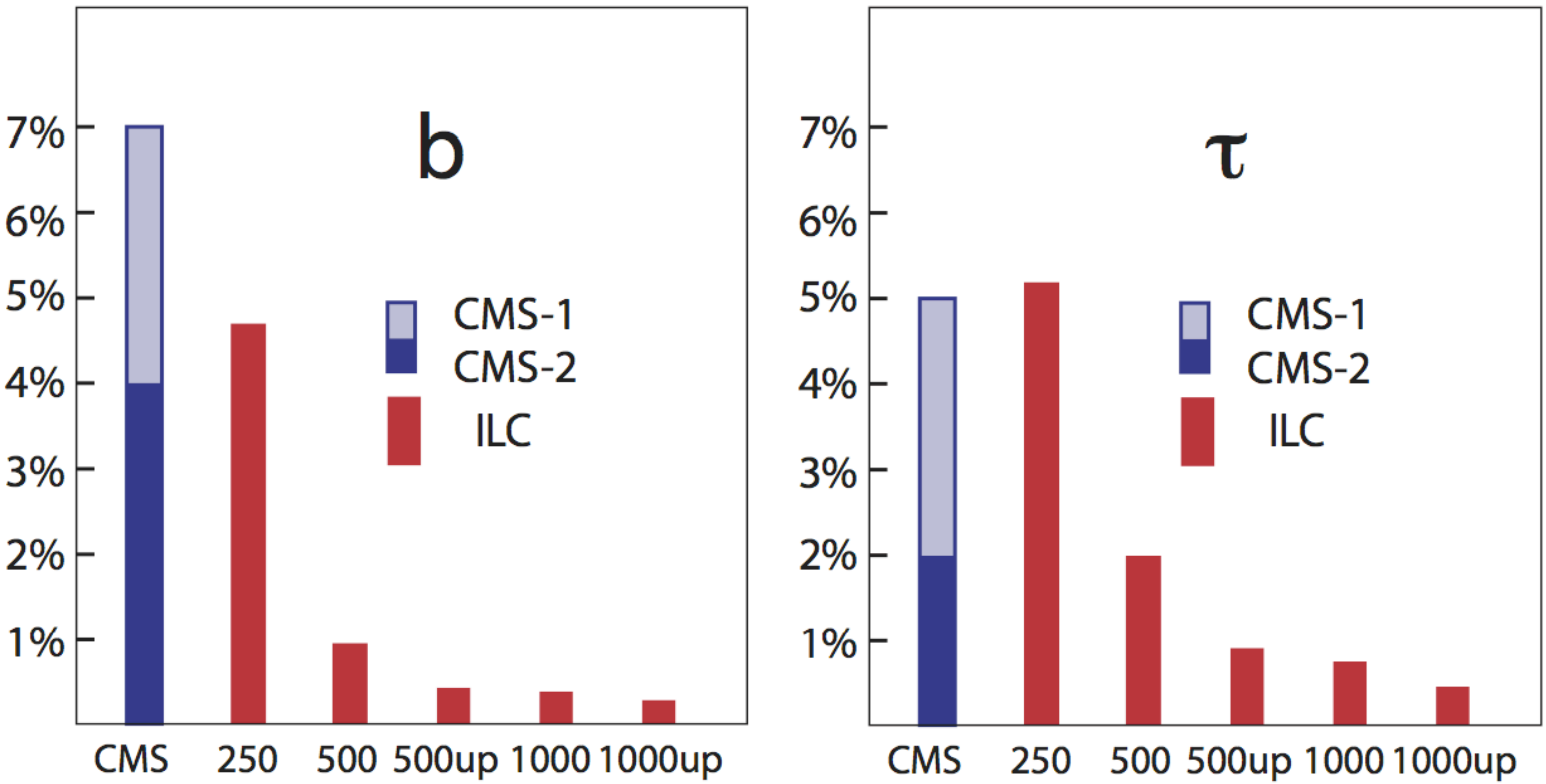
$$BR(h \to WW^*) \ , \ \sigma(e^+e^- \to \nu\overline{\nu}h, h \to b\overline{b}) \ , \ BR(h \to b\overline{b})$$

The first system is available already at 250 GeV. But, it has low statistics. The second is available at 350 GeV and above.

Here are my projections of Higgs coupling accuracy at various proposed stages of the ILC:







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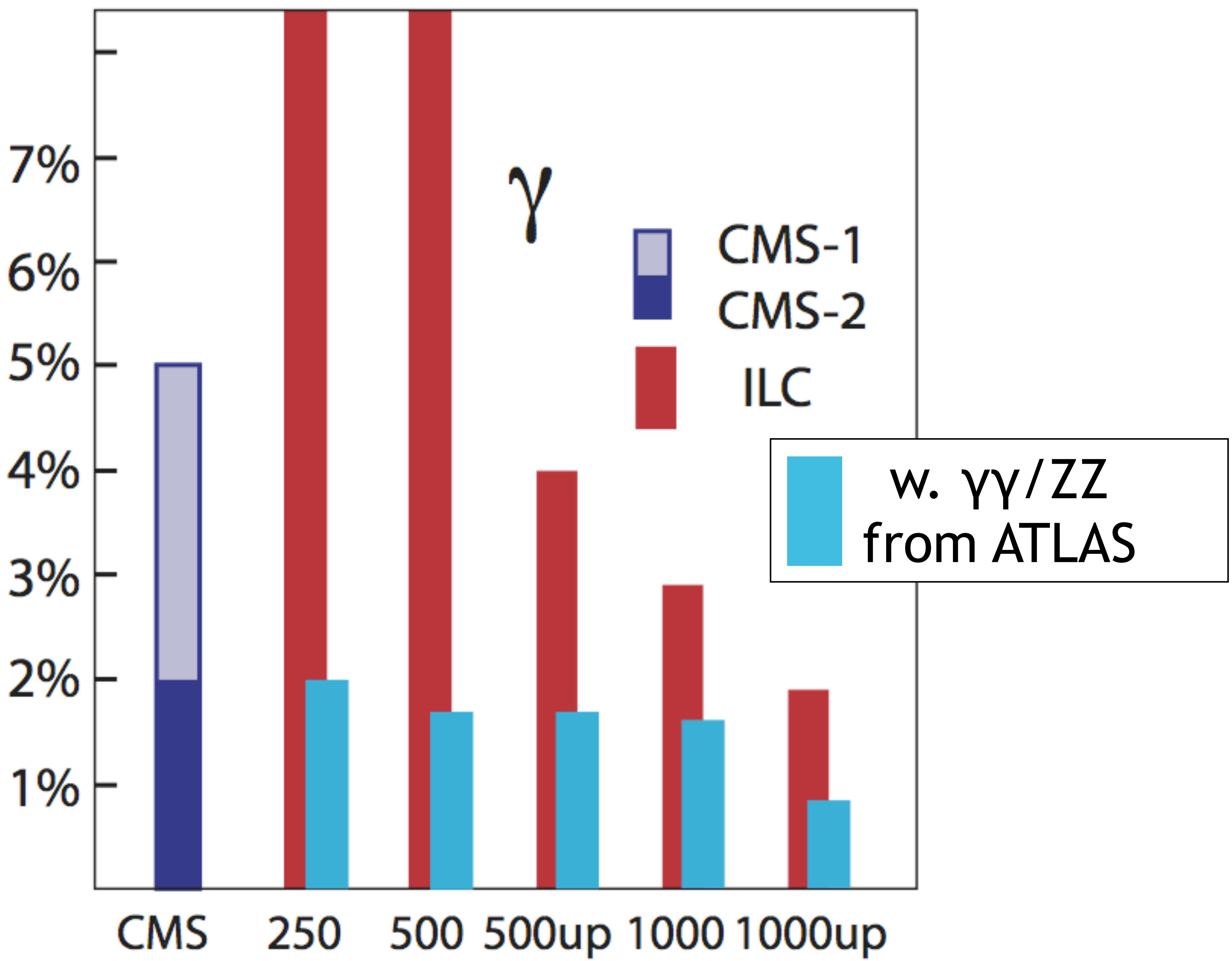

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The ILC is now being considered seriously by the Japanese government for a construction start in the next few years and data in the late 2020’s. It will be a very interesting successor to the LHC. I hope it succeeds.

Whatever the fate of this project, the precision study of the properties of the Higgs boson will be an important part of the future of particle physics, and of your future.

The knowledge is out there. We have much to learn.