The Higgs Boson and Electroweak Symmetry Breaking

3. Models of EWSB

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In this last lecture, I will take up a different topic in the physics of the Higgs field.

In the first lecture, I emphasized that most of the parameters of the Standard Model are associated with the couplings of the Higgs field.   These parameters determine the Higgs potential, the spectrum of quark and lepton masses, and the structure of flavor and CP violation in the weak interactions.

These parameters are not computable within the SM. They are inputs.   If we want to compute these parameters, we need to build a deeper and more predictive theory.

In particular, a basic question about the SM is:

Why is the SU(2)xU(1) gauge symmetry spontaneously broken ?

The SM cannot answer this question. I will discuss:

In what kind of models can we answer this question ?

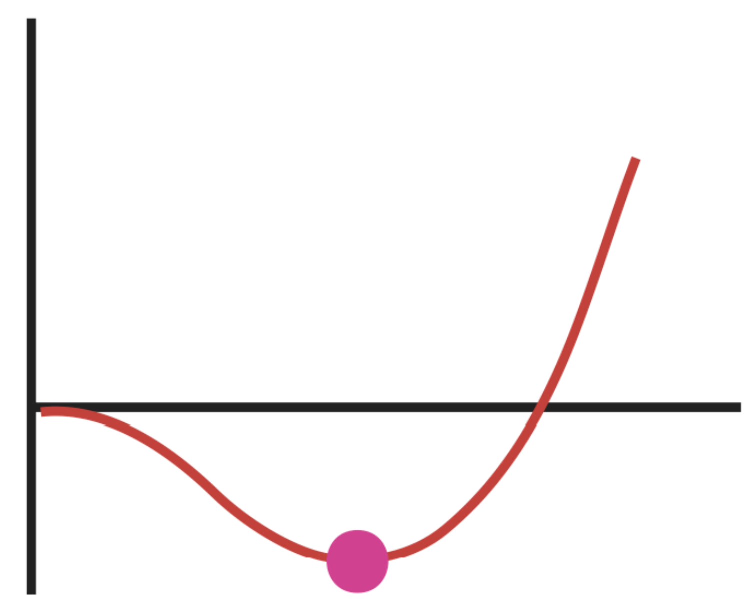
For orientation, I will present the explanation for spontaneous symmetry breaking in the SM.

We have a single Higgs doublet field φ. It has some potential $V(\varphi)$. The potential is unknown, except that it is invariant under SU(2)xU(1). However, if the theory is renormalizable, the potential must be of the form

$$V(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4$$

Now everything depends on the sign of μ^2. If $\mu^2 > 0$ the minimum of the potential is at $\varphi = 0$ and there is no symmetry breaking.

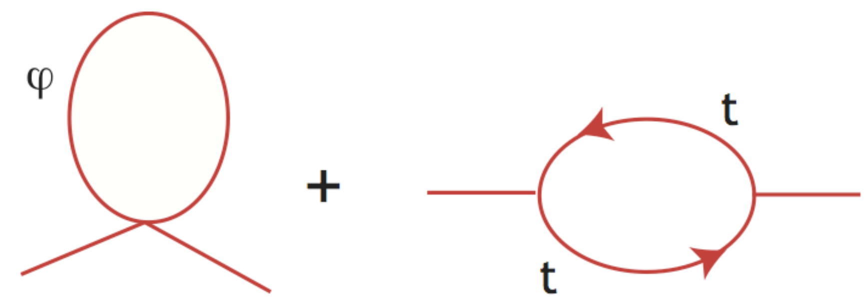
If $\mu^2 < 0$, the potential has the form: and there is a minimum away from 0.



That's it. Don't ask any more questions.

Actually, there is a question that we can ask. If we compute loop corrections to the parameter μ^2, is the requirement $\mu^2 < 0$ stable ? In fact, at 1 loop, we find:

$$\mu^2 = \mu^2_{\rm bare} + \frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \cdots$$



So:

1. The corrections are additive and depend on the ultraviolet cutoff.

2. The corrections are potentially much larger than the needed value $|\mu^2| \approx (100 \text{ GeV})^2$.

3. The corrections come with both signs. The sign of the correction depends on how we cut off the theory.

To compute the sign of μ^2, we need a model with more structure than that in the SM. In particular, we need:

1. There should be a symmetry that forbids additive, divergent corrections to μ^2. The simplest way to achieve this is to require $\mu^2 = 0$ to first approximation.

2. There should be new particles whose loops cancel the quadratic divergences of the diagrams on the previous page.

3. There should be a higher-order calculation that produces a nonzero value $\mu^2 < 0$.

This suggests that we want an effective theory valid at the TeV energy scale that contains:

1. the 4-component Higgs doublet field φ as a field in this theory.

2. new particles that cancel the quadratic divergences in the corrections to μ^2. In particular, there should be top quark partners accessible at the LHC.

3. a symmetry that, when exact, requires $\mu^2 = 0$.

What kind of symmetry will do this ? The symmetry must forbid the Lagrangian term

$$\Delta\mathcal{L} = -\mu^2|\varphi|^2$$

This is not so easy.

Four symmetries are known that can do this:

1. Scale invariance. However, there are no known models. Scale invariance must be broken, and this immediately regenerates the divergences.

2. $\delta\varphi = \epsilon$ This symmetry appears if φ is a Goldstone boson of symmetry breaking at a higher scale.

3. $\delta\varphi = \overline{\epsilon}\psi$ Then the φ mass term is forbidden by a chiral symmetry acting on ψ . This requires supersymmetry.

4. $\delta\varphi = \epsilon_\mu A^\mu$ Then the φ mass term is forbidden by gauge invariance. This structure appears in theories of extra dimensions, with $\varphi = A^5$.

The three classes of models on the previous page have a common feature. They are not simple modifications of the SM. Each class contains a large number of new particles - a new spectroscopy.

Supersymmetry and extra dimensions give profound modifications of our basic picture of space-time geometry.

So, it is all or nothing! Either the SM is the end of the story and we cannot understand anything about the Higgs sector, or our model of the Higgs sector must change our theory of fundamental interactions in an essential way.

In the rest of this lecture, I will discuss the cases of supersymmetry and Goldstone boson Higgs in more detail.

My main goal will be to study the radiative corrections to the Higgs mass. Do these automatically generate $\mu^2 < 0$? The answer will be yes, provided that corrections due to the top quark partners dominate.

Extra-dimensional models are related to Goldstone boson models by AdS/CFT duality. So that case is included implicitly in our discussion.

Supersymmetry is a symmetry that links fermion and boson fields. In principle, we could add to the SM a fermionic partner of the Higgs field and impose the symmetry $\delta\varphi = \overline{\epsilon}\psi$.

Then we will find cancellation of quadratic divergences in 1-loop diagrams, but the divergences will reappear in 2-loop diagrams. To avoid the divergences to all orders, we must build a supersymmetric generalization of the SM.

Supersymmetry can be softly broken by mass terms for the partners without reintroducing the quadratic divergences. This must be done to avoid $m(\widetilde{e}) = m(e)$.

A theory with exact supersymmetry has a charge Q that connects fermion and boson fields. Q must carry spin-1/2 and must commute with the Hamiltonian. Let $Q_{L\alpha}$ be the left-handed spinor part of Q . Then

$$\{Q_{L\alpha}, (Q_{L\beta})^\dagger\} = 2\sigma^\mu_{\alpha\beta} R_\mu$$

The operator R_μ is a spin-1 operator which is strictly nonzero (unless $Q = 0$) and commutes with the Hamiltonian.

However, the Coleman-Mandula theorem states that, if the S-matrix is nonzero, the only such operator is the total energy-momentum P_μ .

Thus, Q_L must act on every field in the model. Every state must have a partner with opposite statistics and spin differing by 1/2.

The simplest supersymmetric theory that contains the SM is called the MSSM - Minimal Supersymmetric Standard Model. It contains:

a complex scalar field for each chiral fermion

$$e_L, e_R \leftrightarrow \widetilde{e}, \widetilde{\overline{e}}$$

a chiral fermion for each gauge boson

$$A^a_\mu \leftrightarrow \lambda^a$$

two Higgs doublets and a chiral fermion partner for each

$$\varphi_u, \varphi_d \leftrightarrow \widetilde{h}_u, \widetilde{h}_d$$

If supersymmetry is exact, loop corrections to the Higgs masses cancel between the SM partners and their partners.

We can add to the theory soft masses terms $m^2|\widetilde{f}|^2$ for the scalars and soft Majorana mass terms $m_\lambda \lambda \cdot \lambda$ for the gauginos and Higgsinos.

$$\lambda \cdot \lambda = \lambda_\alpha \epsilon_{\alpha\beta} \lambda_\beta$$

In a supersymmetric model, the scalar potential and Yukawa terms are unified in the following way: There is a superpotential $W(\phi_i)$, a function of the scalar fields. Then the Lagrangian contains

$$\Delta\mathcal{L} = -\left|\frac{\partial W}{\partial \phi_i}\right|^2 - \frac{1}{2}\psi_i \cdot \psi_j \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

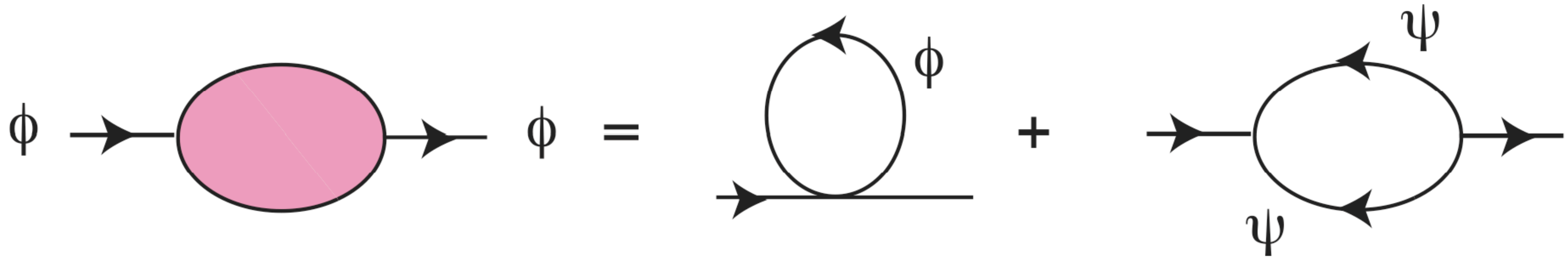
For example, the superpotential $W = \frac{1}{3} g \phi^3$

gives

$$\delta\mathcal{L} = -g^2|\phi^2|^2 - g\phi\lambda \cdot \lambda$$

The same coupling constants appear in the 4-scalar and Yukawa terms.

In this simple theory, we can see a supersymmetry cancellation of corrections to the scalar mass.



$$= (-4ig^2) \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2}$$

$$-\frac{1}{2}(-2ig)(2ig) \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left[\frac{i\sigma \cdot p}{p^2}_{\alpha\beta} \epsilon_{\alpha\gamma}\epsilon_{\beta\delta} \frac{i\sigma \cdot (-p)}{p^2}_{\gamma\delta}\right]$$

$$= 0$$

More generally, it can be shown that all corrections to the superpotential (except for those from field strength renormalization) vanish in any theory with exact supersymmetry.

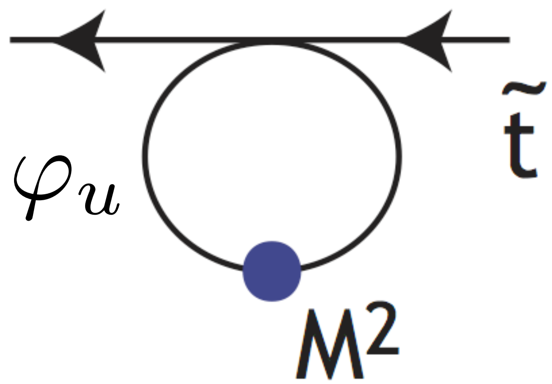
Now add soft supersymmetry breaking terms.

If there is spontaneous supersymmetry breaking anywhere in the theory, this is eventually communicated to the SM fields and generates soft mass terms. Usually, all of the generated scalar masses will be positive.

However, these terms can be renormalized by radiative corrections.

A particularly interesting set of corrections are those proportional to the top quark Yukawa coupling. These involve the three fields $(\widetilde{t}, \widetilde{\overline{t}}, \varphi_u)$.

Loops of each field correct the mass terms for the other two. One representative contribution is



$$= -iy_t^2 \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2} (-iM_{Hu}^2) \frac{i}{p^2}$$

$$= i \frac{y_t^2}{(4\pi)^2} M_{Hu}^2 \log \Lambda^2$$

Comparing to the standard form $-i\delta M_t^2$, this is a negative contribution to M_t^2. More precisely, what we have computed is a renormalization group coefficient

$$\frac{dM_t^2}{d\log Q} = \frac{2}{(4\pi)^2} y_t^2 M_{Hu}^2$$

with the sign such that M_t^2 decreases as $\log Q$ runs to the infrared. If the soft mass is generated positive at short distances, it can become negative at large distances.

Actually, we must consider the full system of soft mass terms for $(\widetilde{t}, \overline{\widetilde{t}}, \varphi_u)$. Any of these fields could develop a negative mass term. The RG equations for these terms are

$$\frac{dM_t^2}{d\log Q} = \frac{2}{(4\pi)^2} \cdot 1 \cdot y_t^2 [M_t^2 + M_{\overline{t}}^2 + M_{Hu}^2 + A_t^2] - \frac{8}{3\pi}\alpha_3 m_3^2 + \cdots$$

$$\frac{dM_{\overline{t}}^2}{d\log Q} = \frac{2}{(4\pi)^2} \cdot 2 \cdot y_t^2 [M_t^2 + M_{\overline{t}}^2 + M_{Hu}^2 + A_t^2] - \frac{8}{3\pi}\alpha_3 m_3^2 + \cdots$$

$$\frac{dM_{Hu}^2}{d\log Q} = \frac{2}{(4\pi)^2} \cdot 3 \cdot y_t^2 [M_t^2 + M_{\overline{t}}^2 + M_{Hu}^2 + A_t^2] + \cdots$$

The numerical coefficients (1,2,3) are the numbers of degrees of freedom circulating in the loop. If the three masses are equal at short distances, the Higgs mass term is the first one pushed to negative values as Q decreases.

I find it remarkable the supersymmetry contains a dynamical theory of SU(2)xU(1) spontaneous symmetry breaking based on the large top quark Yukawa coupling.

Supersymmetric models have many other beautiful features. For example, they naturally incorporate grand unification and cosmic dark matter. Despite the lack of success (so far) in finding supersymmetric particles at the LHC, they remain the most attractive theories of physics beyond the SM.

Next, I will analyze a relatively simple model in which the Higgs multiplet appears as a set of Goldstone bosons.

Consider a strong interaction theory at a very large mass scale (10 TeV), coupled weakly to SU(2)xU(1) gauge bosons. Assume that the theory has a pattern of chiral symmetry breaking that leaves SU(2)xU(1) unbroken. Then the Goldstone bosons will be in complete SU(2)xU(1) multiplets. One of these might be the complex Higgs doublet φ .

Models in which the Higgs multiplet arises in this way, as Goldstone bosons, are called "Little Higgs" models.

Here is a very simple (actually, too simple) realization of a Little Higgs model: Let the global symmetry of the original theory be SU(3)xSU(3), spontaneously broken to SU(3). There are 8 Goldstone bosons. Let SU(2)xU(1) be contained in the unbroken SU(3) subgroup. Then we can describe the Goldstone bosons by a nonlinear Lagrangian with variables

$$V = e^{2i\Pi^a t^a/f} \qquad 2i\Pi^a t^a = \begin{pmatrix} \Phi & H \\ -H^\dagger & \phi \end{pmatrix}$$

H is a 2-component complex scalar field that we can identify with the Higgs field.

The original SU(3)xSU(3) symmetries of the theory forbid mass terms for the Goldstone fields. Actually, either one of the symmetries

$$V \to \Lambda_R V \qquad V \to V\Lambda_L^\dagger$$

is sufficient to forbid these mass terms.

To write a top quark mass term in this theory, we must extend the $(t,b)_L$ multiplet to an SU(3) multiplet

$$\chi_L = \begin{pmatrix} u \\ b \\ U \end{pmatrix}_L$$

Then we can write a Yukawa term

$$\mathcal{L} = -\lambda_1 f \begin{pmatrix} 0 & 0 & \overline{u}_R \end{pmatrix} V \chi_L - \lambda_2 f \overline{U}_R U_L$$

The U_L is an extra SU(2) singlet quark with charge 2/3. I have added another SU(2) singlet quark U_R to allow U_L to obtain a mass.

Notice that the first term respects

$$V \to V \Lambda_L^\dagger \qquad \chi \to \Lambda_L \chi$$

while the second term respects

$$V \to \Lambda_R V$$

The H multiplet can get a mass only when both Yukawa couplings are used.

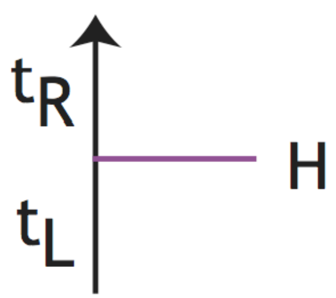
Even when H = 0, one set of quark eigenstates can get a mass. The mass eigenstates are

$$t_L = u_L \qquad t_R = \frac{\lambda_2 u_R - \lambda_1 U_R}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

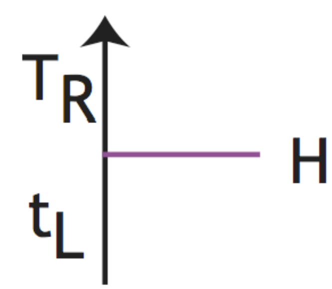
$$T_L = U_L \qquad T_R = \frac{\lambda_1 u_R + \lambda_2 U_R}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

$$m_T = \sqrt{\lambda_1^2 + \lambda_2^2} f$$

The couplings of these fields to H are



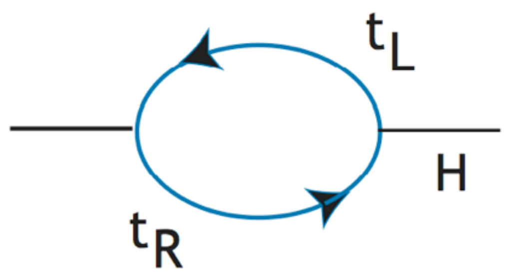
$$= -i\lambda_t$$



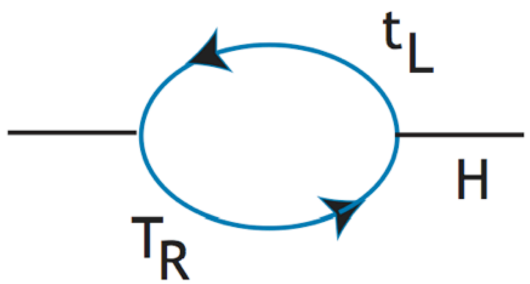
$$= -i\lambda_T$$

$$\lambda_t = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \qquad \lambda_T = \frac{\lambda_1^2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

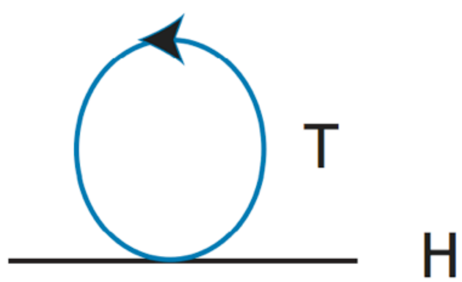
The 1-loop corrections to the H mass term are



$$= -6\lambda_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$$



$$= -6\lambda_T^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_T^2}$$



$$= +6\frac{\lambda_T}{f} \int \frac{d^4k}{(2\pi)^4} \frac{m_T}{k^2 - m_T^2}$$

The third diagram comes from the nonlinear coupling through V.

The quadratic divergences of these diagrams can be written as a sum of terms each of which has only one Yukawa coupling. So, these must add to zero.

The sum of the diagrams gives

$$m_H^2 = -3\frac{\lambda_1^2\lambda_2^2 f^2}{8\pi^2}\log\frac{M^2}{m_T^2} = -3\frac{\lambda_T^2 m_T^2}{8\pi^2}\log\frac{M^2}{m_T^2}$$

The result involves both Yukawa couplings, as required. The negative sign can be traced to the negative contribution of the pure top quark loop, which we saw earlier in this lecture.

The log divergences should be cut off at the strong interaction scale of 10 TeV. This is a chiral perturbation theory logarithm, analogous to the terms $\log\Lambda^2/m_\pi^2$ seen in pion physics.

This mechanism for obtaining a negative Higgs mass term is due to Arkani-Hamed, Cohen, Katz, and Nelson. Similar terms appear in more realistic Little Higgs models.

We have now seen that models in which the Higgs field arises as a Goldstone boson contain a dynamical mechanism for spontaneous symmetry breaking based on the large value of the top quark Yukawa coupling.

The mechanism is completely different from the one in supersymmetry. However, it builds on the same suggestion from the couplings of the Standard Model.

There is a striking contrast between these theories and the Standard Model.

The Standard Model is simple and (so far) successful. However, it is limited. If it is correct, that is the end of our ability to make predictions in particle physics.

The models I have described in this lecture are complex and rather wild. They bring in a new spectroscopy, with new particles that should be discovered at the LHC.

Which path is correct ? We do not know.

The truth is out there. We must keep probing for it.